ANALYTICAL AND NUMERICAL STUDIES ON KICKED BEAMS IN THE CONTEXT OF HALF-INTEGER STUDIES

G. Franchetti

GSI & Goethe U. Frankfurt

F. Asvesta, H. Bartosik, T. Prebibaj

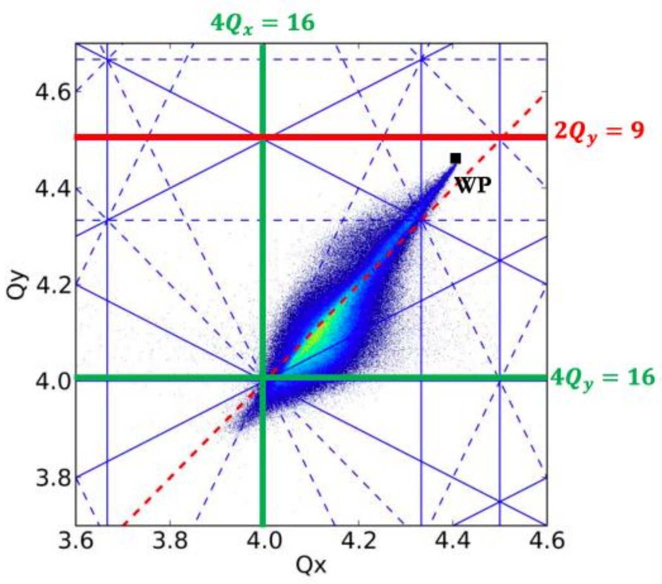
CERN

11.10.2023

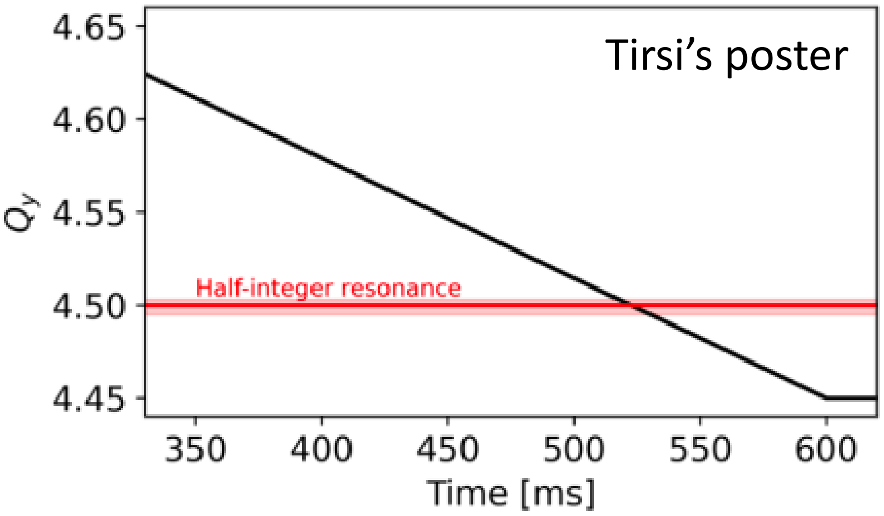
HB 2023

Context → half-integer studies at the CERN-PSB

Studies at the PSB on half-integer resonance



Studies on the effect of space charge on the half-integer



The dynamics of resonance crossing has brought to the attention the interplay of

Chromaticity ⟺ **Space charge** ⟺ **Coherent oscillations**

in a Coasting Beam

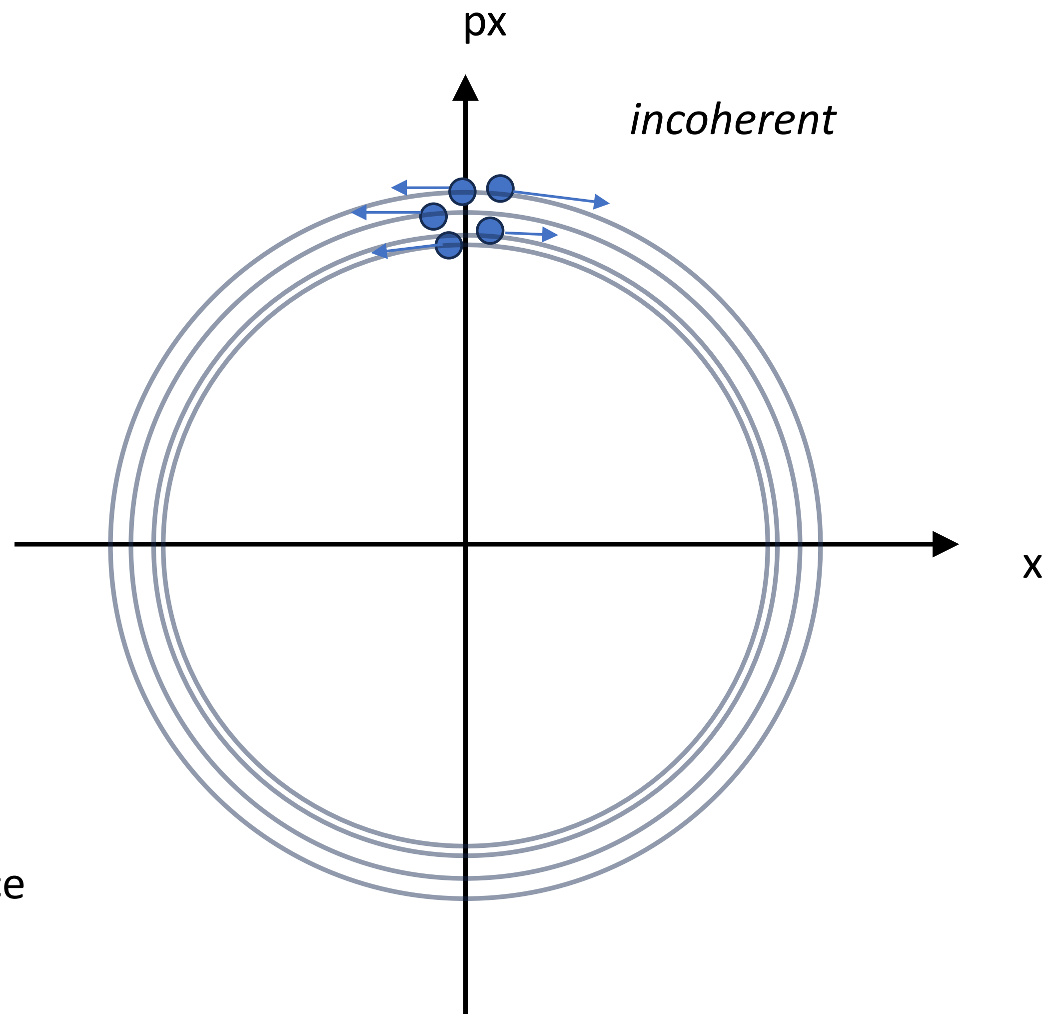
Chromaticity

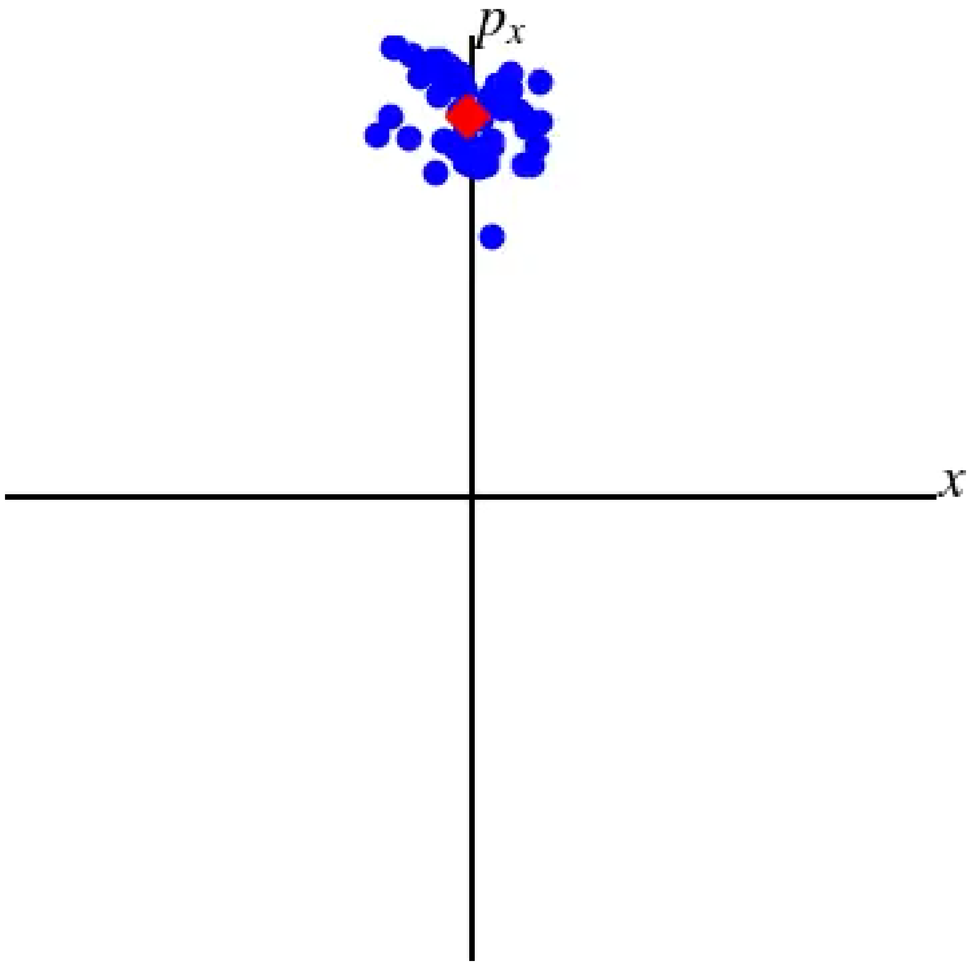
$$\begin{aligned}\langle \delta p \rangle &= 0 \\ \langle \delta p^2 \rangle &= \sigma_p^2\end{aligned}$$

Each particle has a tune-shift as

$$\Delta Q_x = Q_{x0} \xi \delta p$$

ξ Is taken positive for convenience





Betatron amplitude modulated by

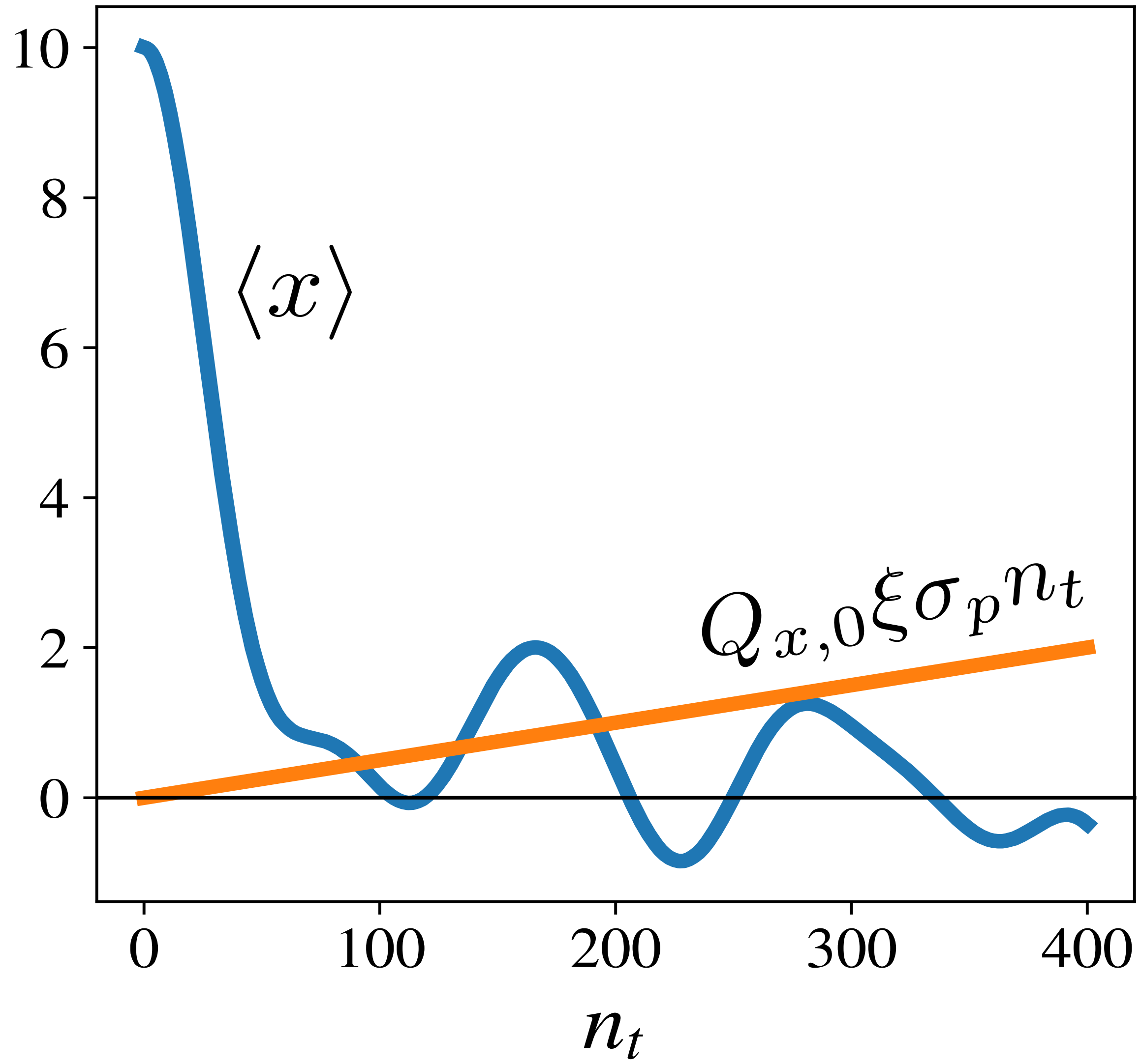
$$\Lambda\left(\frac{Q_{x0}\xi_x\sigma_p}{R}s\right)$$

$$\Lambda(u) = \int \cos(u\lambda)g(\lambda)d\lambda$$

For a Gaussian distribution

$$\Lambda(u) = \exp\left(-\frac{1}{2}u^2\right)$$

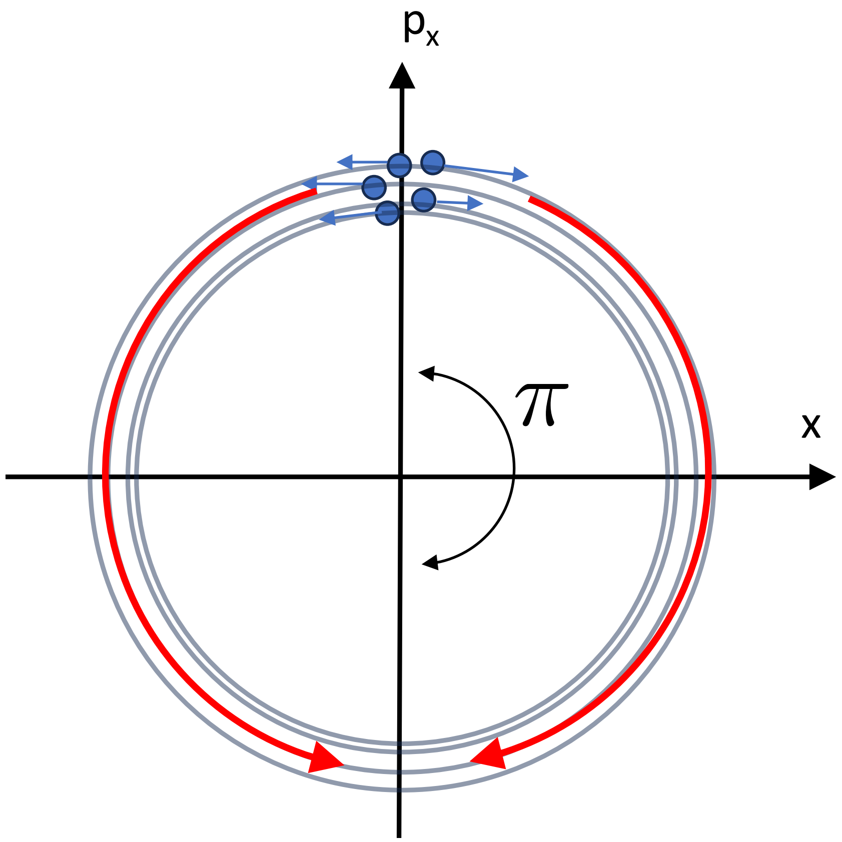
Characteristic scaling



Characteristic scaling

Decoherence has a characteristic scale proportional to

$$Q_{x,0}\xi\sigma_p n_t \simeq 0.5$$



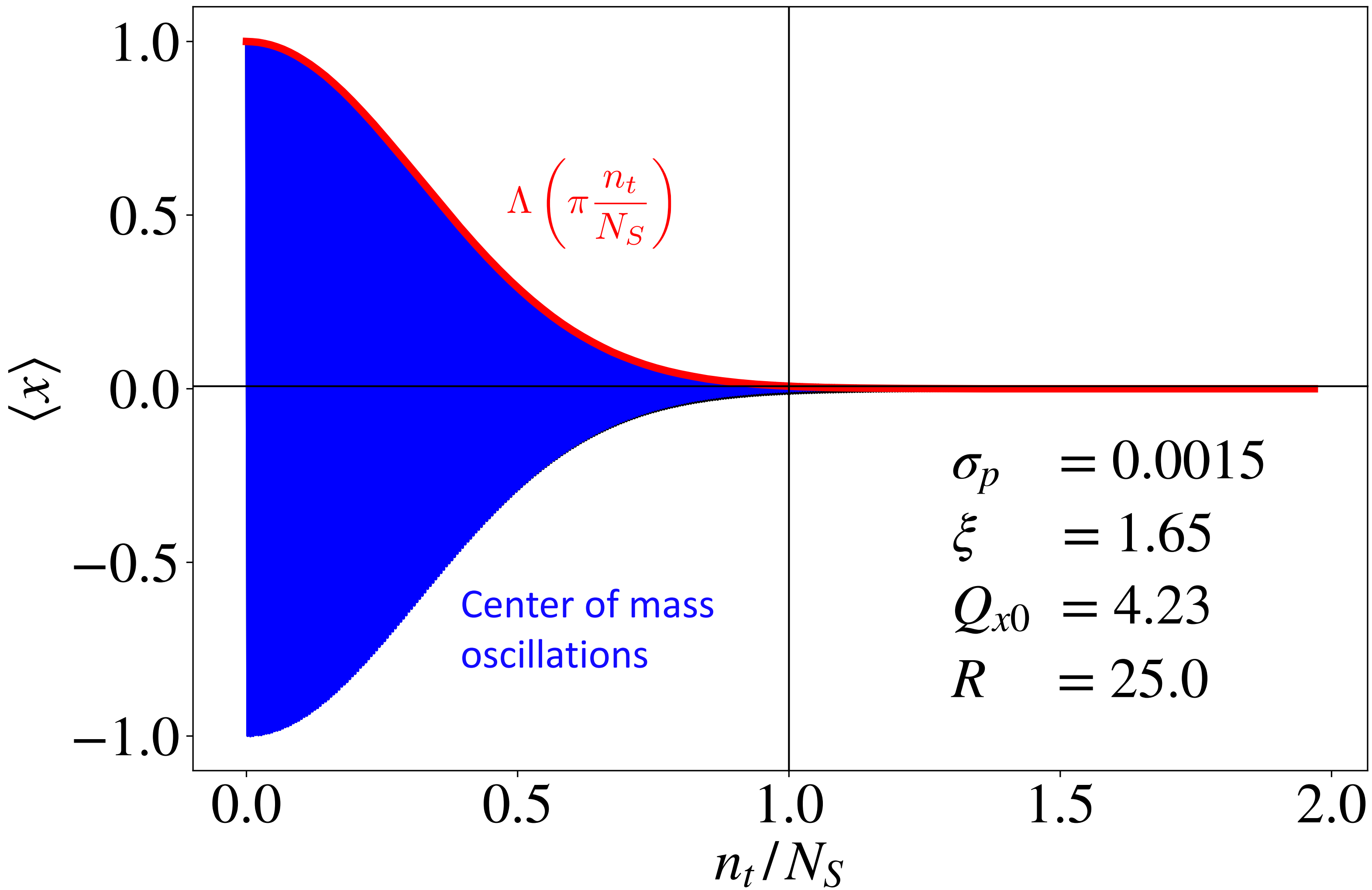
Define the rms chromatic detuning

$$\delta Q_\xi = Q_{x,0}\xi\sigma_p$$

Define the reference turn number

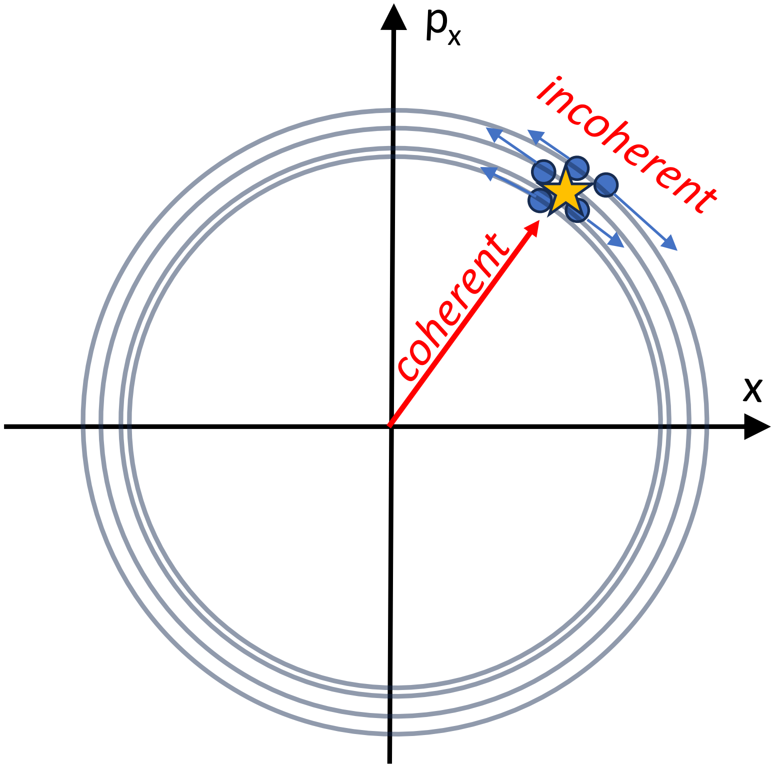
$$N_S = \frac{1}{2\delta Q_\xi}$$

For a Gaussian distribution in $\boldsymbol{\delta}$p/p

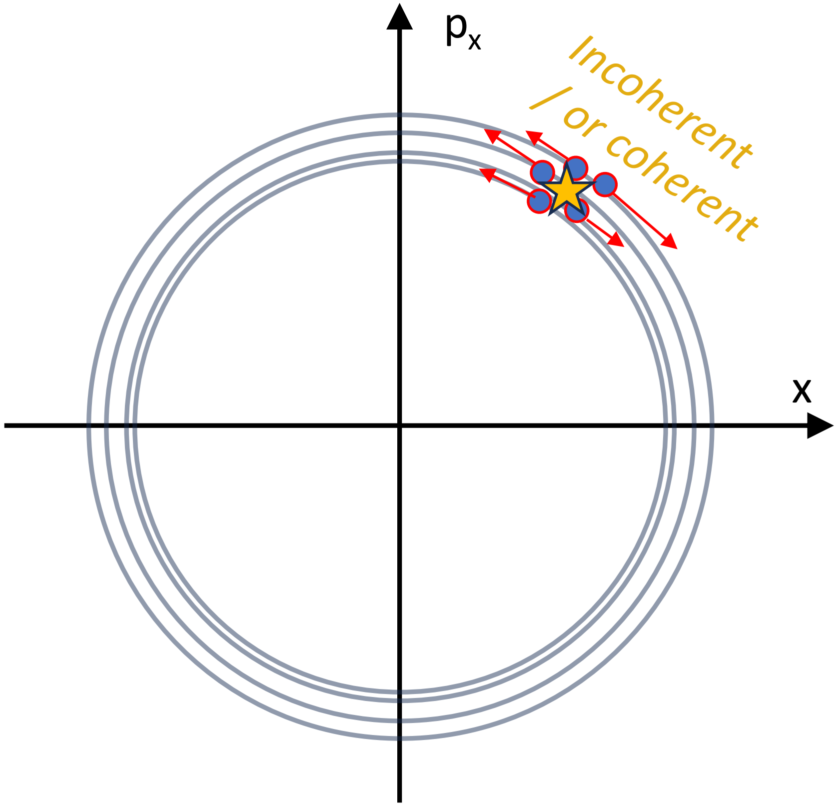


Space charge and chromaticity

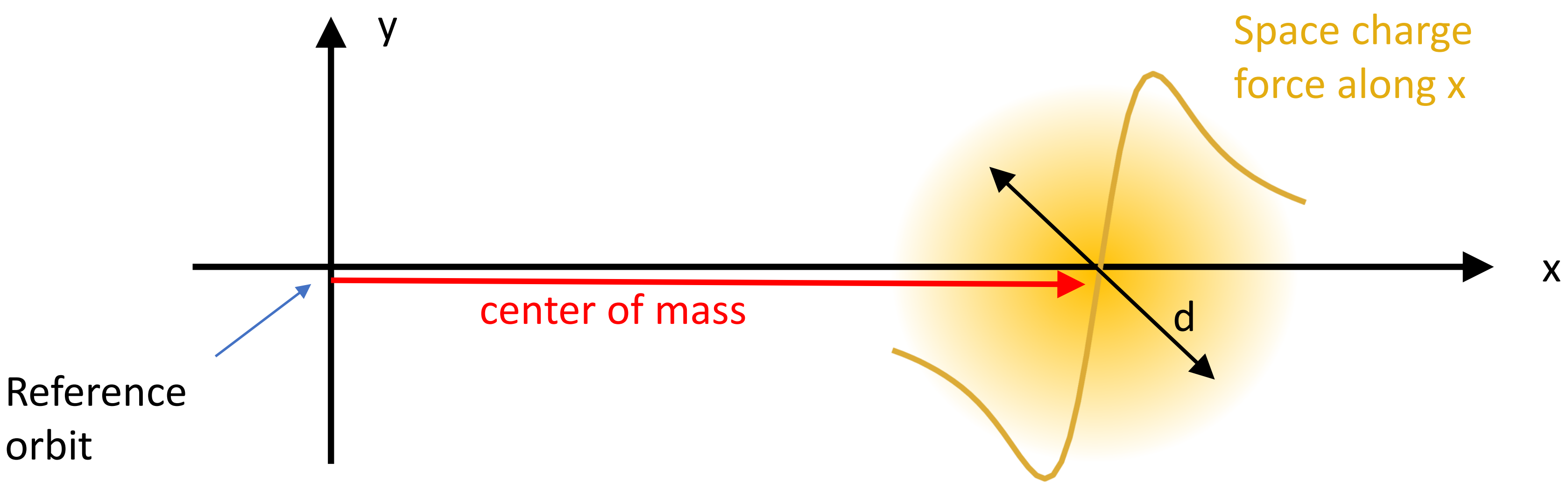
Effect of chromaticity →
Damping by decoherence



The effect of space charge is a defocusing force.
It is zero on the center of mass.

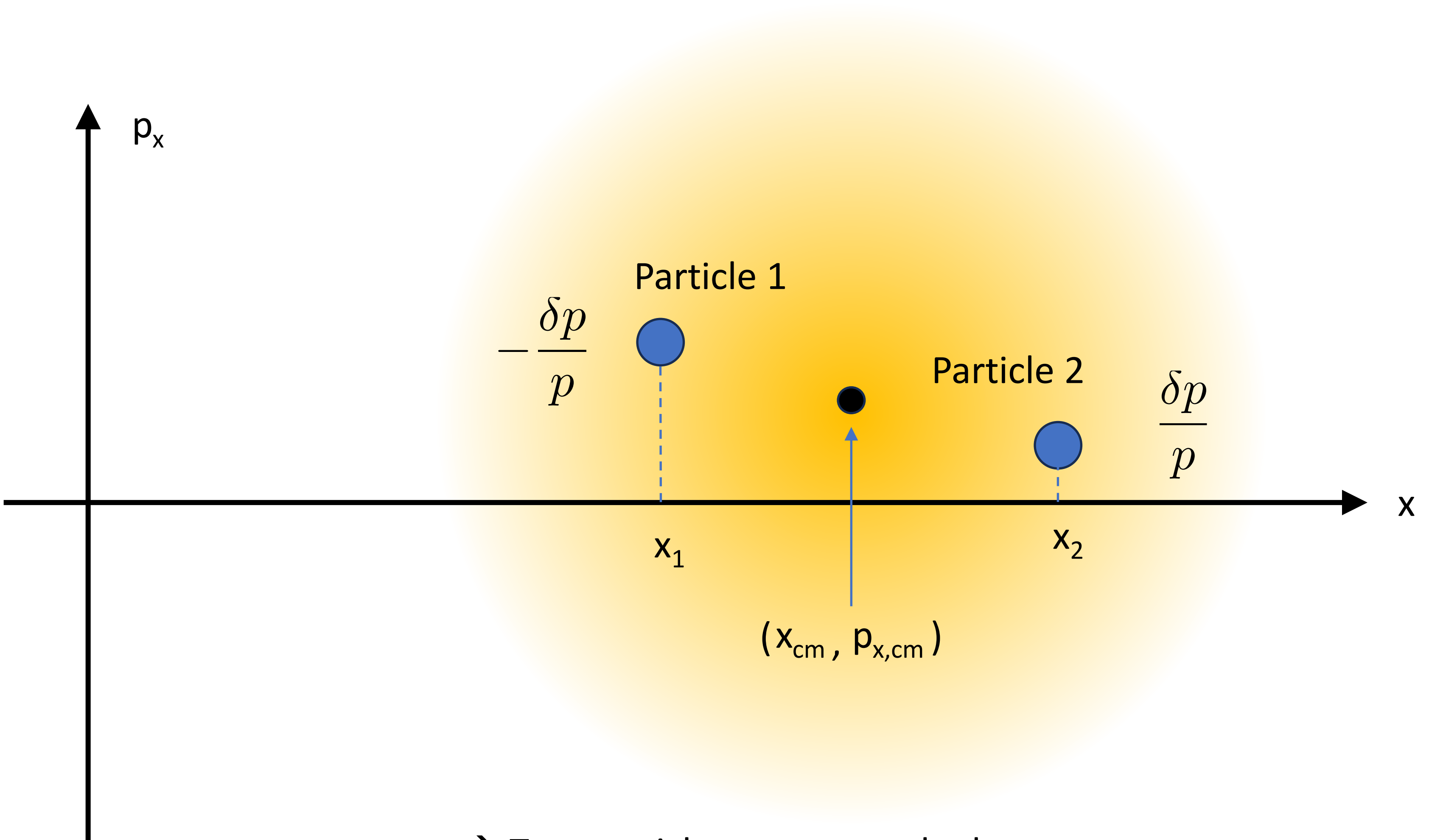


Space charge forces



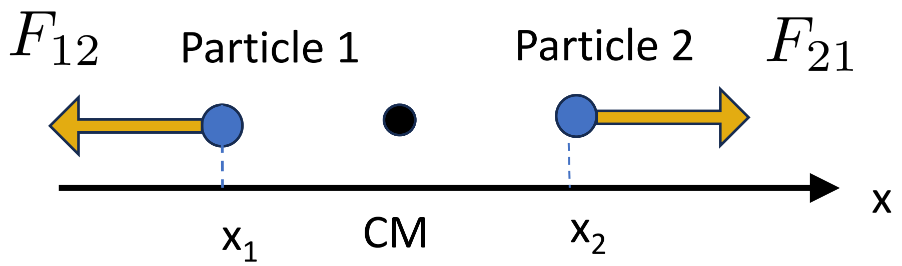
1) A particle *close* to the beam center feels a linear force $\propto r$
2) If it is *away* from it, it decays as $\propto 1/r$
3) "*close*" and "*away*" are is relative to the center of mass and "d"

Two-Particle Model



→ Two particles represent the beam

Two-Particle Model: forces



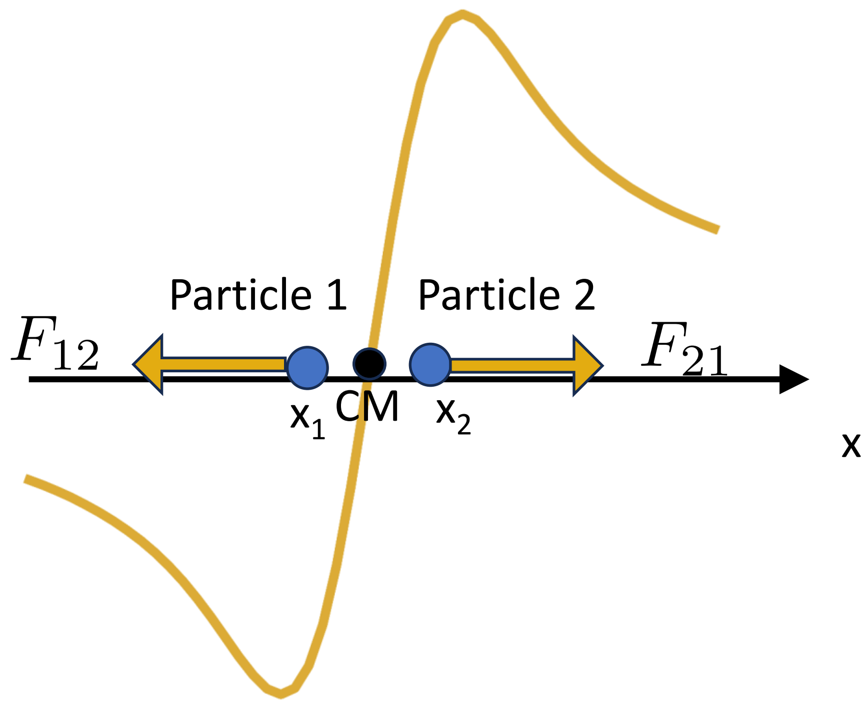
$$F_{12} = \frac{x_1 - x_2}{|x_1 - x_2|} f(x_{12})$$

$$f(x_{12}) = \lambda \frac{|x_1 - x_2|}{d^2 + (x_1 - x_2)^2}$$

d = is a characteristic length
λ = coulomb strength

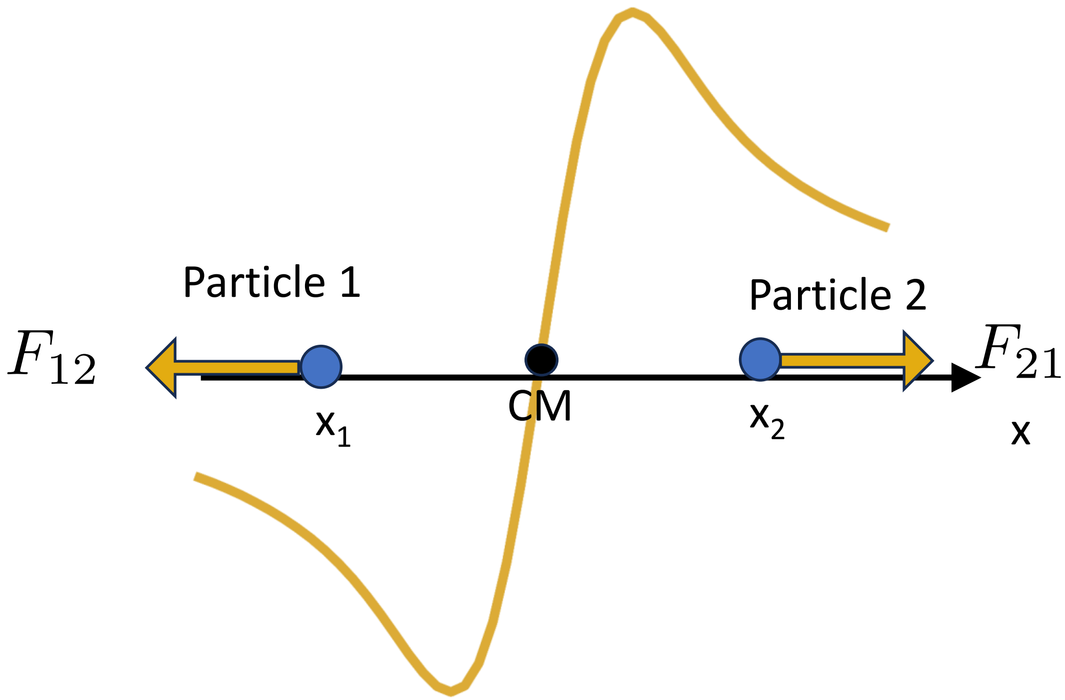
Properties → $F_{12} + F_{21} = 0$

"close" particles



$$f(x_{12}) \simeq \frac{\lambda}{d^2}|x_1 - x_2|$$

"away" particles



$$f(x_{12}) \simeq \lambda \frac{1}{|x_1 - x_2|}$$

Equations of motion

In the reference closed orbit we have

$$\begin{cases} x_1'' + \dfrac{(Q_{x,0}+Q_{x,0}\xi\delta p/p)^2}{R^2}x_1 & = & F_{12} \\ x_2'' + \dfrac{(Q_{x,0}-Q_{x,0}\xi\delta p/p)^2}{R^2}x_2 & = & F_{21} \end{cases}$$

Scaling

$$z = \frac{2}{d}x$$

$$z_{cm} = \frac{z_1 + z_2}{2}$$

This coordinate is the “scaled center of mass”

$$z = z_1 - z_{cm}$$

This coordinate is the “scaled the beam size”

Equations of motion in the scaled coordinates

$$\begin{cases} \ddot{z}_{cm} + k z_{cm} + \Delta k z = 0 \\ \ddot{z} + k z + \Delta k z_{cm} = 2\frac{\lambda}{d^2}\frac{z}{1+z^2} \end{cases}$$

Center of mass

One particle

Focusing of the lattice

Effect of the chromaticity + **δ**p/p

Effect of "space charge"

$$\Delta k = 2k\delta Q_{\xi}/Q_{x,0}$$

No space charge case

$$\begin{cases} \ddot{z}_{cm} + kz_{cm} + \Delta k z = 0 \\ \ddot{z} + kz + \Delta k z_{cm} = 0 \end{cases}$$

Focusing in z, z_{cm} is on the linear coupling resonance

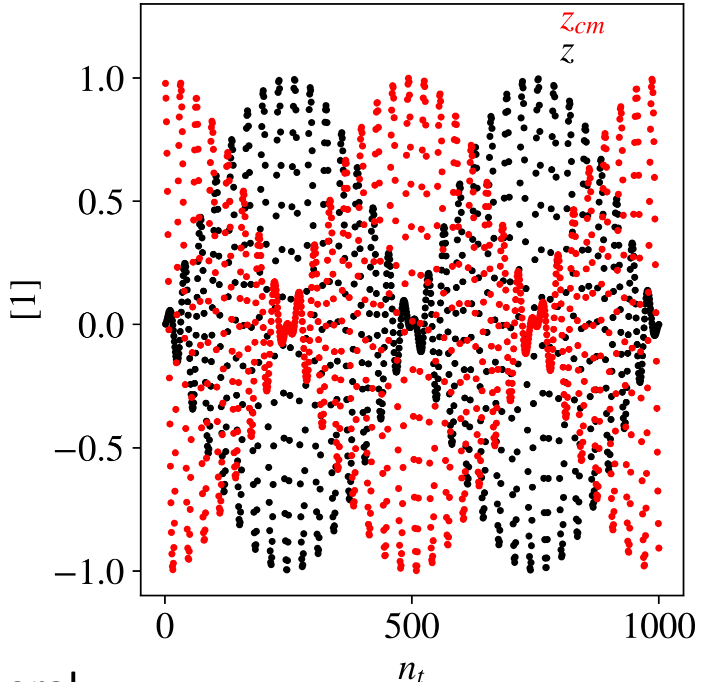
Linear coupling with strength **Δ**k

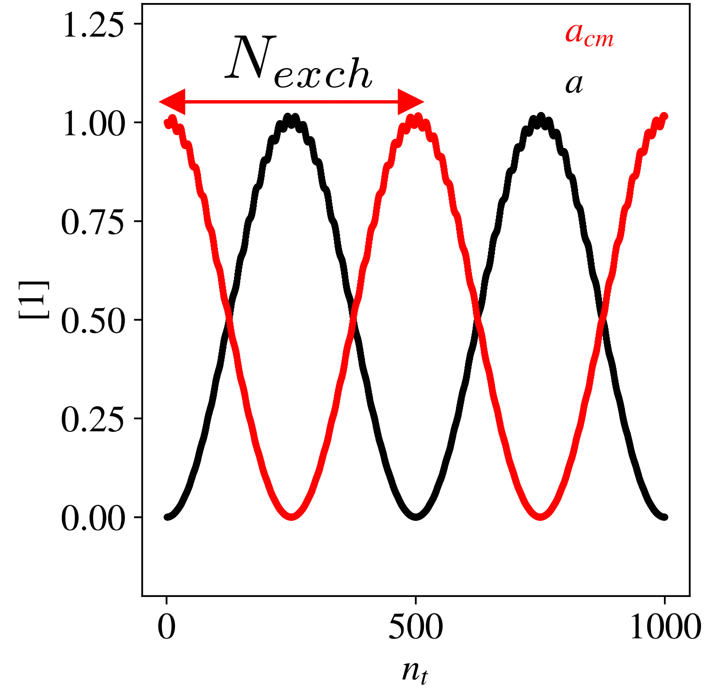
Emittance exchange between the dynamical variables z and z_{cm}

Emittance exchange

Call a, a_{cm} the emittances of z, z_{cm}.

Example $\delta Q_\xi = 10^{-3}$ $N_{exch} = 500$





In general

$$N_{exch} = \frac{1}{2\delta Q_\xi} \quad \Rightarrow \quad N_{exch} = N_S$$

Including space charge

$$\begin{cases} \ddot{z}_{cm} & + & k z_{cm} & + & \Delta k z & = & 0, \\ \ddot{z} & + & k_d z & + & \Delta k z_{cm} & = & -2 \frac{\lambda}{d^2} \frac{z^3}{1+z^2}, \end{cases}$$

with k_d a new focusing strength associated with the incoherent tune-shift

$$k_d = k - 2\frac{\lambda}{d^2} = \frac{(Q_{x,0} + \Delta Q_{x,sc})^2}{R^2}$$

$$\Delta Q_z = \Delta Q_{x,sc} = -\frac{\lambda R^2}{Q_{x,0} d^2}$$

Therefore the linear space charge "detunes" the system from the linear coupling resonance

Partial emittance exchange

Amount of emittance exchange

$$\frac{a_{\max}}{a_{cm,0}} = \left[1 + \left(\frac{k_-}{\Delta k}\right)^2\right]^{-1}$$

Scaled exchange

Number of turns of the exchange periodicity

$$\sqrt{1 + \left(\frac{k_-}{\Delta k}\right)^2} \frac{N_{\mathrm{exch}}}{N_S} = 1$$

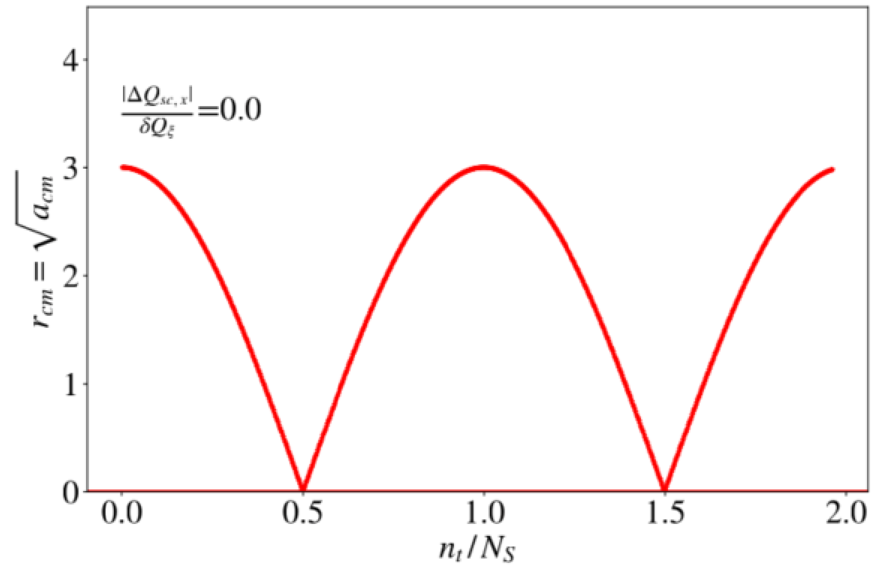
Scaled periodicity

With

$$k_- = \frac{k - k_d}{2} \quad \Rightarrow \quad \left|\frac{k_-}{\Delta k}\right| = \frac{1}{2} \frac{|\Delta Q_{sc}|}{\delta Q_\xi}$$

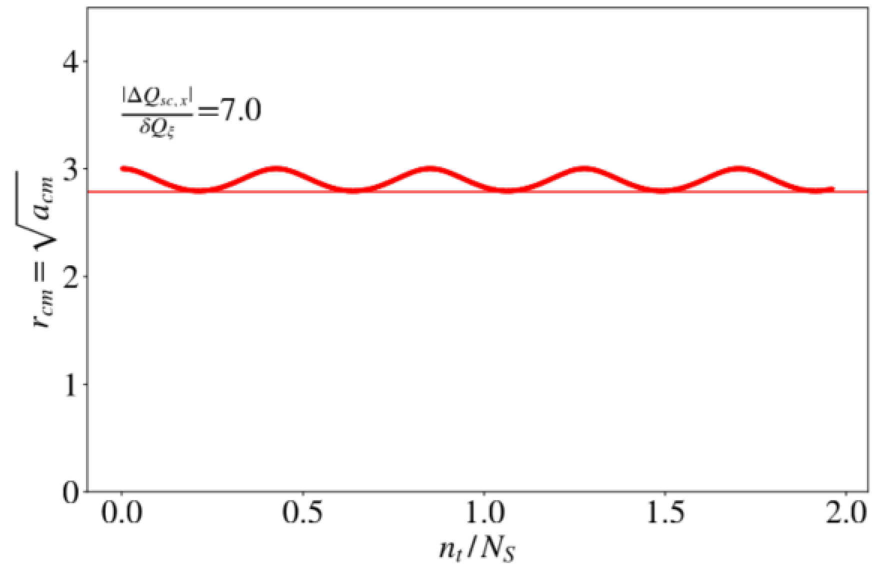
General parameter controlling the process

Summary



No space charge →
1) Full emittance exchange
2) Periodicity $N_{exch} = N_S$

Full dechoerence and re-choerence due to the linear coupling
In a beam → irreversible decoherence



With enough space charge →
1) Partial emittance exchange
2) It depends on $|\Delta Q_{sc}|/\delta Q_\xi$
3) The periodicity scales with N_S

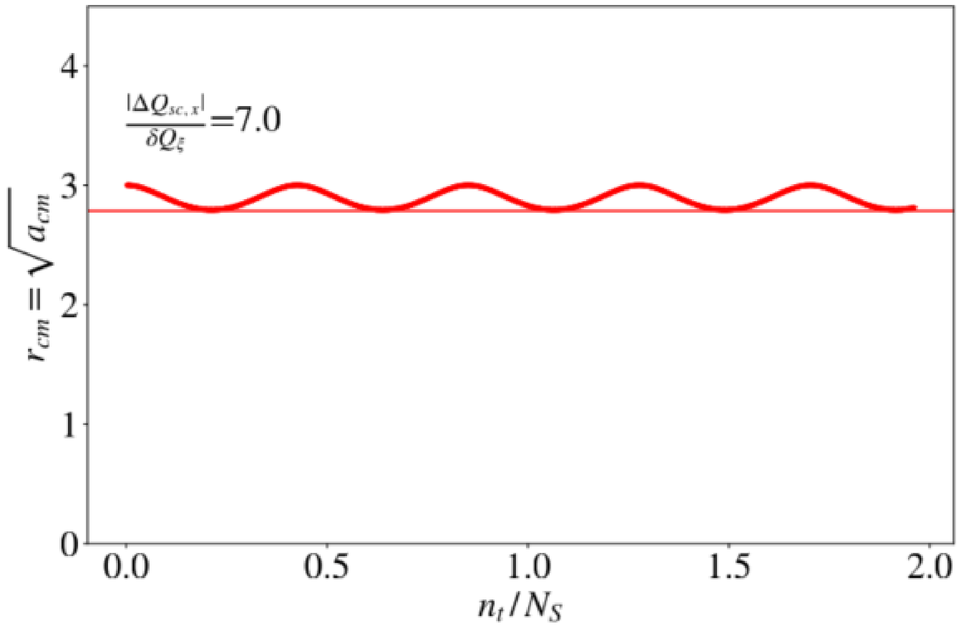
No decoherence:
space charge “detunes from the linear coupling resonance”, → it prevents the decoherence from the chromaticity

Intriguing prospects

If the center of mass oscillates,
it is a measurable quantity

The motion of the center of mass
is of a "coherent" dynamics

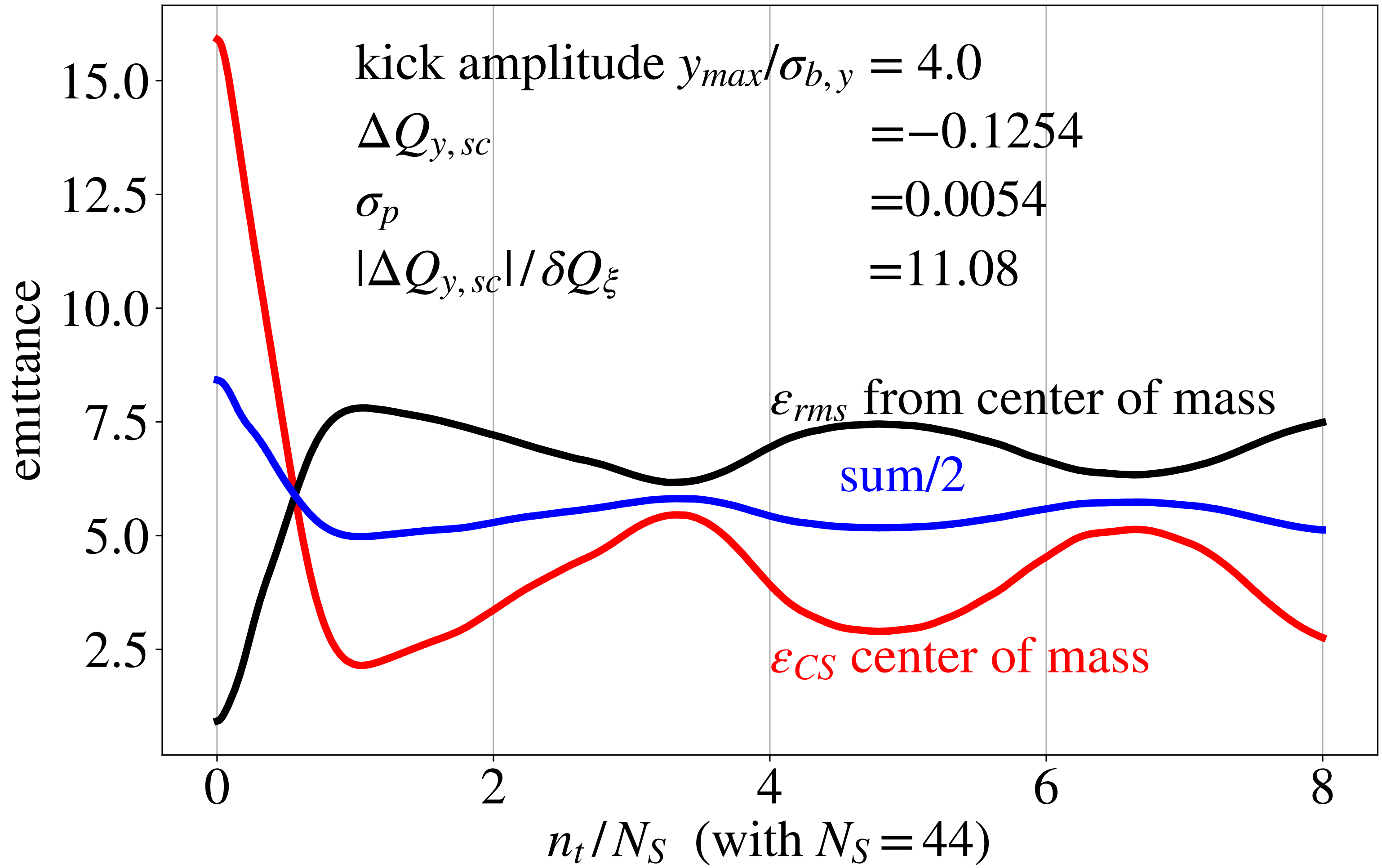
The motion with respect to the
center of mass "may be an
incoherent dynamics"



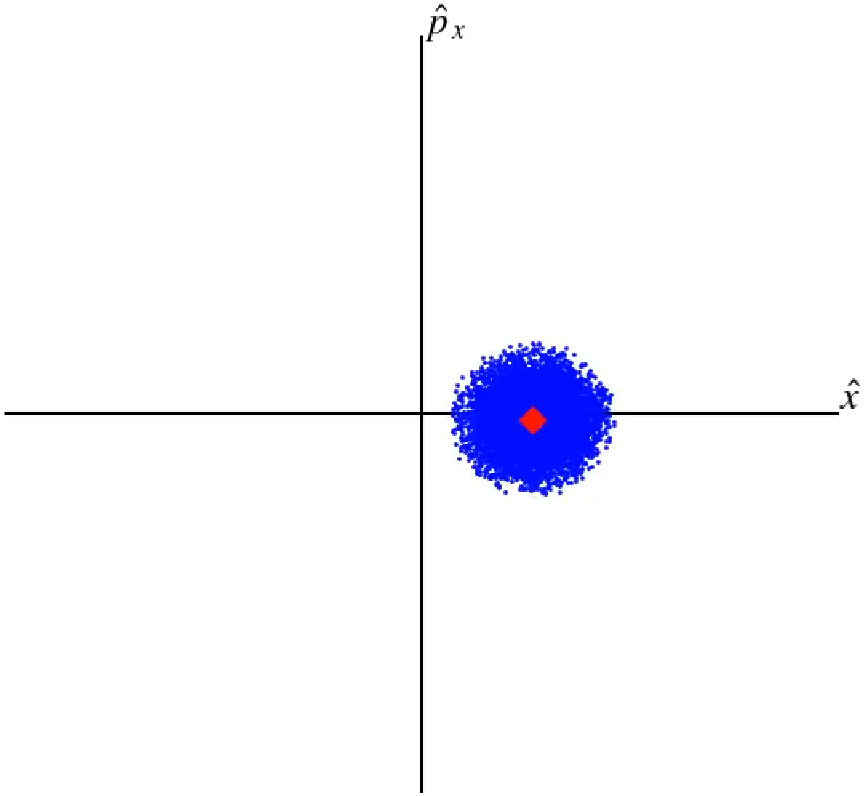
Periodic exchange of energy between two fundamentally different ``modes" of the dynamics.

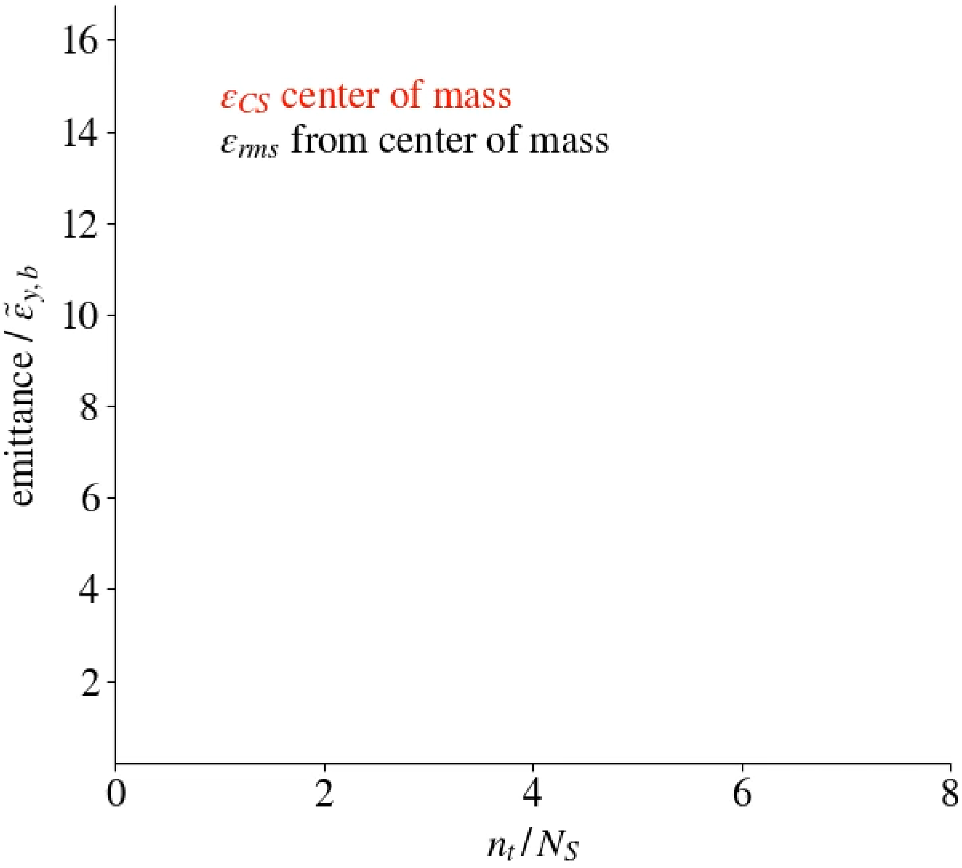
Particle in Cell Simulations of a coasting beam

No impedance or collective effect. Pure direct field with image charge.

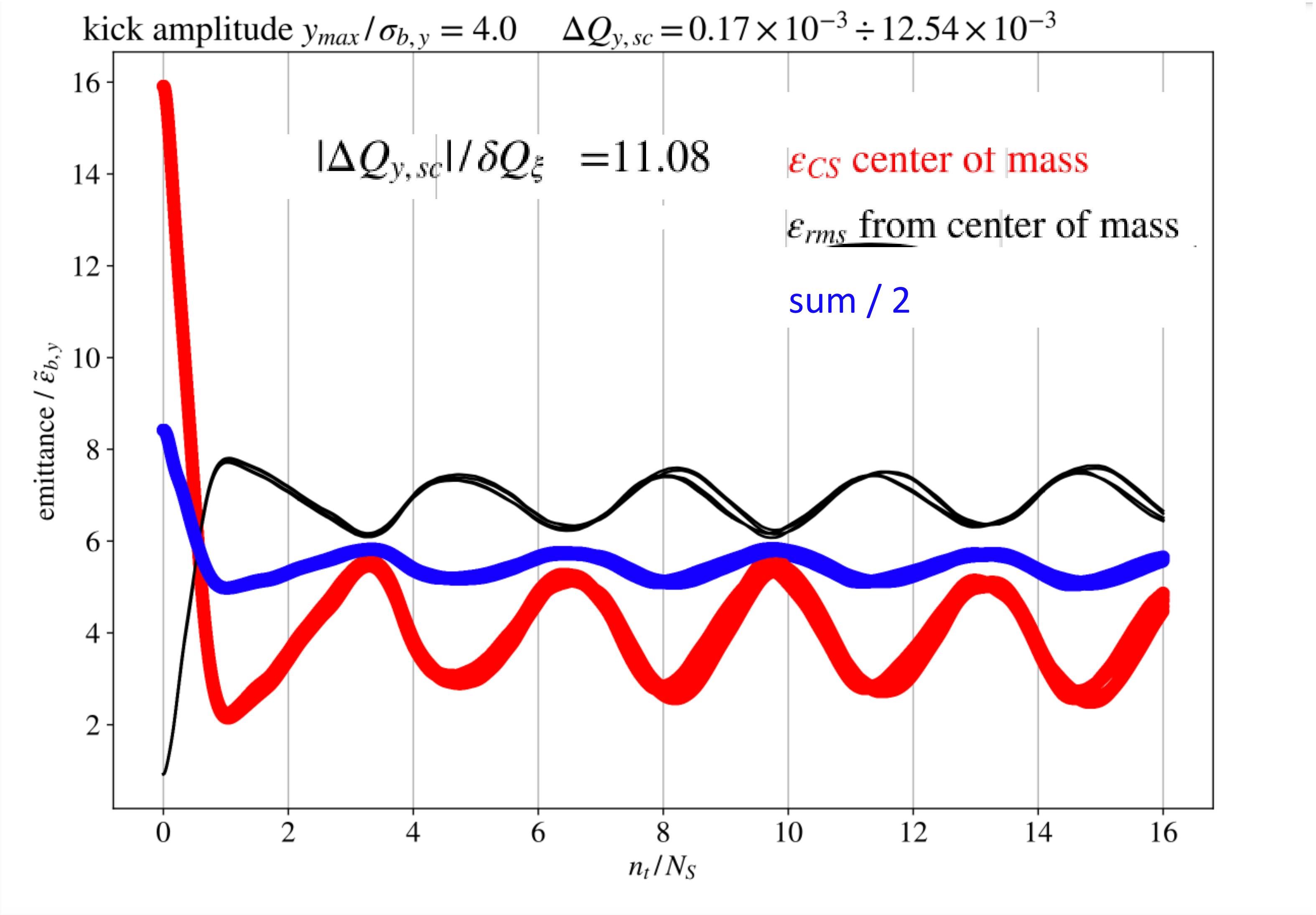


A deeper complexity…

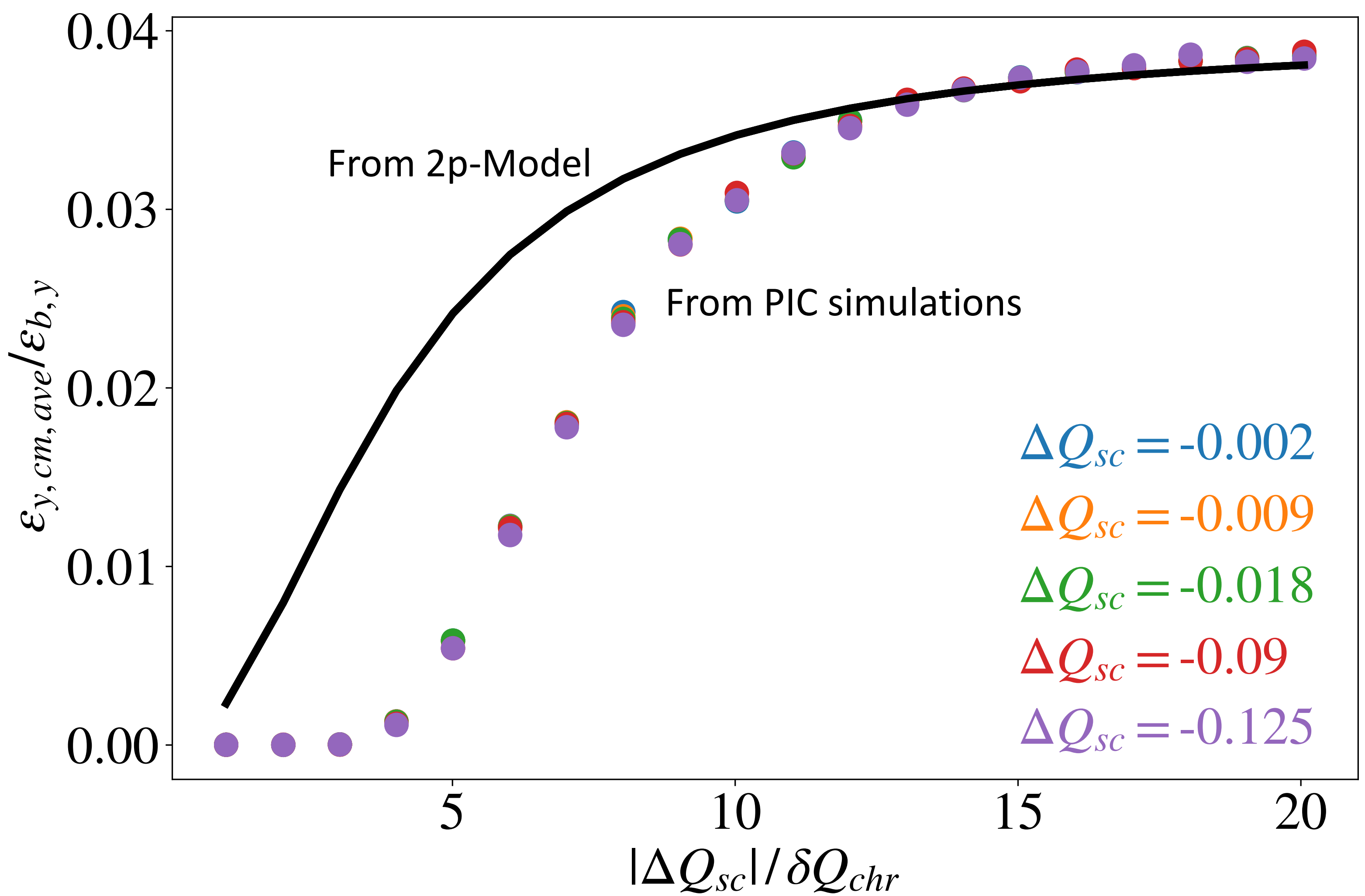




However, the scaling works very well …



Another example: kick amplitude $y_{max} / \boldsymbol{\sigma}_{y,b} = 0.2$



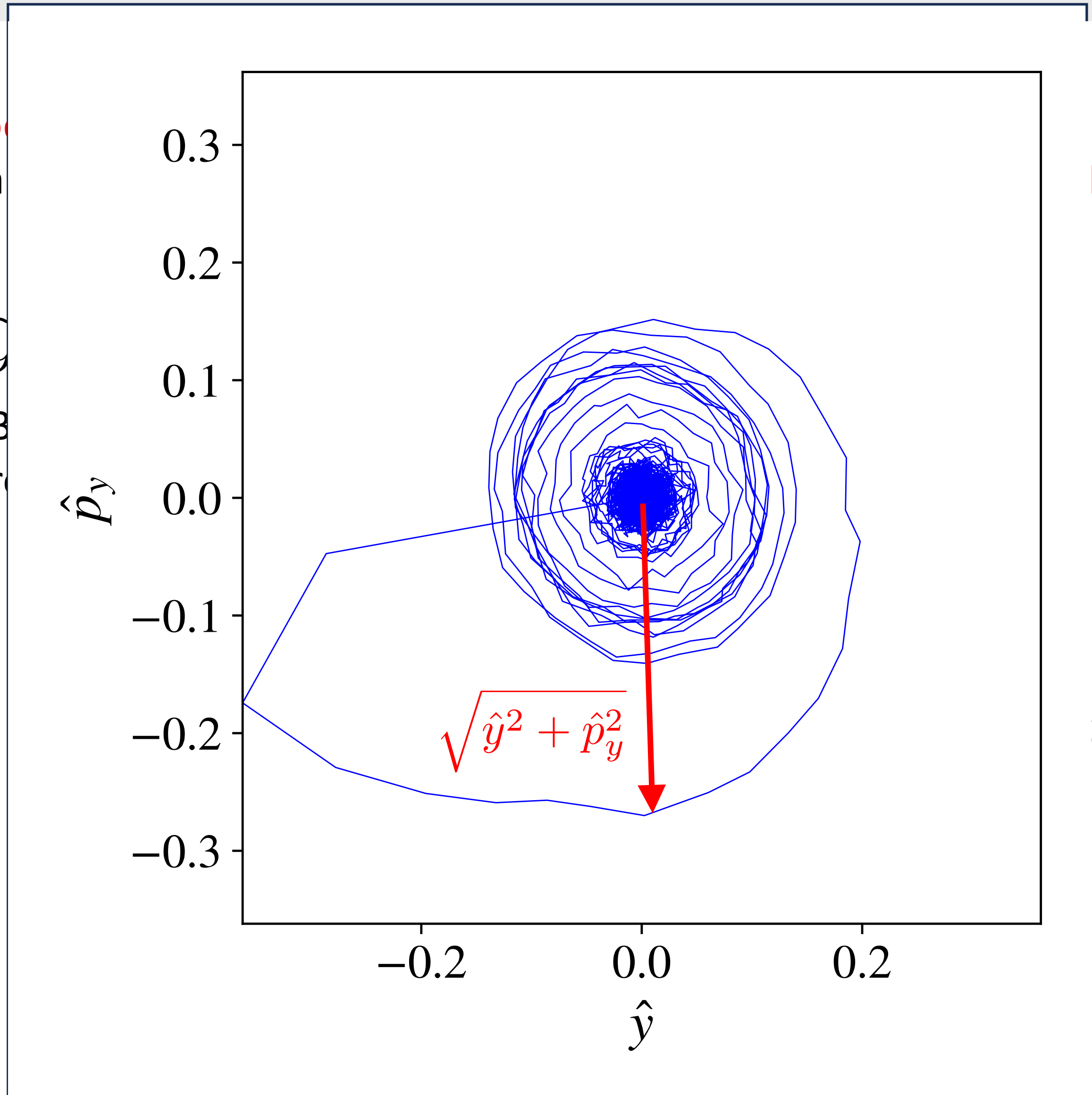
Search of this effect in the CERN-PSB

Make use of a sp center of mass
of a coasting bea RF activated

$n_t \to \hat{y}$ $_t)$

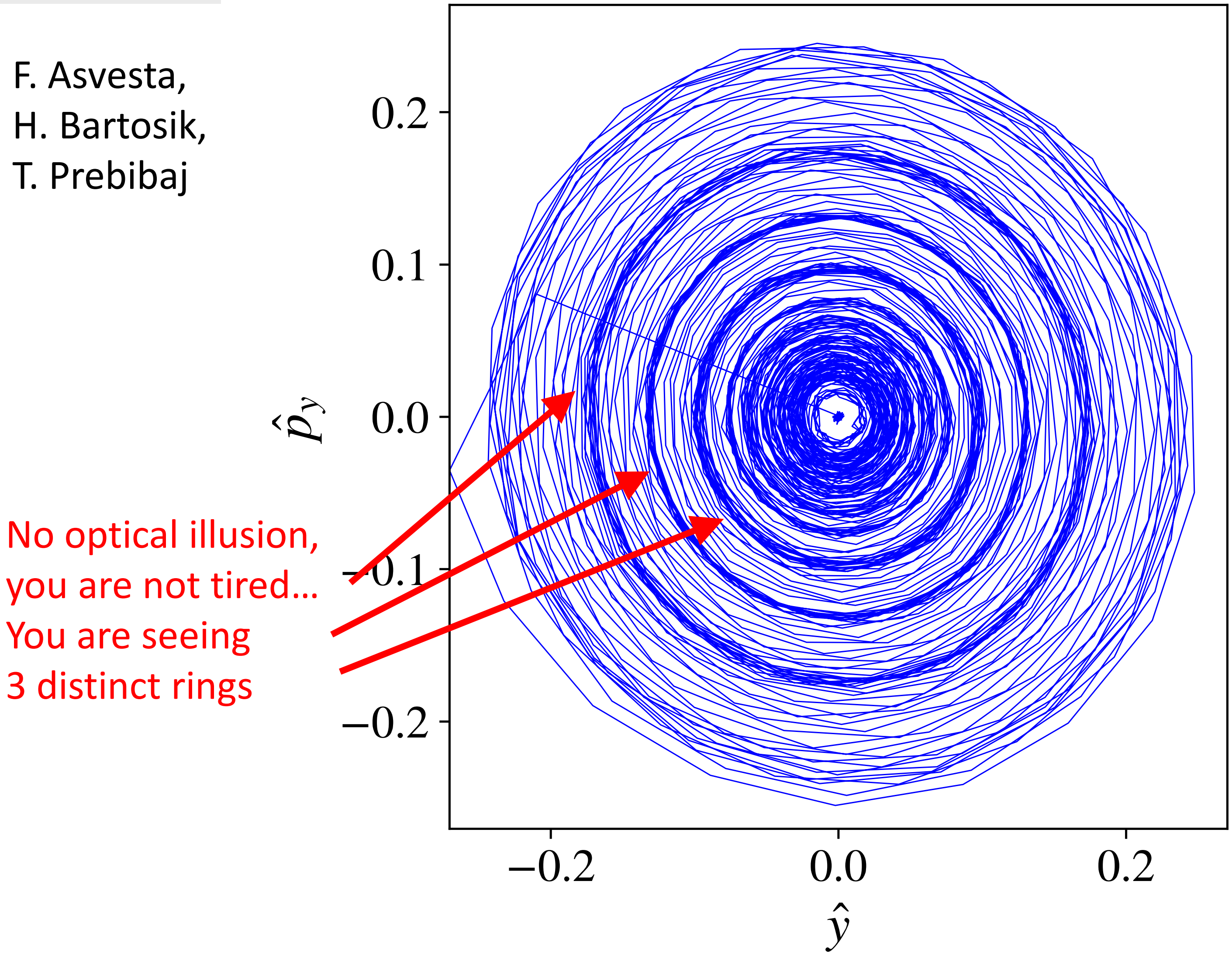
turn B
r

$n_t)$



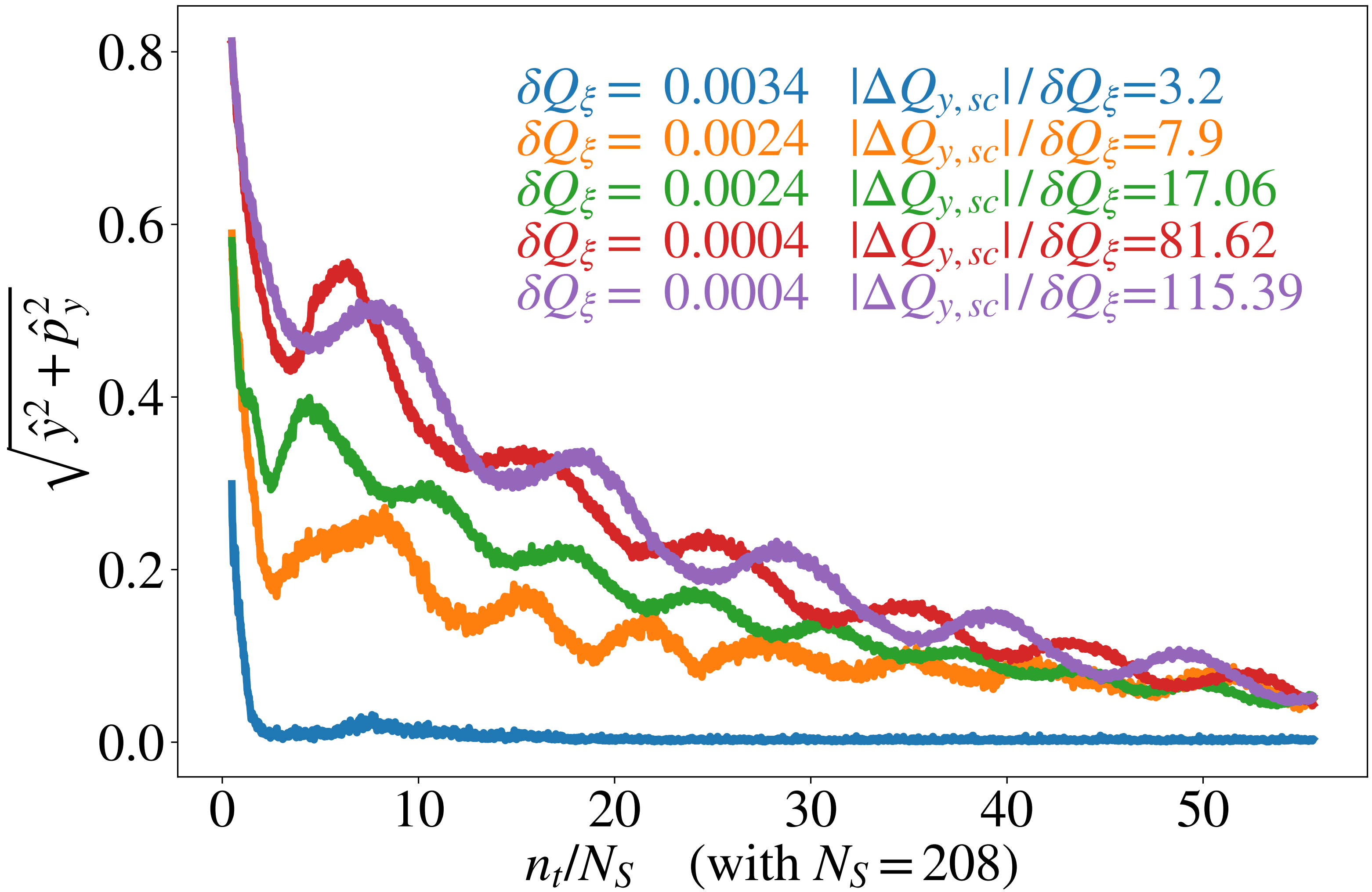
Another example of measurement results

F. Asvesta,
H. Bartosik,
T. Prebibaj



PSB Measurements

F. Asvesta, H. Bartosik, T. Prebibaj



Summary & Outlook

- The joint effect of space charge and chromaticity on a kicked coasting beam has been investigated.
- A two-particle model suggests a linear coupling mechanism between coherent and incoherent dynamics of the kicked system
- This model allows us to retrieve the relevant scaling parameters of the dynamics
- Particle In Cell simulations of a kicked coasting beam follow the same scaling !!
- An experimental campaign at the CERN-PSB has confirmed that the center of mass exhibits ``beating oscillations'' !!
- More studies have to follow to fully interpret the experimental results.

Thank you for your attention

Info on coasting beam measurement

M. Gasior and R. Jones, "High Sensitivity Tune Measurement by Direct Diode Detection", Proc. of DIPAC 2005, Lyon, France, CTWA01, p. 312-314
(link: **https://accelconf.web.cern.ch/d05/PAPERS/CTWA01.pdf**).

- The base-Band Q (BBQ) system is based Direct Diode Detection (3D) method.
- Two peak detectors that are connected to the electrodes of the beam position pick-up (Fig. 1) yield the amplitude modulation envelope of the beam signal (sampling bunch signals close to their maxima at the bunch repetition rate, downmixing the wideband bunch spectrum into the baseband). Capacitors remove the DC content of the peak detector voltages and a differential amplifier (DA) subtracts them. Notch Filter + Band Pass + Gain control.
- By adjusting the gain, the BBQ system is highly sensitive to detect the charge variations even when not bunched (this is NOT mentioned in the paper).

A nice schematic I found on another paper from 2011 (https://cds.cern.ch/record/1372193/files/CERN-BE-2011-016.pdf) in which they mention turn-by-turn resolution of 30nm for the LHC: