An alternative subtraction scheme for NLO QCD calculations

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.

based on
C.H.Chung, M. Krämer, TR (JHEP 1106:144,2011)
C.H.Chung, TR (Phys.Rev. D87 (2013))
TR (Mod. Phys. Lett. A, Vol. 28, No. 23 (2013))
M. Bach, TR (arXiv:1311.5773)
TR (work in progress)

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Aim of the talk

- will not talk for hours on importance of NLO at LHC (a talk for itself); instead:
- give a schematic overview on sources of divergences and treatment for NLO calculations
- explain idea and setup of subtraction scheme(s)

  (although here this feels like bringing owls to Athens...)
- explain idea of new scheme
- show results for a couple of (simple) examples
- (only very few words about parton showers)

Disclaimer !

- most things I talk about have been done $\sim$ 10 years ago
- also, other groups have worked on this
- therefore now: $\Rightarrow$ more of an overview
- recently started to implement this into Herwig [but progress is slow]

1. NLO calculations and subtraction schemes
   - Structure of NLO calculations
   - Subtraction Schemes
2. Nagy Soper subtraction scheme
   - Scheme setup
   - Applications
3. Summary and Outlook

Structure of NLO calculations

General structure of NLO cross sections (1)

Contributions to a fixed order cross section $\sigma(\alpha^{\kappa})$

- Leading order (LO) cross section, contributions to $\mathcal{O}\left(\alpha^{\kappa}\right)$:

$$\sigma_{\text{Born}} = \int d\Gamma_m \, |\mathcal{M}_{\text{Born}}^{(m)}|^2(s)$$

with $d\Gamma_m$ phase space for m particles in final state,
$\mathcal{M}_{\text{Born}}^{(m)}$ matrix element

General structure of NLO cross sections (2) - NLO part

- NLO contributions, to $\mathcal{O}(\alpha^{\kappa+1})$:
  virtual (=loops) and real (additional particle emission)
- virtual corrections:

$$\sigma_{\text{virt}} = \int d\Gamma_m \, 2\, Re(\mathcal{M}^{(m)}_{\text{Born}} \, (\mathcal{M}^{(m)}_{\text{virt}})^*)$$

- emission of additional real particles:

$$\sigma_{\text{Born}+1} = \int d\Gamma_{m+1} \, |\mathcal{M}^{(m+1)}|^2$$

  one more particle in the final state
- correct power counting requires to take both virtual and real diagrams into account, such that

$$\sigma_{\text{NLO}}(\alpha^\kappa) = \sigma^{(m)}_{\text{Born}}(\alpha^\kappa) + \sigma^{(m)}_{\text{virt}}(\alpha^{\kappa+1}) + \sigma^{(m+1)}_{\text{real}}(\alpha^{\kappa+1})$$

NLO corrections - sources of divergences

- NLO calculations involve integrals over undetermined loop momenta

$$\mathcal{M}_{\text{virt}} \propto \int \prod_i d^4 k_i\, F(k_i\, p_1\, p_2, ...)$$

(F: general function of internal and external momenta; depends on Lorentz structure, ...)

- poles for $k \longrightarrow \infty$: ultraviolet divergence
  treated by renormalizing the parameters of the theory (masses, couplings, ...), $\Rightarrow$ not my topic here
- poles for $k \longrightarrow 0$: infrared divergences

$\Rightarrow$ will attack these in the rest of my talk

Infrared divergences

- not everything lost: for well defined variables, infrared singularities cancel for fixed $\mathcal{O}(\alpha^n)$ calculations (Kinoshita, Lee, Nauenberg, 1964)
- (aside: we here assume that the Born cross section σ_{Born} is infrared finite)
- source of infrared divergence: emission of massless particles
- appears as terms $\frac{1}{p_i p_j}$ in matrix elements
- $p_{i,j}$: four momenta; p_i emitter, p_j emitted particle
- $$p_i p_j \;=\; E_i\, E_j\, (1\,-\cos\,\theta_{ij})$$
  for massless particles
- $E_j \to 0$: soft divergence, $\cos\theta_{ij} \to 1$: collinear divergence
- both can appear at the same time: douple poles

Infrared divergences: Treatments

- KNL theorem: infrared divergences cancel between real and vitrual contributions:
  need to have a good (analytical) parametrization
- solution: go from $D = 4$ to $D = 4 - 2\,\varepsilon$ dimensions
- poles then appear as

$$\sigma_{\text{real}} = \frac{A}{\varepsilon^2} + \frac{B}{\varepsilon} + \ldots$$

- A, B depend on the splitting process
- eg in QCD $\tilde{p}_i \rightarrow p_i + p_j$ (omitted color factors etc)

$$q \rightarrow q\,g \quad : \quad \propto \frac{1}{\varepsilon^2} + \frac{3}{2\,\varepsilon}$$

$$g \rightarrow q\,\bar{q} \quad : \quad \propto -\frac{1}{3\varepsilon}$$

- poles arise from **integration** of phase space of p_j
- important: **this behaviour is the same for all processes**

Subtraction Schemes

Dipole subtraction: general idea

- know that pole structure always the same
- can also show: in the singular limits,

$$|\mathcal{M}^{(m+1)}|^2 \longrightarrow D_{ij}(p_i, p_j)\,|\mathcal{M}^{(m)}|^2, \quad D_{ij} \sim \frac{1}{p_i p_j} \qquad (1)$$

- D_{ij}: **dipoles**, contain complete singularity structure
- also means that

$$\int d\Gamma_{m+1} \left(|\mathcal{M}^{(m+1)}|^2 - \sum_{ij} D_{ij} |\mathcal{M}^{(m)}|^2 \right) = \text{finite}$$

- **general idea of dipole subtraction:** make use of (1), shift singular parts from $m+1$ to m particle phase space

Dipole subtraction for total cross sections (1)

Master formula

$$\sigma_{NLO} = \int d\Gamma_m \left(|\mathcal{M}|^2_{\text{Born}} + 2\,\text{Re}(\mathcal{M}_{\text{Born}}\mathcal{M}^*_{\text{virt}}) + \widetilde{F}_{\text{sing}}|\mathcal{M}|^2_{\text{Born}} \right) J^{(m)} + \int d\Gamma_{m+1} \left(|\mathcal{M}|^2_{\text{real}}\, J^{(m+1)} - F_{\text{sing}}|\mathcal{M}|^2_{\text{Born}}\, J^{(m)} \right)$$

$\Rightarrow$ effectively added "0"; both integrals finite (major work: Catani, Seymour, 1996)

- Js: define the quantities which are measured; here you put in guarantee that Born part is infrared finite (eg require that $p_k p_l > p^2_{min}$)
- $\tilde{F} = \int dp_j\, F$; $\Longleftarrow$ this is where all the work is

Dipole subtraction for total cross sections (2)

Master formula in more detail (I,K,P)

$$\sigma = \sigma^{LO} + \sigma^{NLO}$$

$$\sigma^{NLO} = \int_{m+1} d\sigma^R + \int_m d\sigma^V + \int d\sigma^C$$

$$= \int_{m+1} \left(d\sigma^R - d\sigma^A\right) + \int_m \left(d\sigma^{\tilde{A}} + d\sigma^V + d\sigma^C\right),$$

$\Rightarrow$ effectively added "0"; both integrals finite

$$\sigma_m^{\text{NLO}}(s) = \int_m \left\{ |\widetilde{\mathcal{M}}_{\text{virt}}(s;\varepsilon)|^2 + \mathbf{I}(\varepsilon)|\mathcal{M}_{\text{Born}}(s)|^2 + \int_0^1 dx \, \left(\mathbf{K}(x) + \mathbf{P}(x;\mu_F)\right) |\mathcal{M}_{\text{Born}}(x,s)|^2 \right\}$$

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$$\sigma_m^{\mathrm{NLO}}(s) = \int_m \Big\{ |\widetilde{\mathcal{M}}_{\mathrm{virt}}(s;\varepsilon)|^2 + \mathbf{I}(\varepsilon)|\mathcal{M}_{\mathrm{Born}}(s)|^2 + \int_0^1 dx\, \left(\mathbf{K}(x) + \mathbf{P}(x;\mu_F)\right) |\mathcal{M}_{\mathrm{Born}}(x,s)|^2 \Big\}$$

Integrated Dipoles in more details: I, K, P

- in principle, obtain $\int d\Gamma_1 \, D \, = \, \int_0^1 \, dx \, \Big(\mathbf{I}(\varepsilon) \, + \, \tilde{\mathbf{K}}(x,\varepsilon) \Big)$
- $\mathbf{I}(\varepsilon) \, \propto \, \delta(1-x)$: corresponds to loop part
- $\tilde{\mathbf{K}}(x,\varepsilon)$ contains finite parts as well as **collinear singularities**
- latter need to be cancelled by adding **collinear counterterm**

$$\frac{1}{\varepsilon} \left(\frac{4\pi\mu^2}{\mu_F^2} \right)^{\varepsilon} \, P^{qq}(x)$$

  depends on factorization scale μ_F ($P^{qq}(x)$ splitting function)
- PDFs come in again: term already accounted for by folding w PDF, needs to be subtracted

Interlude: energy momentum conservation (1)

...unfortunately, some complications are involved...

- previous slide: add and subtract "0" in terms of

$$\int d\Gamma_m \widetilde{F}_{\text{sing}} |\mathcal{M}^{(m)}_{\text{Born}}|^2 - \int d\Gamma_{m+1} F_{\text{sing}} |\mathcal{M}^{(m)}_{\text{Born}}|^2$$

- addition and subtraction takes place in different phase spaces

$$m \longleftrightarrow m+1$$

- somehow need to define a matching $(m+1) \Rightarrow (m)$
- why ?? want all external particles to be onshell

  here: $p^2 = 0$ for massless particles

Interlude: energy momentum conservation (2)

- example: $q \rightarrow q + g$ splitting

$$m+1 : q(p_i) + g(p_j), \quad m : q(p_{\tilde{i}})$$

- everything onshell: $p_{\tilde{i}}^2 = 0,\ p_i^2 = 0,\ p_j^2 = 0$
- not possible if $p_{\tilde{i}} = p_i + p_j$!!
- $\Rightarrow$ need to redistribute the momenta somehow

$$p_{\tilde{a}}^{(m)} = F\left(p_a^{(m+1)}, p_b^{(m+1)}, \dots\right)$$

- also need to keep total energy/ momentum conserved:

$$\sum_m p_{\tilde{a}} \stackrel{!}{=} \sum_{m+1} p_a$$

(sum over outgoing particles only)

Second ingredient: Parametrization of integration variables

- again: remember you have

$$F_{\text{sing}} \propto D_{ij}, \; \widetilde{F}_{sing} = \int d\Gamma_1 \, D_{ij}, \; d\Gamma_1 \propto d^4 p_j \, \delta(p_j^2)$$

$$\Longrightarrow \widetilde{F}_{sing} \propto \int d^4 p_j \, \delta(p_j^2) \, D_{ij}$$

- 3 free variables (in D dimensions: $D-1$)

  !! need to be written in terms of m particle variables !!
- now all ingredients:
  total energy momentum conservation, onshellness of external particles, need for integration variables

Nagy Soper subtraction scheme

Subtraction schemes

- many different subtraction schemes are around
- best known: Catani, Seymour, 1996 (hep-ph/9605323)
- more and more popular: Frixione, Kunszt, Signer, 1995 (hep-ph/9512328)
- many other (private ones) around...

important message:

poles have to be the same; finite parts can differ

$\Rightarrow$ **behaviour in the singular regions is unique** $\Leftarrow$

NLO at hadron colliders: parton showers

- previous slide: **factorization** in hard (=perturbative) and soft (=PDF) part
- additional ingredient: **parton showers**
- give **probability of additional particle radiation**, attached to hard matrix element ("hardness" guaranteed through cutoff scale)

$\Rightarrow$ **highly important for correct description of processes at Tevatron/ LHC**

- **combining** with NLO: **understood**, but slightly nontrivial
- popular prescriptions: **MC@NLO** (Frixione, Webber 2002), **Powheg** (Frixione, Nason, Oleari 2007)
- **a lot of development in this field, also for even higher orders** [see talks at SM@LHC]

Nagy Soper dipoles: shower algorithm (commercial slide)

- Nagy Soper dipoles: suggested in 2007 (JHEP 0709 (2007)) in the context of parton showers
- "Parton showers with quantum interference"
  $\Rightarrow$ aim is to treat parton showers on a quantum-mechanical level (usual treatment: classical, ie averaging over spins, no interference effects, only leading color)

follow up work: (same authors)

- "Parton showers with quantum interference: Leading color, spin averaged" (JHEP 0803 (2008))
  shows equivalence to standard showers in singular limit
- "Parton showers with quantum interference: Leading color, with spin" (JHEP 0807 (2008))
- "A parton shower based on factorization of the quantum density matrix" (JHEP 06 (2014) 097) !!! IMPLEMENTATION !!!

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More work on this (same authors)

- "Effects on subleading color in a parton shower" (JHEP 07 (2015) 119)
- "Summing threshold logs in a parton shower" (JHEP 10 (2016) 019)
- "What is a parton shower ?" (Phys.Rev.D 98 (2018) 1, 014034)
- "Jets and threshold resummation in Deductor" (Phys.Rev.D 98 (2018) 1, 014035)
- "Parton showers with more exact color evolution" (Phys.Rev.D 99 (2019) 5, 054009)
- "Evolution of parton showers and parton distribution functions" (Phys.Rev.D 102 (2020) 1, 014025)
- "Summations of large logarithms by parton showers" (Phys.Rev.D 104 (2021) 5, 054049)
- "Summations by parton showers of large logarithms in electron-positron annihilation" (arXiv:2011.04777)
- "Multivariable evolution in final state parton shower algorithms" (Phys.Rev.D 105 (2022) 5, 054012)

Nagy Soper subtraction scheme

Main motivation for new scheme

- basic idea: can use the splitting functions in the parton shower as dipole subtraction terms

  $\Rightarrow$ have same behaviour in singular limits
- "turn around" of idea suggested by Nagy, Soper (hep-ph/0503053): use Catani Seymour Dipoles for shower algorithm
- realizations: Catani Seymour Showers in Sherpa (Schumann ea '07), Herwig++ (Plätzer ea '11), ... (Winter ea, Dinsdale ea '07, ...))
- introduce new matching between m and $m+1$ phase spaces

$\Rightarrow$ leads to a much smaller number of subtraction terms

especially important for large number of external particles

$\Rightarrow$ same dipoles in shower and subtraction scheme: facilitates matching with NLO calculations

Interim: Status of NLO tools

(I already apologize for non-completeness of the list)

- two main ingredients, living in different phase spaces:

$$\sigma_{\text{virt}} = \int d\Gamma_m \, 2\, Re(\mathcal{M}_{\text{Born}}^{(m)} \, (\mathcal{M}_{\text{virt}}^{(m)})^*); \quad \sigma_{\text{real}} = \int d\Gamma_{m+1} \, |\mathcal{M}^{(m+1)}|^2$$

- **virtual contribution:** many many [(new) (non) public] tools on the market, within MC or as standalone "virtual" generators (eg Openloops (Pozzorini ea, '12), Golem/Gosam (Cullen ea,'11), Blackhat(Bern ea, '08), aMC@NLO (Frixione ea, '11), and many more)

⇒ this part handled by above tools ("NLO revolution")

- **real emission:**
  (typically) handled by the interfacing Monte Carlo

⇒ this is where new scheme becomes important ⇐

Difference 1: Shifting momenta

- matching between m and $m+1$ particle spaces requires reshuffling of momenta
- for
$$p^{(m)}_{\text{mother}} = p^{(m+1)}_{\text{daughter, 1}} + p^{(m+1)}_{\text{daughter,2}},$$
not all particles can be onshell simultaneously

$\Rightarrow$ need additional spectators to take over additional momenta

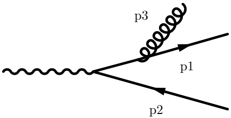
- Catani Seymour: define emitter-spectator pair, momentum goes to 1 additional particle only

$\Rightarrow$ quite easy integrations; however, for increasing number of particles, huge number of transformations necessary

- Nagy Soper:
shift momenta to **all** non-emitting external particles
- number of transformations = number of emitters
- leads to more complicated integrals during framework setup
- in general: # of transformations: CS $\sim N^3_{\text{jets}}/2$, NS$\sim N^2_{\text{jets}}/2$

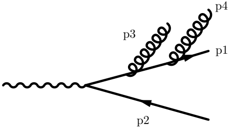
Shifting momenta: Example (1)

$$\gamma^* \longrightarrow q(p_1)\bar{q}(p_2)\, g(p_3) \text{ (@ NLO)}$$



part of Born contribution

real gluon emissions for this diagramm:

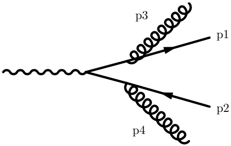


CS: 1 momentum shift/ spectator

p_2, p_3: 2 transformations

NS: 1 total transformation

Shifting momenta: Example (2)



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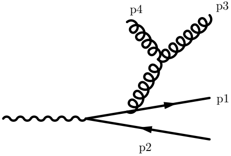


CS: 1 momentum shift/ spectator

p_1, p_2: 2 transformations

NS: 1 total transformation

$\Rightarrow$ from simple counting:

10 transformations using CS vs 5 using NS dipoles !!

of course many more contributions (eg $g \rightarrow q\,\bar{q}$, other Born terms, ...)

Maximal number of transformations

Maximal number of **momentum mappings** using
Catani Seymour or Nagy Soper scheme
counting: consider gluon-splittings only
(maximal number of mappings)

emitter, spectator	**CS**, (ij)	**CS**, k	**NS**, (ij)
fin,fin	$\binom{N'}{2}$	$(N'-2)$	$\binom{N'}{2}$
fin,ini	$\binom{N'}{2}$	2	–
ini,fin	$2\,N'$	$(N'-1)$	$2\,N'$
ini,ini	$2N'$	1	–
total $(\sum_{\text{comb's}}(ij)\times(k))$	$N'^2(N'+3)/2 = (N+1)^2(N+4)/2$		$N'(N'+3)/2 = (N+1)(N+4)/2$
$\sim$	$N^3/2$		$N^2/2$

(N' number of real emission, N number of Born type final state particles)

Final state $g \rightarrow q\,\bar{q}$: Catani Seymour vs Nagy Soper (1)

- $g(\tilde{p}_i) \rightarrow q(p_i) + \bar{q}(p_j)$,
  spectator: any other final state parton, p_k
- Dipole keeping angular correlations

$$\langle\mu|v_{ij}^2|\nu\rangle_{\text{NS, CS}} = -\underbrace{\frac{4\,\pi\,\alpha_s}{\hat{p}_i\,\hat{p}_j}}_{\text{sing}}\left[g^{\mu\nu} + 2\frac{k_\perp^\mu\,k_\perp^\nu}{\hat{p}_i\,\hat{p}_j}\right]$$

$$\textbf{NS:}\ k_\perp = p_i - (1-z)\gamma(y)\tilde{p}_i - \frac{z}{\gamma(y)}\,y\,\tilde{n}$$

$$\textbf{CS:}\ k_\perp = p_i - z\,\tilde{p}_i - y(1-z)\tilde{p}_k$$

- $\tilde{p}_i$, $\tilde{p}_k$: Born-type kinematics, mother parton/ spectator
- y: singular variable
- z: parametrization of angle between $(p_i,\,p_k)$ (CS), $(p_j,\tilde{n})$ (NS)

Final state $g \rightarrow q\,\bar{q}$: Catani Seymour vs Nagy Soper (2)

- Dipole (in terms of integration variables):

$$D^{ij,k}_{\text{NS, CS}} \propto \underbrace{\frac{1}{y}}_{\text{sing}} \left[1 - \frac{z\,(1-z)}{1-\varepsilon}\right]$$

- NS definitions

$$y_{\text{NS}} = \frac{p_i p_j}{(p_i+p_j)Q - p_i p_j}, \quad z_{\text{NS}} = \frac{p_j \tilde{n}}{p_i \tilde{n} + p_j \tilde{n}}$$

$$\tilde{n} = \frac{1+y+\lambda}{2\lambda}\,Q - \frac{a}{\lambda}\,(p_i+p_j), \; \lambda = \sqrt{(1+y)^2 - 4\,a\,y}, \; a = \frac{Q^2}{(p_i+p_j)Q - p_i p_j}$$

- CS definitions:

$$y_{CS} = \frac{p_i p_j}{p_i p_j + p_i p_k + p_j p_k}, \quad z_{CS} = \frac{p_i p_k}{p_i p_k + p_j p_k}$$

Final state $g \to q\,\bar{q}$: Catani Seymour vs Nagy Soper (3)

- CS matching (all other final state particles untouched)

$$\tilde{p}_i = p_i + p_j - \frac{y}{1-y} p_k, \quad \tilde{p}_k^\mu = \frac{1}{1-y} p_k$$

- NS matching

$$\tilde{p}_i = \frac{1}{\lambda}(p_i + p_j) - \frac{1-\lambda+y}{2\,\lambda\,a} Q, \quad \tilde{p}_k^\mu = \Lambda^\mu{}_\nu p_k^\nu \quad \textbf{all fs particles}$$

$$\Lambda^{\mu\nu} = g^{\mu\nu} - \frac{2\,(K+\widetilde{K})^\mu (K+\widetilde{K})^\nu}{(K+\widetilde{K})^2} + \frac{2\,K^\mu \widetilde{K}^\nu}{K^2}, \; K = Q - p_i - p_j, \; \widetilde{K} = Q - \tilde{p}_i$$

- integration measure (identical, same pole structure)

$$[dp_j]_{\text{CS}} = \frac{(2\,\tilde{p}_i \tilde{p}_k)^{1-\varepsilon}}{16\,\pi^2} \frac{d\Omega_{d-3}}{(2\,\pi)^{1-\varepsilon}}\, dz\, dy\, (1-y)^{1-2\,\varepsilon} y^{-\varepsilon}\, [z\,(1-z)]^{-\varepsilon},$$

$$[dp_j]_{\text{NS}} = \frac{(2\,\tilde{p}_i\, Q)^{1-\varepsilon}}{16\,\pi^2} \frac{d\Omega_{d-3}}{(2\,\pi)^{1-\varepsilon}}\, dz\, dy\, \lambda^{1-2\,\varepsilon} y^{-\varepsilon}\, [z\,(1-z)]^{-\varepsilon}$$

Final state $g \to q\,\bar{q}$: Catani Seymour vs Nagy Soper (4)

- result CS

$$\mu^{2\varepsilon}\int[dp_j]\,D^{ij,k} = \frac{\alpha_s}{2\,\pi}\,\frac{1}{\Gamma(1-\varepsilon)}\,T_R\left(\frac{2\mu^2\pi}{\tilde{p}_i\tilde{p}_k}\right)^{\varepsilon}\left[-\frac{2}{3\,\varepsilon}-\frac{16}{9}\right]$$

- result NS

$$\mu^{2\varepsilon}\int[dp_j]\,D^{ij} = T_R\,\frac{\alpha_s}{2\,\pi}\,\frac{1}{\Gamma(1-\varepsilon)}\left(\frac{2\,\pi\,\mu^2}{p_{\tilde{i}}Q}\right)^{\varepsilon}$$

$$\times\left[-\frac{2}{3\,\varepsilon}-\frac{16}{9}+\frac{2}{3}\left[(a-1)\,\ln(a-1)-a\,\ln\,a\right]\right],$$

- for $a=1$, reduces completely to Catani Seymour result
- (reason: $a=1$ implies only 2 particles in the final state, $\tilde{n}\to p_k$, $\Rightarrow$ complete equivalence)
- tradeoff: all final state particles get additional momenta: integral more complicated, but fewer transformations necessary

Difference 2: Combining showers and NLO (1)

Very very short...

- double counting: hard real emissions are described in both shower and "real emission" matrix element
- want:hard: matrix element, soft: shower (always talk about 1 jet)
- can be achieved by adding and subtracting a counterterm

$$-\int_{m+1} d\sigma^{\text{PS}}|_{m+1} + \int_{m+1} d\sigma^{\text{PS}}|_{m}$$

details eg in hep-ph/0204244: "Matching NLO QCD computations and parton shower simulations" (Frixione, Webber), MC@NLO

Difference 2: Combining showers and NLO (2)

- important: have new terms in $m+1$ phase space

$$\int_{m+1}\left(d\sigma^{R}\underbrace{-d\sigma^{A}+d\sigma^{PS}|_{m}}_{=0}-d\sigma^{PS}|_{m+1}\right)$$

- same splitting functions: second and third term cancel !!
  left with

$$\int_{m+1}\left(d\sigma^{R}-d\sigma^{PS}|_{m+1}\right)$$

$\Rightarrow$ improves numerical efficiency

- more details on this, also for MC@NLO vs Powheg:
  S. Hoeche ea, "A critical appraisal of NLO+PS matching methods", JHEP 1209 (2012

Scheme validation (C.H.Chung, M. Krämer, TR, JHEP 1106 (2011) 144; C.H.Chung, TR, Phys Rev D87 (2013))

- **main reason** for improved scaling: **different mapping**
- leads to **more complicated integrated subtraction terms**
- ✓ **all done and veryfied:** (final result independent of subtraction scheme)

Test-processes

- single W at hadron colliders
- Dijet production at lepton colliders
- $p\bar{p} \rightarrow H$ and $H \rightarrow g\,g$
- DIS
- $e\,e \rightarrow$ 3 jets

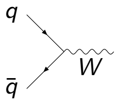
Applications

Single W (easy)

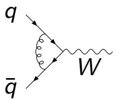
[Deep inelastic scattering]

$e^+ e^- \longrightarrow 3$ **jets** (hard)

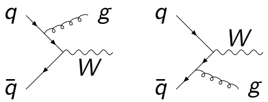
Single W production (slide by C.H. Chung)



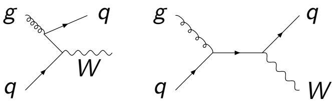
Tree level: $q\bar{q} \longrightarrow W$



Virtual corrections: $q\bar{q} \longrightarrow W$



Real corrections: $q\bar{q} \longrightarrow Wg$



$gq \longrightarrow Wq$ (+ 2 more diagrams)

Subtraction terms: Nagy Soper vs Catani Seymour

NS, CS-NS, CS= NS+CS-NS

- 2 particle phase space (real emission)

$$\mathcal{D}^{14,2} + \mathcal{D}^{24,1} = \underbrace{\frac{1}{4}\frac{1}{9}\sum|\mathcal{M}_{\text{real}}|^2}_{\text{singular}} + \underbrace{\frac{16}{9}g^2\,\alpha_s\,\pi}_{\text{finite}}$$

- 1 particle phase space (virtual contribution)

$$\mathbf{I}(\varepsilon)|\mathcal{M}_b|^2 = \underbrace{\frac{2\alpha_s}{3\pi}\frac{1}{\Gamma(1-\varepsilon)}(-8+\frac{2}{3}\pi^2)|\mathcal{M}_b|^2}_{\text{finite}} - \underbrace{|\widetilde{\mathcal{M}_v}|^2}_{\text{singular (+finite)}}$$

$$\begin{aligned} \mathbf{K}^a(xp_a) &= \frac{\alpha_s}{2\pi}C_F\frac{1}{\Gamma(1-\varepsilon)}\left[-(1-x)\ln x + 2(1-x)\ln(1-x)\right. \\ &+4x\left(\frac{\ln(1-x)}{1-x}\right)_+ - \frac{2x\ln x}{(1-x)_+} - \left(\frac{1+x^2}{1-x}\right)_+\ln\left(\frac{4\pi\mu^2}{2xp_a\cdot p_b}\right) \\ &\left.+(1-x)\right] \end{aligned}$$

compare to Nagy Soper :

pole structure the same, finite terms differ ✓

Subtraction terms: Nagy Soper vs Catani Seymour

NS, CS-NS, CS= NS+CS-NS

- **2 particle phase space (real emission)**

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$$\mathbf{K}^a(xp_a) = \frac{\alpha_s}{2\pi} C_F \frac{1}{\Gamma(1-\varepsilon)}\Big[-(1-x)\ln x + 2(1-x)\ln(1-x) + 4x\left(\frac{\ln(1-x)}{1-x}\right)_+ - \frac{2x\ln x}{(1-x)_+} - \left(\frac{1+x^2}{1-x}\right)_+ \ln\left(\frac{4\pi\mu^2}{2xp_a\cdot p_b}\right) + (1-x)\Big]$$

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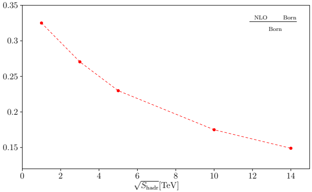
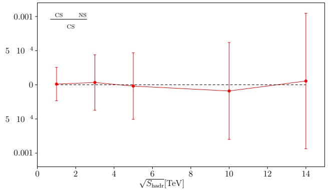
compare to Nagy Soper :
pole structure the same, finite terms differ ✓

Numerical results for single W (slide by C. Chung)

input: $M_W = 80.35$ GeV, PDF $\Rightarrow$ cteq6m, $\alpha_s(M_W) = 0.120299$



$\frac{\sigma_{NLO}-\sigma_{LO}}{\sigma_{LO}}$ as a function of $\sqrt{S}_{\rm hadr}$
corrections up to 30%



relative difference between CS and NS: $\frac{\sigma_{CS}-\sigma_{NS}}{\sigma_{CS}}$
agree on the sub-permill level ✓

difference between schemes:

subtraction in m particle phase space, $\mathbf{K}(x)$ terms

pole structure the same, finite terms shifted around ✓

Deep inelastic scattering (subprocess of...)

- **considered process:**

$$e(p_{in})\,q(p_1)\;\longrightarrow\;e(p_{out})\,q(p_4)\;[g(p_3)]$$

- CS: spectator for final state gluon emission:

  initial state quark

- NS: spectator for final state gluon emission:

  final state lepton

- ("spectator" = spectator in momentum mapping)

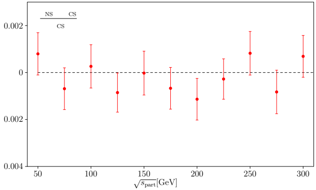
⇒ first nontrivial check of NS scheme ⇐

DIS: Catani Seymour vs Nagy Soper - numerical result

considered process:

$$e(p_{in})\, q(p_1) \longrightarrow e(p_{out})\, q(p_4)\ [g(p_3)]$$

apply both schemes: get the same result



relative difference between CS and NS: $\frac{\sigma_{CS}-\sigma_{NS}}{\sigma_{CS}}$

agree on the sub-permill level ✓

$e^+ e^- \longrightarrow$ 3 jets (1)

- consider process

$$e^+ e^- \longrightarrow q \bar{q} g$$

  at NLO

- real emission contributions:

$$e^+ e^- \longrightarrow q \bar{q} q \bar{q},\; q\bar{q} g g$$

- number of necessary mappings (in total):

$$(8+10)_{CS} \text{ vs } (4+5)_{NS}$$

- 3 different color structures: $C_A\, C_F^2,\; C_A\, C_F\, n_f\, T_R,\; C_A^2\, C_F$
- singular parts: $C_A\, C_F\, n_f\, T_R$: $q\bar{q}\, q\, \bar{q}$ only,
  $C_A^2\, C_F,\; C_A\, C_F^2$: $q\bar{q}\, gg$ only

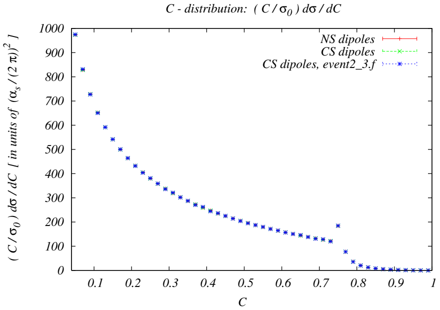
$e^+ e^- \longrightarrow$ 3 jets (2)

- result known for a long time: Ellis ea 1980 (also Kuijf 1991, Giele ea 1992)
- infrared safe observable: C-distribution (Ellis ea 1980),

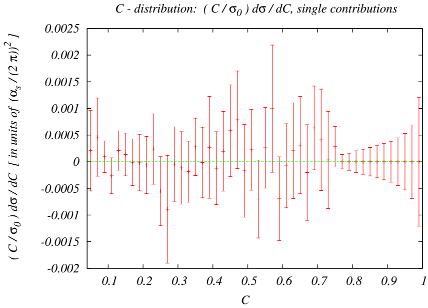
$$C^{(n)} = 3 \left\{ 1 - \sum_{i,j=1,\, i<j}^{n} \frac{s_{ij}^2}{(2\, p_i \cdot Q)\,(2\, p_j \cdot Q)} \right\}$$

- integrals more complicated as more final state particles are involved in mapping

$e^+ e^- \longrightarrow$ 3 jets ($N_C^2 C_F$ component) - numerical result



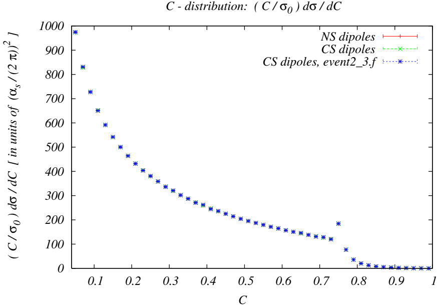
Comparison between NS and CS implementation as well as event2_3.f from M. Seymour



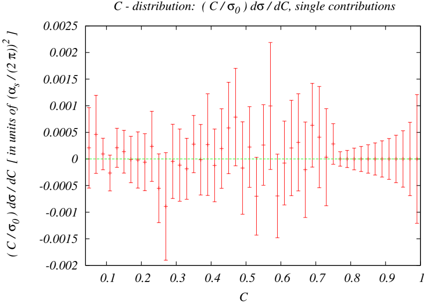
Relative difference between NS and CS implementation

- results agree on the permill level, compatibel with 0 ✓
- remark: for $C > 0.75$, only real emission contributes ⇒ difference exactly 0 (when using the same setup)

$e^+ e^- \longrightarrow 3$ jets ($N_C^2\, C_F$ component) - numerical result



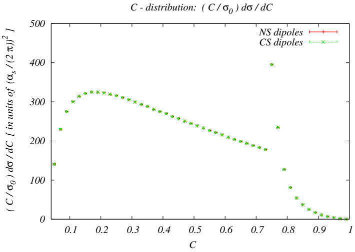
Comparison between NS and CS implementation as well as event2_3.f from M. Seymour



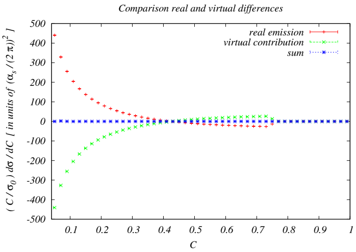
Relative difference between NS and CS implementation

- results agree on the permill level, compatibel with 0 ✓
- remark: for $C > 0.75$, only real emission contributes $\Rightarrow$ difference exactly 0 (when using the same setup)

$e\, e \longrightarrow 3$ jets: all components



sum of all color contributions

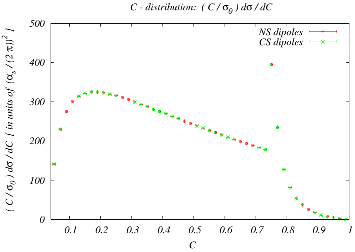


differences between real and virtual contributions from CS and NS dipoles respectively

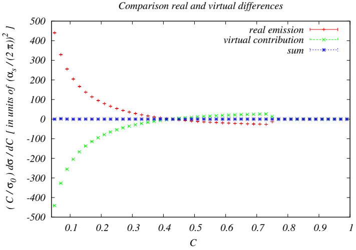
overall agreement on permill level ✓

large differences adding up to 0 ✓

$e\,e \longrightarrow 3$ jets: all components



sum of all color contributions



differences between real and virtual contributions from CS and NS dipoles respectively

overall agreement on permill level ✓

large differences adding up to 0 ✓

Work in progress: implementation into Herwig

- started a long time ago...
- so far: started with the easy processes: Drell Yan, $e^+e^- \to$ dijet
- Drell Yan: OKish for 8/ 13 TeV, with/ without cuts
- means: sub-permill agreement on total cross sections, within errorbars
- for dijet: work in progress [wanted to show some results $\Rightarrow$ lack of time...]
- thanks to S. Plaetzer for support, and COST for several STSMs

Helac implementation

- "Quantifying quark mass effects at the LHC: A study of pp $\rightarrow$ b anti-b b anti-b + X at next-to-leading order", G. Bevilacqua, M. Czakon, M. Kraemer, M. Kubocz, M. Worek, JHEP 07 (2013) 095
- "Complete Nagy-Soper subtraction for next-to-leading order calculations in QCD", G. Bevilacqua, M. Czakon, M. Kubocz, M. Worek, JHEP 10 (2013) 204
  contains speed comparisons, up to factor 2 !
- "Matching the Nagy-Soper parton shower at next-to-leading order", M. Czakon, H. B. Hartanto, M. Kraus, M. Worek, JHEP 06 (2015) 033

Status Quo and Outlook

Status quo (instead of Summary)

- goal: establish NS dipole formalism
- all integrals are done ✓
- all singularity structures checked ✓
- all finite term cross checked ✓

$\Rightarrow$ Massless scheme validated $\Leftarrow$

"Test" processes
(w extensive documentation)

- single W at hadron colliders
- Dijet production at lepton colliders
- $p\bar{p} \rightarrow H$ and $H \rightarrow g\, g$
- DIS
- $e\, e \rightarrow$ 3 jets

Catani Seymour vs Nagy Soper - Summary and upshot

NS upshots:

+ **less transformations** in the subtraction terms (leading to faster codes for higher multiplicity final states)
− **more complicated expressions**, and **numerically evaluable integrals**, in the subtraction terms (already for “easy” processes)
- both due to **different mapping procedure**

⇒ need to test on higher multiplicity final states to see net result ⇐

Outlook

Outlook

- make generically available for application in new higher order calculations
- **implementation in current NLO tool(s)**
- finish interpolation of finite integrals
- (further down the road: extension to massive scheme, combination with parton shower)

! Thanks for listening !

Appendix

Processes at hadron colliders: general

- hadron colliders (as Tevatron, LHC) collide **hadrons**
- QCD: talks about **partons**
- transition: parton distribution functions (PDFs) $f_l(x, \mu_F)$; $l = q, \bar{q}, g$ flavour, x momentum fraction, (μ_F factorization scale)

masterformula

$$\sigma_{\text{hadr}}(p\,\bar{p} \rightarrow X) = \sum_{l_1, l_2} \int dx_1 \int dx_2\, f_{l_1}(x_1) f_{l_2}(x_2)\, \sigma_{\text{part}}(x_1, x_2;\, l_1 l_2 \rightarrow X)$$

- perturbative, nonperturbative part

Catani Seymour Dipoles (the praise)

- Catani, Seymour 1996: suggested dipole subtraction scheme for NLO calculations, massless particles
- became "standard" for many NLO calculations
- paper in general handled as a "toolbox", ie formulas can be extracted without actually understanding how to get there and where they come from
- has been applied in numerous calculations (7. March '12: 798 citations)
- also **very** helpful for (becoming) experts
- follow up work: massive particles (Catani, Dittmaier, Seymour, Trocsanyi, 02), combined with phase space slicing (Nagy, 03)

Dipole subtraction: Real master formula

Real Masterformula $(s = (p_a + p_b)^2)$

$$\sigma(s) = \int_m d\Phi^{(m)}(s) \frac{1}{n_c(a) n_c(b)} |\mathcal{M}^{(m)}|^2(s) F_J^{(m)}$$
$$+ \int d\Phi^{(m+1)}(s) \left\{ \frac{1}{n_c(a) n_c(b)} |\mathcal{M}^{(m+1)}|^2(s)) F_J^{(m+1)} - \sum_{\text{dipoles}} (\mathcal{D} \cdot F_J^{(m)}) \right\}$$
$$+ \int d\Phi^{(m)}(s) \left\{ \frac{1}{n_c(a) n_c(b)} |\mathcal{M}^{(m)}|^2_{1\text{ loop}}(p_a, p_b) + \mathbf{I}(\varepsilon) |\mathcal{M}^{(m)}|^2(s) \right\}_{\varepsilon=0} F_J^{(m)}$$
$$+ \left\{ \int dx_a\, dx_b \delta(x - x_a)\, \delta(x_b - 1) \int d\Phi^{(m)}(x_a p_a, x_b p_b) |\mathcal{M}^{(m)}|^2 (x_a p_a, x_b p_b) \right.$$
$$\left. \times \left(\mathbf{K}^{a,a'}(x) + \mathbf{P}^{a,a'}(x_a p_a, x_b p_b, x; \mu_F^2) \right) \right\} + (a \leftrightarrow b)$$

where all colour/ phase space factors have been accounted for

NS integration measures (1)

Initial state

$$d\xi_p = dx \int_0^1 dy' \int_0^1 dv \frac{(2p_a \cdot p_b)^{1-\varepsilon} x^{\varepsilon-1}}{(4\pi)^2} \frac{\pi^{\varepsilon-\frac{1}{2}}}{\Gamma\left(\frac{1-2\varepsilon}{2}\right)} \times \left[y'(1-y')\right]^{-\varepsilon} \left[v\,(1-v)\right]^{-\frac{1+2\,\varepsilon}{2}} \Theta\left[(1-x)x\right]$$

Final state, $2 \rightarrow 2$ processes

$$d\xi_p = \frac{(2p_\ell \cdot Q)^{1-\varepsilon}}{16} \frac{\pi^{-\frac{5}{2}+\varepsilon}}{\Gamma\left(\frac{1}{2}-\varepsilon\right)} \times \int_0^1 du u^{-\varepsilon}(1-u)^{-\varepsilon} \int_0^1 dx x^{1-2\varepsilon}(1-x)^{-\varepsilon} \int_0^1 dv \left[v\,(1-v)\right]^{-\frac{1+2\varepsilon}{2}}$$

NS integration measures (2)

Final state, $2 \to n$ processes

$$d\xi_p = \frac{(2\,p_{\tilde{i}}Q)^{1-\varepsilon}}{16}\,\frac{\pi^{-\frac{5}{2}+\varepsilon}}{\Gamma\left(\frac{1}{2}-\varepsilon\right)} \int_0^1 du\,u^{-\varepsilon}\int_0^1 dx\,\delta^{1-\varepsilon}\gamma^{1-2\varepsilon}\left[(1-x)(x-x_0)\right]^{-\varepsilon}\int_0^1 dv\,\left[v(1-v)\right]^{-\frac{1+2\,\varepsilon}{2}}.$$

Variables

$$\lambda = \sqrt{(1+y)^2 - 4\,a\,y},\; a = \frac{Q^2}{2\,p_{\tilde{i}}Q},\; y = \frac{\hat{p}_i\hat{p}_j}{p_{\tilde{i}}Q}$$

$$\gamma = \frac{1}{2}\,(1+y+\lambda)\,,\; x_0(y,a) = \frac{1-\lambda+y}{1+\lambda+y},$$

$q \;\to\; q\,g$ for initial state quarks: Catani Seymour (1)

- $q(\tilde{p}_1) \;\to\; q(p_1) + g(p_4)$, q enters hard interaction
- Dipole:

$$D^{14,2} \;=\; -\frac{8\,\pi\,\mu^2\,\alpha_s\,C_F}{s+t+u}\left(\frac{2\,s\,(s+t+u)}{t\,(t+u)} + (1-\varepsilon)\,\frac{t+u}{t}\right)$$

- matching ($\tilde{p}_2 \;=\; p_2$)

$$\tilde{p}_1 \;=\; x\,p_1,\quad x \;=\; 1 - \frac{p_4\,(p_1+p_2)}{(p_1\,p_2)}$$

$$\tilde{p}_k^\mu \;=\; \Lambda^\mu{}_\nu\,p_k^\nu,\quad (k\text{: final state particles})$$

$$\Lambda^{\mu\nu} \;=\; -g^{\mu\nu} - \frac{2\,(K+\widetilde{K})^\mu(K+\widetilde{K})^\nu}{(K+\widetilde{K})^2} + \frac{2\,K^\mu\,\widetilde{K}^\nu}{K^2}$$

$$K = p_1 + p_2 - p_4,\quad \widetilde{K} = \tilde{p}_1 + p_2$$

$q \rightarrow q\,g$ for initial state quarks: Catani Seymour (2)

- integration variables:

$$v\,=\,\frac{p_1\,p_4}{p_1\,p_2},\;x\,=\,1-\frac{p_4\,(p_1+p_2)}{(p_1\,p_2)}$$

- in p_1, p_2 cm system: $E_4\,\rightarrow\,0\;\Rightarrow\;x\,\rightarrow\,1$ (softness)
  $\cos\theta_{14}\,\rightarrow\,1\;\Rightarrow\;v\,\rightarrow\,0$ (collinearity)
- Dipole in terms of integration variables

$$D^{14,2}\,=\,-\frac{8\,\pi\,\alpha_s\,C_F}{v\,x\,s}\left(\frac{1+x^2}{1-x}-\varepsilon(1-x)\right)$$

- integration measure

$$[dp_j]\,=\,\frac{(2\,p_1\,p_2)^{1-\varepsilon}}{16\,\pi^2}\,\frac{d\Omega_{d-3}}{(2\,\pi)^{1-\varepsilon}}\,dv\,dx\,(1-x)^{-2\,\varepsilon}\left[\frac{v}{1-x}\left(1-\frac{v}{1-x}\right)\right]^{-\varepsilon}$$

where $v\,\leq\,1-x$ and all integrals between 0 and 1

$q \to q\,g$ for initial state quarks: Catani Seymour (3)

- result

$$\mu^{2\varepsilon}\int [dp_j]\,D^{14,2} = \frac{\alpha_s}{2\,\pi}\,\frac{1}{\Gamma(1-\varepsilon)}\,C_F\,\left(\frac{2\mu^2\pi}{p_1\,p_2}\right)^{\varepsilon}$$

$$\times\int_0^1 dx\,\left(\mathbf{I}(\varepsilon)\delta(1-x)+\tilde{\mathbf{K}}(x,\varepsilon)\;\underbrace{-\frac{1}{\varepsilon}P^{qq}(x)}_{\text{killed by coll CT}}\right)$$

with

$$\mathbf{I}(\varepsilon) = \frac{1}{\varepsilon^2}+\frac{3}{2\varepsilon}-\frac{\pi^2}{6}$$

$$\mathbf{K}(x) = (1-x)-2\,(1+x)\ln(1-x)+\left(\frac{4}{1-x}\ln(1-x)\right)_+$$

$$P^{qq}(x) = \left(\frac{1+x^2}{1-x}\right)_+ \quad \text{regularized splitting function}$$

$q \to q\,g$ for initial state quarks: Nagy Soper (1)

- $q(\tilde{p}_1) \to q(p_1) + g(p_4)$, q enters hard interaction
- Dipole:

$$D^{14,2} = -\frac{8\,\pi\,\mu^2\,\alpha_s\,C_F}{s+t+u}\left(\frac{2\,s\,u\,(s+t+u)}{t\,(t^2+u^2)} + (1-\varepsilon)\,\frac{u}{t}\right)$$

as CS, same pole structure as CS

- matching, integration variables, integration measure: as Catani Seymour($v \leftrightarrow y$)
- Dipole in terms of integration variables

$$D^{14,2} = -\frac{8\pi\alpha_s\,C_F}{x\,s} \times \left(\frac{1-x-y}{y}(1-\varepsilon) + \frac{2\,x}{y\,(1-x)} - \frac{2\,x\,[2\,y-(1-x)]}{(1-x)[y^2+(1-x-y)^2]}\right)$$

$q \longrightarrow q\,g$ for initial state quarks: Nagy Soper (2)

- result

$$\mu^{2\varepsilon}\int [dp_j]\, D^{14,2} = \frac{\alpha_s}{2\,\pi}\,\frac{1}{\Gamma(1-\varepsilon)}\,C_F\,\left(\frac{2\,\mu^2\pi}{p_1 p_2}\right)^{\varepsilon}$$

$$\times\int_0^1 dx\,\left(\mathbf{I}(\varepsilon)\delta(1-x) + \tilde{\mathbf{K}}(x,\varepsilon)\ \underbrace{-\frac{1}{\varepsilon}P^{qq}(x)}_{\text{killed by coll CT}}\right)$$

with

$$\mathbf{K}(x) =$$

$$(1-x) - 2\,(1+x)\ln(1-x) + \left(\frac{4}{1-x}\ln(1-x)\right)_+ -(1-x)$$

- equivalence of dipoles schemes checked analytically

Catani Seymour subtraction terms for single W (1)

- 2 particle phase space (real emission)

$$\mathcal{D}^{14,2} + \mathcal{D}^{24,1} = \frac{8}{9}\pi\,\alpha_s\,g^2\left(\frac{s^2+(s+t+u)^2}{t\,u}\right)$$
$$= \underbrace{\frac{1}{4}\frac{1}{9}\sum|\mathcal{M}_{\text{real}}|^2}_{\text{singular}} + \underbrace{\frac{16}{9}g^2\,\alpha_s\,\pi}_{\text{finite}}$$

- 1 particle phase space (virtual contribution)

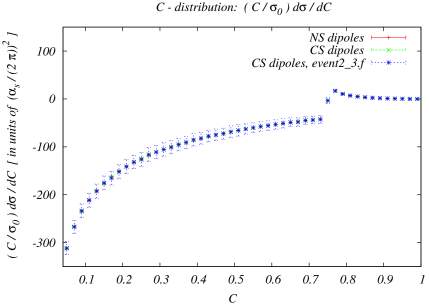
$$\mathbf{I}(\varepsilon)|\mathcal{M}_b|^2 = \underbrace{\frac{2\alpha_s}{3\pi}\frac{1}{\Gamma(1-\varepsilon)}\left(-8+\frac{2}{3}\pi^2\right)|\mathcal{M}_b|^2}_{\text{finite}} - \underbrace{|\widetilde{\mathcal{M}}_v|^2}_{\text{singular (+finite)}}$$

all singular terms will disappear in subtraction

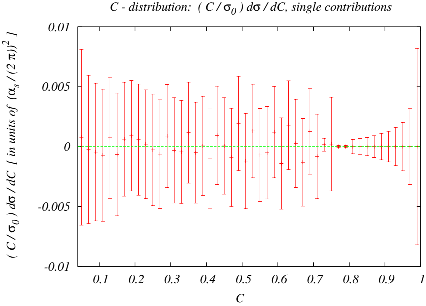
Catani Seymour subtraction terms for single W (2)

$$\begin{aligned}
\mathbf{K}^a(xp_a) &= \frac{\alpha_s}{2\pi} C_F \frac{1}{\Gamma(1-\varepsilon)} \left[\left(\frac{4}{1-x} \ln(1-x) \right)_+ + (1-x) \right. \\
&\quad \left. - 2(1+x)\ln(1-x) - \left(\frac{1+x^2}{1-x} \right)_+ \ln\left(\frac{4\pi\mu^2}{2xp_a \cdot p_b} \right) \right] \\
\mathbf{P}(x,\mu_F^2) &= \frac{\alpha_s}{2\pi} C_F \frac{1}{\Gamma(1-\varepsilon)} \left(\frac{1+x^2}{1-x} \right)_+ \ln\left(\frac{4\pi\mu^2}{\mu_F^2} \right)
\end{aligned}$$

$e^+ e^- \longrightarrow$ 3 jets ($n_f T_R$ component): results



Comparison between NS and CS implementation as well as event2_3.f from M. Seymour



Relative difference between NS and CS implementation

- results agree on the (sub)-percent level, compatibel with 0 ✓ (integration not completely optimized yet)
- remark: for $C > 0.75$, only real emission contributes $\Rightarrow$ difference exactly 0 (when using the same setup)

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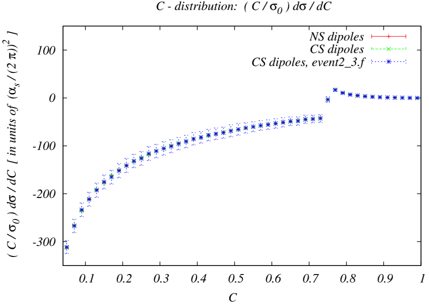

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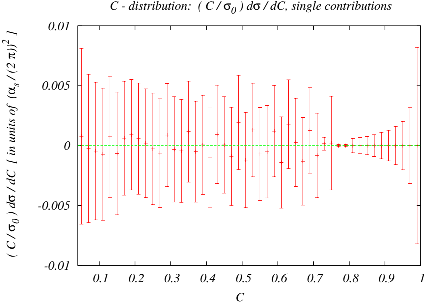
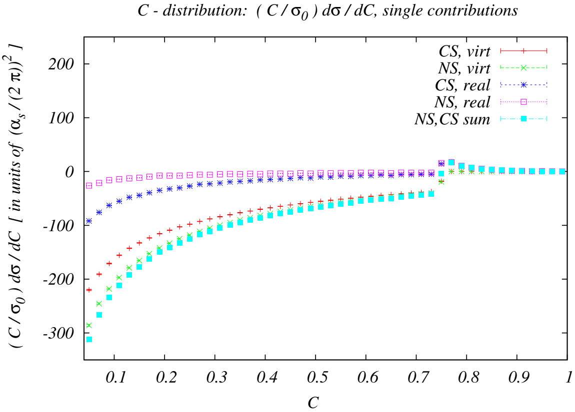


Relative difference between NS and CS implementation

- results agree on the (sub)-percent level, compatibel with 0 ✓ (integration not completely optimized yet)
- remark: for $C > 0.75$, only real emission contributes $\Rightarrow$ difference exactly 0 (when using the same setup)

$e\,e \longrightarrow q\,\bar{q}\,q\,\bar{q}$: single components



real and virtual contributions from CS and NS dipoles respectively as well as sum

What about constant scaling ?? (for the experts...)

(this is typically a question from an FKS user/ author...)

- aMC@NLO (Frixione ea): constant scaling for certain processes
- makes use of symmetries in matrix elements and phase space

⇒ also possible here ⇐

- need to **partition**, but not to **parametrize** à la FKS
  ($\hat{=}$ each partition contains at most one soft/ soft and collinear divergence)
- then dipoles in our scheme which reflect this singularity structure obey **single** mapping ⇒ **constant scaling**

!! very preliminary, no implementation yet...

- details on linear scaling in aMC@NLO: Frederix ea, JHEP 0910 (2009) 003

Recent progress

DIS: Catani Seymour, mapping and integrated dipoles

Mapping

$$\tilde{\mathbf{p}}_1 = x_{43,1} p_1, \quad \tilde{\mathbf{p}}_4 = p_3 + p_4 - (1 - x_{43,1}) p_1$$

$$x_{34,1} = \frac{p_i p_o}{p_3 p_4 + p_1 p_3}$$

Integrated subtraction terms

$$\int_0^1 \mathbf{dx} |\mathcal{M}|^2_{2,\text{tot}} = \int_0^1 \frac{dx}{x} \left\{ -\frac{9}{2} \frac{\alpha_s}{2\pi} C_F \delta(1-x) + K^{\text{eff}}_{\text{fin}}(x) + P^{\text{eff}}_{\text{fin}}(x; \mu_F^2) \right\} |\mathcal{M}|^2_{\text{Born}}(x p_1)$$

$$\mathbf{K}^{\text{eff}}(\mathbf{x}) = \frac{\alpha_s}{2\pi} C_F \left\{ \left(\frac{1+x^2}{1-x} \ln \frac{1-x}{x} \right)_+ + \frac{1}{2} \delta(1-x) + (1-x) - \frac{3}{2} \frac{1}{(1-x)_+} \right\}$$

DIS: Nagy Soper - mapping

Initial state: mapping

$$\mathbf{p_1} = x\,\hat{p}_1,\; \mathbf{p_i} = \hat{p}_i,\; \mathbf{p}^{\mu}_{\mathbf{o,4}} = \mathbf{\Lambda}^{\mu}{}_{\nu}(\mathbf{K},\hat{\mathbf{K}})\hat{p}^{\nu}_{o,4},\, K = x\,\hat{p}_1 + \hat{p}_i,\; \hat{K} = \hat{p}_1 + \hat{p}_i - \hat{p}_3$$

$$x = \frac{\hat{p}_o \cdot \hat{p}_4}{\hat{p}_i \cdot \hat{p}_1}.$$

Final state: mapping

$$\mathbf{p_i} = \hat{p}_i,\; \mathbf{p_1} = \hat{p}_1,\; \mathbf{p_4} = \frac{1}{1-y}\left[\hat{p}_3 + \hat{p}_4 - y\,(\hat{p}_1 + \hat{p}_i)\right],\; \mathbf{p_o} = \frac{\hat{p}_o}{1-y},$$

$$y = \frac{\hat{p}_3 \cdot \hat{p}_4}{\hat{p}_1 \cdot \hat{p}_i}$$

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Nagy Soper - subtraction in the virtual contribution (here:DIS)

⇒ **major difference in integrated subtraction terms**

$$\int_0^1 \mathbf{dx}\,|\mathcal{M}|_2^2 = \int_0^1 dx \left\{ \frac{\alpha_s}{2\,\pi} C_F\,\delta(1-x) \left[-9 + \frac{1}{3}\pi^2 - \frac{1}{2}\mathrm{Li}_2[(1-\tilde{z}_0)^2] + 2\ln 2\,\ln\tilde{z}_0 + 3\ln\tilde{z}_0 + 3\,\mathrm{Li}_2(1-\tilde{z}_0) + \mathbf{I}_{\mathrm{fin}}^{\mathrm{tot},0}(\tilde{\mathbf{z}}_0) + \mathbf{I}_{\mathrm{fin}}^{1}(\tilde{\mathbf{a}}) \right] + K_{\mathrm{fin}}^{\mathrm{tot}}(x;\tilde{z}) + P_{\mathrm{fin}}^{\mathrm{tot}}(x;\mu_F^2) \right\} |\mathcal{M}|^2_{\mathrm{Born}}(x\,p_1),$$

$$\mathbf{K}_{\mathrm{fin}}^{\mathrm{tot}}(\mathbf{x};\tilde{\mathbf{z}}) = \frac{\alpha_s}{2\,\pi} C_F \left\{ \frac{1}{x} \left[2\,(1-x)\,\ln(1-x) - \left(\frac{1+x^2}{1-x}\right)_+ \ln x + 4\,x\left(\frac{\ln(1-x)}{1-x}\right)_+ \right] + \mathbf{I}_{\mathrm{fin}}^{1}(\tilde{\mathbf{z}},\mathbf{x}) \right\},$$

⇒ **contains integrals which need to be evaluated numerically** ⇐

Nagy Soper - integrals to be evaluated numerically

⇒ **Integrals contain nontrivial functions depending on m and $m+1$ four-momenta** ⇐

$$\mathbf{I}_{\text{fin}}^{\text{tot},0}(\tilde{\mathbf{z}}_0) = 2\int_0^1 \frac{dy}{y}\left\{\frac{\tilde{z}_0}{\sqrt{4\,y^2\,(1-\tilde{z}_0)+\tilde{z}_0^2}}\times\ln\left[\frac{2\,z\,\sqrt{4\,y^2\,(1-\tilde{z}_0)+\tilde{z}_0^2}\,(1-y)}{\left(2\,y+\tilde{z}_0-2\,y\,\tilde{z}_0+\sqrt{4\,y^2\,(1-\tilde{z}_0)+\tilde{z}_0^2}\right)^2}\right]+\ln 2\right\}.$$

$$\mathbf{I}_{\text{fin}}^{\mathbf{1}}(\tilde{\mathbf{a}}) = 2\int_0^1 \frac{du}{u}\int_0^1\frac{dx}{x}\times\left[\frac{\mathbf{x}\,(\mathbf{1}-\mathbf{x}+\mathbf{u}\,\mathbf{x}\,[(\mathbf{1}-\mathbf{u}\,\mathbf{x})\,\tilde{\mathbf{a}}+\mathbf{2}])}{\mathbf{k}(\mathbf{u},\mathbf{x},\tilde{\mathbf{a}})}-\frac{1}{\sqrt{1+4\,\tilde{a}_0\,u^2\,(1+\tilde{a}_0)}}\right].$$

$$\mathbf{I}_{\text{fin}}^{\mathbf{1}}(\tilde{\mathbf{z}},\mathbf{x}) = \frac{2}{(1-x)_+}\,\frac{1}{\pi}\int_0^1\frac{dy'}{y'}\left[\int_0^1\frac{dv}{\sqrt{v\,(1-v)}}\,\frac{\tilde{\mathbf{z}}}{\mathbf{N}(\mathbf{x},\mathbf{y}',\tilde{\mathbf{z}},\mathbf{v})}-1\right],$$

DIS: Nagy Soper - variables in integrals to be evaluated numerically

for some integrals, $m+1$ variables have to be reconstructed

⇒ difference wrt standard scheme(s) ⇐

in initial state subtraction terms

$$N = \frac{\hat{p}_3 \cdot \hat{p}_4}{\hat{p}_4 \cdot \hat{Q}} \frac{1}{1-x} + y', \tilde{z} = \frac{1}{x} \frac{p_1 \cdot \hat{p}_4}{\hat{p}_4 \cdot \hat{Q}}$$

$$\hat{p}_3 = \underbrace{\frac{(1-x)(1-y')}{x}}_{\alpha} p_1 + \underbrace{(1-x)\,y'}_{\beta} p_i - k_\perp, \hat{p}_4^\mu = \Lambda^\mu{}_\nu(\hat{K}, K)\hat{p}_4^\nu$$

$$k_\perp^2 = -2\,\alpha\,\beta\,p_1 \cdot p_i,\ k_\perp = -|k_\perp| \begin{pmatrix} 0 \\ 1-2\,v \\ 2\sqrt{v\,(1-v)} \\ 0 \end{pmatrix},$$

in final state subtraction terms

$$k^2(x,u,\tilde{a}) = \left[(1+ux-x)(z-z') + ux\,((1-ux)\,\tilde{a}+1)\right]^2 + 4\,u\,x\,z'\,(1-z)\,(1+u\,x-x)\,((1-ux)\,\tilde{a}+1)$$

$$\tilde{a} = \frac{p_1 \cdot p_o}{p_1 \cdot (p_i - (1-v)\,p_o)}$$

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$\Rightarrow$ difference wrt standard scheme(s) $\Leftarrow$

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$$\hat{\mathbf{p}}_3 = \underbrace{\frac{(1-\mathbf{x})\,(1-\mathbf{y}')}{\mathbf{x}}}_{\alpha}\mathbf{p}_1 + \underbrace{(1-\mathbf{x})\,\mathbf{y}'}_{\beta}\mathbf{p}_i - \mathbf{k}_\perp,\ \hat{\mathbf{p}}_4^\mu = \Lambda^\mu{}_\nu(\hat{\mathbf{K}},\mathbf{K})\hat{\mathbf{p}}_4^\nu$$

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Numerical integrals - approximations through grids/ polynomials (1) (work done by M. Bach)

status as on arXiv:

scheme has in total **7** integrals which need to be evaluated numerically

- **5** depend on **1** external parameter
  (leftover finite terms from singular regions, à la $\varepsilon \times \frac{1}{\varepsilon}$)

$$I_3(a),\; I_{\text{fin}}(a),\; I_{\text{fin}}^{(b)}(a),\; I_{\text{fin}}^{(e)}(a),\; I_{\text{fin}}(\tilde{z}_0)$$

- **2** depend on **2** external parameters
  (finite terms for interference integrals in non-singular regions)

$$I_{\text{fin}}^{(d)}(\tilde{a}, a), I_{\text{fin}}(\tilde{z}_0, x)$$

- (not really a problem though...)

Numerical integrals - approximations through grids/ polynomials (2) (work done by M. Bach)

- **work in progress:** approximate these using polynomials and/ or grids
- all 'one-parameter' integrals: **approximated by polynomials, implemented and checked** (apart from $I_{\text{fin}}(\tilde{z}_0)$)
- agreement for approximation: typically $\mathcal{O}(10^{-6})$ **or better**
- set up for **interpolating grids for others:** on the way

⇒ no additional integrations needed ⇐

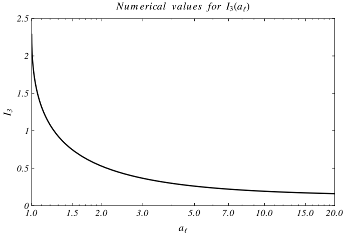
⇒ implementation 'like' Catani Seymour scheme ⇐

(however: the hard part are always the real emissions...)

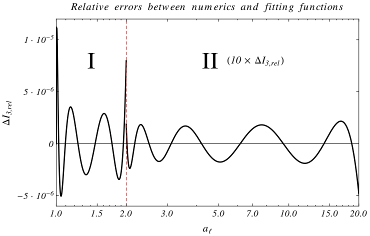
Numerical integrals - approximations through grids/ polynomials (3) (work done by M. Bach)

$$I_3(a_\ell) = -\int_0^{y_{\max}} dy \left[\frac{(\lambda - 1 + y)^2}{4\,y} + 1\right] \frac{(1+y)\ln\, x_0}{\lambda},$$

$$\lambda_\ell(y, a_\ell) = \sqrt{(1+y)^2 - 4\,a_\ell\,y},\;\; x_0(y, a_\ell) = \frac{1-\lambda+y}{1+\lambda+y},$$
$$y_{\max}(a_\ell) = \left(\sqrt{a_\ell} - \sqrt{a_\ell - 1}\right)^2$$



$I_3(a)$



relative error