Triangle singularity in light meson sector

by Mikhail Mikhasenko, ORIGINS Excellence Cluster, Munich, Germany

04.04 at 11h15, Cubotron, 6th floor
Triangle singularity at the light-meson sector

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July 4\textsuperscript{th}, 2022
Standard model of particle physics
One of the most beautiful and elegant(!) theory in physics
**Standard model of particle physics**

One of the most beautiful and elegant(!) theory in physics

Mathematical equations and formulas are shown, related to the Standard Model (SM) of particle physics, including electroweak-Higgs and QCD (Quantum Chromodynamics).

**SM: Electroweak-Higgs & QCD**

Equations and formulas are presented in a natural and readable format.
Standard model of particle physics

One of the most beautiful and elegant(!) theory in physics

$$\mathcal{L}_{\text{SM}} = -\frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - \frac{1}{2} g^{abc} g_{a}^{\mu} g_{b}^{\nu} g_{c}^{\rho} \partial_{\mu} \phi \partial_{\nu} \phi \partial_{\rho} \phi - \frac{1}{2} M^{2} W^{+}_{\mu} W_{\mu}^{\ast} - \frac{1}{2} Z_{\mu} Z^{\ast}_{\mu} + M^{2} Z^{0} Z^{0} - \frac{1}{2} \partial_{\mu} A_{\mu} \partial_{\nu} A^{\nu} - \frac{1}{2} g_{a} g^{ab} F_{ab}^{\mu} F_{\mu}^{\nu} + M^{2} W^{+}_{\mu} W_{\mu}^{\ast} - \frac{1}{2} g^{abc} g_{b}^{\mu} g_{c}^{\nu} F_{a}^{\mu} F^{\nu}_{\ast} - \partial_{\mu} F_{ab}^{\mu} \partial_{\nu} F^{ab}_{\nu} - 2 \partial_{\mu} \partial_{\nu} (W^{\pm}_{\mu} W^{\ast}_{\nu} + W^{\pm}_{\nu} W^{\ast}_{\mu}) - \partial_{\mu} \partial_{\nu} (A_{\mu} A^{\ast}_{\nu} - A_{\nu} A^{\ast}_{\mu}) - 2 i g_{a} g^{abc} (Z^{0}_{\mu} g_{c}^{\rho} F_{ab}^{\rho} - \partial_{\mu} F_{ab}^{\rho} g_{c}^{\rho}) - \partial_{\mu} F_{ab}^{\mu} \partial_{\nu} F^{ab}_{\nu} - \partial_{\mu} F_{ab}^{\mu} \partial_{\nu} F^{ab}_{\nu} - \partial_{\mu} F_{ab}^{\mu} \partial_{\nu} F^{ab}_{\nu} - \partial_{\mu} F_{ab}^{\mu} \partial_{\nu} F^{ab}_{\nu} - \partial_{\mu} F_{ab}^{\mu} \partial_{\nu} F^{ab}_{\nu} - \partial_{\mu} F_{ab}^{\mu} \partial_{\nu} F^{ab}_{\nu} - \partial_{\mu} F_{ab}^{\mu} \partial_{\nu} F^{ab}_{\nu}$$

$$\Rightarrow$$

QCD: Self-couplings of gluons, — color confinement
Standard model of particle physics

One of the most beautiful and elegant(!) theory in physics

SM: Electroweak-Higgs & QCD

QCD: Self-couplings of gluons, — color confinement

Hadronic matter
Standard model of particle physics

One of the most beautiful and elegant(!) theory in physics

\[ \mathcal{L}_{SM} = -\frac{1}{2} \partial_{\mu} g_{a} \partial^{\mu} g_{a} - g_{f} f^{abc} \partial_{\mu} g_{b} \partial^{\mu} g_{c} - \frac{1}{4} f^{abc} f_{abc} \partial^{\mu} g_{a} \partial^{\mu} g_{b} - \partial_{\mu} W_{\mu}^{a} \partial_{\mu} W_{\mu}^{a} - M^{2} W_{\mu}^{a} W_{\mu}^{a} - \frac{1}{2} Z_{\mu}^{0} Z_{\mu}^{0} - \frac{1}{2} Z_{\mu}^{0} Z_{\mu}^{0} - \frac{1}{2} h_{\mu} A_{\mu} - A_{\mu} A_{\mu} - \lambda_{h} (\partial_{\mu} Z_{\mu}^{0}) (W_{\mu}^{a} W_{\mu}^{a} - W_{\mu}^{a} W_{\mu}^{a} - Z_{\mu}^{0} W_{\mu}^{a} W_{\mu}^{a} + Z_{\mu}^{0} W_{\mu}^{a} W_{\mu}^{a} + A_{\mu} A_{\mu} A_{\mu} + A_{\mu} A_{\mu} A_{\mu} + A_{\mu} A_{\mu} A_{\mu}) - \lambda_{s} (A_{\mu} A_{\mu} A_{\mu}) - \lambda_{h} (\partial_{\mu} Z_{\mu}^{0}) (W_{\mu}^{a} W_{\mu}^{a} - W_{\mu}^{a} W_{\mu}^{a} - Z_{\mu}^{0} W_{\mu}^{a} W_{\mu}^{a} + Z_{\mu}^{0} W_{\mu}^{a} W_{\mu}^{a}) + g s_{w}^{2} (A_{\mu} A_{\mu} A_{\mu} + A_{\mu} A_{\mu} A_{\mu} + A_{\mu} A_{\mu} A_{\mu}) + g s_{w} s_{t} (A_{\mu} Z_{\mu}^{0} W_{\mu}^{a} - W_{\mu}^{a} W_{\mu}^{a} - 2 A_{\mu} Z_{\mu}^{0} W_{\mu}^{a} W_{\mu}^{a} - \frac{1}{2} f_{\mu} h_{\mu} h - 2 M^{2} h_{\mu} h - \frac{1}{2} f_{\mu} h_{\mu} h - \frac{1}{2} f_{\mu} h_{\mu} h) - \frac{1}{2} v_{w} f_{\mu} h_{\mu} h - \frac{1}{2} g s_{w}^{2} Z_{\mu}^{0} Z_{\mu}^{0} Z_{\mu}^{0} - \frac{1}{2} g s_{w}^{2} Z_{\mu}^{0} Z_{\mu}^{0} Z_{\mu}^{0} - \frac{1}{2} g (W_{\mu}^{a} \partial_{\mu} \phi^{+} - \phi^{0} \partial_{\mu} \phi^{+} - \frac{1}{2} v_{w} f_{\mu} h_{\mu} h - \frac{1}{2} g s_{w}^{2} Z_{\mu}^{0} Z_{\mu}^{0} Z_{\mu}^{0} - \frac{1}{2} g s_{w}^{2} Z_{\mu}^{0} Z_{\mu}^{0} Z_{\mu}^{0} - \frac{1}{2} g (W_{\mu}^{a} \partial_{\mu} \phi^{+} - \phi^{0} \partial_{\mu} \phi^{+} - \frac{1}{2} v_{w} f_{\mu} h_{\mu} h - \frac{1}{2} g s_{w}^{2} Z_{\mu}^{0} Z_{\mu}^{0} Z_{\mu}^{0} - \frac{1}{2} g s_{w}^{2} Z_{\mu}^{0} Z_{\mu}^{0} Z_{\mu}^{0} - \frac{1}{2} g (W_{\mu}^{a} \partial_{\mu} \phi^{+} - \phi^{0} \partial_{\mu} \phi^{+} - \frac{1}{2} v_{w} f_{\mu} h_{\mu} h - \frac{1}{2} g s_{w}^{2} Z_{\mu}^{0} Z_{\mu}^{0} Z_{\mu}^{0} - \frac{1}{2} g s_{w}^{2} Z_{\mu}^{0} Z_{\mu}^{0} Z_{\mu}^{0} - \frac{1}{2} g (W_{\mu}^{a} \partial_{\mu} \phi^{+} - \phi^{0} \partial_{\mu} \phi^{+} - \frac{1}{2} v_{w} f_{\mu} h_{\mu} h - \frac{1}{2} g s_{w}^{2} Z_{\mu}^{0} Z_{\mu}^{0} Z_{\mu}^{0} - \frac{1}{2} g s_{w}^{2} Z_{\mu}^{0} Z_{\mu}^{0} Z_{\mu}^{0} - \frac{1}{2} g (W_{\mu}^{a} \partial_{\mu} \phi^{+} - \phi^{0} \partial_{\mu} \phi^{+} - \frac{1}{2} v_{w} f_{\mu} h_{\mu} h - \frac{1}{2} g s_{w}^{2} Z_{\mu}^{0} Z_{\mu}^{0} Z_{\mu}^{0} - \frac{1}{2} g s_{w}^{2} Z_{\mu}^{0} Z_{\mu}^{0} Z_{\mu}^{0} - \frac{1}{2} g (W_{\mu}^{a} \partial_{\mu} \phi^{+} - \phi^{0} \partial_{\mu} \phi^{+} - \frac{1}{2} v_{w} f_{\mu} h_{\mu} h - \frac{1}{2} g s_{w}^{2} Z_{\mu}^{0} Z_{\mu}^{0} Z_{\mu}^{0} - \frac{1}{2} g s_{w}^{2} Z_{\mu}^{0} Z_{\mu}^{0} Z_{\mu}^{0} - \frac{1}{2} g (W_{\mu}^{a} \partial_{\mu} \phi^{+} - \phi^{0} \partial_{\mu} \phi^{+} - \frac{1}{2} v_{w} f_{\mu} h_{\mu} h - \frac{1}{2} g s_{w}^{2} Z_{\mu}^{0} Z_{\mu}^{0} Z_{\mu}^{0} - \frac{1}{2} g s_{w}^{2} Z_{\mu}^{0} Z_{\mu}^{0} Z_{\mu}^{0} - \frac{1}{2} g (W_{\mu}^{a} \partial_{\mu} \phi^{+} - \phi^{0} \partial_{\mu} \phi^{+} - \frac{1}{2} v_{w} f_{\mu} h_{\mu} h - \frac{1}{2} g s_{w}^{2} Z_{\mu}^{0} Z_{\mu}^{0} Z_{\mu}^{0} - \frac{1}{2} g s_{w}^{2} Z_{\mu}^{0} Z_{\mu}^{0} Z_{\mu}^{0} - \frac{1}{2} g (W_{\mu}^{a} \partial_{\mu} \phi^{+} - \phi^{0} \partial_{\mu} \phi^{+} - \frac{1}{2} v_{w} f_{\mu} h_{\mu} h - \frac{1}{2} g s_{w}^{2} Z_{\mu}^{0} Z_{\mu}^{0} Z_{\mu}^{0} - \frac{1}{2} g s_{w}^{2} Z_{\mu}^{0} Z_{\mu}^{0} Z_{\mu}^{0} - \frac{1}{2} g (W_{\mu}^{a} \partial_{\mu} \phi^{+} - \phi^{0} \partial_{\mu} \phi^{+} - \frac{1}{2} v_{w} f_{\mu} h_{\mu} h - \frac{1}{2} g s_{w}^{2} Z_{\mu}^{0} Z_{\mu}^{0} Z_{\mu}^{0} - \frac{1}{2} g s_{w}^{2} Z_{\mu}^{0} Z_{\mu}^{0} Z_{\mu}^{0} - \frac{1}{2} g (W_{\mu}^{a} \partial_{\mu} \phi^{+} - \phi^{0} \partial_{\mu} \phi^{+} - \frac{1}{2} v_{w} f_{\mu} h_{\mu} h - \frac{1}{2} g s_{w}^{2} Z_{\mu}^{0} Z_{\mu}^{0} Z_{\mu}^{0} - \frac{1}{2} g s_{w}^{2} Z_{\mu}^{0} Z_{\mu}^{0} Z_{\mu}^{0} - \frac{1}{2} g (W_{\mu}^{a} \partial_{\mu} \phi^{+} - \phi^{0} \partial_{\mu} \phi^{+} - \frac{1}{2} v_{w} f_{\mu} h_{\mu} h - \frac{1}{2} g s_{w}^{2} Z_{\mu}^{0} Z_{\mu}^{0} Z_{\mu}^{0} - \frac{1}{2} g s_{w}^{2} Z_{\mu}^{0} Z_{\mu}^{0} Z_{\mu}^{0} - \frac{1}{2} g (W_{\mu}^{a} \partial_{\mu} \phi^{+} - \phi^{0} \partial_{\mu} \phi^{+} - \frac{1}{2} v_{w} f_{\mu} h_{\mu} h - \frac{1}{2} g s_{w}^{2} Z_{\mu}^{0} Z_{\mu}^{0} Z_{\mu}^{0} - \frac{1}{2} g s_{w}^{2} Z_{\mu}^{0} Z_{\mu}^{0} Z_{\mu}^{0} - \frac{1}{2} g (W_{\mu}^{a} \partial_{\mu} \phi^{+} - \phi^{0} \partial_{\mu} \phi^{+} - \frac{1}{2} v_{w} f_{\mu} h_{\mu} h - \frac{1}{2} g s_{w}^{2} Z_{\mu}^{0} Z_{\mu}^{0} Z_{\mu}^{0} - \frac{1}{2} g s_{w}^{2} Z_{\mu}^{0} Z_{\mu}^{0} Z_{\mu}^{0} - \frac{1}
Conventional hadrons

\[ q \bar{q} \]

\[ q' \bar{q}' \]

\[ q q' q'' \]

meson

baryon

\[ \sim 10 \text{ classes of mesons} \]

\( (\pi, \eta, K, D, D_s, B, B_s, B_c, \phi, \psi, \Upsilon) \)

and

\[ \sim 20 \text{ classes of baryons} \]

\( (N, \Delta, \Lambda_{(b/c)}, \Xi_{(b/c)}, \Omega_{(b/c)}, \ldots) \)

\[ \times \text{ excitement} \]
Variety of the hadronic states

- Many structures are possible
- Exotic states in light sector:
  - Spin-exotic (non-\( q \bar{q} \) quantum numbers) and Crypto exotic (extra-numerous)

Ordinary matter:

- meson
- baryon
- hadronic molecules
Variety of the hadronic states

- Many structures are possible
- Exotic states in light sector:
  - **Spin-exotic** (non-$q\bar{q}$ quantum numbers) and **Crypto exotic** (extra-numerous)

Ordinary matter:

- **Meson**: $u\bar{u}$
- **Baryon**: $d\bar{d}u$
- **Hadronic molecules**: $uu\bar{d}\bar{d}$

Exotic matter:

- **Glueball**
- **Hybrid**: $\bar{u}d$
- **Tetraquark**: $\bar{u}b\bar{d}b$
Experimental situation on (non-strange) Light mesons

Experimental situation on (non-strange) Light mesons


- Many low-lying observed states are well established and in agreement with the QM
- Some have large width and non-trivial appearance in the spectrum due to overlaps and interferences

![Meson Spectrum Diagram](image-url)
Excitation spectrum of a bound system

QED

$\bar{J} = \bar{L}$

$1S$

$2S$

$2P$

Orbital quantum numbers

Energy (GeV)
Excitation spectrum of a bound system

**QED**

\[ \vec{J} = \vec{L} + \vec{S} \]
\[ \vec{S} = \vec{s}_1 + \vec{s}_2 \]

\[ J = \](energy in GeV)

\[ L = 0 \]
\[ L = 1 \]

Orbital quantum numbers
Excitation spectrum of a bound system

QCD

**Triangle singularity**
Excitation spectrum of a bound system

QCD

\[ 2S \rightarrow \pi \]
\[ 2P \rightarrow a_0, a_2, b_1, a_1 \]

\[ J = L + S \]
\[ S = s_1 + s_2 \]

\[ \times 5 \]

\[ \rho \rightarrow \pi, \eta, \eta', K, \bar{K} \]
Excitation spectrum of a bound system

QCD

- QED: hyperfine splitting
- QCD: is far not hyperfine
Excitation spectrum of a bound system

- QED: hyperfine splitting
- QCD: is far not hyperfine
- Example of spin-flip transition:
  $\rho(\uparrow\uparrow) \rightarrow \pi(\uparrow\downarrow)$ transition is a “QCD-cell division”
The plan of the talk

1 Introduction
   - Meson spectrum
   - Mass, width, pole position
   - Experimental setup

2 Tetraquark candidate $a_1(1420)$
   - Observation and interpretations
   - Triangle Singularity in three-body decays, interference

3 Summary
Invariant-mass distribution, resonances

Hadronic state is a particle
- charact. by **mass** (energy) and **width** (lifetime)
Invariant-mass distribution, resonances

Hadronic state is a particle
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- Hadronic states are **resonances** of the hadronic system
- Read $m$, $\Gamma$ from spectrum
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$$E_{\text{pole}} = m_\rho - i\Gamma_\rho / 2$$
Invariant-mass distribution, resonances

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- resonances are poles of scattering amplitude.

Hadronic states are resonances of the hadronic system

$E_{\text{pole}} = m_\rho - i\Gamma_\rho/2$
Resonances are poles of the amplitude

Electric circuit

\[ I_{\text{rms}} = \frac{U}{R + iL\omega - \frac{i}{C\omega}} \]

Scattering

\[ A_{\pi\pi} = \frac{m\Gamma}{m^2 - s - i\Gamma} \]

**Graphs:**
- **Electric Circuit:**
  - $I_{\text{rms}}$ vs. $\omega$ (Mrad/s)
  - Parameters: $R = 5 \, \Omega$, $L = 5 \, \mu\text{H}$, $C = 2 \, \text{nF}$, $U_{\text{rms}} = 5 \, \text{mV}$

- **Scattering:**
  - $A_{\pi\pi}$ vs. $s \equiv m_{\pi\pi}^2$ (GeV$^2$)
  - Parameters: $m_{\rho} = 0.77 \, \text{GeV}$, $\Gamma_{\rho} = 0.15 \, \text{GeV}$
Laboratory to study hadronic excitations

Diffractive reaction

Pion beam scattered off the proton target

High energy guarantees $t$-channel process.

The target provides the gluonic field

$\pi^-$ production has the largest cross section (inelastic)
Laboratory to study hadronic excitations

**Diffractive reaction**

- Pion beam scattered off the proton target
- High energy guarantees $t$-channel process.
- The target provide the gluonic field
- $3\pi$ production has the largest cross section (inelastic)
Laboratory to study hadronic excitations

Diffractive reaction

- Pion beam scattered off the proton target
- High energy guarantees $t$-channel process.
- The target provide the gluonic field
- $3\pi$ production has the largest cross section (inelastic)
COMPASS Experiment

Spectroscopy, Structure functions
$\pi/\mu$ beam, $10^7$ particles per 10s spill

[COMPASS Experiment (NIM A779 (2015) 69-115)]
Understanding of the $3\pi$ spectrum

The results of the main big fit

— 14 interfering waves $\times$ 11 $t'$-slices simultaneously.
Understanding of the $3\pi$ spectrum

The results of the main big fit
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Understanding of the $3\pi$ spectrum

The results of the main big fit
— 14 interfering waves × 11 $t'$-slices simultaneously.

11 resonances are established including a new $a_1(1420)$
Resonance model fit

The main mass-dependent fit

Axial vector $1^{++}$

Non-resonant coherent background

Model curve

Resonances

Nonres. comp.
Resonance model fit

The main mass-dependent fit

**Axial “?” 1++**

- Exotic candidate!

[COMPASS, PRD98 (2018) 092003]
Resonance model fit

The main mass-dependent fit

Non-$q\bar{q}$

1$^{−+}$

[COMPASS, PRD98 (2018) 092003]
$a_1(1420)$ tetraquark candidate

as a resonance in the $3\pi$ system
Observation of the $a_1(1420)$

[COMPASS, PRL 115 (2015) 082001]
Observation of the $a_1(1420)$

- Observation and interpretations

- COMPASS, PRL 115 (2015) 082001

- $a_1(1420)$ resonance

- $J^{PC} = 2^{++}$, $\rho \pi$ D-wave

- $J^{PC} = 1^{++}$, $\rho \pi$ S-wave

- $J^{PC} = 1^{++}$, $f_0 \pi$ P-wave

- Triangle singularity

- Mikhail Mikhasenko (ORIGINS Cluster)

- Triangle singularity

- April 4th, 2022
Observation of the $a_1(1420)$

Not something ordinary

- Too close to the ground state $a_1(1260)$
- Its width is narrower than the ground state
- Close to threshold $K^*\bar{K}$, i.e. $(d\bar{s}) + (\bar{u}s)$, $E_{th} = 1.39$ GeV.
$a_1(1420)$ interpretations

Possible scenaria

- **Pole** in the amplitude – Genuine resonance
- Singularity of the **non-pole** type
$a_1(1420)$ interpretations

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  - $K^*\bar{K}$ molecule [T. Gutsche et al. (2017)]
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- Singularity of the **non-pole** type
  - Interference with background — interplay between distant cuts

![Diagram](https://via.placeholder.com/150)

$a_1(1420)$ interpretations

Possible scenaria

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  - $K^* \bar{K}$ molecule [T. Gutsche et al. (2017)]

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  - Interference with background — interplay between distant cuts
  - **Rescattering** from $K^* \bar{K}$ — Triangle singularity

$a_1(1420)$ interpretations

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- Singularity of the **non-pole** type
  - Interference with background — interplay between distant cuts
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Decay chains, subchannel resonances

- The relaxation via an intermediate meson
- Direct emission of $\rho$-meson
  $\Rightarrow$ resonances in $(\pi\pi)$ spectrum
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Decay chains, subchannel resonances

The relaxation via an intermediate meson

Direct emission of $\rho$-meson

$\Rightarrow$ resonances in ($\pi\pi$) spectrum
Hadronic double-slit experiment

- Several quantum processes lead to the same outcome
- Intermediate states are entangled
Hadronic double-slit experiment

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- Non-perturbative process - an infinite number of barriers
- Cross-channel effect scales with the resonance width
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Coupled channels (schematically)

- $K\bar{K}\pi$ is a possible decay of the same resonance $a_1$
- Two separated problems?
**Coupled channels (schematically)**

- $K\bar{K}\pi$ is a possible decay of the same resonance $a_1$
- Two separated problems? - No, more entangled states (coupled channels)!
- Hadron interaction mixes probabilities
Coupled channels (schematically)

- $K\bar{K}\pi$ is a possible decay of the same resonance $a_1$
- Two separated problems? - No, more entangled states (coupled channels)!
- Hadron interaction mixes probabilities

- Tiny fraction of the $a_1 \rightarrow K\bar{K}\pi$ probability gets into $\pi\pi\pi$,
- However, only above $K^*\bar{K}$ threshold!
The key effect - the triangle rescattering graph

- $f_0$ is a resonance in $(K\bar{K})$ and also in $(\pi\pi)$ system.
- Ordinary $a_1$ decays to $K\bar{K}\pi$ via $K^*\bar{K}$
- $K\bar{K}$ form $f_0$ that decays to $\pi\pi$
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- has a logarithmic singularity (divergence at a single point)
- $A \sim \log(s_0 - m_{3\pi}^2)$ with $s_0$ determined by masses of involved particles.
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- $A \sim \log(s_0 - m_{f_0\pi}^2)$ with $s_0$ determined by masses of involved particles.
Fit with the rescattering model [COMPASS, PRL(2021)]

Fit perfectly describes the intensity and the phase motion

- No shape parameters for the signal component (TS)
- Background with constant phase is needed to shift the amplitude
- TS model shows a comparable quality to the resonance model (BW-model)
Systematic studies

- Neglecting interference of the conjugated decay chains,
- Neglecting the spins of the particles involved,
- Including the excitations $a_1(1640)$ and $a_2(1700)$
- Varying mass and width of the $K^*$ resonance

TS model systematically yields a similar $R^2_{\text{red}}$ as the BW model.
Emerging interpretation [COMPASS, PRL (2021)]

Triangle singularity

The cut-off $K^*\bar{K}$ cut implies $\sim \log(s - s_0)$

$\sim \frac{1}{(s - s_{a_1})}$

$\sim \frac{1}{(s - s_{a_1})}$

$\sim \frac{1}{(s - s_{a_1})}$
Emerging interpretation [COMPASS, PRL (2021)]

- $a_1(1420)$ signal can be described with $a_1(1260)$ as source for the **rescattering** via the triangle diagram $\Rightarrow$ the first clear observation of the TS
- An additional pole is not needed, although, not excluded
Conclusions and outlook

- **Hadron spectroscopy** is a unique tool for understanding the QCD, the theory of matter formation.
- **Diffractive** reaction is a clean setup for measurements of the excitation spectrum.
- **COMPASS** leads the effort of large combined light-quark meson studies.

The story of $a_1(1420)$

- $a_1(1420)$ signal can be described with the ordinary $a_1$ meson as source for the rescattering via the triangle diagram.
- Old theoretical concept, but observed clearly for the first time!
- A small effect, $\sim 1\%$ as could have been anticipated.
- **Peak and phase motion are not unique sign of a resonance!**

Signal in $f_0\pi\ P$-wave $\Rightarrow$ established Triangle Singularity, no need for the tetraquark.
Beyond the light-meson sector
Growing evidence of the exotic states with heavy flavor

- Many candidates have a hadronic threshold in vicinity: (Meson)(Meson) of (Meson)(Baryon)

<table>
<thead>
<tr>
<th>States</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_0(2900), X_1(2900)$ [22,23]</td>
</tr>
<tr>
<td>$\chi_{c1}(3872)$ [7]</td>
</tr>
<tr>
<td>$Z_c(3900)$ [24], $Z_c(4020)$ [25,26], $Z_c(4050)$ [27], $X(4100)$ [28], $Z_c(4200)$ [29], $Z_c(4430)$ [30,31,32,33], $R_{c0}(4240)$ [32]</td>
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<tr>
<td>$Z_{cs}(3985)$ [34], $Z_{cs}(4000)$, $Z_{cs}(4220)$ [35]</td>
</tr>
<tr>
<td>$\chi_{c1}(4140)$ [36,37,38,39], $\chi_{c1}(4274)$, $\chi_{c0}(4500)$, $\chi_{c0}(4700)$ [39]</td>
</tr>
<tr>
<td>$X(4630)$, $X(4685)$ [35], $X(4740)$ [40]</td>
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<tr>
<td>$X(6900)$ [15]</td>
</tr>
<tr>
<td>$Z_b(10610), Z_b(10650)$ [41]</td>
</tr>
<tr>
<td>$P_c(4312)$ [42], $P_c(4380)$ [43], $P_c(4440), P_c(4457)$ [42], $P_c(4357)$ [44]</td>
</tr>
<tr>
<td>$P_{cs}(4459)$ [45]</td>
</tr>
</tbody>
</table>

Can these states (some of) be manifestation of TS?
Pentaquarks in $pJ/\psi$ mass spectrum

- Narrow peaks in $\rightarrow pJ/\psi$
- Right near $\Sigma_c^*+\bar{D}^{*0}$ threshold

\[ \Lambda_b^0 \rightarrow (pJ/\psi) K^- \]

**Diagram:**
- LHCb data
- Total fit
- Background

**Figure:**
- $\Sigma_c^* \bar{D}^0$
- $\Sigma_c^* \bar{D}^{*0}$
- $P_c(4312)^+$
- $P_c(4440)^+$
- $P_c(4457)^+$

**Legend:**
- $D^{(*)0}$
- $\Sigma_c^{(*)+}$
- hadronic molecule
tightly-bound pentaquark
courtesy of D. Dominguez, CERN

[PRL 122 (2019) 22, 222001]
Pentaquarks in $pJ/\psi$ mass spectrum

$\Lambda_b^0 \rightarrow (pJ/\psi) K^-$

- Narrow peaks in $\rightarrow pJ/\psi$
- Right near $\Sigma^*_c + \bar{D}^{*0}$ threshold

![Graph showing weighted candidates vs. $m_{J/\psi}$](image)

Fit with 7 $P_c^+$ [Meng-Lin Du at al., PRL124 (2020) 7, 072001]
Rescattering interpretation of the $P_c$ states [PRL 122 (2019) 22, 222001]

- TS makes a peak above thresholds
- Many (relevant) thresholds $\Lambda_c \bar{D}^0$, $\Sigma_c \bar{D}^0$, $\chi_c N^*$
- An appropriate Triangle Singularity can be found for all peaks(!)
Rescattering interpretation of the $P_c$ states [PRL 122 (2019) 22, 222001]

- TS makes a peak above thresholds
- Many (relevant) thresholds $\Lambda_c \bar{D}^0$, $\Sigma_c \bar{D}^0$, $\chi_c N^*$
- An appropriate Triangle Singularity can be found for all peaks(!)
- BUT, as soon as width of exchange particle is taken into account

⇒ no acceptable description in rescattering picture has been found
Thank you for the attention
Interfering background

Forward-background scattering

[COMPASS data, MM, PhD thesis]

\[
\cos \theta_{GJ} \Rightarrow m_{3\pi} \text{ (GeV)}
\]

The high-energy exchange processes penetrate to the low energy and make resonance characterization difficult.

Mikhail Mikhasenko (ORIGINS Cluster)
**Classical picture of near-mass-shell rescattering**

Imagine cascade reaction $a_1(1260) \rightarrow K^*(892)\bar{K}$, then $K^* \rightarrow K\pi$, and calculate invariant mass of $K$ and $\bar{K}$ for the case when $K$ is parallel to $\bar{K}$.

**Partial form of Landau conditions**

[[Nucl. Phys. 13, 181 (1959)]]:

- All particles in loop are on mass shell.
- The alignment of moments $\vec{p}_K \uparrow \uparrow \vec{p}_{\bar{K}}$.
- $K$ is faster than $\bar{K}$. 