LATTICE QCD OVERVIEW

Attila Pásztor

ELTE Eötvös Loránd University, Budapest and
ELKH-ELTE Theoretical Physics Research Group

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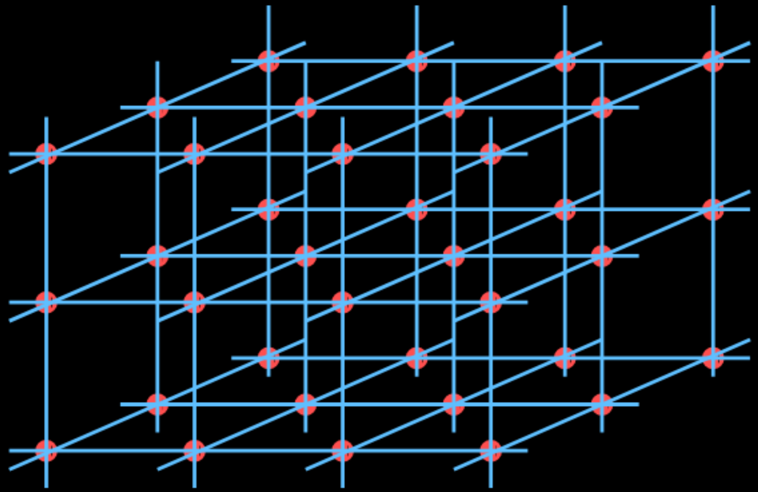


Why is there a lattice plenary at QM?

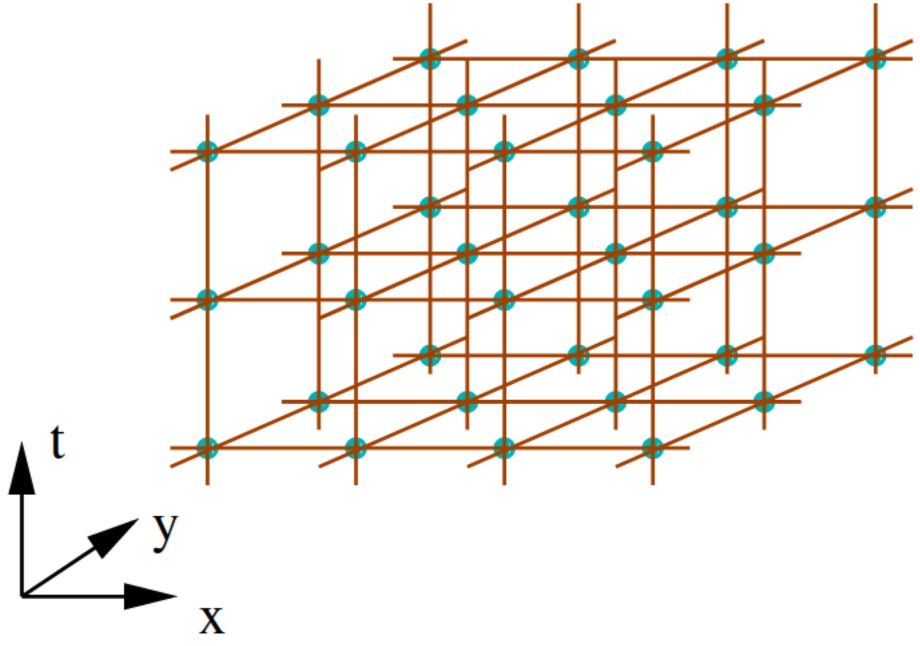
FULLY NON-PERTURBATIVE RESULTS IN FULL QCD ARE VALUEABLE

QUALITY CONTROL
IS ESSENTIAL!

The lattice formulation of QCD

Finite space-time lattice: $N_s^3 N_t \Rightarrow$ finite dimensional integrals

Equilibrium physics: $T = \frac{1}{N_t a}$



1. Continuum limit:

For fixed temperature $a \to 0 \Leftrightarrow N_t \to \infty$

2. Thermodynamic limit:

Size is often measured in units of 1/T

Aspect ratio: $LT = N_s / N_t$

Infinite volume limit: $LT \to \infty$

QCD in a small box is physics, a coarse lattice in a large box is not!

Outline

Two very difficult use cases of lattice QCD that are relevant for heavy ion physics.

1) Nonzero baryochemical potential (main focus of the talk)

1. a) The phase diagram and search for criticality

1. b) The equation of state of a hot-and-dense quark gluon plasma

2) Real time (will be briefly mentioned)

2. a) Real-time properties of heavy quarks at high T

QCD in the grand canonical ensemble

$\hat{p} := \frac{p}{T^4} = \frac{1}{(LT)^3} \log \mathrm{Tr}\left(e^{-(H-\mu_B B-\mu_S S)/T}\right)$ (dimensionless pressure)

$\chi_{ij}^{BS} = \frac{\partial^{i+j}\hat{p}}{\partial\hat{\mu}_B^i \partial\hat{\mu}_S^j}$ $\left(\hat{\mu}_B := \frac{\mu_B}{T}\right)$ (generalized susceptibilities)

DERIVATIVES ⇔ FLUCTUATIONS/CORRELATIONS:

$\chi_1^B \propto \langle B \rangle \propto n_B$; $\chi_2^B \propto \langle B^2 \rangle - \langle B \rangle^2$; $\chi_{11}^{BS} \propto \langle BS \rangle - \langle B \rangle \langle S \rangle$

Lattice QCD at nonzero baryon density

<u>Analytic continuation (ver. 1): Imaginary chemical potential method</u>

Calculate $\langle O \rangle$ at $\mathrm{Im}\mu_B$ ($\mu_B^2 < 0$), extrapolate to $\mu_B^2 > 0$

<u>Analytic continuation (ver. 2): Taylor method</u>

Calculate $\frac{\partial^n}{\partial \mu_B^n} \langle O \rangle$ at $\mu_B = 0$, extrapolate

<u>Reweighting:</u>

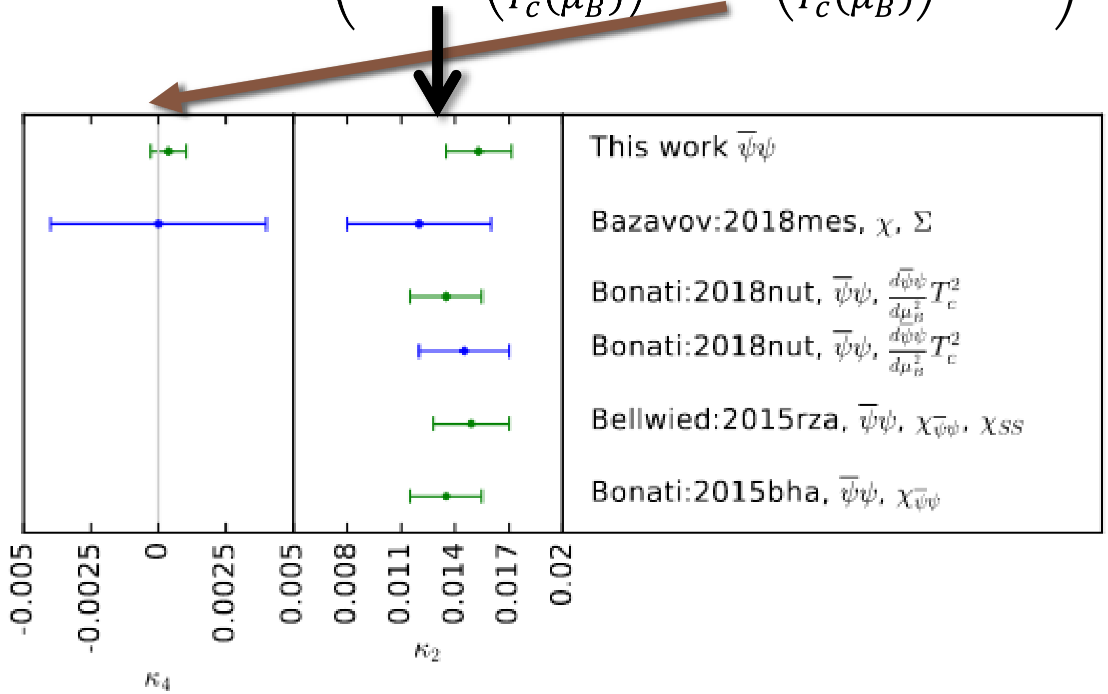
Simulate a different theory, correct the Boltzmann weight in the observable

COST

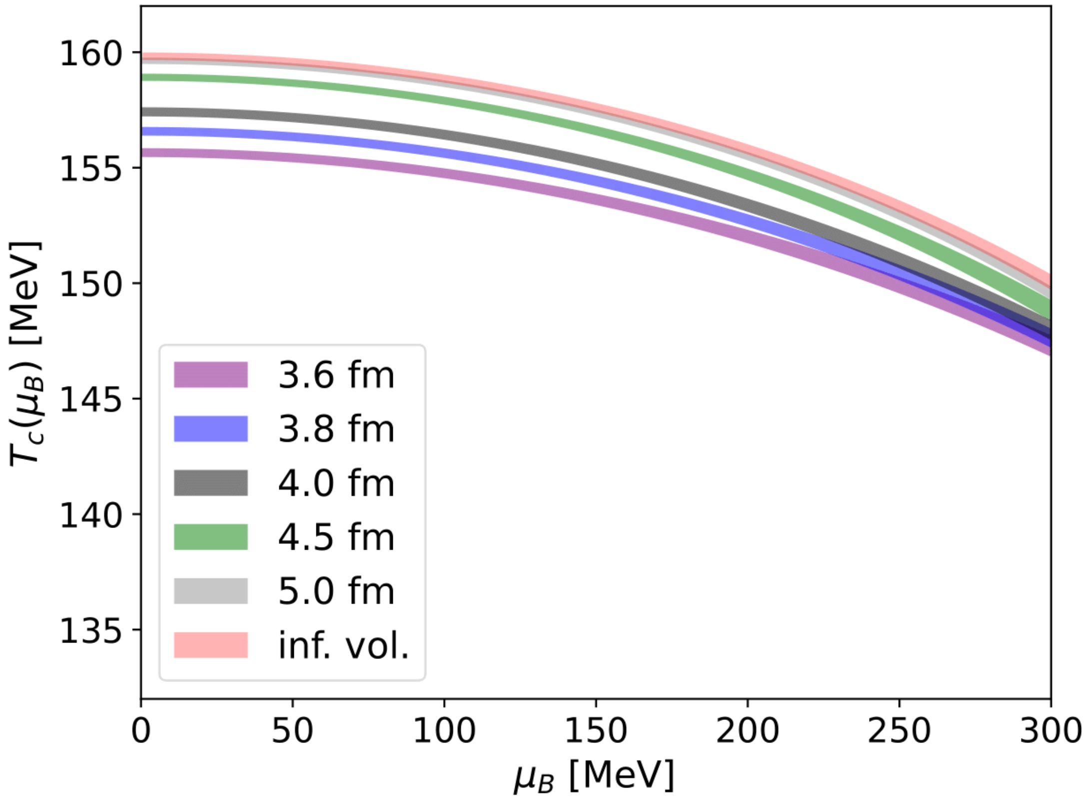
While <u>cut-off</u> and <u>volume</u> effects are important for every lattice result, for $\mu_B > 0$ the way we <u>extrapolate</u> is also an important point of quality control

The phase diagram

$$T_c(\mu_B) = T_c(0)\left(1 - \kappa_2\left(\frac{\mu_B}{T_c(\mu_B)}\right)^2 - \kappa_4\left(\frac{\mu_B}{T_c(\mu_B)}\right)^4 - \cdots\right)$$



[Wuppertal-Budapest, PRL125 (2020)]
Continuum, $\langle S \rangle = 0$, $LT = 4$
$\mu_B > 0$ quantity with good quality control!

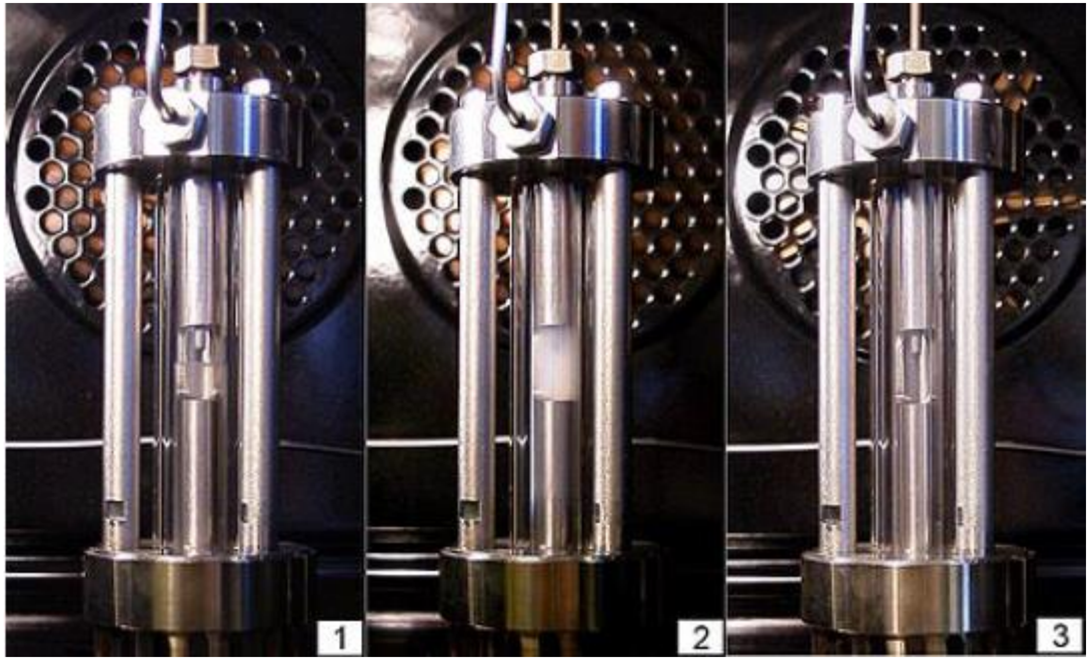


[R. Kara, We 14:20, QCD at finite T and μ]
$N_t = 12$, $\langle S \rangle = 0$, L from small to ∞
Benchmark for effective/functional approaches

These curves contain no info on the order of the transition! How do we search for criticality?

One way: fluctuations

Experiment: tune to criticality



$T < T_c$ $T \approx T_c$ $T > T_c$

HEAT THE SYSTEM

Picture from Wikipedia

Lattice/Taylor: try to see it from far away

$$\chi_n^B = \left(\frac{\partial^n \hat{p}}{\partial \hat{\mu}_B^n}\right)_{\mu_B=0}$$

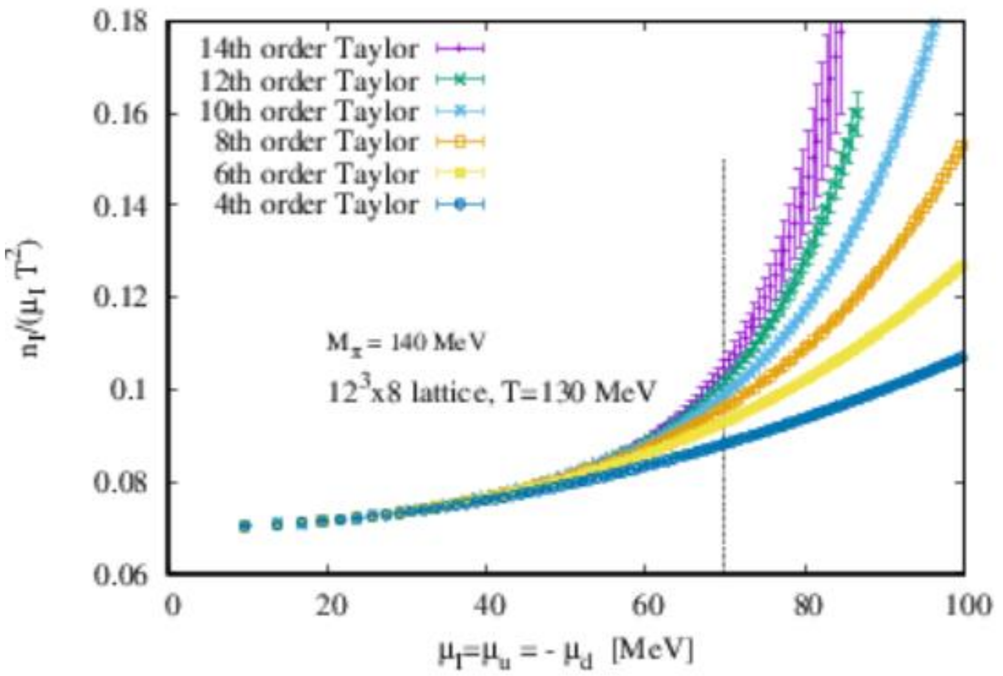
To as large n as possible…

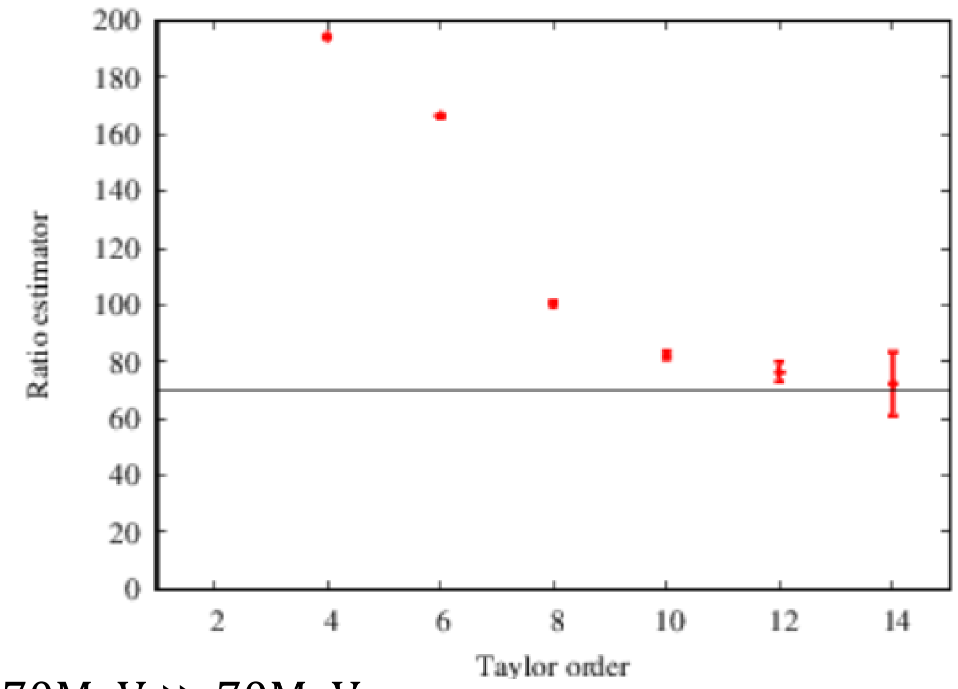
To hopefully see a divergence…

Is this even possible?

A case study: pion condensation [Wuppertal-Budapest, 2308.06105]

- Instead of μ_B, introduce μ_I (prefers π^+ over π^-)
- Second order transition at low T and $\mu_I \approx m_\pi/2 \approx 70\mathrm{MeV}$ [Son&Stephanov, PRL (2001)] [Brandt&Endrődi,





Eventually finds the correct value. 6th order gives $170\mathrm{MeV} \gg 70\mathrm{MeV}$

No high orders in μ_B: analysis of the radius of convergence from Taylor data is premature

Warning: the ratio estimator is not always applicable [Giordano & Pásztor, PRD99(2019)] (here: OK)

More on radius of convergence and analytic structure: **[G. Basar, Tue 16:30] [J. Goshwami, We 15:20]**

The HRG as a non-critical baseline

Hadron resonance gas (HRG) model

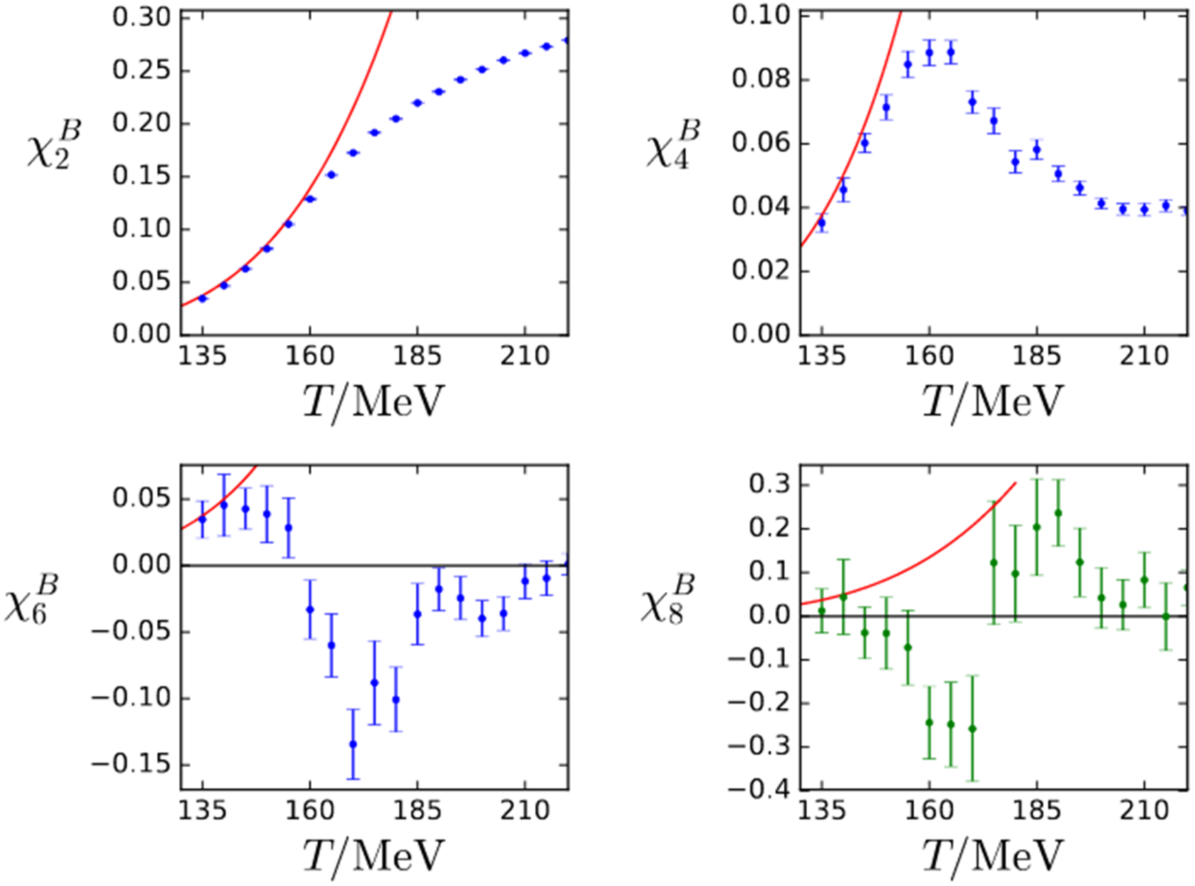
$$p_{QCD} \approx \sum_H p_H^{free}$$

- sum over stable hadrons and resonances
- heavy ion phenomenology uses the HRG as a non-critical baseline
  (non-trivial: see, e.g., [Braun-Munzinger et al, NPA1008(2021)])
- in lattice QCD: can use grand canonical ensemble
- minimum goal: establish deviations from HRG (with good quality control!)

SO, DOES THE HRG DESCRIBE LATTICE DATA ?

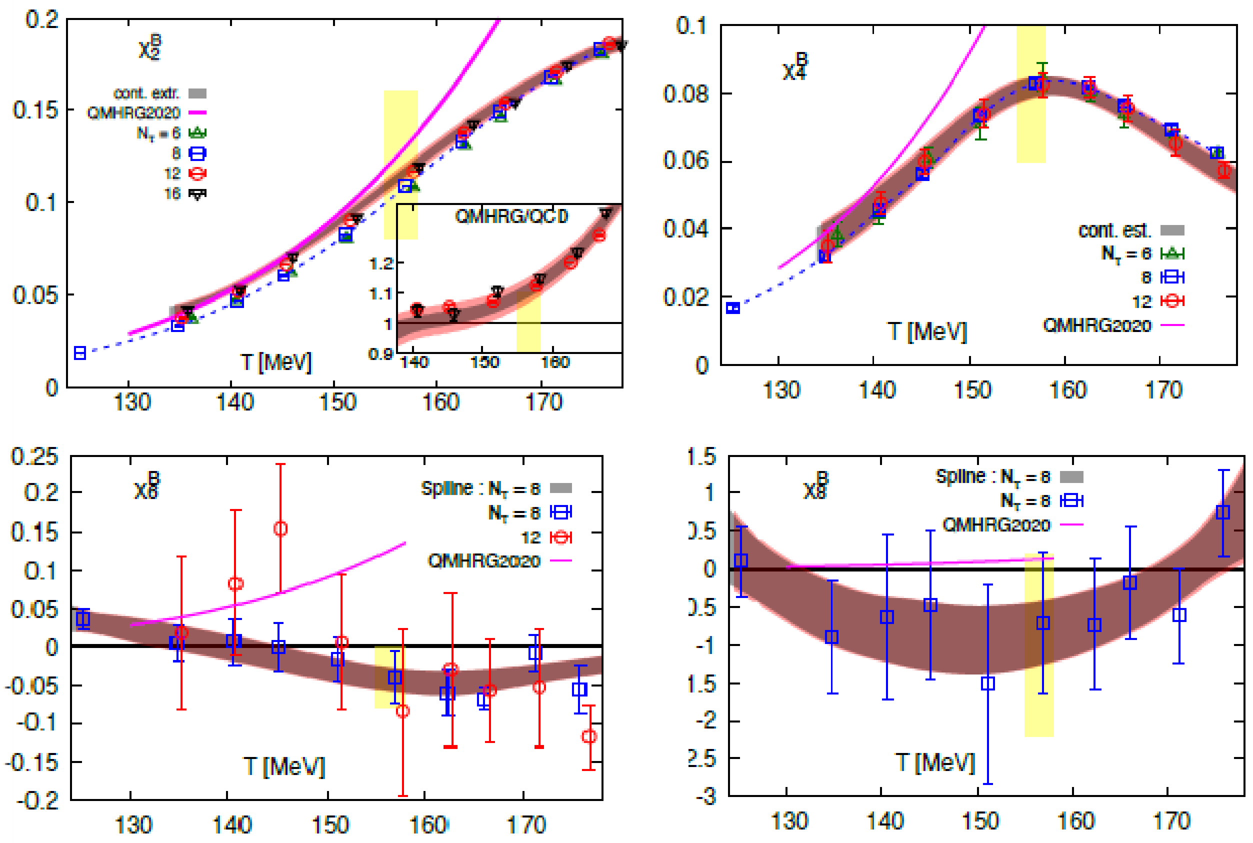
Taylor coefficients of the pressure

From imaginary chemical potential



[Wuppertal-Budapest, JHEP (2018)] (LT=4, N_t=12)

From zero chemical potential

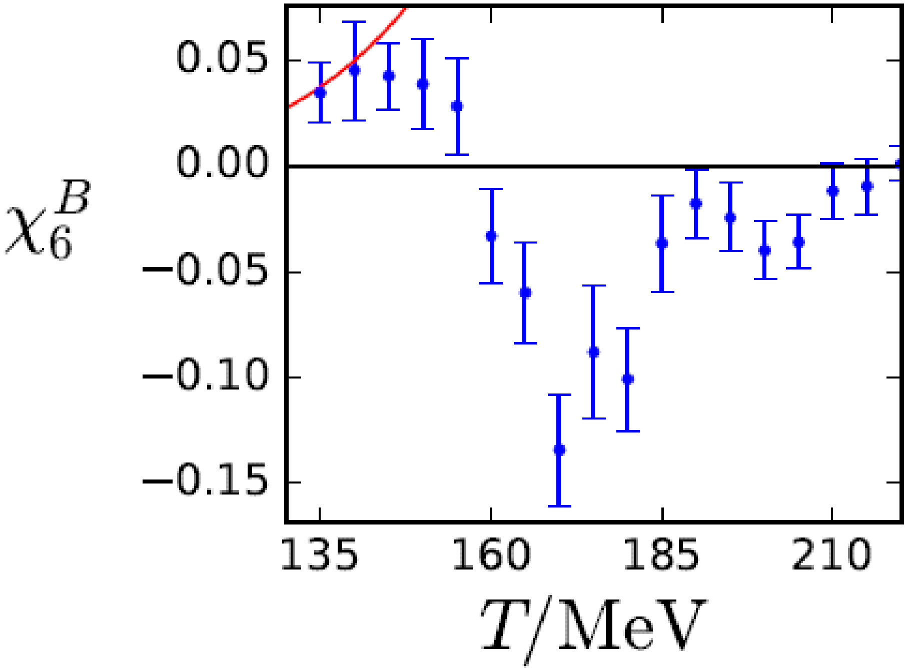


[HotQCD, PRD105 (2022)] (LT=4, N_t=6,8,(12))

[D. Clarke, We 14:40]

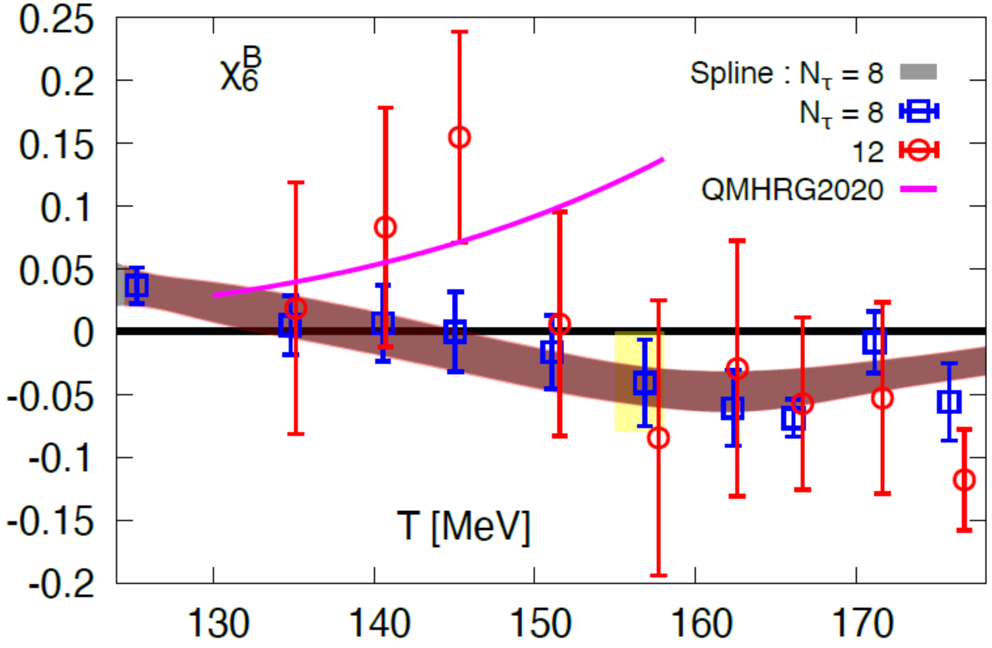
6^{th} order: zoom in to see discrepancies

From imaginary chemical potential



[Wuppertal-Budapest, JHEP (2018)] (LT=4, N_t=12)

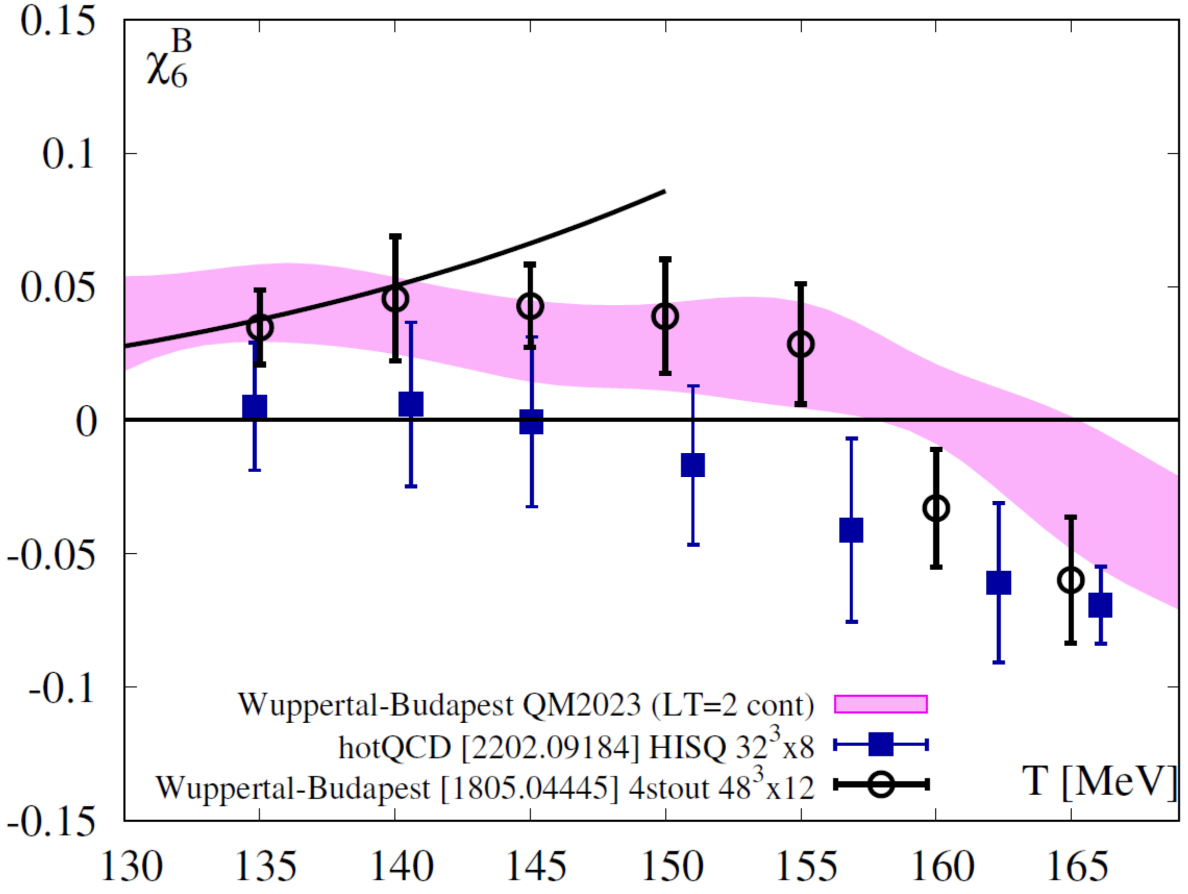
From zero chemical potential



[HotQCD, PRD105 (2022)] (LT=4, N_t=8,(12))
[D. Clarke, We 14:40]

- N_t=12 (left, WB) agrees with the HRG (value and slope) better than N_t=8 (right, HotQCD) at low T
- T=145-155MeV: N_t=12>0 and N_t=8<0

6^{th} order: new dataset



[Sz. Borsányi, Tue 14:50, QCD at finite T and μ]

New dataset:
Taylor, LT=2, continuum (new discretization)

Lower T: cut-off effects dominate
Smaller T means larger a for fixed N_t

5 points at least 1σ below: $\left(\frac{1-0.68}{2}\right)^5 \approx 10^{-4}$

Higher T: finite volume effects dominate
T_c depends on L

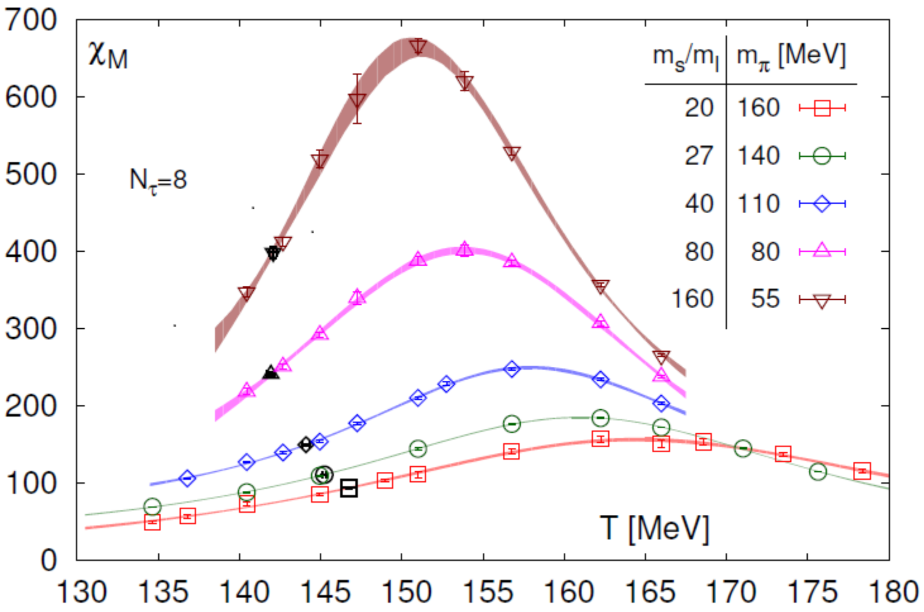
T>145MeV: HRG ≠ LATTICE
T<145MeV: HRG = LATTICE
(within errors)

No sign of a CEP in the Taylor coefficients up to 6^{th} order

Chiral criticality and the equation of state

Smaller-than-physical quark mass @ $\mu_B = 0$

[HotQCD, PRL123 (2019)]

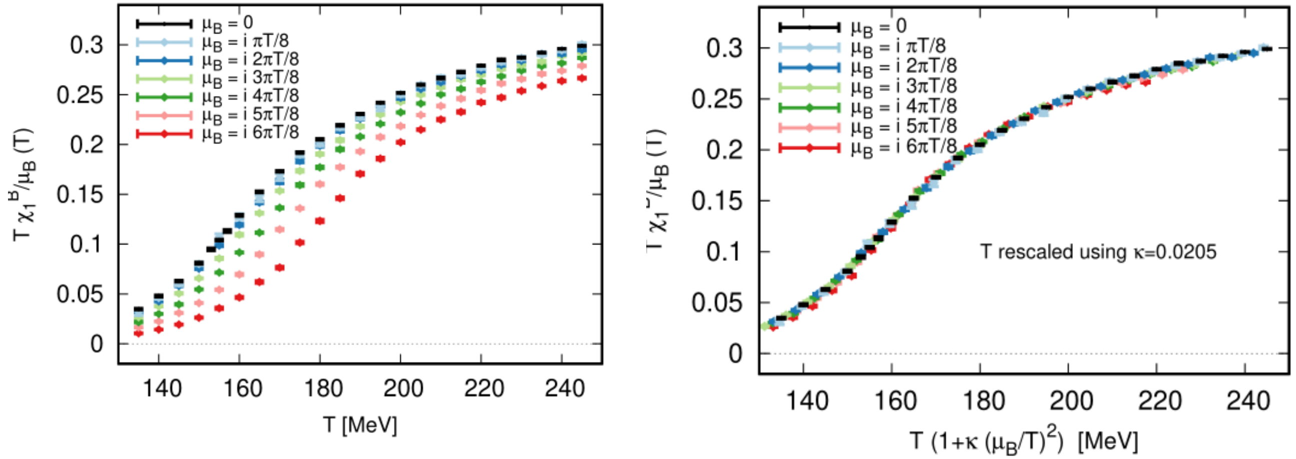


See also [Kotov, Lombardo, Trunin, PLB823 (2021)]:
scaling for heavier-than-physical quark masses

See also [P. Petreczky, We 17:10, QCD at finite T and μ]

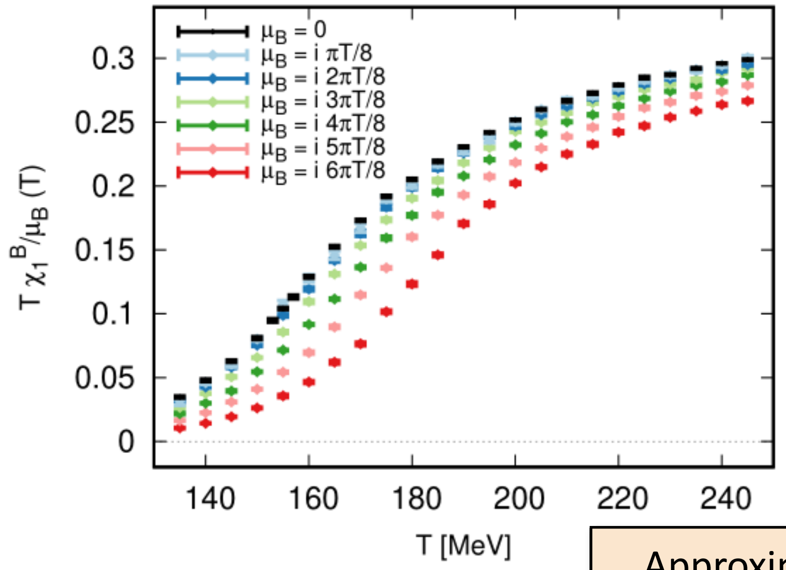
T and μ_B dependence with physical masses

- Empirically: approximate scaling variable $T(1+\kappa_2\hat{\mu}_B^2)$
  $\Rightarrow$ transition not sharpening for small $\hat{\mu}_B^2$
- Collapse predicted by chiral scaling ($\Rightarrow$backup)

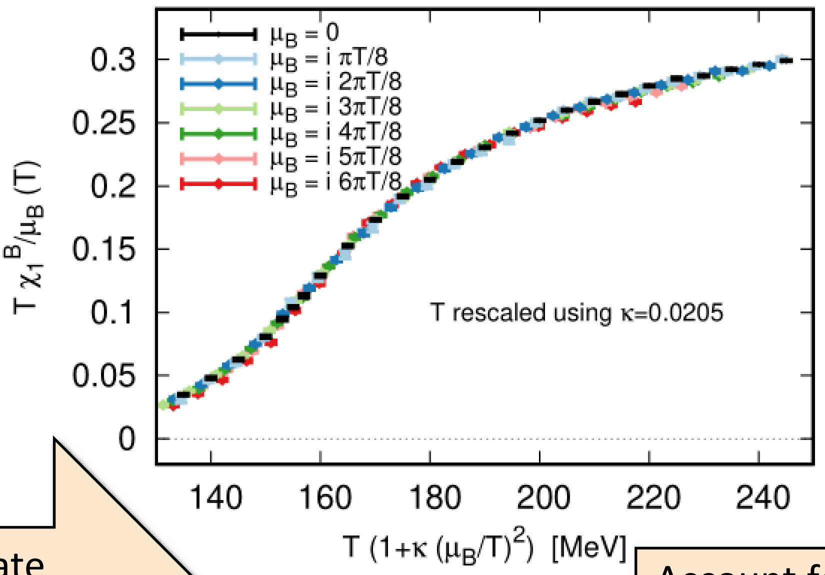


[Wuppertal-Budapest, PRL126 (2021)]

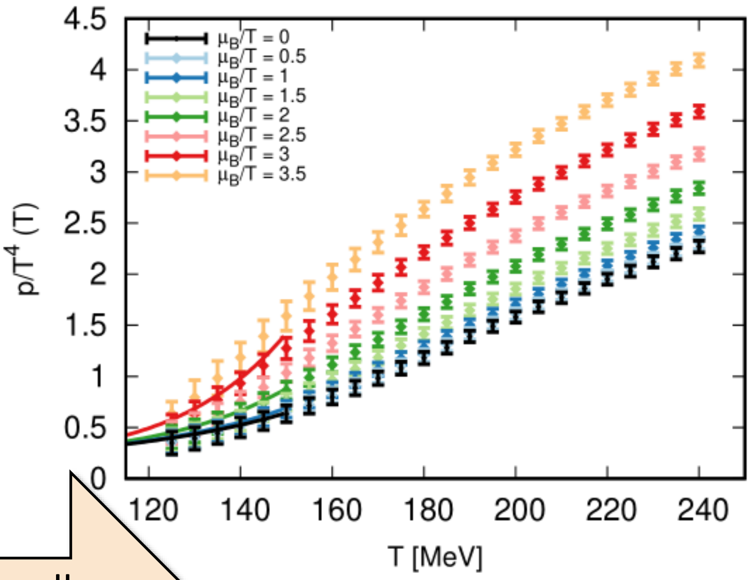
Alternative expansion scheme



Approximate collapse in the $\mathrm{Im}(\mu_B)$ data



Account for small deviations systematically, extrapolate to real μ_B

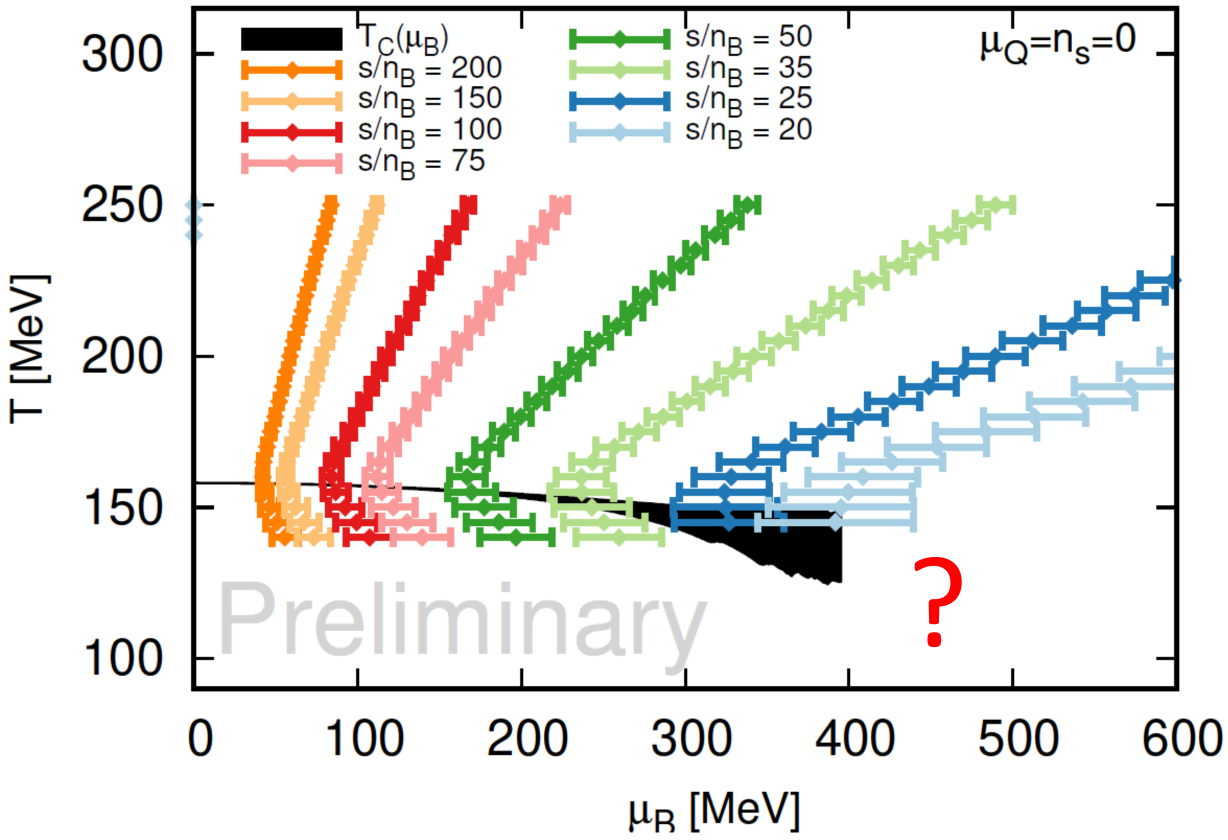


continuum, $LT = 4$, $\mu_S = 0$: [Wuppertal-Budapest, PRL126 (2021)]
continuum, $LT = 4$, $n_S = 0$: [Wuppertal-Budapest, PRD105 (2022)]

Also, small nonzero n_S

Precise EoS from extrapolations

Isentropes (resummation)



RHIC freeze-out [STAR, PRC96 (2017)]

$$\sqrt{s} = 19.6\text{GeV} \leftrightarrow \mu_B \approx 200\text{MeV}$$

$$\sqrt{s} = 11.5\text{GeV} \leftrightarrow \mu_B \approx 300\text{MeV}$$

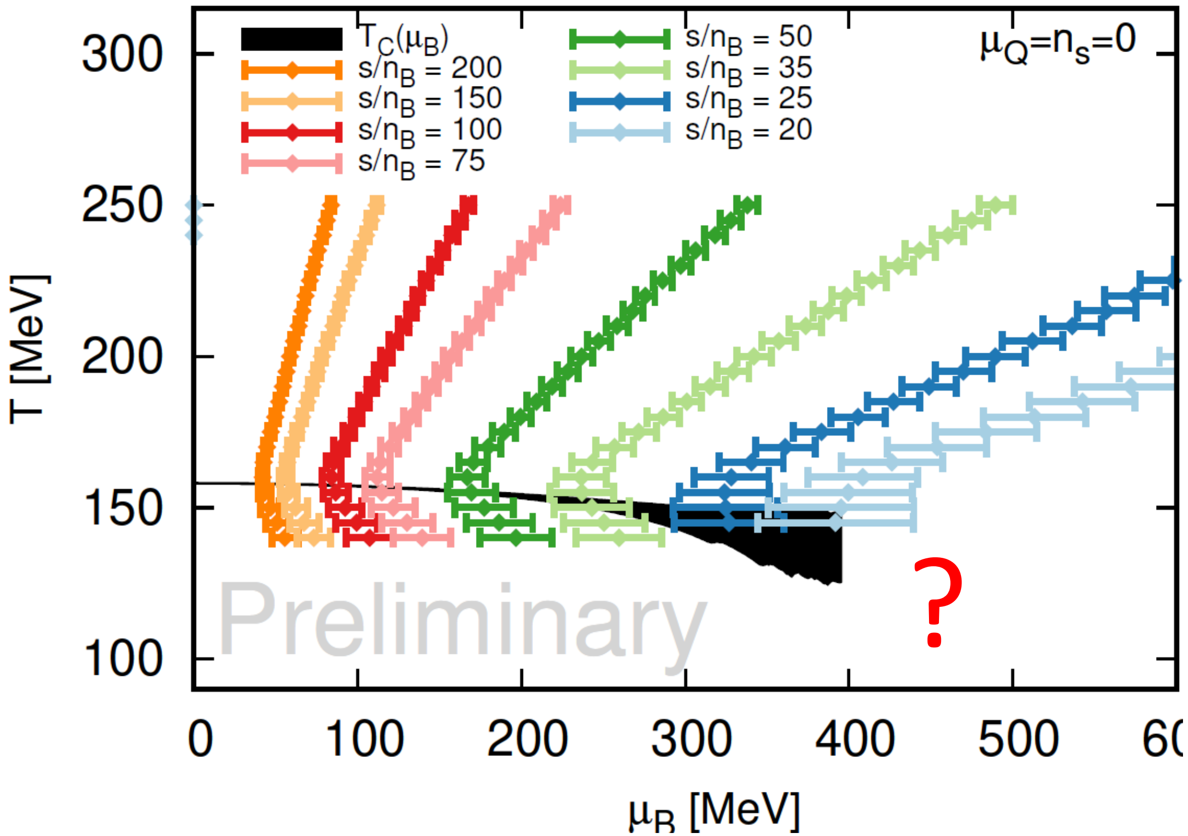
$$\sqrt{s} = 7.7\text{GeV} \leftrightarrow \mu_B \approx 400\text{MeV}$$

No sign of critical lensing within errors

[P. Parotto, Tue 16:30 , QCD at finite T and μ]

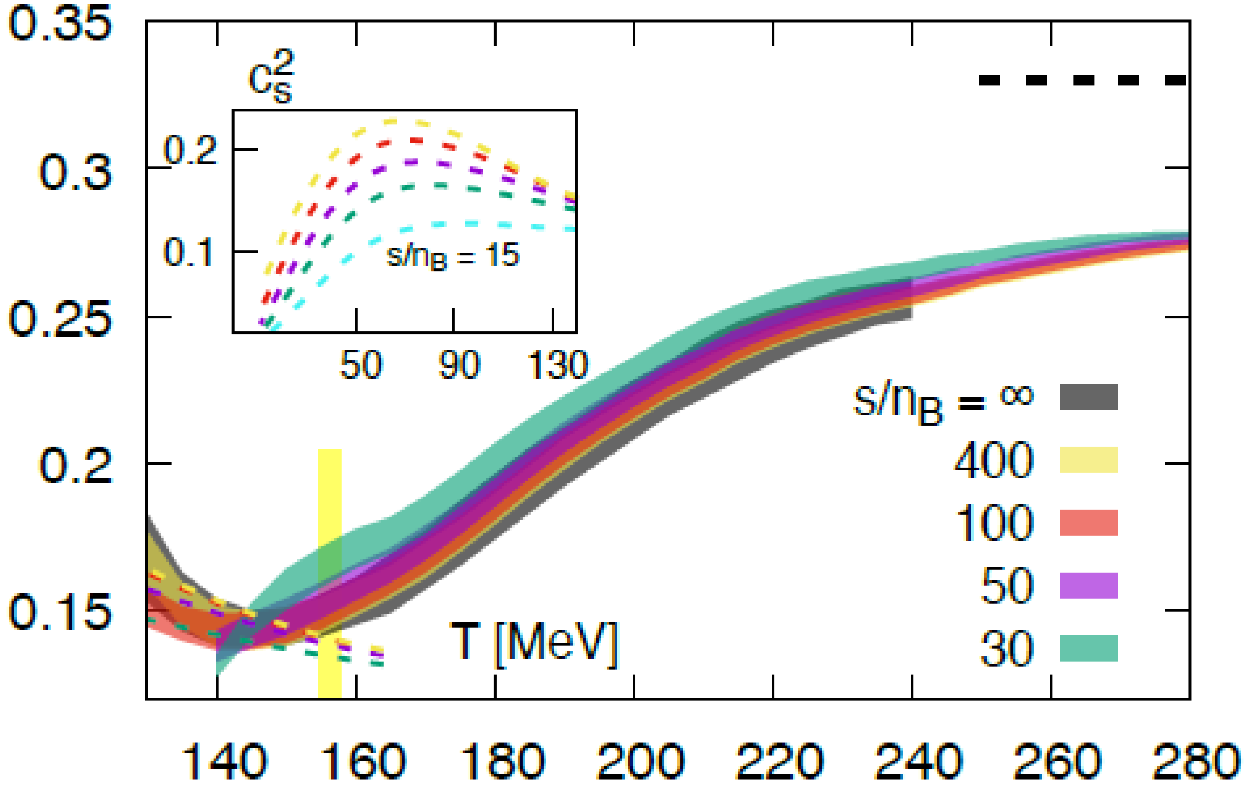
Precise EoS from extrapolations

Isentropes (resummation)



[P. Parotto, Tue 16:30 , QCD at finite T and μ]

Speed of sound on the isentropes (Taylor)



[HotQCD, PRD108 (2023)]
[D. Clarke, We 14:40, QCD at finite T and μ]

More direct methods

Freely tune T and μ_B on the lattice?

Desirable:

No ill-posed analytic continuation

Data closer to conjectured CEP

Common lore:

Impossible

Truth:

Possible (with reweighting), but expensive

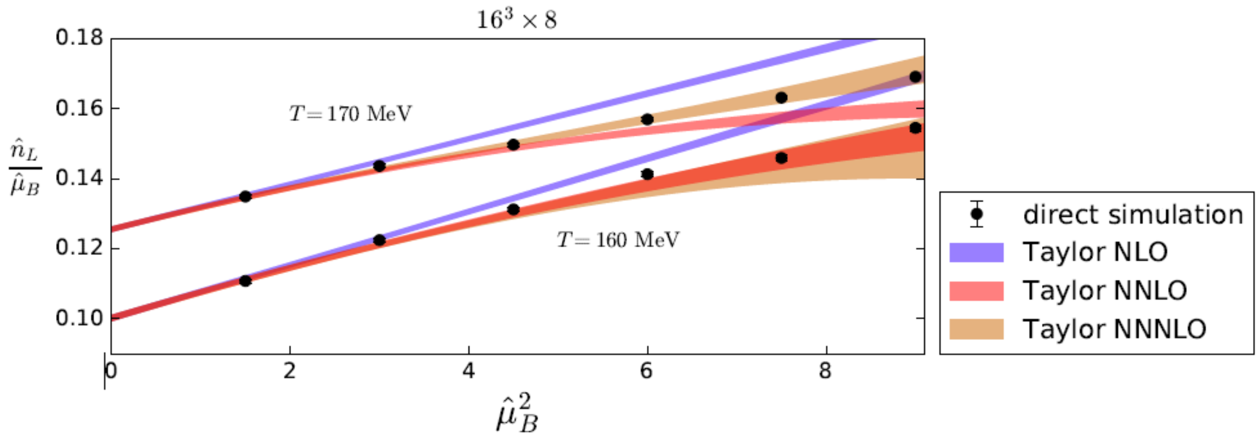
Increasingly more feasible

Technical developments:

[JHEP05 (2020)] [PRD105 (2022)]

[PRD107 (2023)] [2308.06105]

One application: cross-check QGP EoS



[Wuppertal-Budapest, PRD 107 (2023)]

[C.H. Wong, Tue 16:10, QCD at finite T and μ]

For $T \geq 145\text{MeV}$:

4th order Taylor accurate up to $\mu_B = 2T$

Alternative expansion at least up to $\mu_B = 3T$

Future: scan low T and larger μ_B in small volume

Summary on nonzero μ_B

QGP equation of state ☑

- $\mu_B/T<2$ from 4th order Taylor expansion (continuum)
- $\mu_B/T<3\text{-}3.5$ from alternative expansion scheme (continuum)
- Direct simulations agree with extrapolations, provided that the order of expansion is high enough

Search for the CEP ☐

- No solid demonstration of any deviations from the HRG for T<145MeV in cumulants up to 6th order
- No sign of critical lensing in the QGP EoS

Real-time physics

I only have time to advertise two recent papers. Both are about heavy quark physics.

Like at μ_B>0, there is also an analytic continuation problem here. Transport is the most difficult, since it is related to the low frequency (large real-time) behavior

Heavy quark diffusion

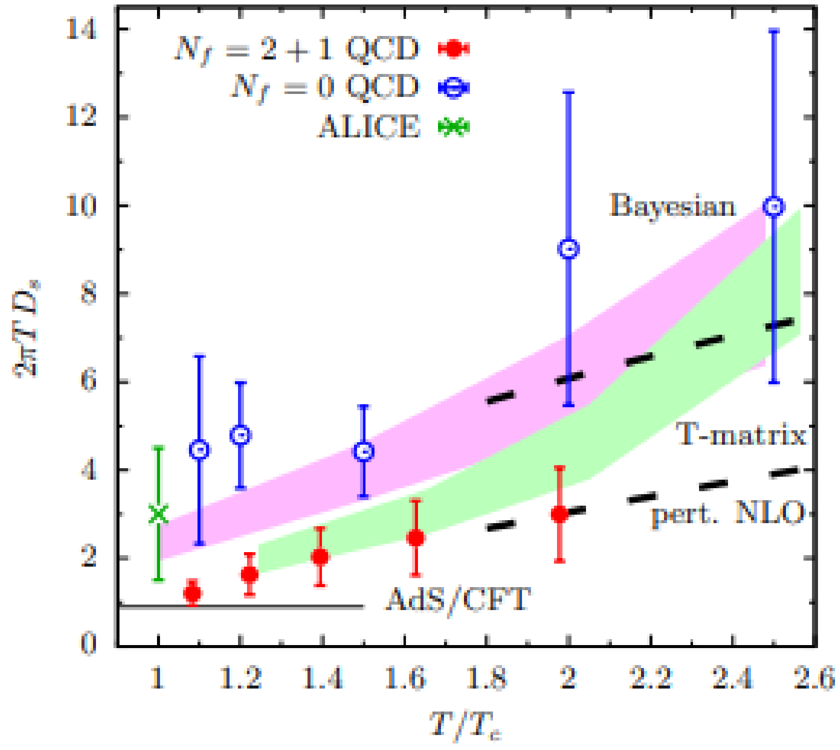
- Previously only available on a pure gluon background
- Now also with dynamical light quarks (m_π=320MeV)

[Altenkort et al, PRL130 (2023)]

- Small value $\Rightarrow$ fast thermalization

[H.T. Shu, Tue 16:50, QCD at finite T and μ]

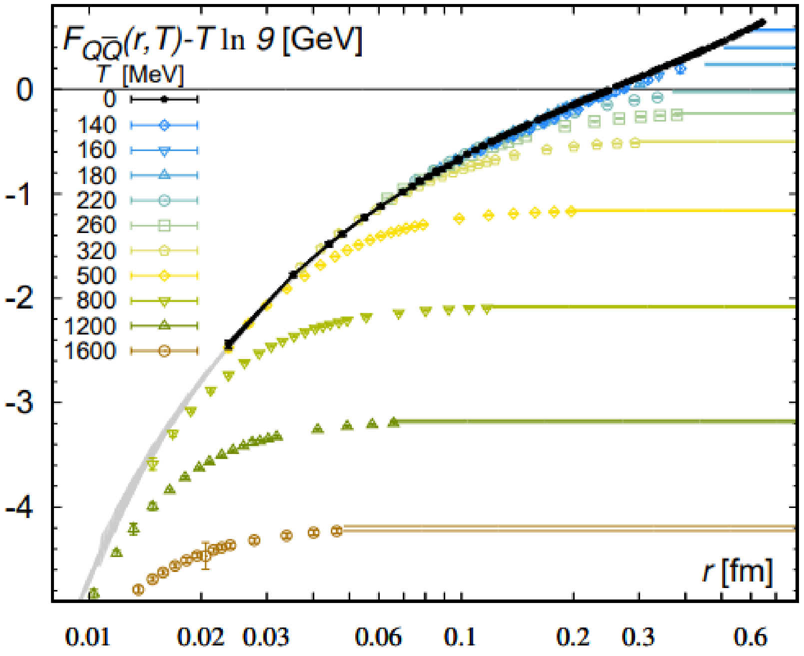
+ new preliminary results: $1/m_Q$ corrections



Real-time potential

Static $Q\bar{Q}$ free energy (Euclidean)

[Bazavov et al, PRD 98 (2018)]



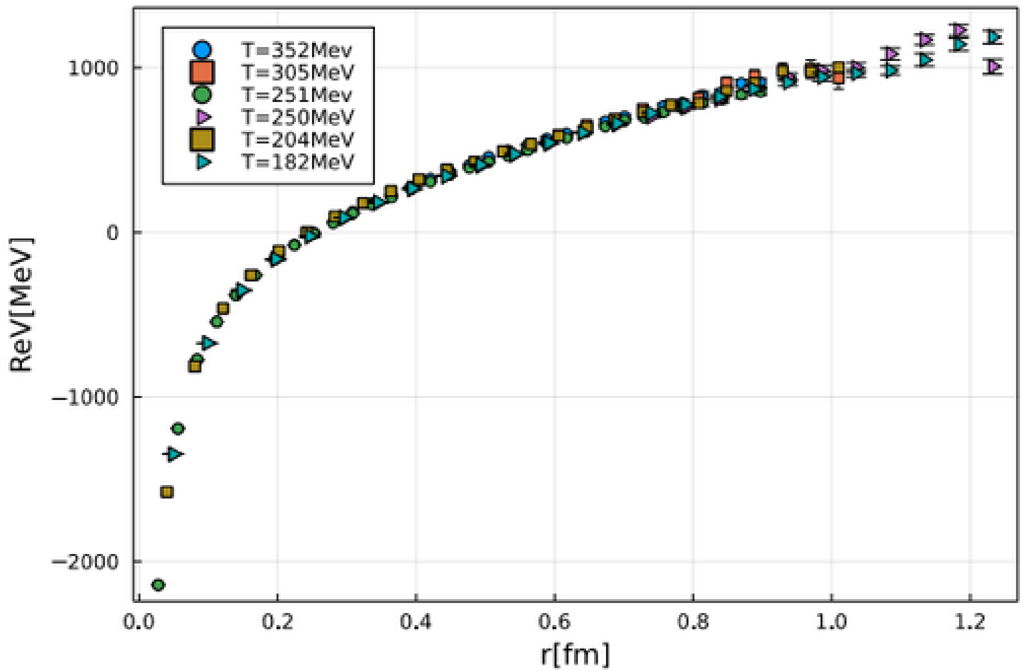
SCREENED, REAL

Recent review: [Bazavov & Weber (2021)]

See also [Wuppertal-Budapest, JHEP04 (2015)]

Real-time $Q\bar{Q}$ potential

[Bazavov et al, 2308.16587]



NOT SCREENED, COMPLEX

See also [Z. Tang, We 17:30, Heavy flavor]

LATTICE TALKS @ QM 2023

Tue 10:00 — X. Yao: Testing Eigenstate Thermalization Hypothesis for Non-Abelian Gauge Theories [New Theory]

Tue 14:50 — Sz. Borsányi: High order fluctuations of conserved charges in the continuum limit [QCD at finite T and μ]

Tue 16:10 — C.H. Wong: Equation of state of a hot-and-dense QGP: lattice simulations at real μ_B vs. extrapolations [QCD at finite T and μ]

Tue 16:30 — P. Parotto: QCD equation of state with improved precision from lattice simulations [QCD at finite T and μ]

Tue 16:30 — G. Basar: Mapping the critical equation of state with resummations [New Theory]

Tue 16:50 — H.-T. Shu: Heavy quark diffusion from 2+1 flavor lattice QCD [QCD at finite T and μ]

We 14:20 — R. Kara: Finite volume effects near the chiral crossover [QCD at finite T and μ]

We 14:40 — D. Clarke: QCD material parameters at zero and non-zero chemical potential from the lattice [QCD at finite T and μ]

We 15:20 — J. Goshwami: Exploring the Critical Points in QCD with Multi-Point Padé and ML Techniques in (2+1)-flavor QCD [Critical Point]

We 17:10 — P. Petreczky: Microscopic encoding of macro universality [QCD at finite T and μ]

We 17:30 — Z. Tang: T-matrix analysis of Wilson line correlators from Lattice QCD [Heavy Flavor]

BACKUP

LATTICE TALKS @ QM 2023 – THE CHIRAL LIMIT

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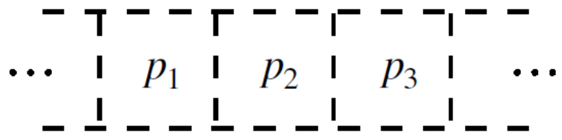
P. Petreczky: Microscopic encoding of macroscopic universality [QCD at finite T and μ]

Connection between thermodynamic divergences in the chiral limit (macroscopic) and the eigenvalues of the Dirac equation (microscopic). [Ding et al, 2305,10916]

LATTICE TALKS @ QM 2023 – REAL TIME PHYSICS



Thermalization of a chain of plaquettes

[X. Yao, PRD128 (2023)]

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Beyond the hadron resonance gas

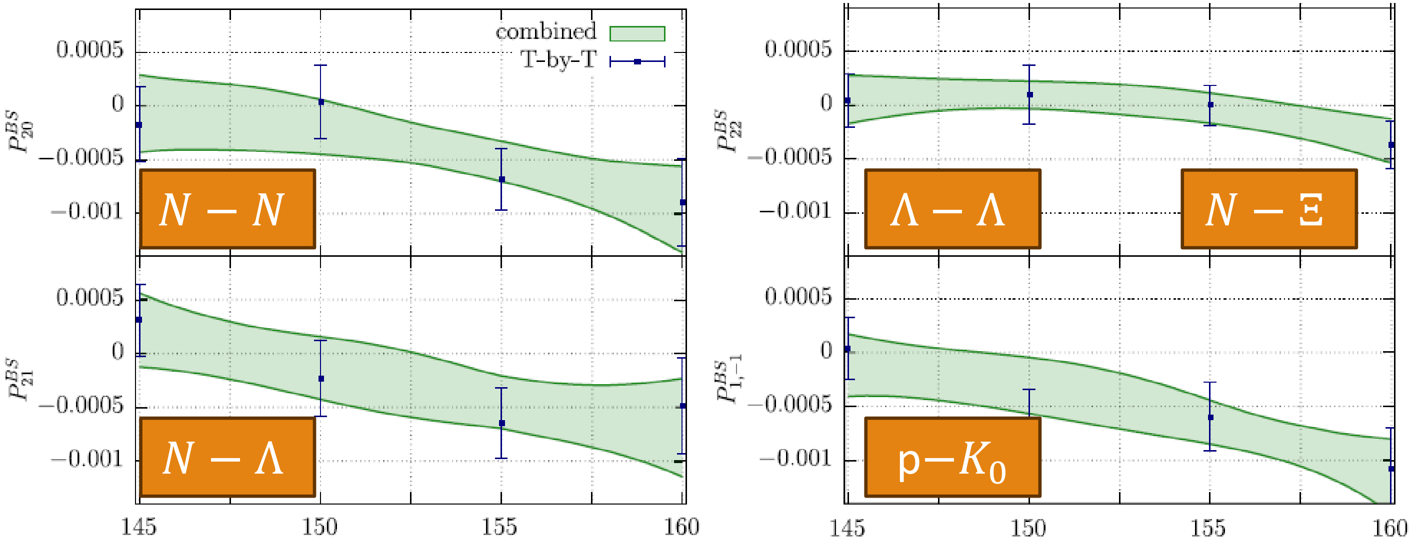
$$\chi_1^B(T,\mu_B,\mu_S) = P_{10}^{BS}(T)\sinh(\hat{\mu}_B) + P_{11}^{BS}(T)\sinh(\hat{\mu}_B - \hat{\mu}_S) + \cdots + 2P_{20}^{BS}(T)\sinh(2\hat{\mu}_B) + \cdots$$

$\{$ N Λ … $\}$ $\{$ N − N … $\}$

S-matrix formalism:
[Dashen et al, PR187 (1969)]

Repulsive interactions ⇒ negative sector
Attractive interactions ⇒ negative sector

Lattice data vs repulsive extensions of HRG:
[Huovinen, Petreczky PLB777 (2017)]
[Vovchenko, Pásztor et al, PLB 775 (2017)]



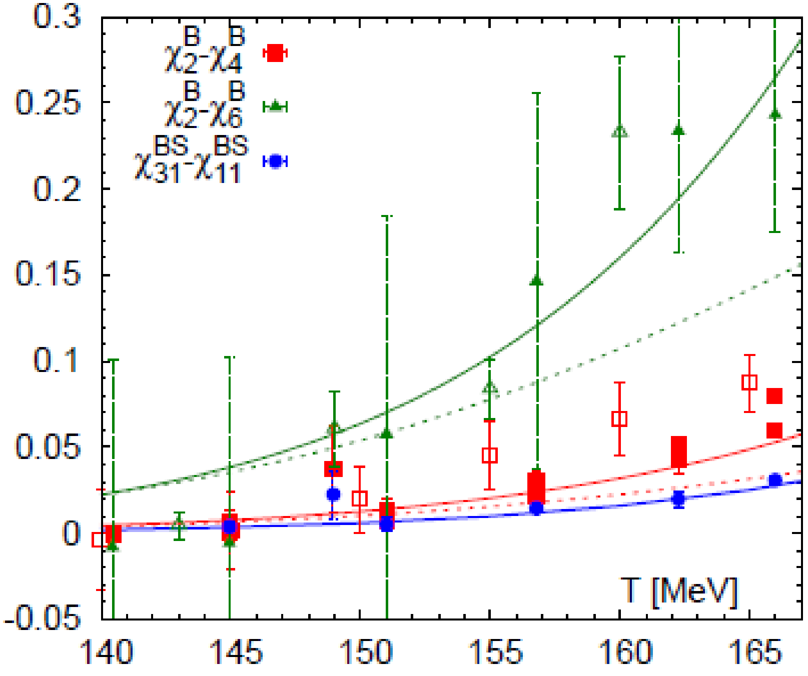
[Wuppertal-Budapest, PRD104(2021)]

Repulsive hadronic models vs lattice data

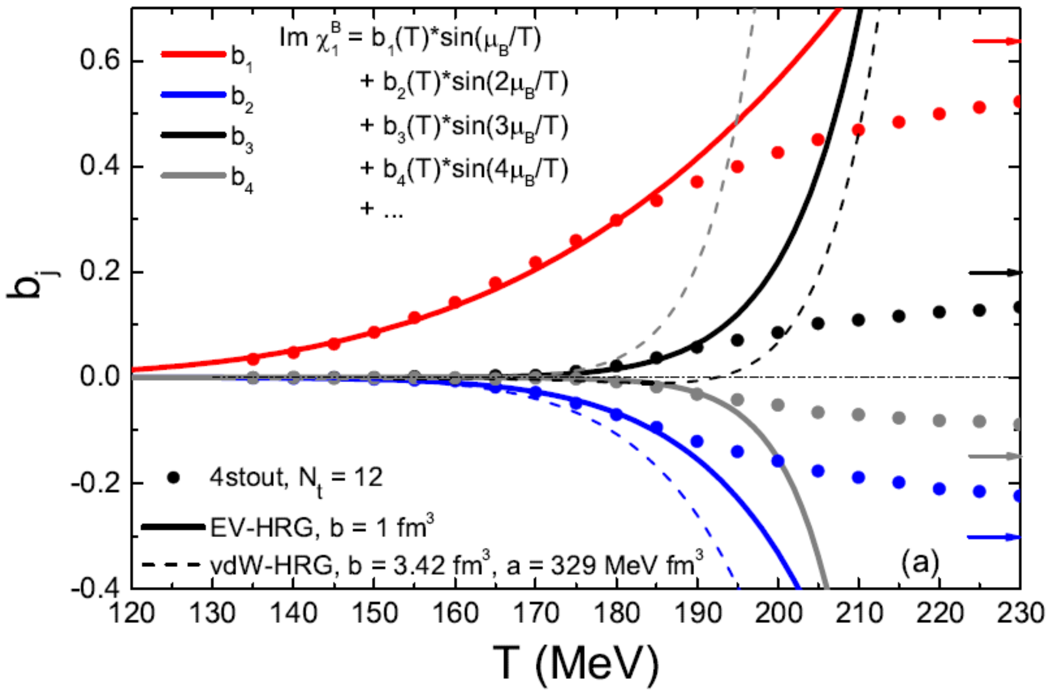
Repulsive core of NN interactions is very well established, and the HRG model does not take it into account at all!

[Huovinen, Petreczky PLB777 (2017)]

[Vovchenko, Pásztor et al, PLB 775 (2017)]



LT=4, N_t=8, Taylor VS repulsive mean field



LT=4, N_t=12, Imμ_B VS excluded volume or VdW HRG

See also [Bellwied et al, PRD 104 (2021)] for a systematic study

O(4) scaling and resummation

Empirical observations from imaginary μ_B data:

- Σ/f_π^4 collapses as a function of $T\left(1+\kappa\left(\frac{\mu_B}{T}\right)^2\right)$ but Σ/T^4 does not
- $\chi_1^B/(\mu_B/T)$ collapses as a function of $T\left(1+\kappa\left(\frac{\mu_B}{T}\right)^2\right)$ but χ_2^B does not

BUT WHY?

One possible explanation is scaling near the chiral limit:

$$p_{QCD}(T,\mu_B,m)-p_{QCD}(0,0,m)\sim f_{sing}(h,t)\sim t^{2-\alpha}F\left(\frac{h}{t^{\beta\delta}}\right)$$ where $h\sim m$ and t $\sim T-T_{ch}(1-\kappa(\mu_B/T_{ch})^2)$

$$\Rightarrow \Sigma_{sing}=m\frac{\partial}{\partial m}f_{sing}=t^{2-\alpha}\frac{h}{t^{\beta\delta}}F'\left(\frac{h}{t^{\beta\delta}}\right)$$

$\Rightarrow$ near T_{ch} near the chiral limit, Σ/f_π^4 is a function of the scaling variables h and t only, while Σ/T^4 is no

$$\Rightarrow \frac{1}{(\mu_B/T_{ch})}\frac{\partial}{\partial(\mu_B/T_{ch})}f_{sing}=(2-\alpha)t^{1-\alpha}F\left(\frac{h}{t^{\beta\delta}}\right)(2\kappa)+t^{1-\alpha-\beta\delta}F'\left(\frac{h}{t^{\beta\delta}}\right)(-\beta\delta)(2\kappa)\coloneqq(2\kappa)G(h,t)$$

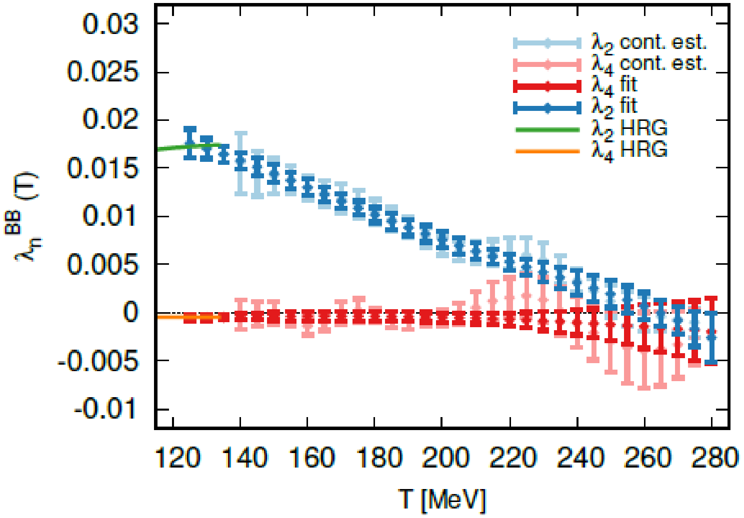
$\Rightarrow$ again, a function of h and t only, while

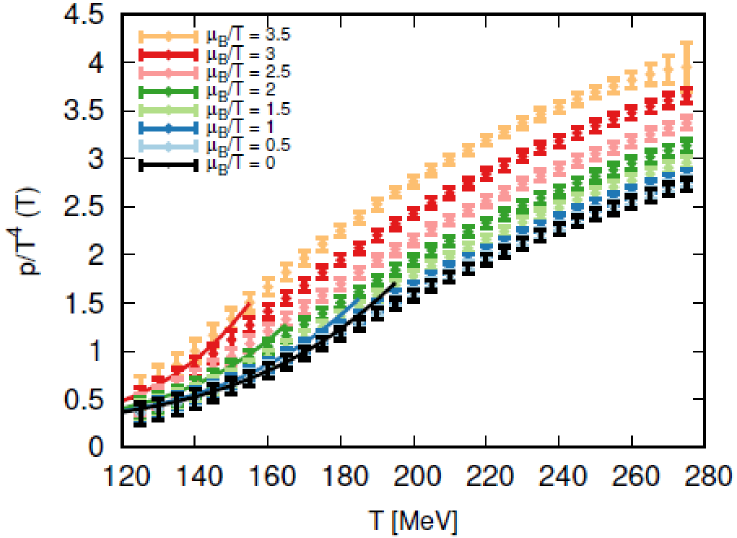
$$\frac{\partial^2}{\partial(\mu_B/T_{ch})^2}f_{sing}=(2\kappa)G(h,t)+\left(\frac{(2\kappa)\mu_B}{T_{ch}}\right)^2\frac{\partial G}{\partial t}$$

$\Rightarrow$ not a function of h and t only

Resummed EoS: some details

- Systematically improvable ansatz: $F(T,\mu_B) = F(T',0) \qquad T' = T(1-\lambda_2(T)\hat{\mu}_B^2 - \lambda_4(T)\hat{\mu}_B^4 - \cdots)$
- This ansatz together with a choice of the observable F defines an extrapolation scheme (resummation)
- A good choice for $\langle S\rangle = 0$ is $F = \frac{c_1^B(T,\hat{\mu}_B)}{c_1^B(T\to\infty,\hat{\mu}_B)}$ where $c_1^B := \left(\frac{d\hat{p}}{d\,\hat{\mu}_B}\right)_{\langle S\rangle=0}$
- The normalization makes sure the infinite temperature behavior is correct
- The ansatz itself exploits the existence of the approximate scaling variable
- Already the leading order, with λ_2 only generates terms to all orders in the Taylor expansion of $\hat{p}$
- Analysis is like the extrapolation of $T_c(\hat{\mu}_B)$
- Result: λ_4 is very small, while λ_2 has a very simple temperature dependence





Equation of state (summary)

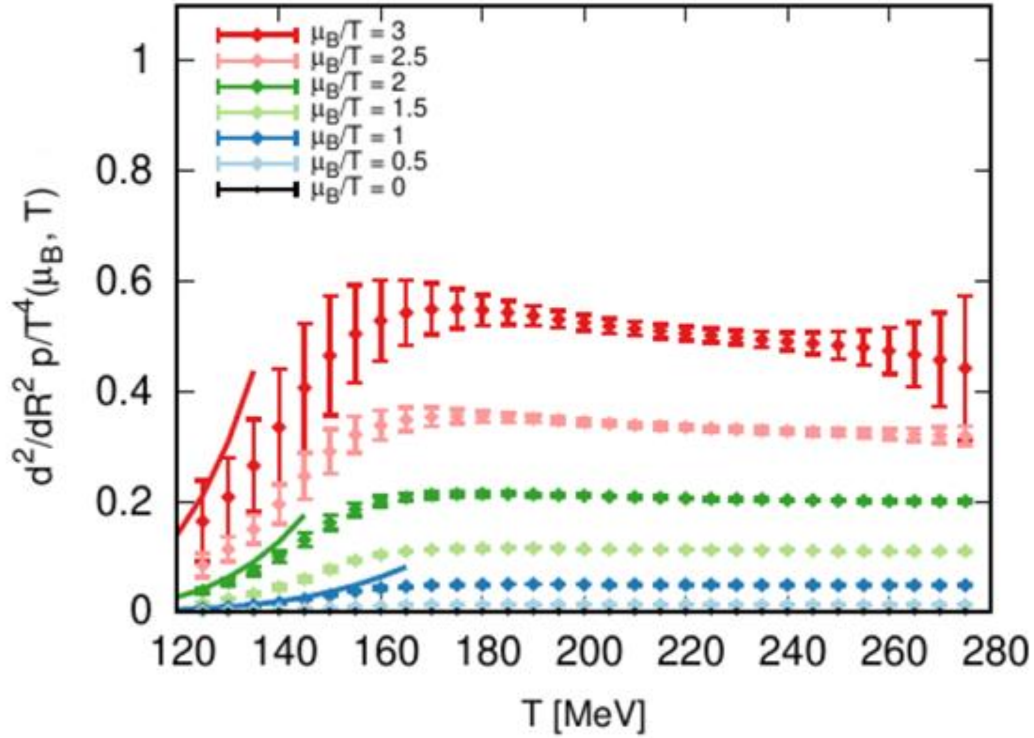
1. Realize the existence of the approximate scaling variable
2. Turn it into a systematically improvable extrapolation ansatz [Borsányi et al, PRL126 (2021)]
3. Validate the scheme by comparison with direct simulation results at non-zero density on finite (but reasonable) lattices [Borsányi et al, PRD107 (2023)]
4. Calculate the coefficients of the validated extrapolation scheme in the continuum in conditions relevant for heavy ion phenomenology. [Borsányi et al, PRD105(2022)]
5. Realize that the finite μ_B part is so precise that the errors are dominated by $\mu_B=0$, so make the $\mu_B=0$ equation of state more precise. **[P. Parotto, Tue 16:30 , QCD at finite T and μ]**

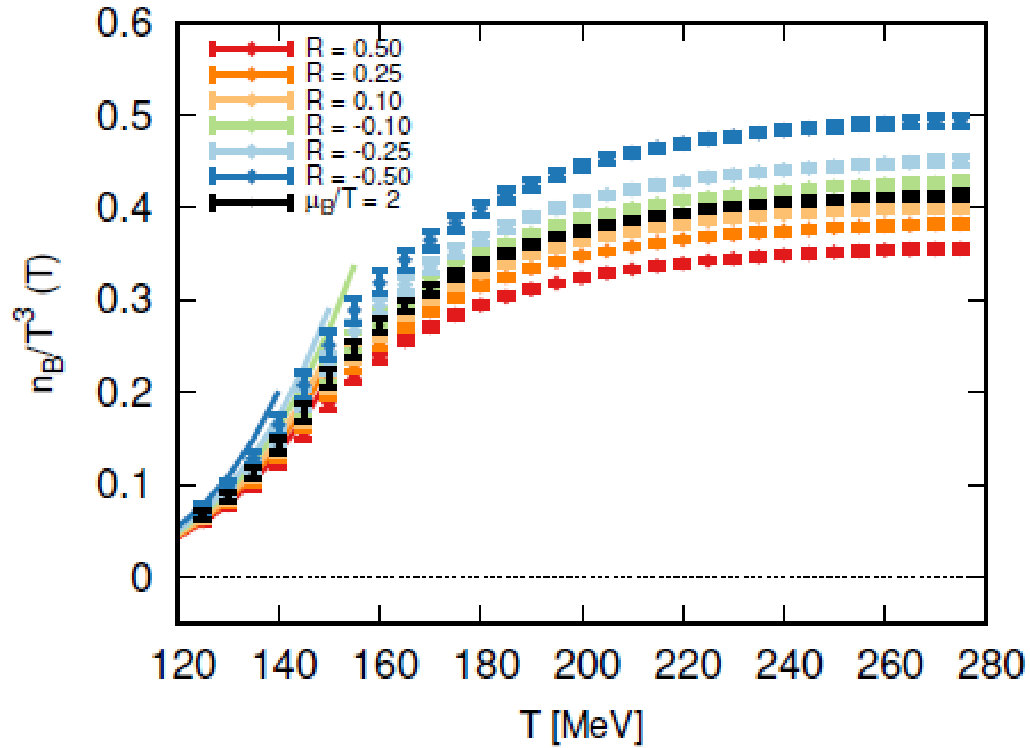
$\Rightarrow$ A PRECISE EQUATION OF STATE FOR THE RHIC BES RANGE

Beyond strangeness neutrality

Makes it possible to take small local fluctuations of strangeness into account in hydrodynamics:

$$\hat{p}(T,\mu_B,R) \approx \hat{p}(T,\mu_B,0) + \frac{1}{2}\frac{d^2\hat{p}}{dR^2}R^2 \quad \text{where} \quad R = \frac{n_S}{n_B}$$





[Borsányi et al, PRD105 (2022)]

Reweighting

Fields: ϕ Target theory: $Z_t = \int D\phi\, w_t(\phi)$ Simulated theory: $Z_s = \int D\phi\, w_s(\phi)$

$$\langle O \rangle_t = \frac{\int D\phi\, w_t(\phi) O(\phi)}{\int D\phi\, w_t(\phi)} = \frac{\int D\phi \frac{w_t(\phi)}{w_s(\phi)} w_s(\phi) O(\phi)}{\int D\phi \frac{w_t(\phi)}{w_s(\phi)} w_s(\phi)} = \frac{\left\langle \frac{w_t}{w_s} O \right\rangle_s}{\left\langle \frac{w_t}{w_s} \right\rangle_s} \quad \text{and} \quad \frac{Z_t}{Z_s} = \left\langle \frac{w_t}{w_s} \right\rangle_s$$

Two problems (usually exponentially hard in the volume) can arise:

- sign problem: $\frac{w_t}{w_s} \in \Rightarrow$ large signal to noise ratios

- overlap problem: tails of $\mathrm{P}\left(\frac{w_t}{w_s}\right)$ do not decay fast enough $\Rightarrow$ potentially incorrect results

Two choice of w_s that eliminate this overlap problem:

- phase reweighting: $w_s = e^{-S_{YM}} |\det M| \quad \Rightarrow \quad \frac{Z_t}{Z_s} = \left\langle e^{i\,\theta} \right\rangle_s$

- sign reweighting: $w_s = e^{-S_{YM}} |\operatorname{Re} \det M| \quad \Rightarrow \quad \frac{Z_t}{Z_s} = \langle \pm \rangle_s$

Staggered rooting and low T difficulties

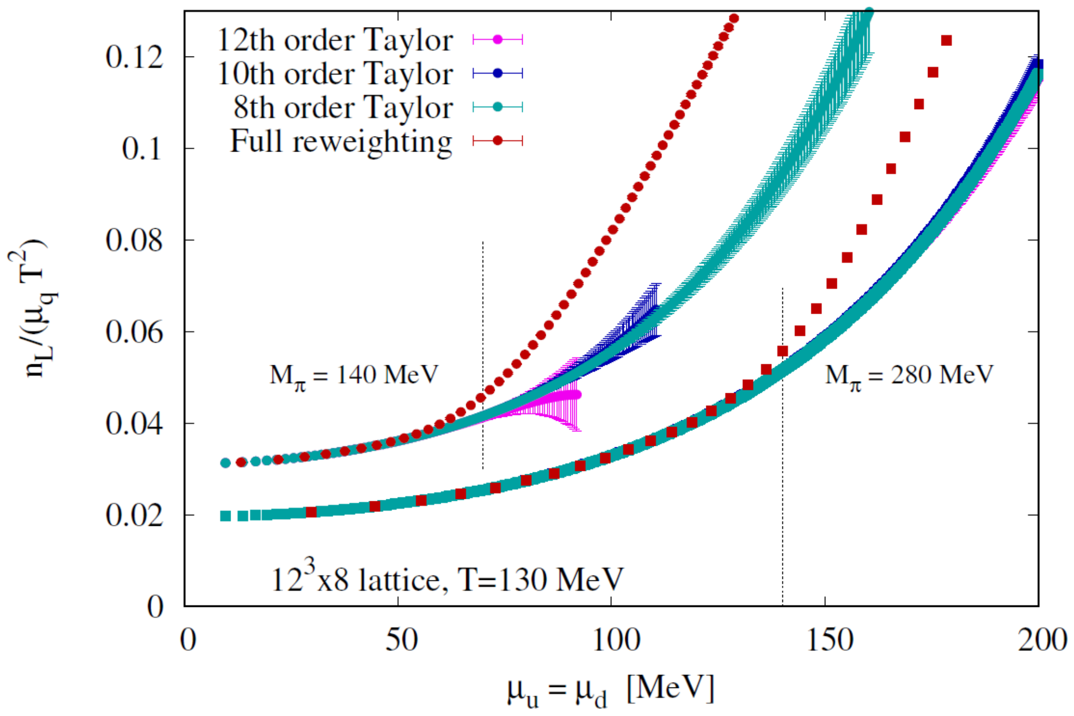
Say I want N_f=2+1 with staggered: $Z = \int DU (\det M_{ud}(U,\mu))^{\frac{1}{2}} (\det M_s(U))^{\frac{1}{4}} e^{-S_{YM}(U)}$

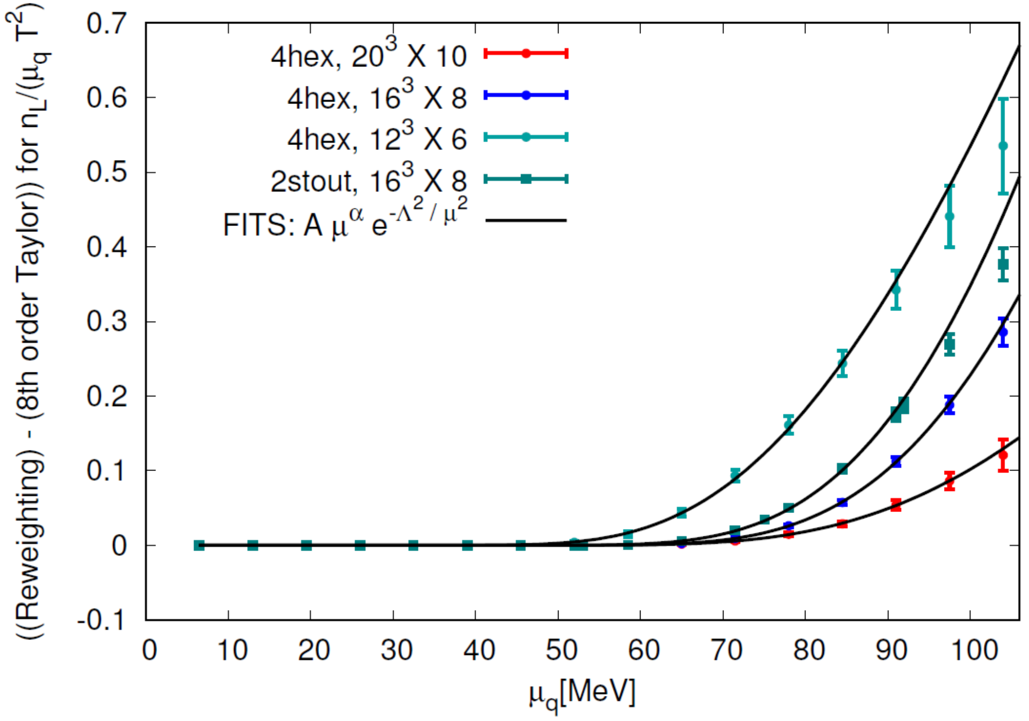
Determinant complex, so sqrt ambiguous. Standard choice: continuously connect to the positive root at μ=0

We empirically observe that this leads to non-analytic behavior (essential singularity) at μ=0

The non-analytic part is suppressed for $\mu < m_\pi$

The amplitude of the non-analytic part decreases with the lattice spacing

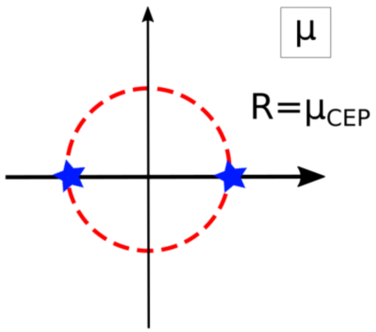


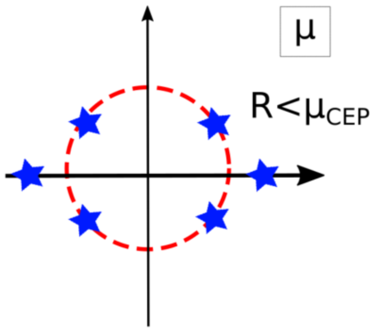


Radius of convergence

$$\hat{p} = \hat{p}(T, \mu_B = 0) + \frac{1}{2}\chi_2^B \hat{\mu}_B^2 + \frac{1}{4!}\chi_4^B \hat{\mu}_B^4 + \cdots \quad \text{converges for } |\hat{\mu}_B| < R =?$$

Motivation: Inside the radius of convergence of the Taylor expansion there can be no singularities in the complex μ_B plane, and thus also no CEP on the real μ_B line





- For a long time (≈15years) ratio estimators were used
- For complex singularities (expected, e.g., for $T \approx T_{crossover}$) doesn't converge

[Vovchenko et al, PRD97 (2018)] [Giordano & Pásztor, PRD99(2019)]

- There are also possible issues with lattice artefacts

[Giordano et al, PRD101 (2020)] [Borsányi et al, 2308.06105]

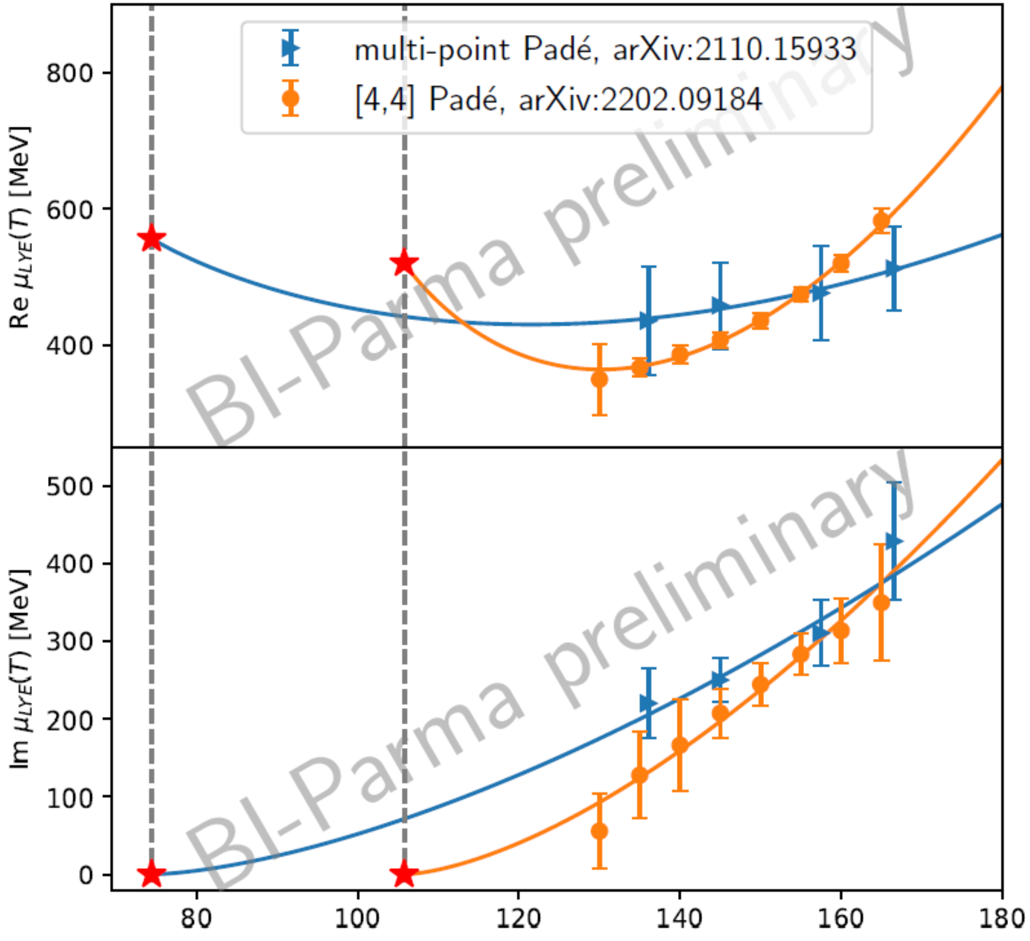
- For reliable estimation, needs many more orders
- Higher orders not available in the continuum
- Can be phenomenologically estimated from O(4) scaling + other assumptions

[Mukherjee & Skokov, PRD103 (2021)]

$\Rightarrow$ All current lattice estimates of R should be considered preliminary/exploratory estimates, with inadequate quality control ($\Rightarrow$ MORE WORK)

CEP at nonzero μ_B? (Parma-Bielefeld)

[J. Goshwami, We 15:20]



The basic idea

- crossover ⇔ critical point at complex $\mu_B=\mu_{LYE}$ (Lee-Yang edge)
- near CEP: μ_{LYE} moves to the real line
- find μ_{LYE} by analytic continuation, extrapolate T dependence

Datasets

Blue: $N_t=6$, imaginary μ_B Orange: $N_t=8$, Taylor

- Extends the Z_2 scaling near CEP all the way to $T_c(\mu_B=0)$, where O(4) chiral scaling is likely relevant
- Radius of convergence @ crossover is likely almost T indep.>
  - Method without truncation errors on a coarse lattice: [Giordano et al, PRD101 (2020)]
  - Phenomenological analysis assuming O(4) scaling: [Mukherjee & Skokov, PRD103 (2021)]
- Puzzle: as data becomes more like HRG (low T), the system looks more critical (smaller $\mathrm{Im}\mu_{LYE}$)?
- Deviations from the HRG are probably cut-off effects
- Systematics of the blue and orange points?