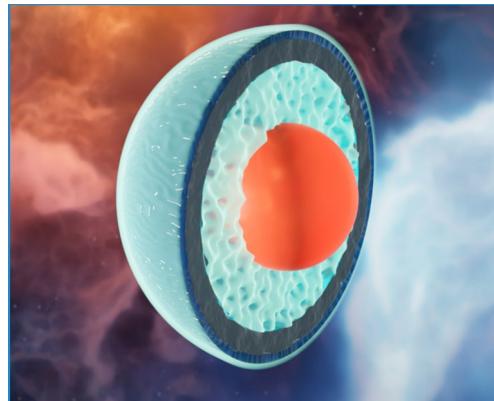




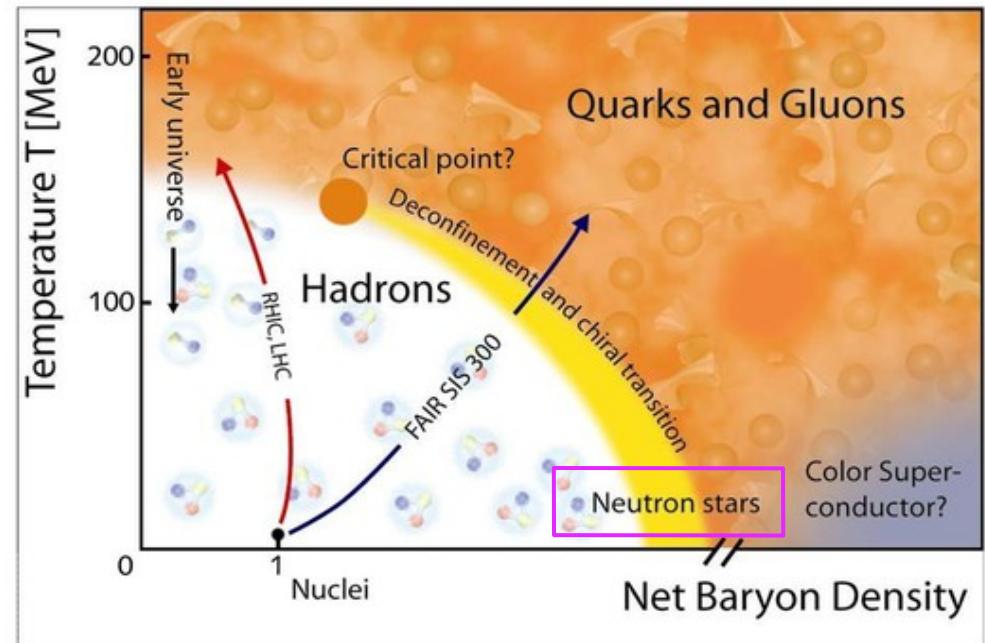
# Quark Matter and Nuclear Astrophysics



Quark Matter 2023 – Houston, Texas, USA  
4 September 2023

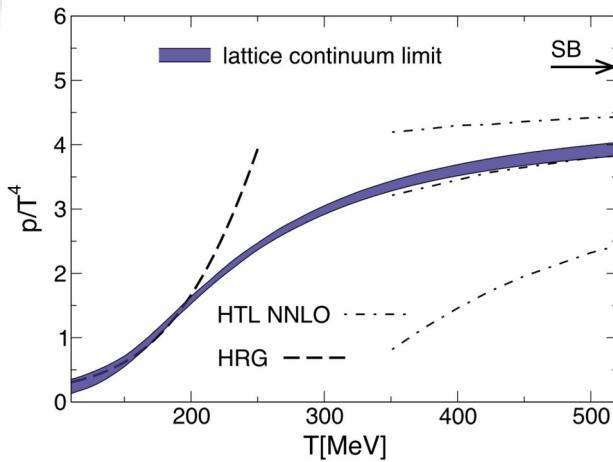
Tyler Gorda  
TU Darmstadt

# Does deconfined Cold Quark Matter occur in nature?



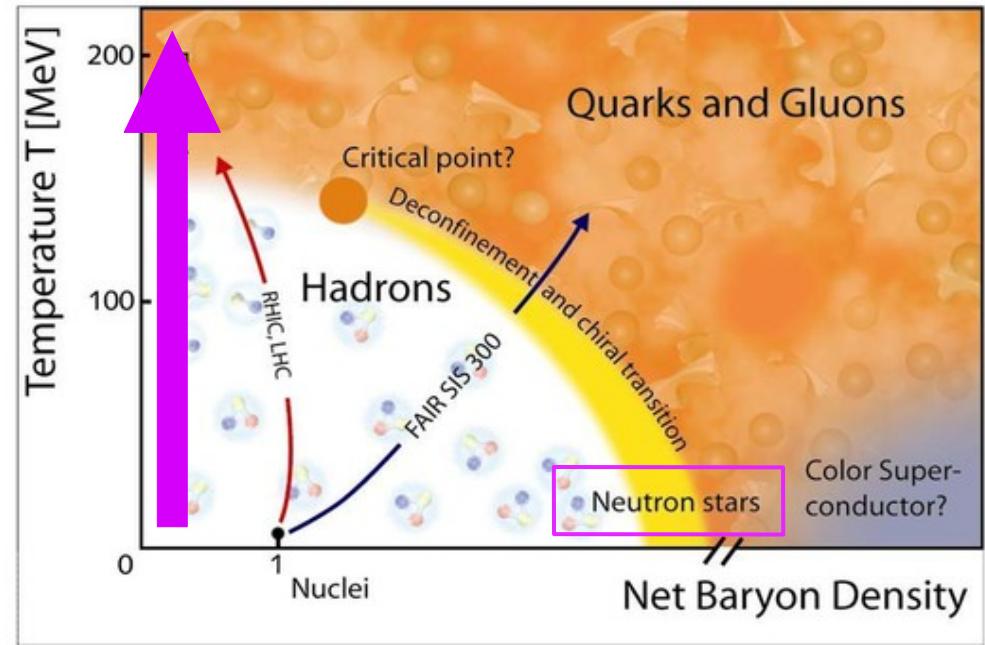
Compressed Baryonic Matter (CBM) experiment

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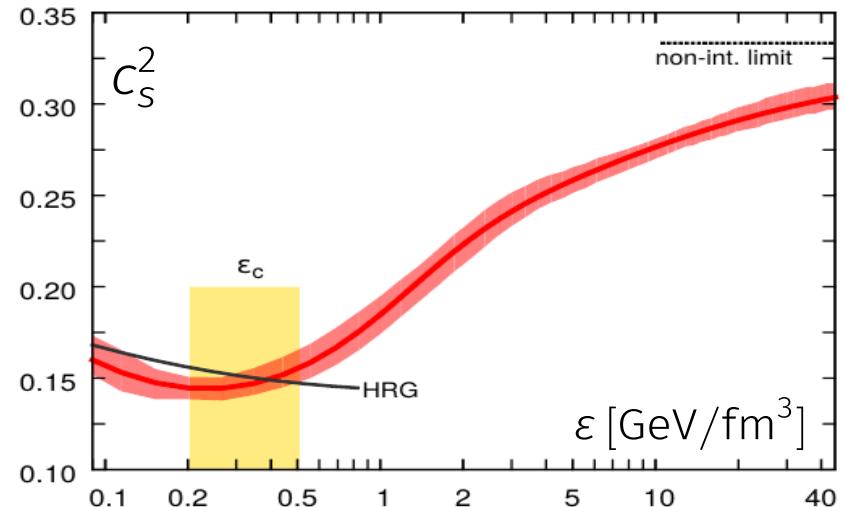
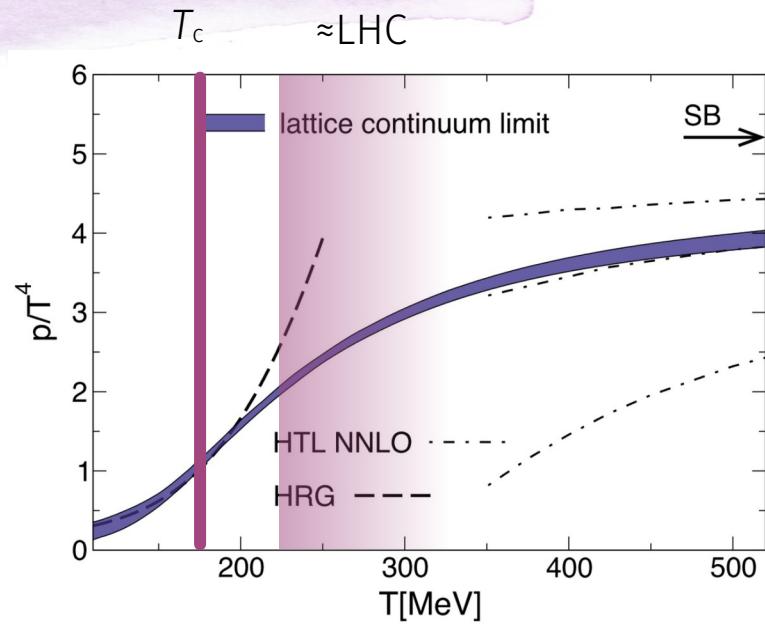
Borsanyi, Fodor, Hoelbling, Katz, Krieg, Szabo  
Phys. Lett. B 370 (2014)

Let's take inspiration from high- $T$ .  
**How do we know what we know?**



Compressed Baryonic Matter (CBM) experiment

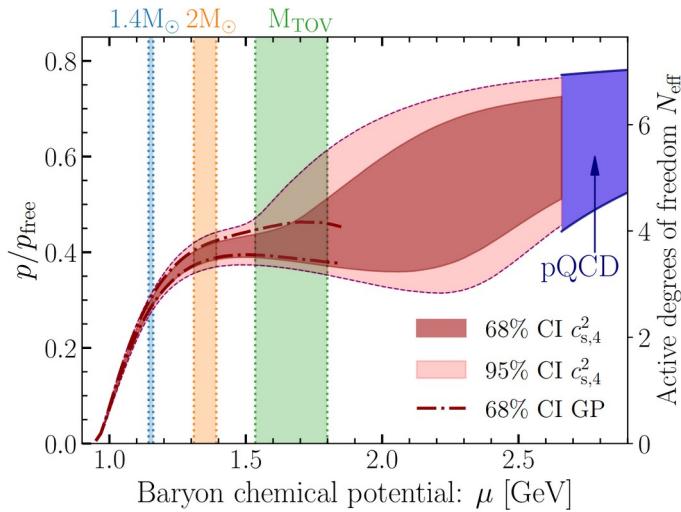
# High- $T$ : Compare Lattice result to asymptotics



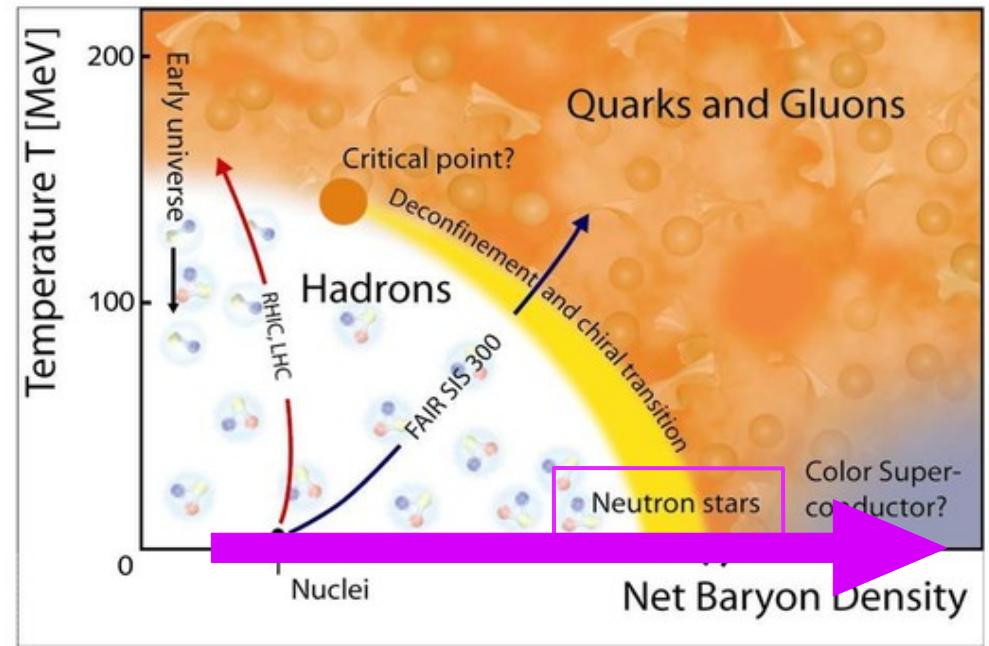
Here we know the full thermodynamic answer. Then compare Lattice result to controlled hadronic and partonic/quark calculations

HotQCD Phys.Rev.D 90 (2014), Borsanyi, Fodor, Hoelbling, Katz, Krieg, Szabo Phys. Lett. B 370 (2014) Gardim, Giacalone, Luzum, Ollitrault, Nature Physics 16, 615-619 (2020)

# Does deconfined Cold Quark Matter occur in nature?

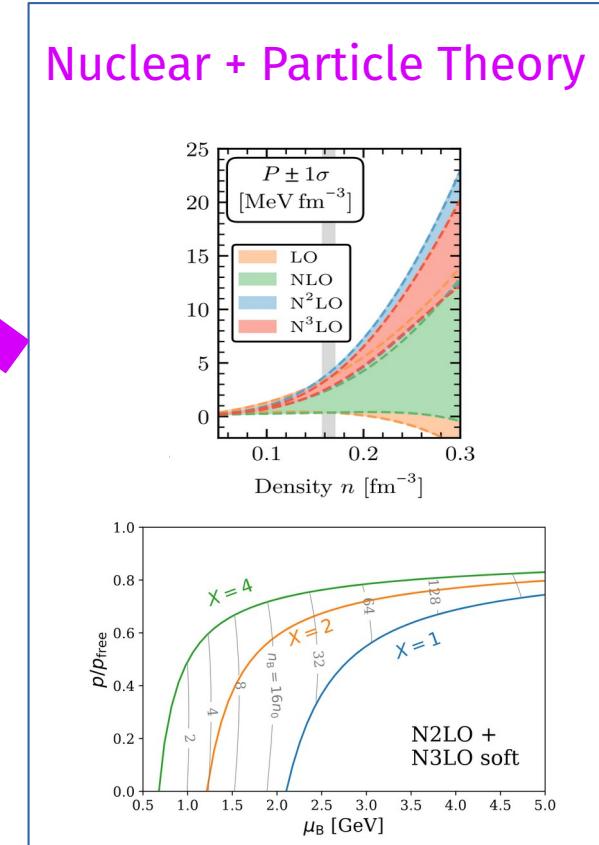
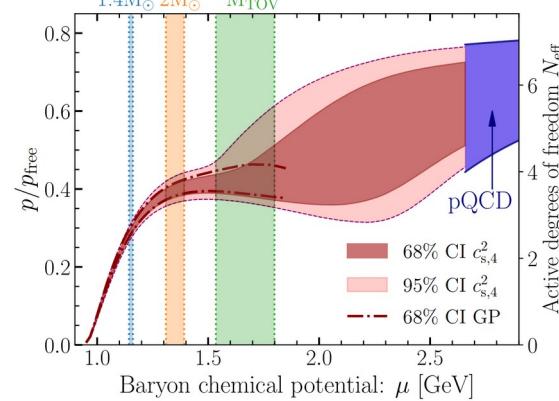
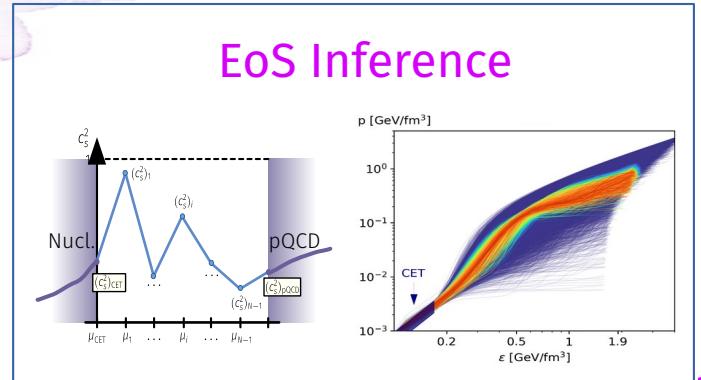
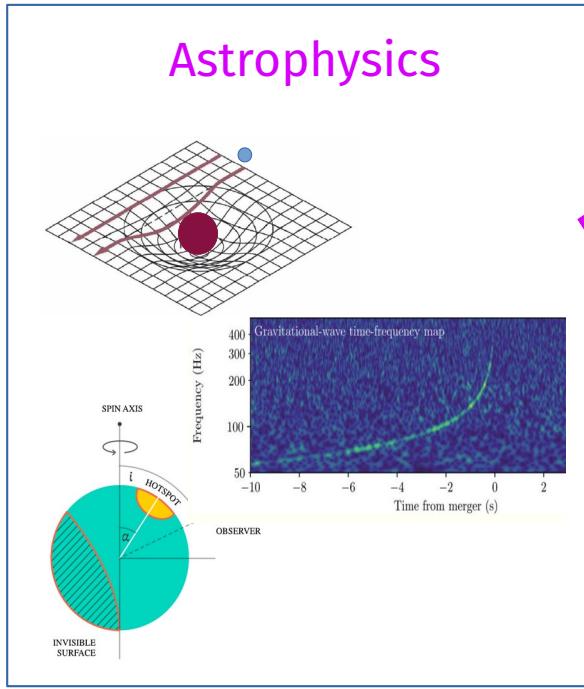


At  $T = 0$ , no lattice, but we have **astrophysics**; and calculations in **nuclear and particle theory**

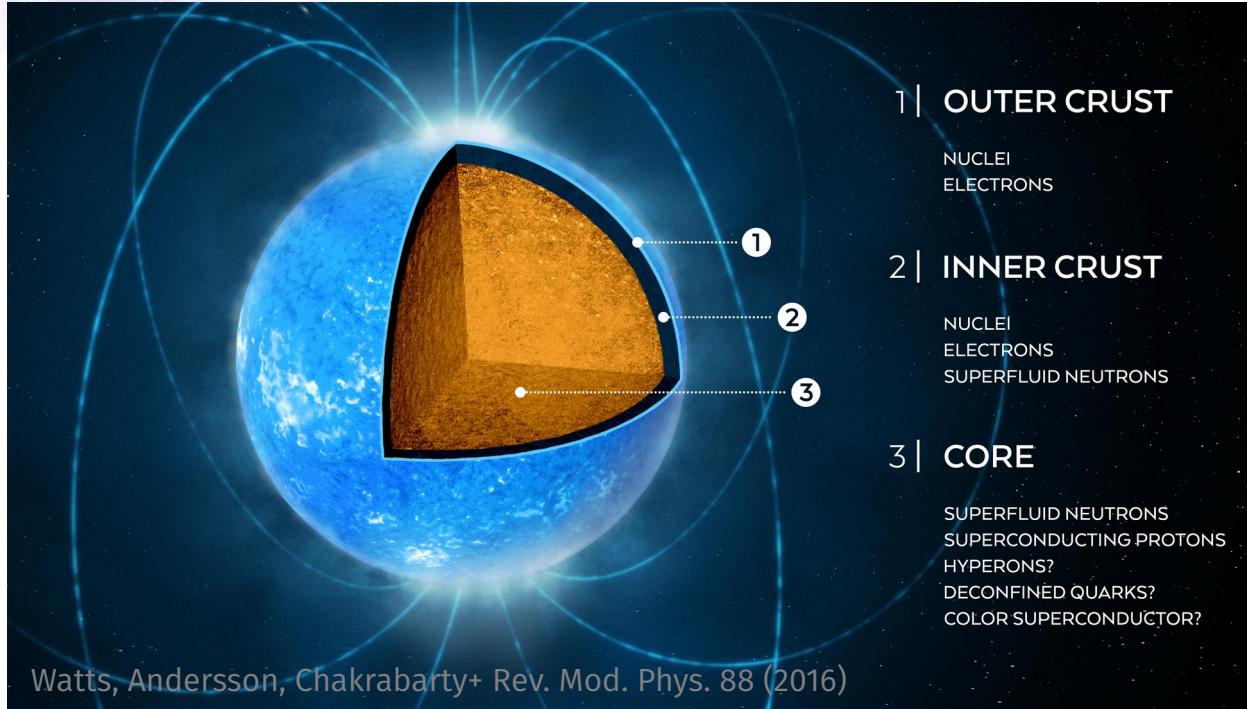


Compressed Baryonic Matter (CBM) experiment

# $T = 0$ is a synthesis of theory and experiment



# What is a Neutron Star?

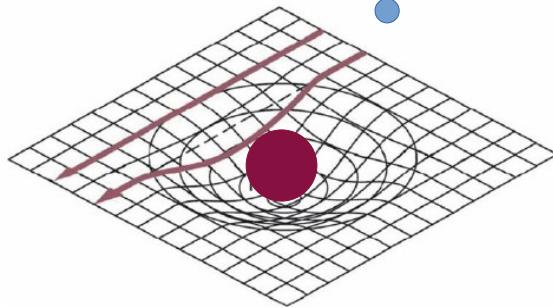


$$\text{Mass } \lesssim 2M_{\odot}, \quad 11 \text{ km} \lesssim R \lesssim 13 \text{ km}, \quad T \lesssim \text{keV} \sim 10^7 \text{ K}.$$

Collapsed remnant of dead stars, held from collapse by **repulsive** nuclear/QCD forces

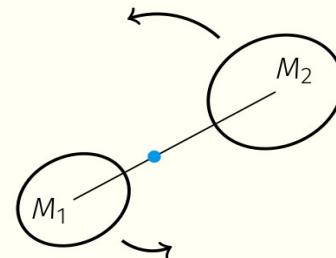
**Cleanest probes** of structure/bulk properties when they are in **binary system with a companion**

# Neutron stars in binaries tell us about structure



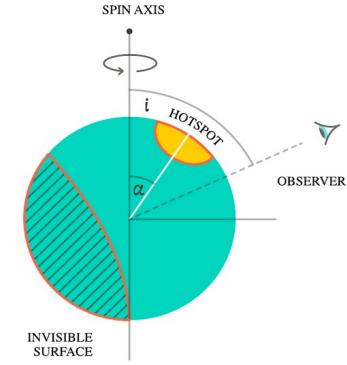
Masses

Demorest, Pennucci, Ransom, Roberts, Hessels. Nature 467 (2010) pp. 1081-1083;  
Antoniadis+ Science 340 (2013) p. 6131;  
Cromartie+ (NANOGrav). Nature Astron. 4.1 (2019).  
E. Fonseca+ Astrophys. J. Lett. 915.1 (2021)



Deformabilities

Abbott+ (LIGO Scientific, Virgo) PRL 119 (2017); PRL 121 (2018); PRX 9 (2019).

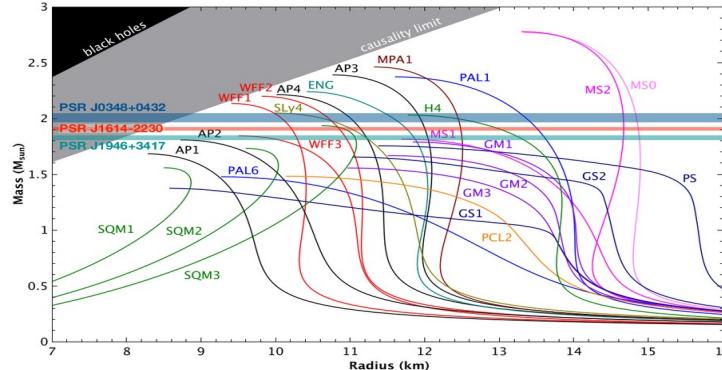


Radii, compactness

Miller+ Astrophys. J. Lett. 918.2 (2021), p. L28.  
Riley+ Astrophys. J. Lett. 918.2 (2021) p. L27.

# Neutron stars in binaries tell us about structure

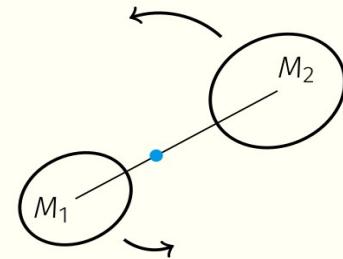
$$M_{\text{TOV}} \geq \begin{cases} 1.97 \pm 0.04 M_{\odot} & \text{PSR J1614-2230} \\ 2.01 \pm 0.04 M_{\odot} & \text{PSR J0348+0432} \\ 2.08 \pm 0.07 M_{\odot} & \text{PSR J0740+6620} \end{cases}$$



Norbert Wex

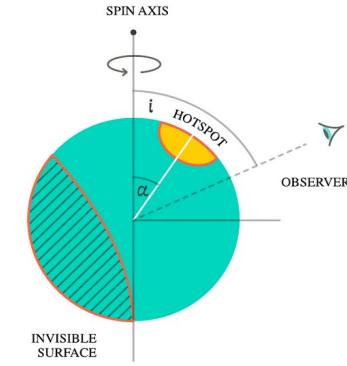
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Demorest, Pennucci, Ransom, Roberts, Hessels. Nature 467 (2010) pp. 1081-1083;  
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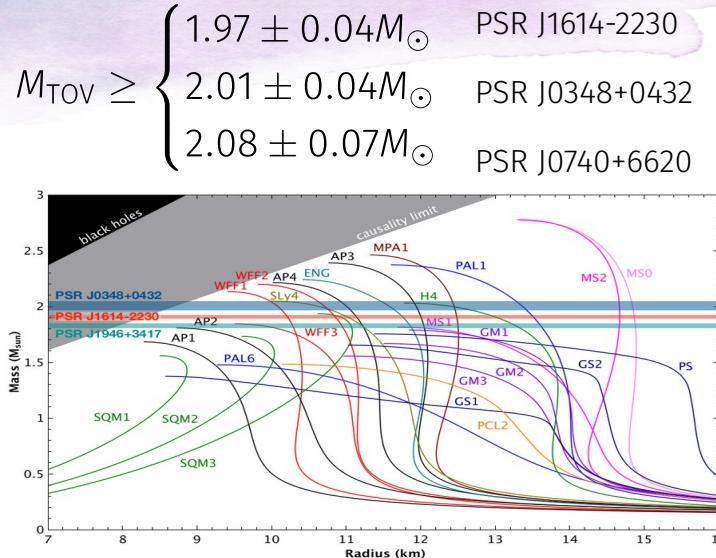
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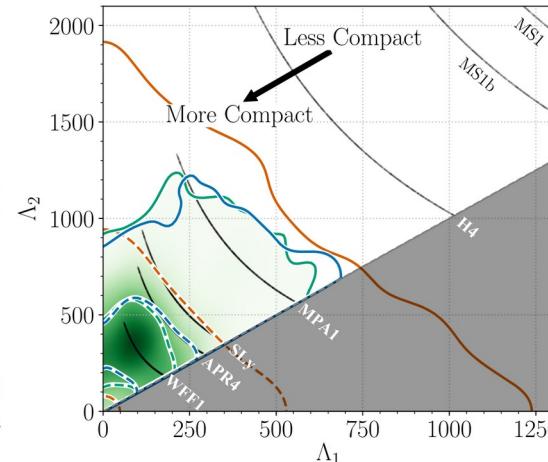


Norbert Wex

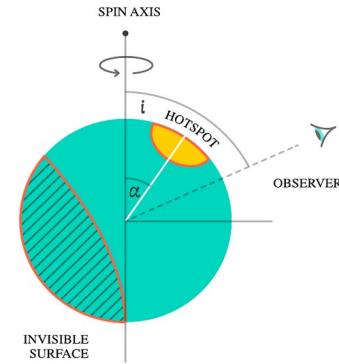
Masses

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 E. Fonseca+ Astrophys. J. Lett. 915.1 (2021)

$$\Lambda(M) \equiv |Q_{ij}/\mathcal{E}_{ij}| M^5$$



Deformabilities



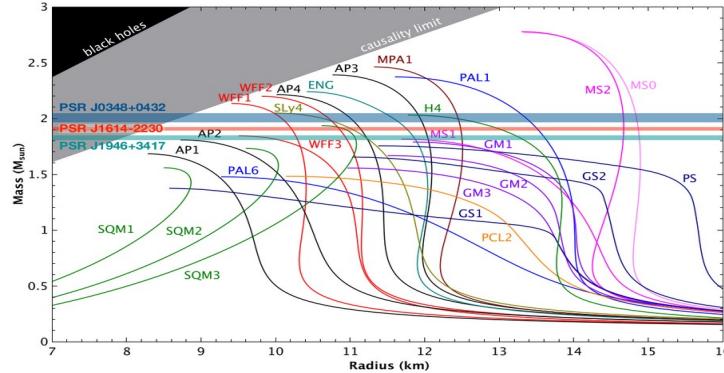
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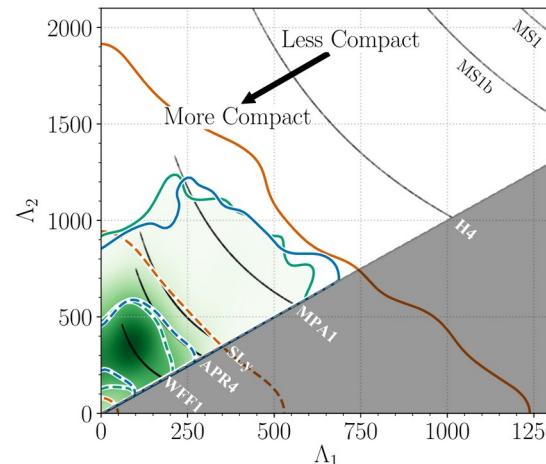


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 E. Fonseca+ Astrophys. J. Lett. 915.1 (2021)

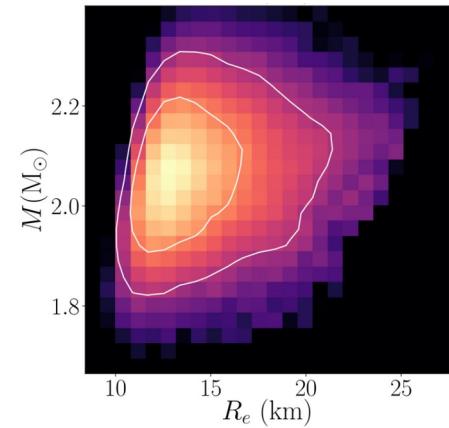
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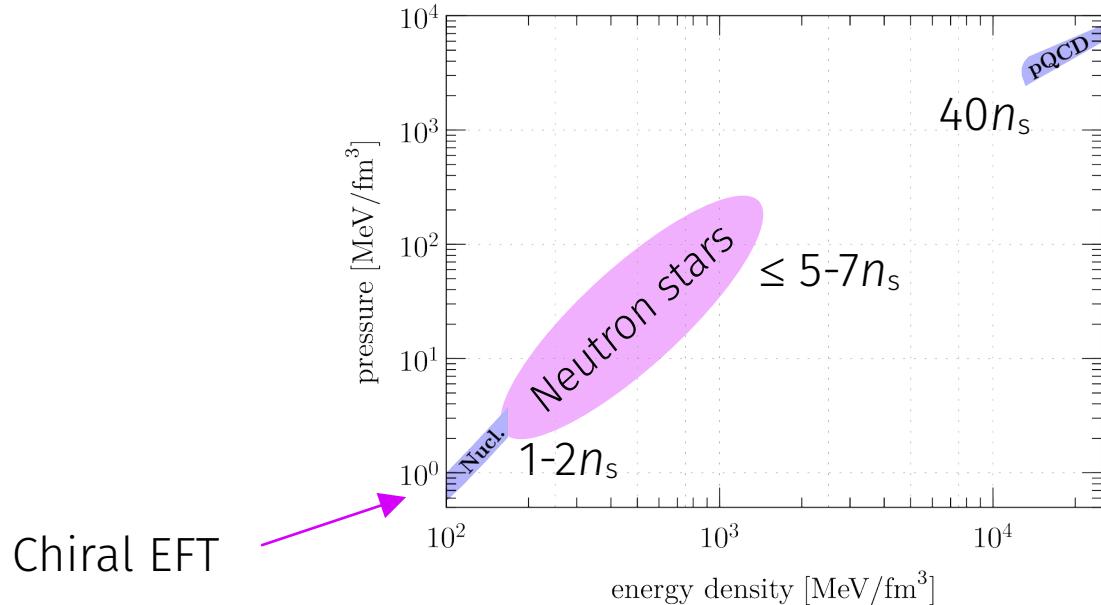
Miller+ Astrophys. J. Lett. 918.2 (2021), p. L28.  
 Riley+ Astrophys. J. Lett. 918.2 (2021) p. L27.



Radii, compactness

Use these data in place of lattice results

# Theory predicts EoS at low and high Density



Kurkela, Romatschke, Vuorinen, PRD 81, 105021 (2010),  
Gorda, Kurkela, Romatschke, Säppi, Vuorinen, PRL 121, 202701 (2018),  
Gorda, Kurkela, Paatelainen, Säppi, Vuorinen, PRL 127, 162003 (2021), PRD 104, 074015 (2021),  
Gorda, Paatelainen, Säppi, Seppänen 2307.08734.

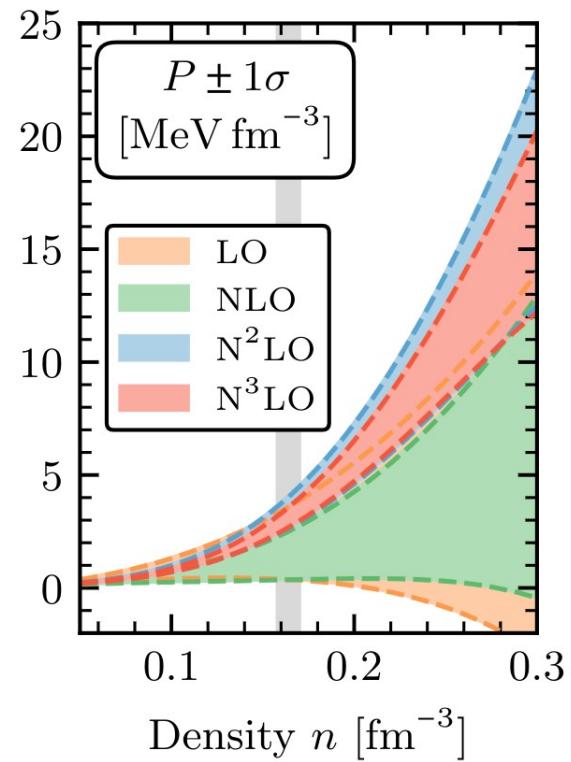
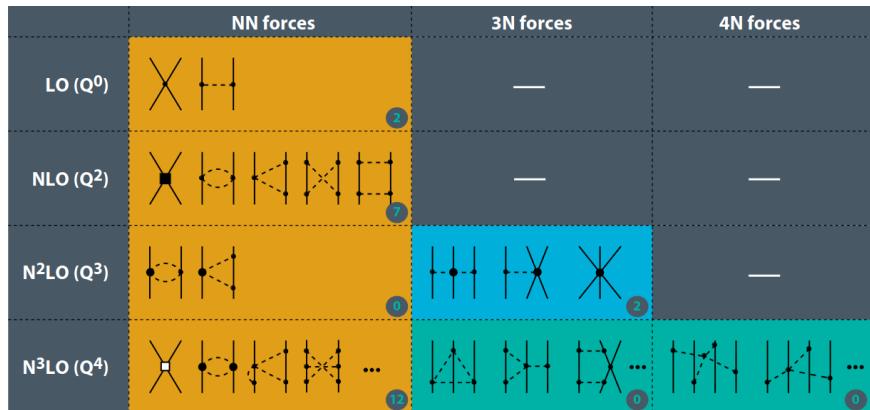
Hebeler, Lattimer, Pethick, Schwenk. *Astrophys. J.* 773 (2013);  
Tews, Krüger, Hebeler, Schwenk, *PRL* 110, 032504 (2013)  
Lynn, Tews, Carlson+ *PRL* 116, 062501 (2016),  
Drischler, Hebeler, Schwenk, *PRL* 122, 042501 (2019),  
Drischler, Furnstahl, Melendez, Phillips, *PRL* 125, 202702 (2020),  
Keller, Hebeler, and Schwenk, *PRL* 130, 072701 (2023).

# Hadronic matter: Chiral EFT

Describes **massive nucleons** interacting via pion exchange and contact interactions. EFT terms dictated by **chiral symmetry**

Calibrated by **nuclear data**

**Uncertainty estimates** by e.g. Gaussian process regression and naturalness arguments



Drischler, Holt, Wellenhofer, Ann.Rev.Nucl.Part.Sci. 71 (2021) 403-432

# Quark matter: perturbative QCD (recent developments)

Describes (nearly) massless quarks, gluons

interacting. Quarks are approximately free, up to  
 $[O(20\%)]$  perturbative corrections

$$\frac{p}{p_0} = 1 + a_1 \alpha_s(\bar{\Lambda}) + a_2 \alpha_s^2(\bar{\Lambda}) + a_3 \alpha_s^3(\bar{\Lambda}) + \dots$$

free quark gas\*

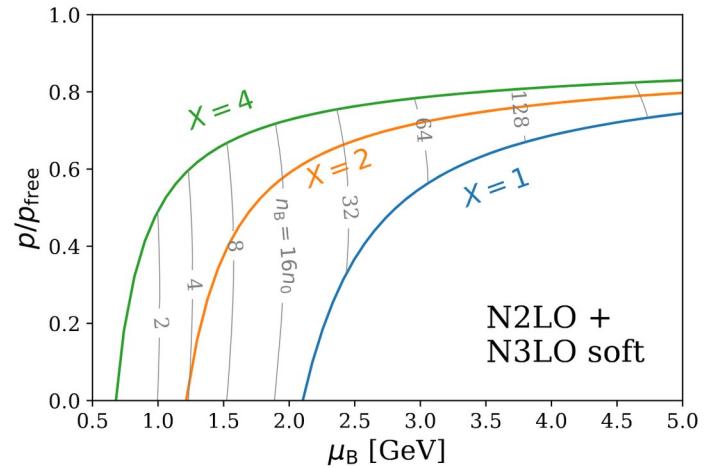
\* $(p_0 \propto \mu^4, p_{\text{pairing}} \propto \mu^2 \Delta^2)$

Freedman, McLerran, PRD 16 (1977)

Gorda, Kurkela, Romatschke, Säppi, Vuorinen, PRL 121 (2018) – ongoing

Alford, Schmitt, Rajagopal, Schäfer, Rev. Mod. Phys. 80, 1455 (2008)

Calibrated by collider data.



Gorda, Kurkela, Paatelainen, Säppi, Vuorinen,  
PRL 127, 162003 (2021), PRD 104, 074015 (2021),

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Alford, Schmitt, Rajagopal, Schäfer, Rev. Mod. Phys. 80, 1455 (2008)

Calibrated by collider data.

In last two years, the **full structure of the N3LO pressure** computation has been made clear

Gorda, Kurkela, Paatelainen, Säppi, Vuorinen, PRL 127, 162003 (2021), PRD 104, 074015 (2021),  
Gorda, Paatelainen, Säppi, Seppänen 2307.08734

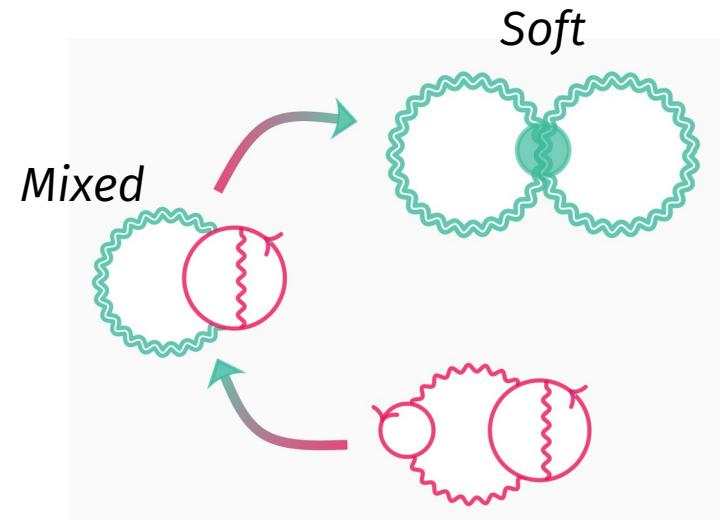
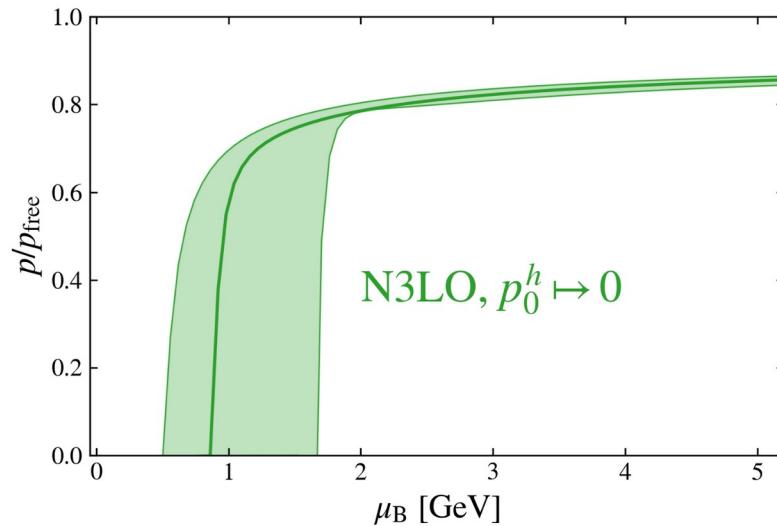


Figure: Saga Säppi

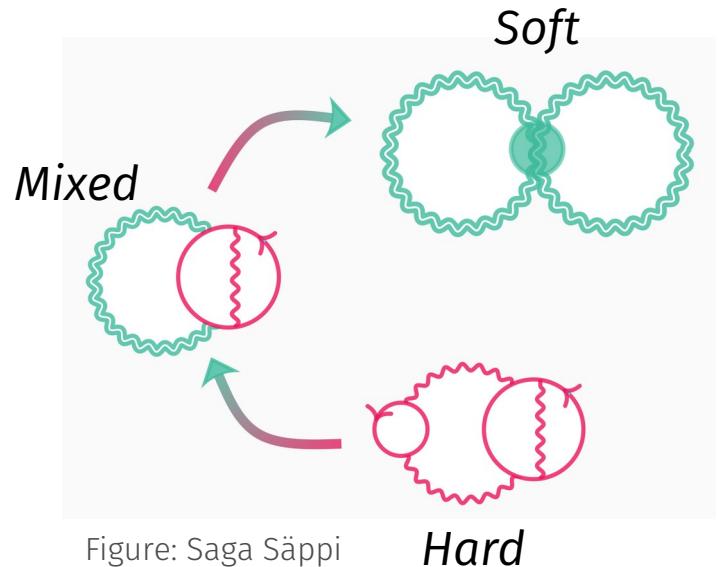
**Soft:** 2 interacting gluons **screened at LO**  
**Mixed:** 1 gluon **screened at NLO**  
**Hard:** gluons are **unscreened**

# Quark matter: perturbative QCD (recent developments)



Gorda, Paatelainen, Säppi, Seppänen 2307.08734

**Soft** and **Mixed** contributions now computed.  
Result is extremely **well converged**



**Soft:** 2 interacting gluons **screened at LO**  
**Mixed:** 1 gluon **screened at NLO**  
**Hard:** gluons are **unscreened**

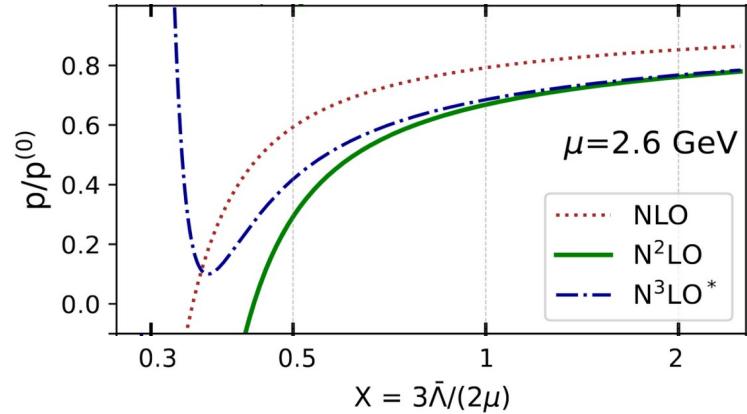
# Quark matter: perturbative QCD (recent developments)

Recent Bayesian Uncertainty estimation

$$\frac{p}{p_0} = 1 + a_1 \alpha_s(\bar{\Lambda}) + a_2 \alpha_s^2(\bar{\Lambda}) + a_3 \alpha_s^3(\bar{\Lambda}) + \dots$$

Scale-variation  
uncertainty

Missing higher-order  
(truncation) uncertainty

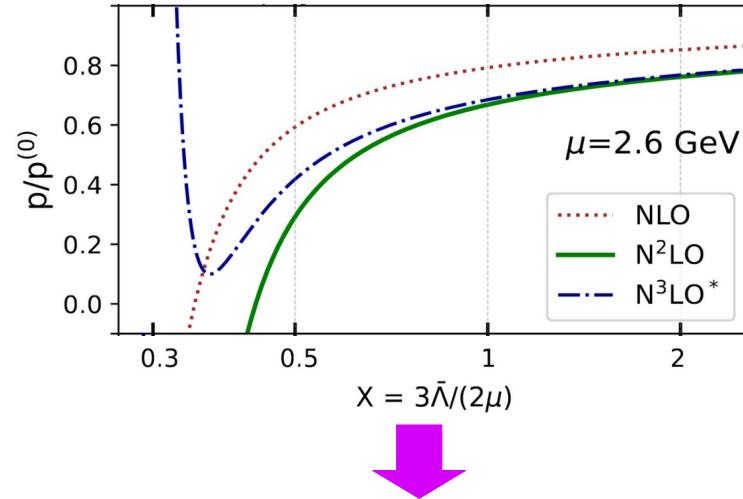


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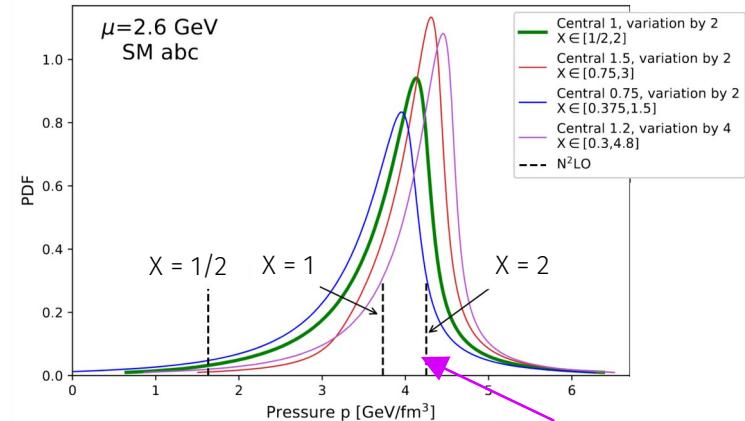
Scale-variation uncertainty      Missing higher-order (truncation) uncertainty



**Machine-learning based** Bayesian interpretation of these uncertainties. Perturbative series modeled as draws from a statistical model of convergent series, trained with available terms

Cacciari & Houdeau, JHEP 09, (2011), M. Bonvini, Eur. Phys. J. C 80, 989 (2020), Duhr, Huss, Mazeliauskas, Szafron, JHEP 122, (2021)

Gorda, Komoltsev, Kurkela, Mazeliauskas, JHEP 06 (2023)



Favors **better converged  $X$**

# Chiral EFT + pQCD + Thermodynamics constrain extreme EoSs

## 1. Stability

$$\partial_\mu^2 \Omega(\mu) \leq 0 \implies \partial_\mu n(\mu) \geq 0$$

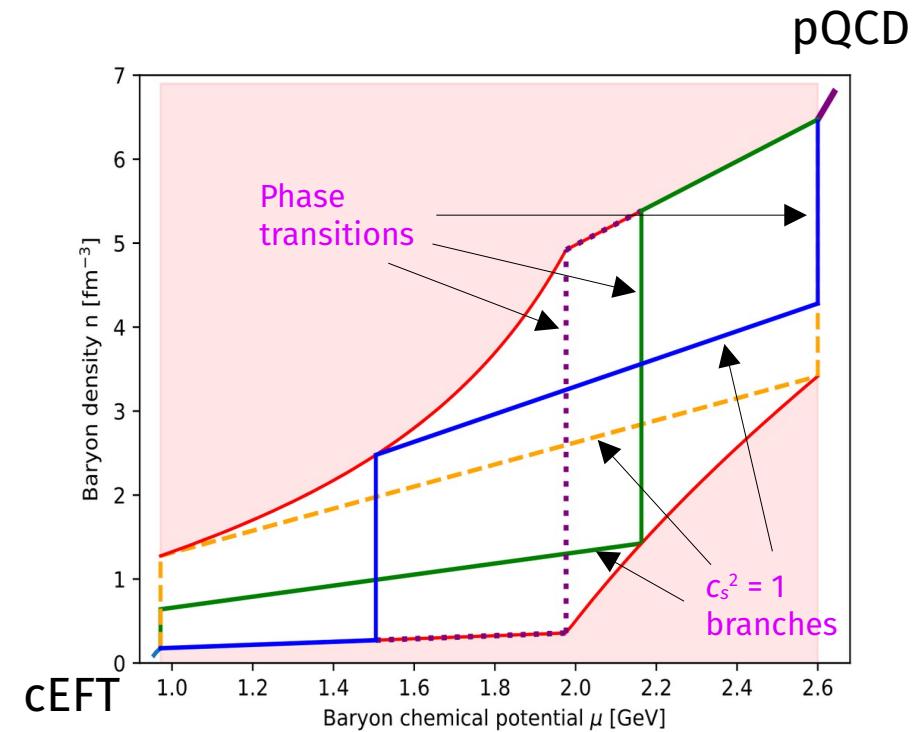
## 2. Causality

$$c_s^{-2} = \frac{\mu}{n} \frac{\partial n}{\partial \mu} \geq 1 \implies \partial_\mu n(\mu) \geq \frac{n}{\mu}$$

## 3. Consistency

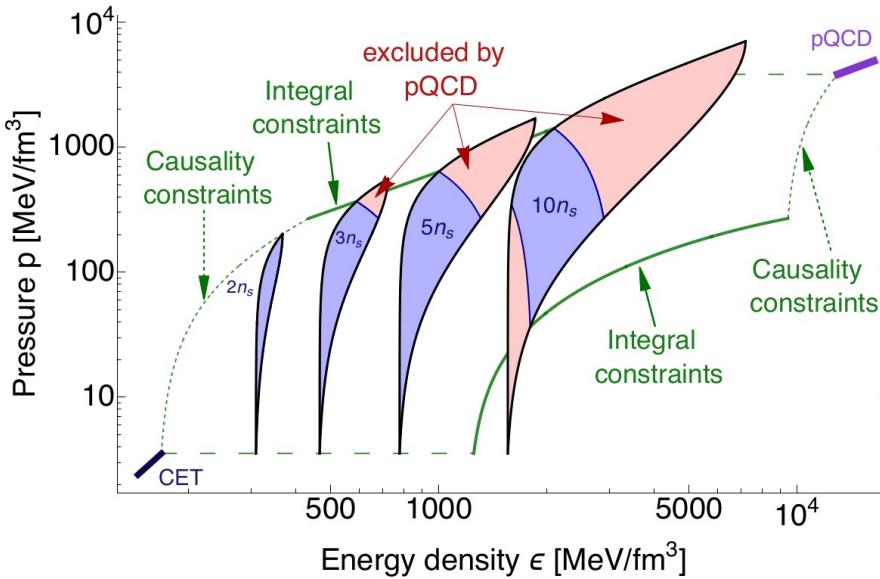
$$\int_{\mu_{\text{CET}}}^{\mu_{\text{QCD}}} d\mu n(\mu) = p_{\text{QCD}} - p_{\text{CET}} \quad \text{Fixed!}$$

“integral constraints”



Komoltsev and Kurkela, PRL 128 (2022)

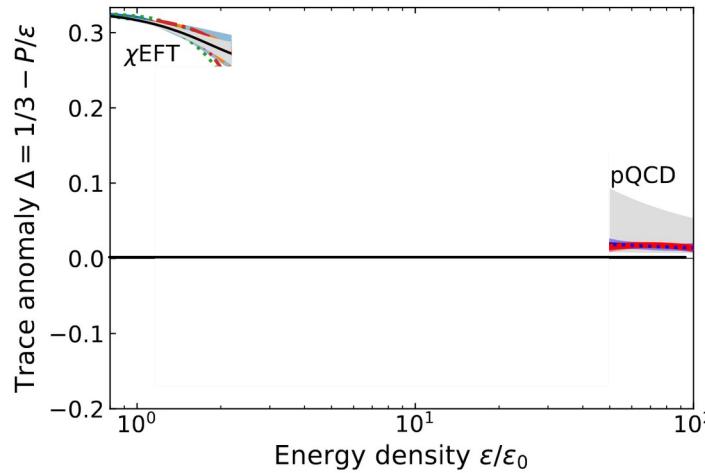
# Chiral EFT + pQCD + Thermodynamics constrain extreme EoSs



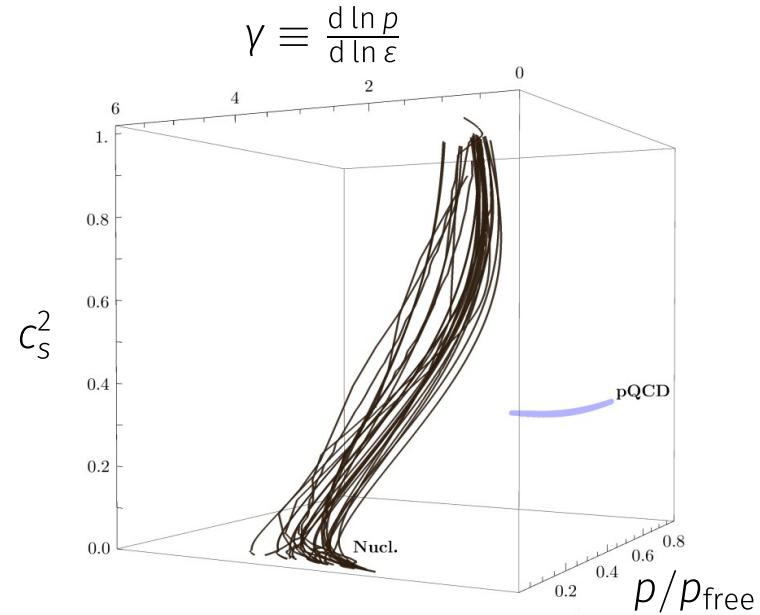
Region of  $(\epsilon, p)$  at fixed  $n$  constrained by general principles

# Hadronic and Quark matter are different

Quark matter is **approximately conformal**, hadronic matter is **non-conformal**. This leads to different thermodynamics:



ADAPTED from Fujimoto, Fukushima, McLerran, Praszalowicz,  
PRL 129 (2022) 25, 252702

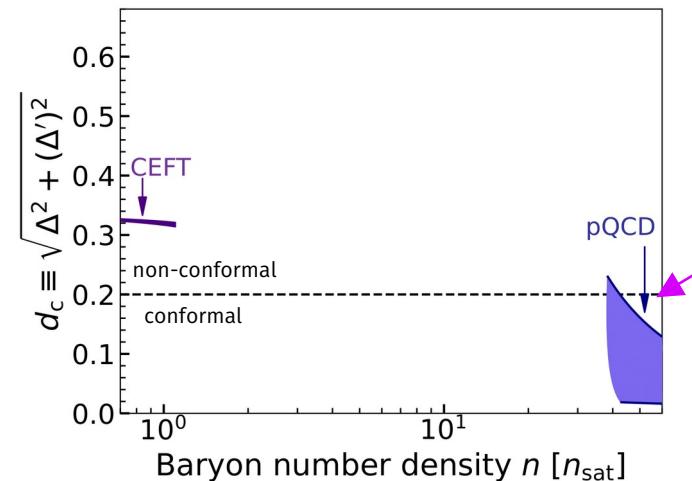


Annala, Gorda, Kurkela, Näättilä, Vuorinen Nat. Phys. 16  
(2020)

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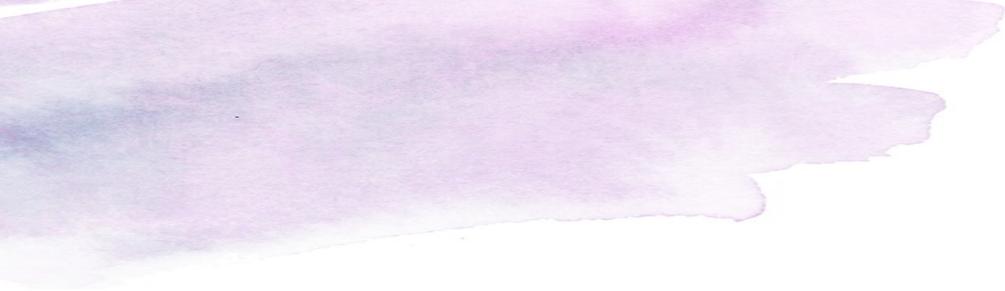
Quark matter is **approximately conformal**, hadronic matter is **non-conformal**. This leads to different thermodynamics:

Combine **trace anomaly**  $\Delta$  and **its rate of change**  
 $\Delta' \equiv d \ln \Delta / d \ln \varepsilon$ ,  
so not purely local measure



Average of chiral EFT and pQCD as dividing line

Annala, Gorda, Hirvonen, Komoltsev, Kurkela, Näyttälä, Vuorinen  
2303.11356



Putting everything together...

# Bayesian EoS inference setup

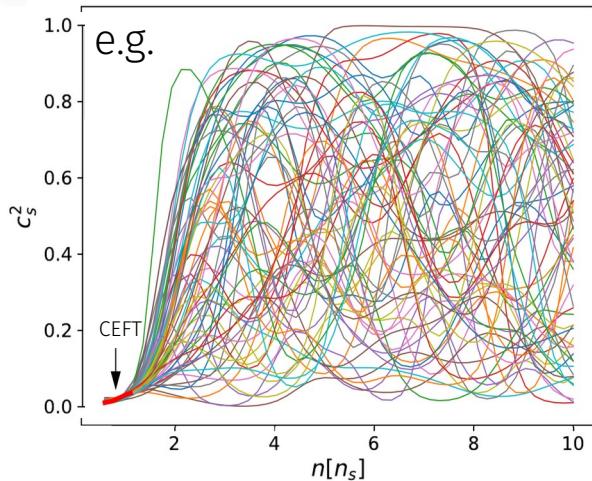
The diagram illustrates the Bayesian inference process for an Equation of State (EoS). It features a central equation:  $P(\text{EoS} | \text{data}) = \frac{P(\text{EoS})P(\text{data} | \text{EoS})}{P(\text{data})}$ . Two arrows point to the terms in the numerator: one from the text "EoS Prior" above the first term, and another from the text "Likelihood of EoS given data + theory" above the second term.

$$P(\text{EoS} | \text{data}) = \frac{P(\text{EoS})P(\text{data} | \text{EoS})}{P(\text{data})}$$

EoS Prior

Likelihood of EoS  
given data + theory

# Bayesian EoS inference setup



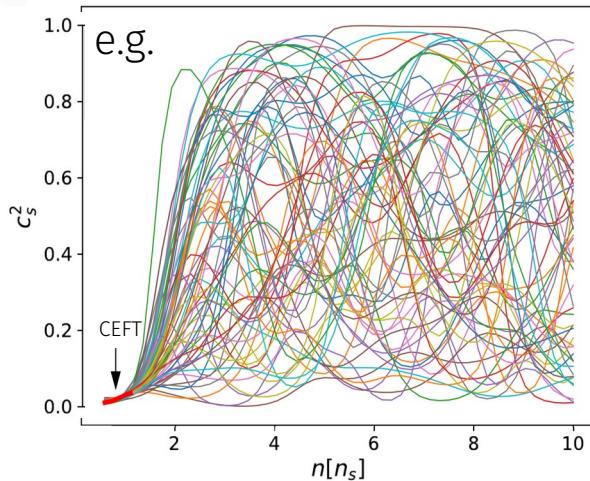
Following Landry & Essick Phys. Rev. D 99 (2019)

Gorda, Komoltsev, Kurkela, ApJ 950 (2023)

Likelihood of EoS  
given data + theory

$$P(\text{EoS} \mid \text{data}) = \frac{P(\text{EoS})P(\text{data} \mid \text{EoS})}{P(\text{data})}$$

# Bayesian EoS inference setup

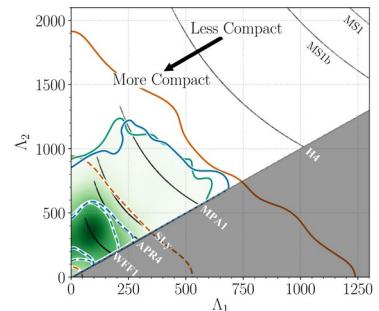
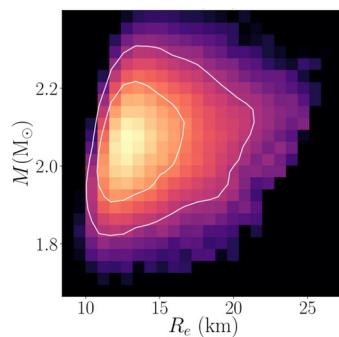
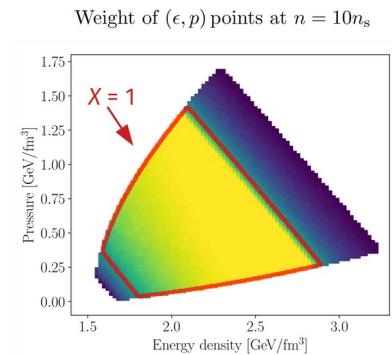


Following Landry & Essick Phys. Rev. D 99 (2019)

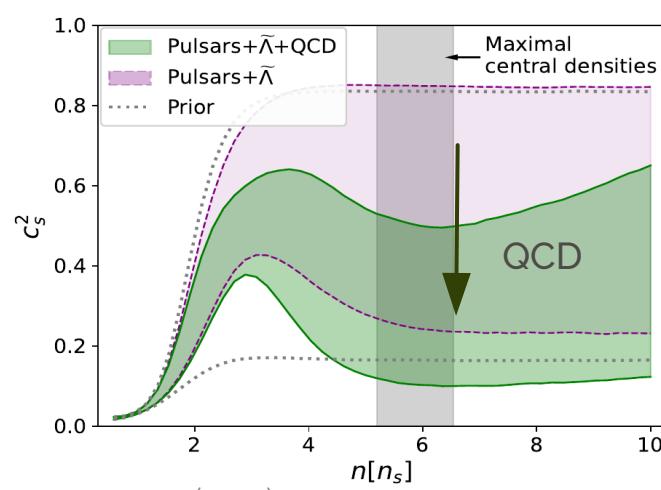
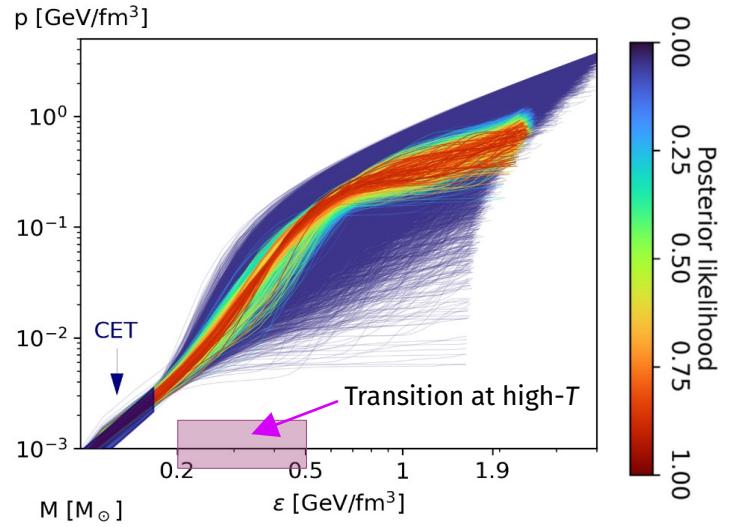
Gorda, Komoltsev, Kurkela, ApJ 950 (2023)

$$P(\text{EoS} | \text{data}) = \frac{P(\text{EoS})P(\text{data} | \text{EoS})}{P(\text{data})}$$

Likelihood of EoS  
given data + theory



# Bayesian EoS inference results 1a



Gorda, Komoltsev, Kurkela, ApJ 950 (2023)

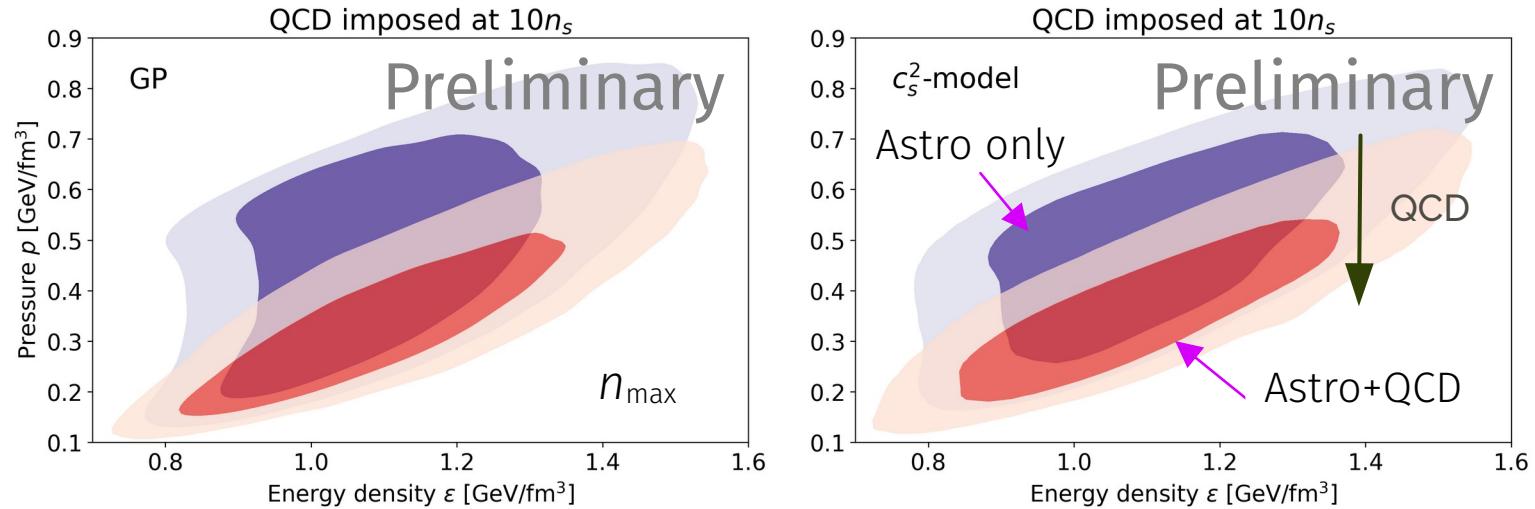
Stiff→soft transition, with softening driven by QCD input

(Note no assumption about extremeness of EoS extension beyond  $10n_s$  is used!)

See also

Annala, Gorda, Kurkela, Näättilä, Vuorinen Nat. Phys. 16 (2020),  
Altiparmak, Ecker, Rezzolla, ApJ.Lett. 939 (2022);  
Ecker & Rezzolla, ApJ.Lett. 939 (2022); Fujimoto, Fukushima, McLellan,  
Praszalowicz, PRL 129 (2022); Marczenko, McLellan, Redlich, Sasaki, PRC  
107 (2023);

# Bayesian EoS inference results 1b

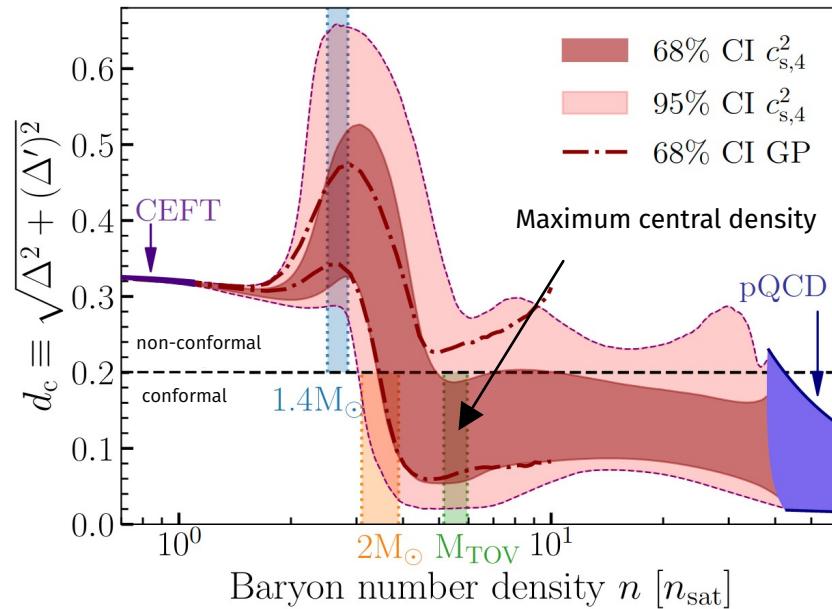


Ongoing with Gorda, Komoltsev, Kurkela, Margueron, Somasundaram, Tews

Softening **does not depend strongly on prior** between  $n_{\max}$  and  $10n_s$

Models from  
Somasundaram, Tews, Margueron Phys. Rev. C 107, 025801  
(2023), Phys.Rev.C 107, L052801 (2023),  
Gorda, Komoltsev, Kurkela, ApJ 950 (2023).

# Bayesian EoS inference results 2



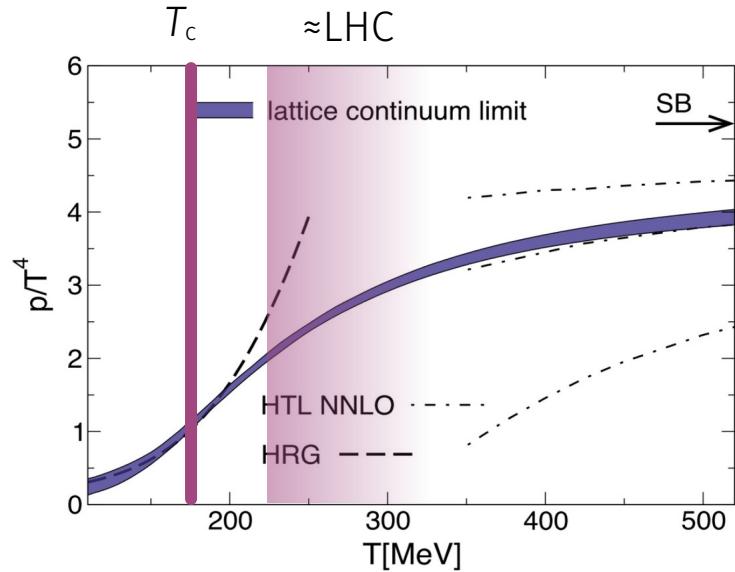
Annala, Gorda, Hirvonen, Komoltsev, Kurkela, Näyttälä, Vuorinen 2303.11356

Clear **non-conformal→conformal transition** within the cores of **stable neutron stars**

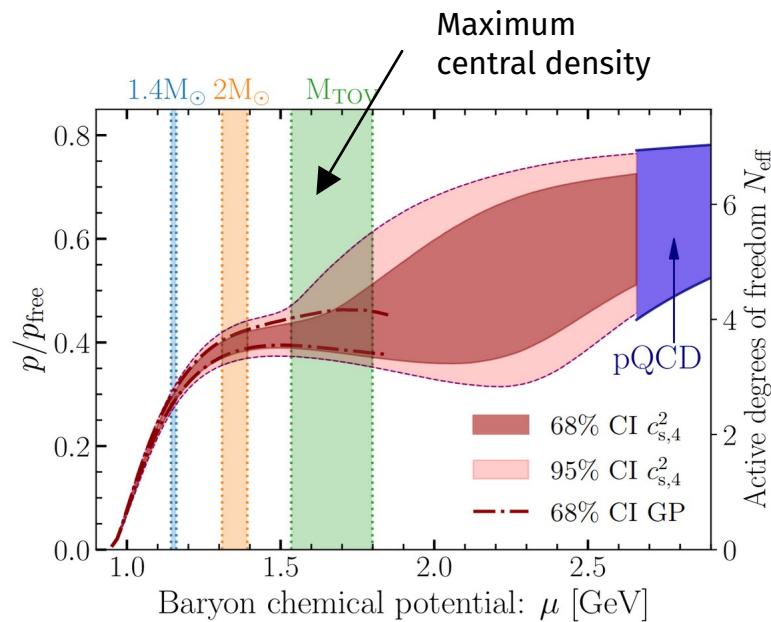
See also

Annala, Gorda, Kurkela, Näyttälä, Vuorinen Nat. Phys. 16 (2020),  
Altiparmak, Ecker, Rezzolla, ApJ.Lett. 939 (2022);  
Ecker & Rezzolla, ApJ.Lett. 939 (2022); Fujimoto, Fukushima, McLellan,  
Praszalowicz, PRL 129 (2022); Marczenko, McLellan, Redlich, Sasaki, PRC  
107 (2023);

# Bayesian EoS inference results 3



Number of degrees of freedom **consistent with deconfined quark matter**

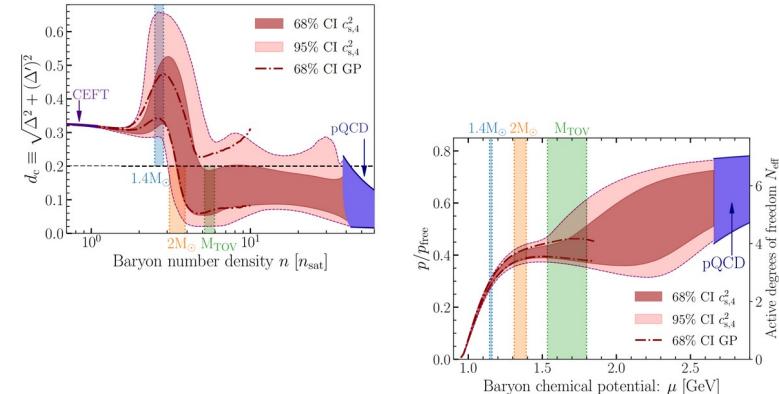
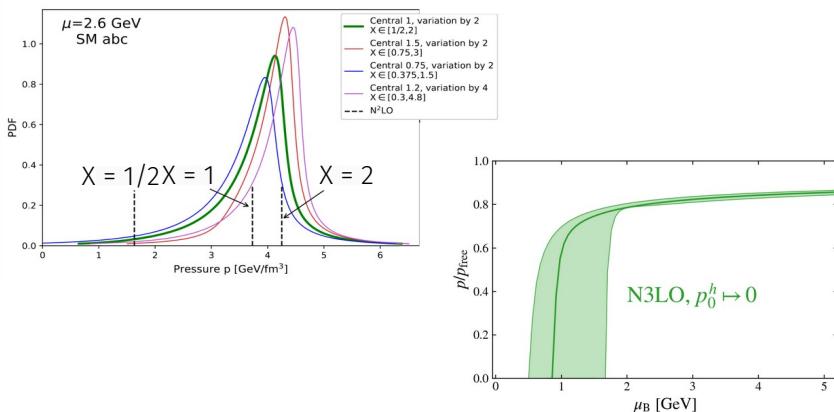


# Takeaways

- ✓ Robust thermodynamic evidence for hadronic $\rightarrow$ quark transition probed within the most massive neutron stars.

Evidence for cold quark matter created in nature.

- ✓ Astrophysics + nuclear-theory + particle-theory input are all essential in this conclusion



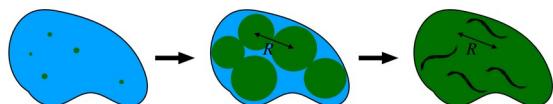
- ✓ Improvements in pQCD calculations and analysis
  - Statistical analysis of pQCD truncation errors favors higher pressures, which are better converged
  - Soft + mixed pQCD EoS tightly converged; hard sector remains

# What's next?

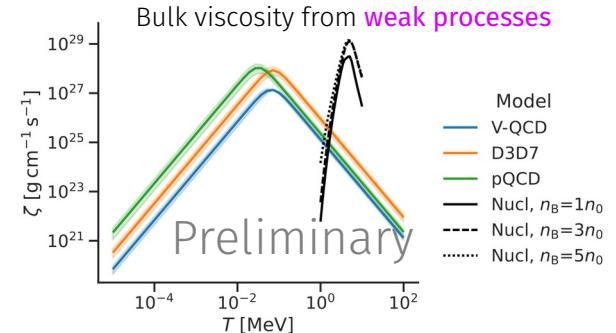
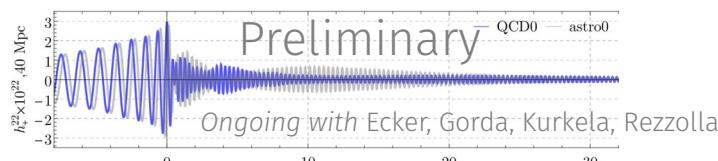
? Other lines of evidence of deconfinement/quark behavior?

- At high- $T$ , thermodynamics is just one approach
- Ongoing work on **transport** calculations

Ongoing with Cruz Rojas, Gorda, Hoyos, Jokela, Järvinen, Kurkela, Paatelainen, Säppi, Vuorinen



Casalderrey-Solana, Mateos, Sanchez-Garitaonandia 2210.03171



? Robust (post-)merger signals of deconfinement?

- MHz gravitation waves from bubble nucleation in remnant?
- Post-merger frequency spectra shift from QCD input? (ongoing)

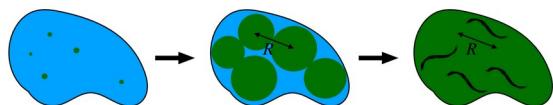
? Can similar analyses be performed at nonzero  $T$ , to apply statistical EoS inference throughout the phase diagram? (See Jamie Karthein's talk on Thursday)

# What's next?

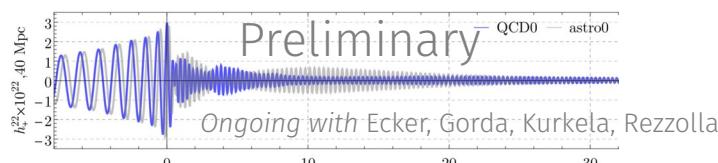
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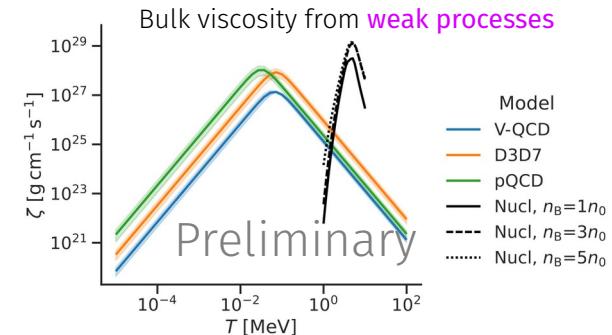
Ongoing with Cruz Rojas, Gorda, Hoyos, Jokela, Järvinen, Kurkela, Paatelainen, Säppi, Vuorinen



Casalderrey-Solana, Mateos, Sanchez-Garitaonandia 2210.03171



? Can **similar analyses** be performed at **nonzero  $T$** , to apply statistical EoS inference throughout the phase diagram? (See Jamie Karthein's talk on Thursday)



? Robust **(post-)merger signals** of deconfinement?

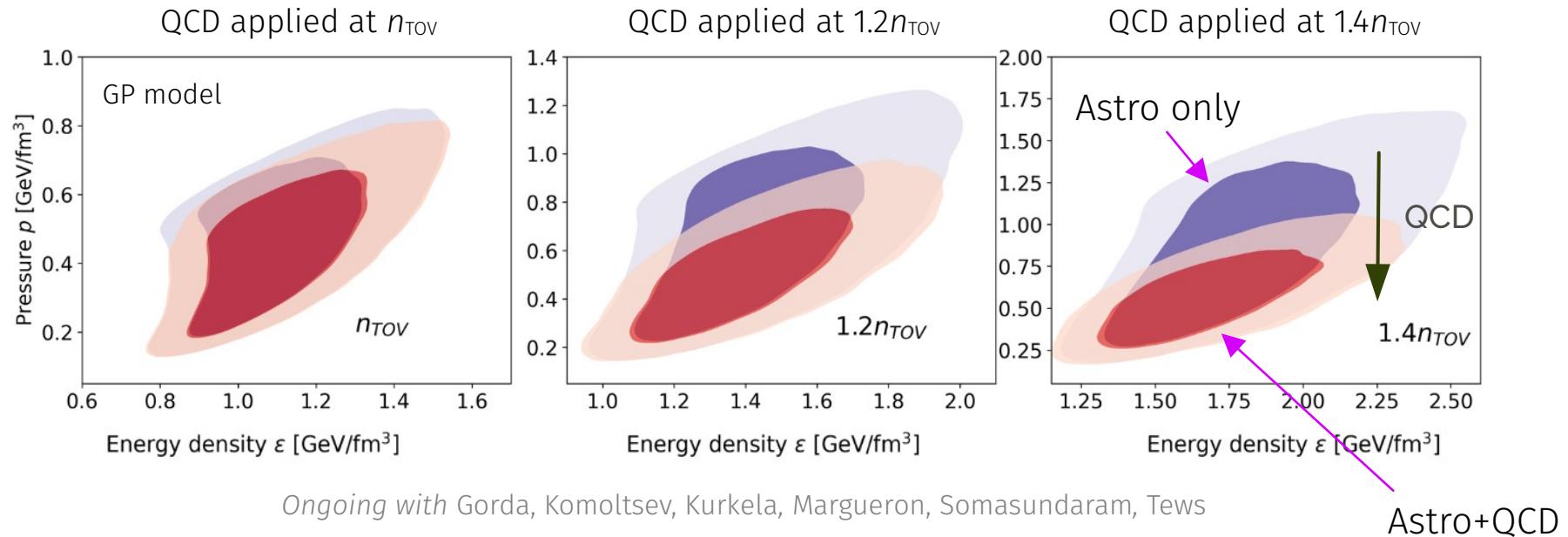
- MHz gravitation waves from bubble nucleation in remnant?
- Post-merger frequency spectra shift from QCD input? (ongoing)

Thanks for your attention!



*Here there be details...*

# Softening must happen near highest densities in NSs

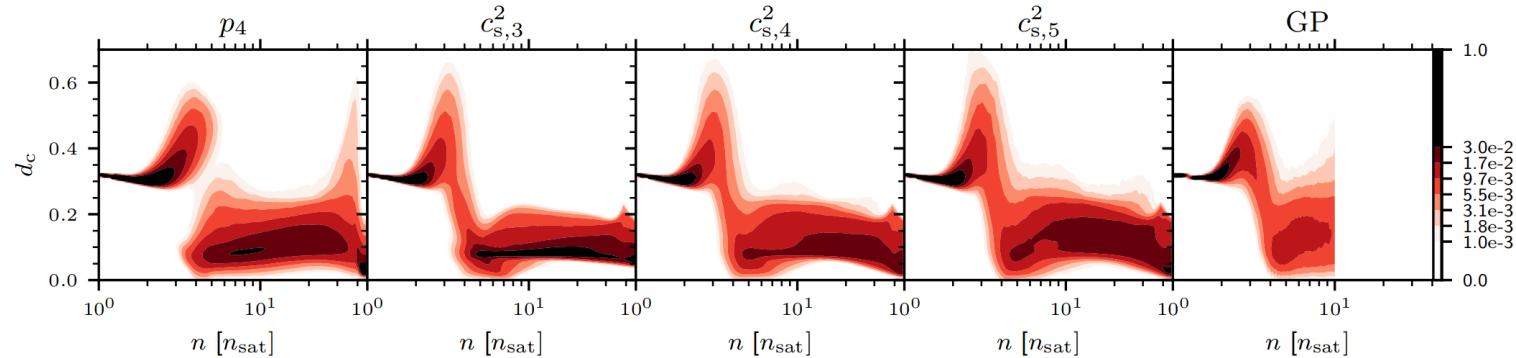


**Maximally general construction** still shows strong softening just beyond  $n_{TOV}$

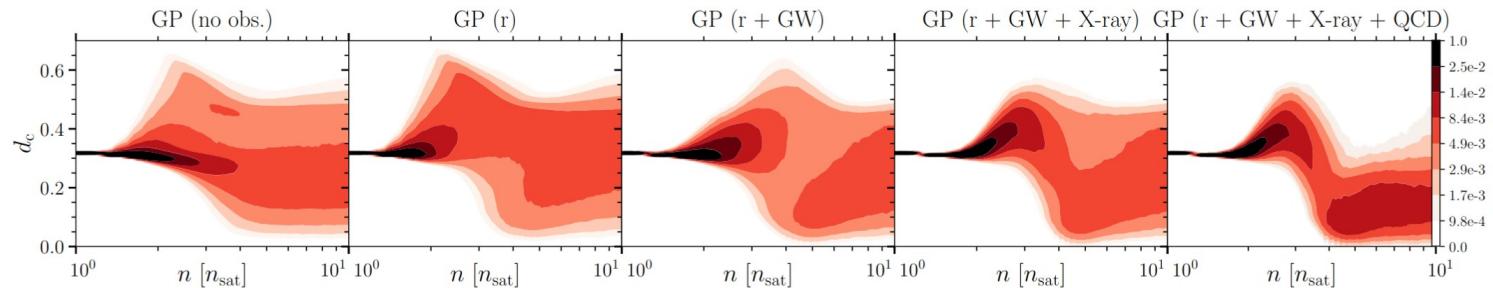
Compare to  
Somasundaram, Tews, Margueron Phys.Rev.C 107, L052801 (2023),  
Essick, Legred, Chatzioannou, Han, Landry Phys.Rev.D 108 (2023) 4,  
043013, Brandes, Weise, Kaiser 2306.06218.

# Dissecting conformal analysis

Robust to interpolants:



X-ray, QCD input very important:



# The $abc$ model of convergent series

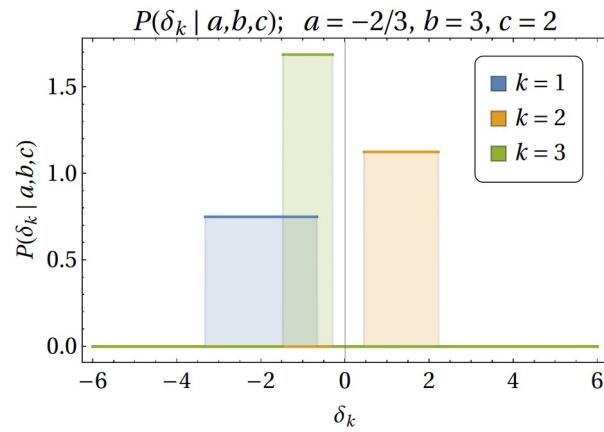
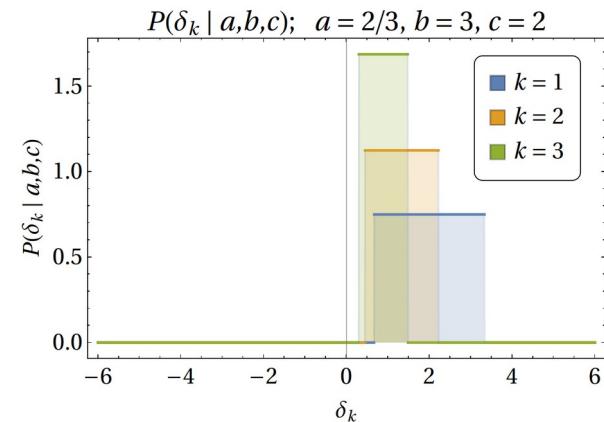
Normalize sequence by LO term

$$\delta_k \equiv \frac{\alpha^k O_k}{O_0} \quad (\delta_0 = 1)$$

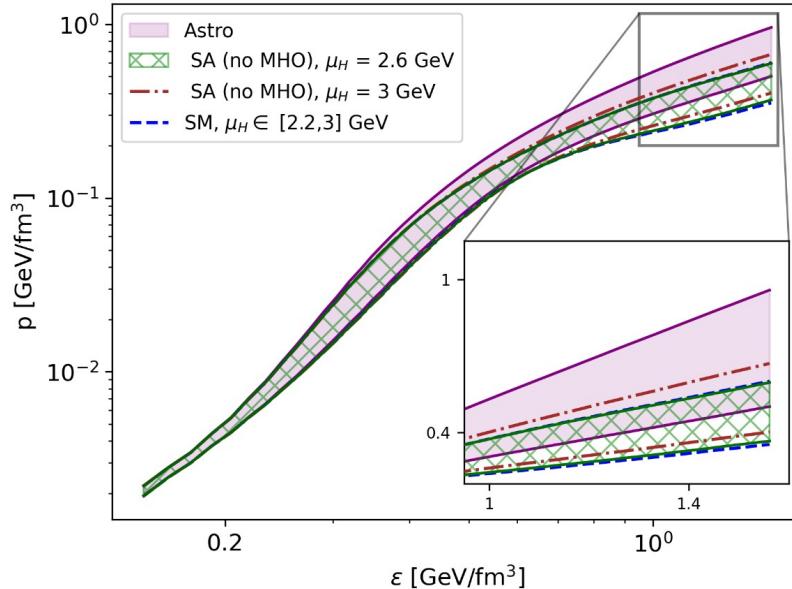
Model assumes  $\delta_k$  is bounded by some geometric series defined by  $(a, b, c)$

$$(-c + b)a^k < |\delta_k| < (c + b)a^k,$$

Flat likelihoods taken for  $\delta_k$  satisfying this inequality. (\*Model also specifies a prior for  $a, b, c$  which favor smaller values of  $|a|, b, c$ )



# Details of statistical pQCD treatment don't affect NS inference



Gorda, Komoltsev, Kurkela, Mazeliauskas JHEP 06 (2023)

# The HARD diagrams

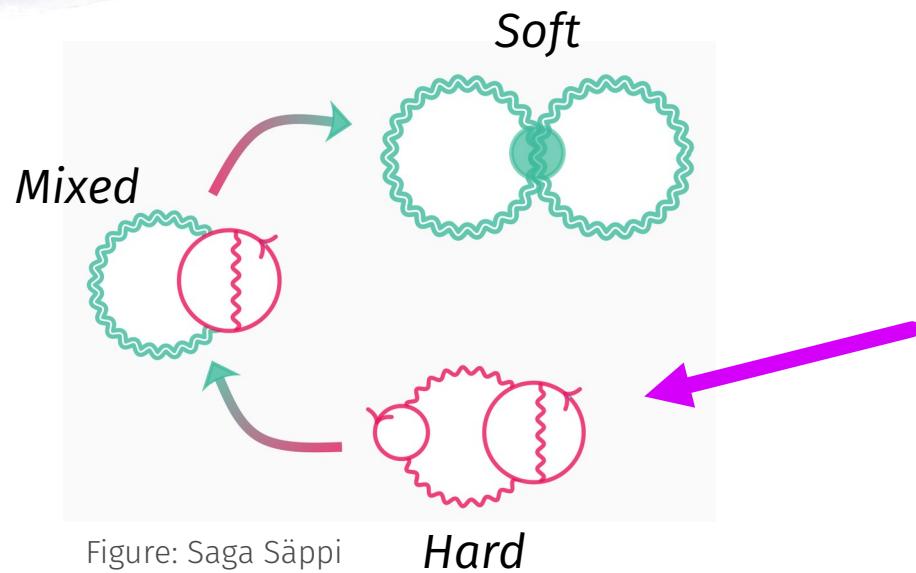


Figure: Saga Säppi

**Soft:** 2 interacting gluons **screened at LO**  
**Mixed:** 1 gluon **screened at NLO**  
**Hard:** gluons are **unscreened**

$$= \text{(diagram 1)} + \text{(diagram 2)} + \text{(diagram 3)} + \text{(diagram 4)} + \text{(diagram 5)} + \text{(diagram 6)}$$
$$+ \text{(diagram 7)} + \text{(diagram 8)} + \text{(diagram 9)} + \text{(diagram 10)} + \text{(diagram 11)} + \text{(diagram 12)}$$
$$+ \text{(diagram 13)} + \text{(diagram 14)} + \text{(diagram 15)} + \text{(diagram 16)} + \text{(diagram 17)} + \text{(diagram 18)}$$
$$+ \text{(diagram 19)} + \text{(diagram 20)} + \text{(diagram 21)} + \text{(diagram 22)} + \text{(diagram 23)} + \text{(diagram 24)}$$
$$+ \text{(diagram 25)} + \text{(diagram 26)} + \text{(diagram 27)} + \text{(diagram 28)} + \text{(diagram 29)} + \text{(diagram 30)}$$
$$+ \text{(diagram 31)} + \text{(diagram 32)} + \text{(diagram 33)} + \text{(diagram 34)} + \text{(diagram 35)} + \text{(diagram 36)}$$
$$+ \text{(diagram 37)} + \text{(diagram 38)} + \text{(diagram 39)} + \text{(diagram 40)} + \text{(diagram 41)} + \text{(diagram 42)}$$
$$+ \text{(diagram 43)} + \text{(diagram 44)} + \text{(diagram 45)} + \text{(diagram 46)} + \text{(diagram 47)} + \text{(diagram 48)}$$
$$+ \text{(diagram 49)} + \text{(diagram 50)} + \text{(diagram 51)} + \text{(diagram 52)} + \text{(diagram 53)} + \text{(diagram 54)}$$
$$+ \text{(diagram 55)} + \text{(diagram 56)} + \text{(diagram 57)} + \text{(diagram 58)} + \text{(diagram 59)} + \text{(diagram 60)}$$
$$+ \text{(diagram 61)} + \text{(diagram 62)} + \text{(diagram 63)} + \text{(diagram 64)} + \text{(diagram 65)} + \text{(diagram 66)}$$
$$+ \text{(diagram 67)} + \text{(diagram 68)} + \text{(diagram 69)} + \text{(diagram 70)} + \text{(diagram 71)} + \text{(diagram 72)}.$$