QCD at Finite Temperature and Density - Equation of State

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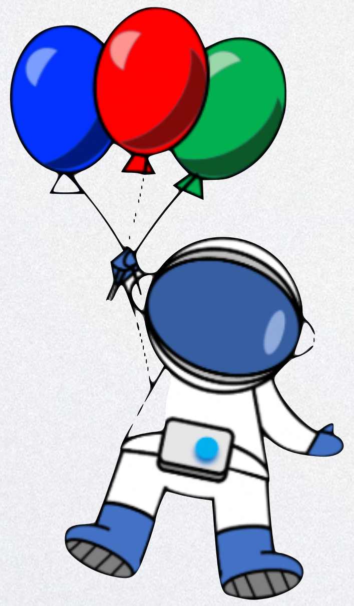
NSF ASCEND Postdoctoral Fellow



Quark Matter 2023, Houston

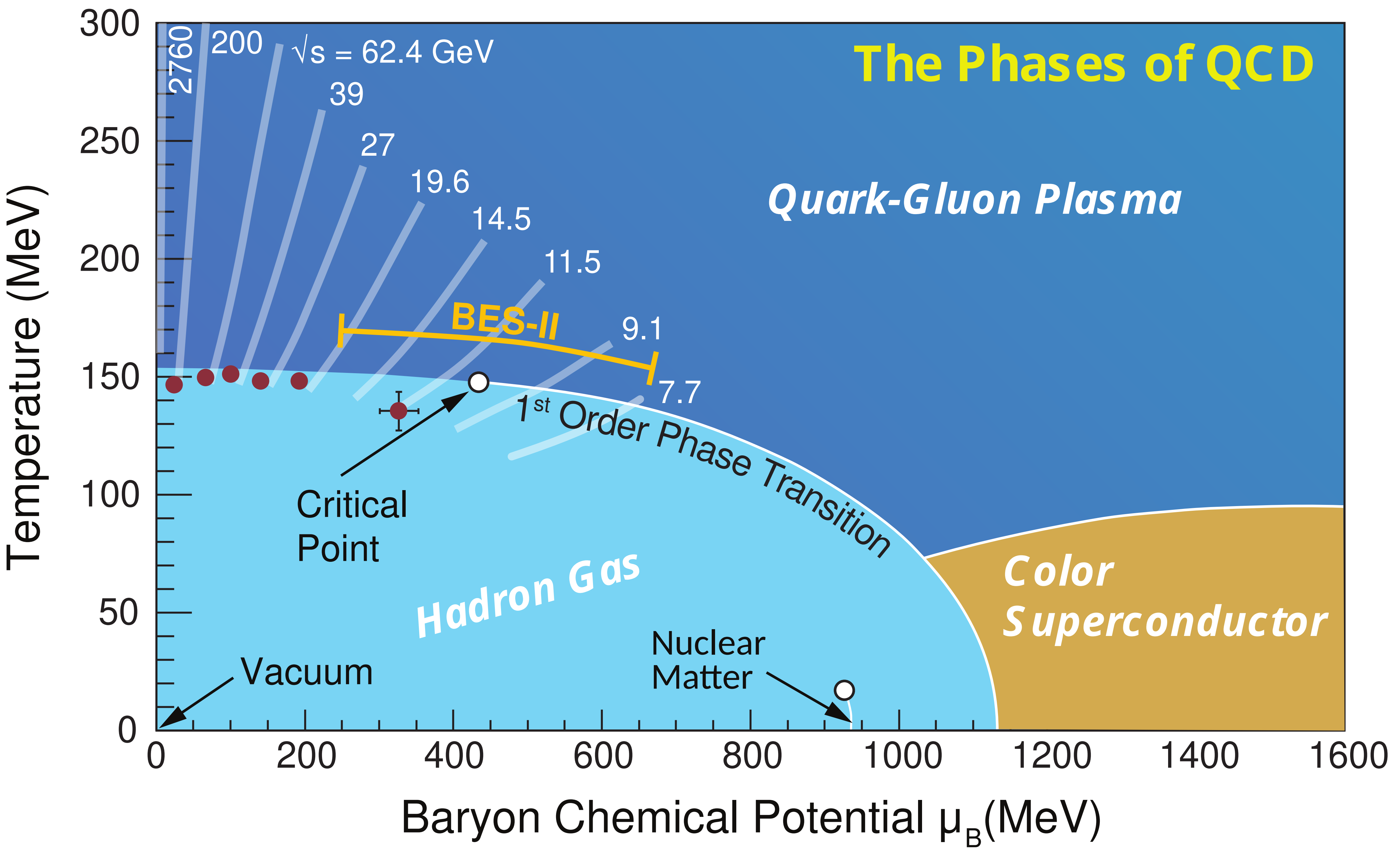
- Familiar QCD phase diagram shows the features of strongly interacting matter under conditions of heat & compression

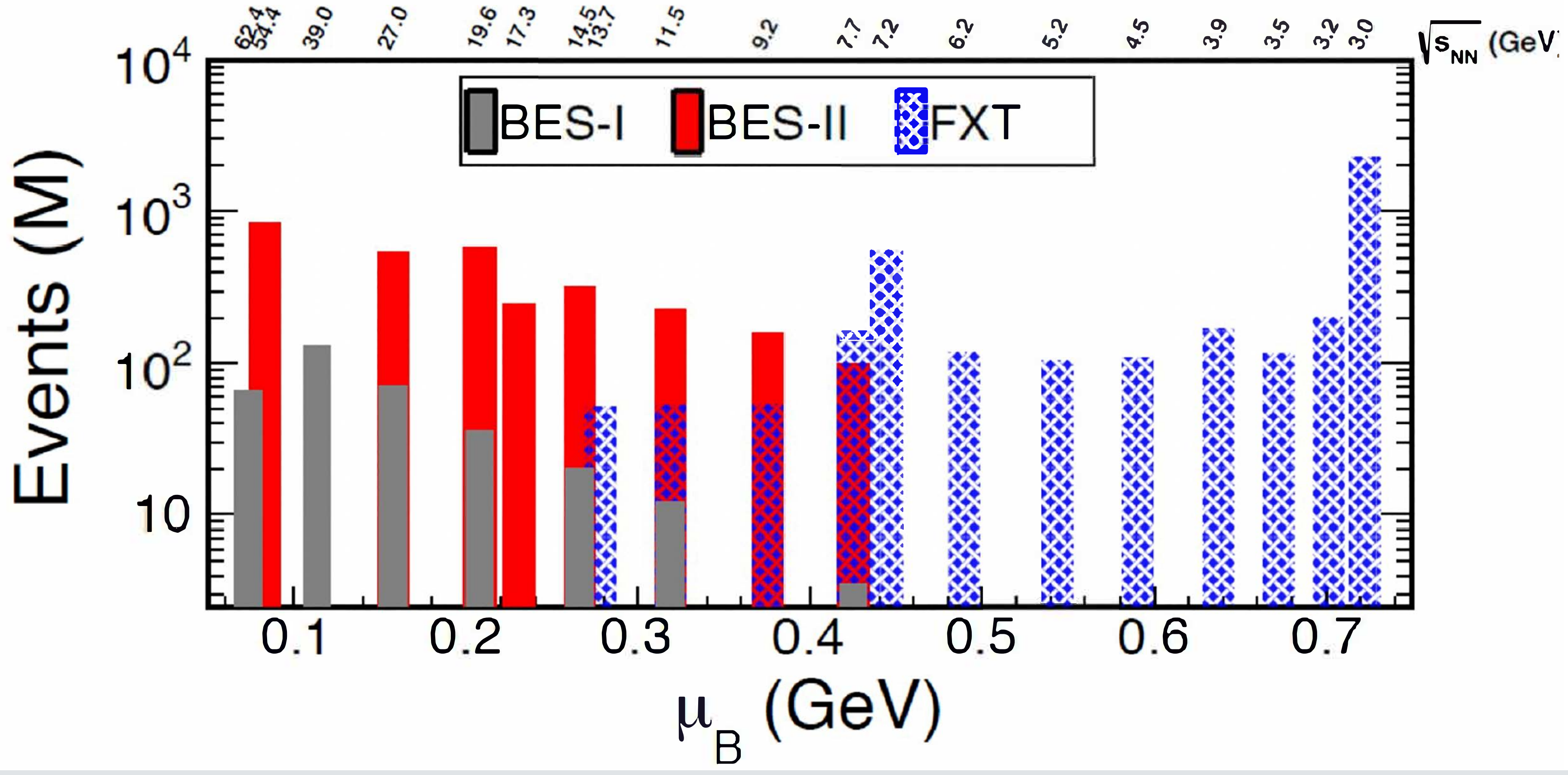
- Experimental program from RHIC covers large range in μ_B

- Understand phase diagram, including transition line $T(\mu_B)$

  → knowledge of the equation of state (EoS)

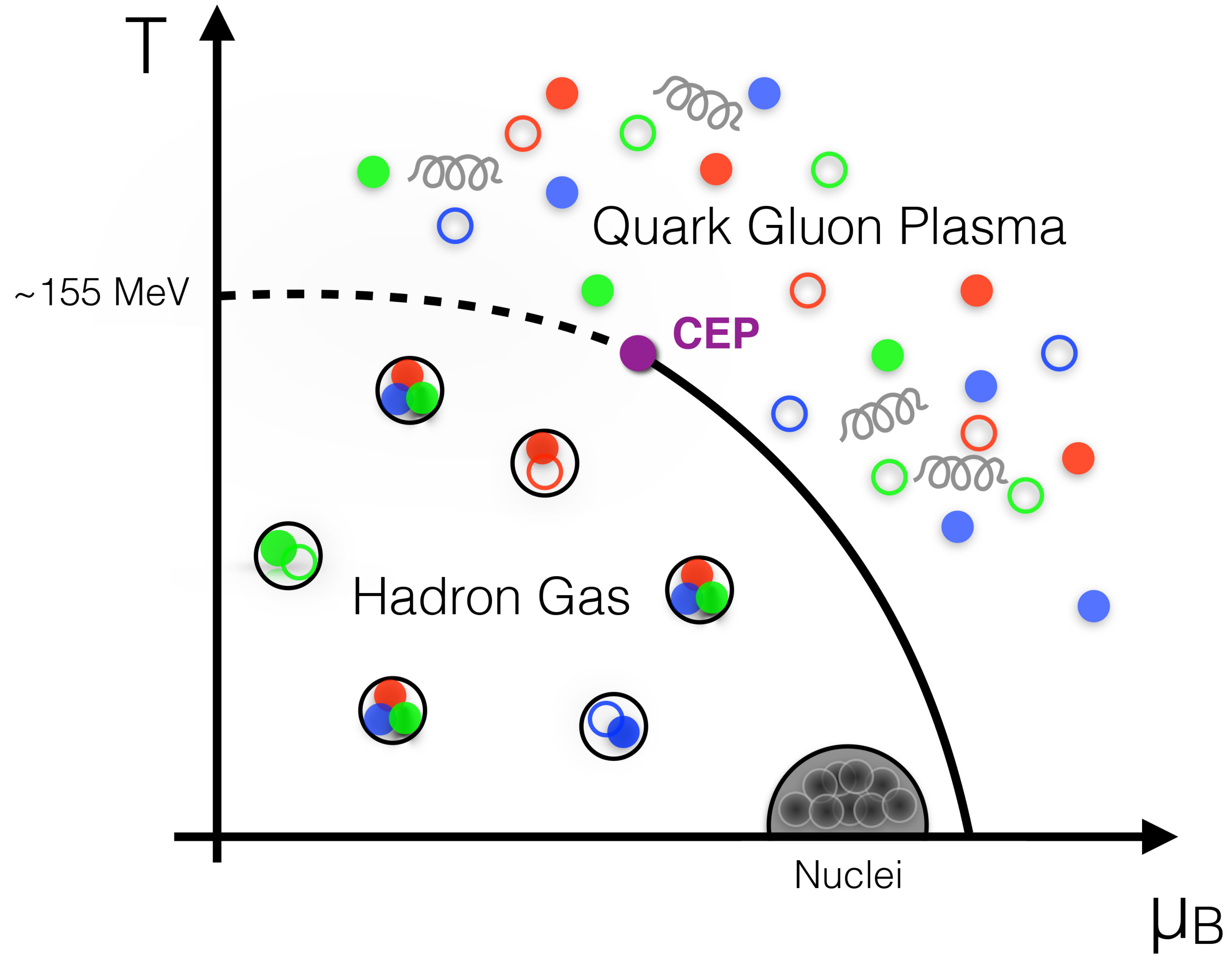
- Equilibrium thermodynamic quantities





Equation of State: Crucial Thermodynamic Input

- Knowledge of the equation of state of strongly-interacting matter in equilibrium is crucial for:
  - Fluctuations, via derivatives of the pressure
  - The hadronic spectrum, i.e. the species present at freeze-out, via partial pressures
  - Hadronic transport simulations
  - Hydrodynamic simulations
  - Neutron star merger simulations
  - The interior composition of neutron stars
  - The behavior of the speed of sound
  - …



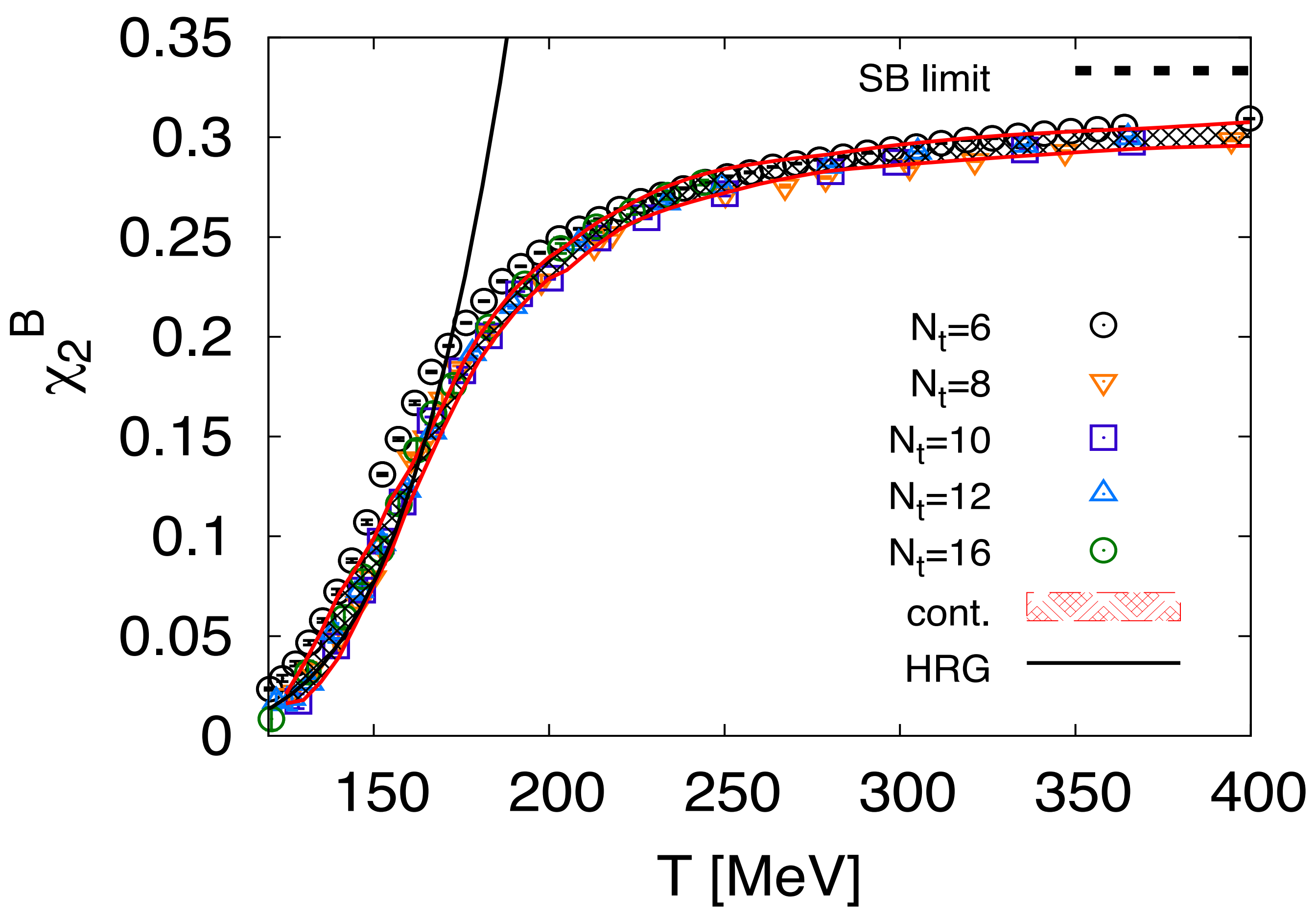
Fluctuations & Equation of State

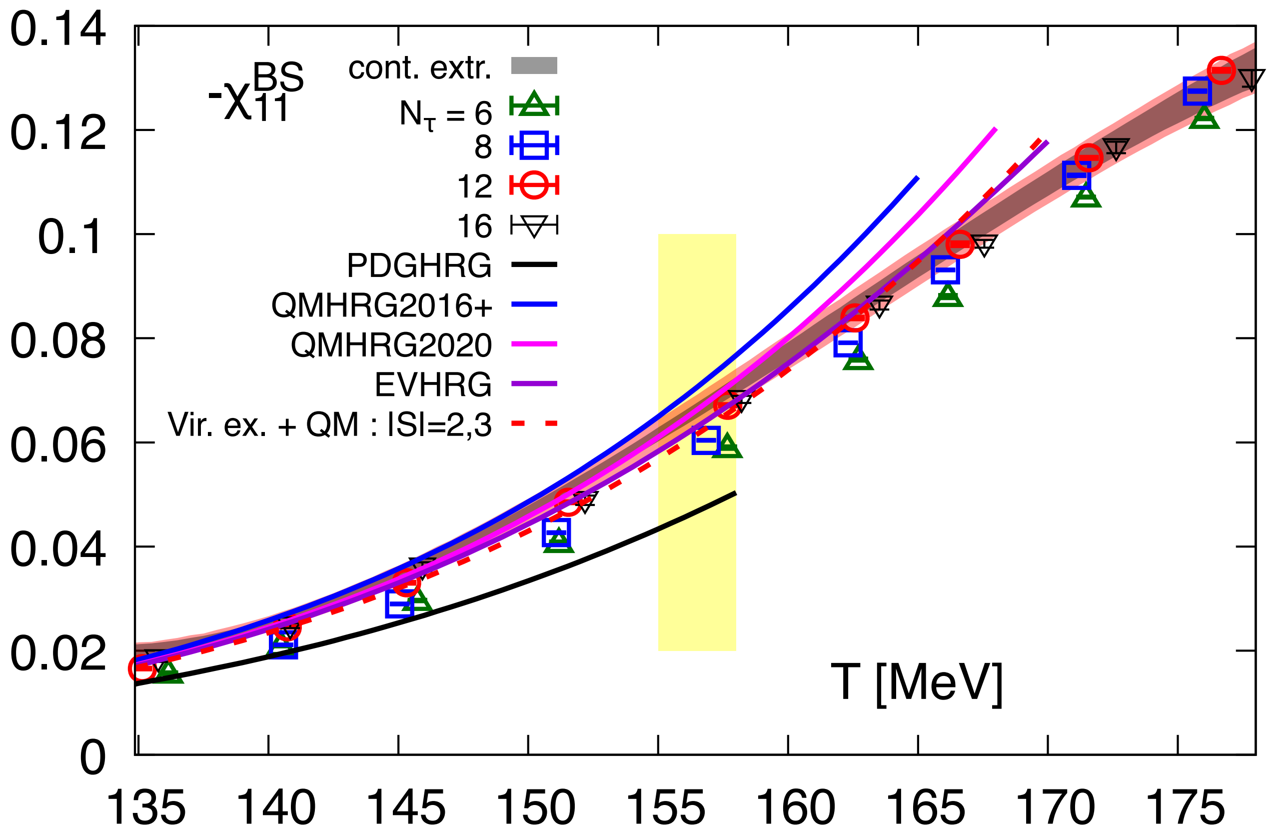
➤ Equilibrium fluctuations are calculated from knowledge of the equation of state

$$\chi^{BSQ}_{lmn} = \frac{\partial^{l+m+n}(p/T^4)}{(\partial\mu_B/T)^l(\partial\mu_S/T)^m(\partial\mu_Q/T)^n}$$

$$M = \chi_1 \qquad \sigma^2 = \chi_2$$

$$S = \chi_3/\chi_2^{3/2} \qquad \kappa = \chi_4/\chi_2^2$$





$$\frac{p(T,\mu_B)}{T^4} = \frac{p(T,0)}{T^4} + \sum_{n=1}^{\infty}\frac{1}{(2n)!}\left.\frac{\mathrm{d}^{2n}(p/T^4)}{d(\frac{\mu_B}{T})^{2n}}\right|_{\mu_B=0}\left(\frac{\mu_B}{T}\right)^{2n} = \sum_{n=1}^{\infty}\frac{1}{(2n)!}\chi^B_{2n}\left(\frac{\mu_B}{T}\right)^{2n}$$

➤ Taylor expansion coefficients give the fluctuations & correlations of conserved charges

S. Borsanyi et al JHEP (2012), D. Bollweg et al PRD (2021)

Equation of State Definition

➤ Equilibrium thermodynamics calculated from first principles lattice QCD computations are well-established with good agreement amongst techniques

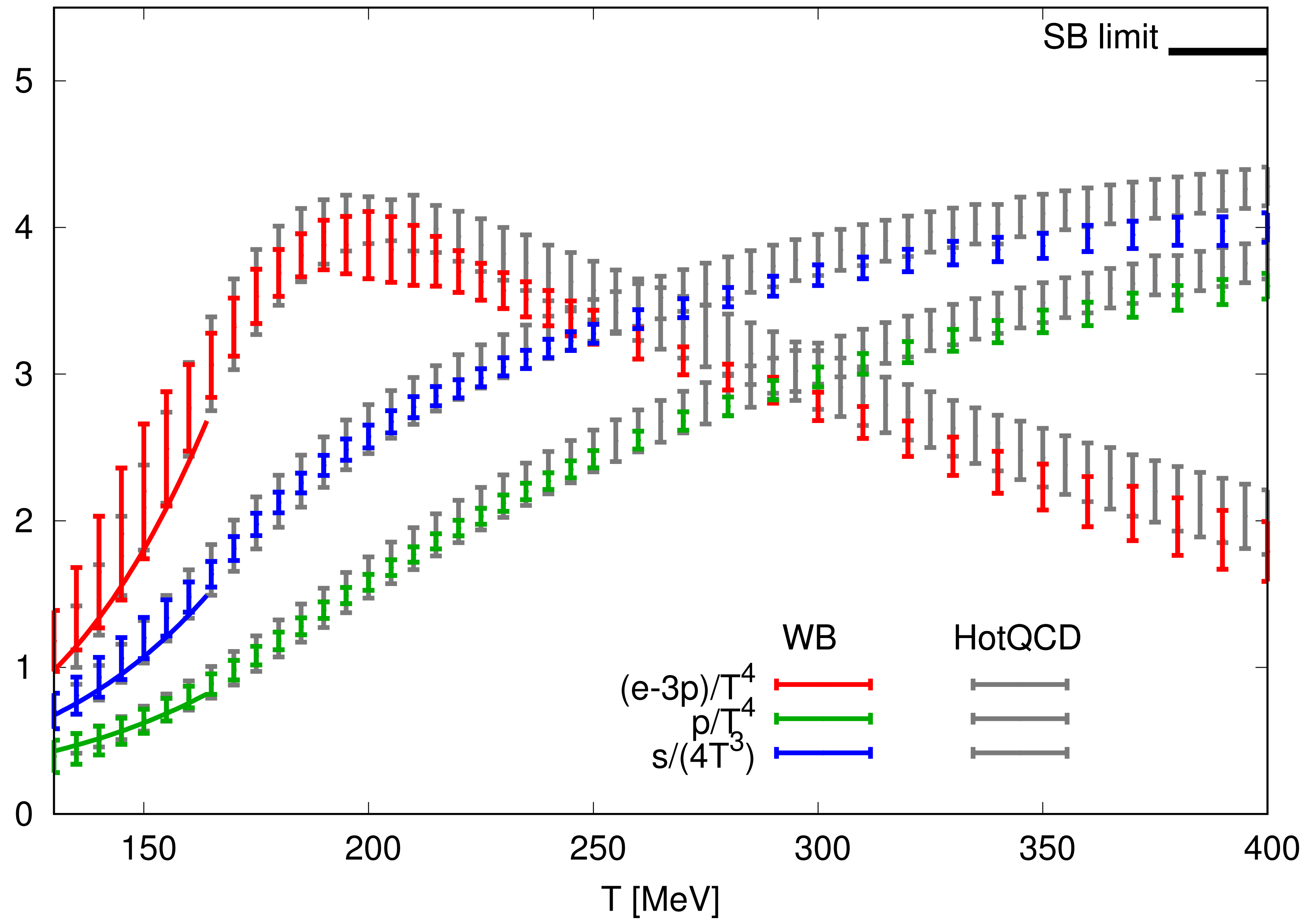
$$\frac{p(T,\mu_B)}{T^4} = \frac{p(T,0)}{T^4} + \sum_{n=1}^{\infty} \frac{1}{(2n)!} \frac{\mathrm{d}^{2n}(p/T^4)}{d(\frac{\mu_B}{T})^{2n}}\bigg|_{\mu_B=0} \left(\frac{\mu_B}{T}\right)^{2n} = \sum_{n=1}^{\infty} \frac{1}{(2n)!} \chi_{2n}^B \left(\frac{\mu_B}{T}\right)^{2n}$$

Charge density: $\frac{n_i}{T^3} = \frac{1}{T^3}\left(\frac{\partial p}{\partial \mu_i}\right)\bigg|_{T,\mu_j}$

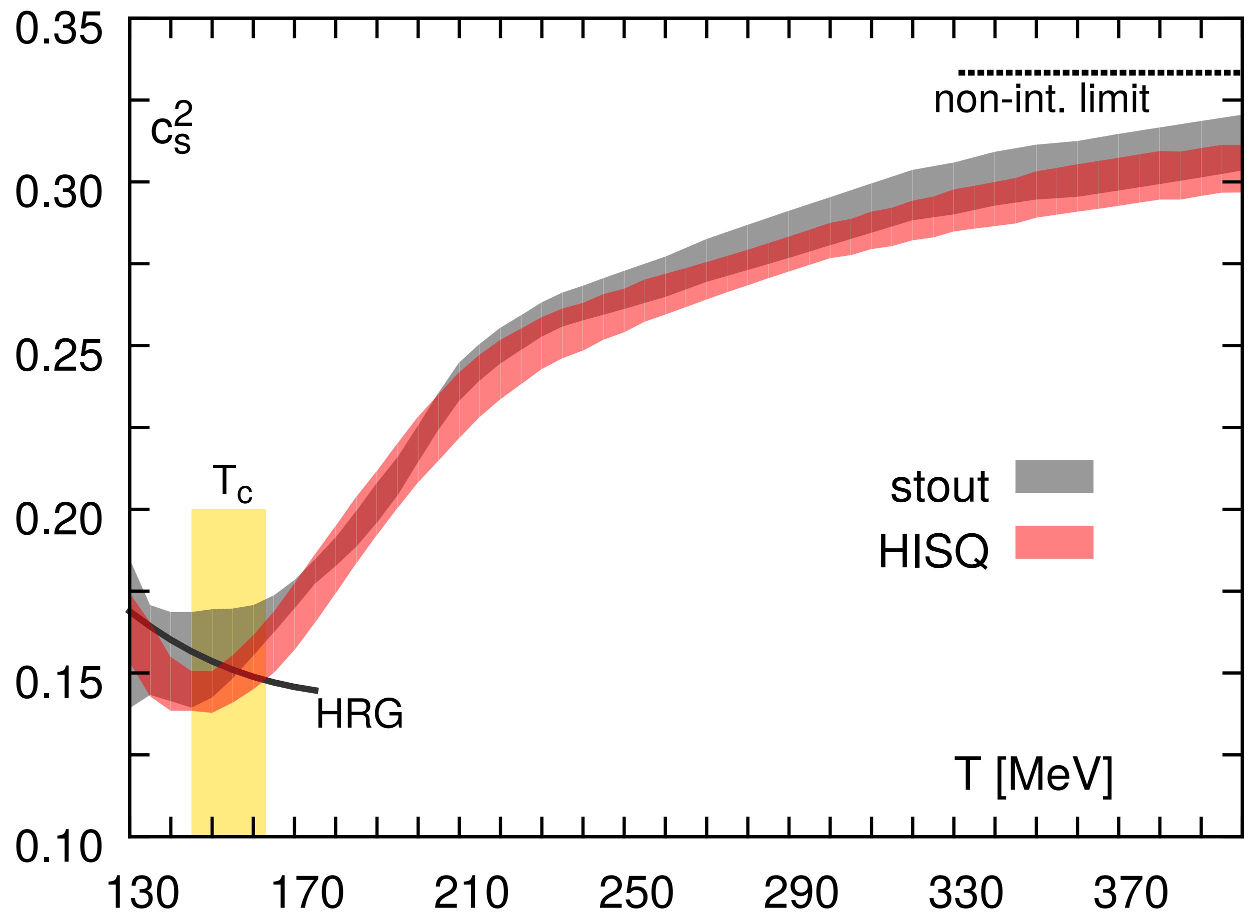
Energy density: $\frac{\epsilon}{T^4} = \frac{s}{T^3} - \frac{p}{T^4} + \sum_i \frac{\mu_i}{T}\frac{n_i}{T^3}$

Entropy density: $\frac{s}{T^3} = \frac{1}{T^3}\frac{\partial p}{\partial T}\bigg|_{\mu_i}$

Trace anomaly: $\frac{I}{T^4} = \frac{\epsilon - 3p}{T^4}$



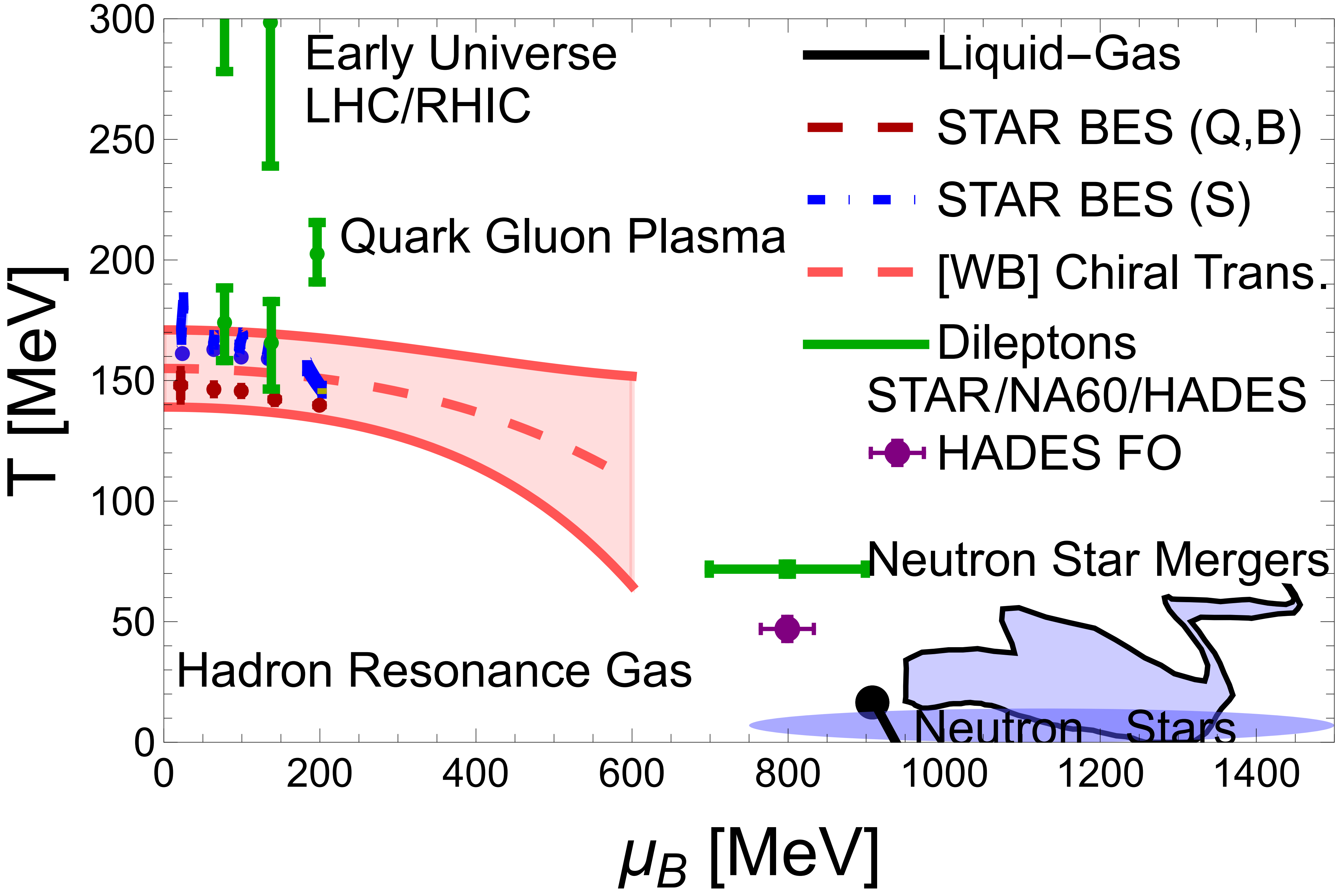
Sound speed: $c_s^2 = \frac{\partial p}{\partial \epsilon}$



A. Bazavov PRD (2014), S. Borsanyi PLB (2014)

QCD Phase Diagram - What We Know

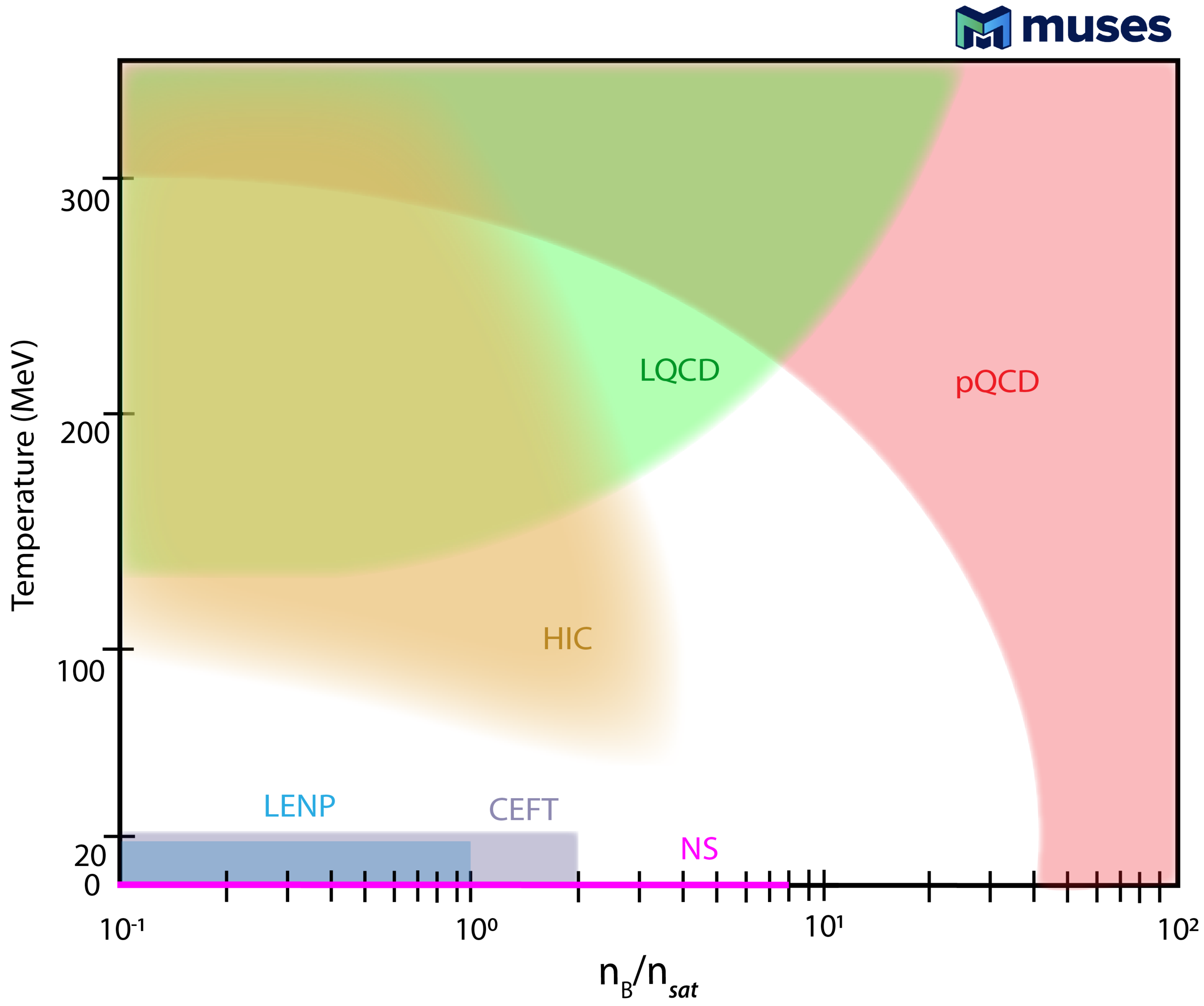
- Besides results at zero/small μ_B from lattice QCD describing HIC matter, the dense matter EoS is under study
- Many open questions remain about the phase diagram of QCD:
  - Is there a critical point? If so, where?
  - Where is the transition line at high density?
  - What are the degrees of freedom in the vicinity of the phase transition?



*A. Lovato, T. Dore, R.D. Pisarski, **J.M. Karthein** et al, arXiV: 2211.02224*

QCD Phase Diagram - How We Know It

- Constraints on the equation of state from first principles results + astrophysical observations + terrestrial experiments
  - Low density, high temperature regime: lattice QCD (sign problem) & heavy-ion collisions
  - Low temperature, high density regime: Chiral EFT, low energy nuclear experiments, neutron stars & their mergers
  - Asymptotic regime: pQCD



R, Kumar, V. Dexheimer, **J.M. Karthein** et al, arXiv:2303.17021

QCD Phase Diagram - How We Fill in the Gaps

- Because we are interested in the transition region of the phase diagram, we must extend across these regimes to fill out the phase diagram
- Approach of MUSES collaboration: merge lattice QCD equation of state with other effective theories
    - Careful study of their respective ranges of validity
    - Constrain the parameters to reproduce known limits
    - Test different possibilities to validate/ exclude them

muses

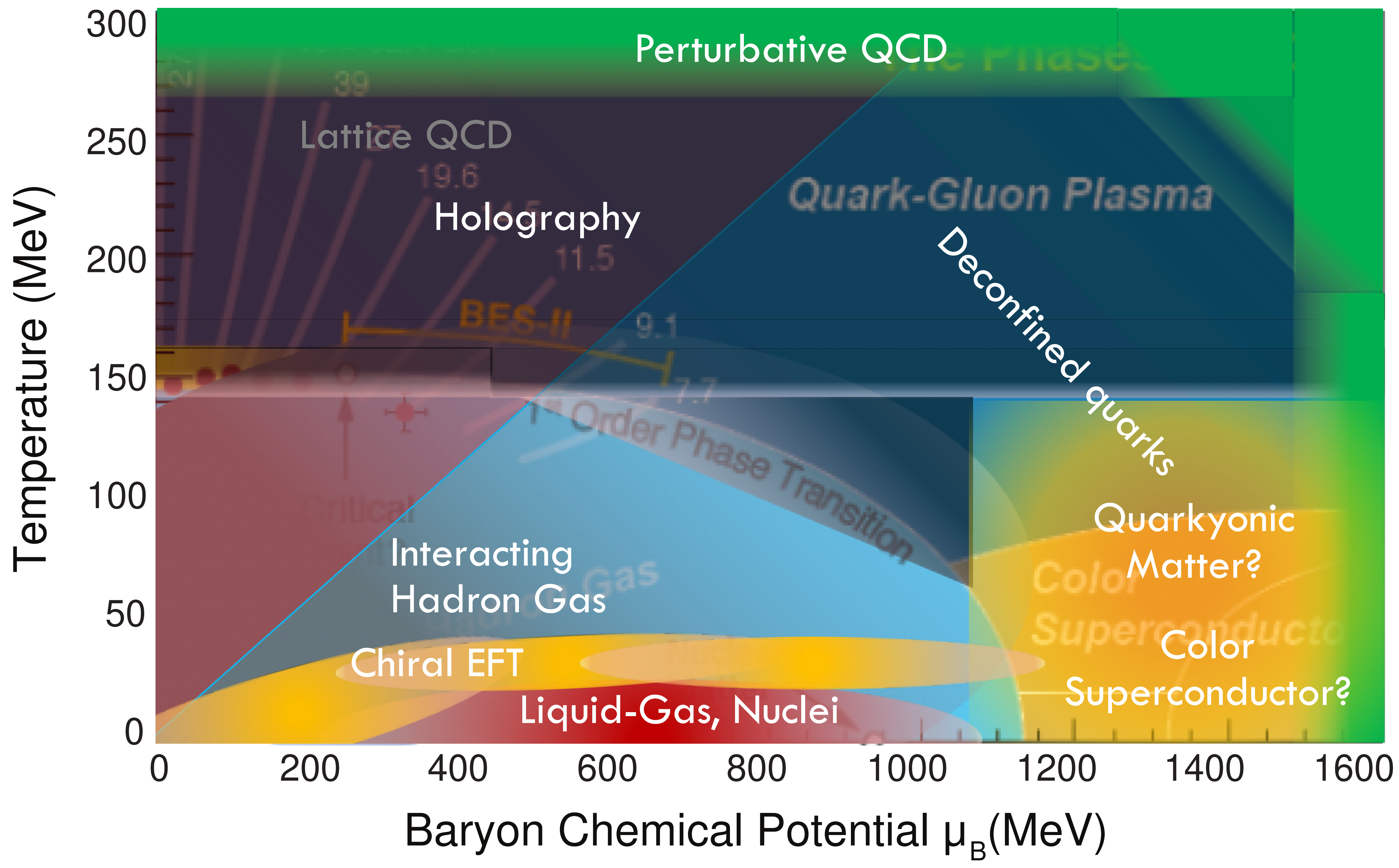


Figure by C. Ratti

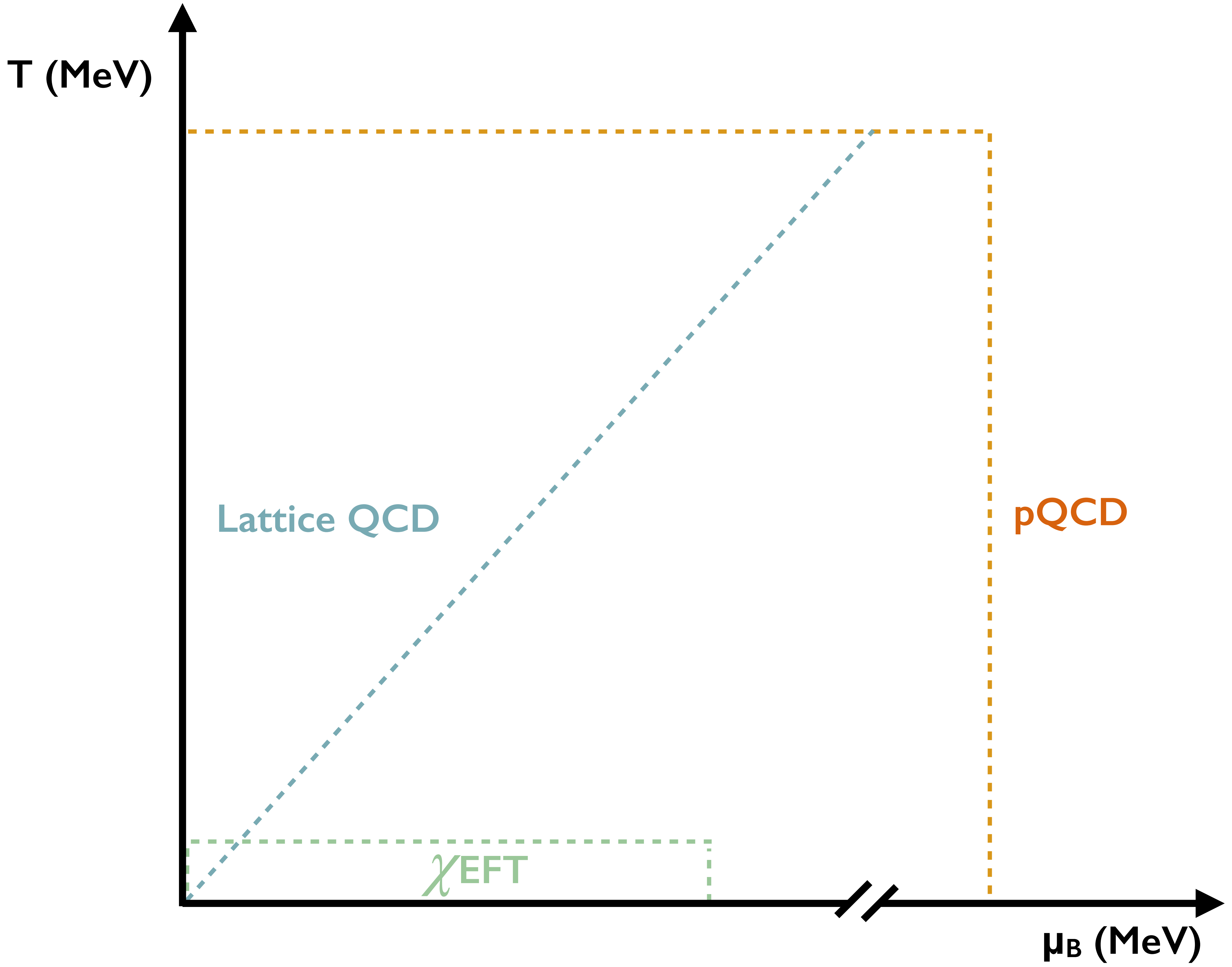
Outline

I. QCD Phase Diagram along $\mu_B = 0$

- Lattice QCD
- Hadronic Gas
- EoS for Heavy-ion Collisions

II. QCD Phase Diagram along $T = 0$

- Chiral EFT & pQCD
- Extracting dense matter EoS
- Neutron stars & heavy-ion collisions



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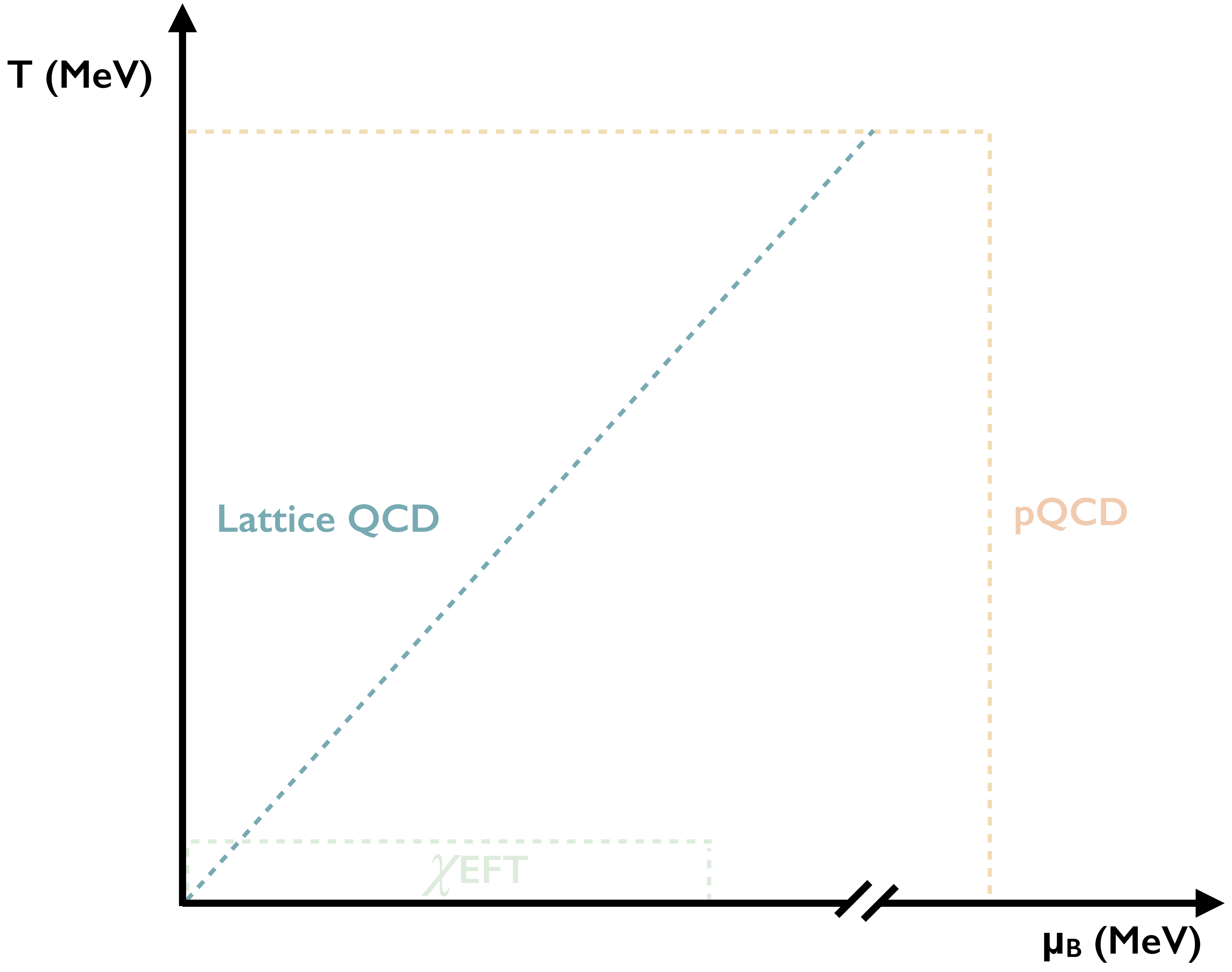


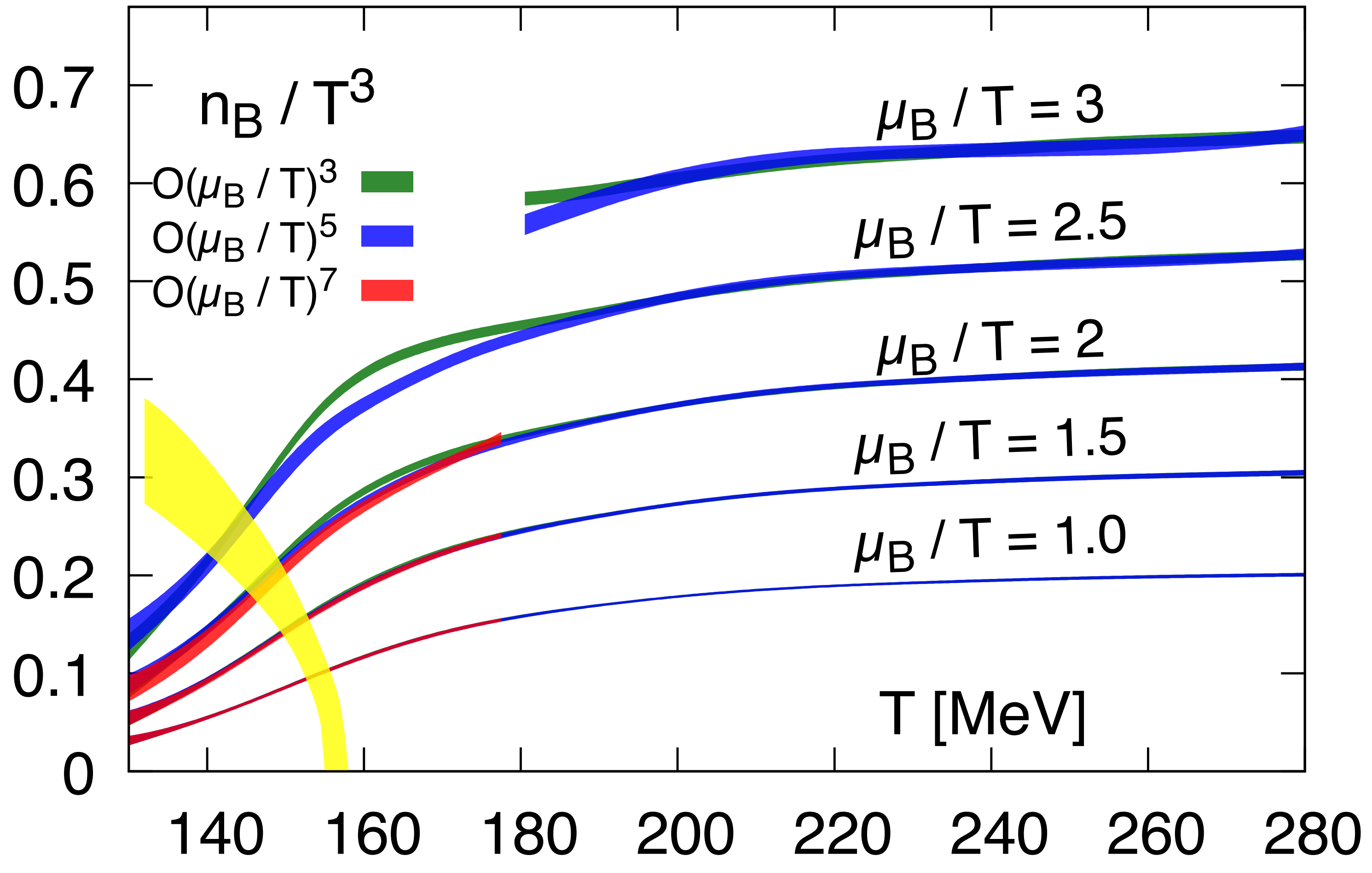
Lattice QCD Results

A. Pásztor Mon., P. Parotto Tues., C.H. Wong Tues., R. Kara Wed., D. Clarke Wed.

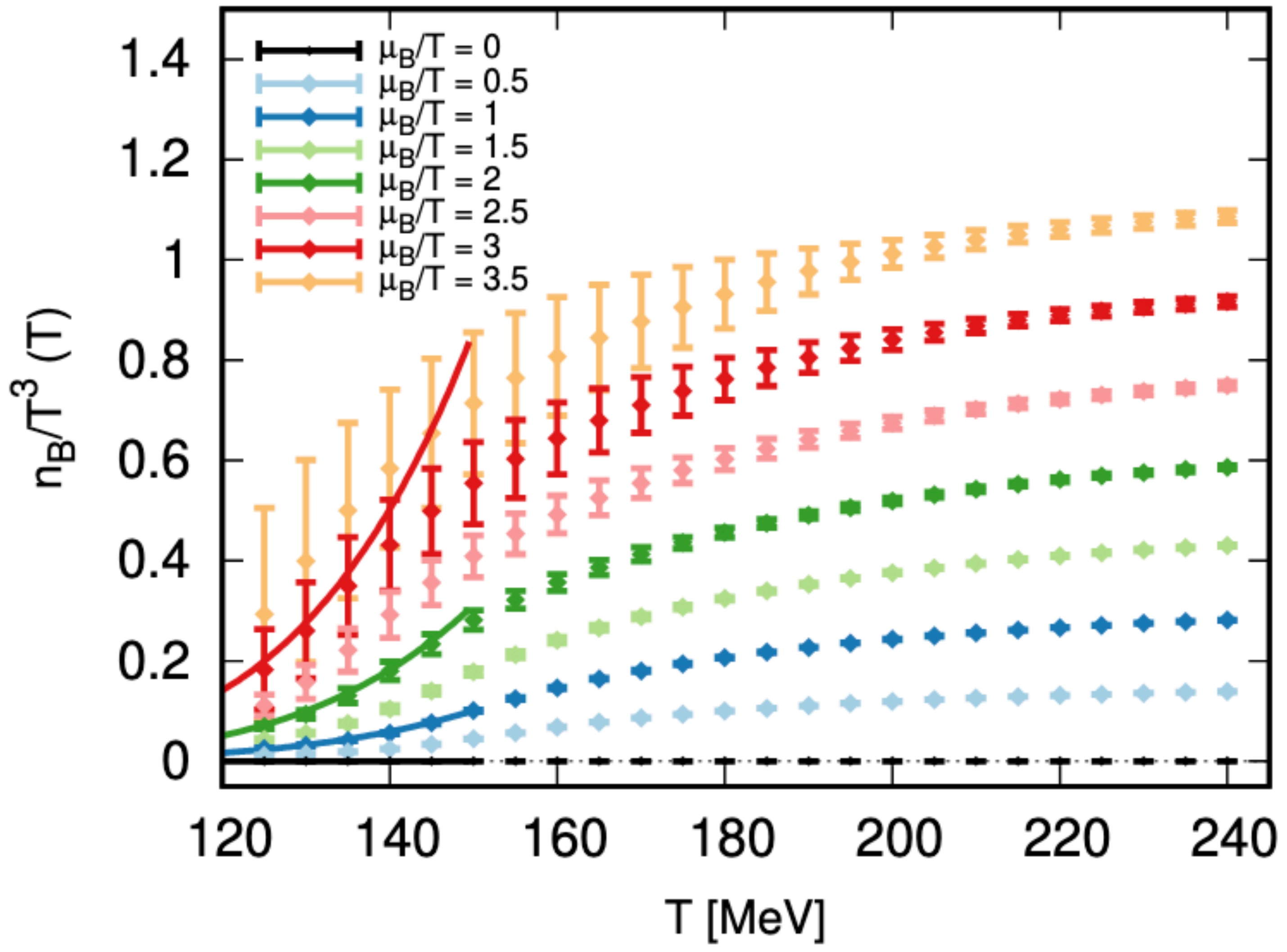
- State-of-the-art lattice calculations on the QCD equation of state for first-principles ground truth for zero and low to moderate μ_B
  - Continuum limit for traditional methods
  - New methods being explored

Extrapolated to finite μ_B | Direct μ_B simulations

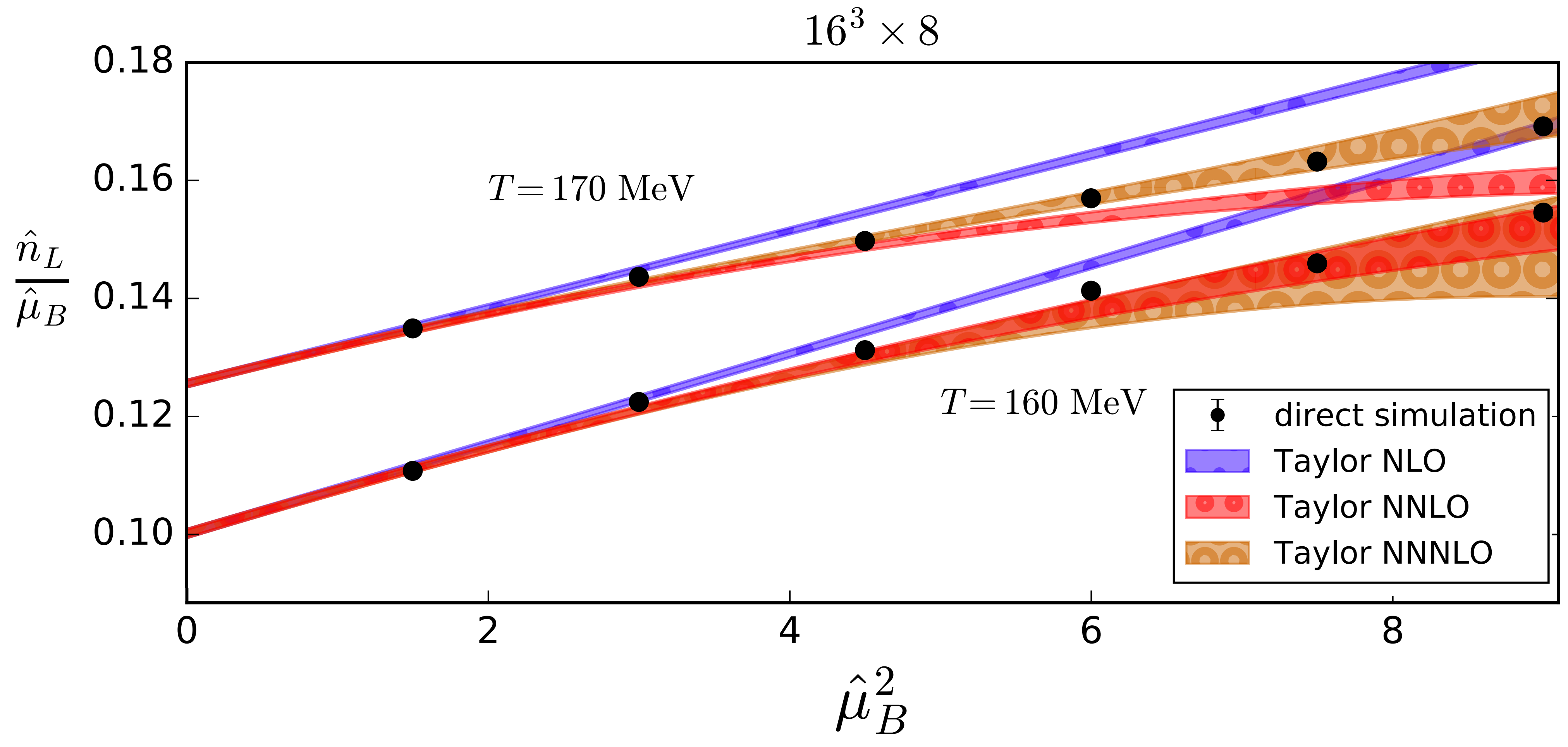
HotQCD: Taylor method



WB: $i\mu_B$, alternative expansion method



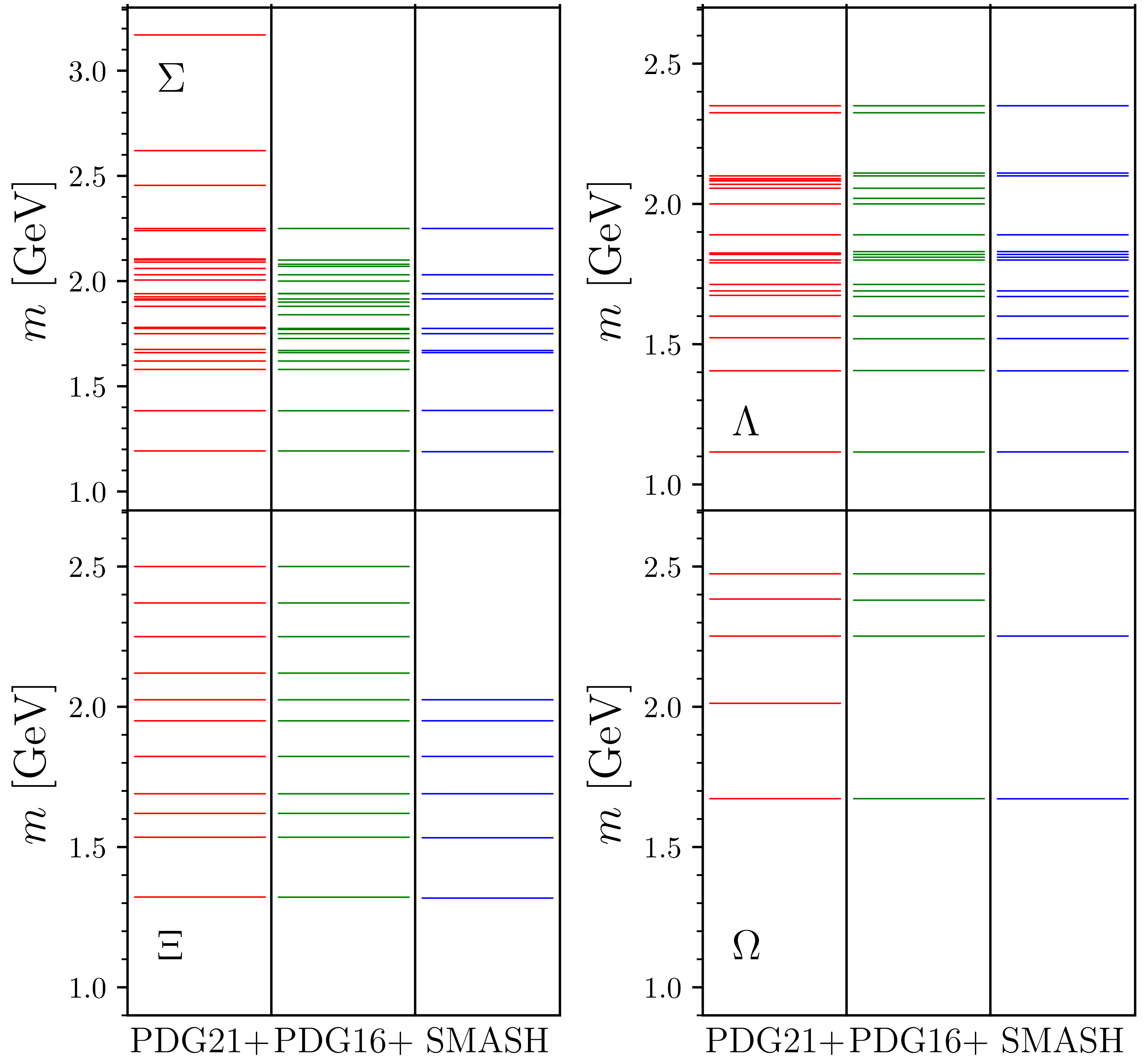
WB: reweighting method



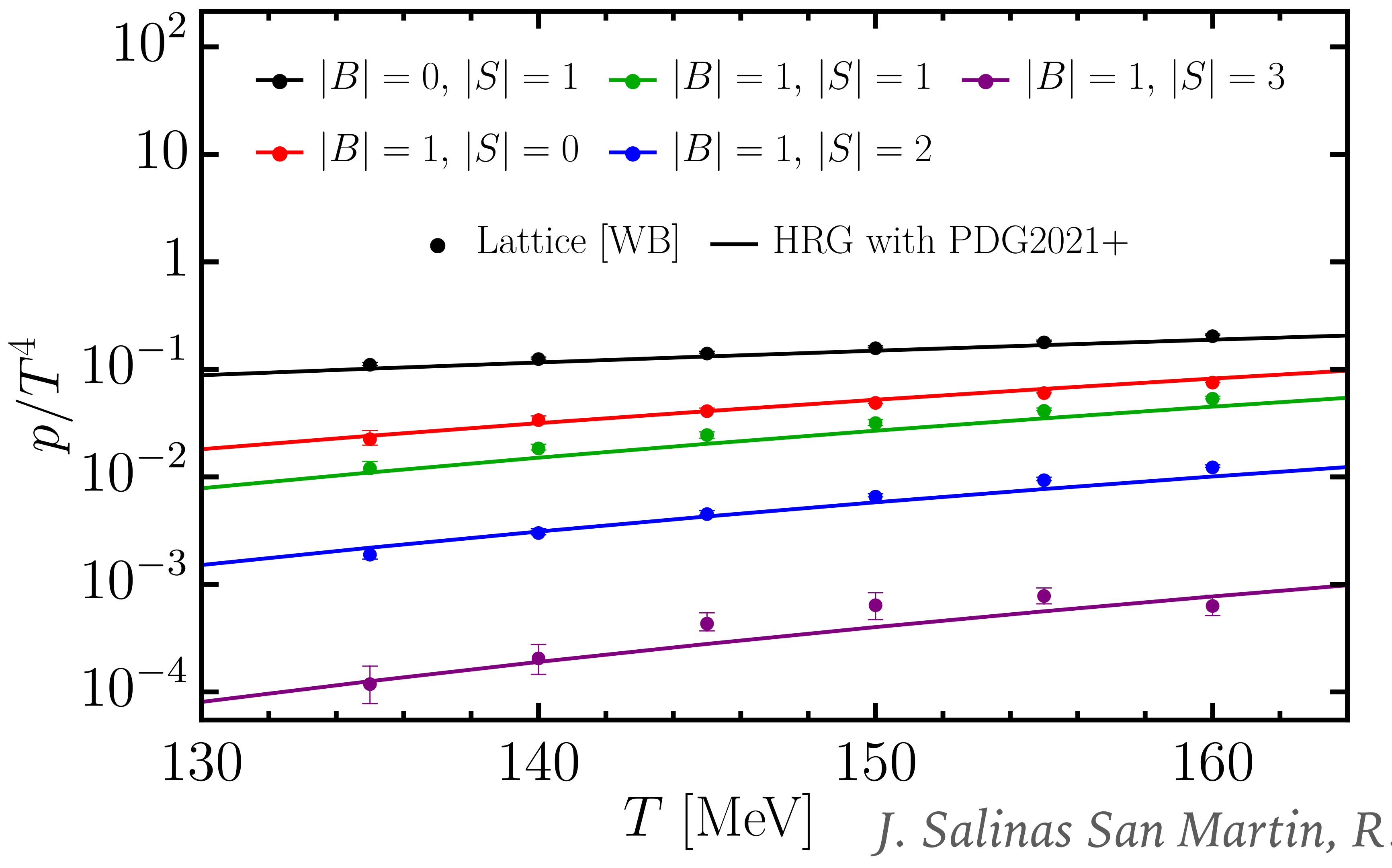
Hadron Resonance Gas Update

J. Salinas San Martin Poster

- The low temperature thermodynamics is well-described by the Hadron Resonance Gas model but hadronic spectrum still unknown
  - Update list of resonances from Particle Data Group (PDG)
  - Improved agreement with lattice when including more states: PDG2021+
  - Decays compatible with SMASH



Updates mainly to strange sector, including newly measured Ω baryon



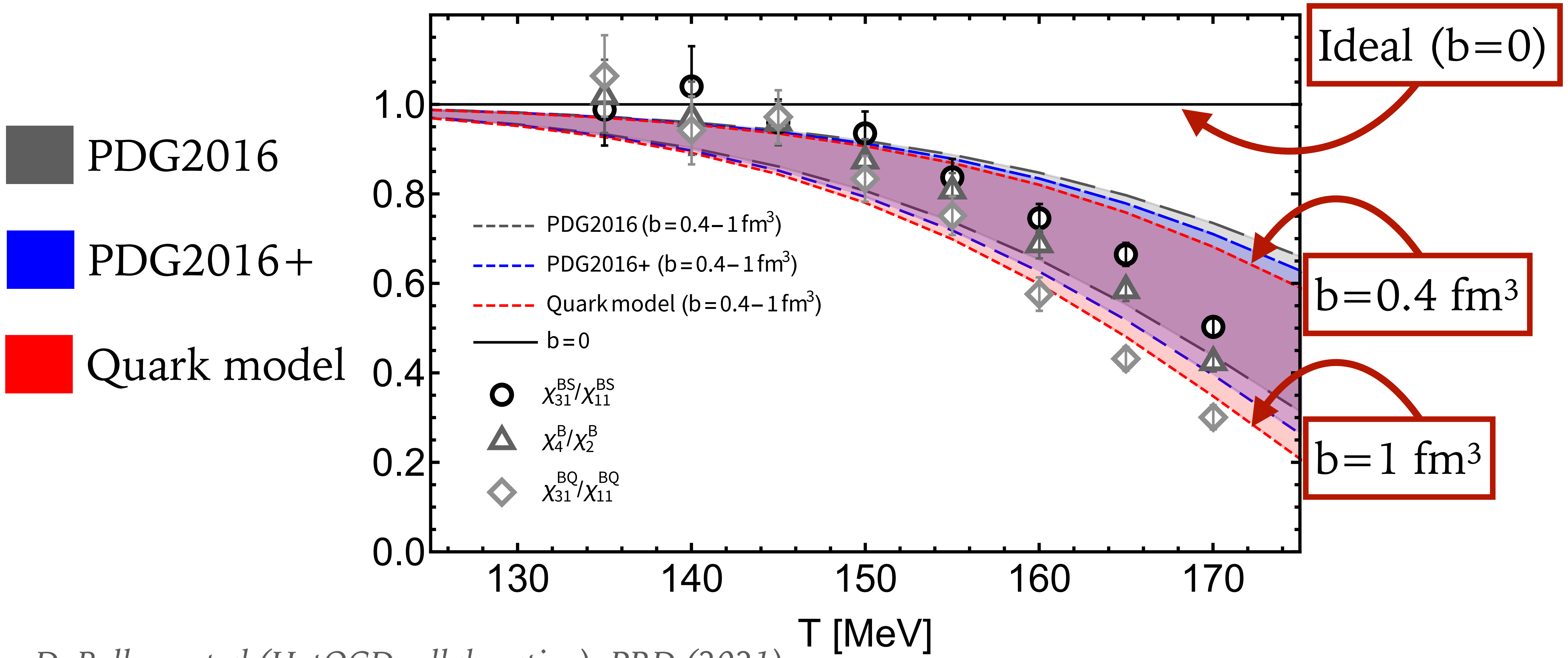
B-field effects: G. Mukherjee Poster

J. Salinas San Martin, R. Hirayama, J. Hammelmann, ***J.M. Karthein*** *et al, arXiv:2309.01737*

Interacting Hadron Resonance Gas

- Probe the hadronic phase: beyond additional states, also investigate interactions that improve agreement with lattice
- Include repulsive interactions for baryons & antibaryons:

$$p^{\rm EV}_{B(\bar{B})} = \frac{T}{b} W[b \sum_{i \in B} \frac{m_i^2 T^2}{2\pi^2} K_2(m_i/T) \exp(\pm \mu_i/T)] = \frac{T}{b} W(\varkappa_B)$$



Need specific quantities that are sensitive to excluded volume:

$$\frac{\chi_4^B}{\chi_2^B} = \frac{\chi_{31}^{BS}}{\chi_{11}^{BS}} = \frac{\chi_{31}^{BQ}}{\chi_{11}^{BQ}} = $$

$$= \frac{1 - 8\,W(\varkappa_B) + 6[W(\varkappa_B)]^2}{[1 + W(\varkappa_B)]^4}$$

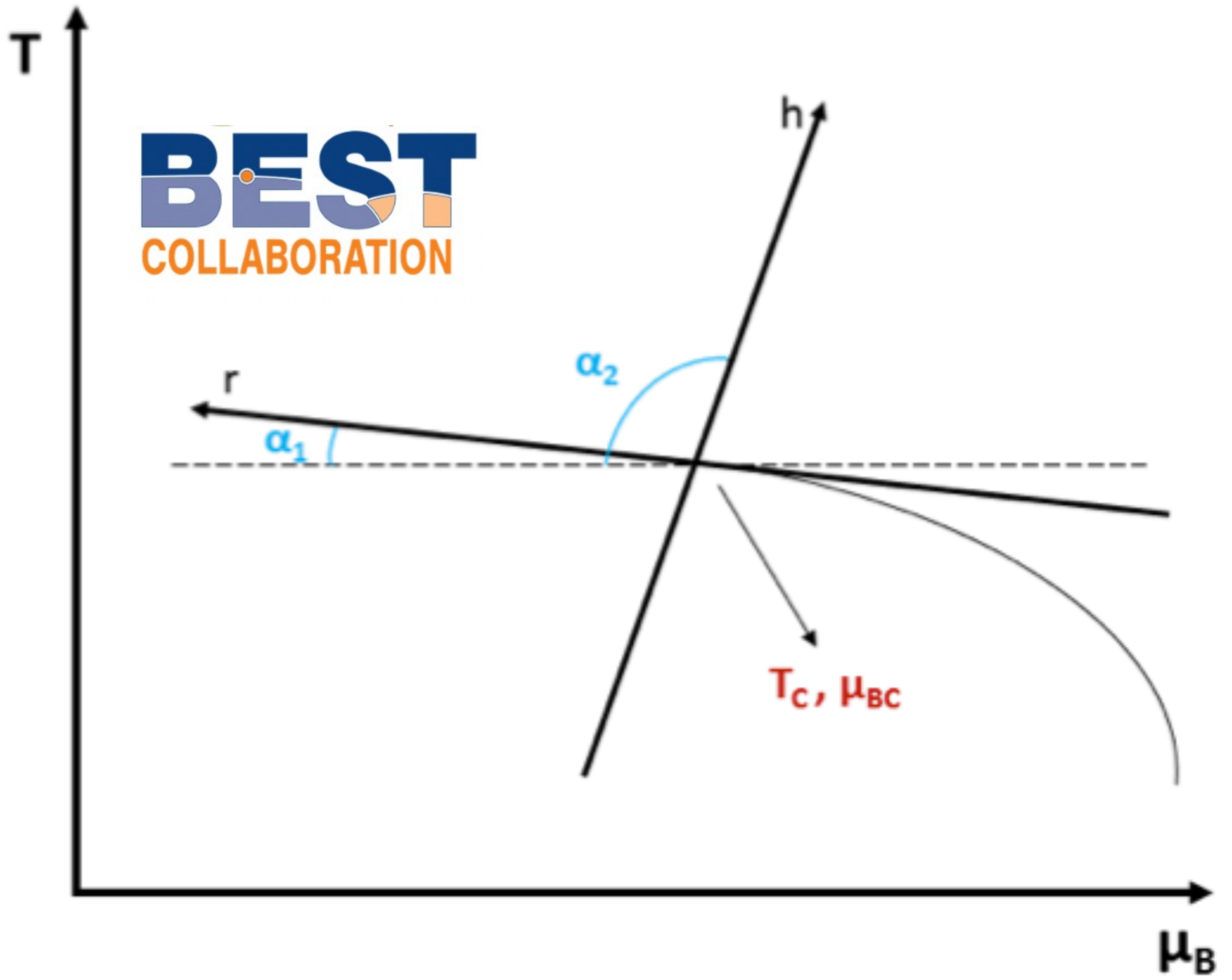
- weak dependence on particle spectrum
- identical EV corrections

See also: D. Bollweg et al (HotQCD collaboration), PRD (2021)

***J.M. Karthein**, V. Koch, C. Ratti, V. Vovchenko, PRD (2021)*

EoS for BES

➤ Combine Lattice + HRG equation of state and incorporate universal scaling features into the QCD phase diagram from the 3D Ising Model equation of state



$$(\mathbf{r}, \mathbf{h}) \longleftrightarrow (\mathbf{T}, \mu_\mathbf{B}): \quad \frac{T - \mathbf{T_C}}{\mathbf{T_C}} = \mathbf{w}\,(r\rho\,\sin\alpha_1 + h\,\sin\alpha_2)$$

$$\frac{\mu_B - \mu_\mathbf{BC}}{\mathbf{T_C}} = \mathbf{w}\,(-r\rho\,\cos\alpha_1 - h\,\cos\alpha_2)$$

➤ Reconstruct the pressure via Taylor expansion coefficients from Lattice QCD

$$T^4 c_n^{\mathrm{LAT}}(T) = T^4 c_n^{\mathrm{Non-Ising}}(T) + c_n^{\mathrm{Ising}}(T)$$

$$P(T, \mu_B) = T^4 \sum_n c_n^{\mathrm{Non-Ising}}(T) \left(\frac{\mu_B}{T}\right)^n + P_{\mathrm{crit}}^{\mathrm{QCD}}(T, \mu_B)$$

➤ Reduce free parameters by imposing constraints from Lattice

$$T = T_0 + \kappa T_0 \left(\frac{\mu_B}{T_0}\right)^2 + O(\mu_B^4). \qquad \alpha_1 = \tan^{-1}\left(2\frac{\kappa}{T_0}\mu_{BC}\right)$$

➤ Further constrain with future experimental data

See also: G. Basar Tues., T. Welle Poster

P. Parotto et al, PRC (2020),
***J. M. Karthein** et al, EPJ+ (2021)*

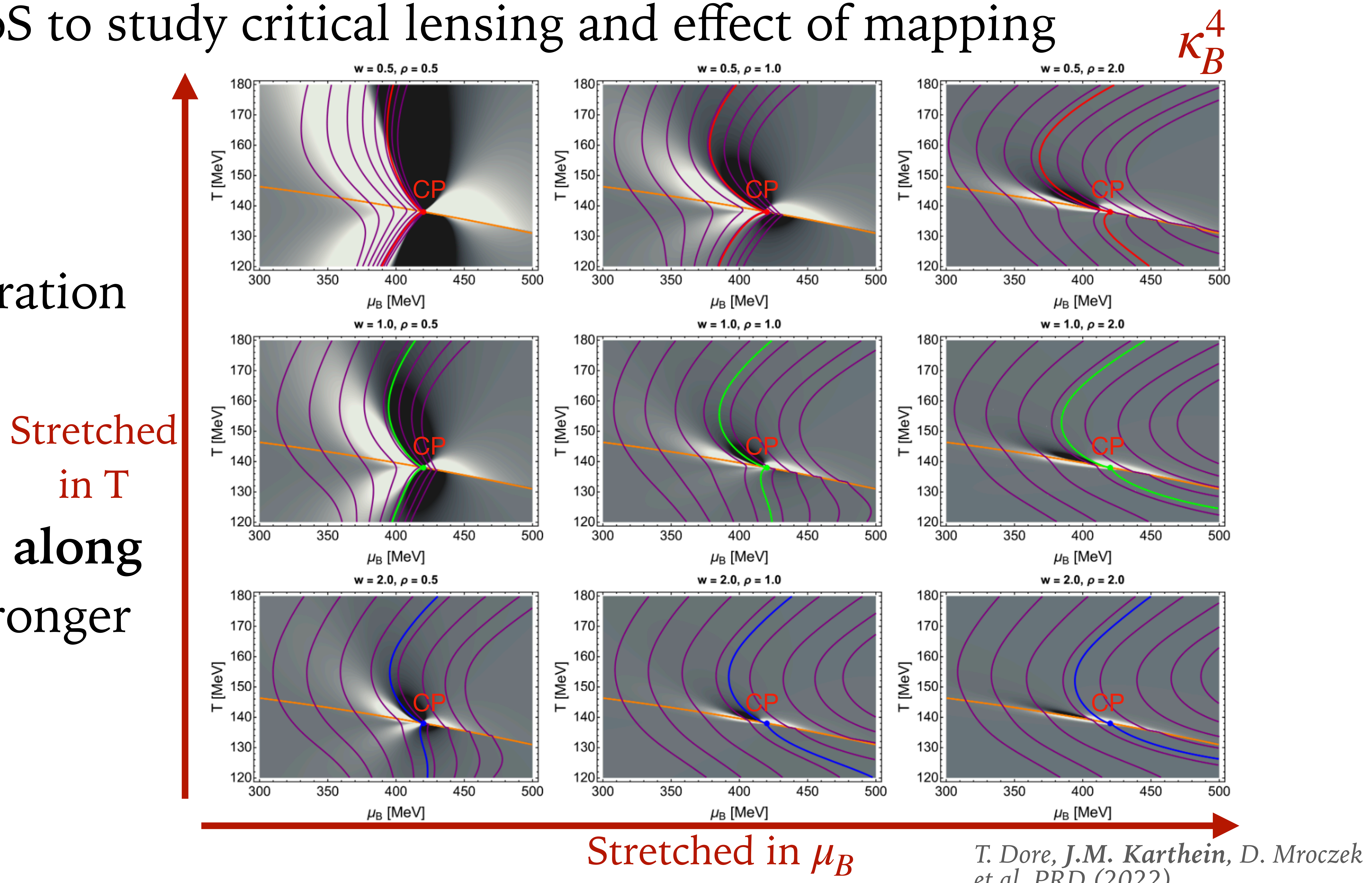
BEST EoS Used to Calculate in Equilibrium: κ_B

- Calculate κ_B^4 from BEST EoS to study critical lensing and effect of mapping parameters

- Small $w, \rho \rightarrow$ smaller separation

$$\frac{d\mu_B}{d(s/n)} \sim (w\rho)r$$

- Critical regions **extending along the T-direction** show a stronger signal and lensing effect



T. Dore, **J.M. Karthein**, D. Mroczek et al, PRD (2022)

BEST EoS Used to Calculate in Equilibrium: ω_p

M. Pradeep Tues.

- Use mapping from BEST EoS to calculate particle multiplicity fluctuations

$$\omega_{4p,\,\sigma} = \frac{6\,(2\tilde{\lambda}_3^2 - \tilde{\lambda}_4)}{T^2 n_p}\,\xi^7 \left(d_p\, g_p \int_{\mathbf{k}} \frac{v_{\mathbf{k}}^{p\,2}}{\gamma_{\mathbf{k}}^{p}} \right)^4$$

  - Update equilibrium estimates from 2010 with input from universality, including new equilibrium results for correlation length to $\mathcal{O}(\epsilon^2)$: $\xi^2(M,t) = R^{-2\nu} g_\xi(\theta)$

$$g_\xi(\theta) = g_\xi(0)\left(1 - \frac{5}{18}\epsilon\theta^2 + \left[\frac{1}{972}(24I - 25)\theta^2 + \frac{1}{324}(4I + 41)\theta^4\right]\epsilon^2\right), \quad I \equiv \int_0^1 \frac{\ln[x(1-x)]}{1 - x(1-x)}dx$$

  - Use in combination with known Ising fluctuations to extract the higher point couplings for the critical equation of state

$$\kappa_{n+1}^{\mathrm{eq}} \propto \left(\frac{\partial^n M^{\mathrm{eq}}(t,h)}{\partial h^n}\right)_t$$

$$\kappa_2 = \langle \sigma_V^2 \rangle = VT\,\xi^2\,; \qquad \kappa_3 = \langle \sigma_V^3 \rangle = 2\lambda_3 V T^2\,\xi^6$$

$$\kappa_4 = \langle \sigma_V^4 \rangle_c \equiv \langle \sigma_V^4 \rangle - 3\langle \sigma_V^2 \rangle^2 = 6VT^3\left[\,2(\lambda_3\xi)^2 - \lambda_4\,\right]\xi^8$$

C. Athanasiou, K. Rajagopal, M. Stephanov, PRD (2010)
***J.M. Karthein**, M. Pradeep, K. Rajagopal, M. Stephanov, Y. Yin, to appear*

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  - Updates: $\xi, \lambda_3, \lambda_4$ (dimensionless, ξ-independent: $\tilde{\lambda}_3 = \lambda_3 T^{1/2}\xi^{3/2}$, $\tilde{\lambda}_4 = \lambda_4 T\xi$)

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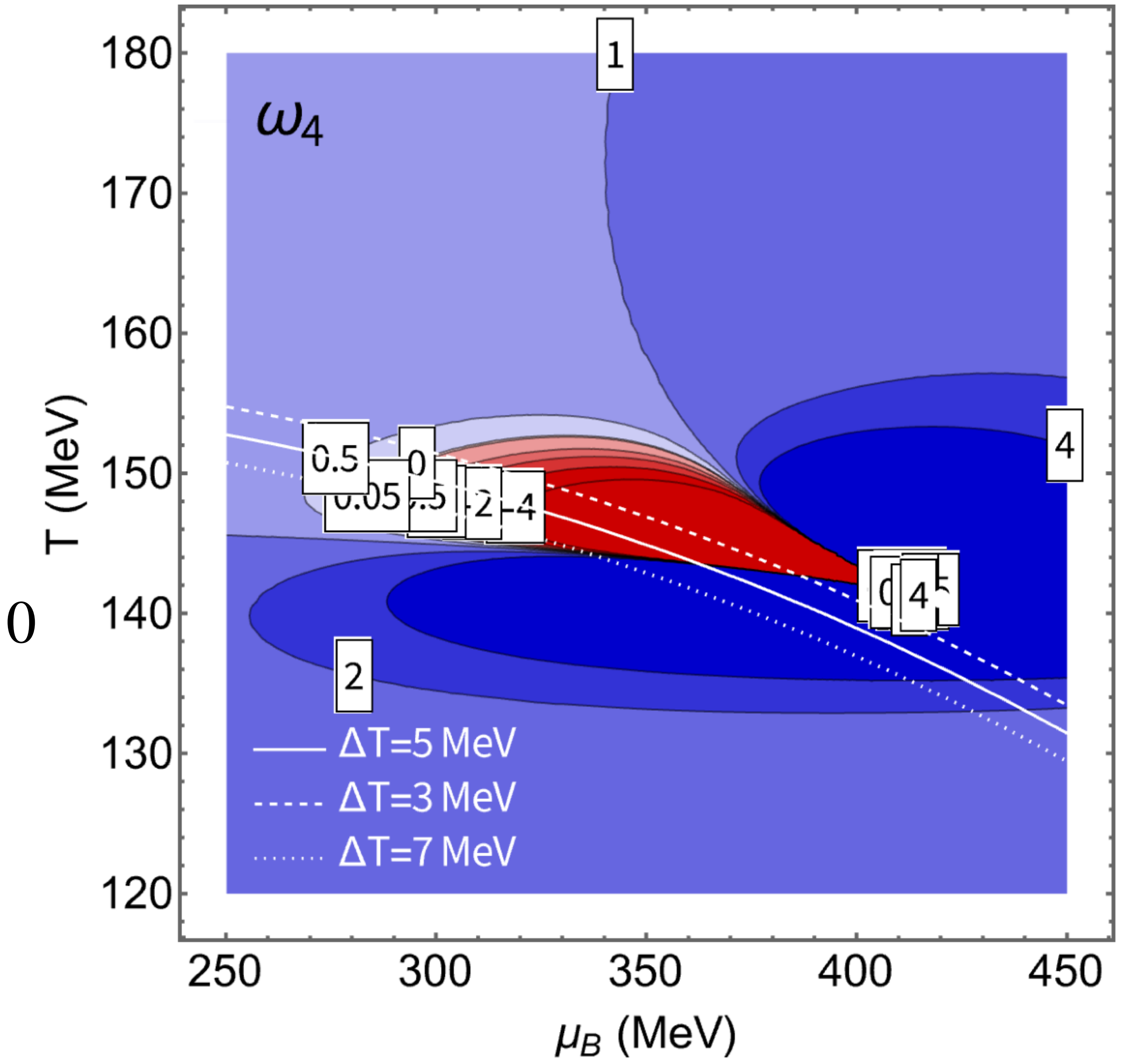
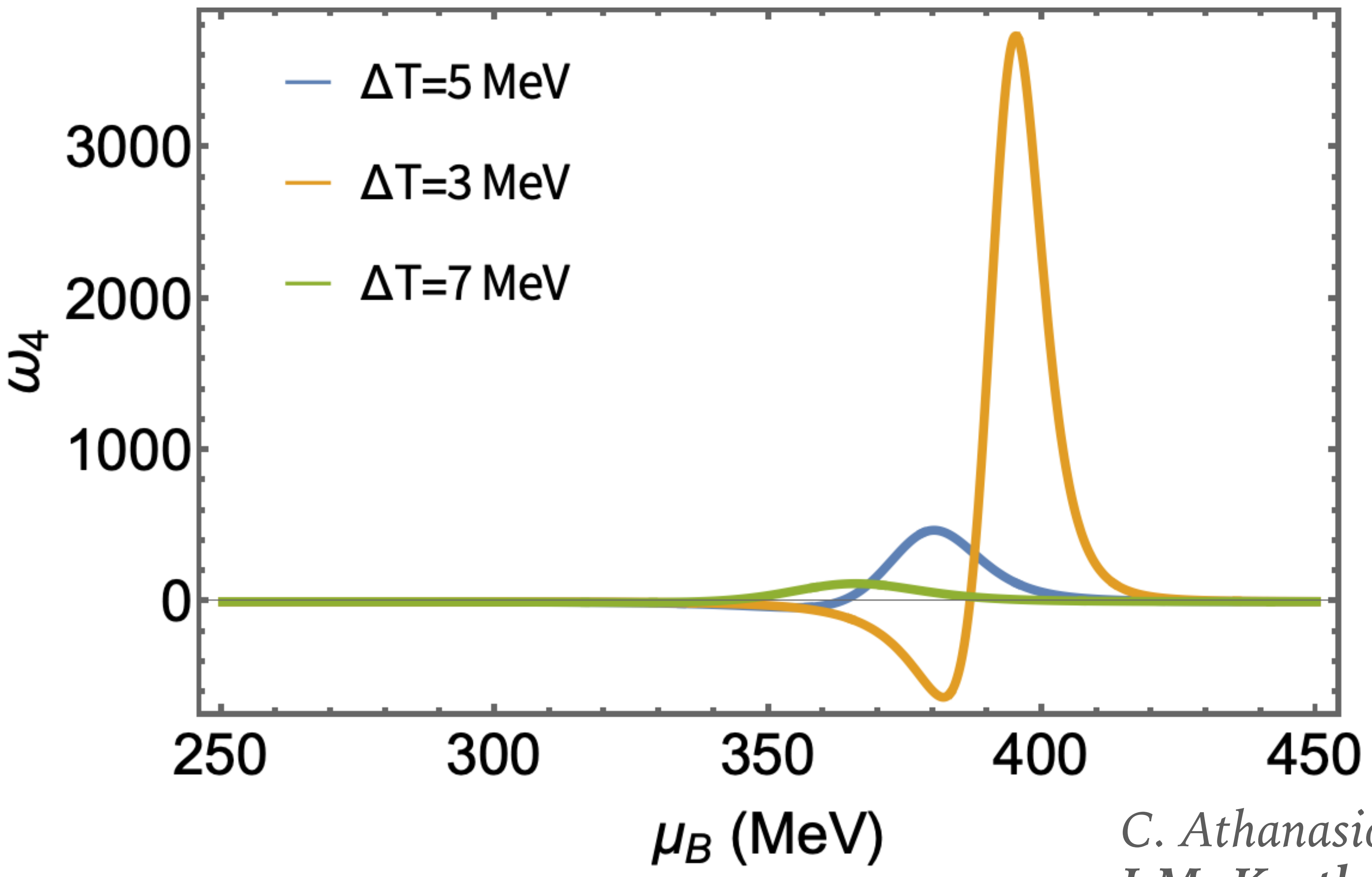
$\mu_{B,C} = 420\,\mathrm{MeV}$

$T_C = 141\,\mathrm{MeV}$

$w = 8, \rho = 0.2$

$\Delta\alpha = \alpha_2 - \alpha_1 = -10$





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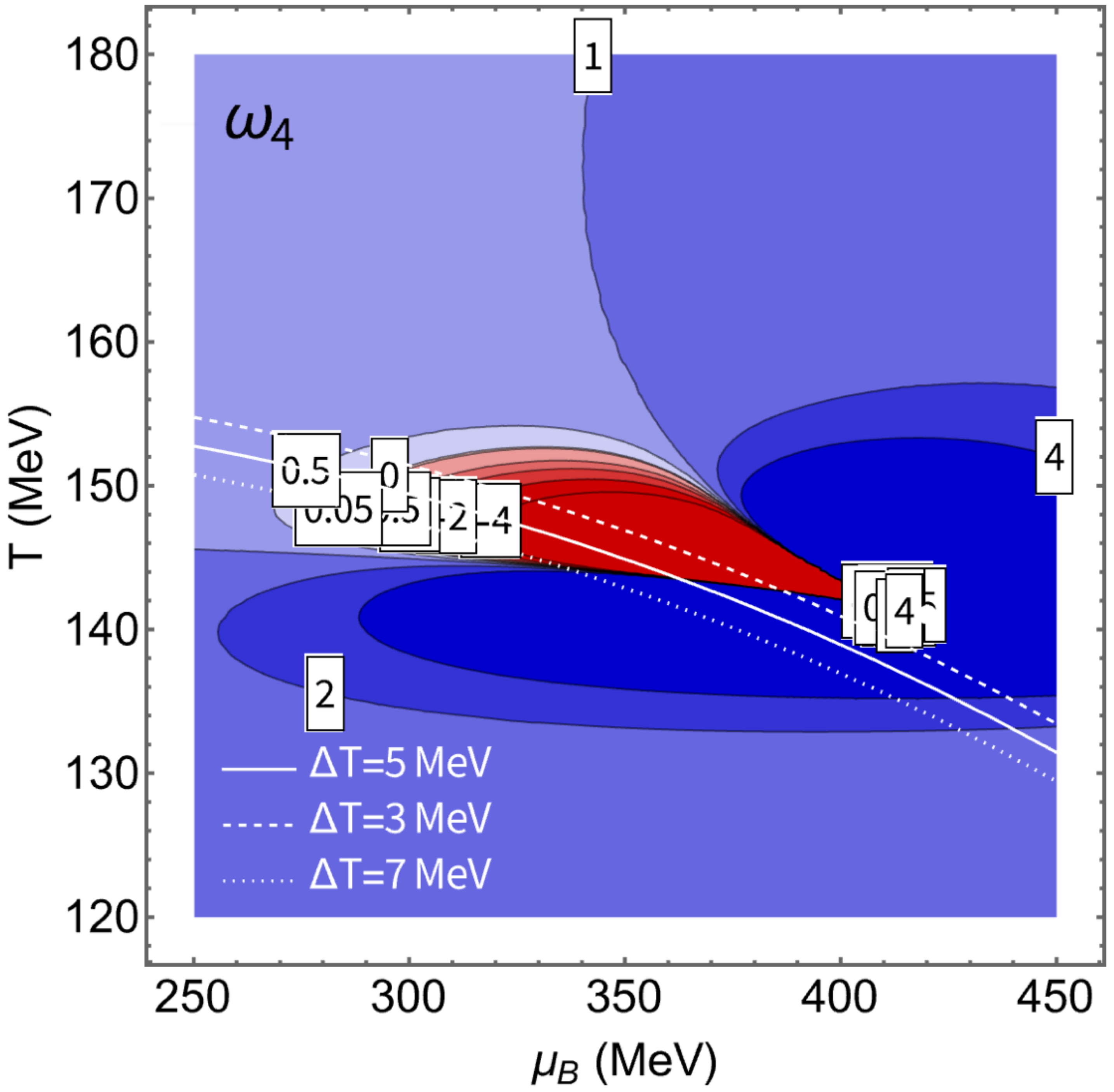
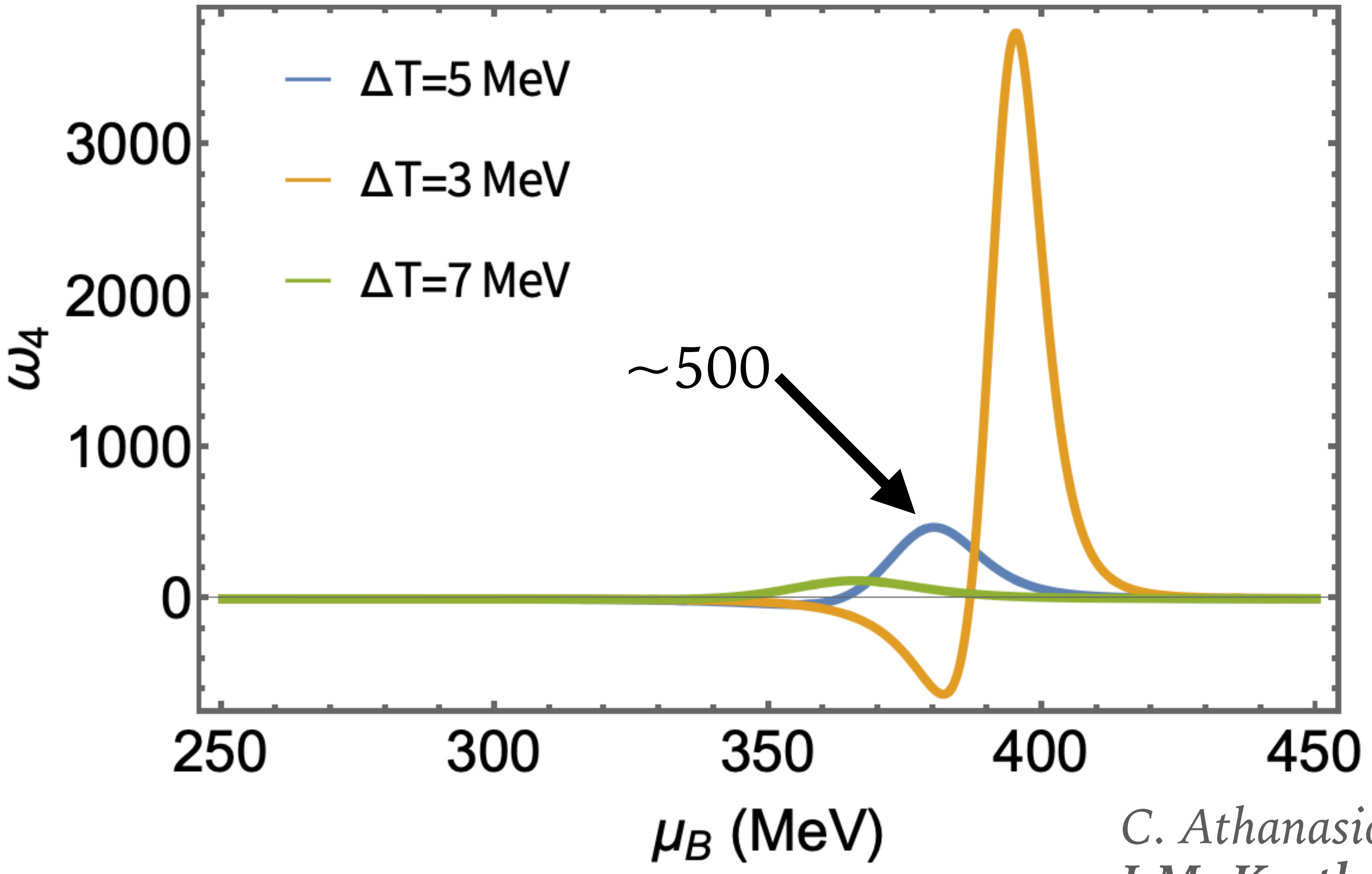




Utilize to make out-of-equilibrium estimates: M. Pradeep Tues.

C. Athanasiou, K. Rajagopal, M. Stephanov, PRD (2010)
***J.M. Karthein**, M. Pradeep, K. Rajagopal, M. Stephanov, Y. Yin, to appear*

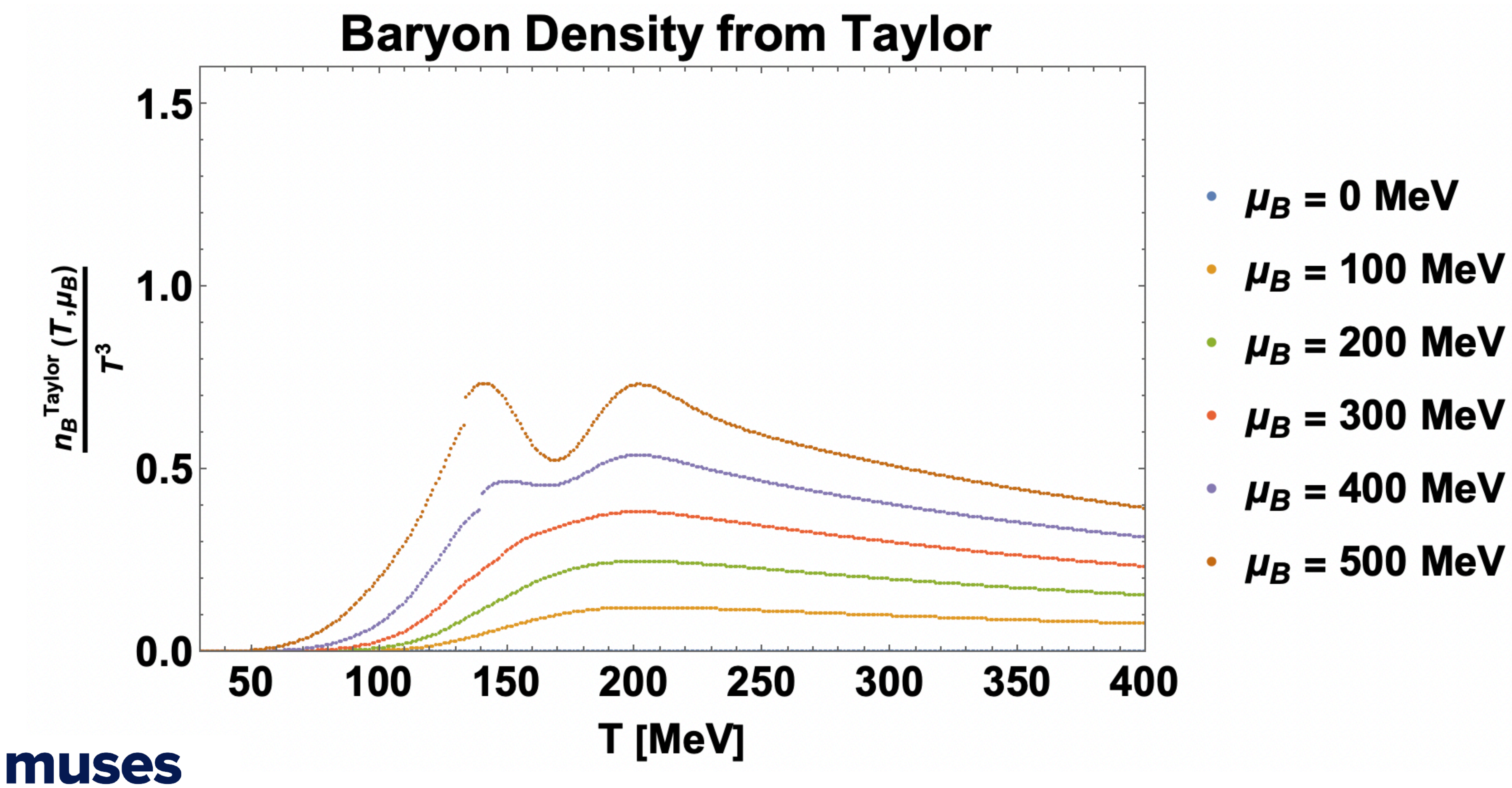
BEST EoS Becomes MUSES Ising-AltExs EoS

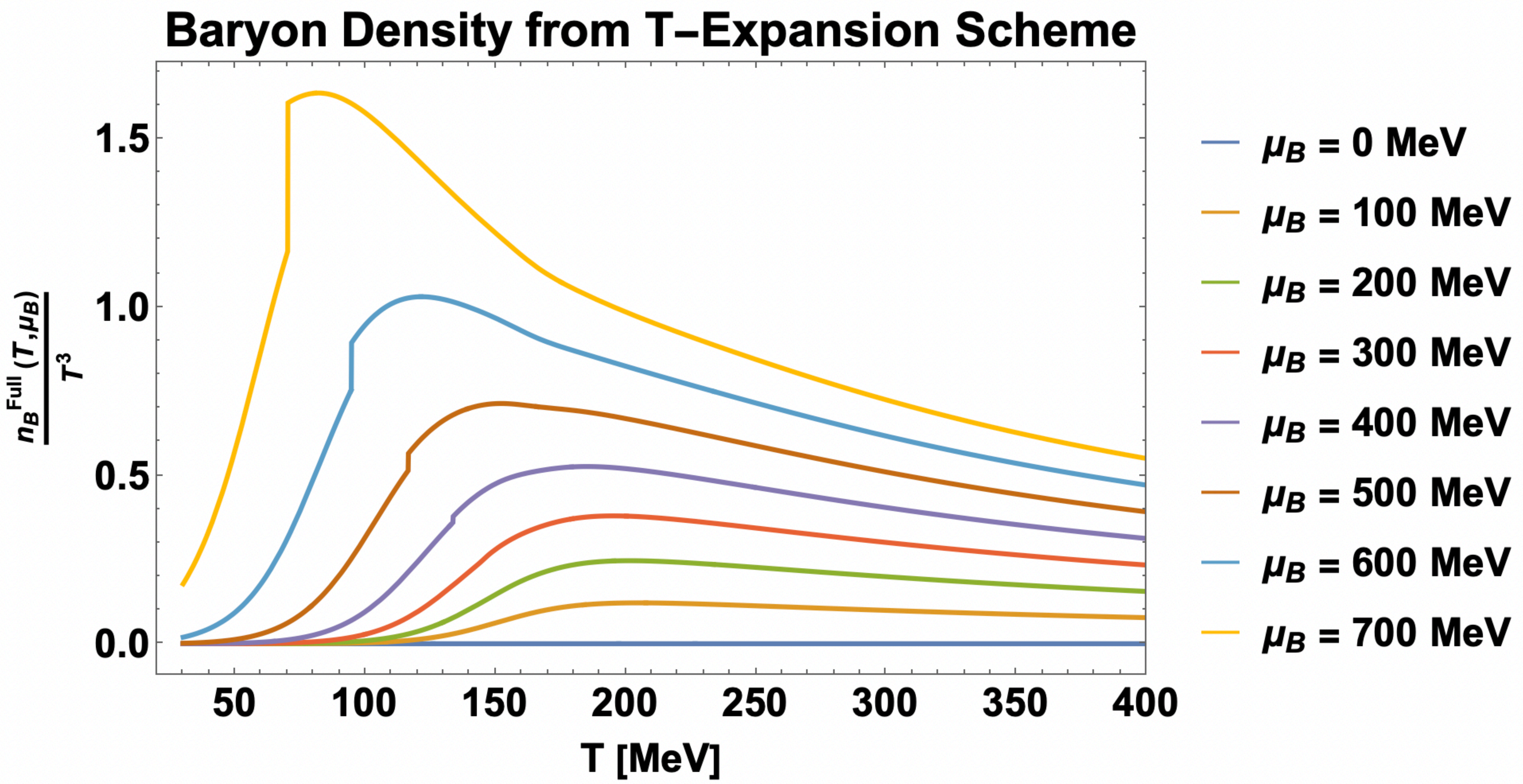
A. Pásztor Mon., M. Kahangirwe Tues.

- Initial formulation from Taylor expansion limited to $\mu_B \leq 450$ MeV
  - Utilize new lattice QCD results from an alternative expansion scheme to cover a larger μ_B range in the phase diagram

$$T'(T,\hat{\mu}_B) = T\left(1 + \kappa_2^{BB}(T)\hat{\mu}_B^2 + \kappa_4^{BB}(T)\hat{\mu}_B^4 + \mathcal{O}(\hat{\mu}_B^6)\right)$$

$$\kappa_2^{BB}(T) = \frac{1}{6T}\frac{\chi_4^B(T)}{\chi_2^{B\prime}(T)} \qquad \kappa_4^{BB}(T) = \frac{1}{360\chi_2^{B\prime}(T)^3}\left(3\chi_2^{B\prime}(T)^2\chi_6^B(T) - 5\chi_2^{B\prime\prime}(T)\chi_4^B(T)^2\right)$$





muses

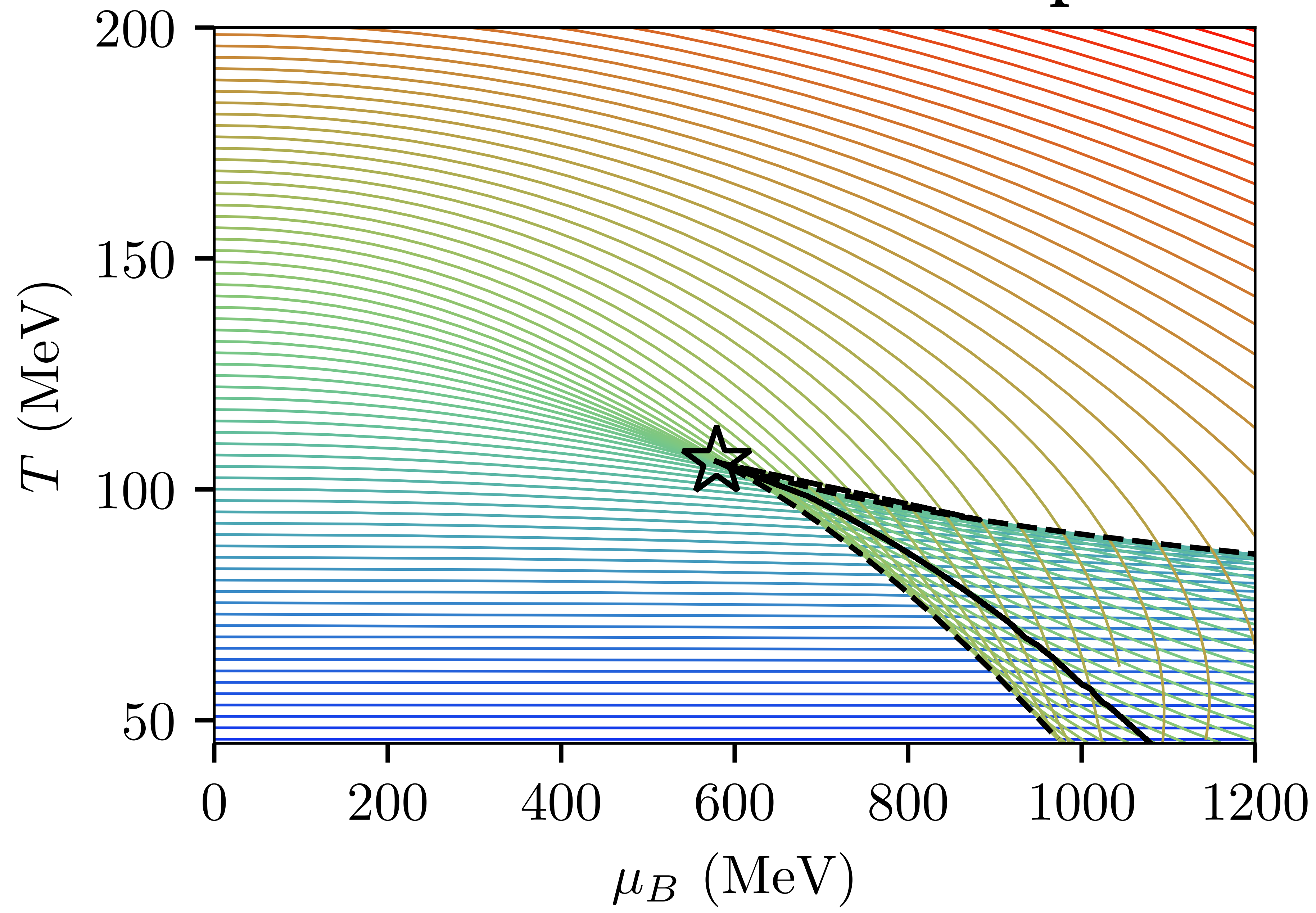
Holographic Equation of State for QCD

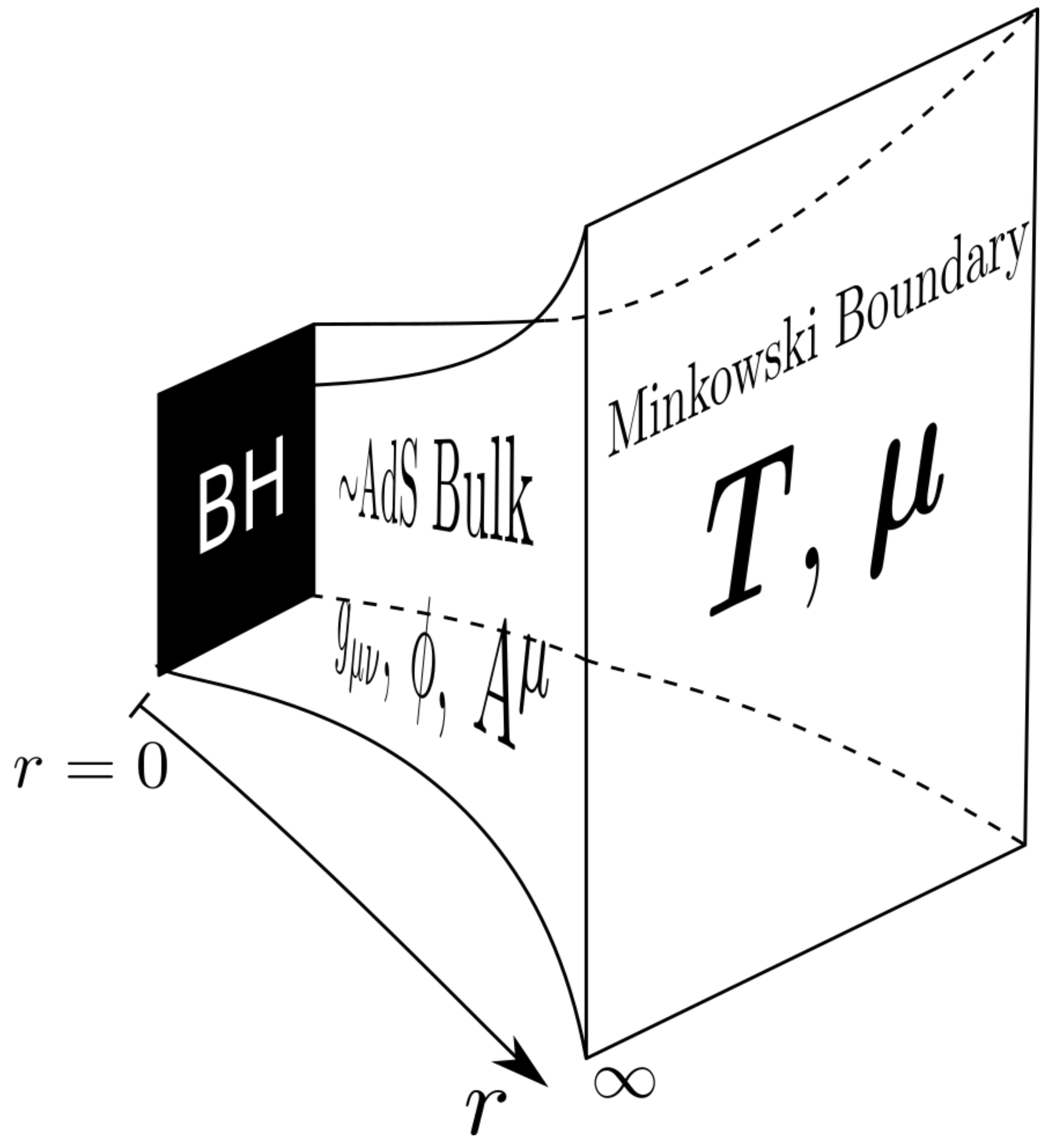
M. Hippert Tues., J. Grefa Wed.

- Alternatively, study critical features in the equation of state via the strongly-coupled black-hole-engineering approach

$$S = \frac{1}{2\kappa_5^2}\int_{\mathcal{M}_5} d^5x\sqrt{-g}\left[R - \frac{(\partial_\mu\phi)^2}{2} - V(\phi) - \frac{f(\phi)F_{\mu\nu}^2}{4}\right]$$

  - Constrained to mimic the lattice QCD equation of state at zero density
  - Provides an estimate for the location of the critical point





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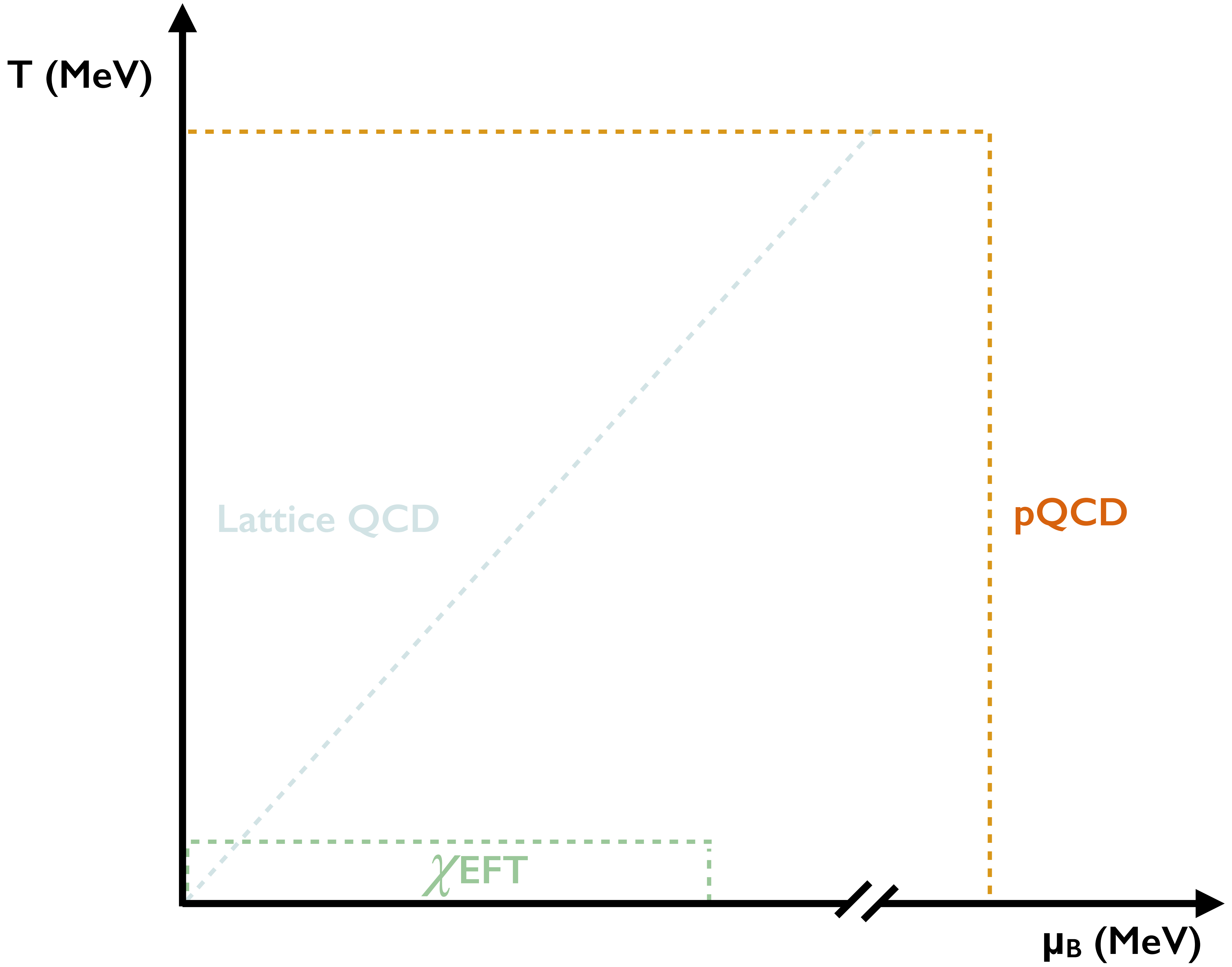
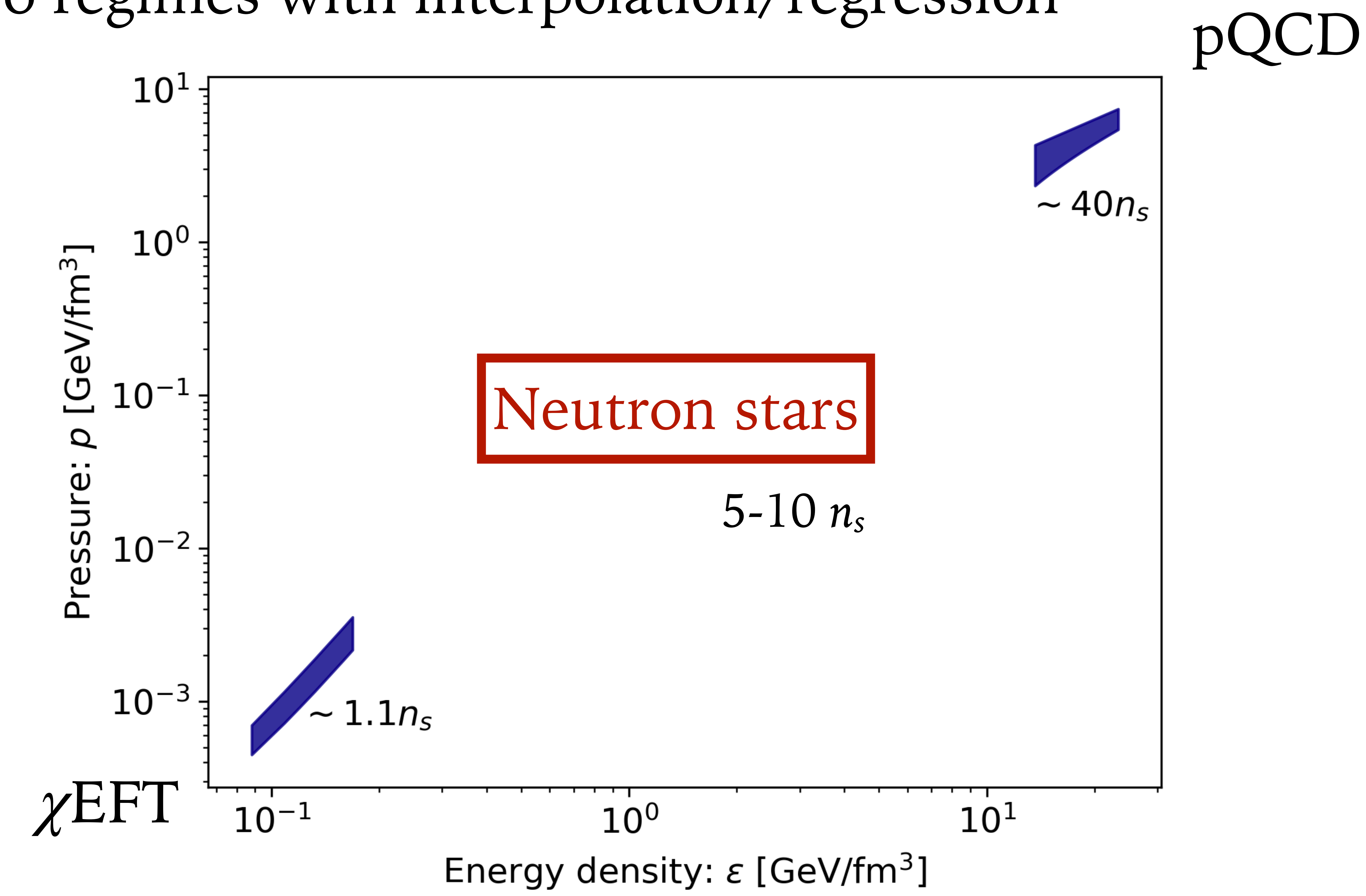
- Neutron stars & heavy-ion collisions



First-principles Dense Matter Equation of State

T. Gorda Mon.

- Theory predicts $T = 0$ equation of state around nuclear densities and at asymptotically high densities, while neutron stars live in between
  - Connect the two regimes with interpolation/regression

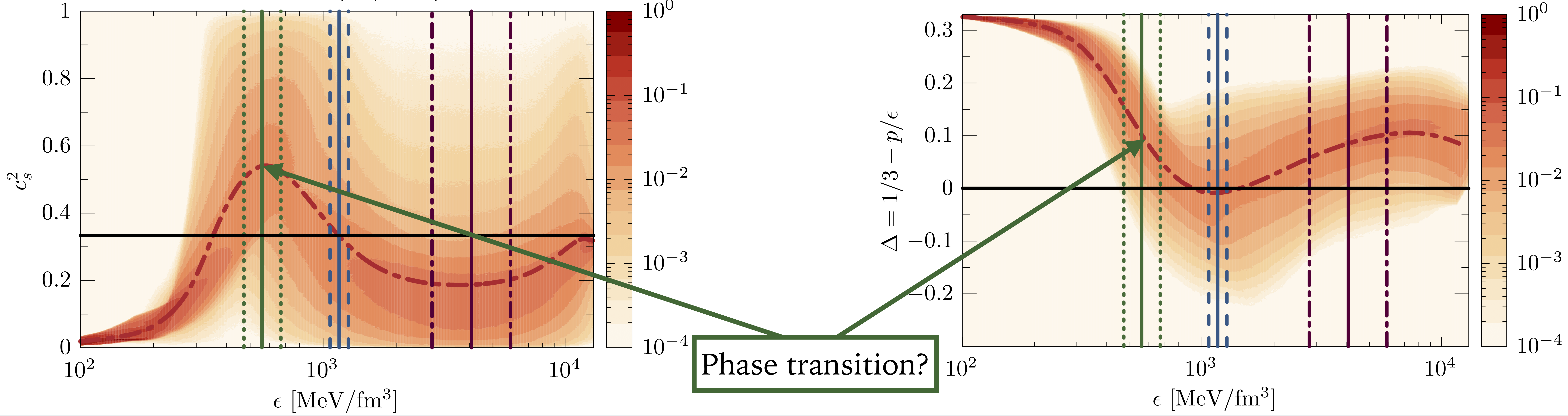


Further Thermodynamics of Dense Matter EoS

C. Sasaki Wed.

- Probe the neutron star equation of state with behavior of further quantities that affect the mass-radius curves
    - Does c_s^2 exceed 1/3 conformal value?
    - Is there a phase transition to quark matter within neutron stars?

$$c_s^2(\mu) = \frac{(\mu_{i+1} - \mu)\, c_{s,i}^2 + (\mu - \mu_i)\, c_{s,i+1}^2}{\mu_{i+1} - \mu_i}$$



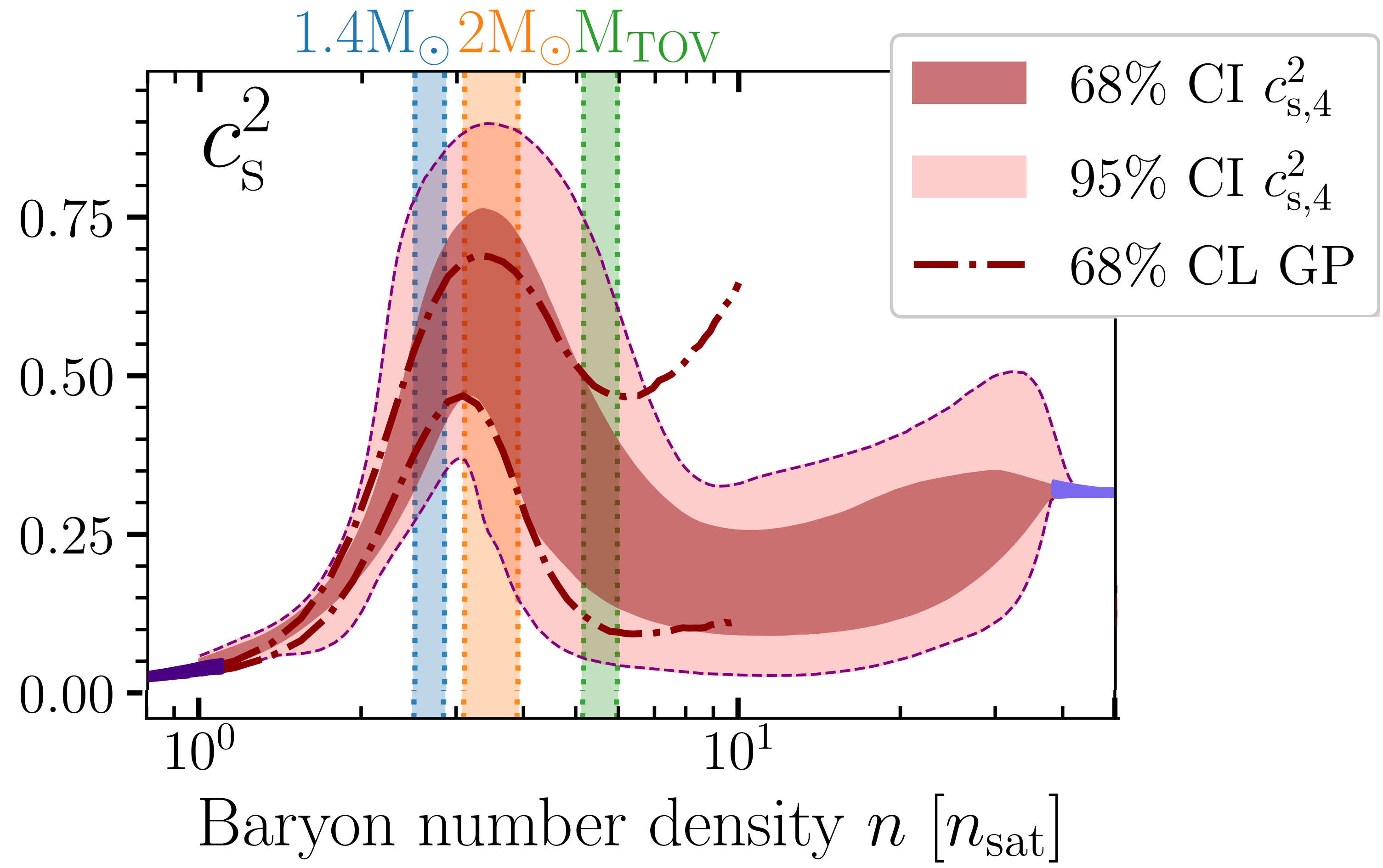
Bayesian Analysis for Neutron Star EoS

T. Gorda Mon., D. Mroczek Wed.

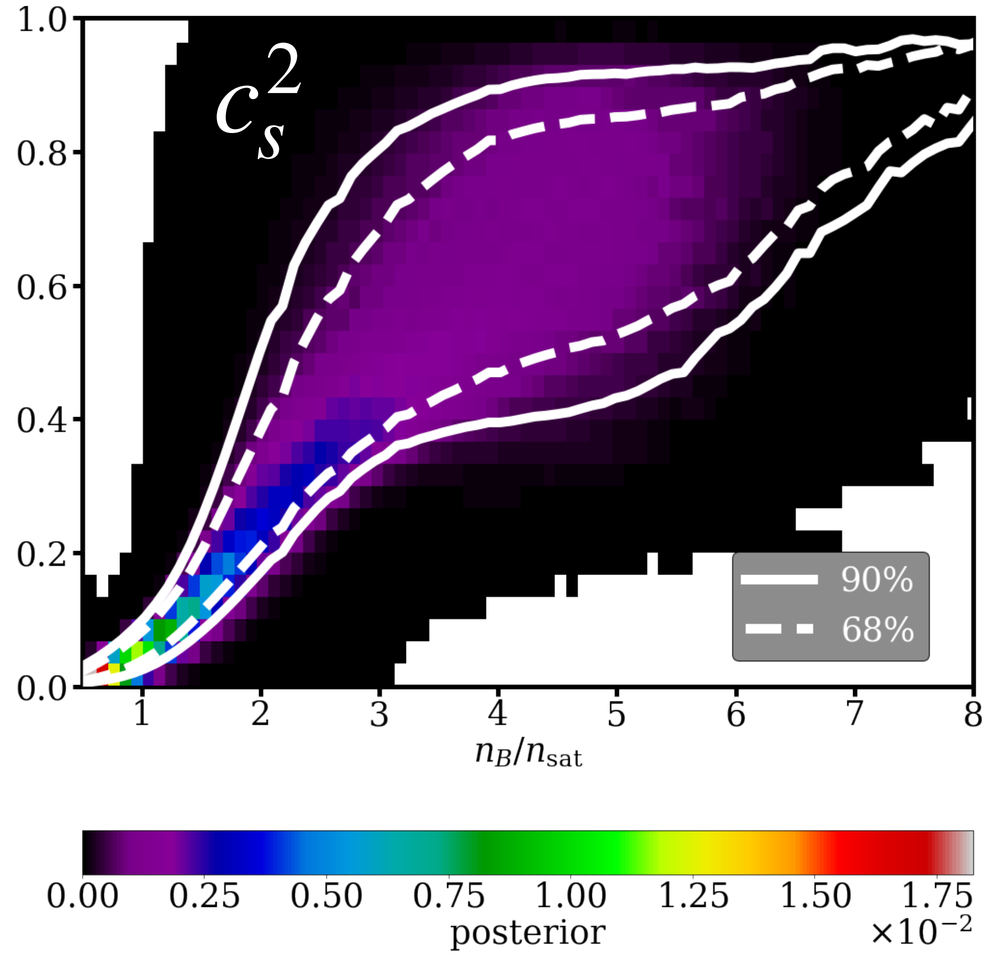
- Extract the dense matter equation of state via Bayesian studies

$$P(\mathrm{EoS}|\mathrm{data}) = \frac{P(\mathrm{data}|\mathrm{EoS})P(\mathrm{EoS})}{P(\mathrm{data})}$$

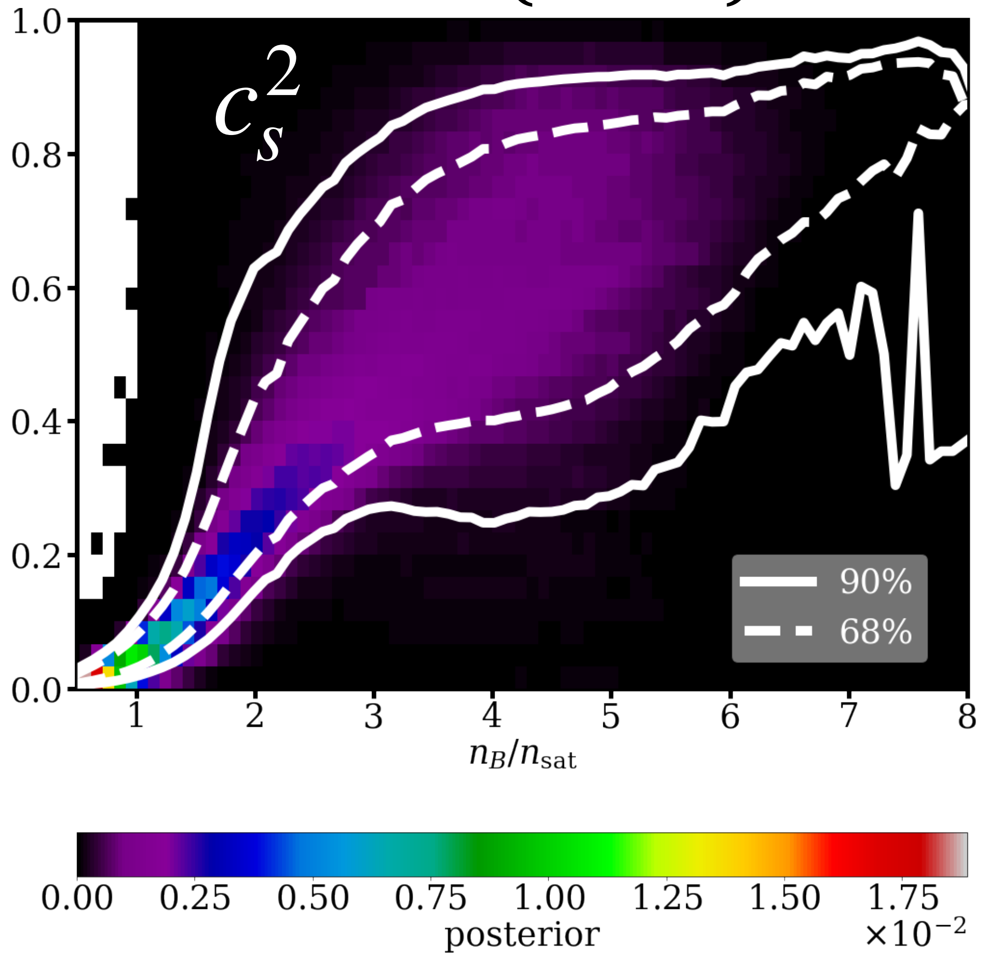
  - Phenomenological piecewise polytropic EoSs, non-parametric Gaussian Process (GP) or modified Gaussian Process (mGP) EoSs
  - Very sensitive to observational constraints/priors



Nucleonic EoS (GP)



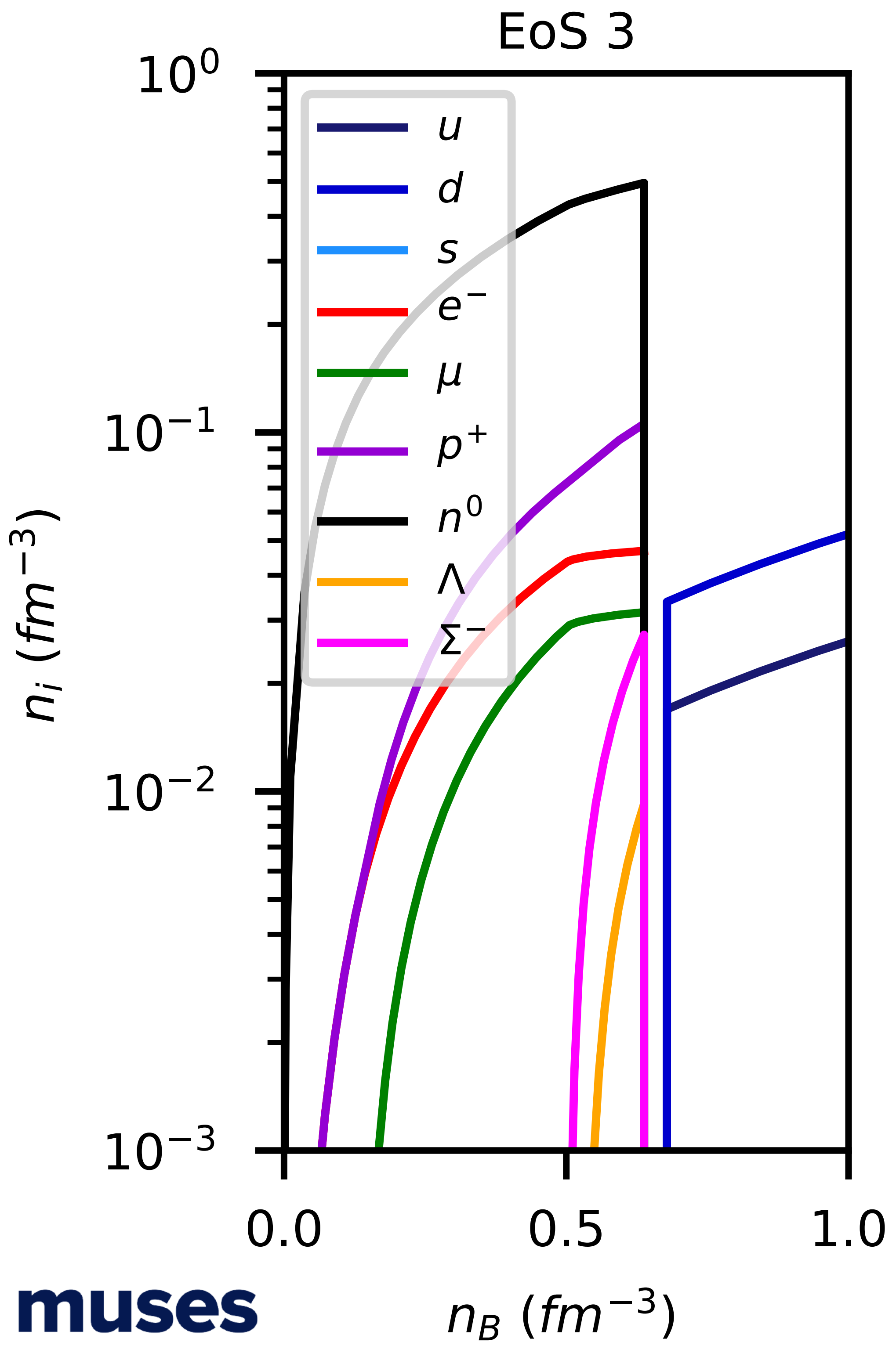
Nucleonic + exotic D.O.F. EoS (mGP)

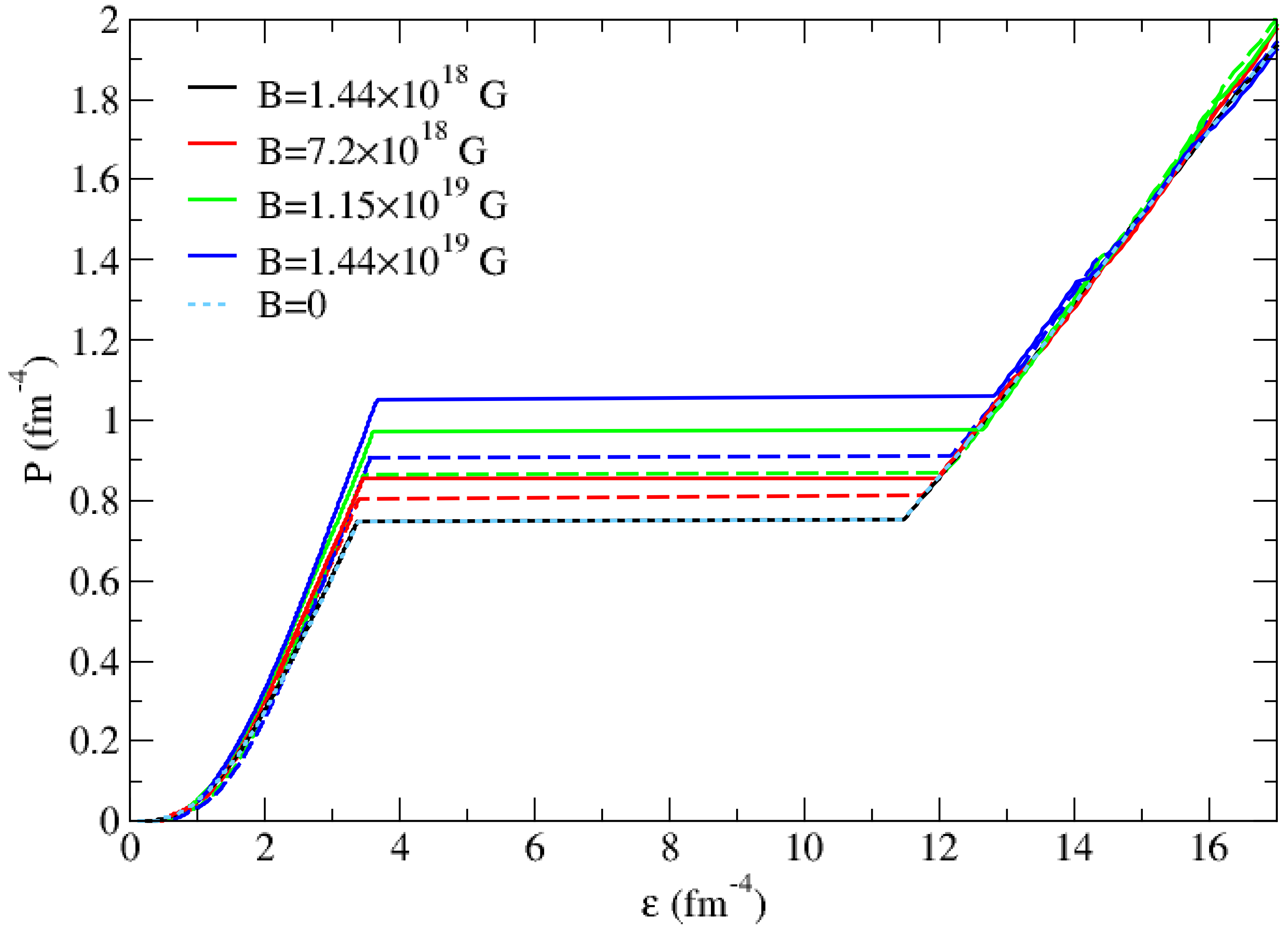


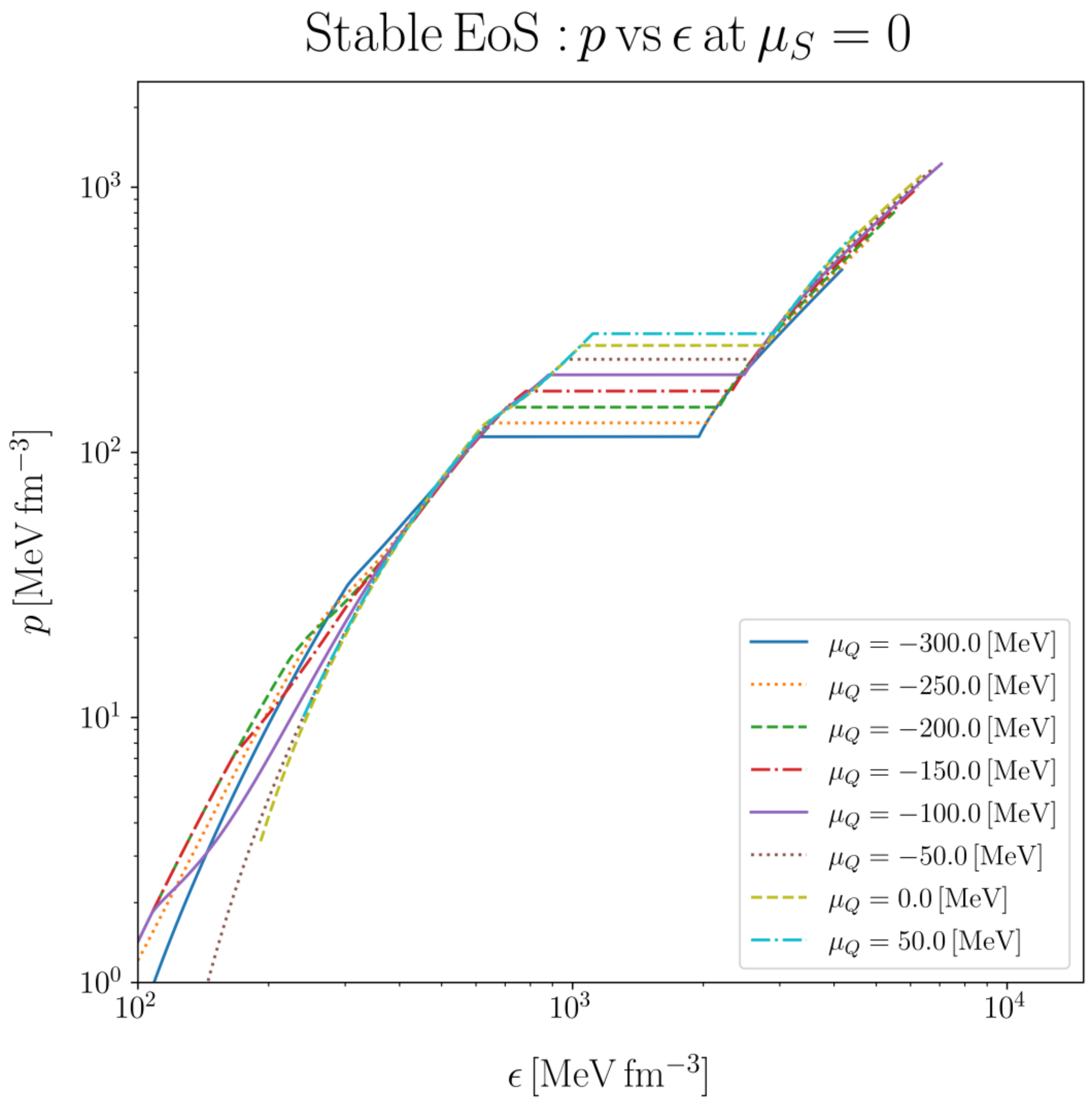
Chiral Mean Field Model

A. Clevinger Wed., V. Dexheimer Wed., N. Cruz Camacho Poster

- Increase phase diagram coverage with model equation of state with three-flavor chiral Lagrangian for hadronic matter
  - Include additional effects: hyperons/delta/quarks, magnetic field, chemical potentials

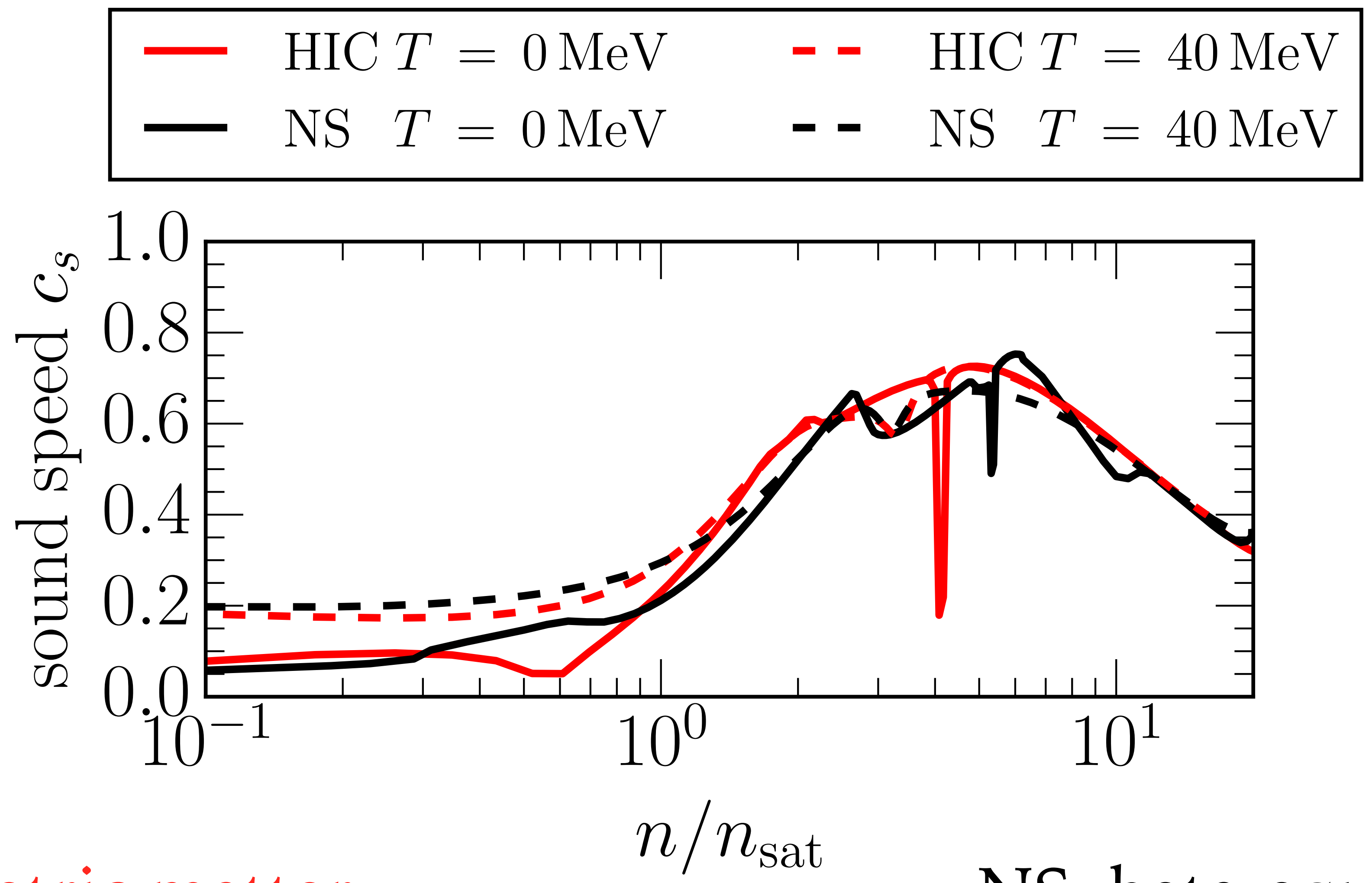






muses

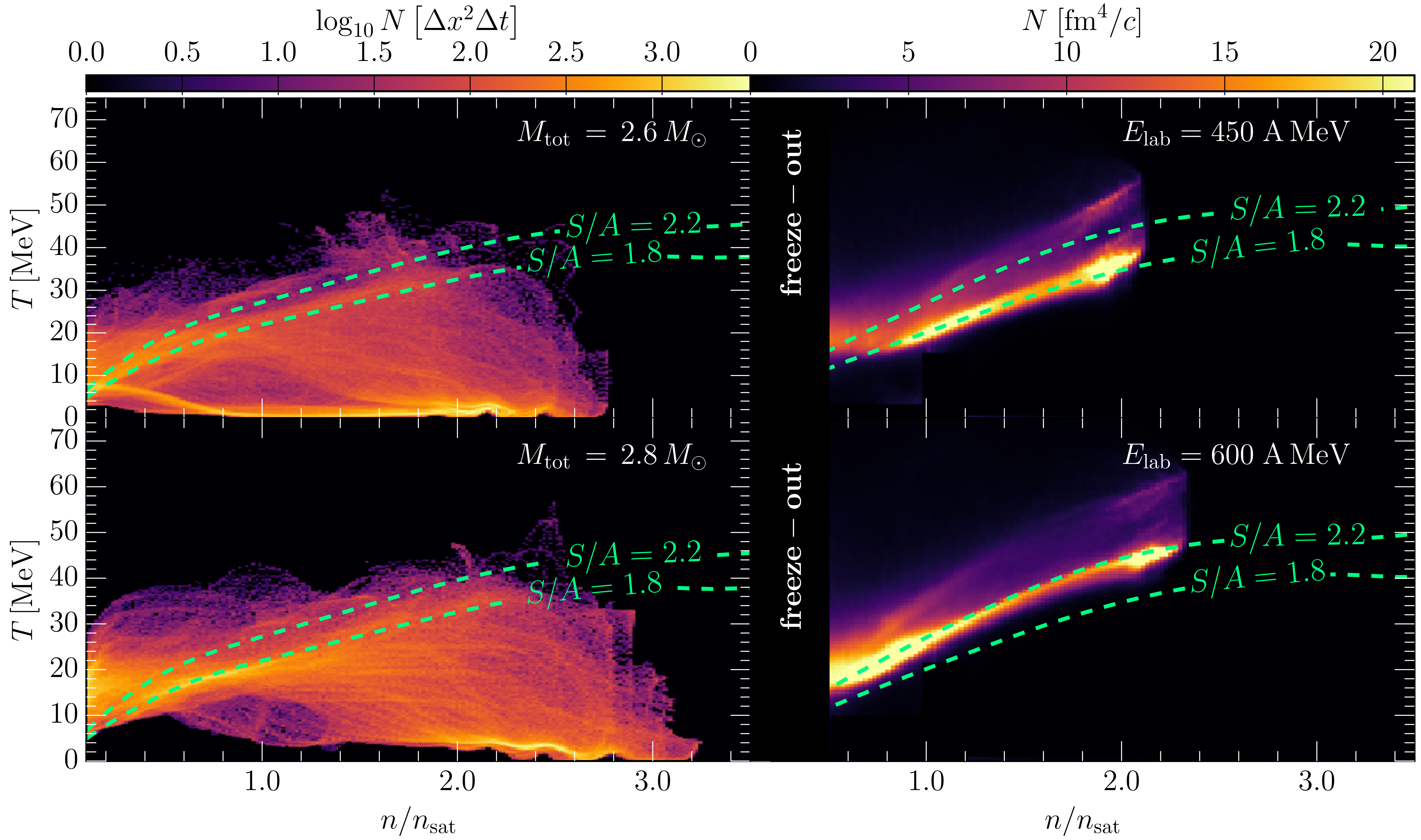
- Utilize the Chiral Mean Field model to study strongly-interacting matter in both binary neutron star mergers and heavy-ion collisions with $\sqrt{s_{NN}} \sim 1.5$ GeV
  - Similar behavior for sound speed (EoS)
  - Isentropic lines probe both types of matter



HIC: symmetric matter, strangeness neutral

NS: beta-equilibrated, charge neutral

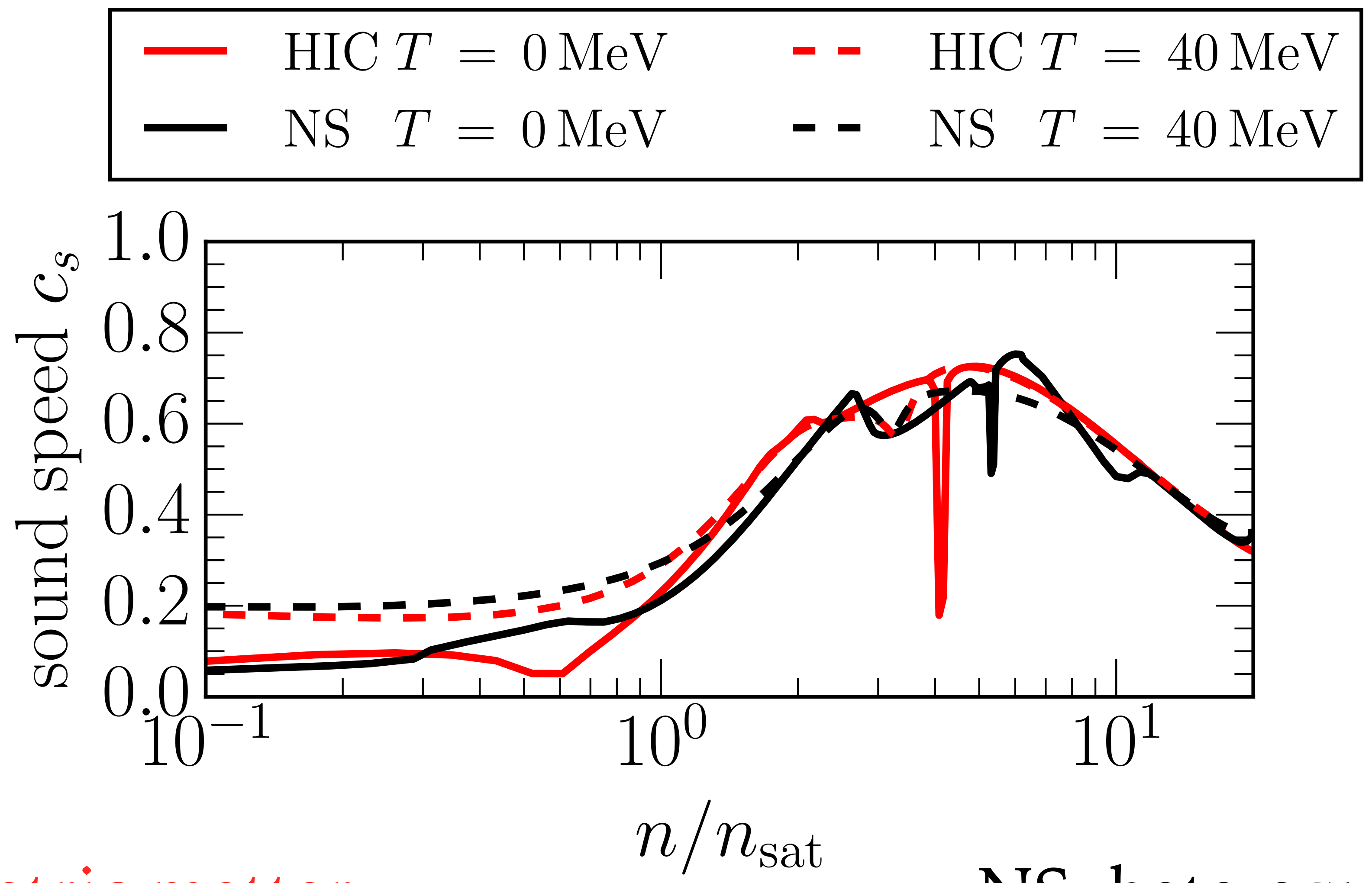
Simulations: BNSM v. HIC



E. Most, A. Motornenko et al, PRD (2023)

Neutron Star Mergers & Heavy-ion Collisions

- Utilize the Chiral Mean Field model to study strongly-interacting matter in both binary neutron star mergers and heavy-ion collisions with $\boxed{\sqrt{s_{NN}} \sim 1.5 \text{ GeV}}$
  - Similar behavior for sound speed (EoS)
  - Isentropic lines probe both types of matter



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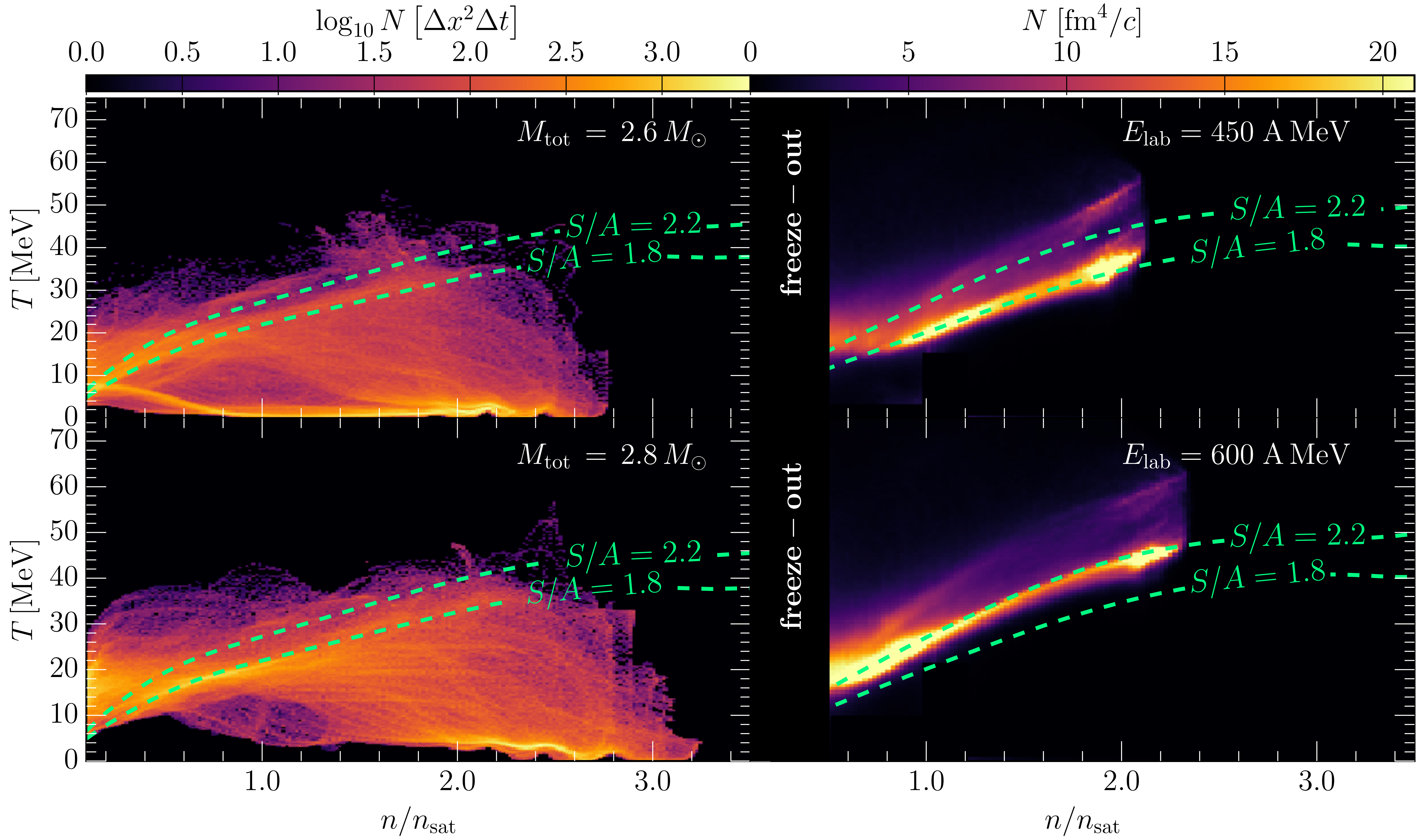

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Impact of HIC & NS on QCD Equation of State

- Many more works impacting the equation of state discussed at this conference!
- Flow - L. Du Tues., Z. Liu Tues., D. Almaalol Wed., I. Karpenko Wed., X. Liu Wed., S. Rav Sharma Poster
- Femtoscopic/hyperonic correlations - M. Grunwald Wed., H. Yu Wed., Posters: M. Stefaniak, M. Sharma, A. Jinno, N. Schild, A.A. Riedel, J. Ditzel, B. Heybeck, X. Li, S. Glaessel
- Sound speed - C. Bernardes Wed., N. Yao Poster
- Low energy heavy-ion collisions - D. Neff Tues., C. Hoehne Wed., M. Kohl Poster
- …

Conclusions

- Exciting era in which we can combine high energy nuclear physics & nuclear astrophysics

We seek knowledge of QCD matter, not HIC or NS matter!

- MUSES will provide custom equations of state that cover any desired portion of the phase diagram with user-chosen parameters

- Precise first principles results continue to help extend coverage of the phase diagram

- We look forward with anticipation to an even brighter era ahead with many new measurements/observations to come

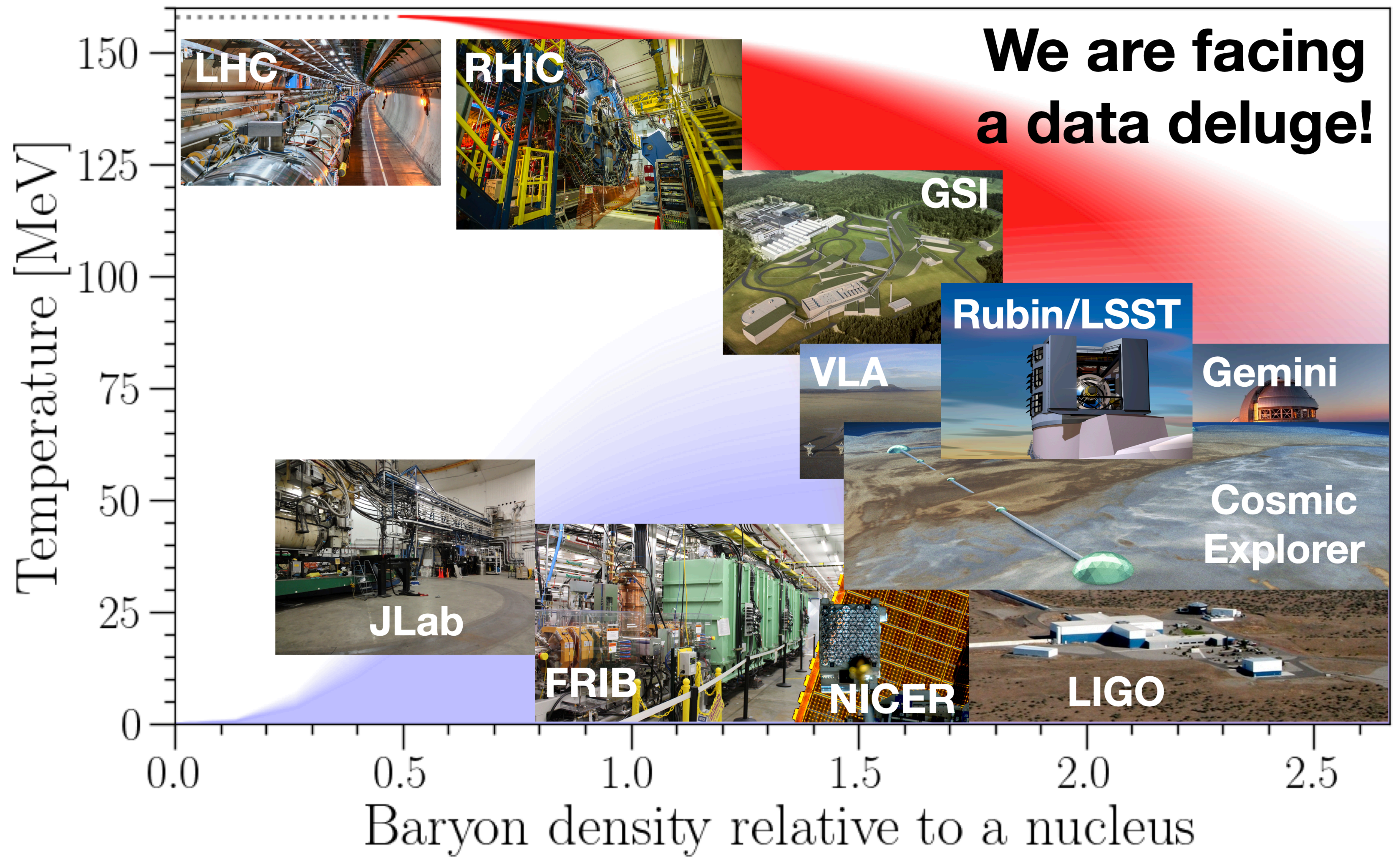

Figure by C. Ratti

ASCEND Fellow Outreach

- NuSTEAM Summer Program
  - Undergraduates spend 6 weeks in Houston on research and lectures & 2 weeks at BNL with poster presentation
  - My role ('21 & '22): professional development lecturer & mentor
- Filling the G-A-P-S
  - Graduate school application workshop series for junior & senior undergraduates at MIT, spin off of MIT LEAPS
  - My role ('22-'23): one-on-one review of materials & train graduate students to teach the workshops
- Postdoc Mentor for prospective NSF ASCEND fellows