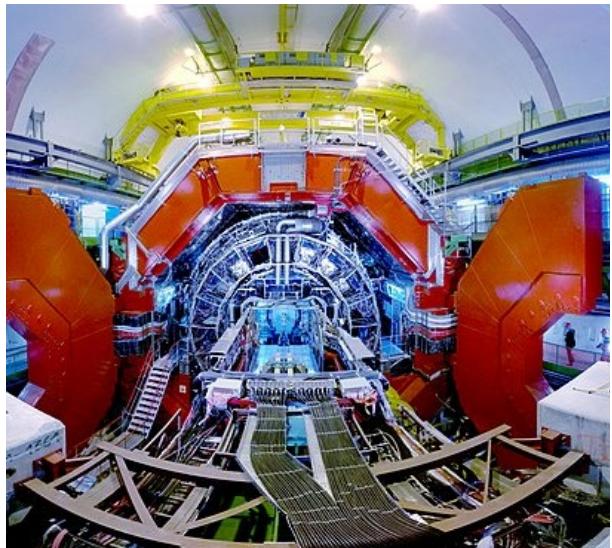
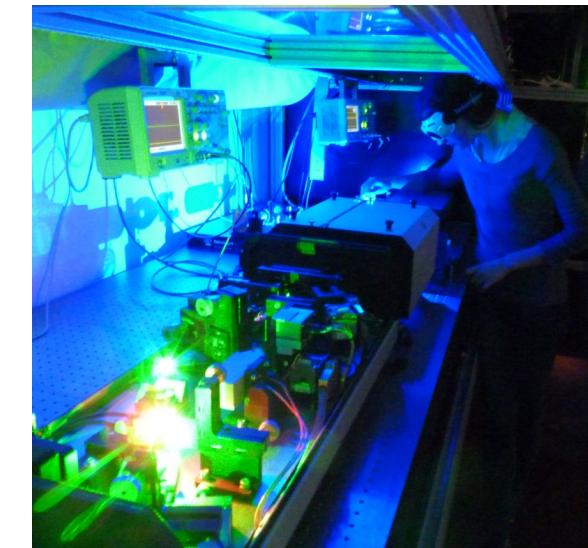


# What ultracold atoms tell us about the real-time dynamics of QCD in extreme conditions



Jürgen Berges  
Institute for Theoretical Physics  
Heidelberg University

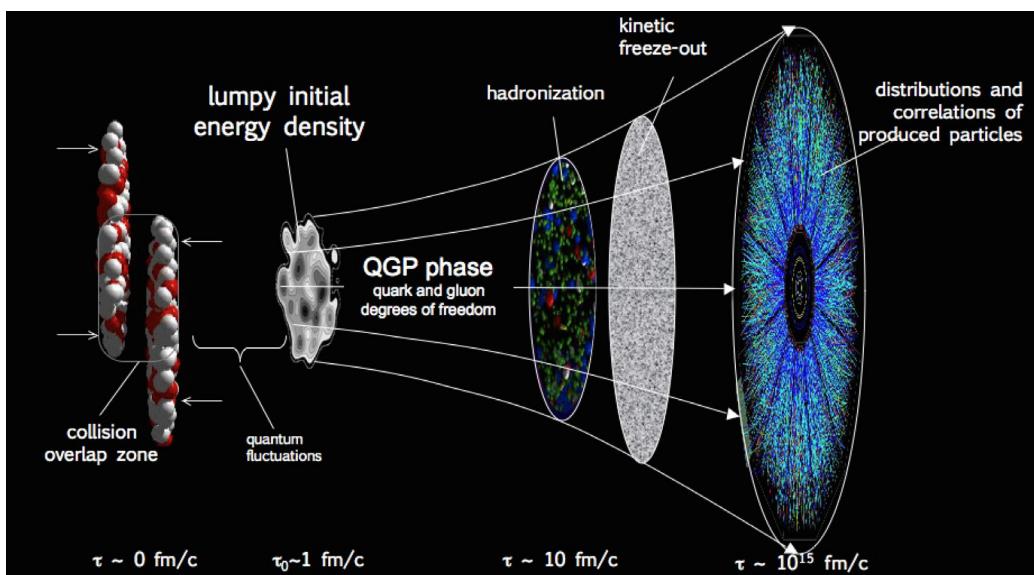
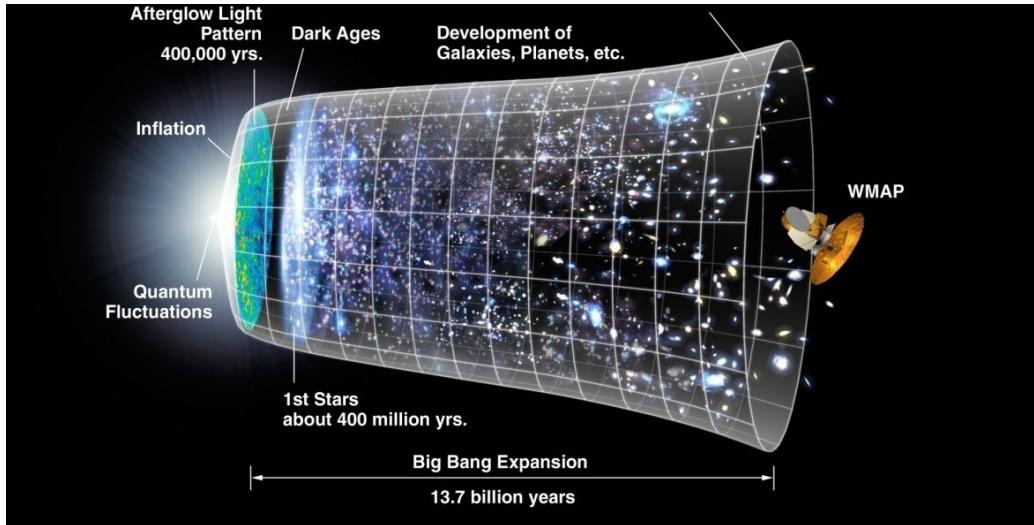
Quark Matter, September 2023



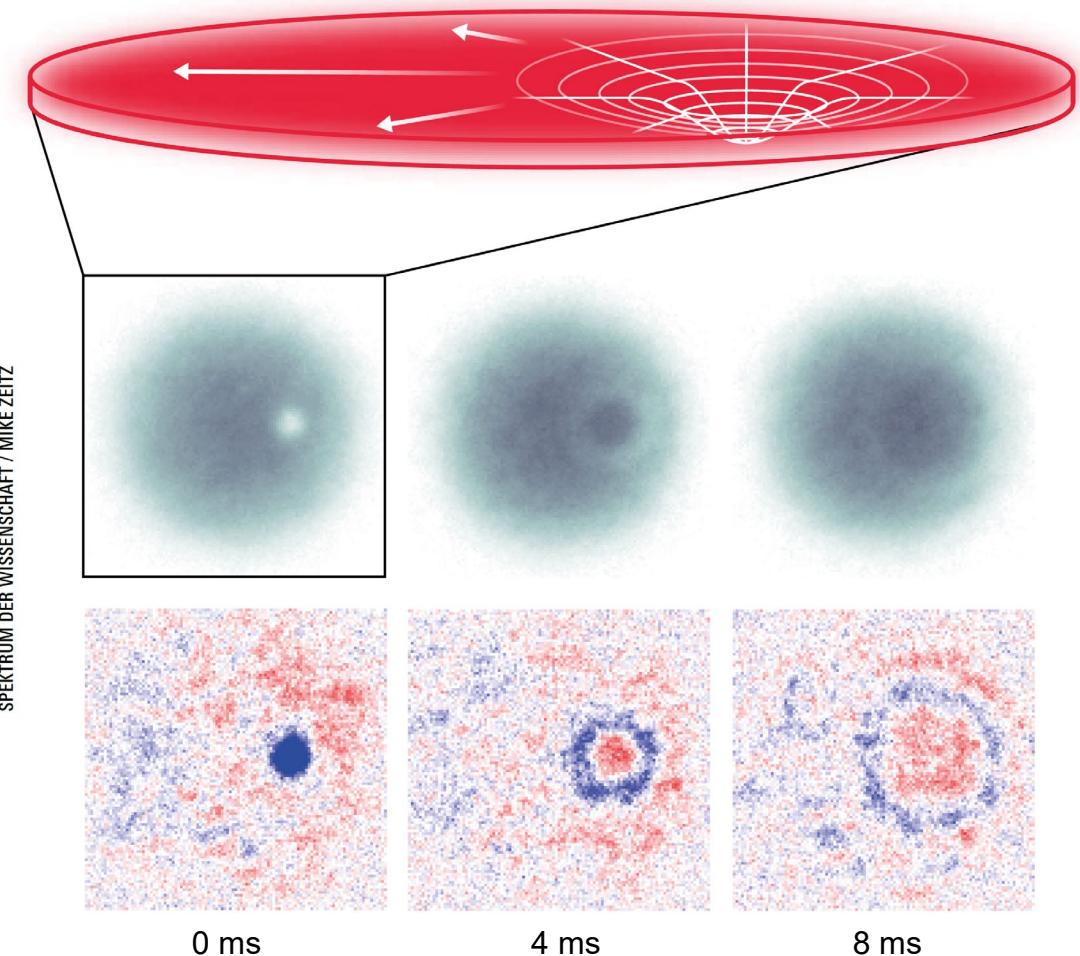
STRUCTURES  
CLUSTER OF  
EXCELLENCE

# Simulating quantum systems by other quantum systems

## ➤ Early Universe/Heavy-ion collisions



## ➤ Quantum simulations with ultracold quantum gases

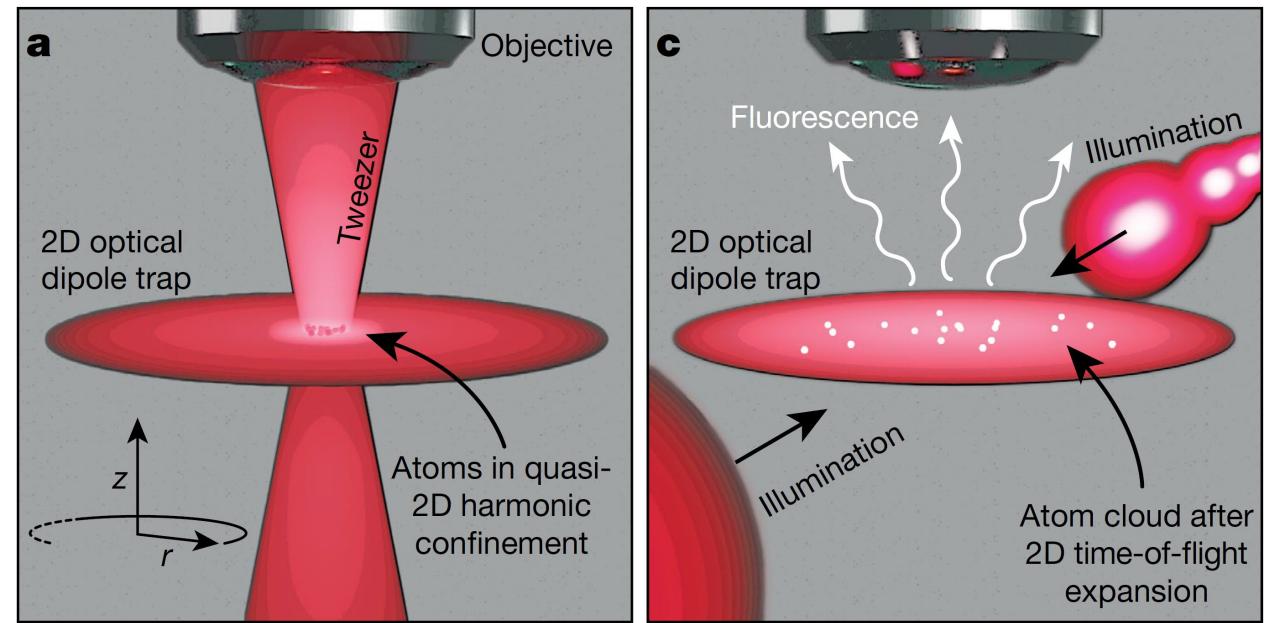
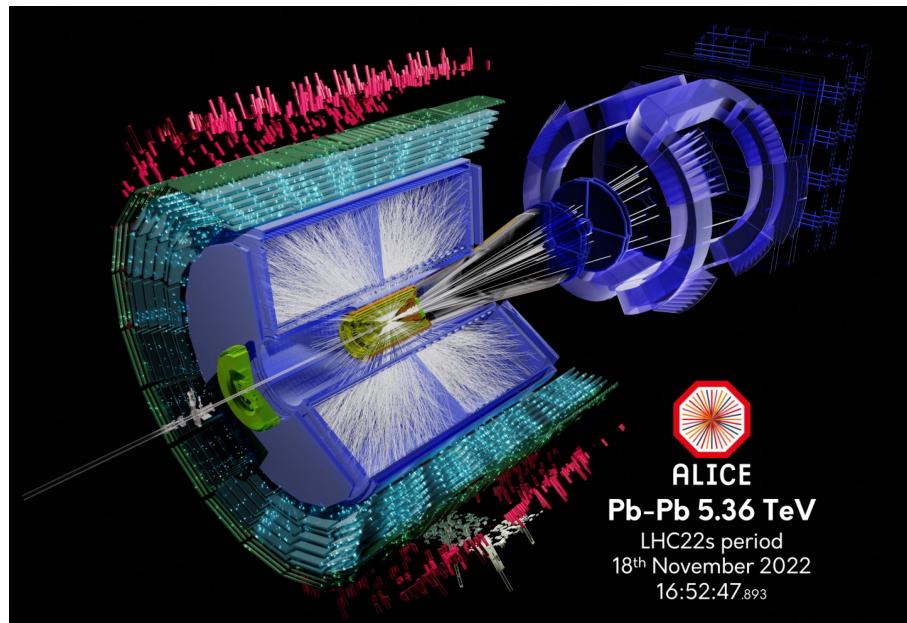


Viermann et al., Nature (2022) 611; Spektrum der Wissenschaft 6.23

# High experimental control of quantum simulators

Quantum simulators with ultracold atoms may help understanding the much more complex heavy-ion collisions:

- Table-top experiments with Bosons/Fermions as quantum resources
- Dimensionality of space, symmetry group, field content can be modified
- Interaction strength (and sign) tunable
- Well-controlled initial state preparation
- *Momentum and space imaging, time-resolved data*



Holten et al., Nature 606 (2022) 287

# Quantum simulation of gauge field dynamics

## ➤ State-of-the-art examples:

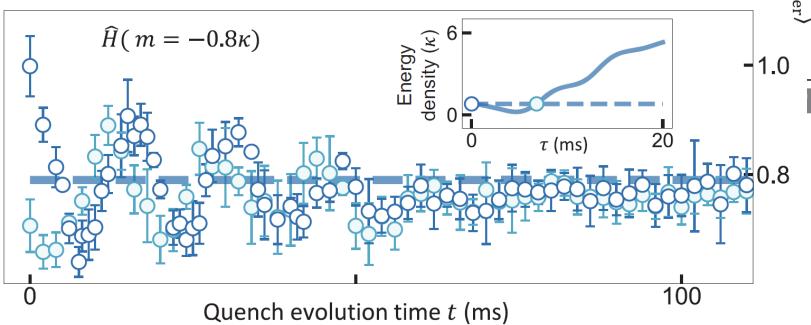
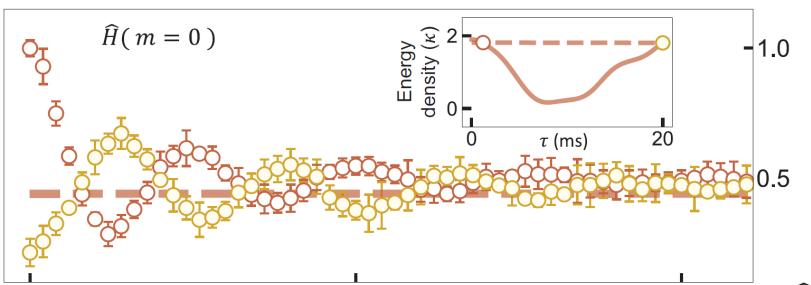
1+1 space-time dimensions

### Thermalization dynamics of a gauge theory on a quantum simulator

*Science* **377**, 311–314 (2022)

Zhao-Yu Zhou<sup>1,2,3,4\*</sup>†, Guo-Xian Su<sup>1,2,3,4</sup>†, Jad C. Halimeh<sup>5</sup>, Robert Ott<sup>6</sup>, Hui Sun<sup>1,2,3,4</sup>, Philipp Hauke<sup>5</sup>, Bing Yang<sup>3,7</sup>‡, Zhen-Sheng Yuan<sup>1,2,3,4,8</sup>, Jürgen Berges<sup>6</sup>, Jian-Wei Pan<sup>1,2,3,4,8</sup>

U(1) lattice gauge theory, 71 sites  $^{87}\text{Rb}$

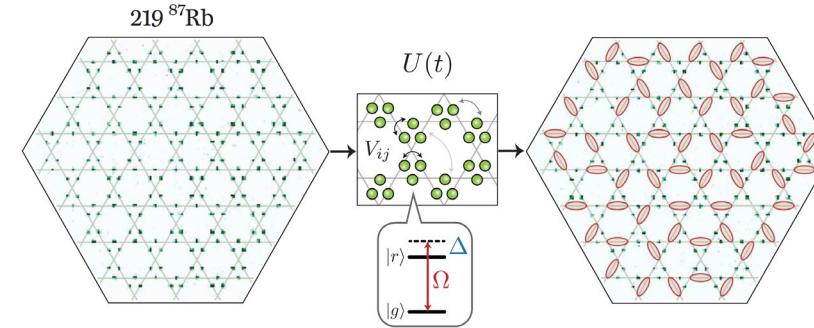


2+1 space-time dimensions

### Probing topological spin liquids on a programmable quantum simulator

$\sim \mathbb{Z}_2$  gauge theory

G. Semeghini<sup>1</sup>, H. Levine<sup>1</sup>, A. Keesling<sup>1,2</sup>, S. Ebadi<sup>1</sup>, T. T. Wang<sup>1</sup>, D. Bluvstein<sup>1</sup>, R. Verresen<sup>1</sup>, H. Pichler<sup>3,4</sup>, M. Kalinowski<sup>1</sup>, R. Samajdar<sup>1</sup>, A. Omran<sup>1,2</sup>, S. Sachdev<sup>1,5</sup>, A. Vishwanath<sup>1,\*</sup>, M. Greiner<sup>1,\*</sup>, V. Vuletić<sup>6,\*</sup>, M. D. Lukin<sup>1,\*</sup>

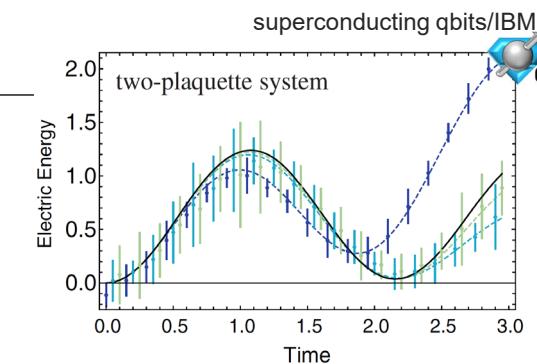
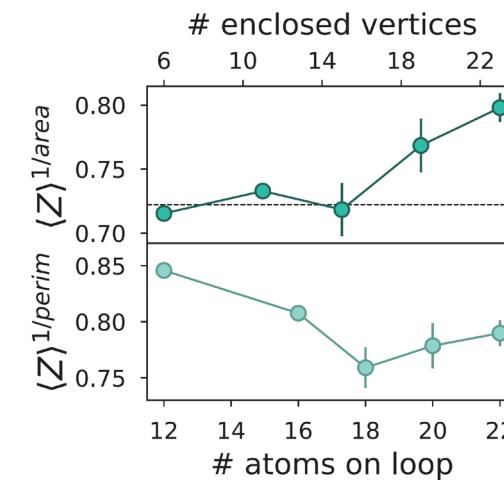


PHYSICAL REVIEW D **103**, 094501 (2021)

### Suggestion Trailhead for quantum simulation of SU(3) Yang-Mills lattice gauge theory in the local multiplet basis

Anthony Ciavarella<sup>1,\*</sup>, Natalie Klco<sup>2,†</sup> and Martin J. Savage<sup>1,‡</sup>

*Science* **374**, 1242–1247 (2021)



Martinez et al., *Nature* **534** (2016); Klco et al., *PRA* **98** (2018); Kokail et al., *Nature* **569** (2019); Lu et al., *PRA* **100** (2019); Mil et al., *Science* **367** (2020); Surace et al., *PRX* **10** (2020); Yang et al., *Nature* **587** (2020); Mueller et al., *PRD* **102** (2020); Atas et al., *Nat. Com.* **12** (2021); Klco et al., *PRD* **101** (2020); Rahman et al., *PRD* **104** (2021); *PRD* **106** (2022); Nguyen, *PRX Quantum* **3** (2022); Mueller et al., *arXiv*: 2210.03089; Farrell et al., *PRD* **107** (2023); Belyansky et al., *arXiv*: 2307.02522; Ciavarella, *arXiv*: 2307.05593;... Recent review: Bauer, Davoudi, Klco, Savage, *Nature Rev. Phys.* **5** (2023)

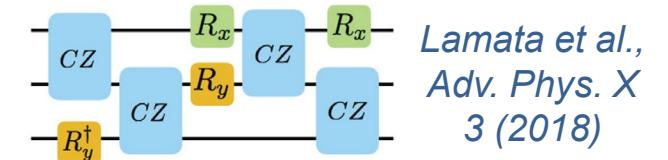
# Analog vs. digital quantum simulations

- **Digital quantum simulation:** programmable

→ see also talk on *Quantum Computing by Savage*

$$e^{-iHt} \underset{l}{\simeq} \left( e^{-iH_1 t/l} \cdots e^{-iH_M t/l} \right)^l + \sum_{i < j} \frac{[H_i, H_j] t^2}{2l} , \quad H = \sum_k^M H_k$$

Trotterized unitary evolution, ‘full state tomography’



sequence of gates to reproduce  $H$

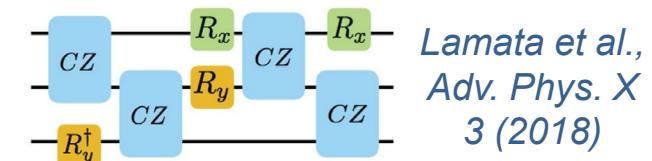
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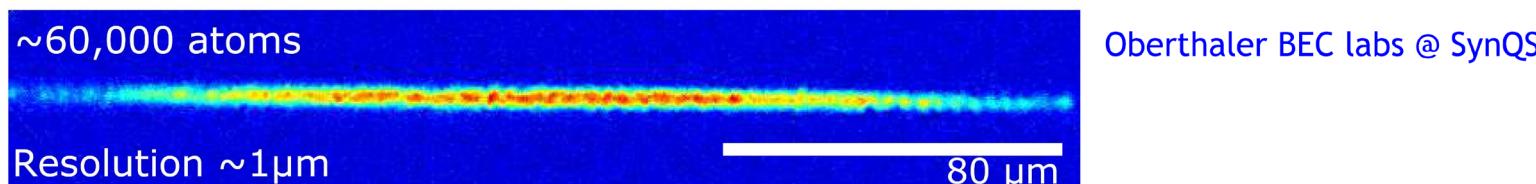
$$e^{-iHt} \underset{l}{\approx} \left( e^{-iH_1 t/l} \cdots e^{-iH_M t/l} \right)^l + \sum_{i < j} \frac{[H_i, H_j] t^2}{2l} , \quad H = \sum_k^M H_k$$

Trotterized unitary evolution, 'full state tomography'



sequence of gates to reproduce  $H$

- Analog quantum simulation: large-scale systems with excellent coherence available in the laboratory

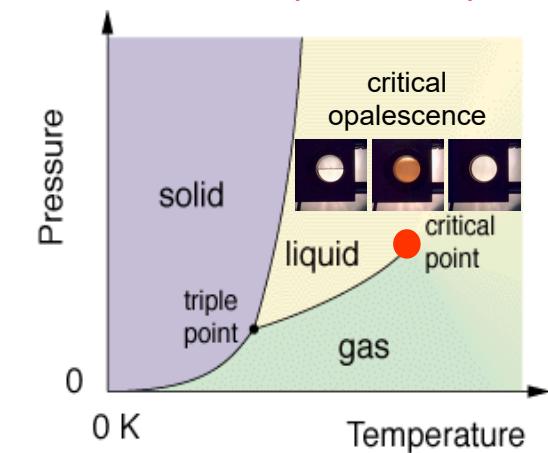


Similar  $H$  within same renormalization/universality class

tremendous simplification for quantum simulations

→ **Universality far from equilibrium?**

**Universality in equilibrium**  
→ QCD critical point, compare:

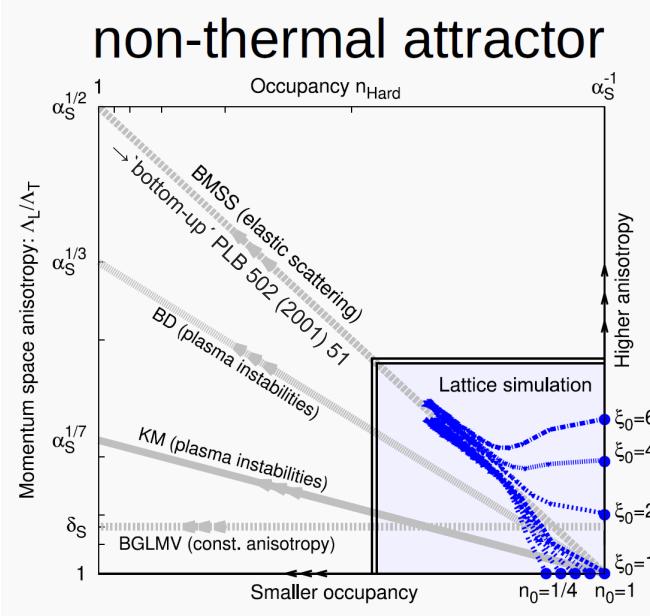


<http://ltl.tkk.fi/research/theory/TypicalPD.gif>  
<https://wiki.brown.edu/confluence/display/physlecdemo/4C50.10+Critical+Opalescence>

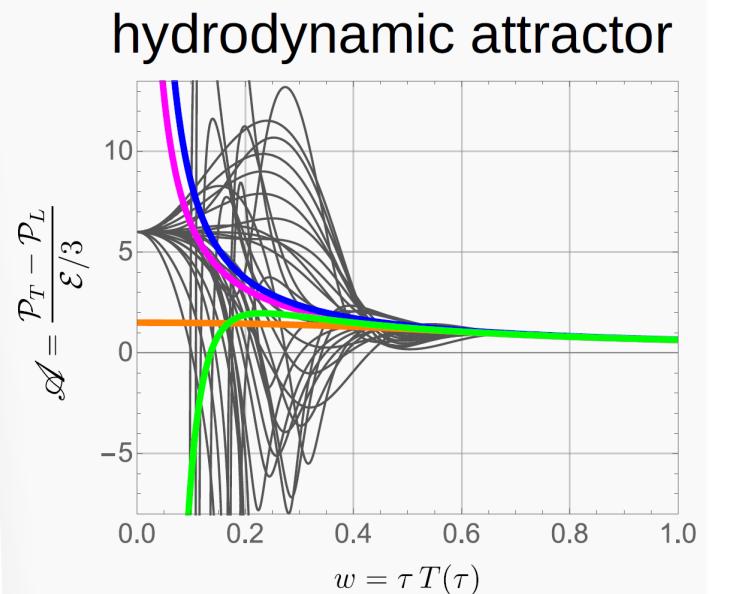
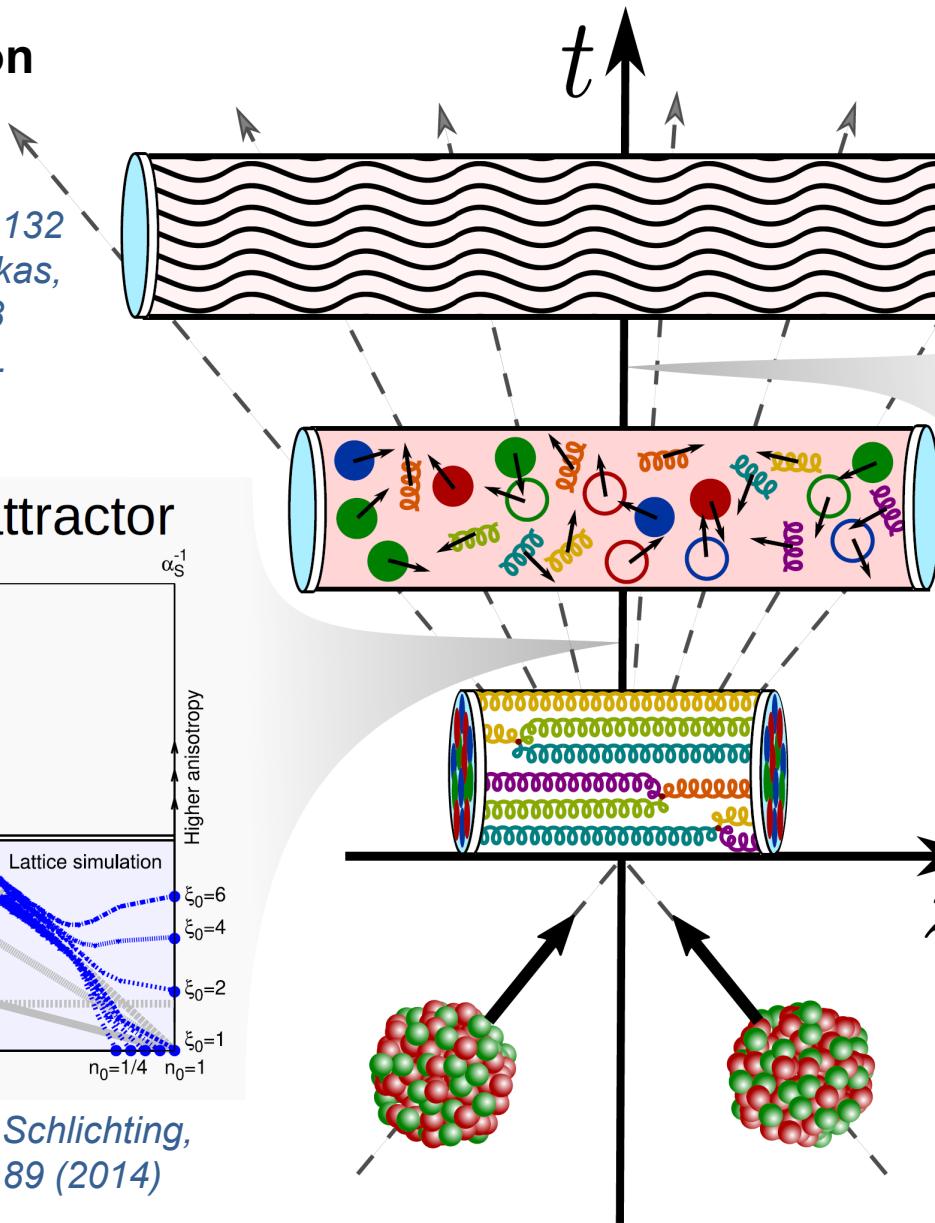
# Effective loss of information: attractors in the QGP evolution

## ➤ Schematic QGP evolution

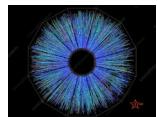
Recent reviews: Jankowski, Spaliński, Prog. Part. Nucl. Phys. 132 (2023); Berges, Heller, Mazeliauskas, Venugopalan, Rev. Mod. Phys. 93 (2021); Schlichting, Teaney, Annu. Rev. Nucl. Part. Sci. 69 (2019); ...



Berges, Boguslavski, Schlichting, Venugopalan, PRD 89 (2014)

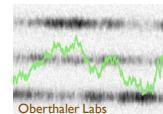
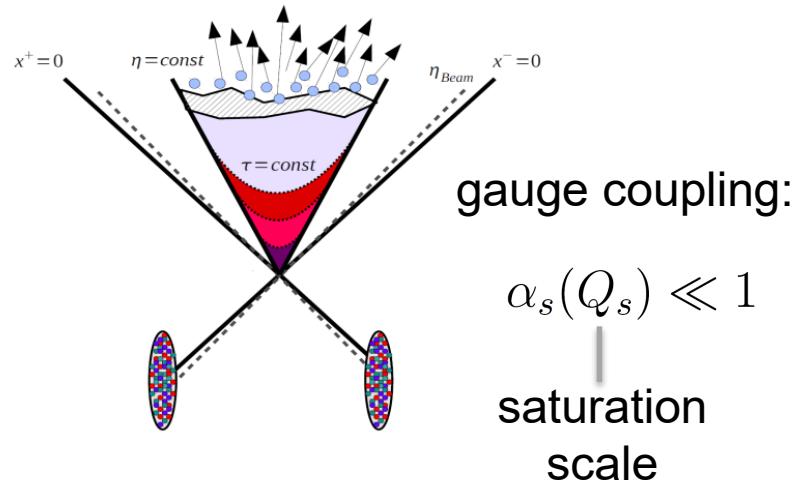


# Dictionary: far-from-equilibrium initial conditions



## Heavy-ion collisions

'Color Glass Condensate':  
McLerran, Venugopalan,...



## Ultracold atoms

'Bose-Einstein condensation':  
Cornell, Ketterle, Wieman



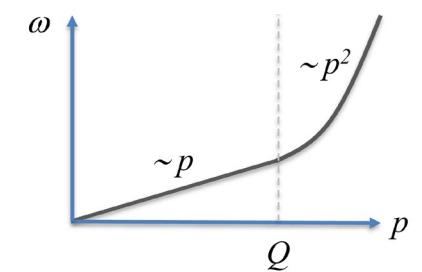
typical scattering length:  $a \simeq 5 \text{ nm}$   
typical bulk density:  $n \simeq 10^{14} \text{ cm}^{-3}$

diluteness:  $\zeta \simeq 10^{-3}$

$$\zeta = \sqrt{n a^3} \ll 1$$

coherence length $^{-1}$ :

$$Q = \sqrt{16\pi n a}$$



**Strongly correlated initial condition for subsequent equilibration process: over-occupation**

$$\frac{\langle A^2 \rangle(Q_s)}{\langle A^2 \rangle_{\text{eq}}(T)} \sim \frac{1}{\alpha_s}$$

$$(\hbar = c = k_B = 1)$$

$$\frac{\langle \phi^2 \rangle(Q)}{\langle \phi^2 \rangle_{\text{eq}}(T)} \sim \frac{1}{\zeta}$$

# Pre-equilibrium dynamics: universality far from equilibrium

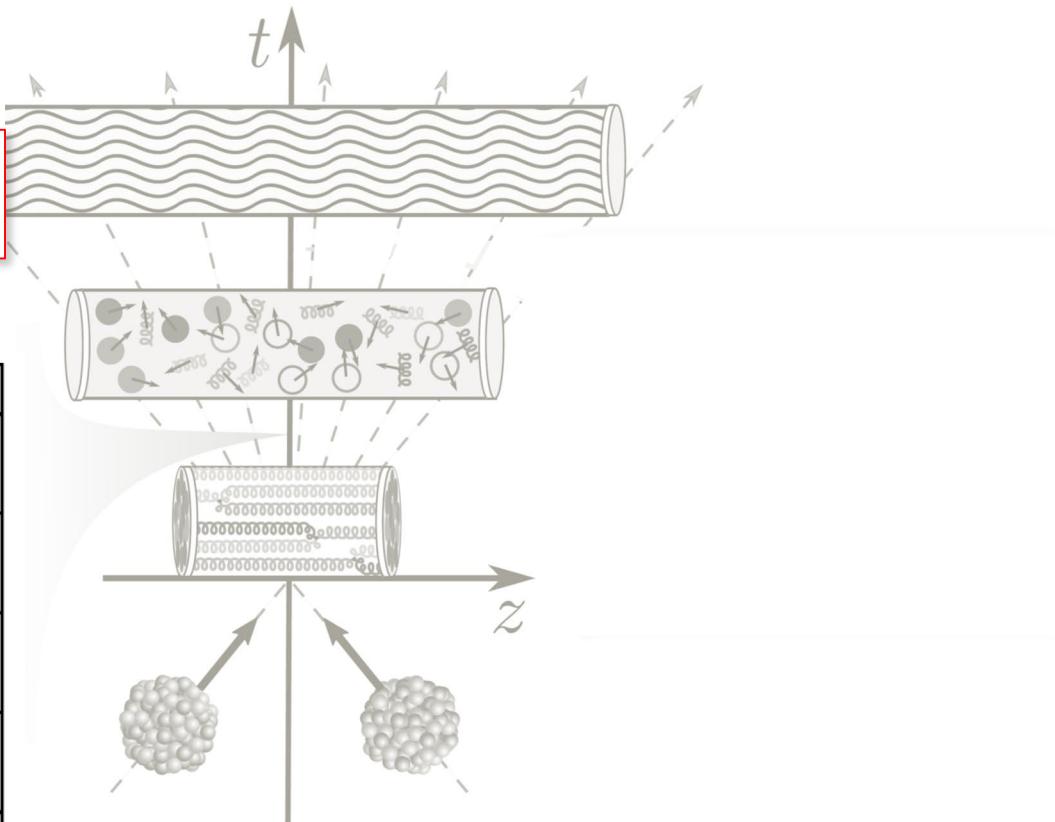
➤ Non-thermal attractor: self-similar scaling

Universal scaling form of distribution function:

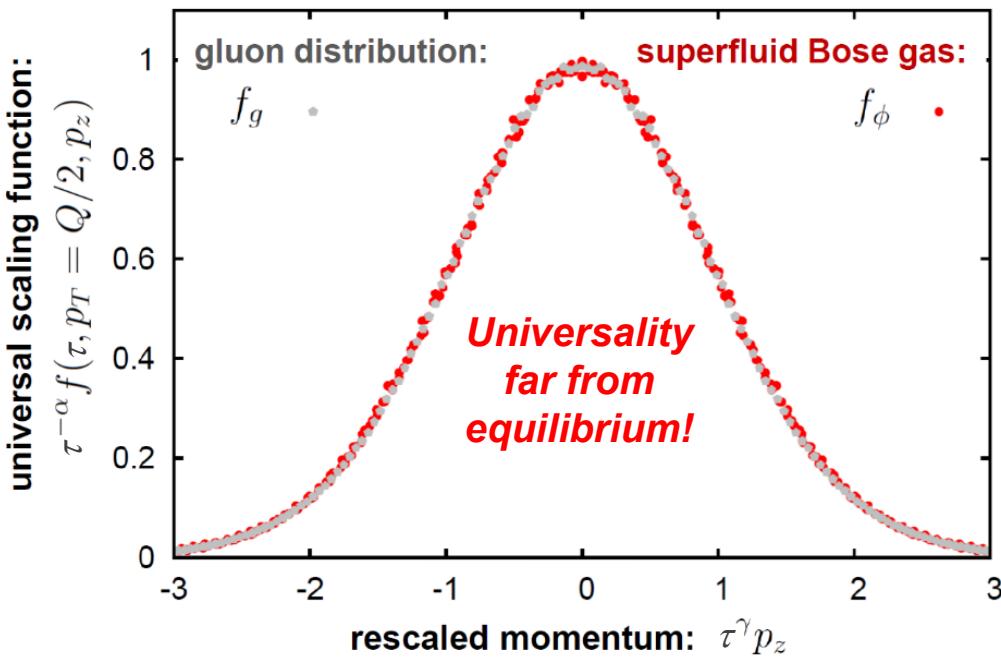
$$\tau = \sqrt{t^2 - z^2}$$

**universal scaling exponents**

$$f(p_T, p_z, \tau) = (Q\tau)^\alpha f_S((Q\tau)^\beta p_T, (Q\tau)^\gamma p_z)$$



Classical-statistical comparison (expanding/relativistic):



Berges, Boguslavski, Schlichting, Venugopalan,  
PRL 114 (2015)

QCD/Cold atom prescaling: Heller, Mazeliauskas, Preis, arXiv:2307.07545; Brewer, Scheihing-Hitschfeld, Yin, JHEP 05 (2022); Mikheev, Mazeliauskas, Berges, PRD 105 (2022); Mazeliauskas, Berges, PRL 122 (2019); Schmied, Mikheev, Gasenzer, PRL 122 (2019)... Cf. hydro attractor: Alalawi, Strickland, JHEP 143 (2022), Brewer, Yan, Yin, PLB 816 (2021); Almaalol, Kurkela, Strickland, PRL 125 (2020); Kurkela, van der Schee, Wiedemann, Wu, PRL 124 (2020)...

# Pre-equilibrium dynamics: universality far from equilibrium

➤ Non-thermal attractor: self-similar scaling

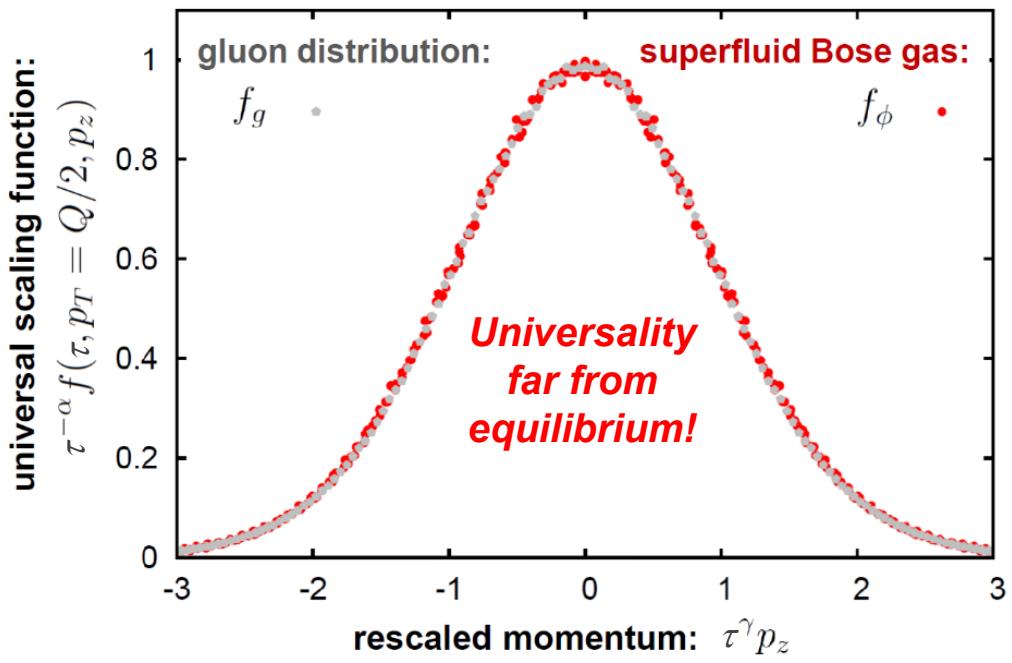
Universal scaling form of distribution function:

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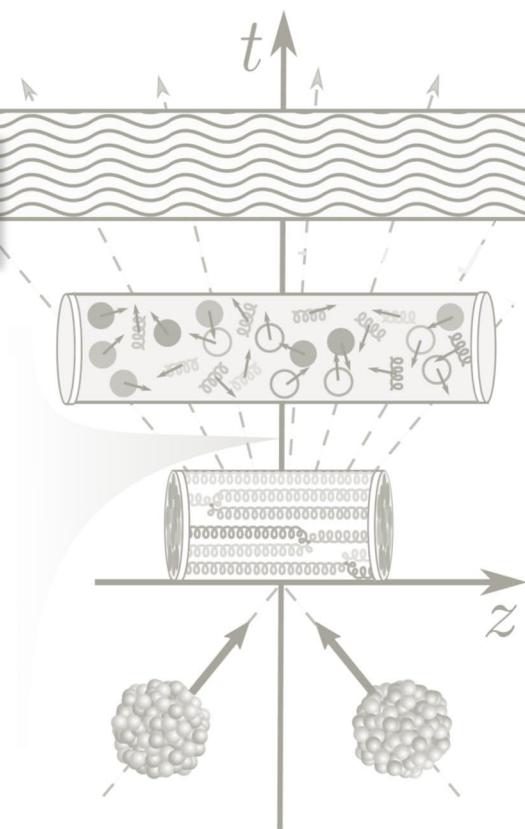
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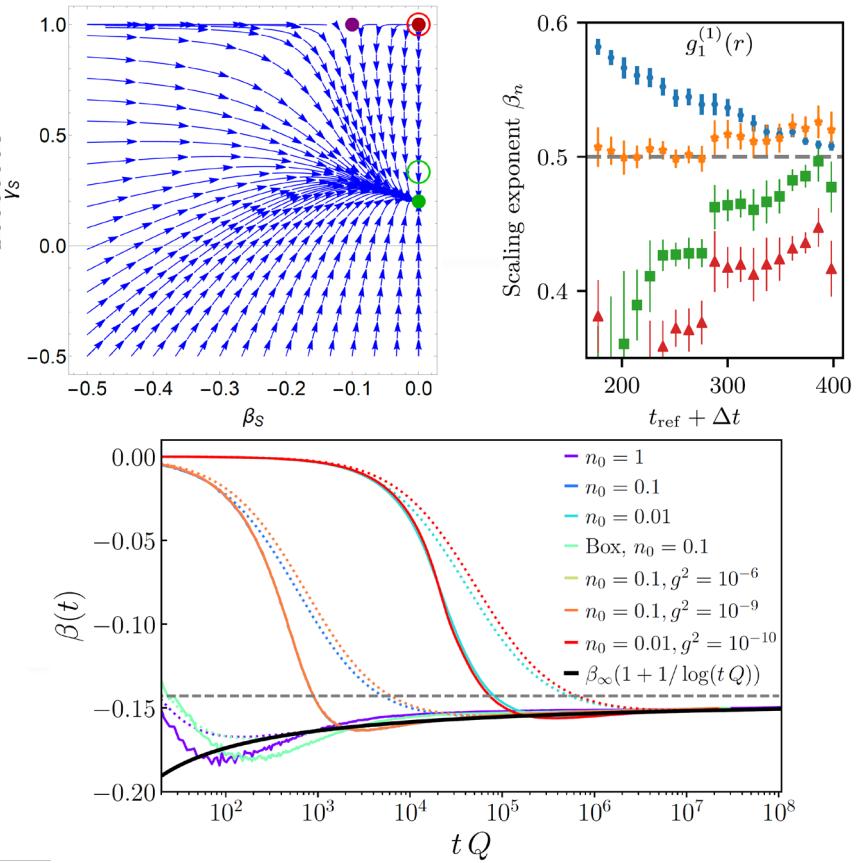
Classical-statistical comparison (expanding/relativistic):



Berges, Boguslavski, Schlichting, Venugopalan,  
PRL 114 (2015)



➤ Approach to scaling (prescaling):

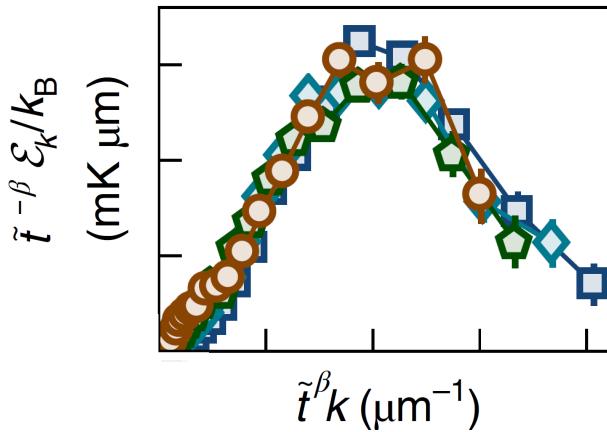
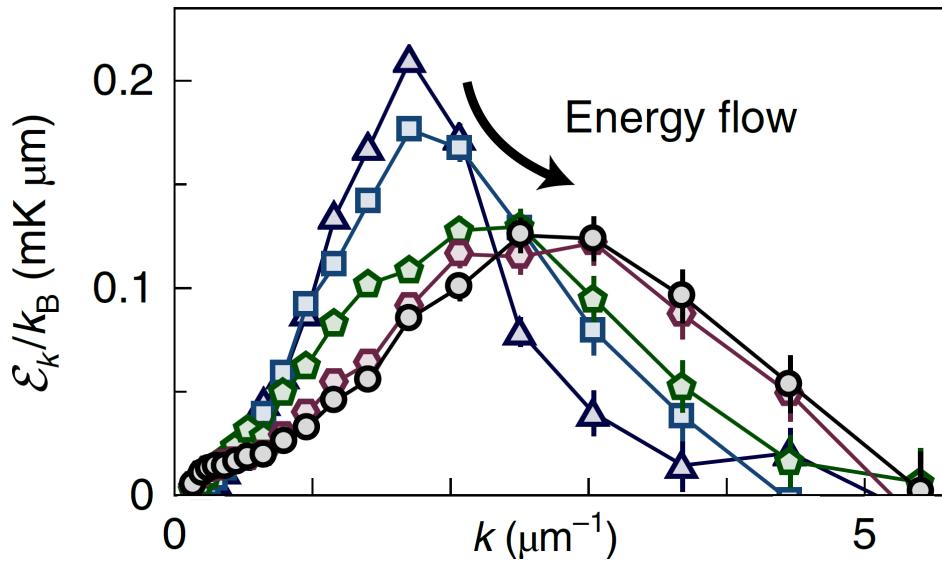


QCD/Cold atom prescaling: Heller, Mazeliauskas, Preis, arXiv:2307.07545; Brewer, Scheihing-Hitschfeld, Yin, JHEP 05 (2022); Mikheev, Mazeliauskas, Berges, PRD 105 (2022); Mazeliauskas, Berges, PRL 122 (2019); Schmied, Mikheev, Gasenzer, PRL 122 (2019)... Cf. hydro attractor: Alalawi, Strickland, JHEP 143 (2022), Brewer, Yan, Yin, PLB 816 (2021); Almaalol, Kurkela, Strickland, PRL 125 (2020); Kurkela, van der Schee, Wiedemann, Wu, PRL 124 (2020)...

# Pre-equilibrium dynamics: cold-atom simulator

- Bose gas experiment in 3+1 space-time dimensions

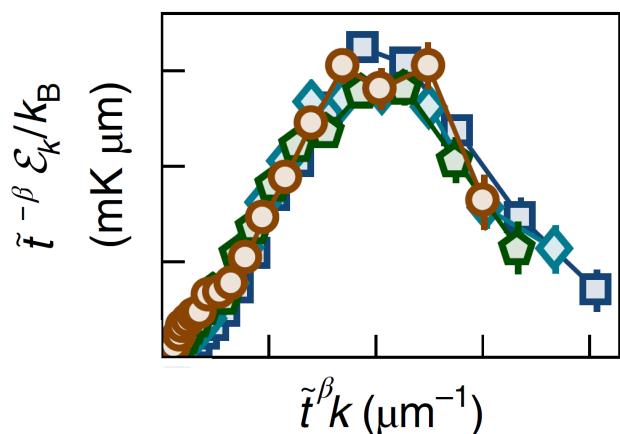
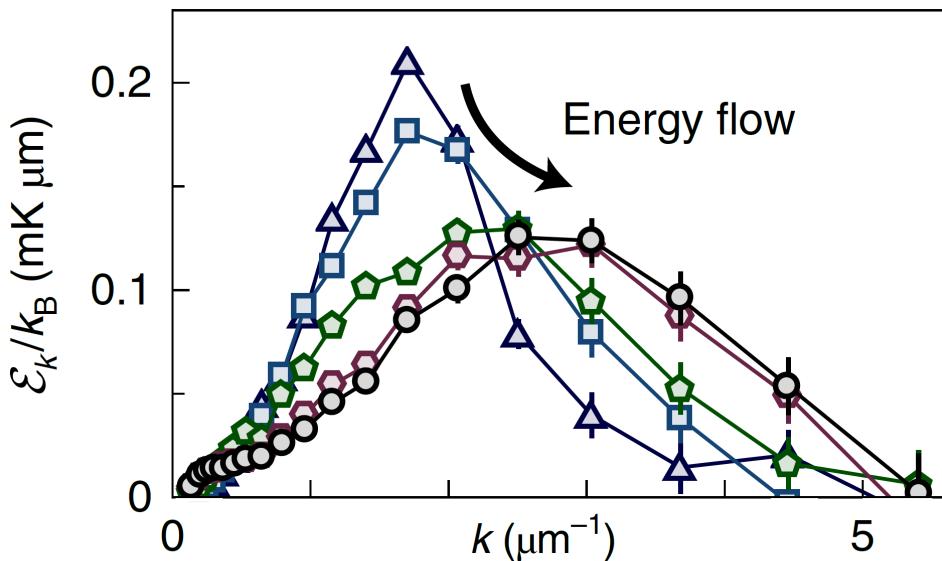
*Glidden et al., Nature Physics 17 (2021)*



# Pre-equilibrium dynamics: cold-atom simulator

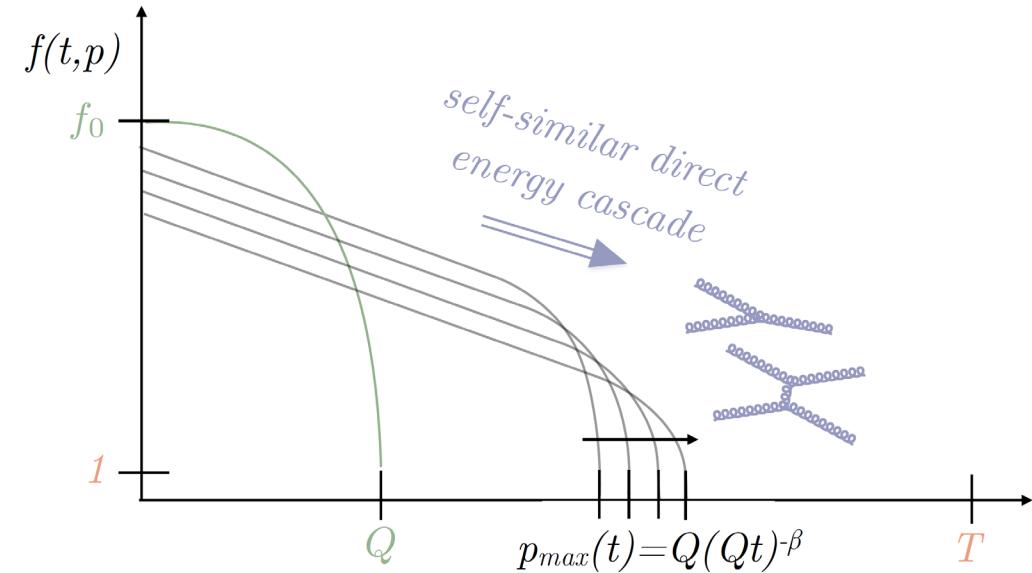
- Bose gas experiment in 3+1 space-time dimensions

*Glidden et al., Nature Physics 17 (2021)*



- Evolution of over-occupied plasma (non-expanding)

*Schlichting, Teaney, Annu. Rev. Nucl. Part. Sci. 69 (2019)*



- Scaling exponent for energy transport:

*Bose gas experiment (nonrel.):*

$$\beta = -0.14(2)$$

( $\approx -1/6?$ )

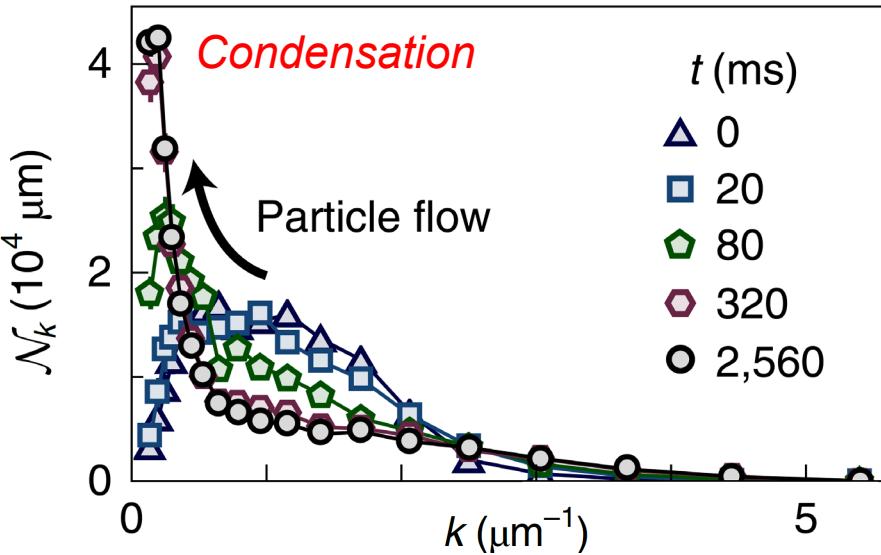
*QCD prediction (rel.):*

$$\beta = -1/7 \approx -0.14$$

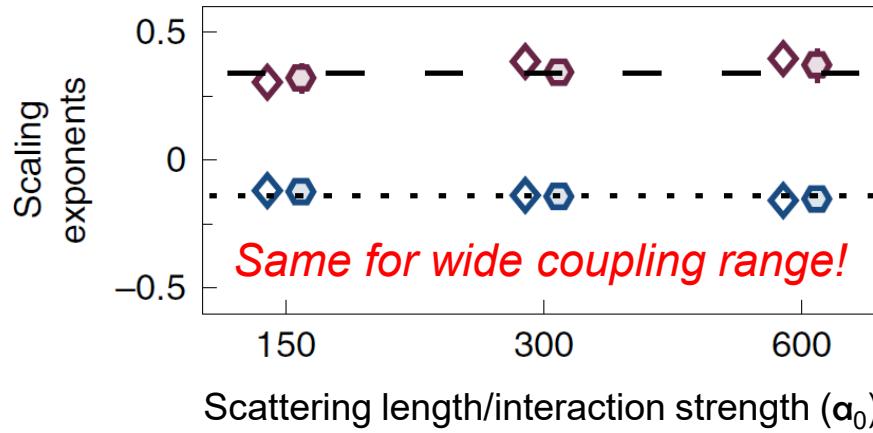
QCD: Schlichting, PRD 86 (2012); Kurkela, Moore, PRD 86 (2012)... Inflationary cosmology/  
Cold atoms: Micha, Tkachev, PRL 90 (2003); Piñeiro Orioli, Boguslavski, Berges, PRD 92  
(2015); Chantesana, Piñeiro Orioli, Gasenzer, PRA (2019)...

# Pre-equilibrium condensation!

- Far-from-equilibrium Bose gas experiment

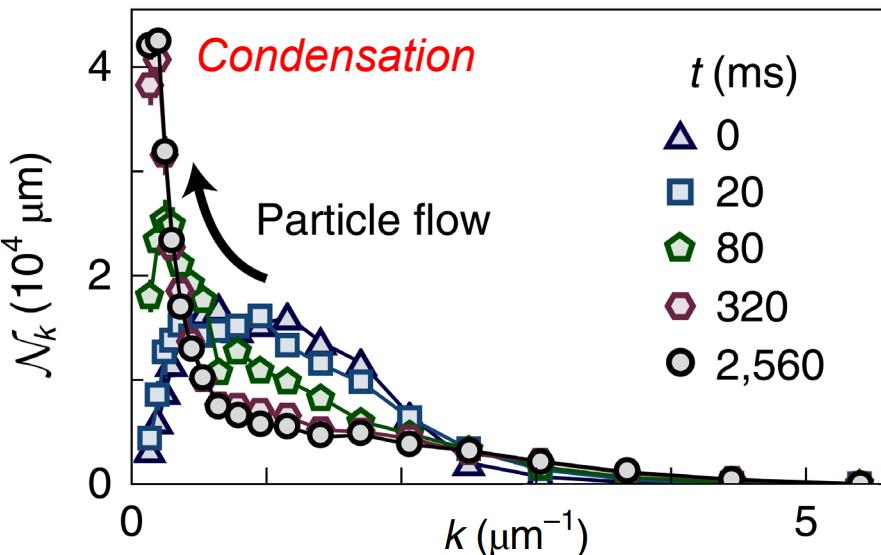


Glidden et al., Nature Physics 17 (2021) 457; Prüfer et al., Nature 563 (2018) 217; Erne et al., Nature 563 (2018) 225

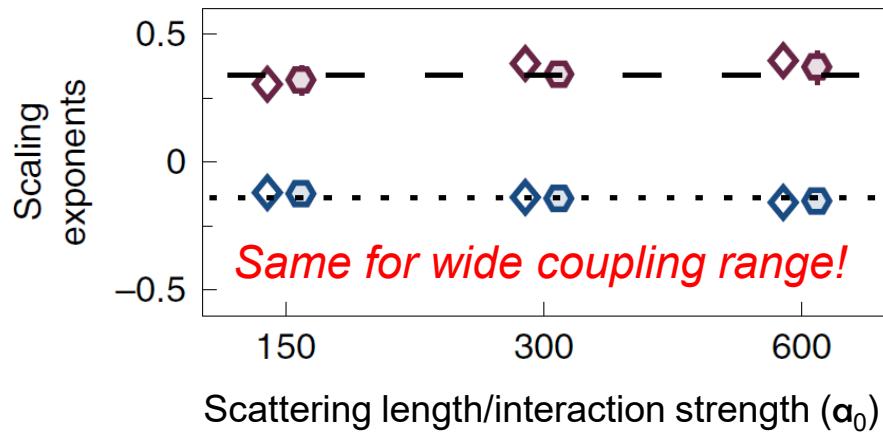


# Pre-equilibrium condensation!

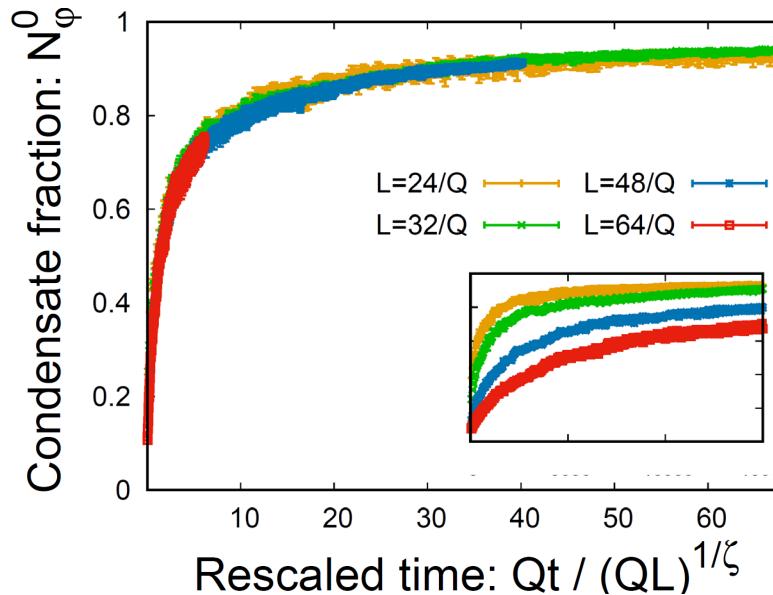
- Far-from-equilibrium Bose gas experiment



Glidden et al., Nature Physics 17 (2021) 457; Prüfer et al., Nature 563 (2018) 217; Erne et al., Nature 563 (2018) 225



- Evolution of over-occupied QCD plasma (classical-statistical)



Spatial Polyakov loop:

$$\mathcal{P} e^{-i g \int_0^L \mathcal{A} dx}$$

$$= e^{i\phi}$$

gauge-invariant eigenvalue  $\phi$

- Scaling exponent for condensation:

Bose gas experiment:

$$\zeta = 0.34(4)$$

QCD prediction:

$$\zeta = 0.34(3)$$

QCD: Berges, Boguslavski, de Bruin, Butler, Pawłowski, arXiv:2307.13669; Berges, Boguslavski, Mace, Pawłowski, PRD 102 (2020); Boguslavski, Kurkela, Lappi, Peuron, NPA 1005 (2021); Blaizot, Liao, Mehtar-Tani, NPA 961 (2017); Kurkela, Moore, PRD 86 (2012); Blaizot, Gelis, Liao, McLerran, Venugopalan, NPA 873 (2012) ...  
Cosmology/Cold atom: Berges, Sexty, PRL 108 (2012); Davis et al., arXiv:1601.06197 ...

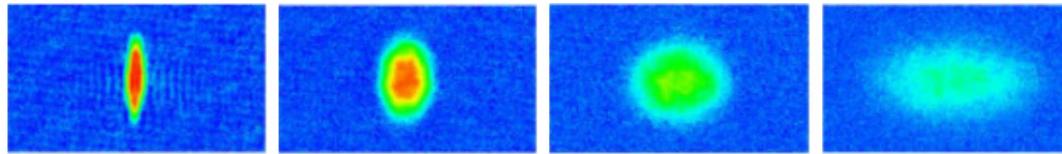
# Superfluid hydrodynamics and origin of collective behavior in small systems

- Quark gluon plasma and strongly coupled Fermi gases best fluids observed

*Review: Adams, Carr, Schäfer, Steinberg, Thomas, New J. Phys. 14 (2012)*

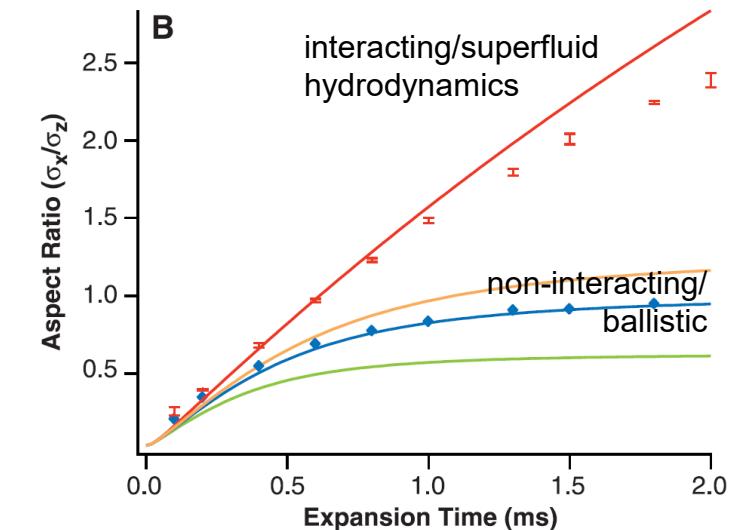
*small  $\eta/s$*

- Hydrodynamic flow signaled by geometry inversion:



*~ $10^5$  atoms: superfluid hydrodynamics*

*O'Hara, Hemmer, Gehm, Granade, Thomas, Science 298 (2002)*



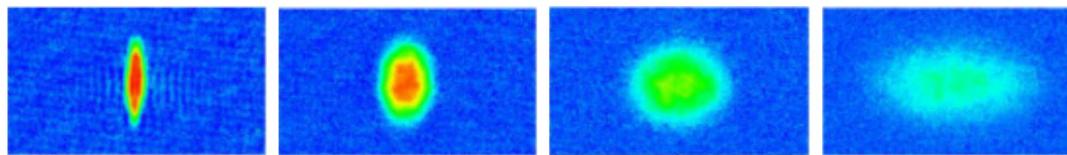
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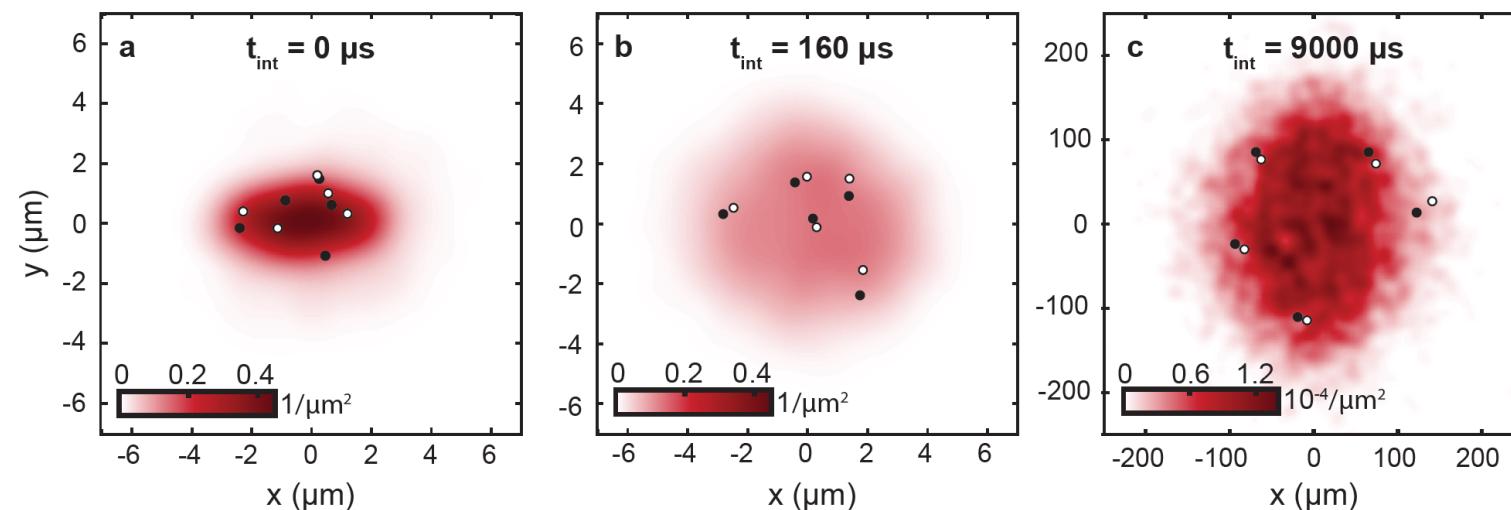
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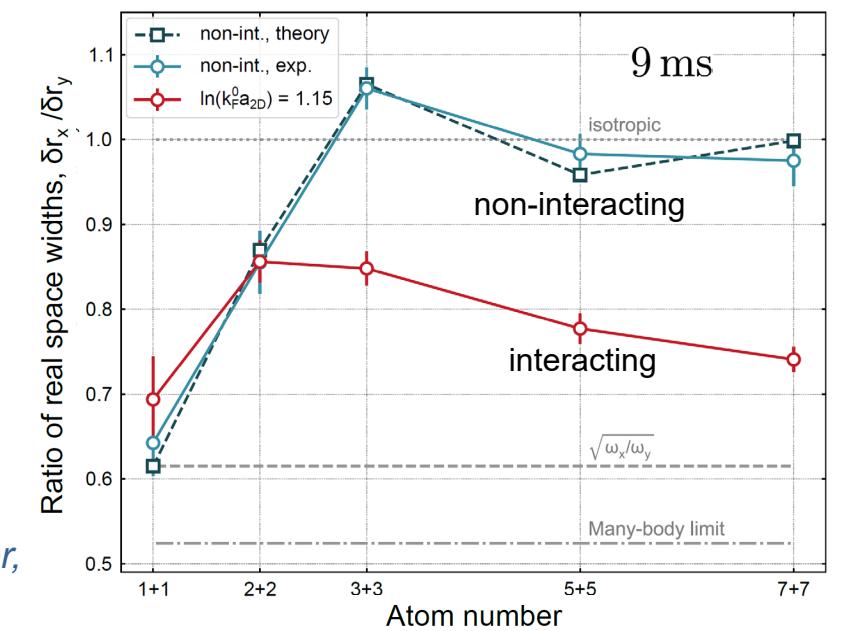
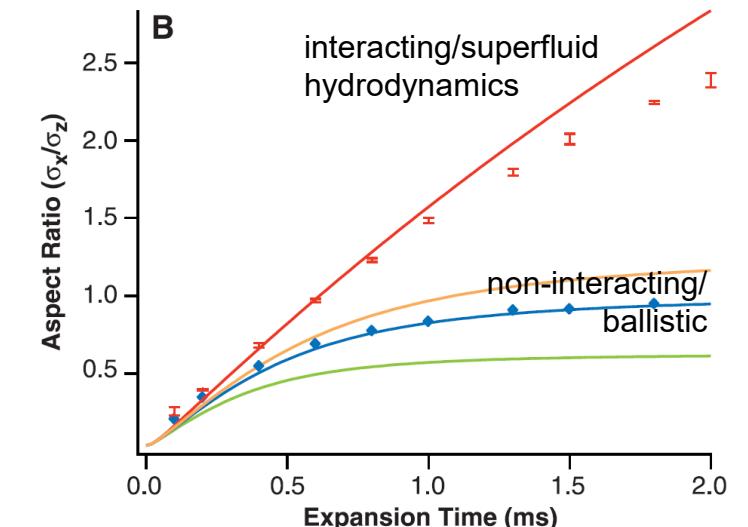
~ $10^5$  atoms: superfluid hydrodynamics

O'Hara, Hemmer, Gehm, Granade, Thomas, Science 298 (2002)

**Emergence of hydrodynamic flow already for few atoms:**



Brandstetter, Lunt, Heintze, Giacalone, Heyen, Gałka, Subramanian, Holten, Preiss, Flörchinger, Jochim, arXiv:2308.09699; Flörchinger, Giacalone, Heyen, Tharwat, PRC 105 (2022)



# Pre-equilibrium collective phenomena: new insights into outstanding questions?

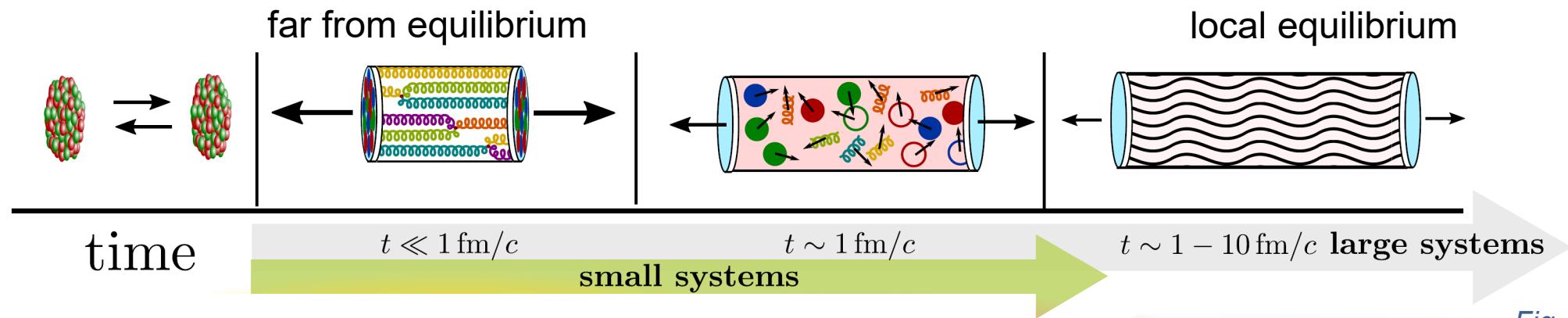
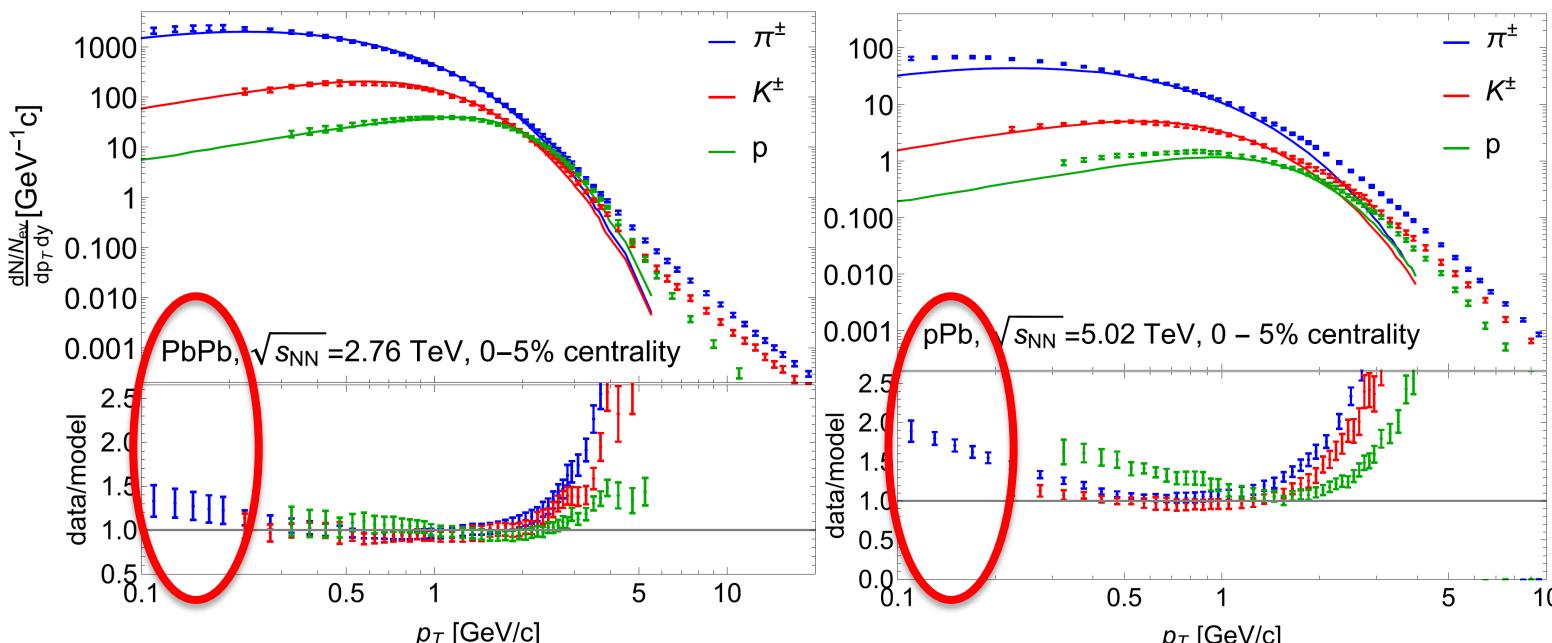


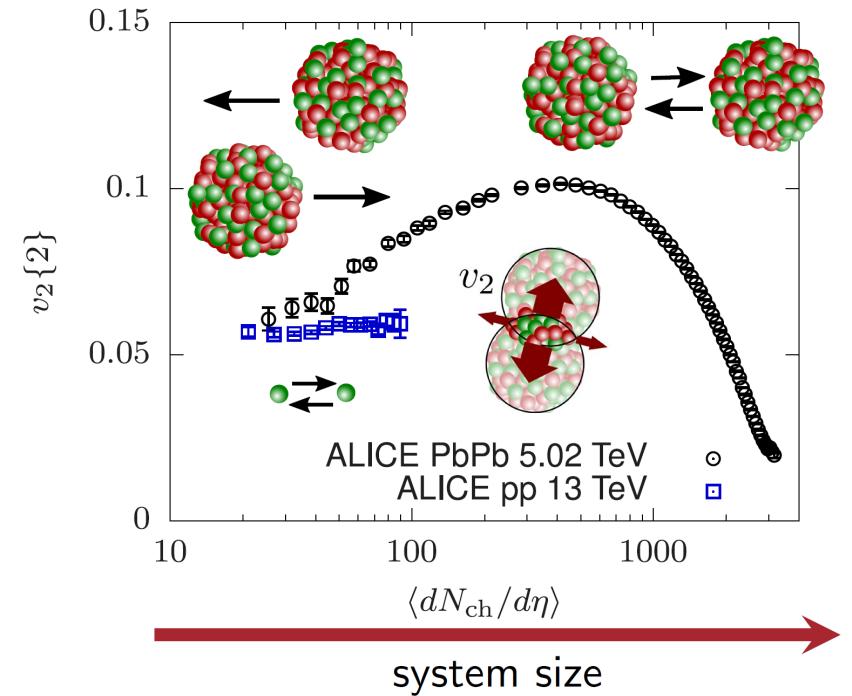
Fig. Mazeliauskas

- Enhanced low- $p_T$  yields from emergent condensates/superfluidity?



Nijs, van der Schee, Gürsoy, Snellings, PRC 103 (2021)

- Origin of collectivity in small systems?



# Pre-equilibrium collective phenomena: effective theories from quantum simulation

## ➤ Close to equilibrium

Hydrodynamics with collective fields

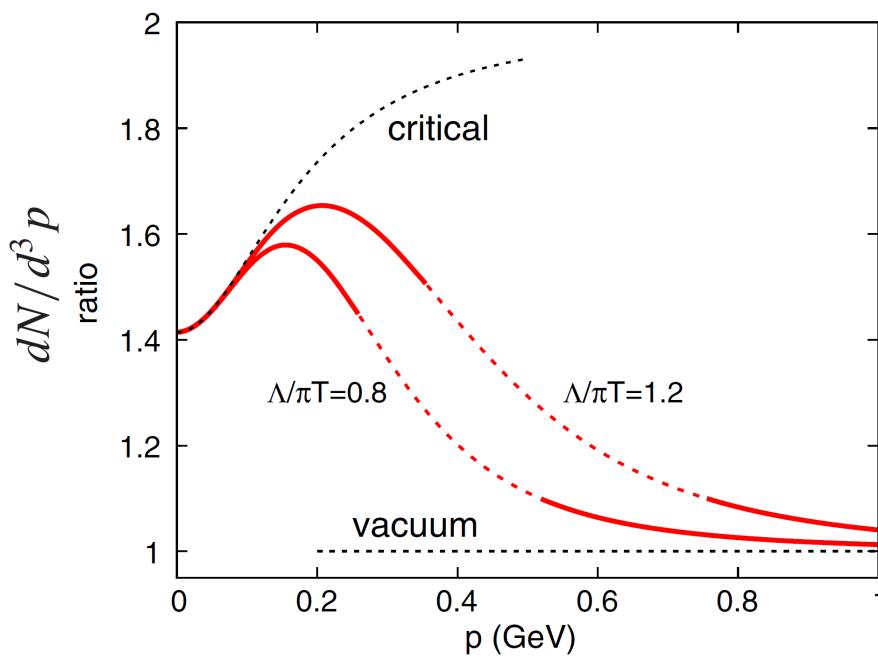
*Hohenberg, Halperin, Rev. Mod. Phys. 49 (1977)*

Review: Bluhm et al., EMMI, arXiv:2001.08831

e.g. excess of low-momentum pions  
from QCD chiral critical fluctuations:

Grossi, Soloviev, Teaney, Yan, PRD 104 (2021),  
102 (2020); ...

## ➤ Far from equilibrium?



# Pre-equilibrium collective phenomena: effective theories from quantum simulation

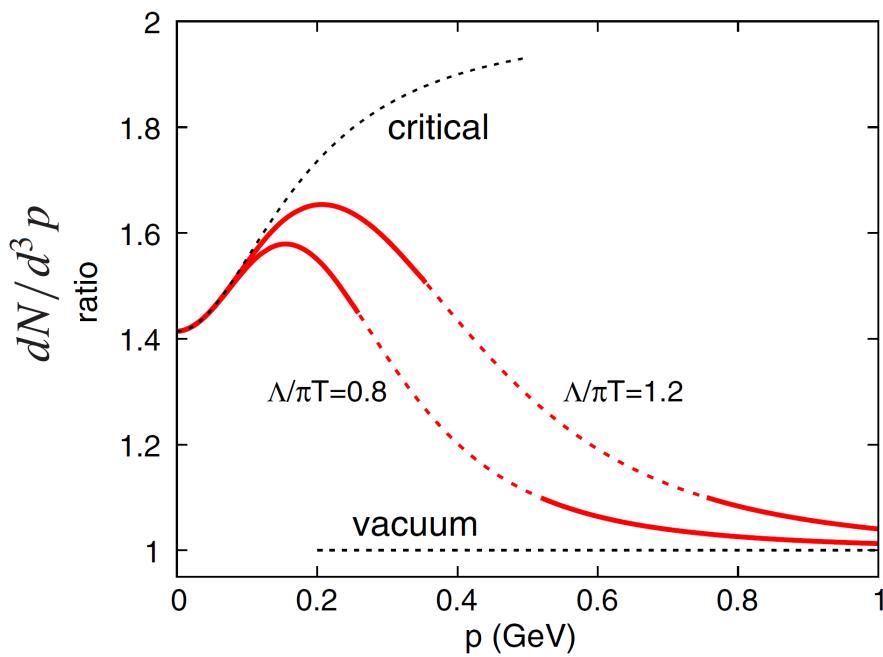
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Review: Bluhm et al., EMMI, arXiv:2001.08831

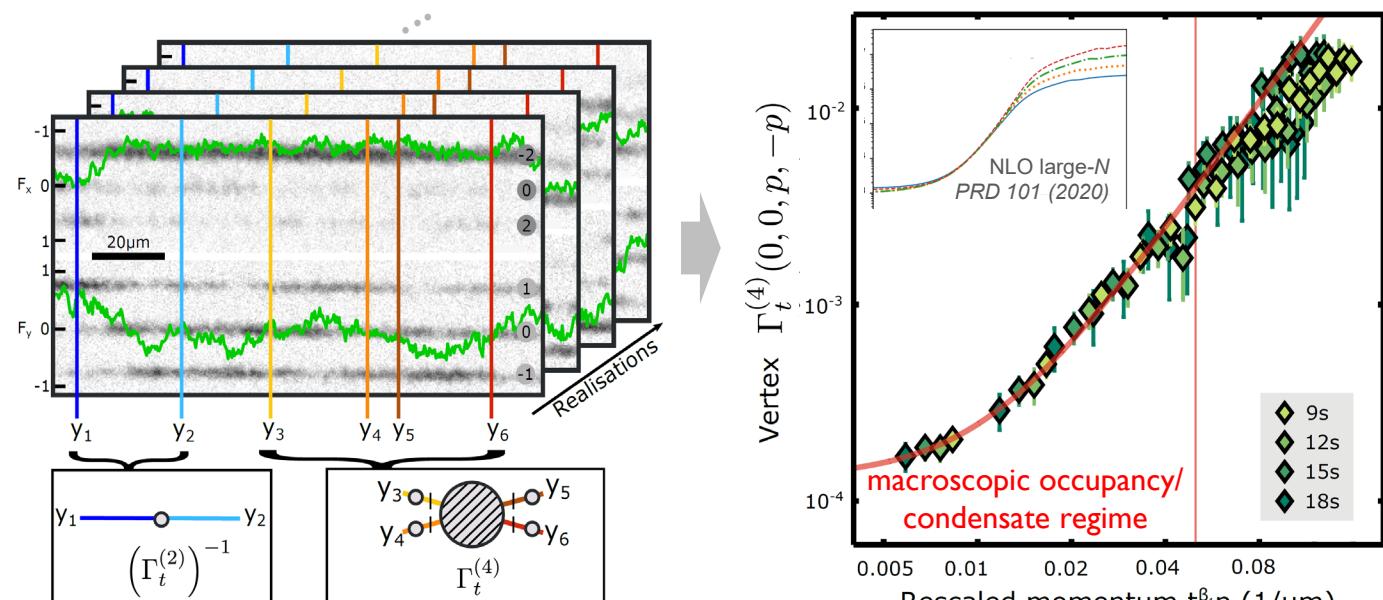
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102 (2020); ...*



## ➤ Far from equilibrium?

Effective kinetic theory for condensation from cold-atom simulator



$$\partial_t G_t^{(2)} = \frac{1}{(\Gamma_t^{(2)})^{-1}} \quad \text{exact} \quad \checkmark$$

$$G_t^{(2)}(p) \sim f_p$$

$$\partial_t f_p = \int_{q,r,s} |T_{pqrs}|^2 \delta(p+q-r-s) \times ((f_p + 1)(f_q + 1)f_r f_s - f_p f_q(f_r + 1)(f_s + 1))$$

$$\sim g \Gamma_t^{(4)}(p, q, r, s)$$

*Prüfer et al., Nature Physics 16 (2020) 1012; Zache et al., PRX 10 (2020);  
Ott, Zache, Berges, SciPost Phys. 14 (2023)*

# Further topics/Outlook

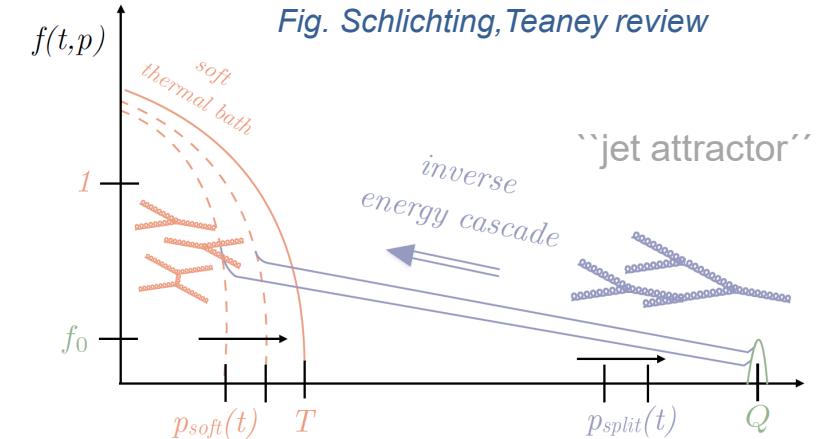
## ➤ Quantum simulation of hard probes/open systems in heavy-ion collisions

*de Jong, Metcalf, Mulligan, Płoskoń, Ringer, Yao, PRD 104 (2021); Barata, Du, Li, Qian, Salgado, PRD 106 (2022); Yao, arXiv:2205.07902;...*

→ impurity dynamics/driven condensates in ultracold quantum gases

*e.g. Cetina, Jag, Lous, Fritzsche, Walraven, Grimm, Levinsen, Parish, Schmidt, Knap, Demler, Science 354 (2016) / Clark, Gaj, Feng, Chin, Nature 551 (2017)*

*Fig. Schlichting, Teaney review*



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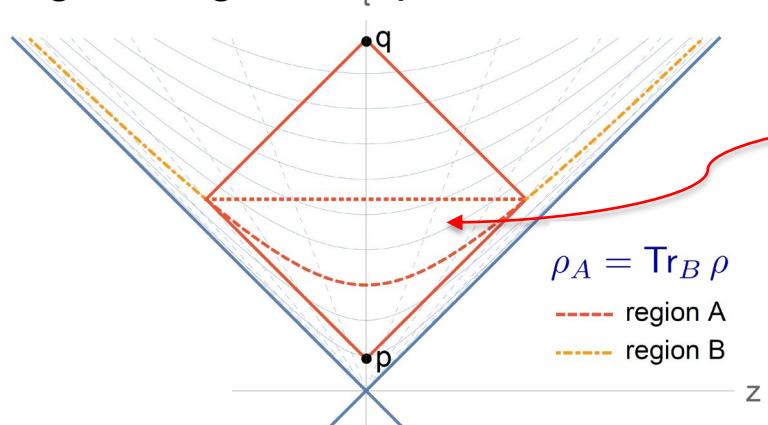
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- Entanglement contribution to entropy for thermalization/small systems

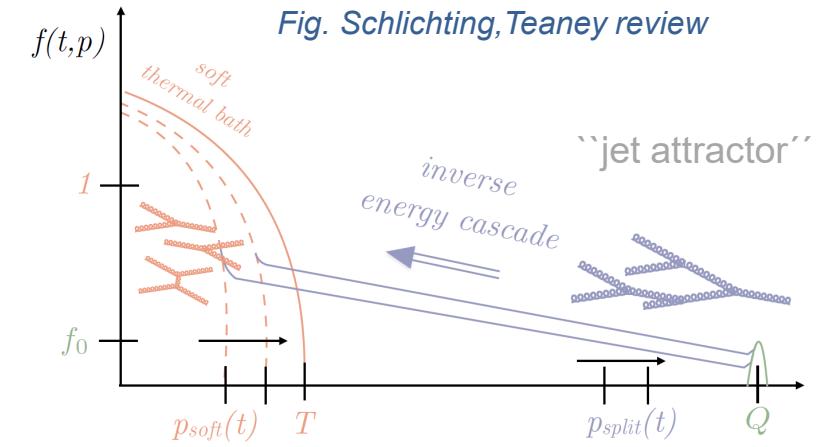
*cf. Eisert, Friesdorf, Gogolin, Nature Phys. 11 (2015)*

→ e.g. emergent temperature for reduced density operator of causal region:

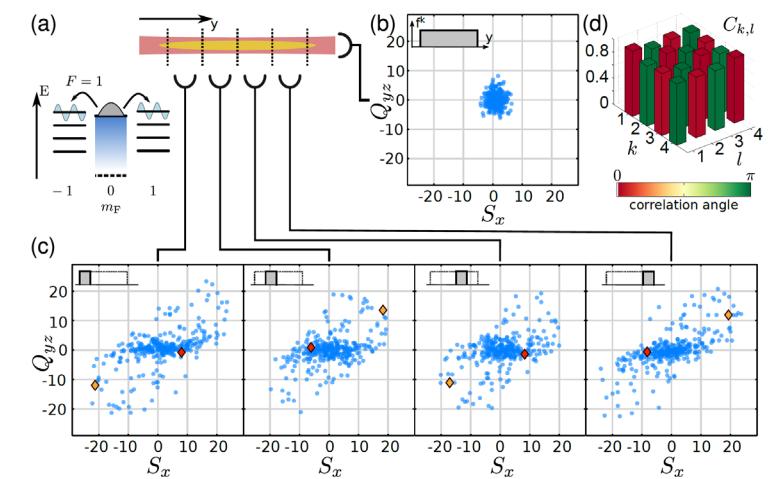


compare Hawking-Unruh temperature for accelerated observer:  $T(x) = \hbar c / (2\pi x)$

*Berges, Flörchinger, Venugopalan, PLB 778 (2018); JHEP 1804 (2018)*



→ space-time resolved entanglement detection:



*Kunkel, Prüfer, Lannig, Strohmaier, Gärttner, Strobel, Oberthaler, PRL 128 (2022)*

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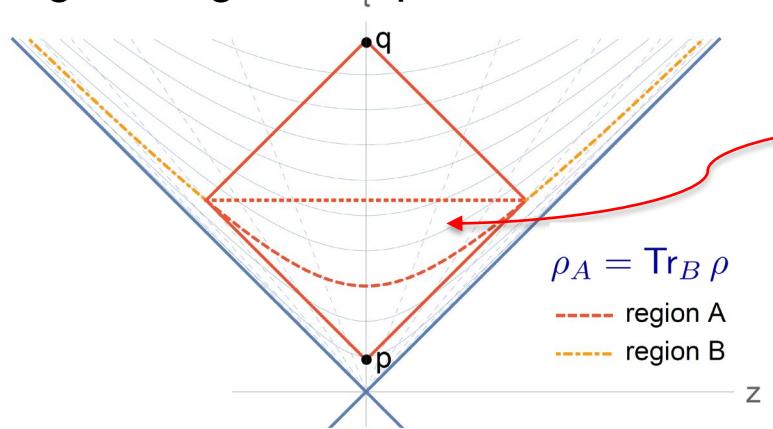
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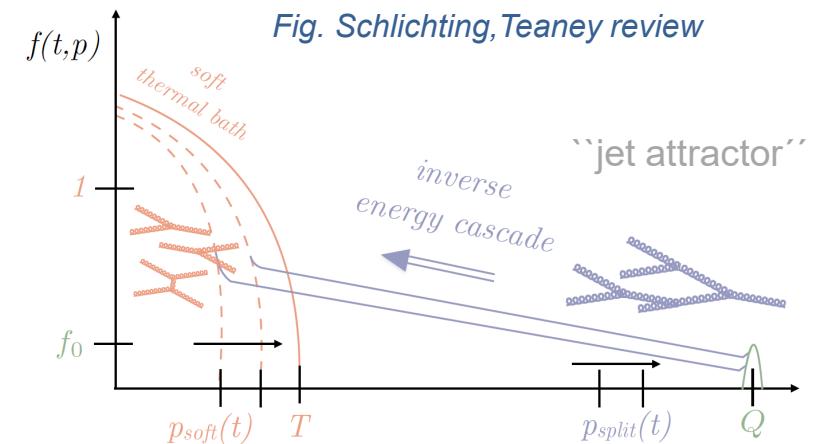
$$T(\tau) = \frac{\hbar}{2\pi\tau}$$

compare Hawking-Unruh temperature for accelerated observer:  $T(x) = \hbar c/(2\pi x)$

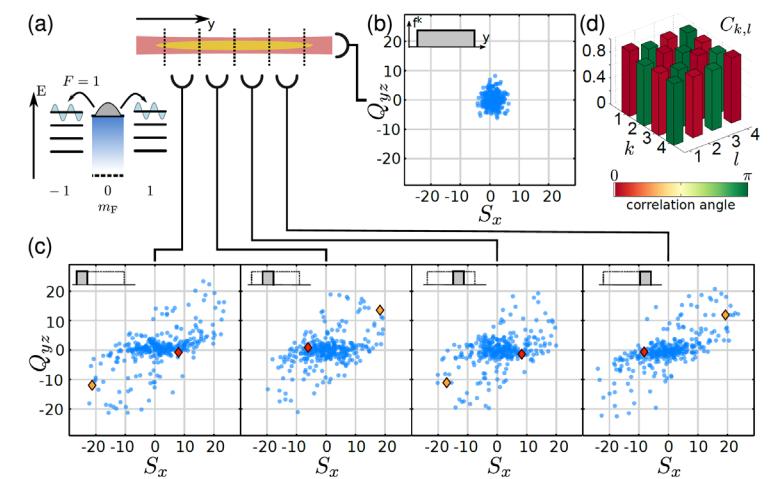
*Berges, Flörchinger, Venugopalan, PLB 778 (2018); JHEP 1804 (2018)*

## ➤ Real-time quantum simulation of hadronization models (Schwinger model)

... *cf. Bauer, Davoudi, Klco, Savage, Nature Rev. Phys. 5 (2023)*



→ space-time resolved entanglement detection:



*Kunkel, Prüfer, Lannig, Strohmaier, Gärttner, Strobel, Oberthaler, PRL 128 (2022)*