EXCITED HADRON CHANNELS IN HADRONIZATION

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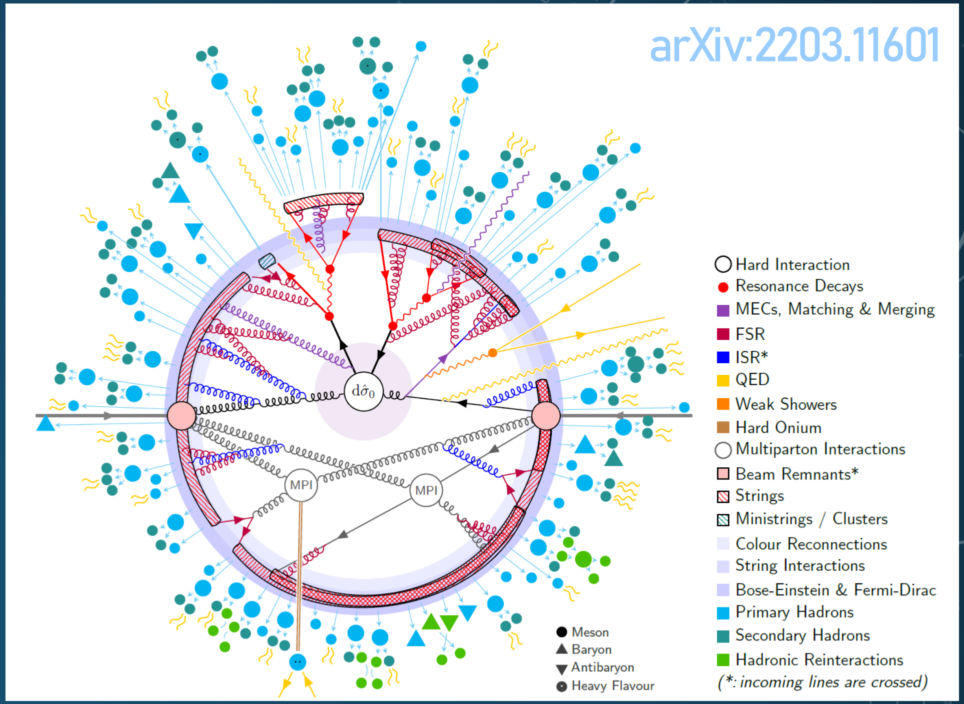
JETSCAPE

NSF

HADRONIZATION

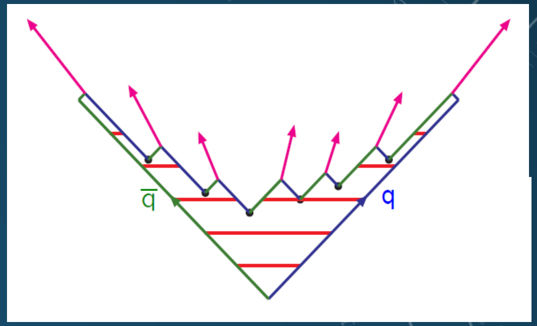
- Renewed interest in Hadronization
- New experimental information, in particular in the HF sector
- Hadronic resonances play an important role in many aspects of heavy ion collisions. We expect this to be also the case for hadronization.



- In this talk we study the inclusion of *meson* resonances into quark recombination.
- Outlook on two more topics:
  - Polarized mesons from orbital angular momentum
  - Hadronic rescattering of hadrons from hard processes

HYBRID HADRONIZATION

- A hybrid of string fragmentation and recombination.
- Interpolates smoothly in between, two limits:
  - Dilute systems → Dominance of string fragmentation
  - Dense systems → Dominance of quark recombination



K. C. Han, R. J. Fries, C. M. Ko, Jet Fragmentation via Recombination of Parton Showers, Phys.Rev.C 93, 045207 (2016)

- Monte Carlo implementation available, e.g implemented in JETSCAPE since v2.0.

- Necessary ingredients: probabilities for coalescence of quarks based on their phase space coordinates.
- Need to compute the necessary probabilities for meson resonances.

SETTING UP THE PROBLEM

- Quarks/antiquarks = wave packets in phase space
- For simplicity: Gaussian wave packets around centroid phase space coordinates $(\vec{r}_i, \vec{p}_i)$, of given width δ. Color and spin information might be available (otherwise treated statistically).

Attractive force

- Short range interaction modeled by isotropic harmonic oscillator potential of width $1/\nu$.
- Use the Wigner formalism in phase space. We need angular momentum eigenstates.

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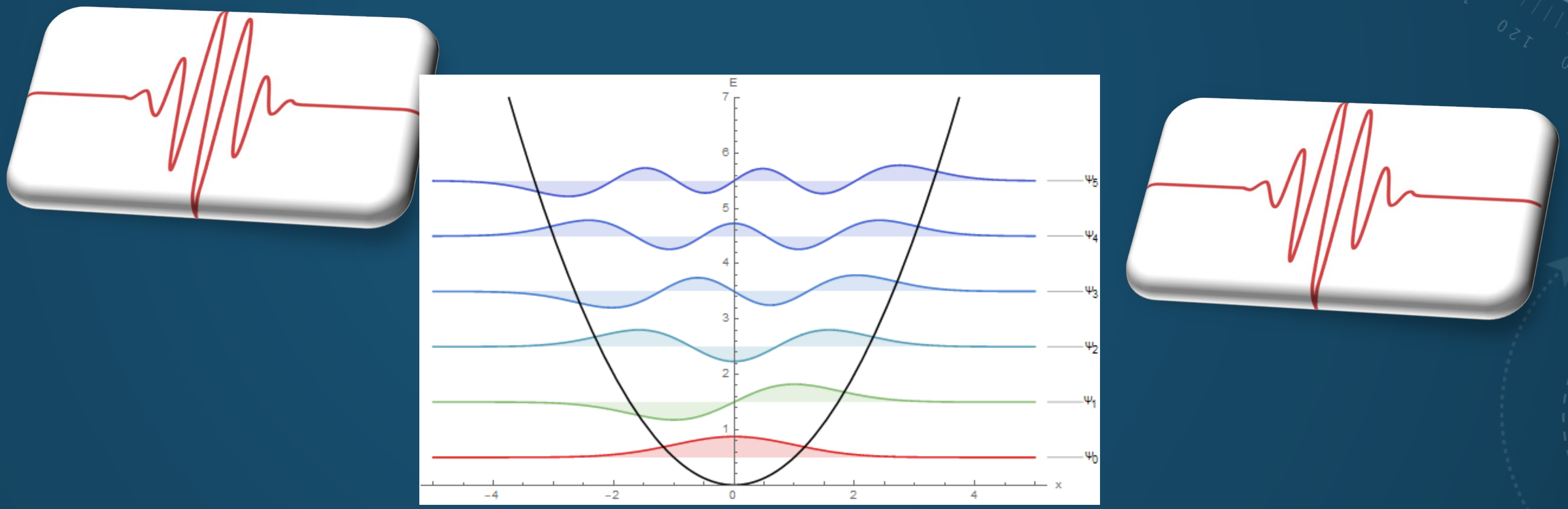


- Short range interaction modeled by isotropic harmonic oscillator potential of width $1/\nu$.
- Use the Wigner formalism in phase space. We need angular momentum eigenstates.

3D-HARMONIC OSCILLATOR IN PHASE SPACE

- Wigner distribution in phase space for given wave functions ψ_1, ψ_2:

$$W_{\psi_2,\psi_1}(\mathbf{r},\mathbf{q}) = \int \frac{d^3\mathbf{r}'}{(2\pi\hbar)^3} e^{\frac{i}{\hbar}\mathbf{r}'\cdot\mathbf{q}}\, \psi_2^*\left(\mathbf{r}+\frac{1}{2}\mathbf{r}'\right)\psi_1\left(\mathbf{r}-\frac{1}{2}\mathbf{r}'\right)$$

- (Diagonal) results known for the 3-D harmonic oscillator: S. Shlomo, M. Prakash, *Phase space distribution of an N –dimensional harmonic oscillator*, Nucl. Phys. A 357,157 (1981); formally known but hard to use.
- In 2-D: R. Simon, G. S. Agarwal, *Wigner representation of Laguerre-Gaussian beams*, Opt. Lett. 25, 1313 (2000); results are in closed form and elegant!

- Recalculate Wigner distributions using an expansion of angular momentum eigenstates in products of 1D-states.

3D-HARMONIC OSCILLATOR IN PHASE SPACE

- Use the well-studied 1D-phase space distributions to build the 3D ones

$$W_{kl}(\mathbf{r},\mathbf{q}) = \sum_{\substack{n_1,n_2,n_3\\ n_1',n_2',n_3'}} D_{kl}\binom{n_1,n_2,n_3}{n_1',n_2',n_3'} W_{n_1'n_1}(r_1,q_1) W_{n_2'n_2}(r_2,q_2) W_{n_3'n_3}(r_3,q_3)$$

Radial quantum number k,
angular momentum quantum number l

Three off-diagonal 1-D Wigner distributions

$$D_{kl}\binom{n_1,n_2,n_3}{n_1',n_2',n_3'} = \frac{1}{2l+1}\sum_m C^*_{klm,n_1'n_2'n_3'} C_{klm,n_1n_2n_3}$$

Averaging magnetic quantum numbers m, since not interested in polarization.

Expansion coefficients for angular momentum eigenstates in terms of products of 1-D states

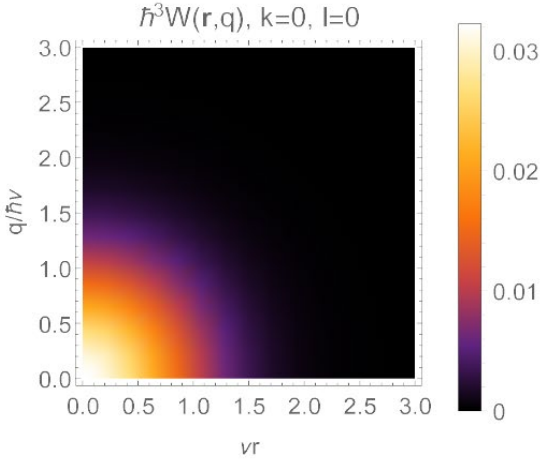
- The off-diagonal 1-D Wigner distributions are known [T. Curtright, T. Uematsu, C. K. Zachos, J. Math. Phys. 42 (2001)]

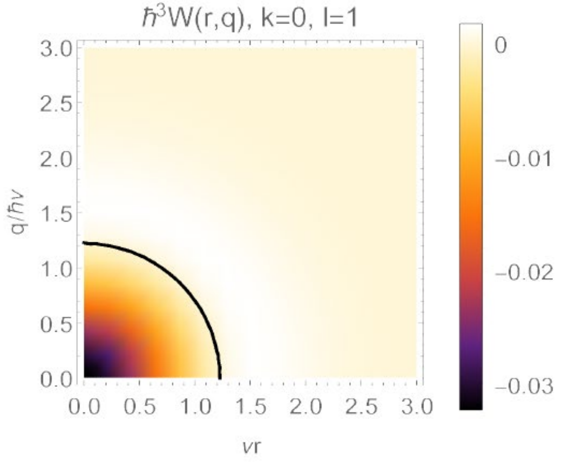
$$W_{n'n}(x,q) = \frac{(-1)^{n'}}{\pi\hbar}\sqrt{\frac{n'}{n}}\, u^{\frac{n-n'}{2}} e^{-u/2} e^{-i(n-n')\zeta} L_{n'}^{(n-n')}(u)$$

$$u = 2\left(q^2/(\hbar^2\nu^2) + \nu^2 x^2\right) \qquad \tan\zeta = q/(\hbar\nu^2 x)$$

WIGNER DISTRIBUTIONS

- Recall that Wigner distributions can be negative.
- When summed over m, they only depend on magnitudes of position and momentum, and the relative angle θ between.
- Examples of a few lowest states

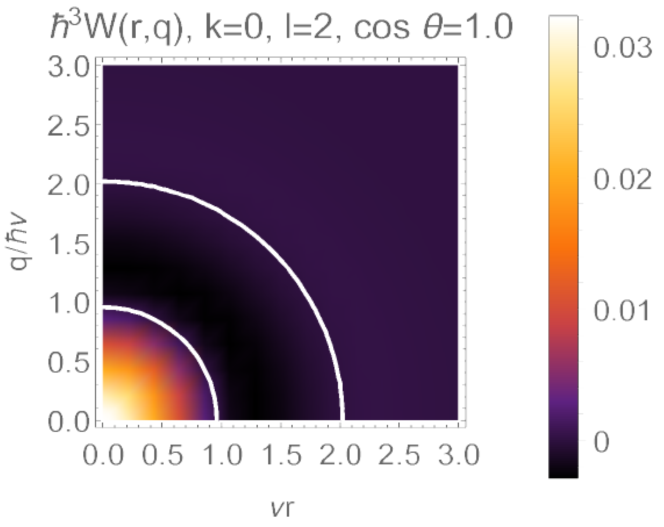


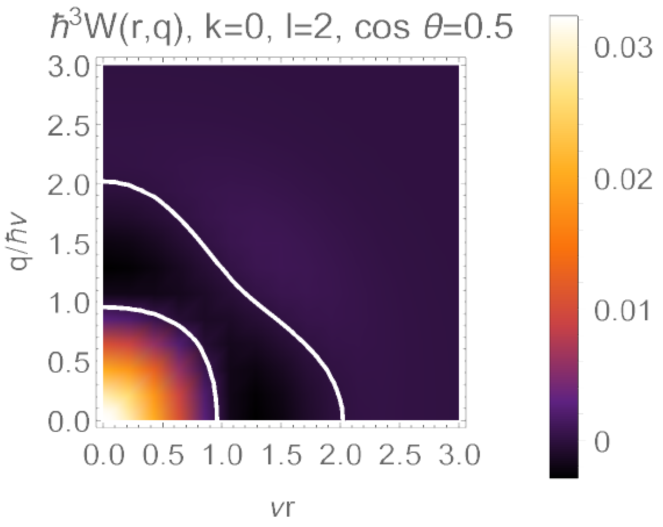


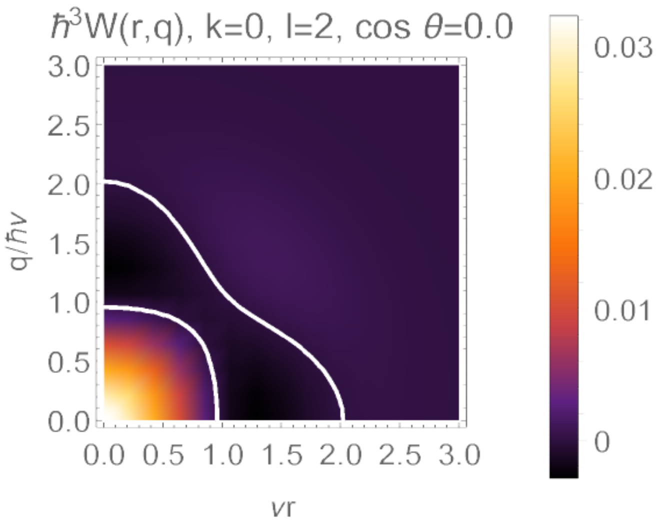
$$W_{00} = \frac{1}{\pi^3\hbar^3} e^{-\frac{q^2}{\hbar^2\nu^2} - \nu^2 r^2},$$

$$W_{01} = W_{00}\left(-1 + \frac{2}{3}\nu^2 r^2 + \frac{2}{3}\frac{q^2}{\hbar^2\nu^2}\right),$$

$$W_{02} = W_{00}\left(1 + \frac{4}{15}\nu^4 r^4 - \frac{4}{3}\nu^2 r^2 + \frac{16}{15}\frac{r^2 q^2}{\hbar^2} - \frac{8}{15}\frac{(\mathbf{r}\cdot\mathbf{q})^2}{\hbar^2} - \frac{4}{3}\frac{q^2}{\hbar^2\nu^2} + \frac{4}{15}\frac{q^4}{\hbar^4\nu^4}\right)$$







COALESCENCE

- Probability for coalescence of Gaussian wave packets using the Wigner distributions.

$$\tilde{\mathcal{P}}_{klm,\mathbf{P}_f} = (2\pi\hbar)^6 \int d^3\mathbf{x}_1 d^3\mathbf{x}_2 d^3\mathbf{k}_1 d^3\mathbf{k}_2 \tilde{W}_{\mathbf{P}_f}(\mathbf{K})\, W_{klm}\left(\Delta\mathbf{x}, \Delta\mathbf{k}\right) W_1(\mathbf{x}_1, \mathbf{k}_1) W_2(\mathbf{x}_2, \mathbf{k}_2)$$

$$\mathcal{P}_{kl} = \sum_m \int d^3\mathbf{P}_f \tilde{\mathcal{P}}_{klm,\mathbf{P}_f}$$

Wigner for center of mass motion.

Bound state Wigner distribution; only depends on relative phase space corrdinates

Wigner distributions of two Gaussian wave packets.

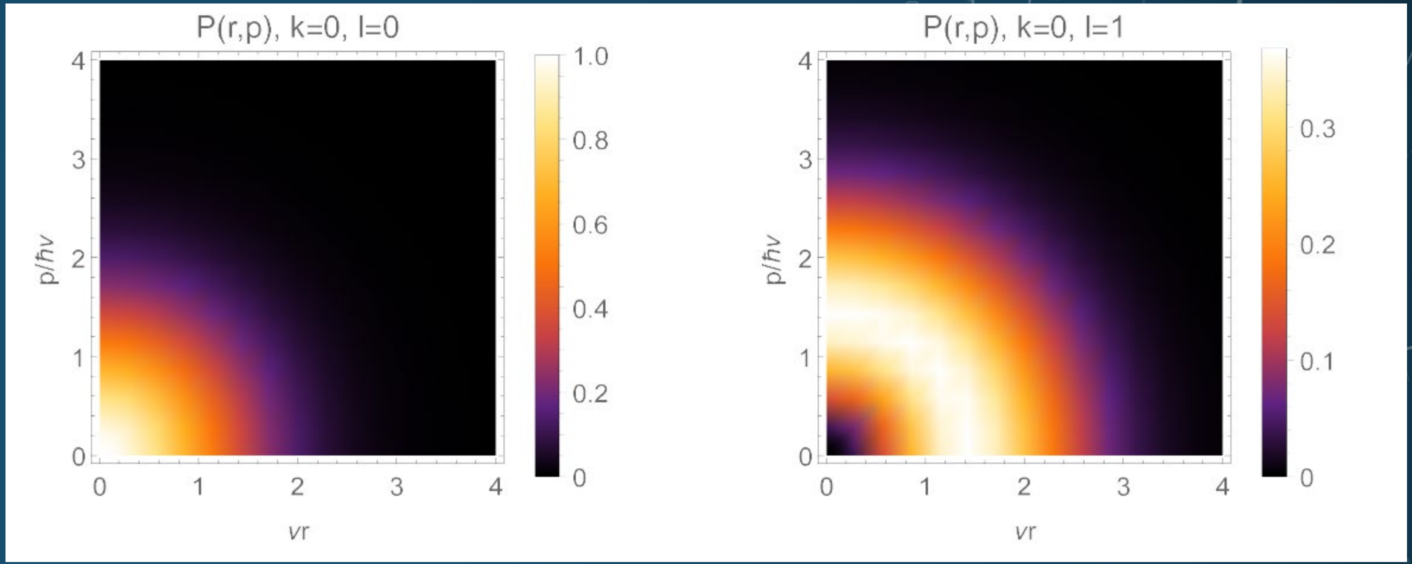
- Again sum over m, since we are not interested in polarization here (see remark later).

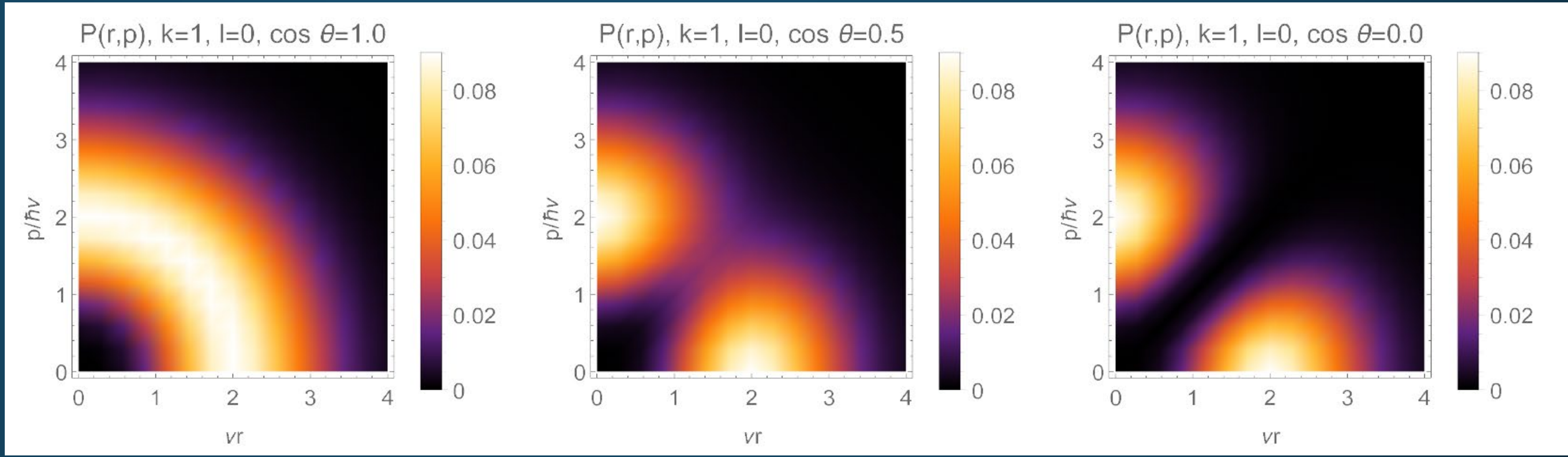
- Results discussed here for $1/\nu = 2\delta$ (relation between quark wave packet width δ and harmonic oscillator length scale $1/\nu$).

See M. Kordell, R. J. Fries, C. M. Ko, Annals Phys. 443 (2022) 168960 for full results.

COALESCENCE PROBABILITIES

- Probabilities depend on the relative coordinates of the wave packet centroids, called r and q here.
- θ = angle between r and q.





COALESCENCE PROBABILITIES

- Probabilities can be written in terms of just two variables: total phase space distance squared v and total angular momentum squared t.

$$v = \frac{\nu^2 r^2}{2} + \frac{p^2}{2\hbar^2\nu^2},$$
$$t = \frac{1}{\hbar^2}\left[p^2 r^2 - (\mathbf{p}\cdot\mathbf{r})^2\right] = \frac{1}{\hbar^2} L^2$$

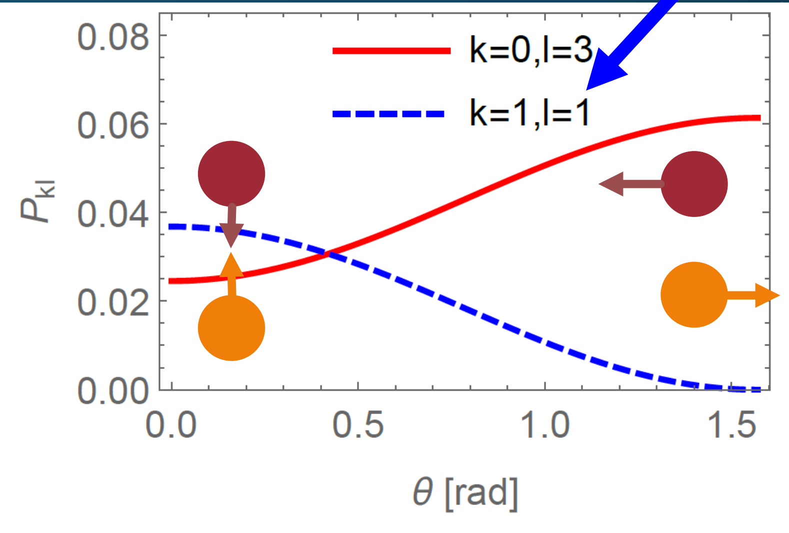
$$\mathcal{P}_{00} = e^{-v},$$
$$\mathcal{P}_{01} = e^{-v} v,$$
$$\mathcal{P}_{02} = \frac{1}{2} e^{-v}\left(\frac{2}{3}v^2 + \frac{1}{3}t\right)$$
$$\mathcal{P}_{10} = \frac{1}{2} e^{-v}\left(\frac{1}{3}v^2 - \frac{1}{3}t\right)$$

- If summed over states with the same energy, the probabilities are simply Poissonian given by phase space distance
  - Same as in 1-D

$$\sum_{2k+l=N} \mathcal{P}_{kl} = e^{-v}\frac{v^N}{N!}$$

- Energy degeneracy broken by orbital angular momentum of the quarks. t makes an intuitive connection between the relative angular momentum of the incoming quarks and the quantum number l of their bound state.

Both are states with N=3



EXCITED MESONS AND THEIR DECAYS

- We include excited mesons up to $N = k + 2l = 4$.
- Hybrid Hadronization uses PYTHIA 8 for decays: available excited states are limited, but the user can easily add more.
- Many more resonances in the PDG -> add to the code
- Add as of yet unconfirmed bound states: extrapolate unknown properties.

Light/Strange $\mathbf{n = 1}$ $\mathbf{(k = 0)}$											
		L=0		**L=1**		**L=2**		**L=3**		**L=4**	
L=J S=0	$I=1$	xx1 1^1S_0 0^{-+}	π	10xx3 1^1P_1 1^{+-}	$b_1(1235)$	10xx5 1^1D_2 2^{-+}	$\pi_2(1670)$	10xx7 1^1F_3 3^{+-}	$b_3(2013)^\dagger$	10xx9 1^1G_4 4^{-+}	$\pi_4(2306)^\dagger$
	$I=\frac{1}{2}$		K		K_{1B}		$K_2(1770)$		$K_{3B}(2157)^\dagger$		$K_{4B}(2485)^\dagger$
	$I=0$		η		$h_1(1415)$		$\eta_2(1870)$		$h_3(2234)^\dagger$		$\eta_4(2547)^\dagger$
	$I=0$		$\eta'(958)$		$h_1(1170)$		$\eta_2(1645)$		$h_3'(2011)^\dagger$		$\eta_4'(2320)^\dagger$
J=L+1 S=1	$I=1$	xx3 1^3S_1 1^{--}	$\rho(770)$	xx5 1^3P_2 2^{++}	$a_2(1320)$	xx7 1^3D_3 3^{--}	$\rho_3(1690)$	xx9 1^3F_4 4^{++}	$a_4(1970)$	xx8‡ 1^3G_5 5^{--}	$\rho_5(2350)$
	$I=\frac{1}{2}$		$K^*(892)$		$K_2^*(1430)$		$K_3^*(1780)$		$K_4^*(2045)$		$K_5^*(2380)$
	$I=0$		$\phi(1020)$		$f_2'(1525)$		$\phi_3(1850)$		$f_4(2300)$		$\phi_5(2584)^\dagger$
	$I=0$		$\omega(782)$		$f_2(1270)$		$\omega_3(1670)$		$f_4(2050)$		$\omega_5(2323)^\dagger$
J=L S=1	$I=1$			20xx3 1^3P_1 1^{++}	$a_1(1260)$	20xx5 1^3D_2 2^{--}	$\rho_2(1715)^\dagger$	20xx7 1^3F_3 3^{++}	$a_3(2072)^\dagger$	20xx9 1^3G_4 4^{--}	$\rho_4(2376)^\dagger$
	$I=\frac{1}{2}$				K_{1A}		$K_2(1820)$		$K_{3A}(2160)^\dagger$		$K_{4A}(2453)^\dagger$
	$I=0$				$f_1(1420)$		$\phi_2(1835)^\dagger$		$f_3(2173)^\dagger$		$\phi_4(2464)^\dagger$
	$I=0$				$f_1(1285)$		$\omega_2(1733)^\dagger$		$f_3'(2087)^\dagger$		$\omega_4(2389)^\dagger$
J=L-1 S=1	$I=1$			10xx1 1^3P_0 0^{++}	$a_0(1450)$	30xx3 1^3D_1 1^{--}	$\rho(1700)$	30xx5 1^3F_2 2^{++}	$a_2(1918)^\dagger$	30xx7 1^3G_3 3^{--}	$\rho_3(2113)^\dagger$
	$I=\frac{1}{2}$				$K_0^*(1430)$		$K^*(1680)$		$K_2^*(1897)^\dagger$		$K_3^*(2092)^\dagger$
	$I=0$				$f_0(1710)$		$\phi_1(1931)^\dagger$		$f_2(2129)^\dagger$		$\phi_3(2310)^\dagger$
	$I=0$				$f_0(1370)$		$\omega(1650)$		$f_2'(1889)^\dagger$		$\omega_3(2101)^\dagger$

PDG states

Unconfirmed states

EXCITED MESONS AND THEIR DECAYS

- Use scaling laws for masses

$$M^2 = M_0^2 + \alpha k$$
$$M^2 = M_0^2 + \beta l\,,$$

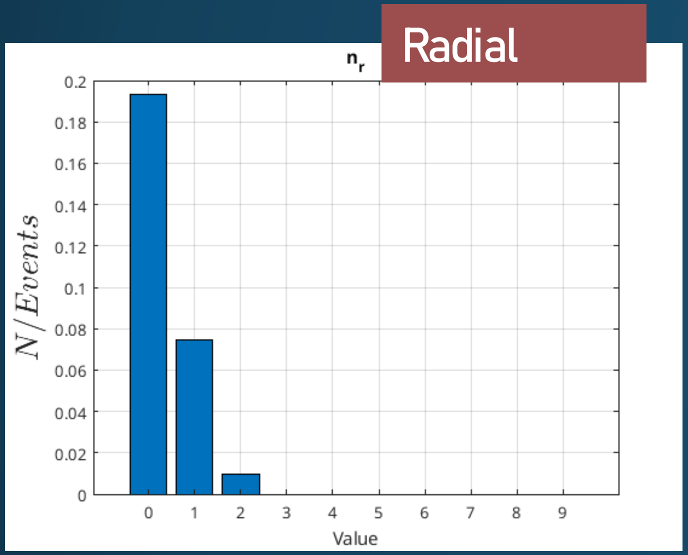
A. V. Anisovich, V. V. Anisovich, A. V. Sarantsev, *Systematics of qqbar states in the (n,M^2) and (J,M^2) planes*, Phys. Rev. D 62, 051502 (2000)

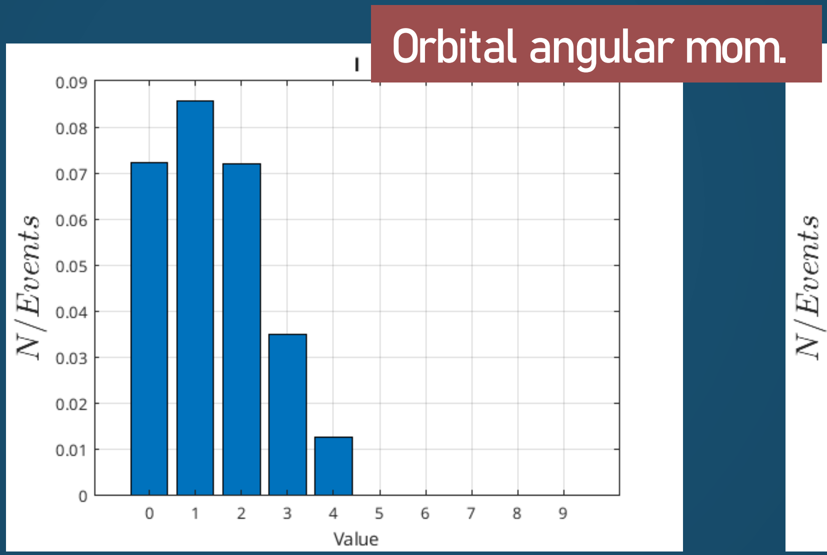
- Decays from minimum set of assumptions: Flavor/OZI, G-parity, phase space weights (up to 5-particle decay) [R. H. Milburn, Rev. Mod. Phys. 27, 1 (1955)], isospin algebra

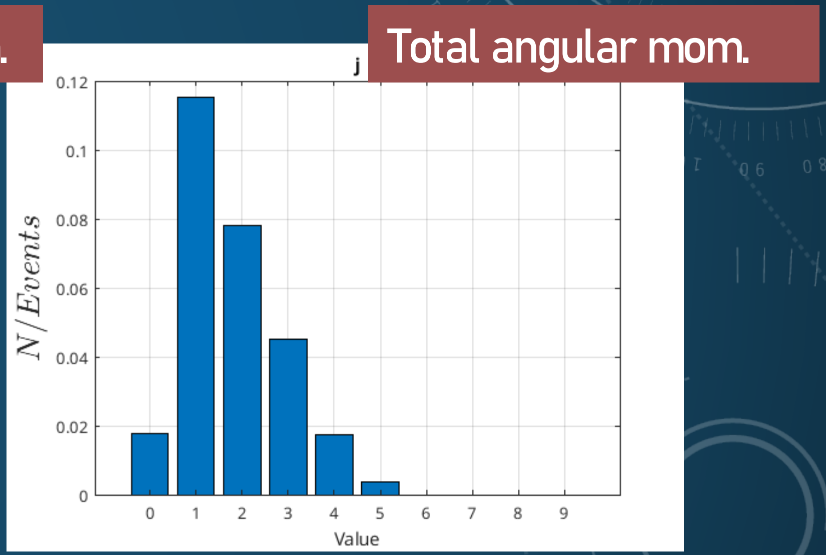
20qq7 (S=1,L=3)		1^3^F_3										
20217		a_3(2072)+	a_3(2072)-	0.3121569535	pi+pi+pi-	211 211 -211	0.2081046356	pi+pi0pi0	211 111 111	0.1370660614	pi+pi+pi+pi-pi-	211 211 211 -2
20117		a_3(2072)0		0.4162092713	pi+pi0pi-	211 111 -211	0.1040523178	pi0pi0pi0	111 111 111	0.3015491729	pi+pi+pi0pi-pi-	211 211 111 -2
20327		K_3A(2160)+	K_3A(2160)-	0.02969148171	K+pi0	321 111	0.05938385418	K0pi+	311 211	0.1882397025	K+pi+pi-	321 211 -211
20317		K_3A(2160)0	K_3A(2160)bar0	0.02969148171	K0pi0	311 111	0.05938385418	K+pi-	321 -211	0.1882397025	K0pi+pi-	311 211 -211
20337		f_3(2173)		0.0473472313	pi+pi-	211 -211	0.02367326055	pi0pi0	111 111	0.3715918033	pi+pi+pi-pi-	211 211 -211 -
20227		f_3'(2087)		0.05917570104	pi+pi-	211 -211	0.0295874067	pi0pi0	111 111	0.3644947569	pi+pi+pi-pi-	211 211 -211 -

FIRST TEST: ABUNDANCES OF RESONANCES

- Use parton states from running PYTHIA 8 e+e- at 91 GeV
- Hybrid Hadronization with $N = 4$, no decays;
- Spin treated statistically, color flow from PYTHIA.



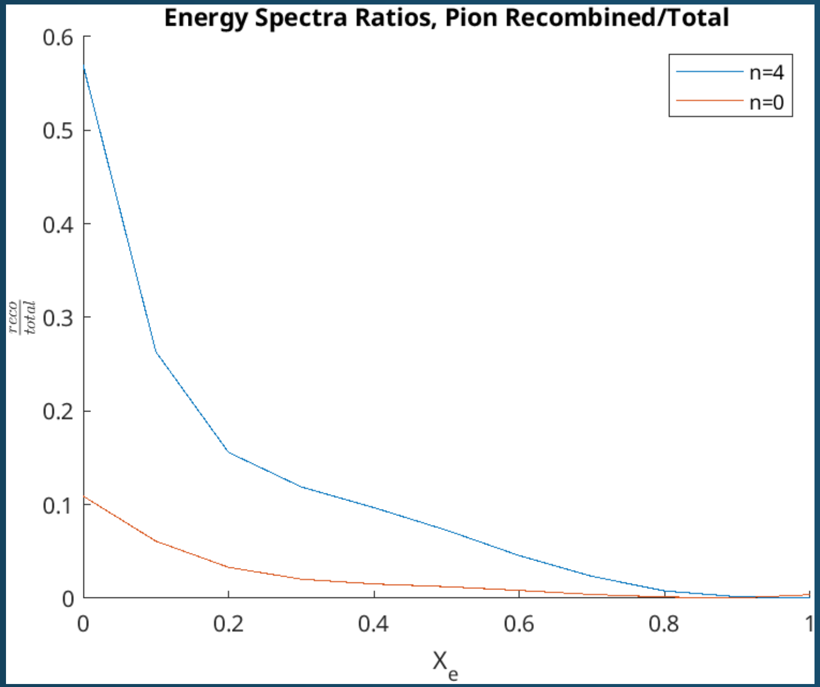


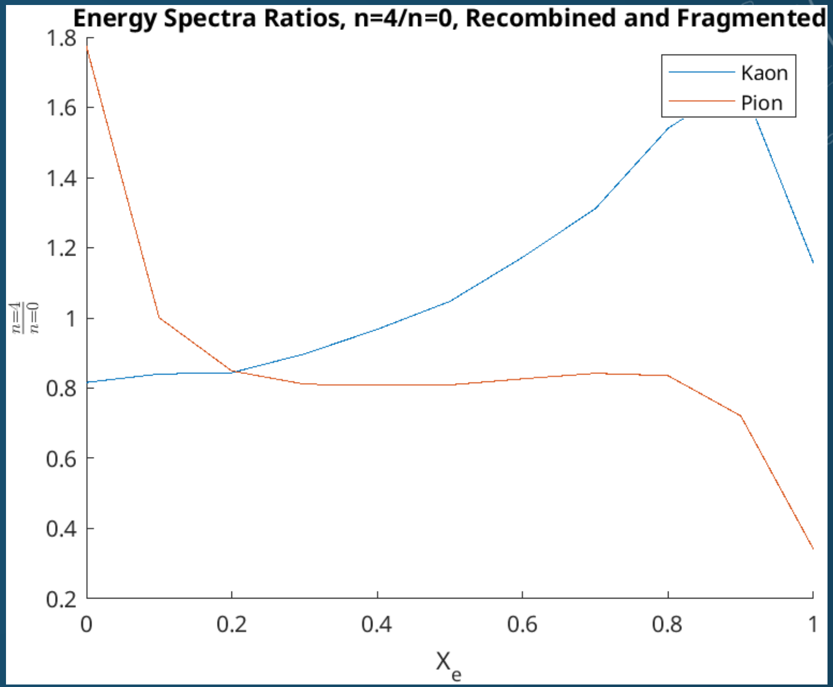


(Hadrons from recombination only)

FIRST TEST: SPECTRA

- e+e- at 91 GeV, $N = 4$, decays=on
- Left panel: $N = 4$ vs $N = 0$ means more recombination, less fragmentation





- Right panel: Competing mechanisms: increasing N increases low momentum decay products, also allows for more recombination at large x_e.

SIDE NOTE: POLARIZATION

- What if we don't sum over magnetic quantum numbers and ask for the polarization of the meson?
- Probabilities are sensitive to the angular momentum component L_z.
- Conclusion: if the collective motion of the quarks at carries net orbital angular momentum, hadronization can give you correspondingly polarized p- and d-wave mesons.

$$P_{011} = e^{-v}\left(\frac{1}{2}v_T + \frac{L_z}{2\hbar}\right)$$

$$P_{011} = e^{-v}v_L$$

$$P_{01\,-1} = e^{-v}\left(\frac{1}{2}v_T - \frac{L_z}{2\hbar}\right)$$

L_z selects a preferred polarization of the meson

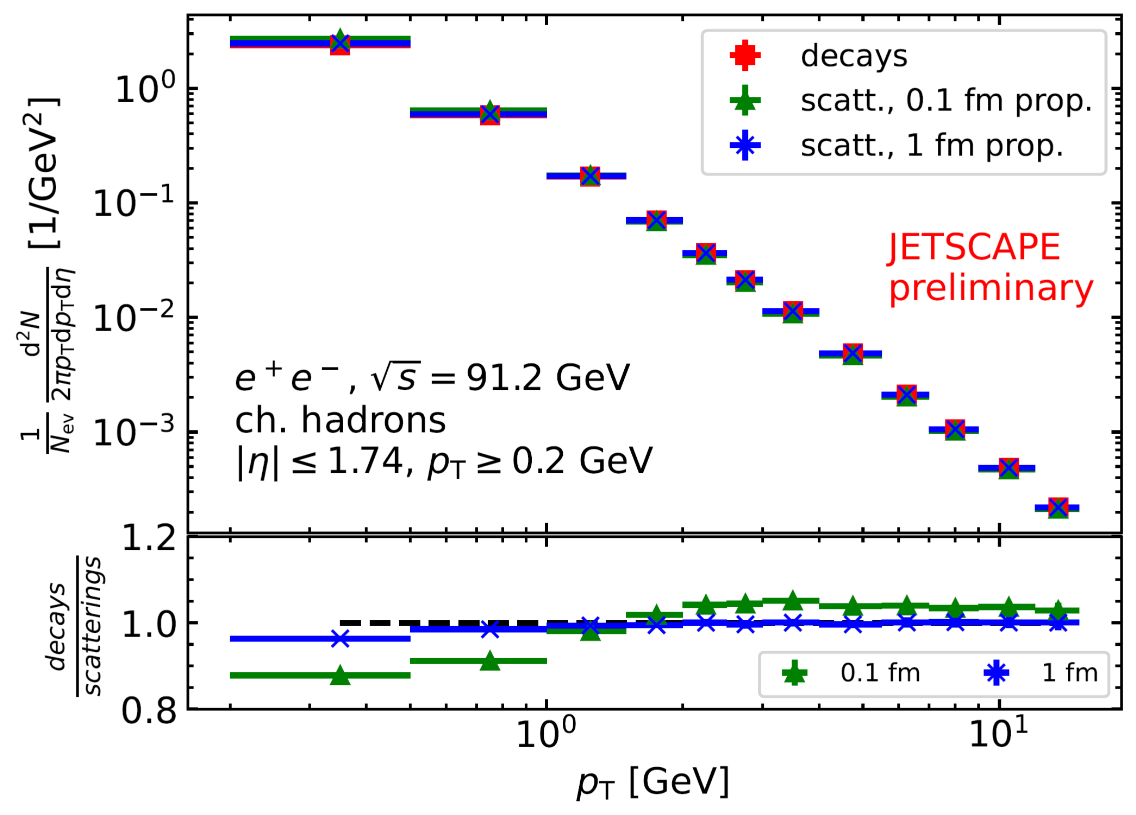
v_T, v_L: squared phase space distance perpendicular and parallel to the quantization axis.

PREVIEW: HADRONIC RESCATTERING FOR HARD PROBES

- Hybrid Hadronization with excited states and other improvements will be released with JETSCAPE 3.6
- One other new trick: hadrons from Hybrid Hadronization/hard probes can now feed into the SMASH hadronic afterburner
- Allows for systematic studies of hadronic rescattering for Hard Probes.
- See poster by Hendrik Roch (530).

e^+e^- with SMASH afterburner

Decays only vs decays+rescattering (two different scenarios)



Rescattering -> Flow-like distortion of the spectrum

SUMMARY

- We have laid the foundation to include excited meson states into the recombination formalism:
  - Coalescence probabilities for Gaussian wave packets have been calculated.
  - Suitble excited meson lists have been curated.

- First test in e^+e^-: $\boldsymbol{p}$- and $\boldsymbol{d}$-wave mesons are important hadronization channels.
- Explore effects in systems beyond e^+e^-

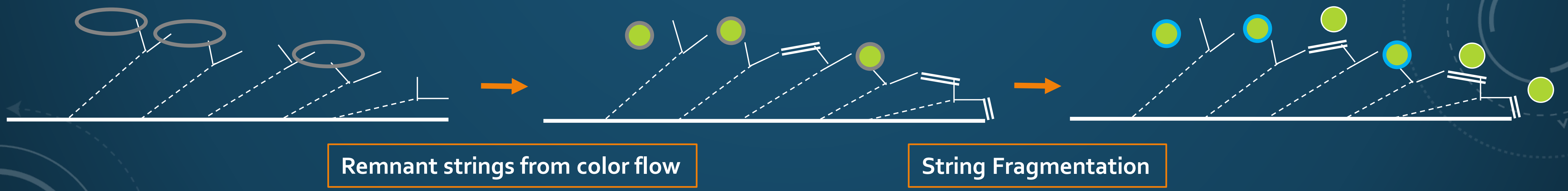
- Adding baryons: tedious but doable

- Novel manifestation of polarization effects from orbital angular motion of quarks?
- SMASH now accepts Hybrid Hadronization hadrons in JETSCAPE: hadronic rescattering for hard probes

BACKUP

JETS IN HYBRID HADRONIZATION

- Decay gluons provisionally into qqbar pairs (gluons whose quarks don't recombine are later reformed)
- Go through all possible quark pairs/triplets, compute the recombination probability and sample it. Recombine the pair/triplet if successful.
- Rejected partons again form acceptable string systems (only color singlets removed!)
- Remnant strings are fragmented by PYTHIA 8.



3D-HARMONIC OSCILLATOR IN PHASE SPACE

- Express angular momentum eigenstates through an expansion in products of 1D-states

Radial, orbital angular momentum and magnetic quantum nubmers

$$\Psi_{klm}(\mathbf{r}) = \sum_{n_1 n_2 n_3} C_{klm,n_1 n_2 n_3} \Phi_{n_1 n_2 n_3}(\mathbf{r})$$

Three 1D-quantum numbers

- Tedious but straight forward, e.g. for $k = 0$:

$$C_{0lm,n_1 n_2 n_3} = \sqrt{\frac{(l+m)!(l-m)!}{2^{2l} n_1! n_2! n_3! (2k+2l-1)!!}}\, 2^{n_3} i^{n_2} \binom{n_2}{\kappa}\, {}_2F_1(-\kappa, -n_1; 1-\kappa+n_2; -1)$$

$$\kappa = \frac{1}{2}(l+m-n_3)$$

- Two conditions for non-zero coefficients:

$$N \equiv n_1 + n_2 + n_3 = 2k + l$$

$$l + m - n_3 = 0 \bmod 2$$

Condition for matching energy of states