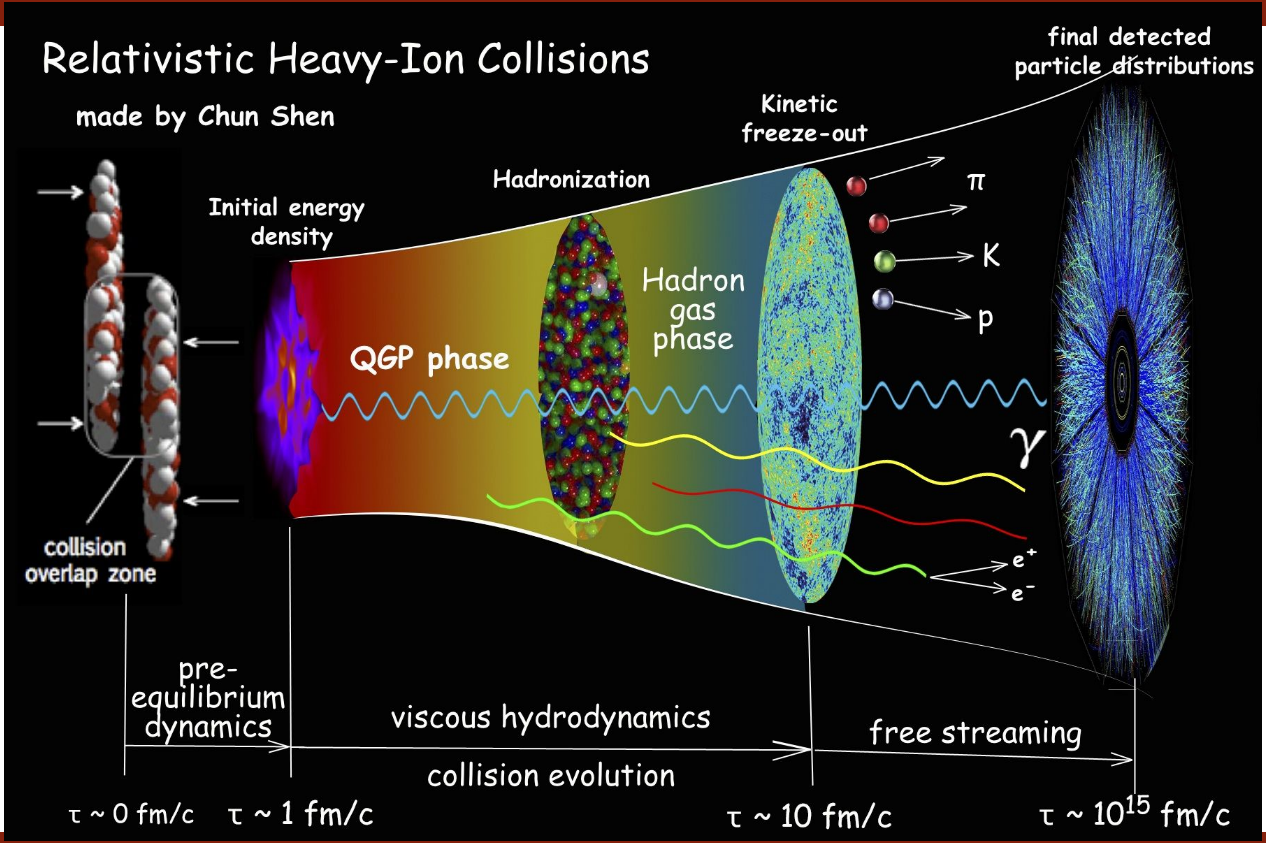
Adiabatic Hydrodynamization

A Natural Framework to Find and Describe Prehydrodynamic Attractors

Rachel Steinhorst

with Bruno Scheihing-Hitschfeld and Krishna Rajagopal

Hydrodynamization in Heavy Ion Collisions



Hydrodynamization in Heavy Ion Collisions

- How can we describe early out-of-equilibrium pre-hydro stage?
  - QCD Kinetic Theory
  - Holography
  - Glasma
- Many descriptions have been shown to have "attractor" solutions
  - See e.g. Kurkela, van der Schee, Wiedemann, Wu, arXiv:1907.08101
- Adiabatic Hydrodynamization framework: understand attractors in kinetic theory as the time-dependent ground state of an evolving effective Hamiltonian, long before hydrodynamization
  - Brewer, Yan, Yin, arXiv:1910.00021
  - Brewer, Scheihing-Hitschfeld, Yin, arXiv:2203.02427

Kinetic Theory and Rescaling

$$\frac{\partial f}{\partial \tau} + \frac{p_\perp}{p}\nabla_{x_\perp} f + \frac{p_\eta}{\tau p}\frac{\partial f}{\partial \eta} + \frac{p_\eta}{\tau}\frac{\partial f}{\partial p_\eta} = \frac{g_s^4 N_c^2}{4\pi} l_{Cb}[f]\left(q\nabla_p^2 f + \lambda \nabla_p \cdot (\hat{p} f(1+f))\right)$$

collision kernel assuming small-angle scattering

where $q = \int_p f(1+f)$ and $\lambda = \int_p \frac{2f}{p}$

- Goal: dynamically rescale *f* and **p** to write theory as $H_{\text{eff}}\, w = -\partial_\tau w$ in such a way that H_{eff} is gapped and w decays to a ground state "attractor"

- System well described by adiabatic approximation if

$$\delta_A \equiv \left| \frac{<\phi_n|\partial_\tau|\phi_0>}{\epsilon_n - \epsilon_0} \right| \ll 1$$

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neglect transverse expansion

assume boost invariance

collision kernel assuming small-angle scattering

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- Previous work: longitudinally expanding, overoccupied approximation (early times) [Brewer, Scheihing-Hitschfeld, Yin, arXiv:2203.02427]
  - Found analytic expression eigenstates; scaling such that $\partial_\tau|\phi_0>=0$
- One step forward, one step back: keep full small-angle collision kernel, but neglect longitudinal term and take $f \ll 1$.
- Suppose: $f(p,\tau) = A(\tau)w(p/D(\tau),\tau) = A(\tau)w(\chi,\tau)$. Then

$$H_{\text{eff}} w = -\partial_\tau w = \frac{\dot{A}}{A} w - \frac{\dot{D}}{D}\chi\partial_\chi w - \frac{q}{\chi^2 D^2}(2\chi\partial_\chi w + \chi^2\partial_\chi^2 w) - \frac{\lambda}{\chi^2 D}(2\chi w + \chi^2 \partial_\chi w)$$

Kinetic Theory and Rescaling

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previous work: simplified collision kernel

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for now: neglect
longitudinal expansion

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Optimizing Adiabaticity

$$H_{\text{eff}} = \frac{\dot{A}}{A} - \frac{\dot{D}}{D}\chi\partial_\chi - \frac{q}{\chi^2 D^2}(2\chi\partial_\chi + \chi^2\partial_\chi^2) - \frac{\lambda}{\chi^2 D}(2\chi + \chi^2\partial_\chi)$$

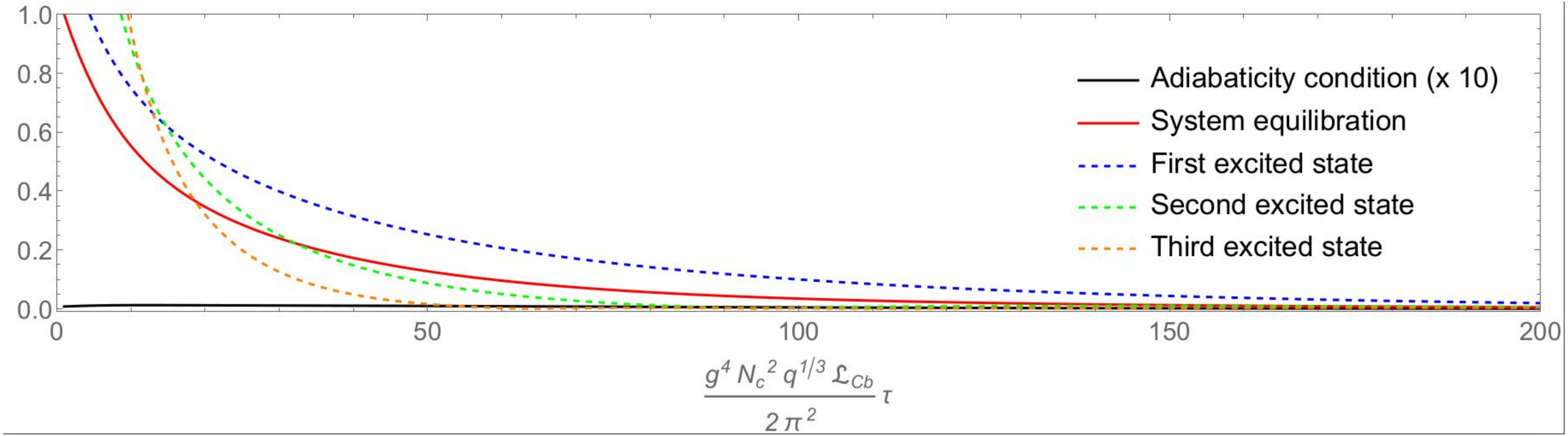
- Express H_{eff} in terms of convenient basis:

$$\psi_L^{(n)} = p_n(\chi), \quad \psi_R^{(n)} = p_n(\chi)e^{-\chi}$$

Note: p_n are generalized Laguerre polynomials with α=2

- To find an adiabatic rescaling, write down δ_A and at each time step minimize over $\ddot{D}$
- Given $f(p, t=0) = \sum_n b_n(t=0)\psi_R^{(n)}$, simultaneously solve for *D(t)* and $w_n(t)$

Optimizing Adiabaticity



As expected, we have a picture in which slow modes dominate before the system has fully thermalized. The excited states decay sequentially, and the evolution is extremely close to being exactly adiabatic.

Return to Original Kinetic Theory

$$\frac{\partial f}{\partial \tau} + \frac{p_z}{\tau}\frac{\partial f}{\partial p_z} = \frac{g_s^4 N_c^2}{4\pi} l_{Cb}[f]\left(q\nabla_p^2 f + \lambda \nabla_p \cdot (\hat{p} f(1+f))\right)$$

- Bring back longitudinal expansion and quadratic terms in f.
- Suppose

$$f(p,\tau) = A(\tau) w(p_\perp / B(\tau), p_z / C(\tau), \tau) = A(\tau) w(\zeta, \xi, \tau)\,.$$

Then $$H_{\text{eff}} = \frac{\dot{A}}{A} - \frac{\dot{B}}{B}\zeta\partial_\zeta + (\frac{\dot{C}}{C} + 1)\xi\partial_\xi - q\tau\left[\frac{1}{B^2}\left(\frac{1}{\zeta}\partial_\zeta + \partial_\zeta^2\right) + \frac{1}{C^2}\partial_\xi^2\right]$$
$$- \frac{\lambda\tau}{p}\left(2(1+Aw) + (1+2Aw)\zeta\partial_\zeta + (1+2Aw)\xi\partial_\xi\right)$$

Optimizing Adiabaticity

$$H_{\text{eff}} = \frac{\dot{A}}{A} - \frac{\dot{B}}{B}\zeta\partial_\zeta + (\frac{\dot{C}}{C} + 1)\xi\partial_\xi - q\tau\left[\frac{1}{B^2}\left(\frac{1}{\zeta}\partial_\zeta + \partial_\zeta^2\right) + \frac{1}{C^2}\partial_\xi^2\right]$$

$$- \frac{\lambda\tau}{B\zeta}(2(1+Aw) + (1+2Aw)\zeta\partial_\zeta + (1+2Aw)\xi\partial_\xi)$$

Note: using approximation $p \approx p_\perp = B\zeta$ for convenience

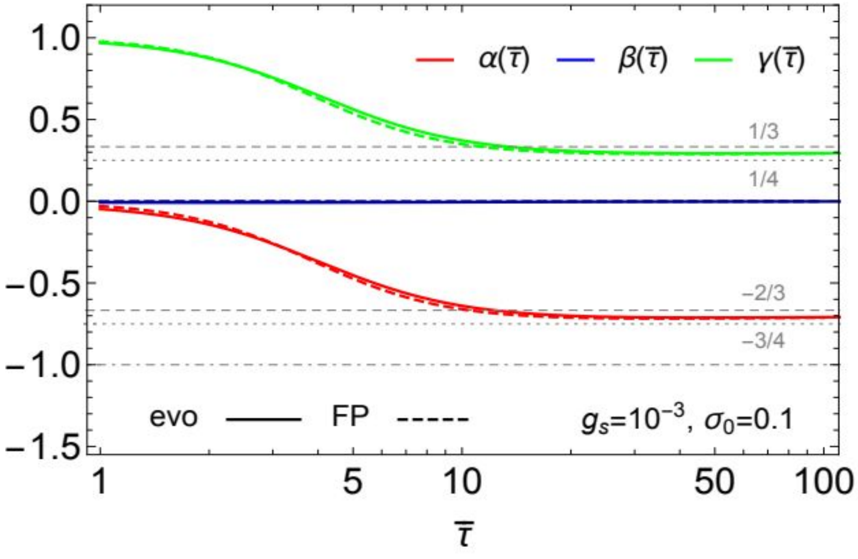
- Express H_{eff} in terms of convenient basis:

$$\psi_{nm}^{(R)} = p_n(\zeta)q_m(\xi)e^{-(\xi^2/2+\zeta)}, \quad \psi_n^{(L)} = p_n(\zeta)q_m(\xi)$$

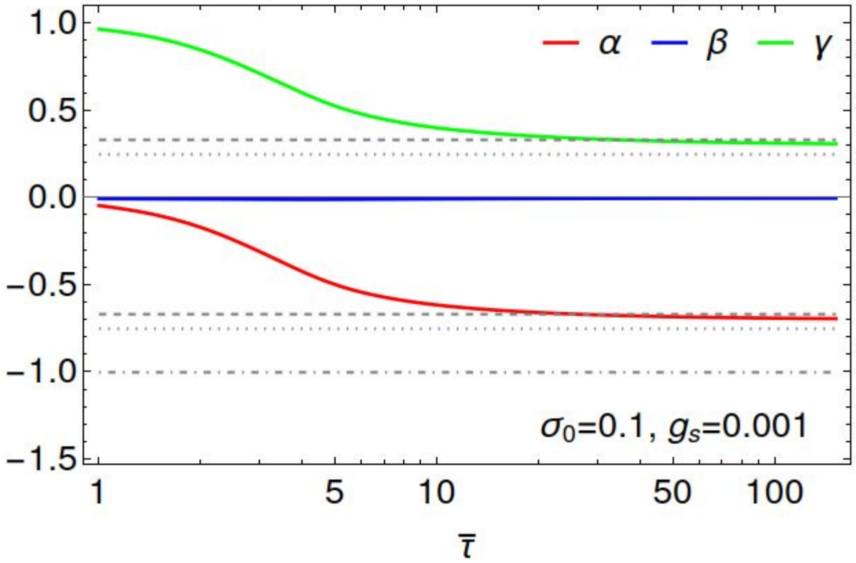
- To find an adiabatic rescaling, write down δ_A and at each time step minimize over $\ddot{B}$ and $\ddot{C}$
- Given $f(p,\tau_0) = \sum_{nm} w_{nm}(\tau_0)\psi_R^{(n,m)}$, simultaneously solve for B, C, and w_n dynamically

Comparison with Previous Results (Part 1)

Brewer, Scheihing-Hitschfeld, Yin, arXiv:2203.02427

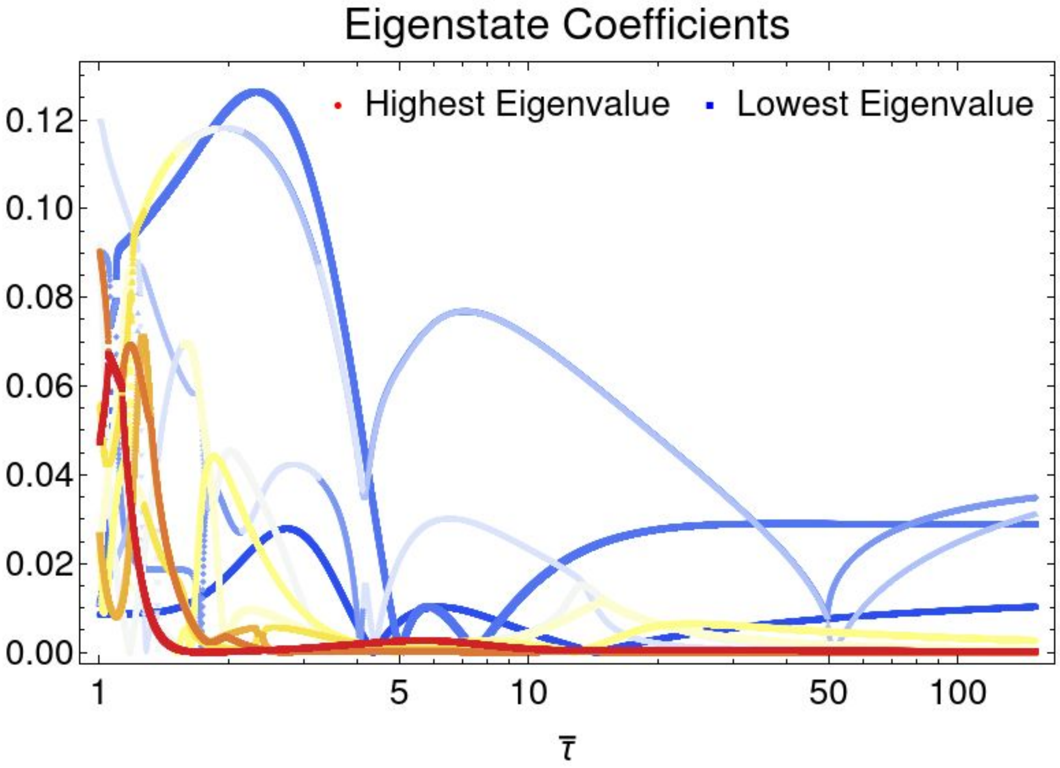


This work:



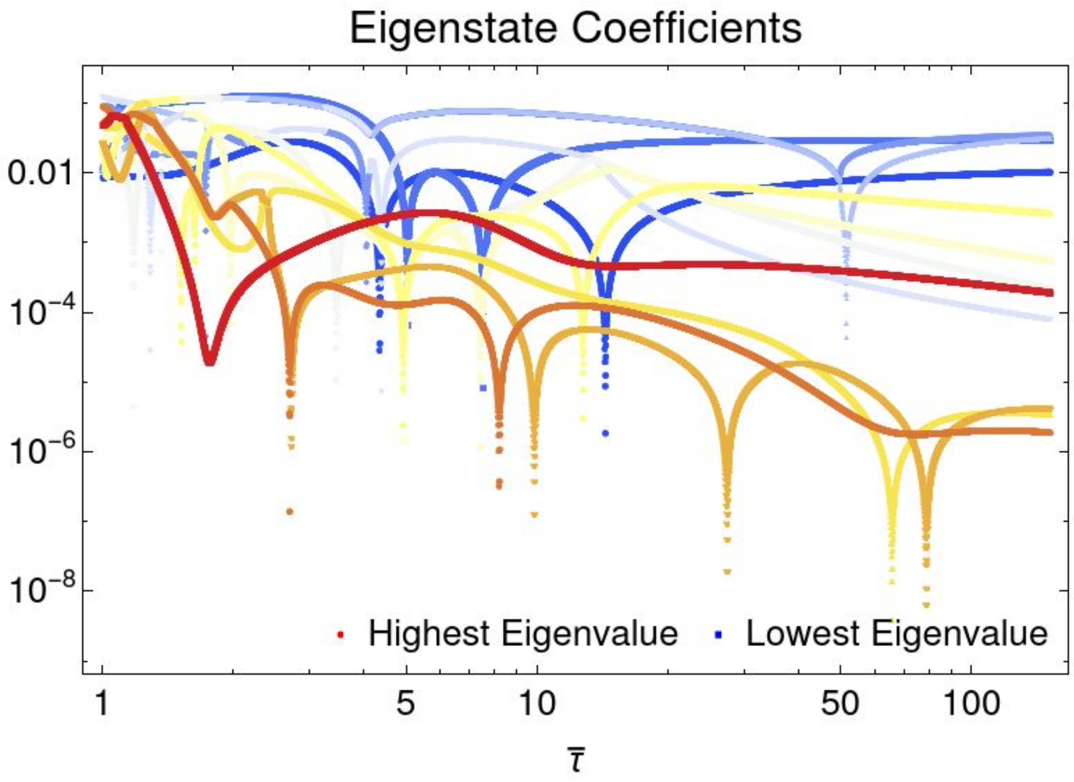
where $\alpha = \frac{\tau}{A}\partial_\tau A\,,\ \ \beta = -\frac{\tau}{B}\partial_\tau B\,,\ \ \gamma = -\frac{\tau}{C}\partial_\tau C$

Dominance of Low-Energy Modes



Evolution is not strictly adiabatic, but low energy modes in the "most adiabatic" frame still quickly dominate

Dominance of Low-Energy Modes



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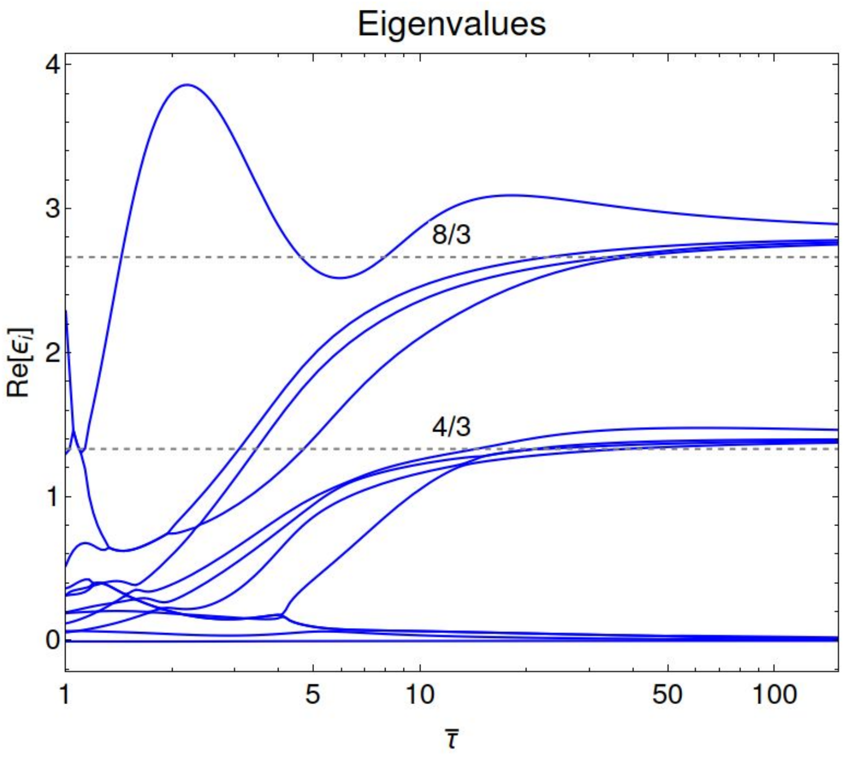
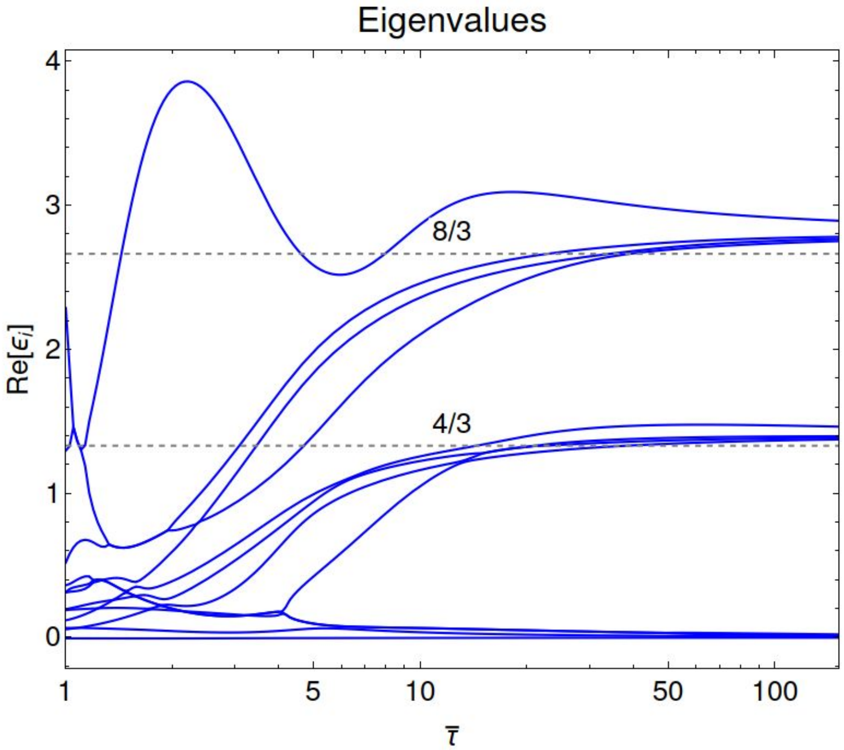
Dominance of Low-Energy Modes



- Not adiabatic because the ground state alone does not dominate, but there is clear separation between low-energy and high-energy modes
- 3 clusters in energy correspond to 3 longitudinal basis states

Comparison with Previous Results (Part 2)



Energy gap between each cluster matches earlier prediction:

$$\mathcal{E}_n = 2n(1 - \gamma)$$

[Brewer, Scheihing-Hitschfeld, Yin, arXiv:2203.02427]

Summary

- We can find an adiabatic frame for a kinetic equation whose effective eigenstates are not known analytically
- We can reproduce behavior of scaling exponents by searching for exponents which maximize adiabaticity
- As in previous work, adiabatic approach explains how attractor behavior arises long before hydro
- By including transverse modes and not assuming $f \gg 1$, this work demonstrates the robustness of adiabatic hydrodynamization

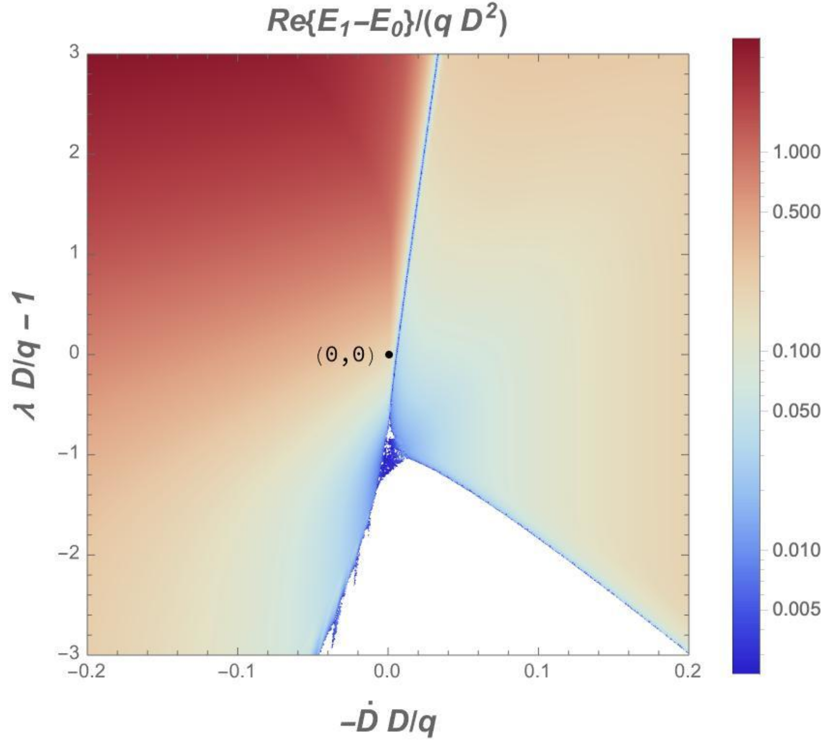
Next Steps

- Extending this analysis to more general kinetic equations
  - Transverse expansion
  - 1-to-2 scattering
- Using attractors as an ingredient in Bayesian analyses of heavy ion collision data (e.g. Trajectum)

Backup Slides

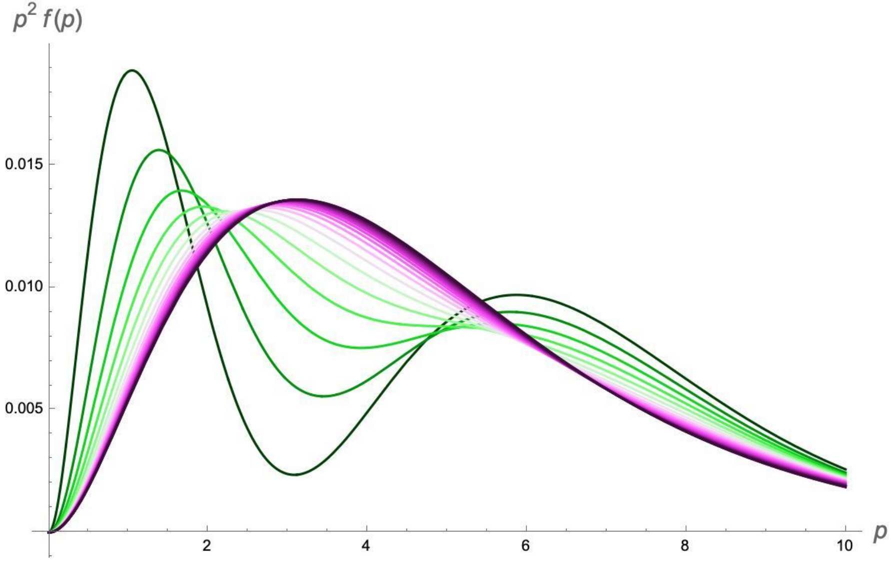
The Energy Gap as a Function of *D*

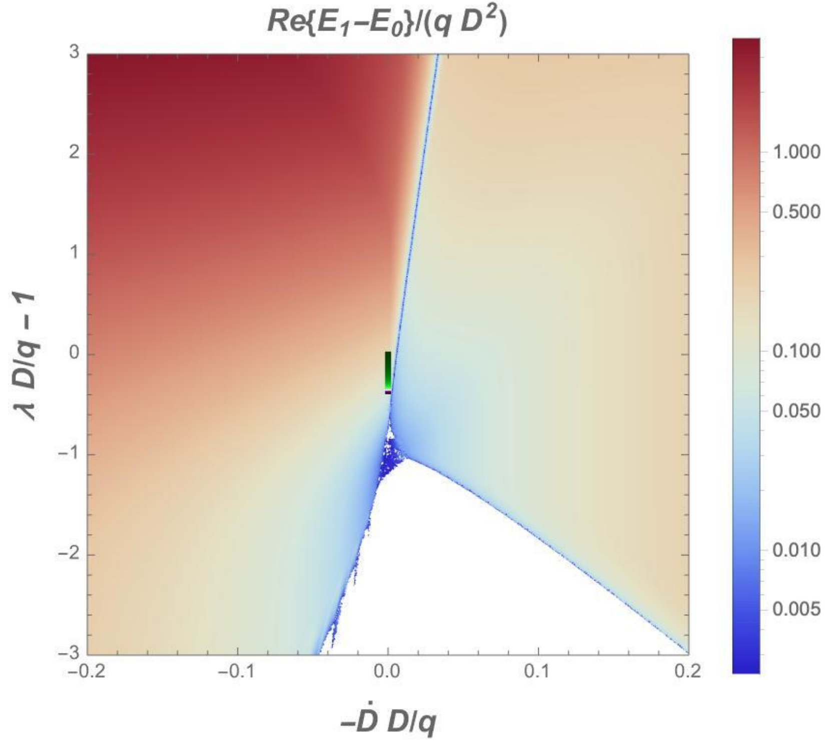
- The energy gap above the instantaneous ground state depends on the choice of the scaling variable *D*.
- The evolution of the system will traverse a 1D path on this parameter space
    - How adiabatic is the evolution on these paths?



Example path #1

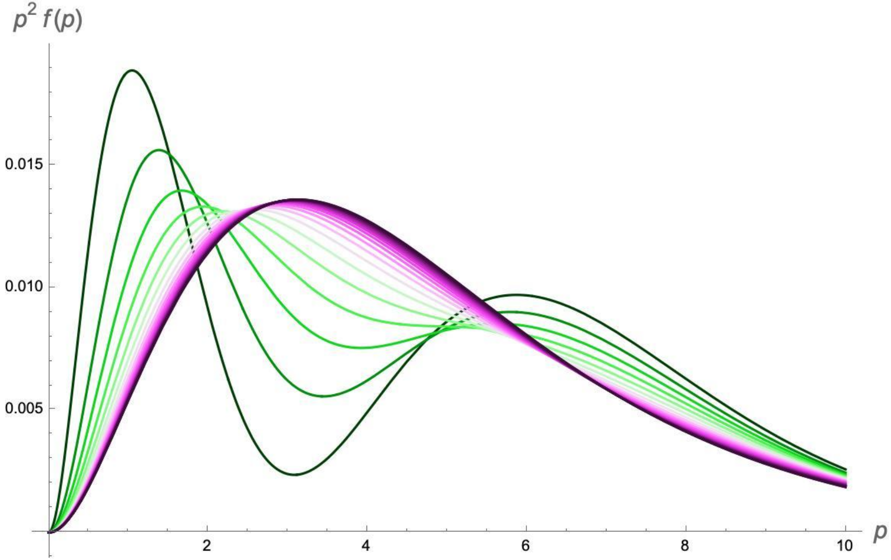
Given some choice of initial distribution function, we can test difference possible choices for *D(t)*. **One choice: *D* constant**

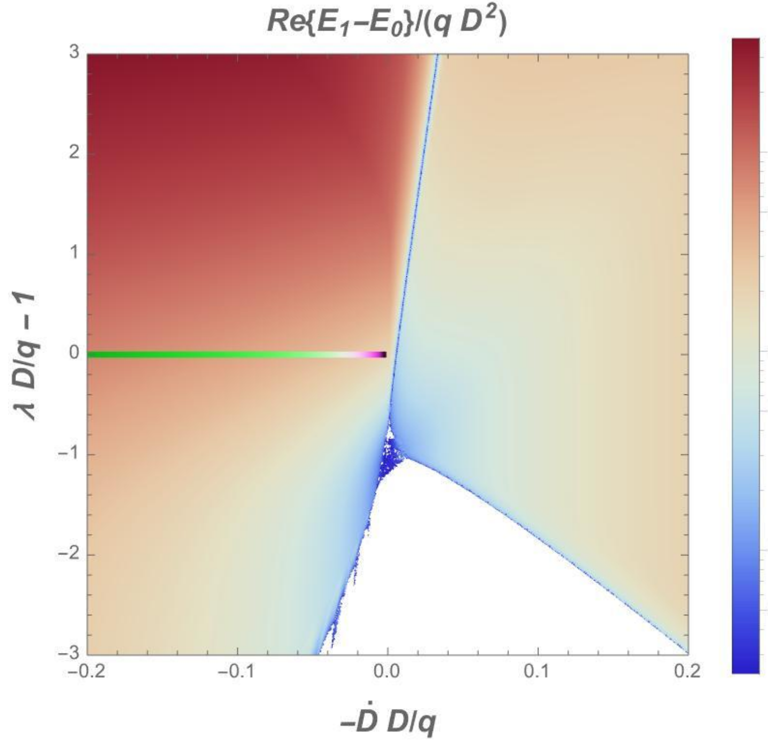




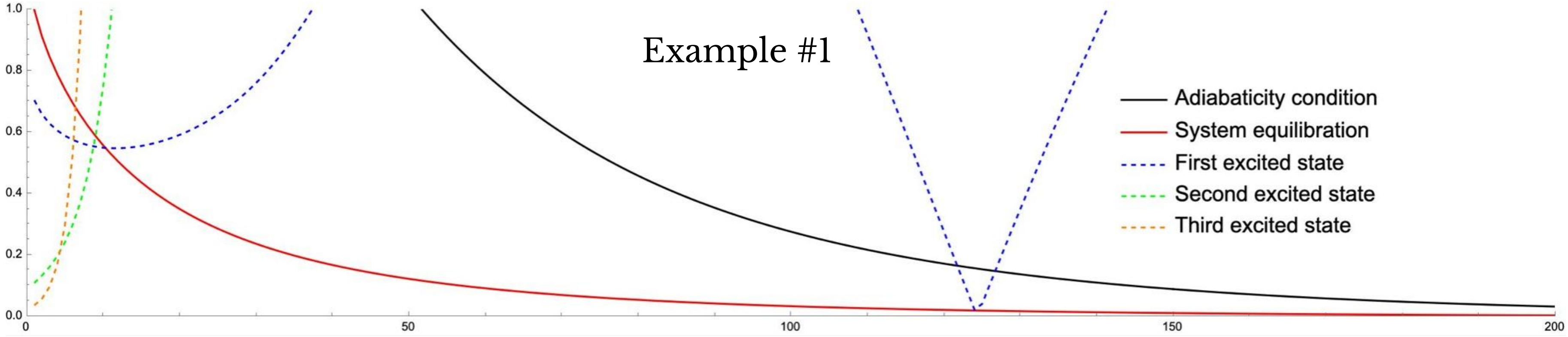
Example path #2

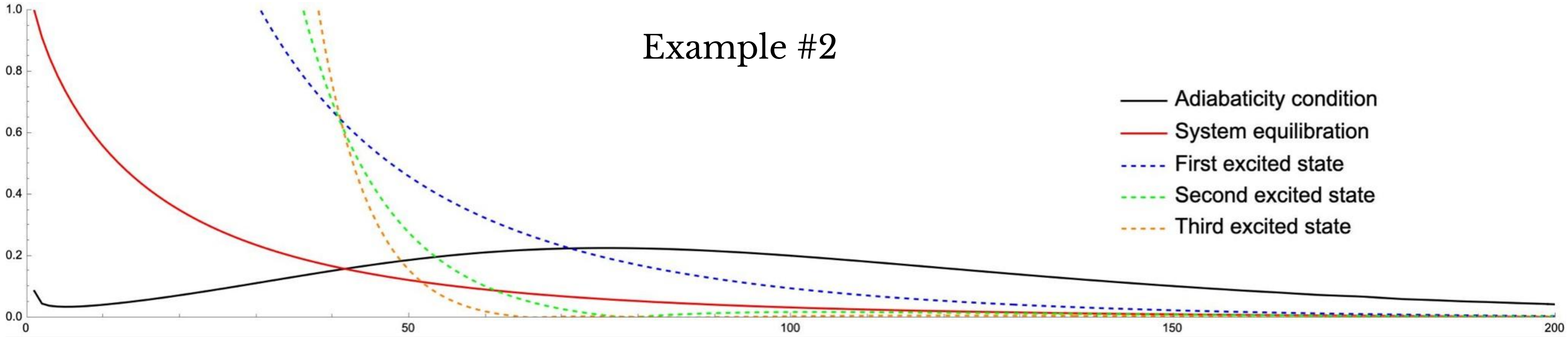
Another possible choice: $D(t) = q/\lambda(t)$





Adiabaticity for example paths





Optimizing Adiabaticity

Choosing D to maximize adiabaticity