

# AMY Lorentz invariant parton cascade

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# Motivation

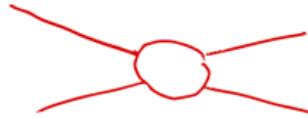
- ▶ kinetic theory good candidate for describing small and large collision systems  
applicability of hydro in small systems questionable
- ▶ phenomenological successes of kinetic theory:
  - ▶ rapid equilibration  
A. Kurkela, Nucl. Phys. A 956 (2016) 136 & A. Kurkela, Y. Zhu, Phys. Rev. Lett. 115 (2015) no.18, 182301
  - ▶  $v_2$  in small collision systems via escape mechanism  
Kurkela, Wiedemann, Wu, Phys. Lett. B 783 (2018) 274 & Kurkela, Mazeliauskas, Törnkvist, JHEP 11 (2021), 216
  - ▶ calculation of photon emission, transport properties, thermalisation of hard partons, ...
- ▶ would like to have MC event generator for apples-to-apples comparisons to data
- ALPACA

# The AMY effective kinetic theory

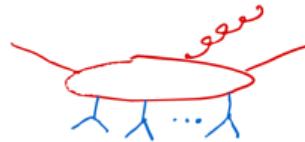
- ▶ effective kinetic theory of QCD at high temperature  $\rightarrow g(T) \ll 1$

Arnold, Moore, Yaffe, JHEP 0301 (2003) 030

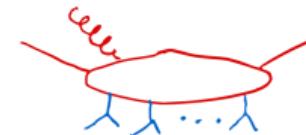
- ▶ partons are quasi-free and quasi-massless particles
- ▶ correctly describes dynamics of hard modes  $\mathcal{O}(T)$   $\rightarrow$  typical momenta in thermal system
- ▶ Boltzmann equation:  $(\partial_t + \mathbf{v} \cdot \nabla_{\mathbf{x}})f(\mathbf{x}, \mathbf{p}, t) = -C[f]$   
dynamics contained in collision kernel  $C[f]$
- ▶ 2 types of processes:



$2 \leftrightarrow 2$



" $1 \rightarrow 2$ "



" $2 \rightarrow 1$ "

- ▶ interaction rates depend on effective masses  $m_{\text{eff}}$  and temperature  $T_*$

# ALPACA – AMY Lorentz-invariant PArtion CAscade

Kurkela, Törnvist, Zapp, arXiv:2211.15454

- ▶ evolves parton ensemble according to AMY collision kernels
- ▶ Lorentz-invariant due to ordering in Lorentz scalar  $\tau$

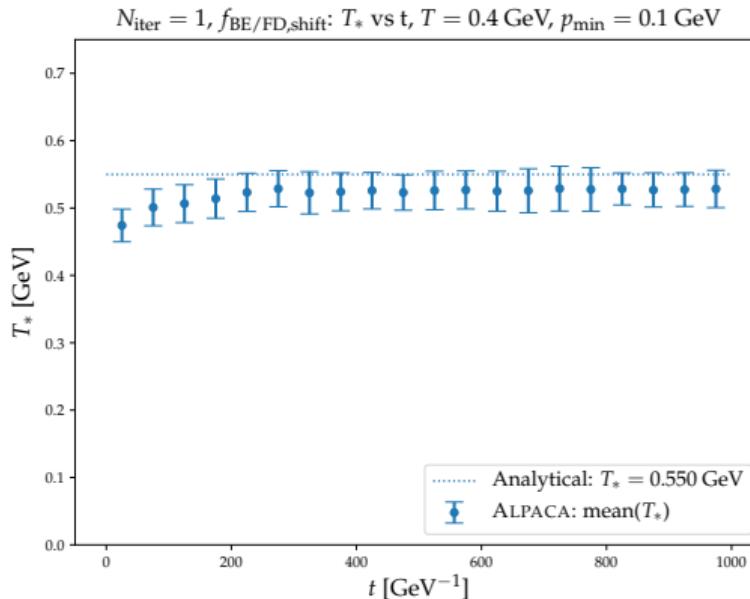
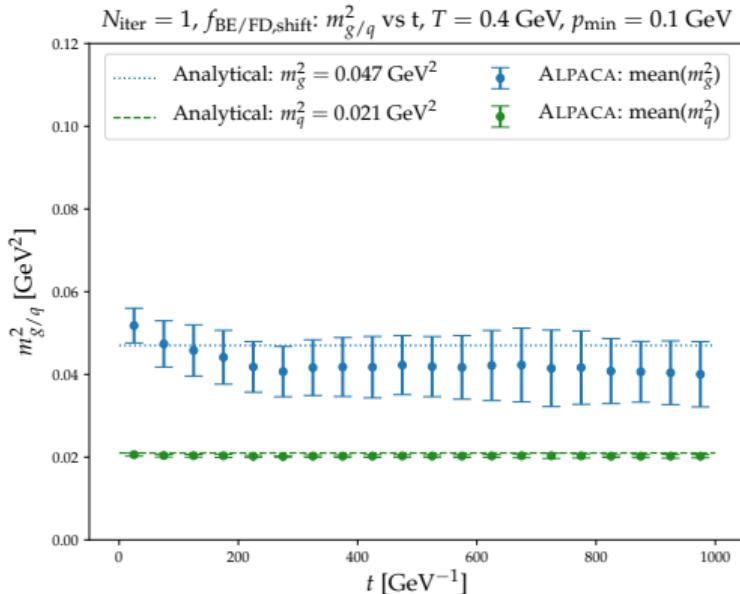
Peter, Behrens, Noack, Phys. Rev. C 49 (1994) 3253 & Borchers, Meyer, Gieseke, Martens, Noack, Phys. Rev. C 62 (2000) 064903

- ▶ elastic scattering and merging: interaction when invariant distance  $d_{ij} < \sqrt{\sigma/\pi}$   
black disk approximation
- ▶ splitting: analogous to parton shower (evolving in time)
- ▶ effective masses and temperature: obtained by summing over nearby partons

$$m_{\text{eff}}^2 \propto \int \frac{d^3 p}{(2\pi)^3} \frac{f(p)}{|p|} \quad \text{and} \quad T_* \propto \frac{1}{m_{\text{eff}}^2} \int \frac{d^3 p}{(2\pi)^3} f(p)[1 + f(p)]$$

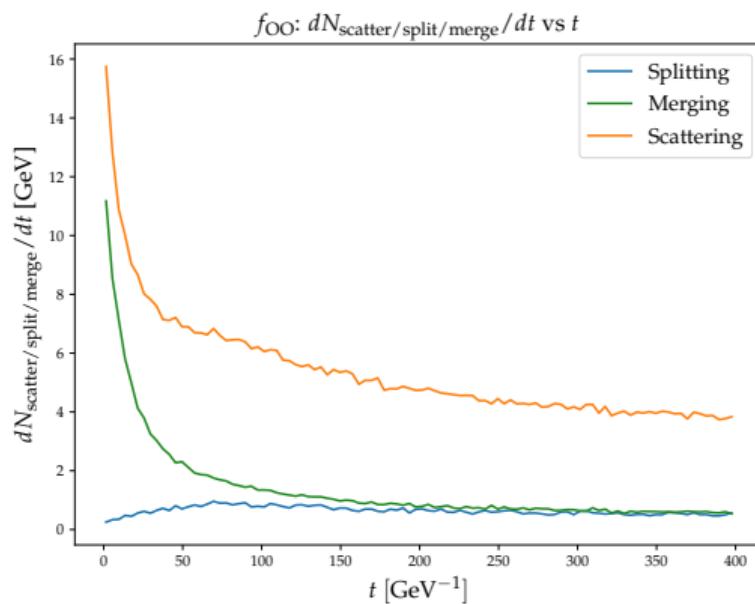
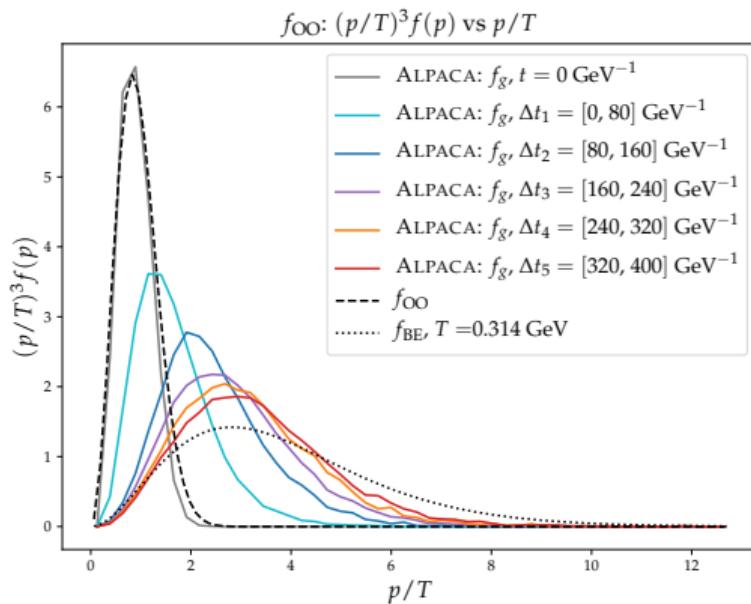
- ▶ infra-red momentum cut-off  $p_{\min}$ : have to regularise divergent splitting/merging rates  
cancellation not automatic as in Boltzmann equation

# Effective masses and temperature



- ▶ agrees well with expectation in thermal equilibrium

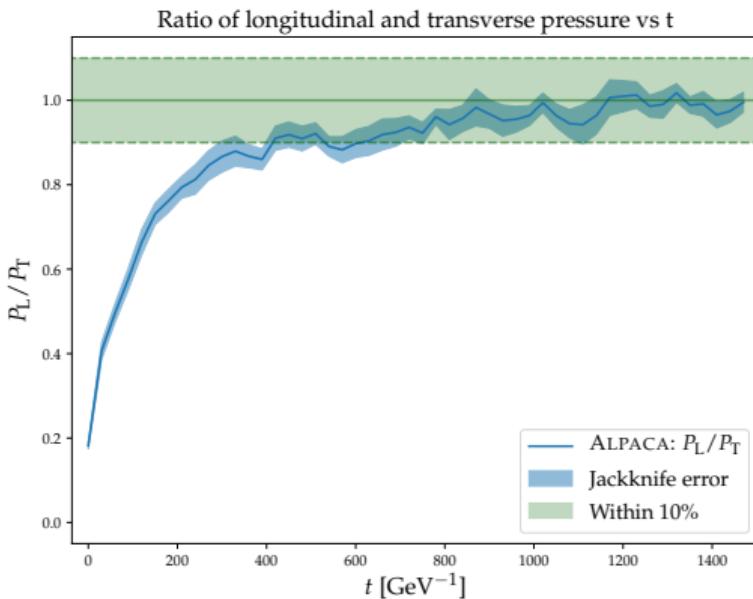
# Thermalisation in ALPACA



- ▶ agrees well with expectation from direct solution of Boltzmann equation

Fu, Ghiglieri, Iqbal, Kurkela, Phys. Rev. D 105 (2022) no.5, 054031

# Isotropisation in ALPACA



► CGC-inspired anisotropic initial condition

$$f_0 = \frac{AQ_0}{\sqrt{p_\perp^2 + \xi^2 p_z^2}} \exp\left(-\frac{2(p_\perp^2 + \xi^2 p_z^2)}{3Q_0^2}\right)$$

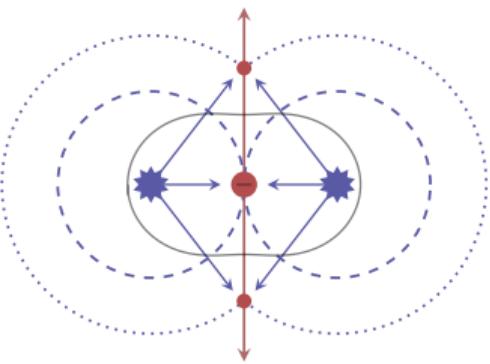
$$A = 1, \xi = 4, Q_0 = 1.8 \text{ GeV}, \lambda = 1$$

Kurkela, Zhu, Phys. Rev. Lett. 115 (2015) no.18, 182301

► system isotropises as expected

# Azimuthal anisotropy in small systems – escape mechanism

Kurkela, Wiedemann, Wu, Phys. Lett. B 783 (2018) 274



- ▶ local isotropisation due to scattering induces global anisotropy
- ▶ requires only  $\mathcal{O}(1)$  scattering per particle
- ▶ also observed in transport code AMPT

He, Edmonds, Lin, Liu, Molnar, Wang, Phys. Lett. B 753 (2016) 506

## ▶ single hit calculation in AMY EKT:

- ▶ calculates first correction to free streaming
- ▶ calculates  $v_2$  from energy flow → only elastic scattering needed
- ▶ CGC inspired initial condition
- ▶ finds sizeable  $v_2$  in small collision systems

Kurkela, Mazeliauskas, Törnkvist, JHEP 11 (2021), 216

corresponds to one scattering per particle

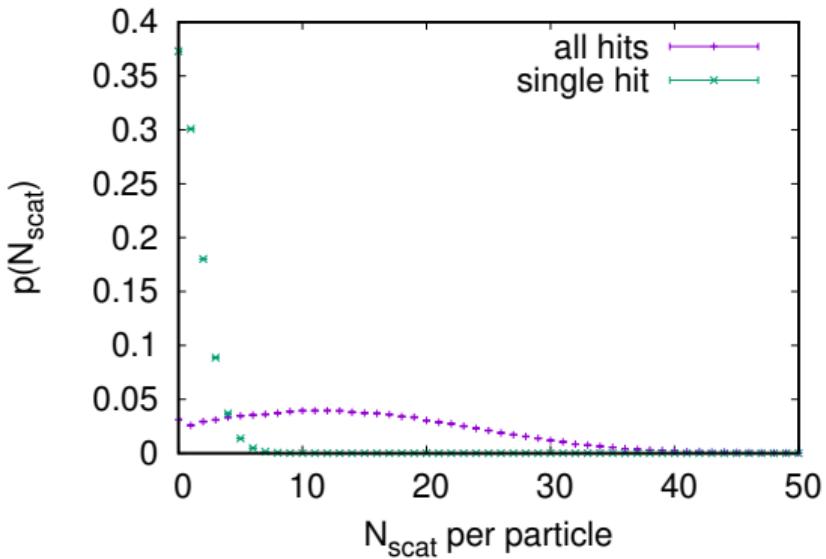
# Comparing ALPACA and single hit calculation

- ▶ **first observation:** ALPACA gives a few to more than 10 scatterings per particle
- ▶ qualitative behaviour does not depend on details of scattering dynamics
- ▶ simple toy model for illustration purposes:
  - ▶ same initial condition as full calculation:

$$f_0 = \frac{A Q(x_\perp)}{\sqrt{p_\perp^2 + \xi^2 p_z^2}} \exp\left(-\frac{2(p_\perp^2 + \xi^2 p_z^2)}{3Q^2(x_\perp)}\right) \left[1 + \epsilon \frac{x_\perp^n}{R_0^n} \cos(n\phi_x)\right] \quad \text{with} \quad Q(x_\perp) = Q_0 \exp\left(-\frac{x_\perp^2}{4R_0^2}\right)$$

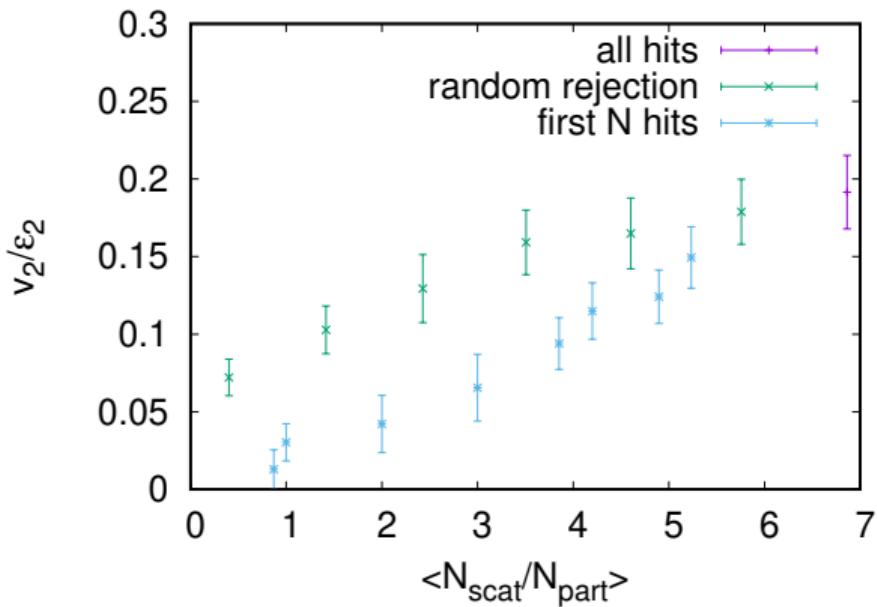
- ▶ gluons only
- ▶ only elastic scattering
- ▶ cross section: constant
- ▶ differential cross section:  $\frac{d\sigma}{dt} \propto \frac{1}{(t - \mu^2)^2}$

# Number of scatterings per particle



- ▶ very broad distribution
- ▶ two ways of obtaining  $\langle N_{\text{scat}} \rangle \approx 1$ :
  - ▶ stop after first scattering → "first hit"
  - ▶ randomly reject scatterings → "single hit"
- ▶ in single hit particles can have several interactions
- residual difference to single-hit calculation

## $v_2$ and number of scatterings

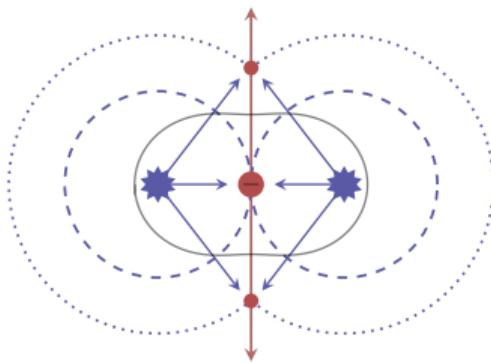


$$\sigma = 25 \text{ GeV}^{-2}, A = 0.3, Q_0 = 3 \text{ GeV}, R_0 = 5 \text{ GeV}^{-1},$$
$$\tau_0 = 1 \text{ GeV}^{-1}, \xi = 4, \epsilon = 0.5, \mu = 1 \text{ GeV}$$

- ▶ single hit  $\neq$  first hit
- ▶ consistent with finding in analytical calculation that  $v_2$  builds up late
- ▶  $v_2$  takes several scatterings to build up
- ▶ distribution in  $N_{\text{scat}}$  per particle very broad
- ▶ not most realistic parameters

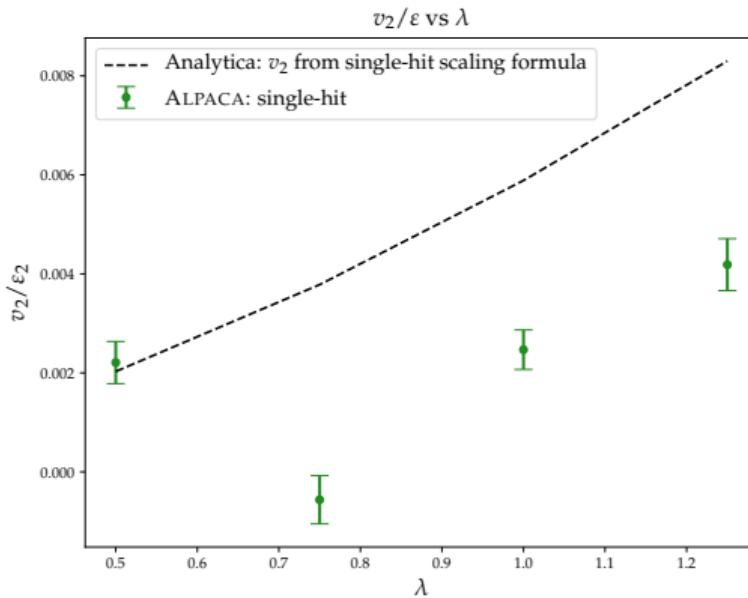
for demonstration only

# The effect of interactions at a distance



- ▶ in single-hit calculation: scattering is local
- ▶ maximises  $v_2$  from escape mechanism
  - incoming momenta in c.m.s. always horizontal
- ▶ in ALPACA: particles interact at a distance
- ▶ scattering averages over finite size space-time volume
- ▶ expect reduction of  $v_2$

# The effect of interactions at a distance



- ▶ **preliminary results!**
- ▶ consistent with expectation for small coupling
- ▶ ALPACA values fall below expectation for increasing coupling
- ▶ consistent with smearing out of effect

# Conclusions

## ALPACA

- ▶ Lorentz-invariant parton cascade implementing AMY dynamics
- ▶ validated for thermal equilibrium
- ▶ validation for thermalisation well under way
- ▶ first results for small collision systems → consistent with single-hit calculation for  $\lambda$

## comparison to single-hit calculation

- ▶ ALPACA typically gives more than one interaction per particle for same initial condition
- ▶ single hit  $\neq$  first hit
- ▶  $v_2$  increases with number of hits
- ▶ ALPACA: scatterings average over finite space-time volume → reduces  $v_2$
- ▶ *generic feature of parton cascades*

# Acknowledgment

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# Bonus material

# Effective kinetic theory – I

Arnold, Moore, Yaffe, JHEP 0301 (2003) 030

## Starting point

- ▶ EKT for weakly coupled QCD plasma
- ▶  $g(T) \ll 1$
- ▶  $T$ : hard scale;  $gT$ : soft scale

## an obvious objection

- ▶ QGP at accessible energies not weakly coupled
- ▶ maybe

not clear any more whether it supports quasi-particle description

- ▶ this EKT very successfully describes approach to hydrodynamic behaviour

# Effective kinetic theory – II

- ▶ weakly coupled QCD plasma consists of quasi-particles

quarks & gluons

- ▶ typical momenta  $\mathcal{O}(T)$
- ▶ thermal mass  $\mathcal{O}(gT)$  due to colour screening
- ▶ propagate as nearly free particles
  - ▶ soft (small angle) scattering rate  $\mathcal{O}(g^2 T)$
  - ▶ hard (large angle) scattering rate  $\mathcal{O}(g^4 T)$

*t-, u- and s-channel diagrams contribute at leading order*

- ▶ hard near-collinear splitting/merging rate  $\mathcal{O}(g^4 T)$
- ▶ formation time of near-collinear splitting/merging process:  $\mathcal{O}(g^2 T)$
- ▶ additional soft scatterings during this time
- ▶ interference → LPM effect

# Elastic scattering ( $2 \leftrightarrow 2$ )

$$C_a^{2 \leftrightarrow 2}[f] = \frac{1}{4|\mathbf{p}|\nu_a} \sum_{b,c,d} \int dPS |\mathcal{M}_{ab \rightarrow cd}(\mathbf{k}, \mathbf{p}', \mathbf{k}')|^2 (2\pi)^4 \delta^{(4)}(P + K - P' - K') \\ \times \left\{ f_a(\mathbf{p})f_b(\mathbf{k})[1 \pm f_c(\mathbf{p}')][1 \pm f_d(\mathbf{k}')] - f_c(\mathbf{p}')f_d(\mathbf{k}')[1 \pm f_a(\mathbf{p})][1 \pm f_b(\mathbf{k})] \right\}$$

- ▶ gain and loss term
- ▶ Bose enhancement & Pauli blocking
- ▶ matrix elements
  - ▶ can use vacuum results for sufficiently hard momentum transfers  $q$
  - ▶ for small  $q$  replace  $q^2 \rightarrow (q^2 + 2\xi^2 m_{\text{eff}}^2)$  in divergent denominators
    - only in isotropic case, in anisotropic case soft instabilities appear

York, Kurkela, Lu, Moore, Phys. Rev. 89 (2014) no.7, 074036

$$m_{\text{eff}}^2 \propto \int \frac{d^3 p}{(2\pi)^3} \frac{f(\mathbf{p})}{|\mathbf{p}|}$$

# Near-collinear splitting/merging

$$\begin{aligned} C_a^{1 \leftrightarrow 2}[f] = & \frac{(2\pi)^3}{2|\mathbf{p}|\nu_a} \sum_{b,c} \int d\mathbf{p}' d\mathbf{k}' \delta(|\mathbf{p}| - \mathbf{p}' - \mathbf{k}') \gamma_{bc}^a(\mathbf{p}, \mathbf{p}'\hat{\beta}, \mathbf{k}'\hat{\beta}) \\ & \left\{ f_a(\mathbf{p})[1 \pm f_b(\mathbf{p}'\hat{\beta})][1 \pm f_c(\mathbf{k}'\hat{\beta})] - f_b(\mathbf{p}'\mathbf{p})f_c(\mathbf{k}'\hat{\beta})[1 \pm f_a(\hat{\beta})] \right\} \\ & + \frac{(2\pi)^3}{|\mathbf{p}|\nu_a} \sum_{b,c} \int d\mathbf{k} d\mathbf{p}' \delta(|\mathbf{p}| + \mathbf{k} - \mathbf{p}') \gamma_{ab}^c(\mathbf{p}'\mathbf{p}, \hat{\beta}, k\hat{\beta}) \\ & \left\{ f_a(\mathbf{p})f_b(k'\hat{\beta})[1 \pm f_c(p'\hat{\beta})] - f_c(p'\mathbf{p})[1 \pm f_a(\hat{\beta})][1 \pm f_b(k\hat{\beta})] \right\} \end{aligned}$$

small transverse momenta in splitting/merging process integrated over

## ► splitting/merging rates

- include multiple scattering & LPM effect
- implicitly depend on distribution functions  $f$
- given by solution to linear integral equation

# Solving Boltzmann equation

- ▶ traditional Monte Carlo approach: discrete time steps
- ▶ violates Lorentz invariance
- ▶ no-interaction theorem:

The only Lorentz-invariant Hamiltonian theory of  $N$  particles moving in a  $6N$ -dimensional phase space is the free theory.

Currie, Jordan, Sudarshan, Rev. Mod. Phys. 35 (1963) 350

- ▶ solution: go to  $8N$  dimensional phase space by allowing particles to go classically off-shell

Peter, Behrens, Noack, Phys. Rev. C 49 (1994) 3253

- ▶ implemented in PCPC

Borchers, Meyer, Gieseke, Martens, Noack, Phys. Rev. C 62 (2000) 064903

and ALPACA

Kurkela, Törnvist, Zapp, arXiv:2211.15454

## Lorentz-invariant cascade in practice

- ▶ positions and momenta of the particles functions of a Lorentz-invariant parameter  $s$

$$\frac{dx_i(s)}{ds} = \{\mathcal{H}, x_i\} = -\frac{\partial \mathcal{H}}{\partial p_i} \quad ; \quad \frac{dp_i(s)}{ds} = \{\mathcal{H}, p_i\} = +\frac{\partial \mathcal{H}}{\partial x_i}.$$

- ▶ interactions ordered frame independently in  $s$
- ▶ invariant distance of two particles i and j

$$d_{ij}^2 = - \left( x_\mu - \frac{xp}{p^2} p_\mu \right) \left( x^\mu - \frac{xp}{p^2} p^\mu \right)$$

with  $x = x_i - x_j$  and  $p = p_i + p_j$

- ▶ compare  $d_{ij}^2$  to scattering cross section to decide whether pair interacts