



Baryon number fluctuations at high baryon density

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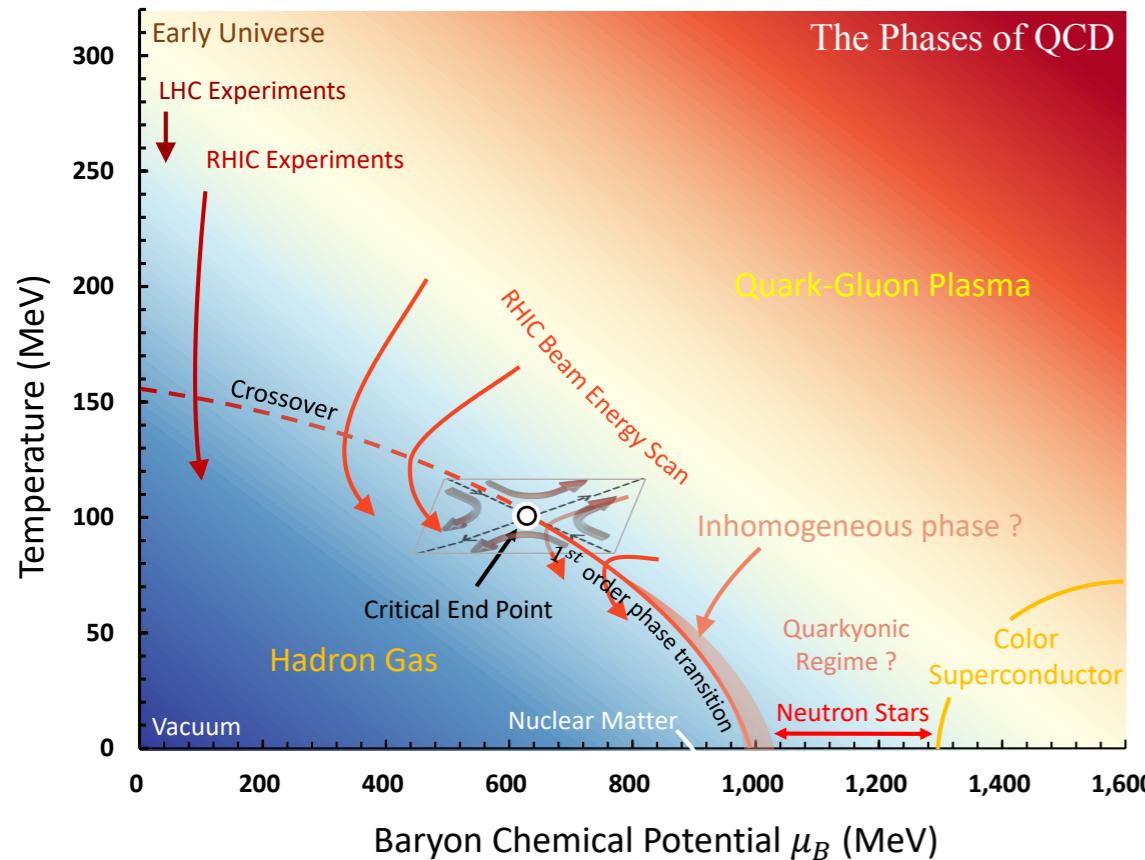


Based on :

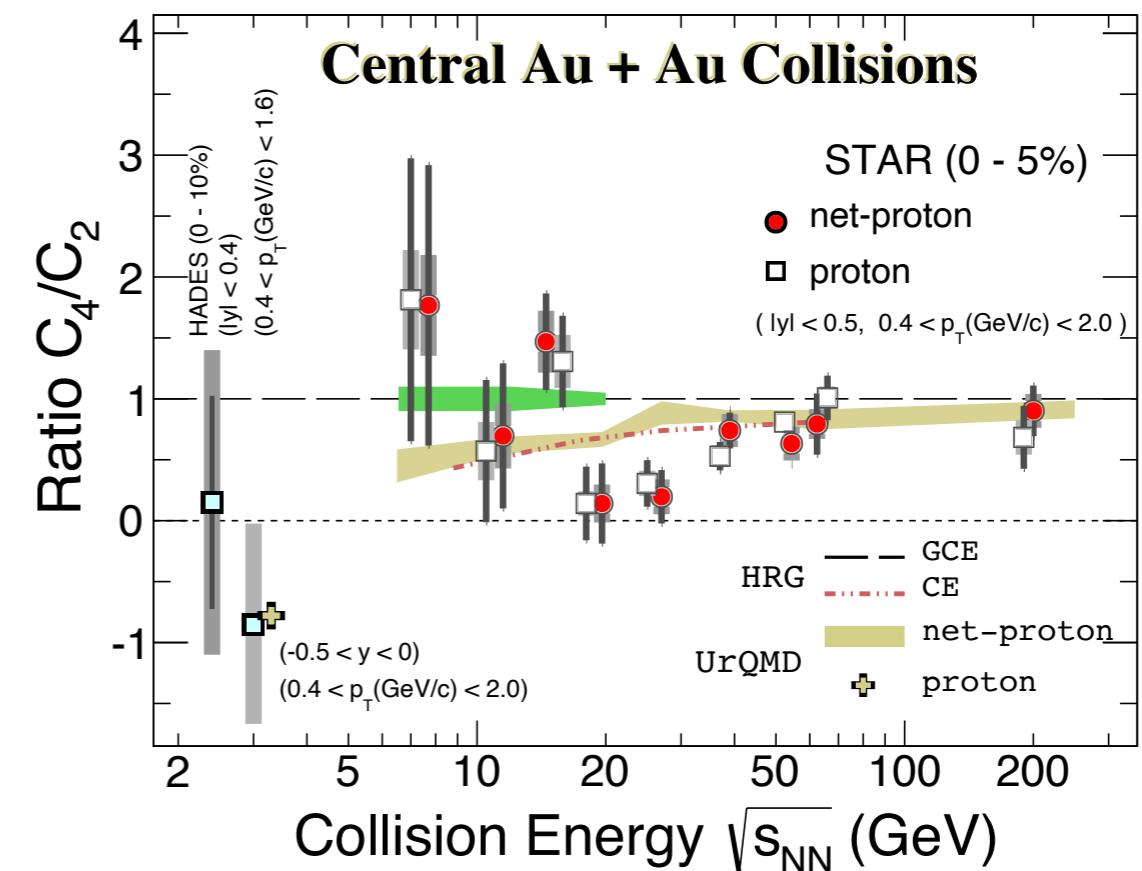
WF, Xiaofeng Luo, Jan M. Pawłowski, Fabian Rennecke, Shi Yin, *Ripples of the QCD Critical Point*, arXiv: 2308.15508;
WF, Xiaofeng Luo, Jan M. Pawłowski, Fabian Rennecke, Rui Wen, Shi Yin, *Hyper-order baryon number fluctuations at finite temperature and density*, PRD 104 (2021) 094047, arXiv: 2101.06035.

QCD phase structure

QCD phase diagram



Fluctuations measured by STAR

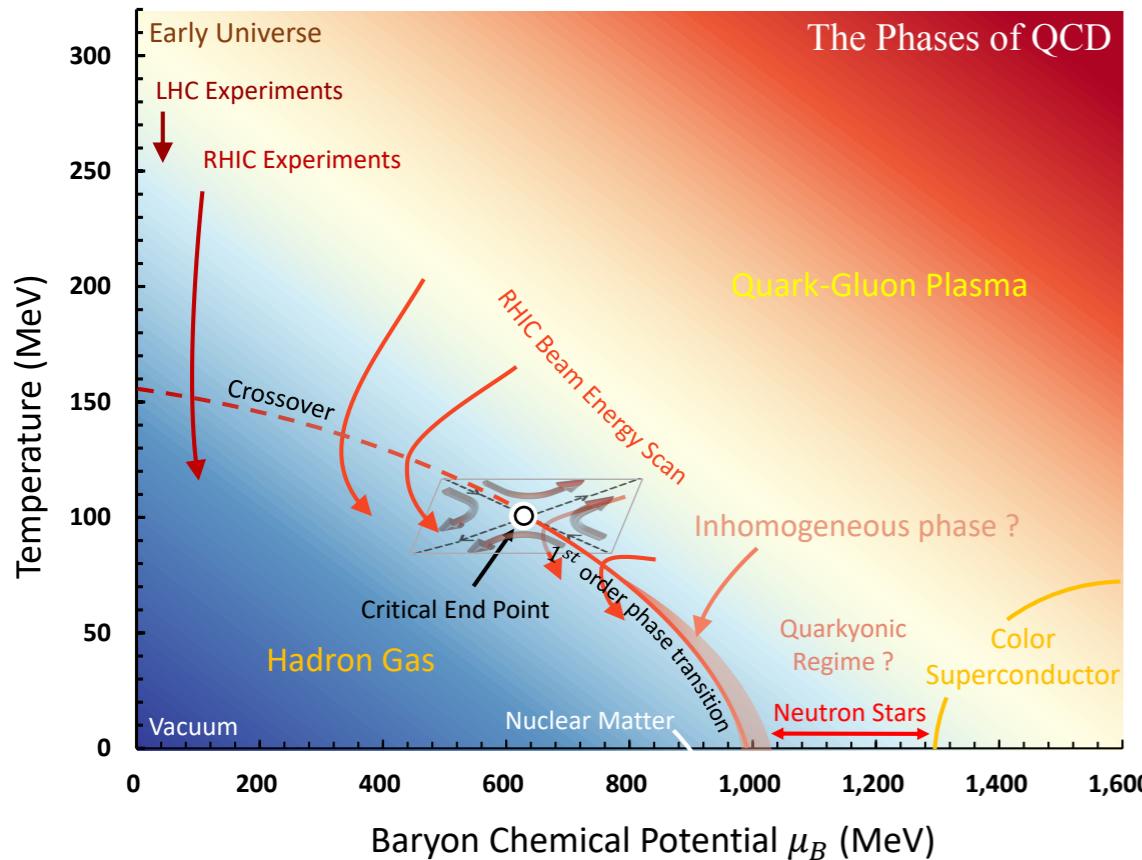


J. Adam *et al.* (STAR), *PRL* 126 (2021), 092301;
M. Abdallah *et al.* (STAR), *PRC* 104 (2021), 024902;
M. Abdallah *et al.* (STAR), *PRL* 128 (2022) 20, 202303

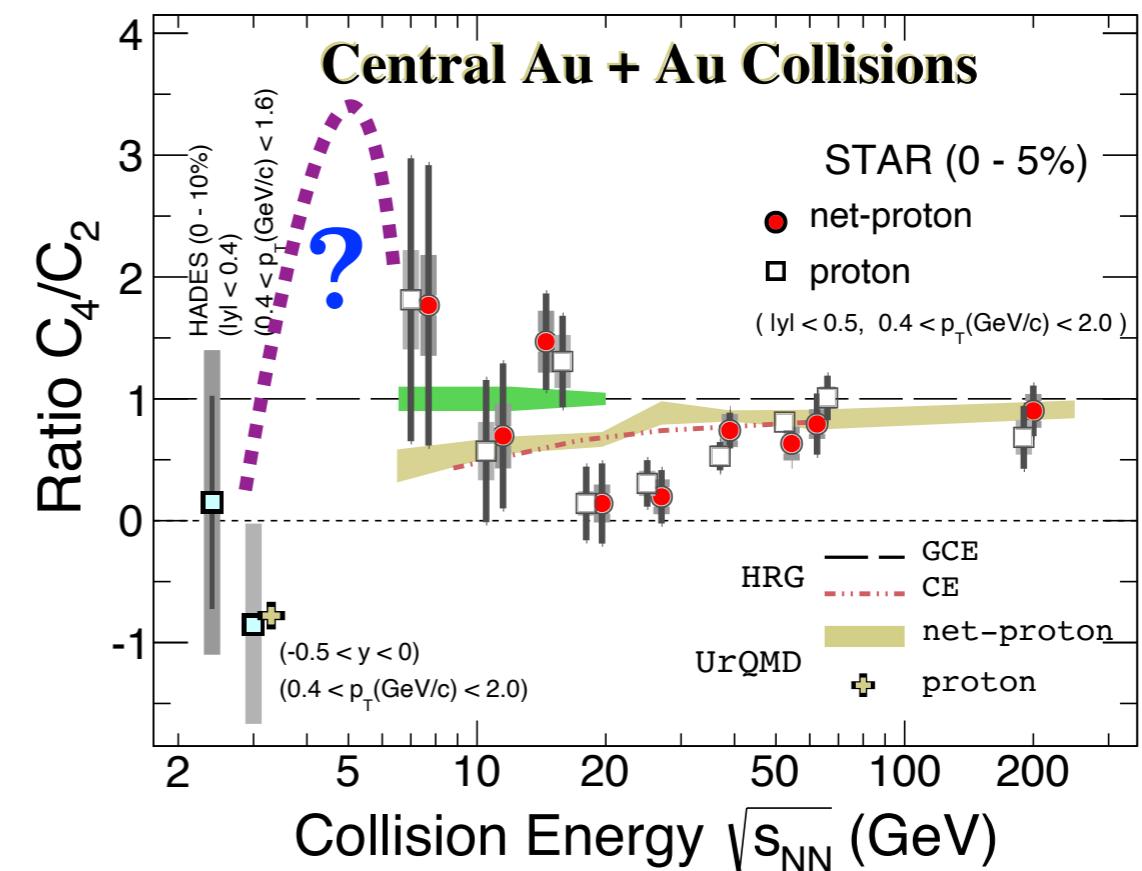
- The non-monotonicity of the kurtosis is observed with 3.1σ significance.
- Is there a “peak” structure in the regime of low colliding energy?

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- Is there a “peak” structure in the regime of low colliding energy?

Outline

- * **Introduction**
- * **Recent advance of QCD phase structure from functional QCD**
- * **Baryon number fluctuations at high density**
- * **Ripples of the QCD critical point**
- * **Summary**

First-principles QCD within fRG

QCD flow equation:

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2} \left(\text{Diagram A} - \text{Diagram B} - \text{Diagram C} + \frac{1}{2} \text{Diagram D} \right)$$

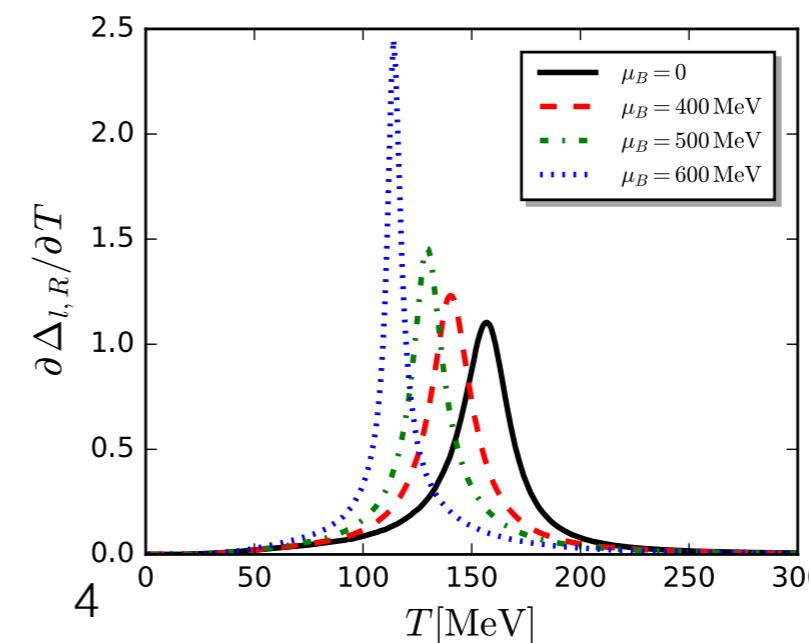
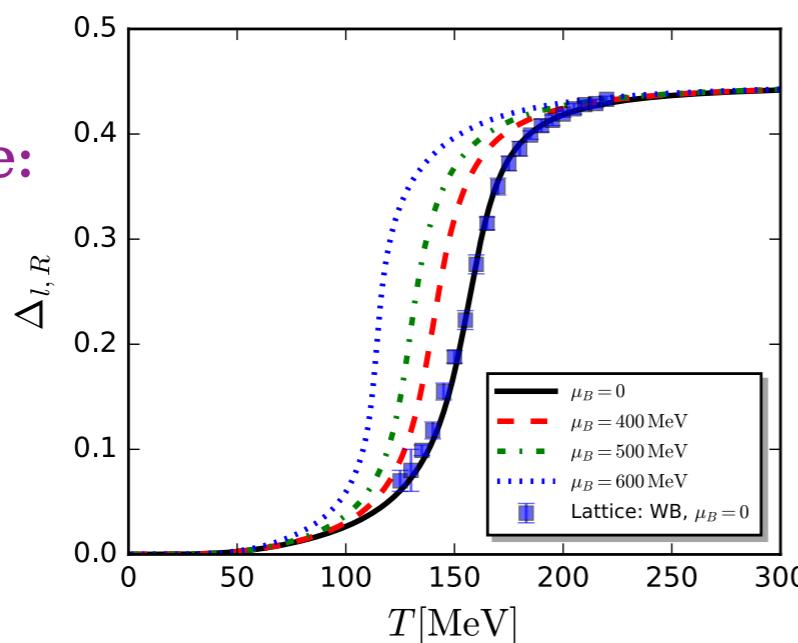
Glue sector:

$$\begin{aligned} \partial_t \text{Diagram E} &= \tilde{\partial}_t \left(\frac{1}{2} \text{Diagram F} + \frac{1}{2} \text{Diagram G} - \text{Diagram H} - \text{Diagram I} \right) \\ \partial_t \text{Diagram J} &= \tilde{\partial}_t \left(\text{Diagram K} \right) \\ \partial_t \text{Diagram L} &= \tilde{\partial}_t \left(\text{Diagram M} - \text{Diagram N} - \text{Diagram O} + \frac{1}{2} \text{Diagram P} \right) \\ \partial_t \text{Diagram Q} &= \tilde{\partial}_t \left(\text{Diagram R} + \text{Diagram S} \right) \end{aligned}$$

Matter sector:

$$\begin{aligned} \partial_t \text{Diagram T} &= \tilde{\partial}_t \left(\text{Diagram U} + \text{Diagram V} \right) \\ \partial_t \text{Diagram W} &= \tilde{\partial}_t \left(\text{Diagram X} + \text{Diagram Y} - \frac{1}{2} \text{Diagram Z} \right) \\ \partial_t \text{Diagram AA} &= \tilde{\partial}_t \left(\text{Diagram BB} + \text{Diagram CC} + \text{Diagram DD} \right) \\ \partial_t \text{Diagram EE} &= \tilde{\partial}_t \left(\text{Diagram FF} + \text{Diagram GG} + \text{Diagram HH} \right) \end{aligned}$$

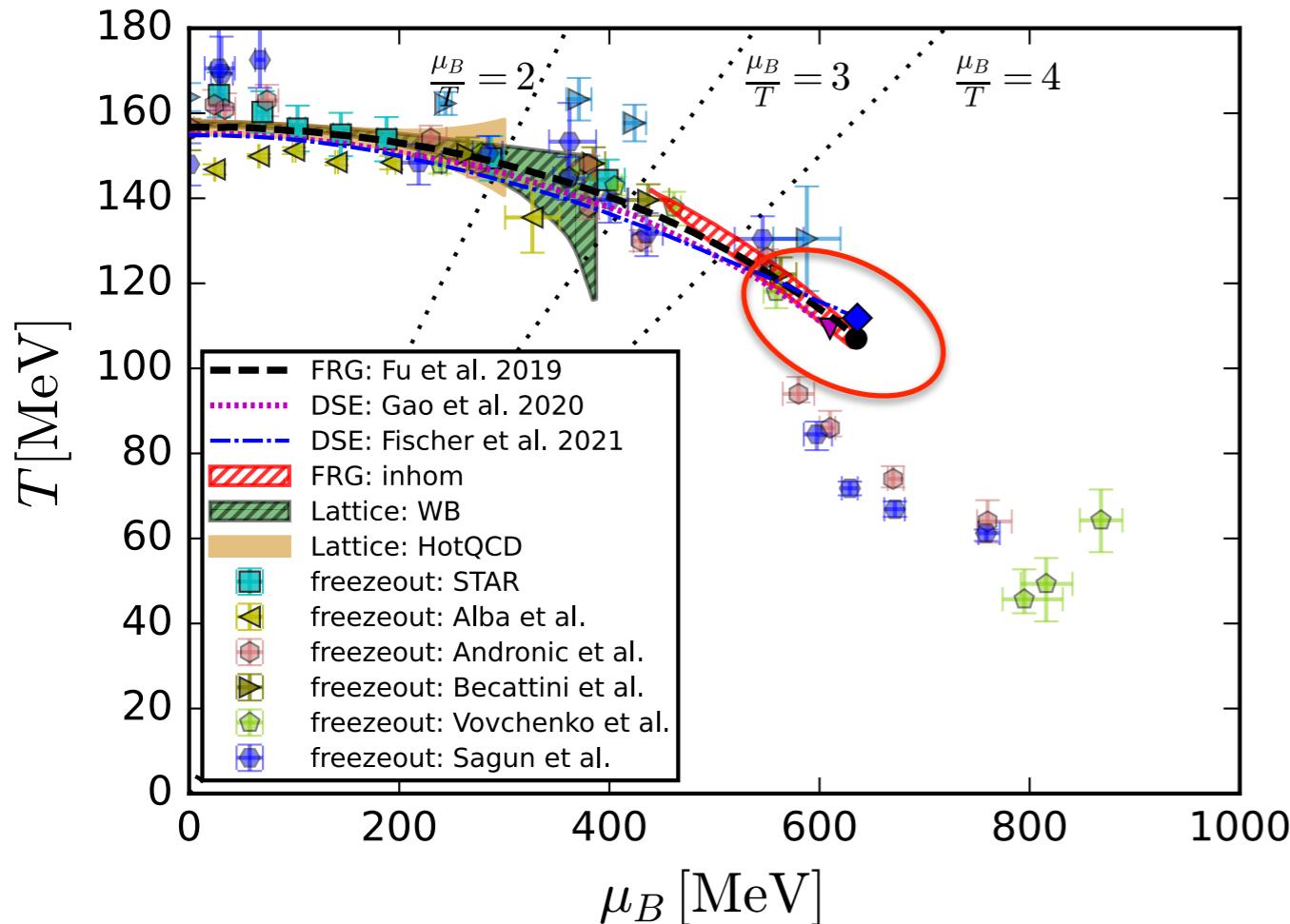
quark condensate:



fRG:WF, Pawłowski, Rennecke,
PRD 101 (2020) 054032

Lattice: Borsanyi *et al.* (WB),
JHEP 09 (2010) 073

CEP from first-principles functional QCD



Estimates of the location of CEP from first-principles functional QCD:

fRG:

$$\bullet \quad (T, \mu_B)_{\text{CEP}} = (107, 635) \text{ MeV}$$

fRG: WF, Pawłowski, Rennecke, *PRD* 101 (2020), 054032

DSE:

$$\nabla \quad (T, \mu_B)_{\text{CEP}} = (109, 610) \text{ MeV}$$

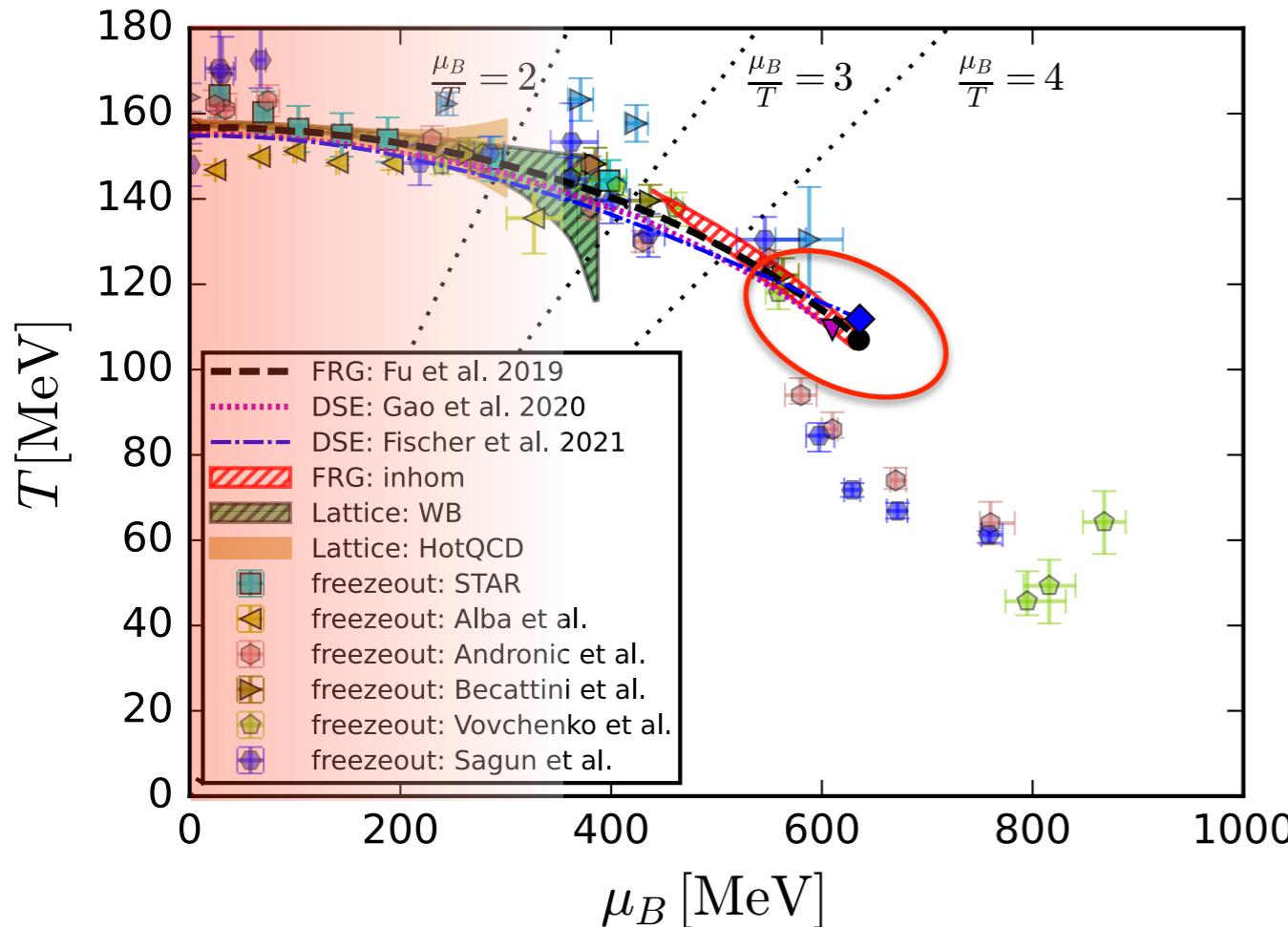
DSE (fRG): Gao, Pawłowski, *PLB* 820 (2021) 136584

$$\diamond \quad (T, \mu_B)_{\text{CEP}} = (112, 636) \text{ MeV}$$

DSE: Gunkel, Fischer, *PRD* 104 (2021) 5, 054022

- No CEP observed in $\mu_B/T \lesssim 2 \sim 3$ from lattice QCD. Karsch, *PoS CORFU2018* (2019) 163
- Recent studies of QCD phase structure from both fRG and DSE have shown convergent estimate for the location of CEP: $600 \text{ MeV} \lesssim \mu_{B,\text{CEP}} \lesssim 650 \text{ MeV}$.

CEP from first-principles functional QCD



Passing through strict benchmark tests in comparison to lattice QCD at vanishing and small μ_B .

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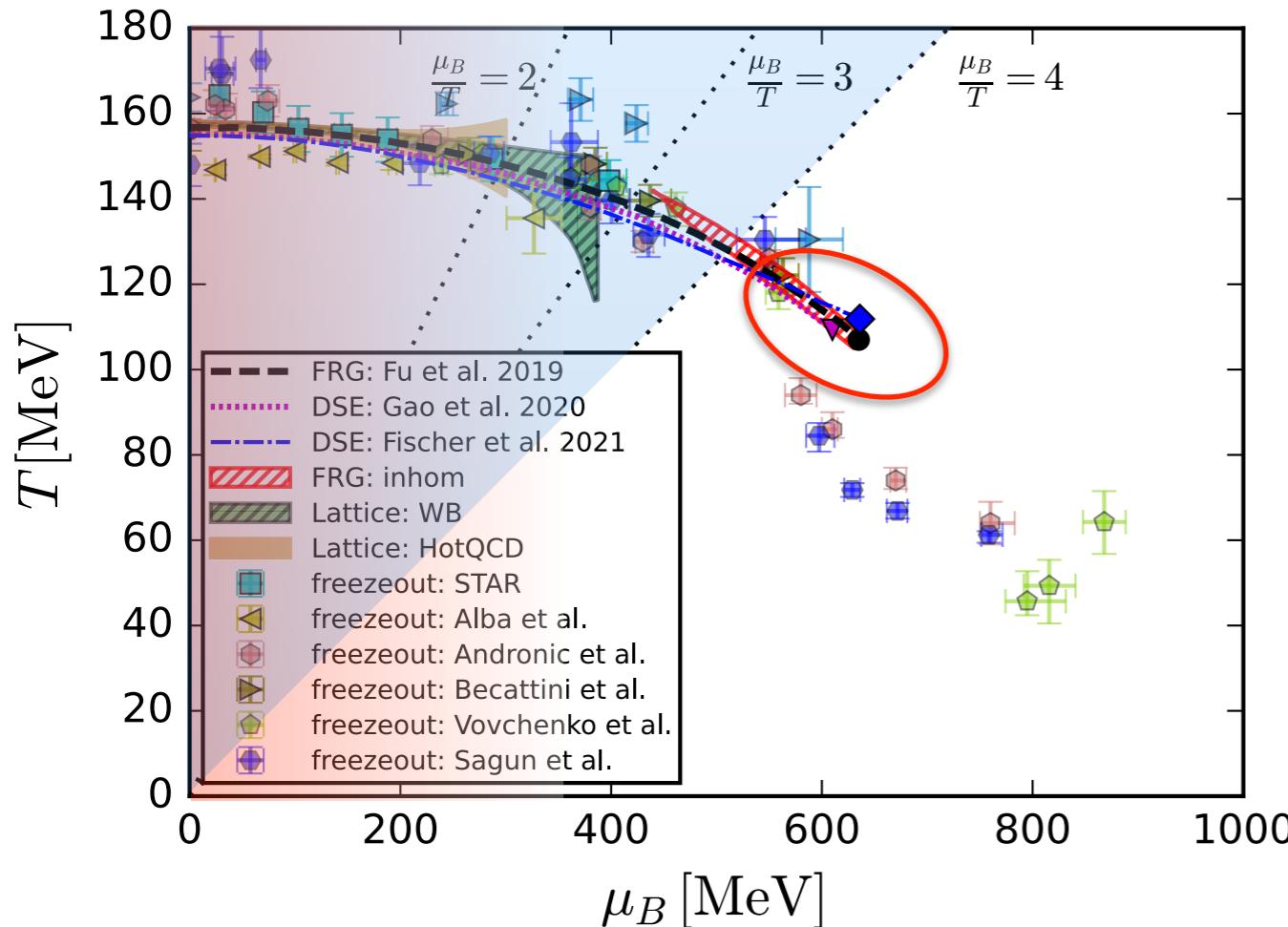
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CEP from first-principles functional QCD



Passing through strict benchmark tests in comparison to lattice QCD at vanishing and small μ_B .

Regime of quantitative reliability of functional QCD with $\mu_B/T \lesssim 4$.

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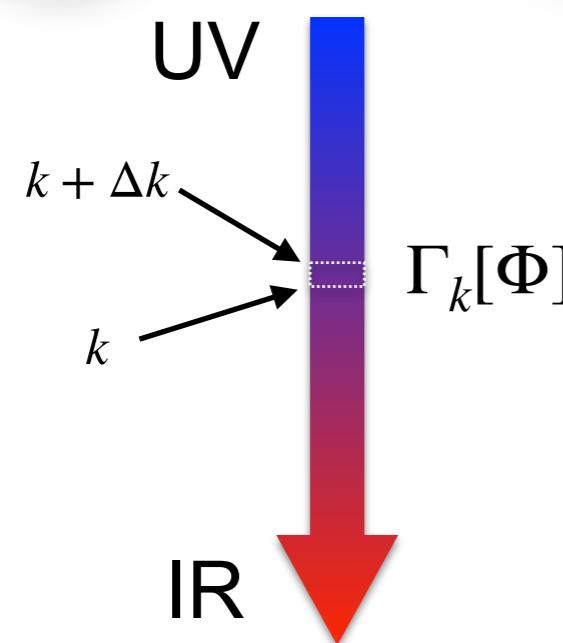
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QCD-assisted LEFT

QCD flow equation:

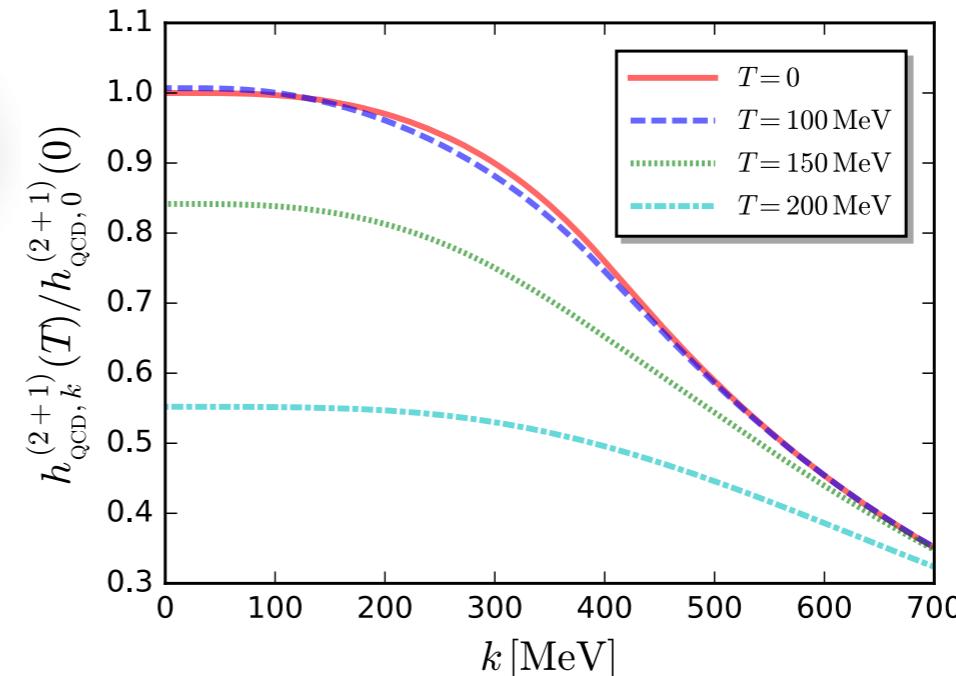
$$\partial_t \Gamma_k[\Phi] = \frac{1}{2} \left(\text{orange loop} - \text{dotted loop} - \text{black loop} + \frac{1}{2} \text{blue loop} \right)$$



LEFT flow equation:

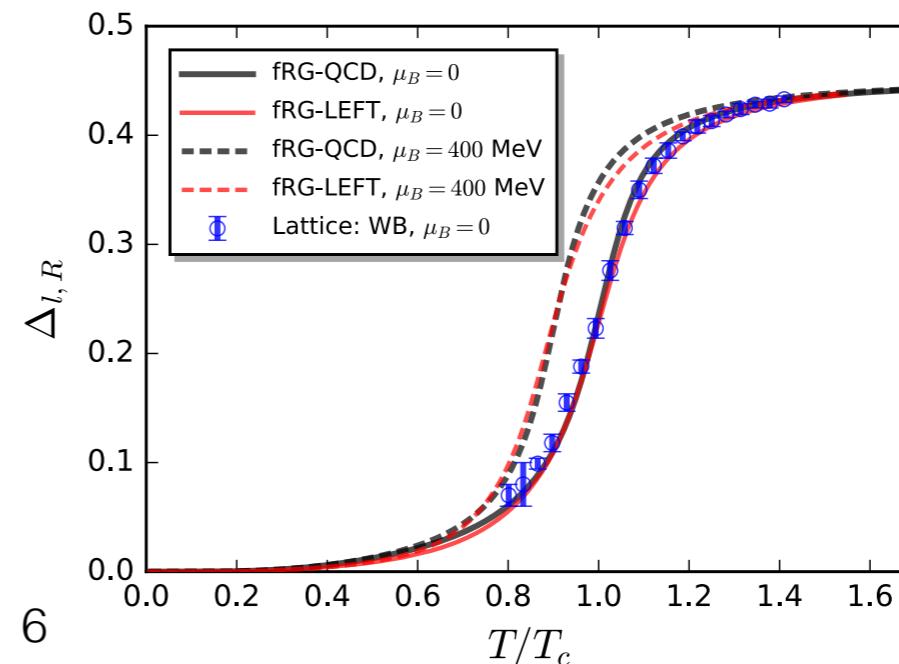
$$\partial_t \Gamma_k[\Phi] = - \text{quark loop} + \frac{1}{2} \text{meson loop}$$

- Yukawa couplings obtained in QCD inputted in QCD-assisted LEFT



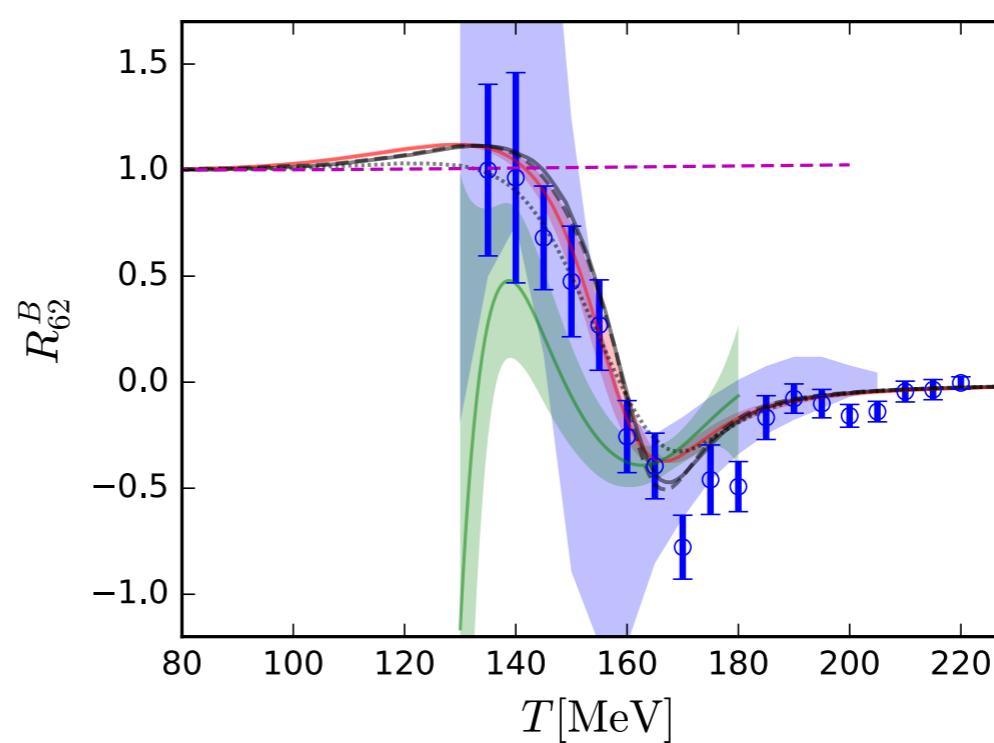
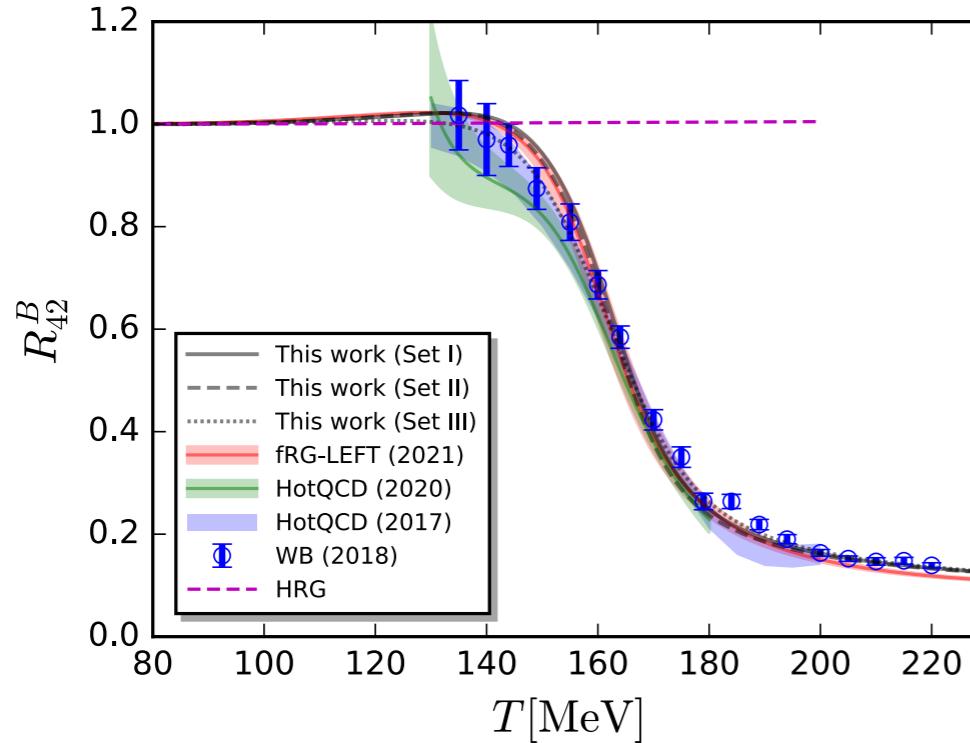
WF,
Pawlowski,
Rennecke, *PRD*
101 (2020)
054032

- Chiral condensates in QCD and QCD-assisted LEFT in agreement



WF, Luo,
Pawlowski,
Rennecke, Yin,
arXiv:
2308.15508

Baryon number fluctuations



fRG: WF, Luo, Pawłowski, Rennecke, Yin, arXiv: 2308.15508;
 WF, Luo, Pawłowski, Rennecke, Wen, Yin, PRD 104 (2021) 094047

HotQCD: A. Bazavov *et al.*, arXiv: PRD 95 (2017), 054504; PRD 101 (2020), 074502

WB: S. Borsanyi *et al.*, arXiv: JHEP 10 (2018) 205

baryon number fluctuations

$$\chi_n^B = \frac{\partial^n}{\partial(\mu_B/T)^n} \frac{p}{T^4}$$

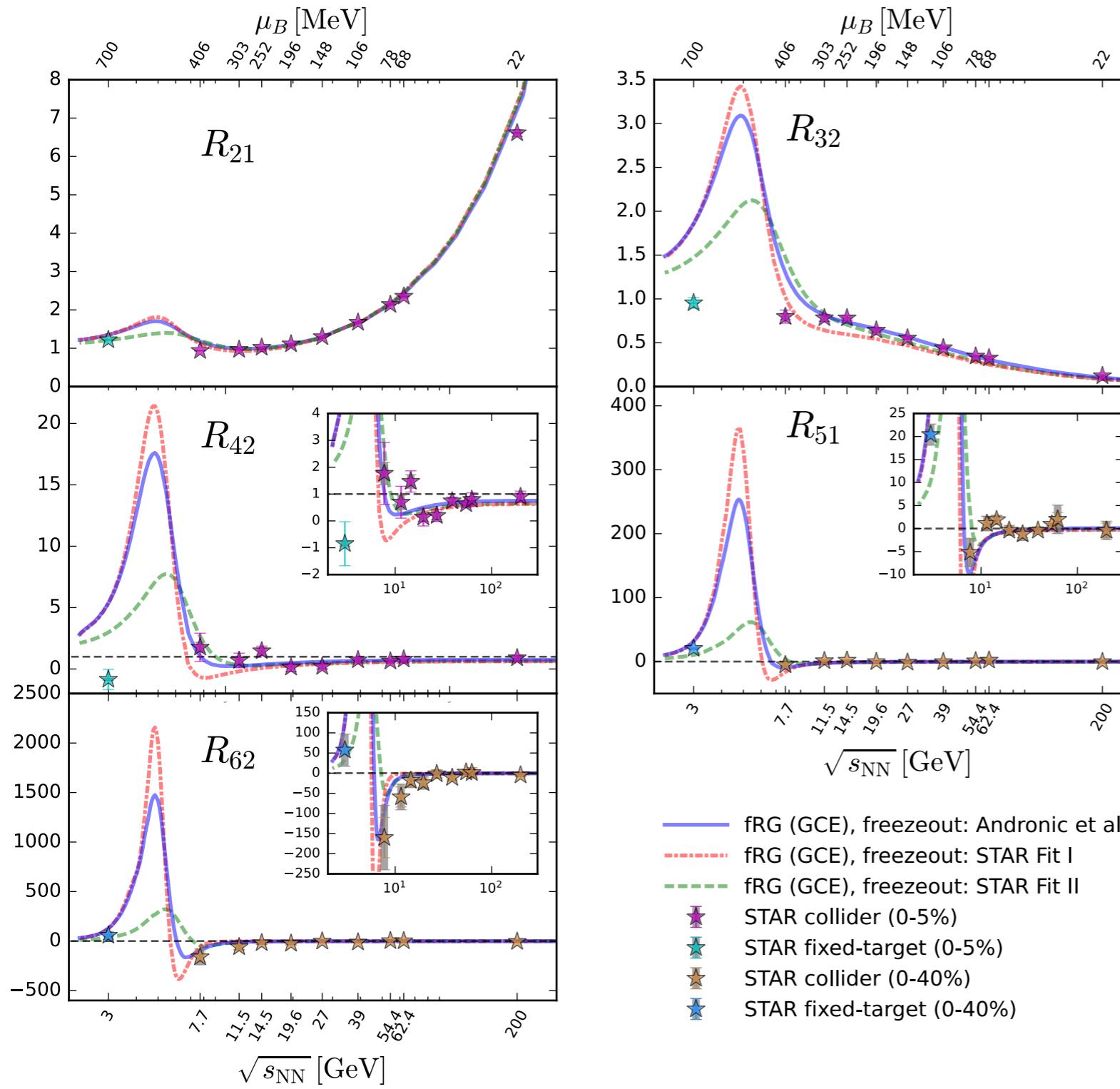
$$R_{nm}^B = \frac{\chi_n^B}{\chi_m^B}$$

relation to the cumulants

$$\frac{M}{VT^3} = \chi_1^B, \frac{\sigma^2}{VT^3} = \chi_2^B, S = \frac{\chi_3^B}{\chi_2^B \sigma}, \kappa = \frac{\chi_4^B}{\chi_2^B \sigma^2},$$

- In comparison to lattice results and our former results, the improved results of baryon number fluctuations at vanishing chemical potential in the QCD-assisted LEFT are convergent and consistent.

Grand canonical fluctuations at the freeze-out



STAR: Adam *et al.* (STAR), *PRL* 126 (2021) 092301;
Abdallah *et al.* (STAR), *PRL* 128 (2022) 202303;
Aboona *et al.* (STAR), *PRL* 130 (2023) 082301

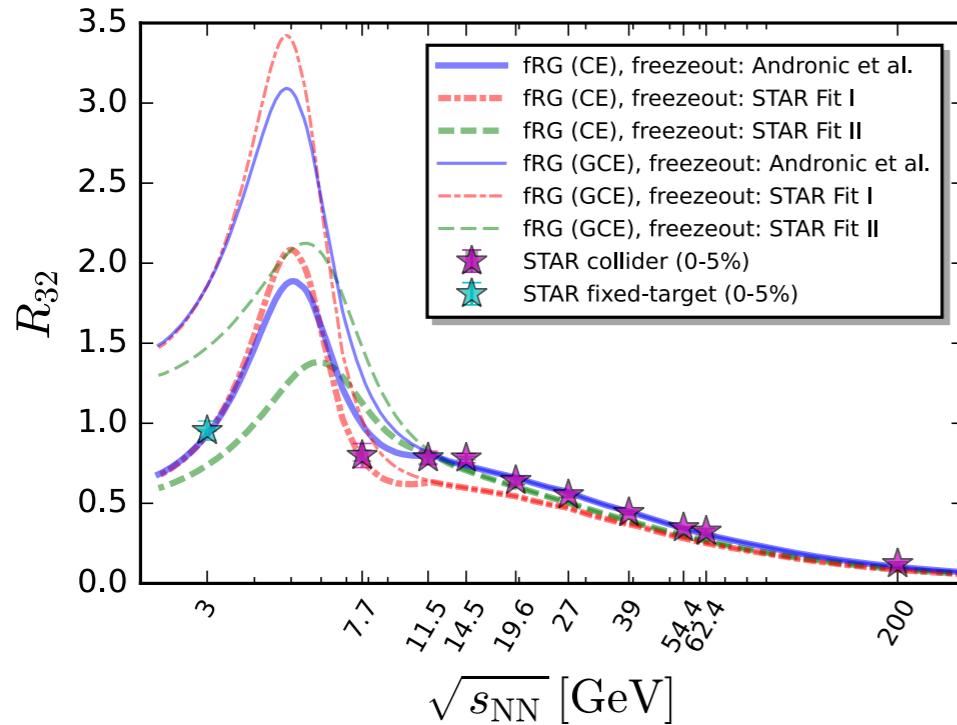
fRG: WF, Luo, Pawłowski, Rennecke, Yin, arXiv:
[2308.15508](https://arxiv.org/abs/2308.15508)

- Results in fRG are obtained in the QCD-assisted LEFT with a CEP at $(T_{\text{CEP}}, \mu_{B_{\text{CEP}}}) = (98, 643)$ MeV.
- Peak structure is found in $3 \text{ GeV} \lesssim \sqrt{s_{\text{NN}}} \lesssim 7.7 \text{ GeV}$.
- Agreement between the theory and experiment is worsening with $\sqrt{s_{\text{NN}}} \lesssim 11.5 \text{ GeV}$.
- Effects of global baryon number conservation in the regime of low collision energy should be taken into account.

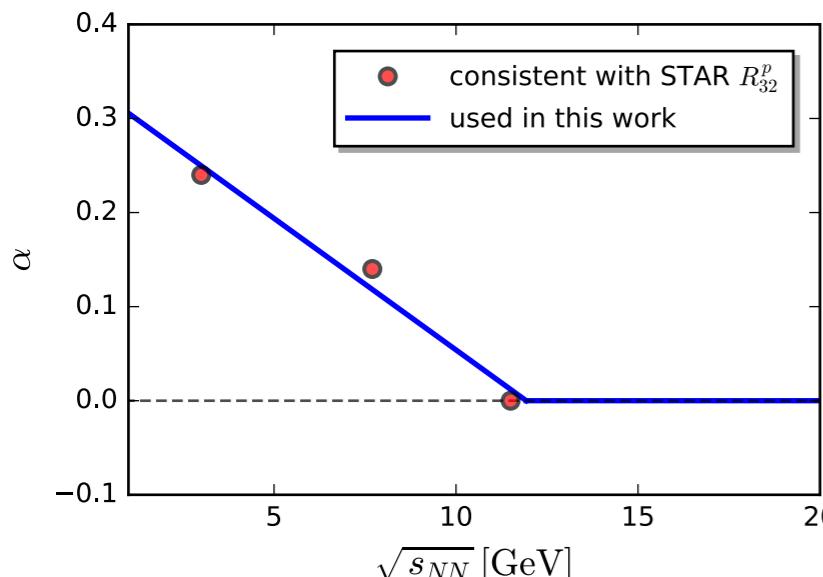
Caveat:

Fluctuations of baryon number in theory are compared with those of proton number in experiments.

Canonical corrections with SAM



- Experimental data R_{32} is used to constrain the parameter α in the range $\sqrt{s_{\text{NN}}} \lesssim 11.5$ GeV.
- We choose the simplest linear dependence



$$\alpha(\bar{s}) = a \left(1 - \sqrt{\bar{s}}\right) \theta(1 - \bar{s})$$

$$a = 0.33, \quad \sqrt{\bar{s}} = \frac{\sqrt{s_{\text{NN}}}}{11.9 \text{ GeV}}$$

SAM:

- We adopt the subensemble acceptance method (SAM) to take into account the effects of global baryon number conservation:

$$\alpha = \frac{V_1}{V}$$

V_1 : the subensemble volume measured in the acceptance window, V : the volume of the whole system.

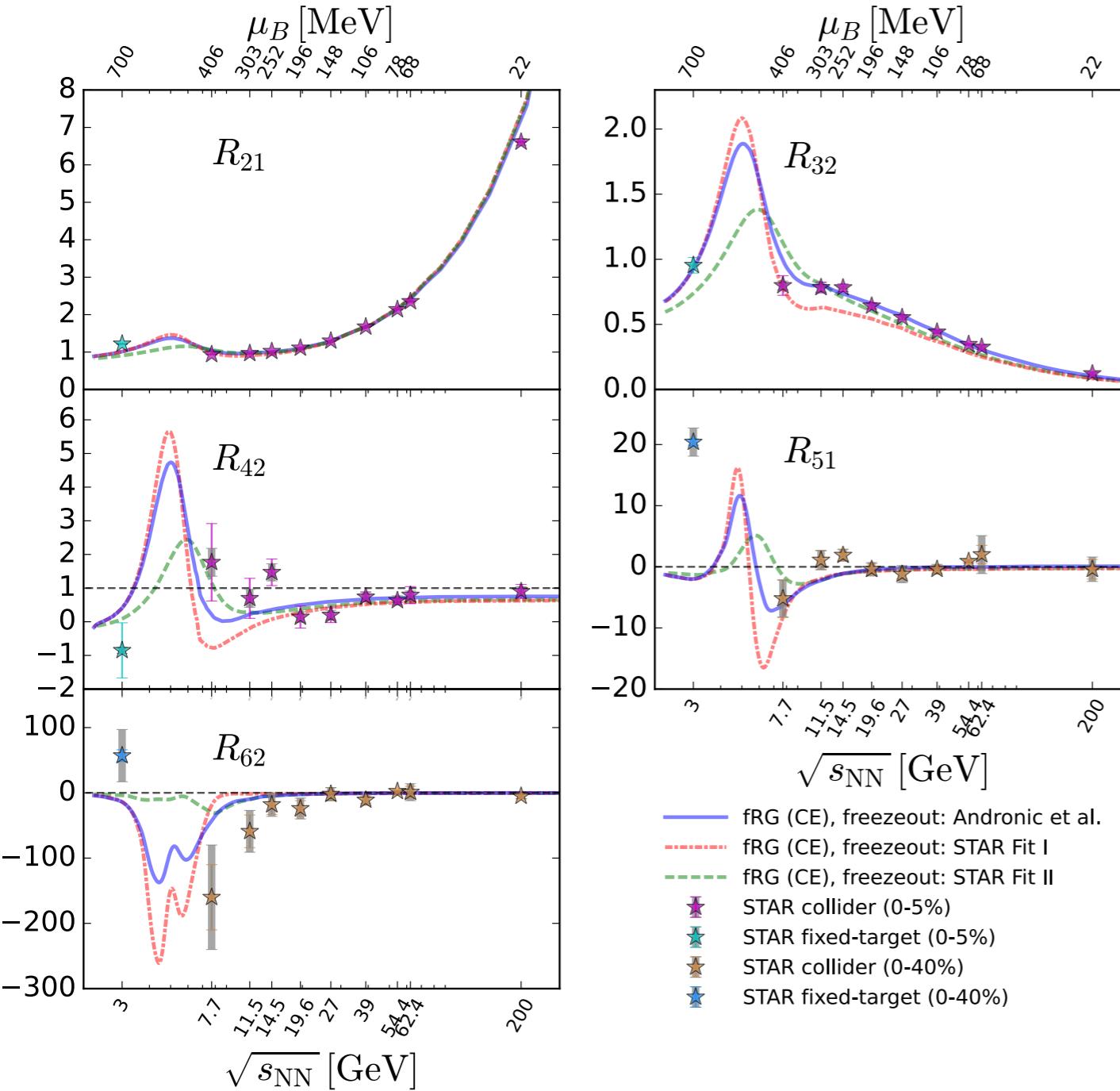
- fluctuations with canonical corrections are related to grand canonical fluctuations as follows:

$$\bar{R}_{21}^B = \beta R_{21}^B, \quad \bar{R}_{32}^B = (1 - 2\alpha) R_{32}^B,$$

$$\bar{R}_{42}^B = (1 - 3\alpha\beta) R_{42}^B - 3\alpha\beta(R_{32}^B)^2$$

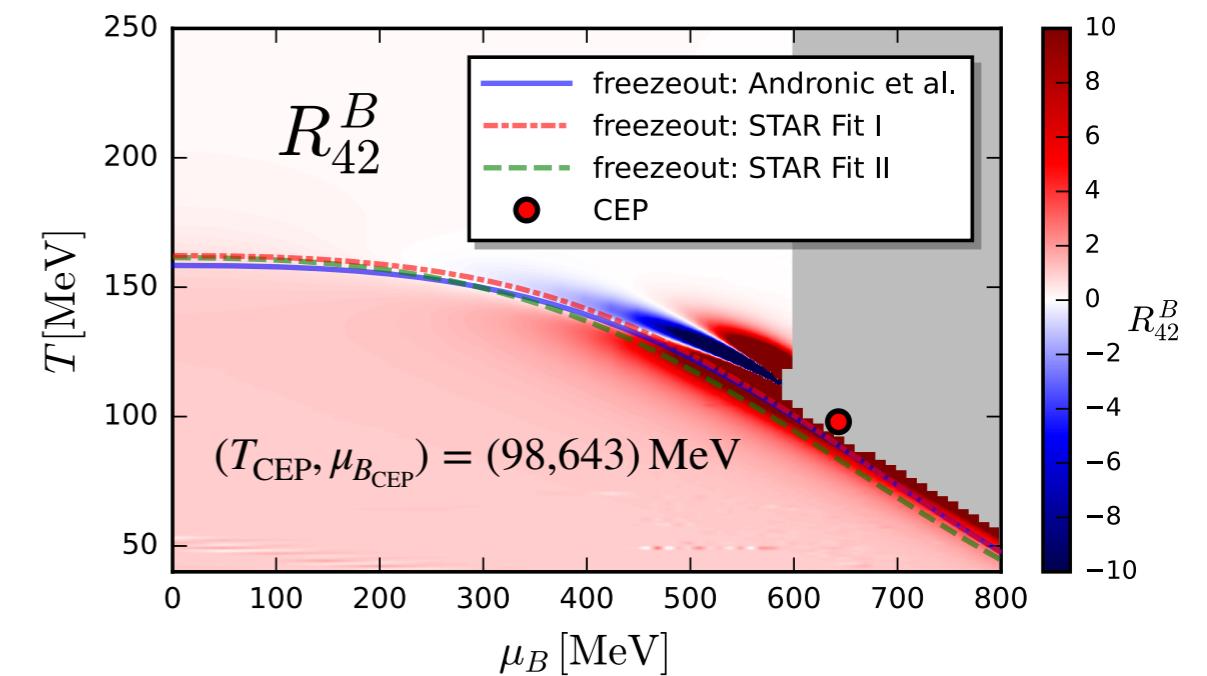
SAM: Vovchenko, Savchuk, Poberezhnyuk, Gorenstein, Koch , PLB 811 (2020) 135868

Canonical fluctuations at the freeze-out



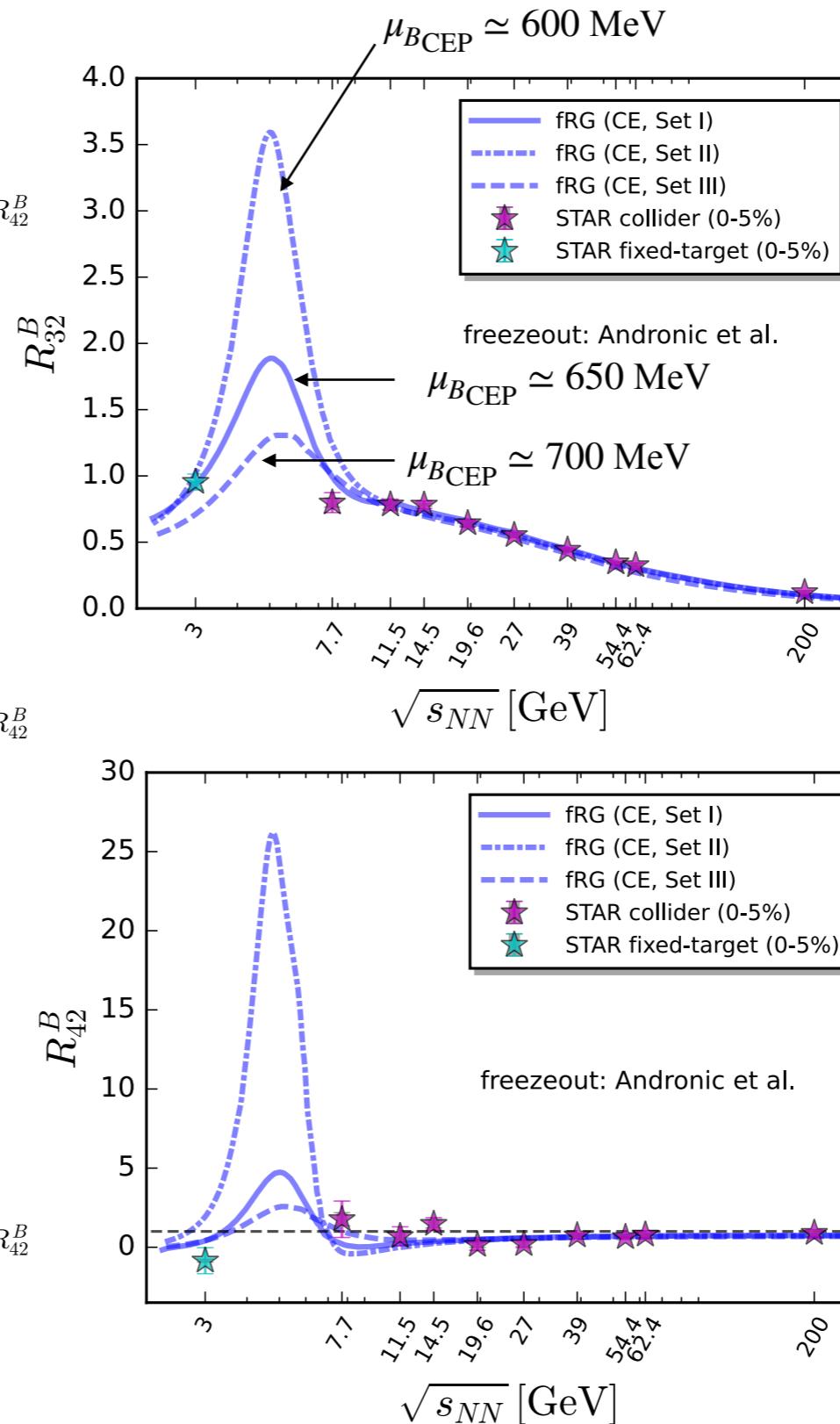
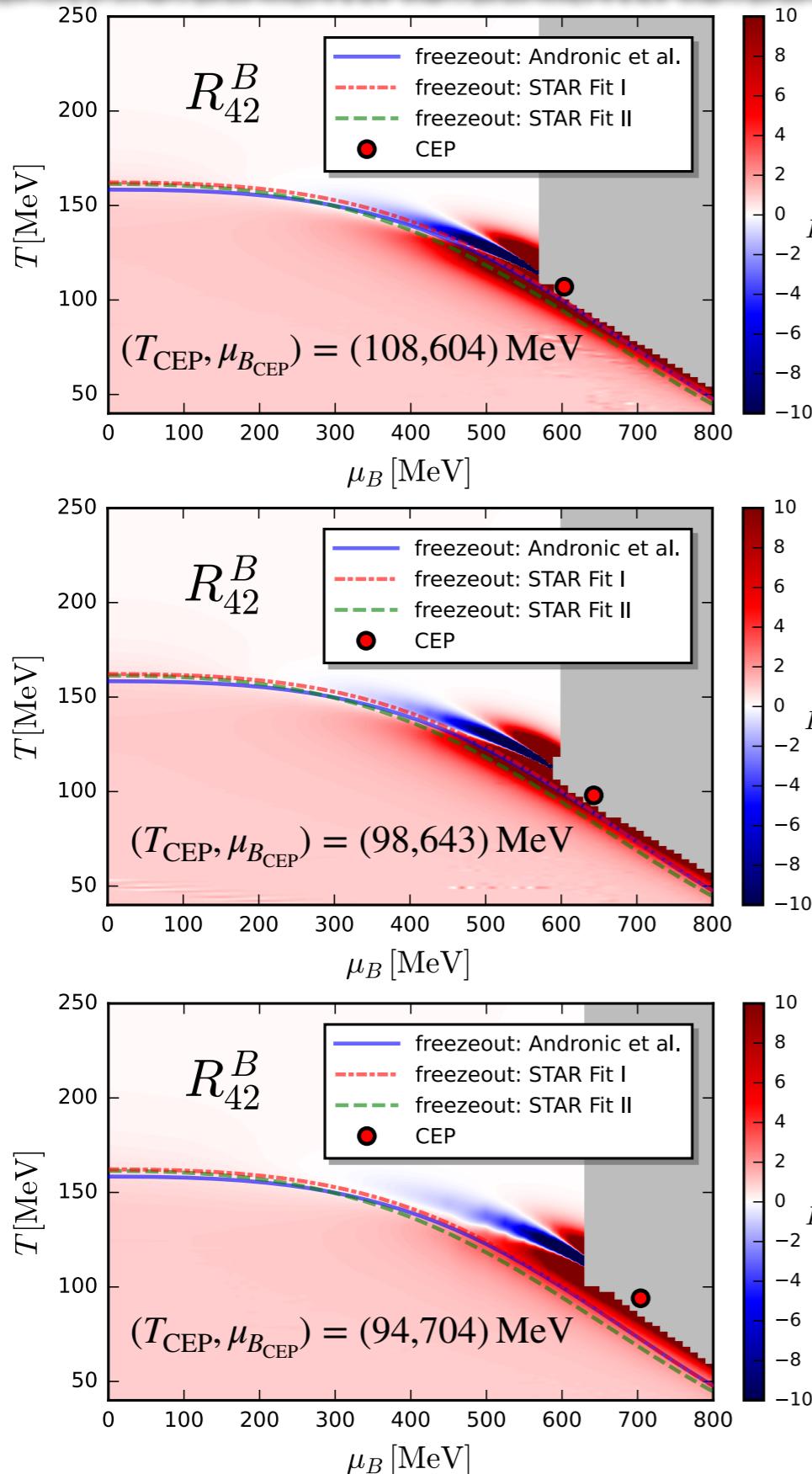
STAR: Adam *et al.* (STAR), *PRL* 126 (2021) 092301;
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Aboona *et al.* (STAR), *PRL* 130 (2023) 082301

fRG: WF, Luo, Pawłowski, Rennecke, Yin, arXiv:
2308.15508



- Peak structure is found in $\sqrt{s_{\text{NN}}} \lesssim 7.7$ GeV.
- Position of peak in R_{42} is $\mu_{B_{\text{peak}}} = 536, 541$ and 486 MeV for the three freeze-out curves, significantly smaller than $\mu_{B_{\text{CEP}}} = 643$ MeV.

Dependence on the location of the CEP



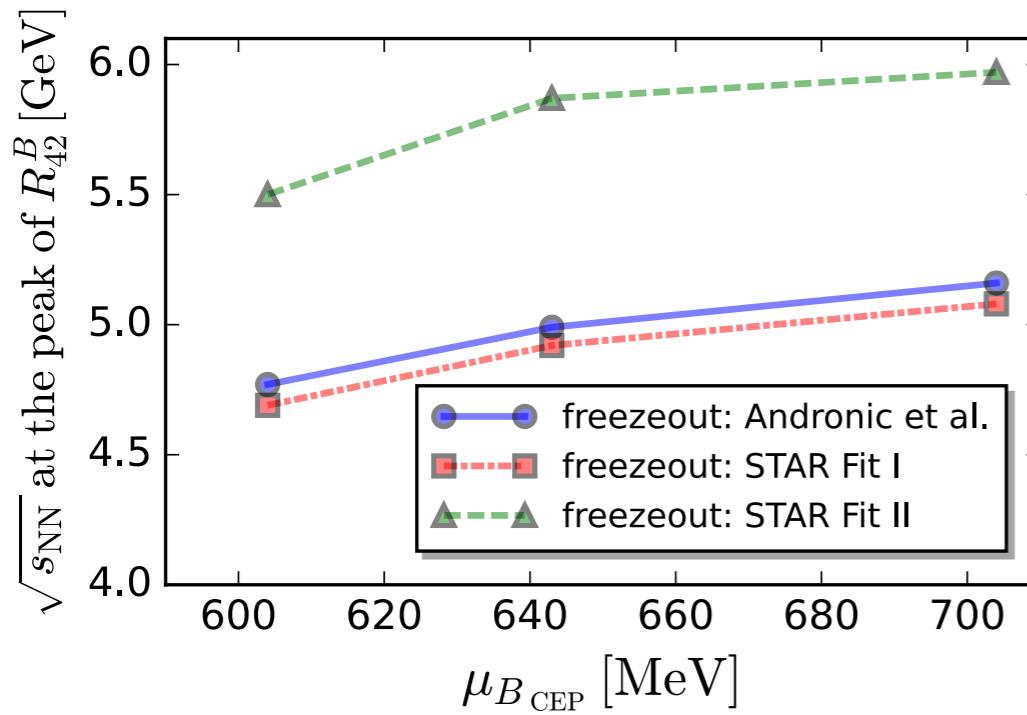
STAR: Adam *et al.* (STAR),
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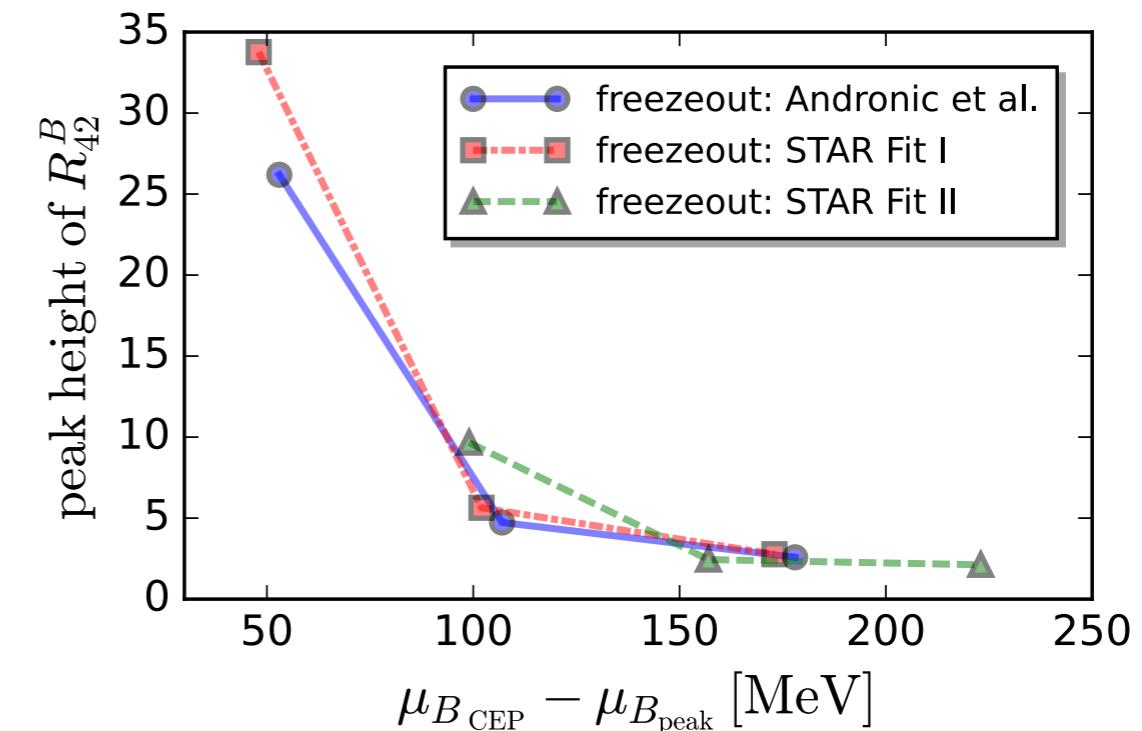
- Position of the peak is insensitive to the location of CEP.
- Height of peak decreases as CEP moves towards larger μ_B .

Ripples of the QCD critical point

Position of peak:



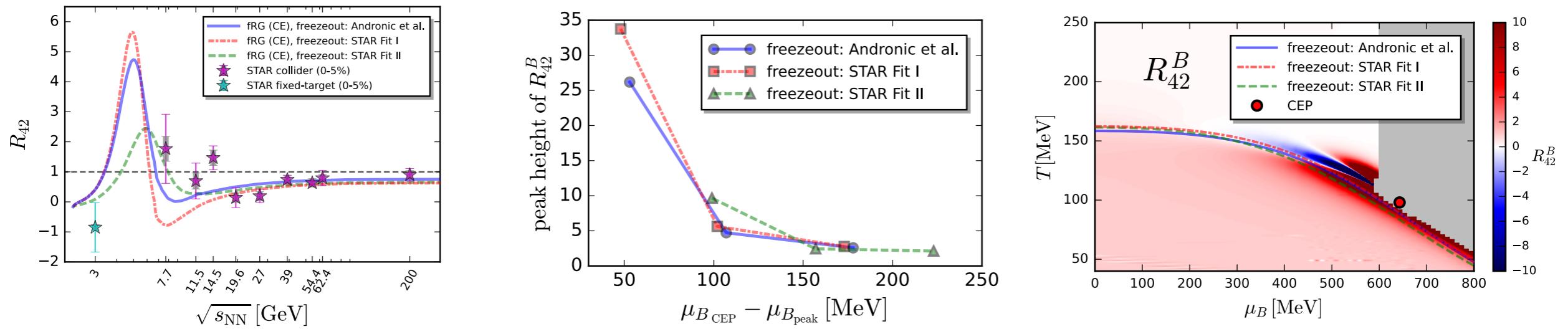
Height of peak:



fRG: WF, Luo, Pawłowski, Rennecke, Yin, arXiv: 2308.15508

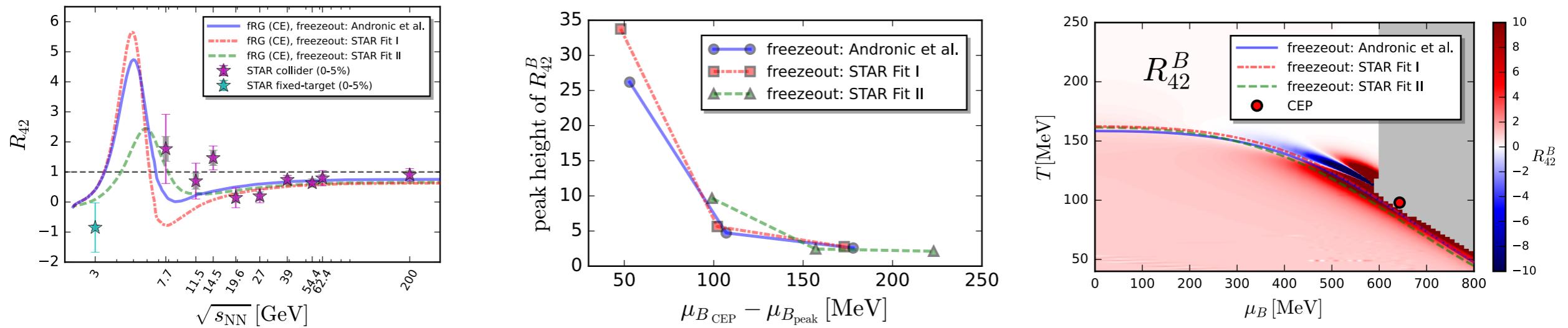
- Note that the ripples of CEP are far away from the critical region characterized by the universal scaling properties, e.g., the critical exponents.
- But, the information of CEP, such as its location and properties, etc., is still encoded in the ripples.

Summary



- ★ A prominent peak structure is found in baryon number fluctuations in the collision energy range of $3 \text{ GeV} \lesssim \sqrt{s_{NN}} \lesssim 7.7 \text{ GeV}$.
- ★ The height of peak depends on the location of the CEP, while its position is more sensitive to the location of the freeze-out curve.
- ★ Information of the peak, i.e., the ripples of CEP can be used to reconstruct the location and properties of CEP.

Summary

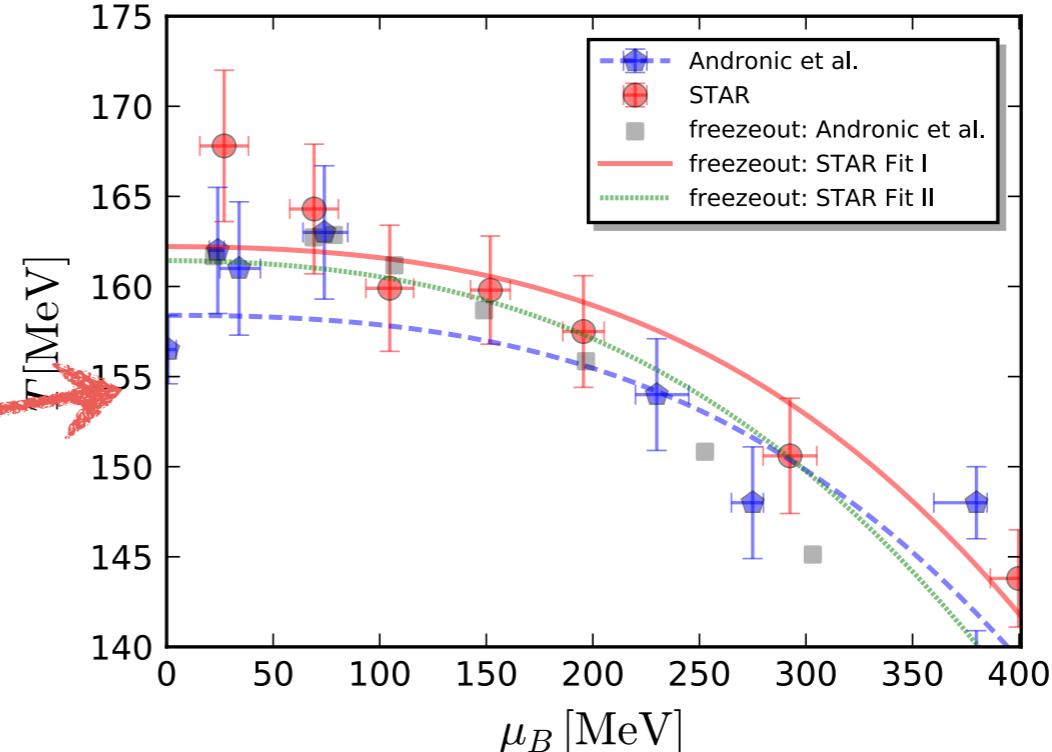
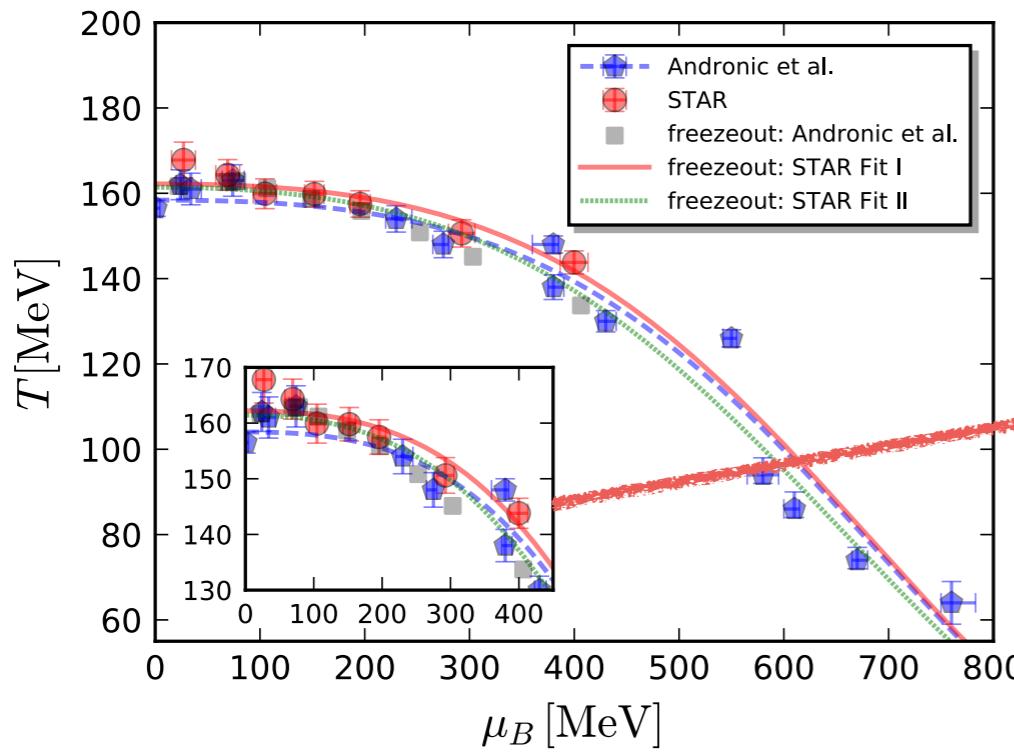


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Thank you very much for your attentions!

Backup

Determination of the freeze-out curve



three freeze-out curves

1. freeze-out: Andronic *et al.*

Andronic, Braun-Munzinger, Redlich, *Nature* 561 (2018) 7723, 321

2. freeze-out: STAR Fit I

L. Adamczyk *et al.* (STAR), *PRC* 96 (2017), 044904

3. freeze-out: STAR Fit II

neglecting first two at low μ_B and the last one

$$\mu_{B_{CF}} = \frac{a}{1 + 0.288\sqrt{s_{NN}}} ,$$

$$T_{CF} = \frac{T_{CF}^{(0)}}{1 + \exp(2.60 - \ln(\sqrt{s_{NN}})/0.45)}$$

all data points

- freeze-out curve should not rise with μ_B
- convexity of the freeze-out curve

Functional renormalization group

Functional integral with an IR regulator

$$Z_k[J] = \int (\mathcal{D}\hat{\Phi}) \exp \left\{ -S[\hat{\Phi}] - \Delta S_k[\hat{\Phi}] + J^a \hat{\Phi}_a \right\}$$

$$W_k[J] = \ln Z_k[J]$$

regulator:

$$\Delta S_k[\varphi] = \frac{1}{2} \int \frac{d^4 q}{(2\pi)^4} \varphi(-q) R_k(q) \varphi(q)$$

flow of the Schwinger function:

$$\partial_t W_k[J] = -\frac{1}{2} \text{STr} \left[(\partial_t R_k) G_k \right] - \frac{1}{2} \Phi_a \partial_t R_k^{ab} \Phi_b$$

Legendre transformation:

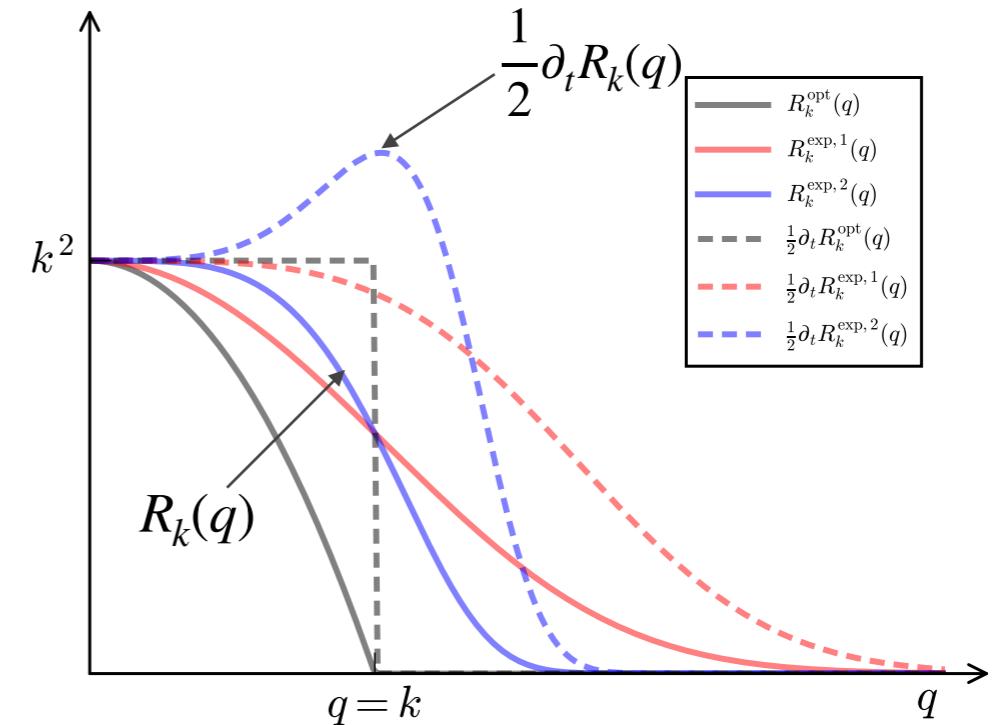
$$\Gamma_k[\Phi] = -W_k[J] + J^a \Phi_a - \Delta S_k[\Phi]$$

flow of the effective action:

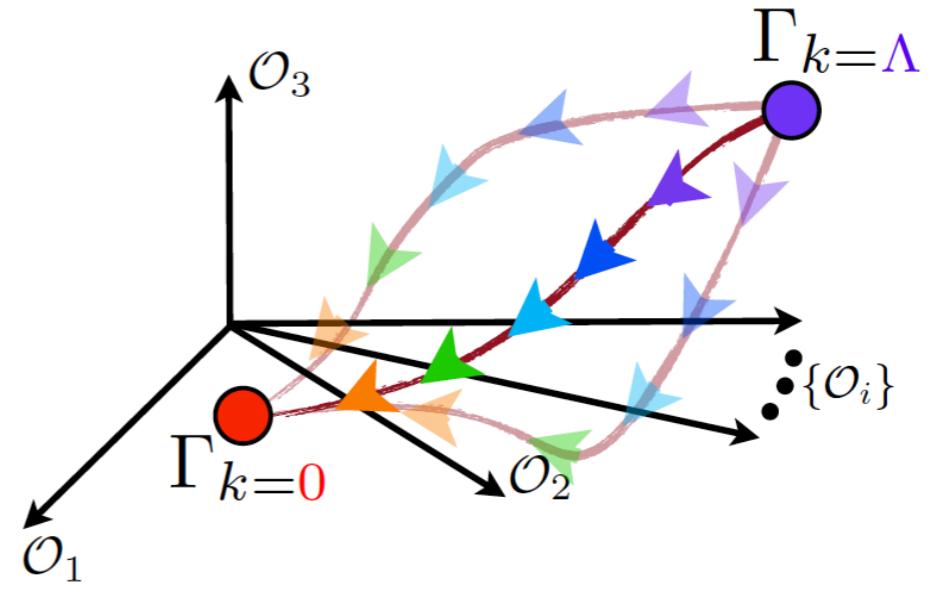
$$\partial_t \Gamma_k[\Phi] = \frac{1}{2} \text{STr} \left[(\partial_t R_k) G_k \right] = \frac{1}{2}$$

Wetterich formula

C. Wetterich, *PLB*, 301 (1993) 90

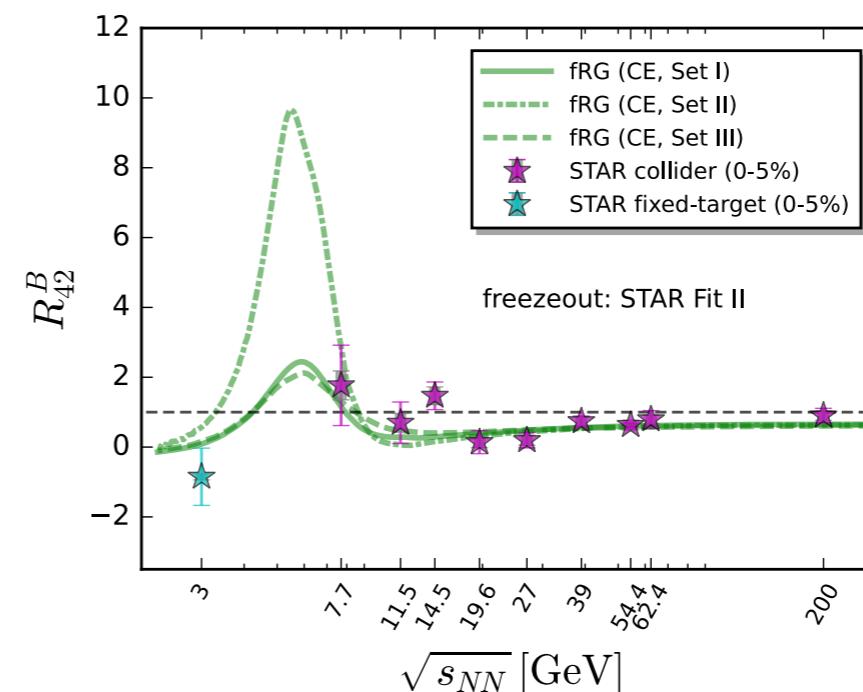
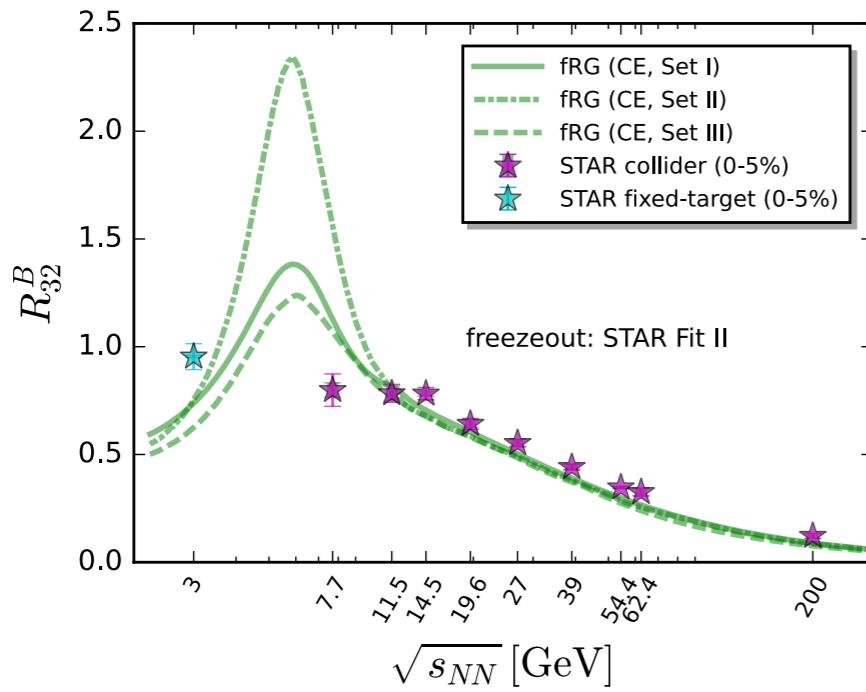
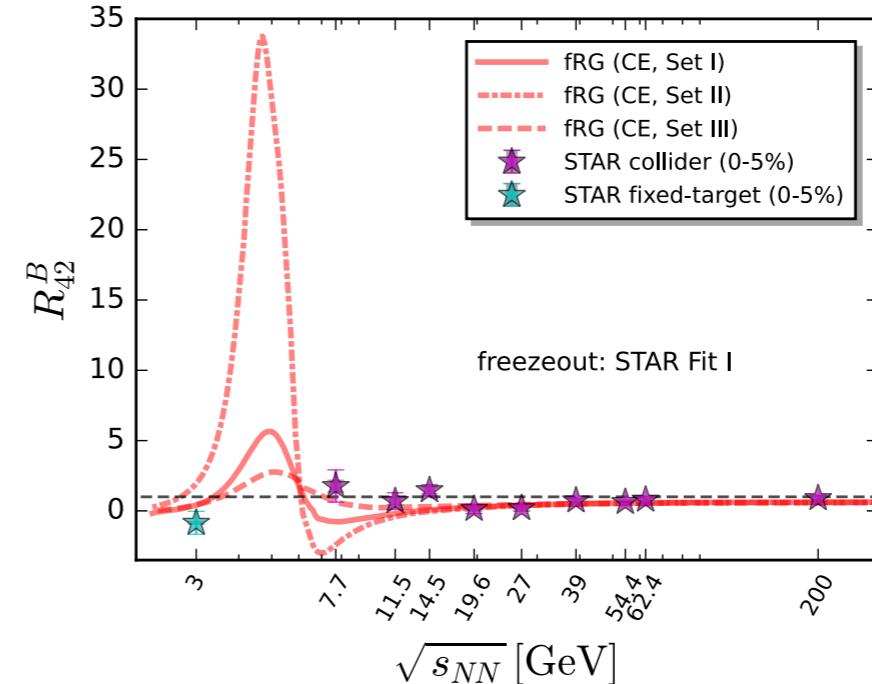
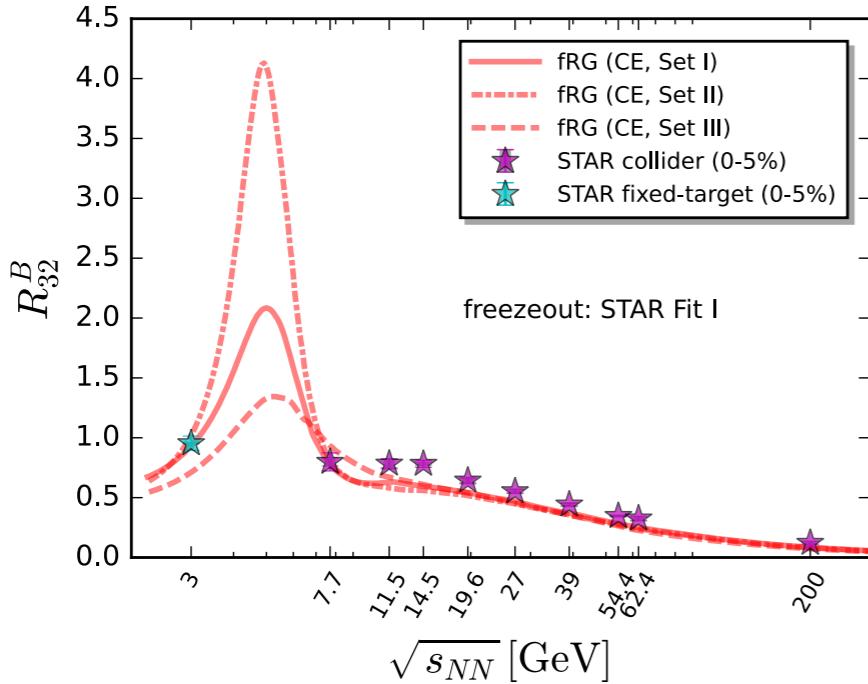


$$G_{k,ab} = \gamma^c{}_a \left(\Gamma_k^{(2)}[\Phi] + \Delta S_k^{(2)}[\Phi] \right)_{cb}^{-1},$$



Review: WF, *CTP* 74 (2022) 097304,
arXiv: 2205.00468 [hep-ph]

Dependence of the location of CEP



STAR: Adam *et al.* (STAR),
PRL 126 (2021) 092301

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