

Location of the QCD critical point predicted by holographic Bayesian analysis

[arXiv:2309.00579](https://arxiv.org/abs/2309.00579)

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In collaboration with J. Grefa, I. Portillo, J. Noronha,
J. Noronha-Hostler, C. Ratti, R. Rougemont and M. Trujillo



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Nucleus-Nucleus Collisions



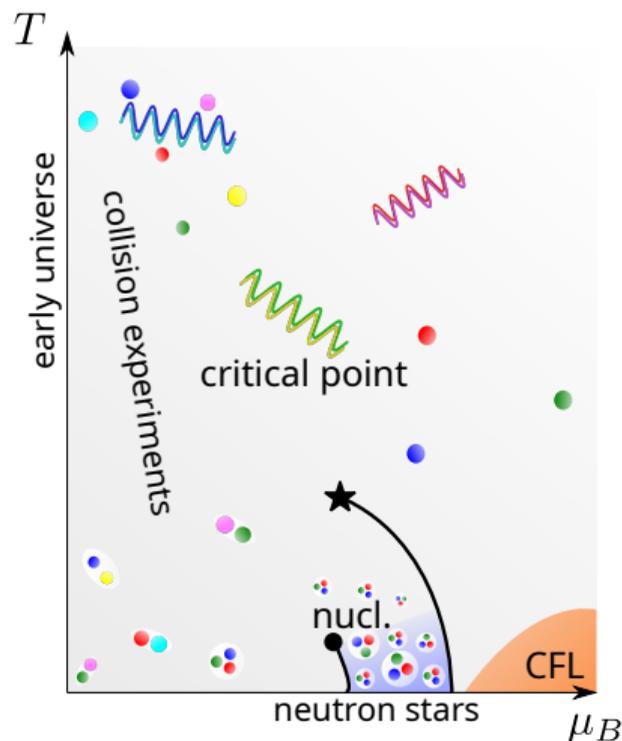
Illinois Center for Advanced Studies of the Universe

Introduction

- Lattice QCD: equation of state for $\mu_B/T \lesssim 3.5$.

Borsányi, Fodor, Guenther et al., PRL **126** (2021)

A. Pasztor — Monday 4pm

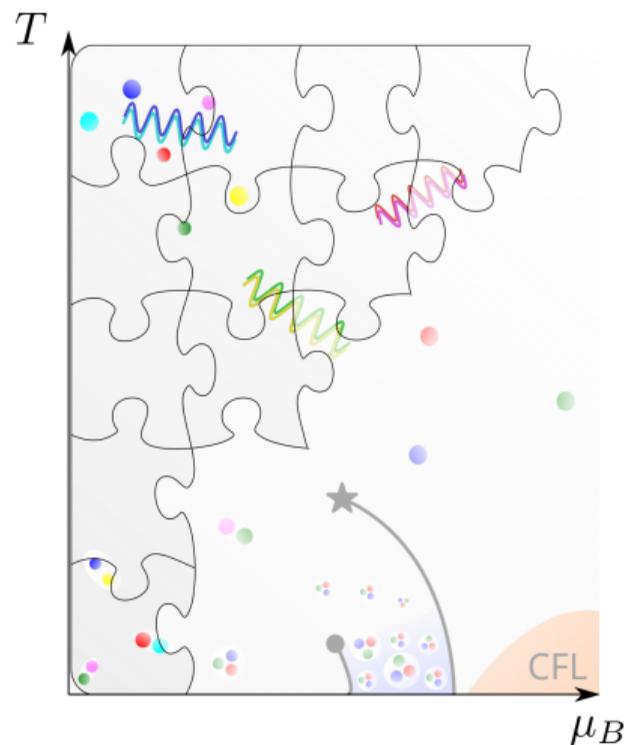


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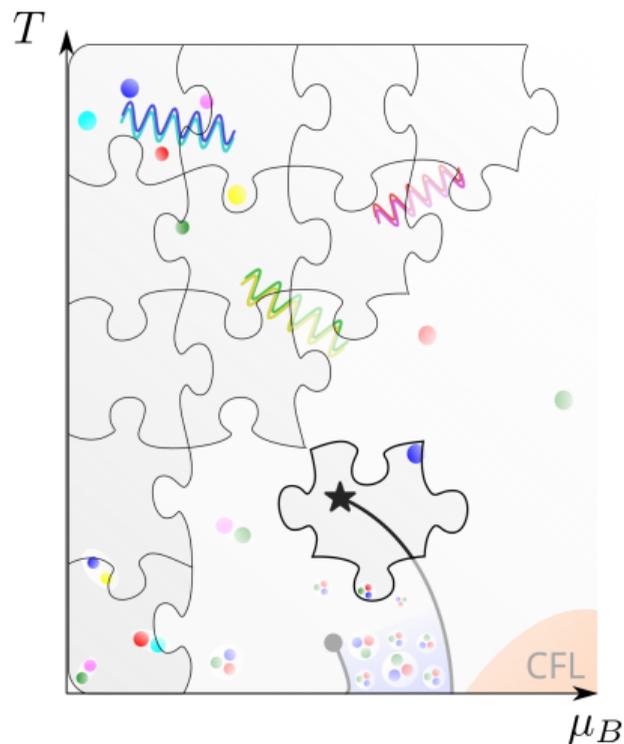
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Borsányi, Fodor, Guenther et al., PRL **126** (2021)

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- What can lattice results tell us about phase diagram/critical point?



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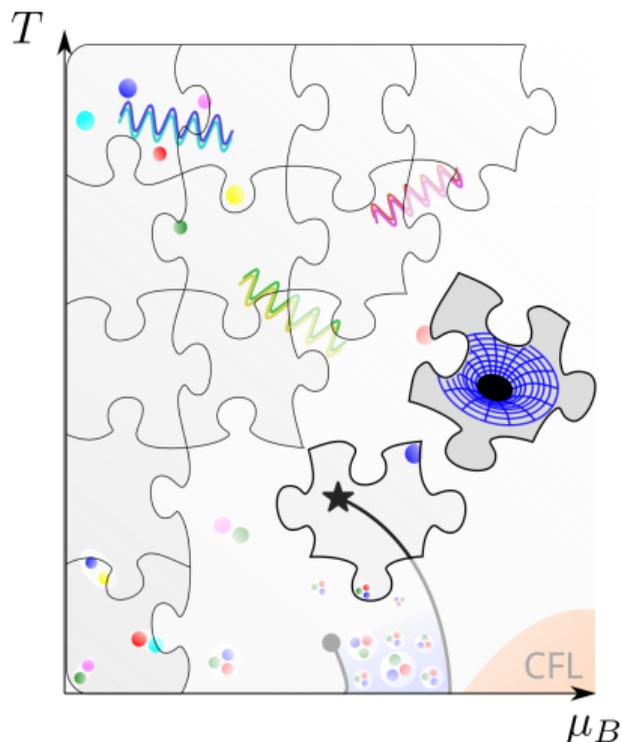
- What can lattice results tell us about phase diagram/critical point?
- Extrapolation: good description of QGP required.
- “Black-hole engineering”: Tweak holographic model to reproduce lattice QCD results.

S. S. Gubser and A. Nellore, PRD **78** (2008)

O. DeWolfe, S. S. Gubser and C. Rosen, PRD **83** (2011)

R. Critelli, J. Noronha, J. Noronha-Hostler, I. Portillo,
C. Ratti, R. Rougemont, PRD **96** (2017)

J. Grefa, J. Noronha, J. Noronha-Hostler, I. Portillo,
C. Ratti, R. Rougemont, PRD **104** (2021)



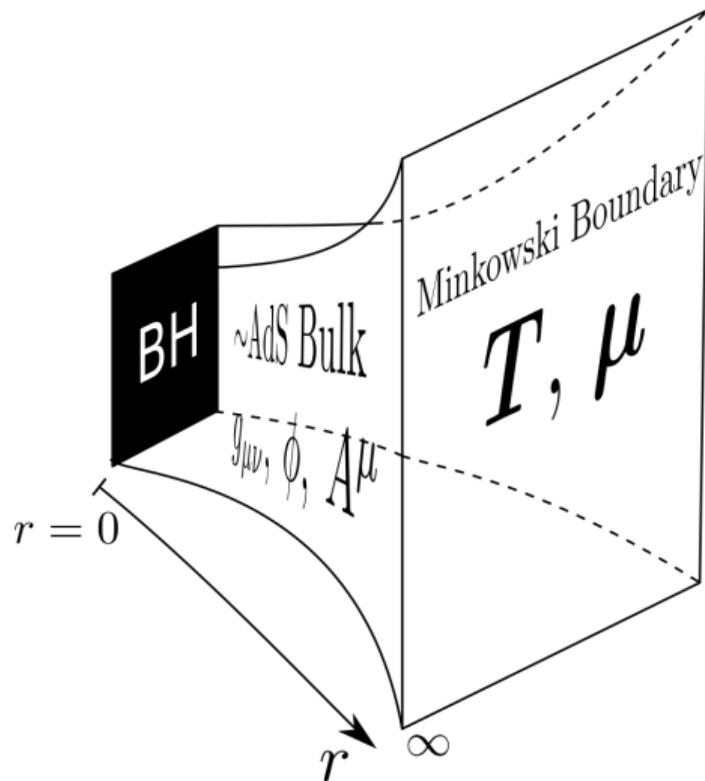
Modeling the QGP with dual black holes

- 5D bulk: Classical gravity with asymptotically Anti-deSitter (AdS_5) geometry.
- 3+1D Boundary: Strongly coupled fluid in Minkowski spacetime.

J. M. Maldacena, *Adv. Theor. Math. Phys.* **2** (1998)

- AdS radius $r \sim$ RG energy scale.
Black-hole horizon at $r = 0 \sim$ thermal state (Hawking temperature).
- Strongly coupled, nearly inviscid behavior of the QGP.

P. Kovtun, D. T. Son, A. O. Starinets, *PRL* **94** (2005)



Einstein-Maxwell-Dilaton model

- Breaking of conformal symmetry: dilaton field ϕ .
- Dual to baryon chemical potential μ : Abelian gauge field A^μ .

- Action:

$$S = \frac{1}{2\kappa_5^2} \int_{\mathcal{M}_5} d^5x \sqrt{-g} \left[R - \frac{(\partial_\mu \phi)^2}{2} - V(\phi) - \frac{f(\phi) F_{\mu\nu}^2}{4} \right],$$

- Two potentials, $V(\phi)$ and $f(\phi)$, tweaked to fit lattice QCD results.

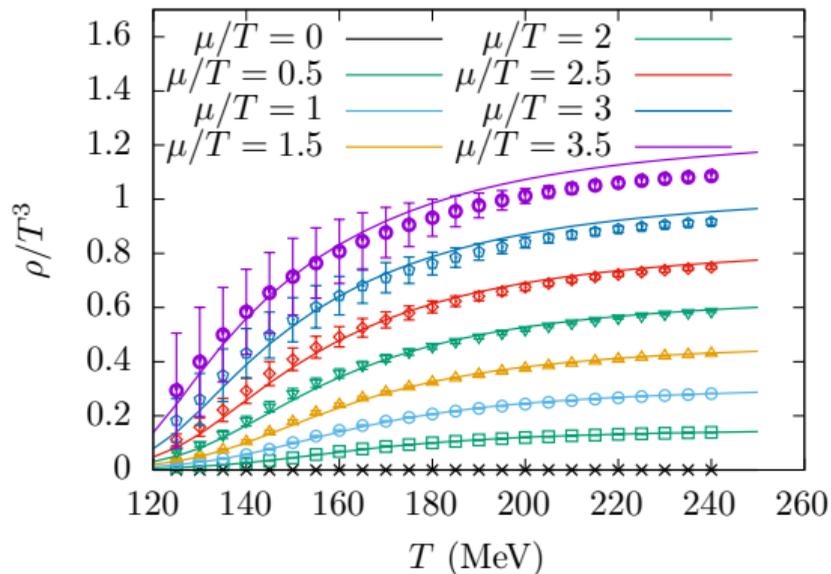
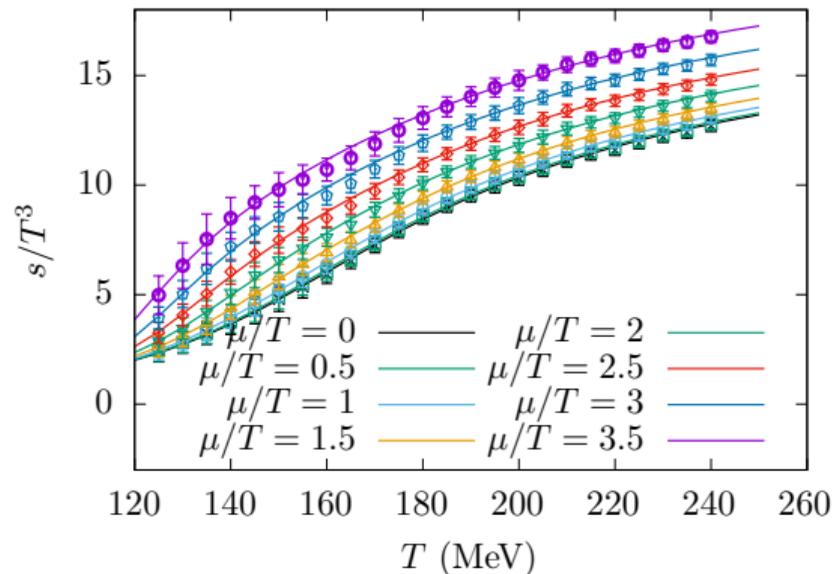
S. S. Gubser and A. Nellore, PRD **78** (2008)

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R. Critelli, J. Noronha, J. Noronha-Hostler, I. Portillo, C. Ratti, R. Rougemont, PRD **96** (2017)

J. Grefa, J. Noronha, J. Noronha-Hostler, I. Portillo, C. Ratti, R. Rougemont, PRD **104** (2021)

Finite-density EoS



R. Critelli, J. Noronha, J. Noronha-Hostler, I. Portillo, C. Ratti, R. Rougemont, PRD **96** (2017)
 Borsányi, Fodor, Guenther et al., PRL **126** (2021)

J. Grefa — Wednesday 5:30pm

- Powerful, flexible model capable of describing crossover region and beyond.

Limitations

- No asymptotic freedom, no quasi-particle excitations.
- Poor description of hadronic phase.

Advantages

- Able to handle out-of-equilibrium physics and predict transport properties.

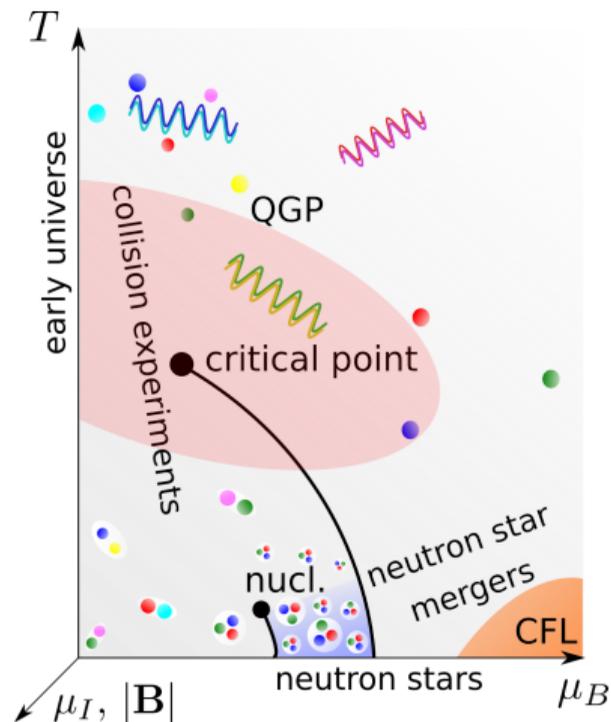
S. S. Gubser, A. Nellore, S. S. Pufu and F. D. Rocha, PRL **101**, (2008)

- Bulk viscosity (non-conformal effect) compatible with heavy-ion analyses.

J. Grefa, M. Hippert, J. Noronha, J. Noronha-Hostler, I. Portillo, C. Ratti and R. Rougemont, PRD **106** (2022)

- Can predict critical point and first-order line.

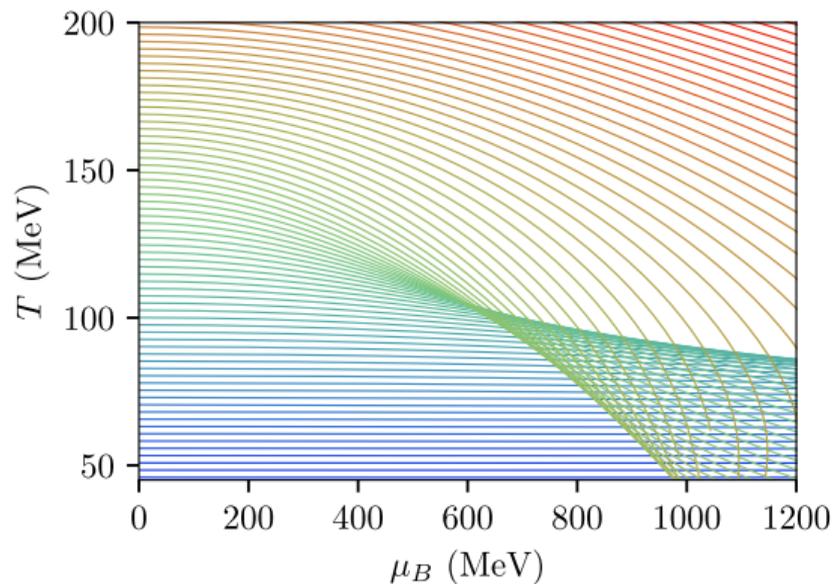
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J. Grefa — Wednesday 5:30pm

Phase diagram

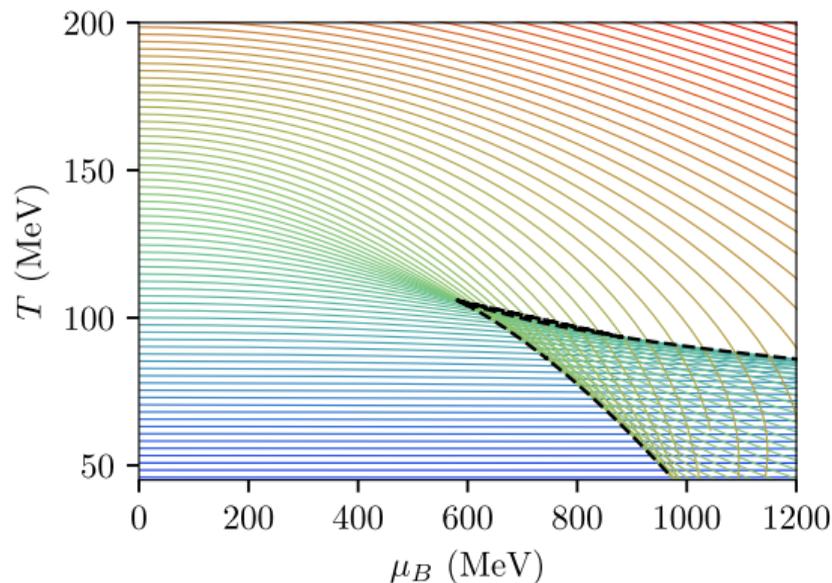
- Dilaton and electric fields at horizon: ϕ_0 and Φ_1 fully specify the physical state.
- Lines of constant ϕ_0 can cross.



MH, J. Grefa, T.A. Manning, J. Noronha, J. Noronha-Hostler, I. Portillo, C. Ratti, R. Rougemont, M. Trujillo, [arXiv:2309.00579](https://arxiv.org/abs/2309.00579).

Phase diagram

- Dilaton and electric fields at horizon: ϕ_0 and Φ_1 fully specify the physical state.
- Lines of constant ϕ_0 can cross.
- Metastable states, spinodal lines.

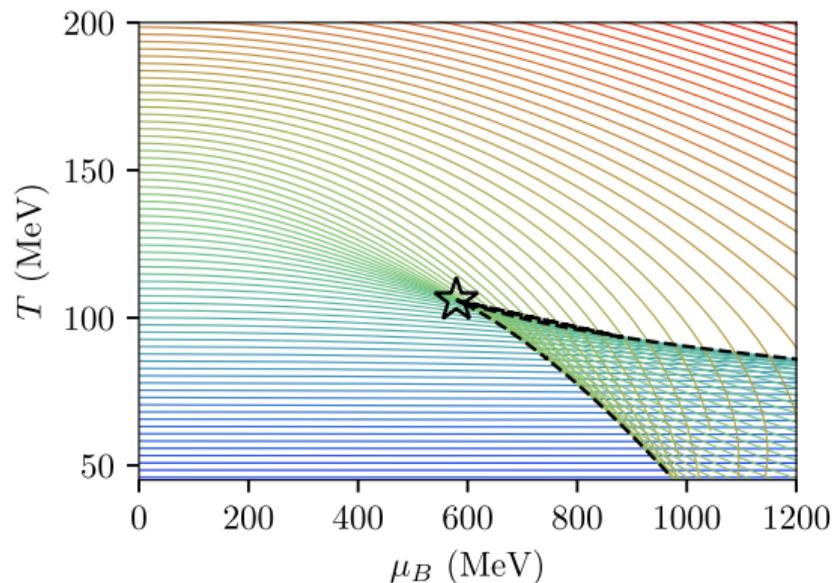


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Phase diagram

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- Critical point: where crossings start.

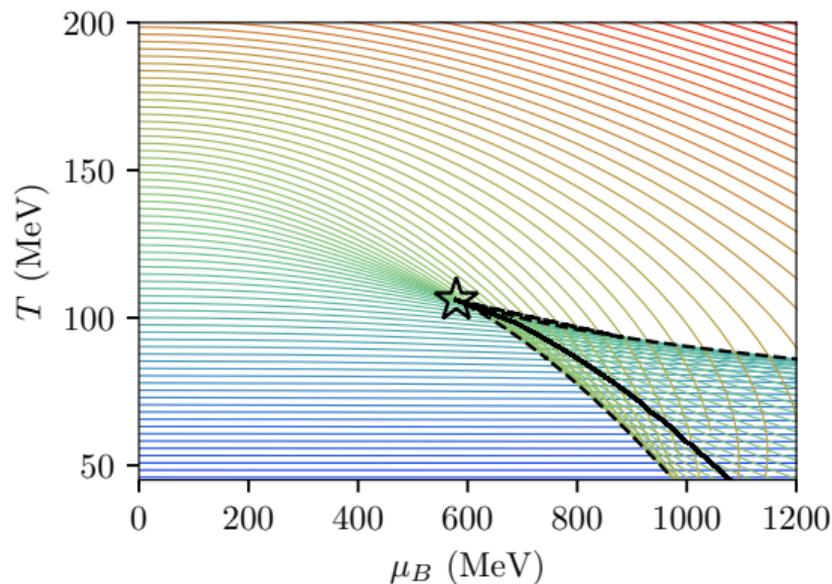
Fast algorithm to find CP!



MH, J. Grefa, T.A. Manning, J. Noronha, J. Noronha-Hostler, I. Portillo, C. Ratti, R. Rougemont, M. Trujillo, [arXiv:2309.00579](https://arxiv.org/abs/2309.00579).

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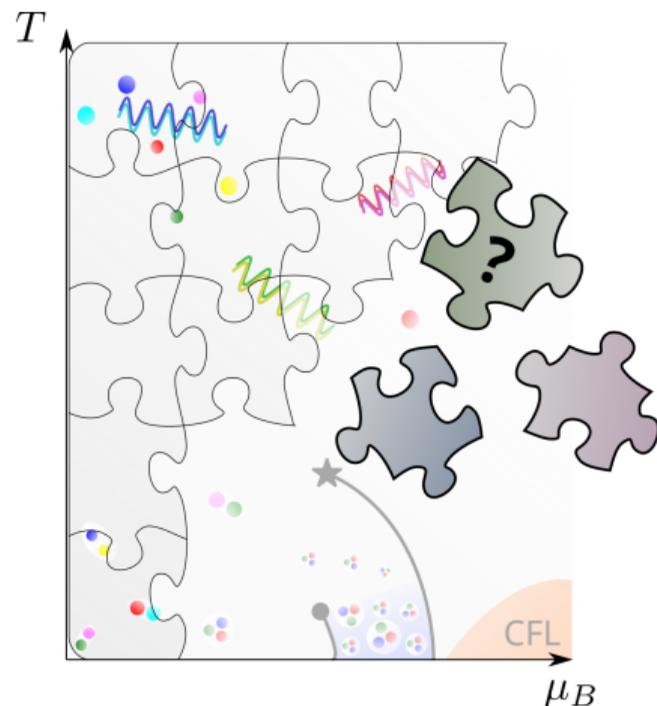
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Fast algorithm to find CP!
- Maxwell construction: first-order line.



MH, J. Grefa, T.A. Manning, J. Noronha, J. Noronha-Hostler, I. Portillo, C. Ratti, R. Rougemont, M. Trujillo, [arXiv:2309.00579](https://arxiv.org/abs/2309.00579).

Bayesian black-hole engineering

- What scenarios described by model compatible with the lattice results + error bars?
- Systematic scan over possible extrapolations to higher densities.
- Use Bayesian inference tools.



MH, J. Grefa, T.A. Manning, J. Noronha, J. Noronha-Hostler, I. Portillo, C. Ratti, R. Rougemont, M. Trujillo, [arXiv:2309.00579](https://arxiv.org/abs/2309.00579).

Assigning probabilities

Bayes' Theorem

$$\underbrace{P(\text{model} \mid \text{results})}_{\text{posterior } \mathcal{P}} \times P(\text{results}) = \underbrace{P(\text{results} \mid \text{model})}_{\text{likelihood } \mathcal{L}} \times \underbrace{P(\text{model})}_{\text{prior knowledge}}$$

Gaussian Likelihood

$$\mathcal{L} = \exp \left\{ -\frac{1}{2} \boldsymbol{\delta x}^T \boldsymbol{\Sigma}^{-1} \boldsymbol{\delta x} - \frac{1}{2} \log \det \boldsymbol{\Sigma} + \text{constant} \right\}$$

- $\boldsymbol{\delta x}$: deviation for $s(T)$ and $\chi_2^{(B)}(T)$ at $\mu = 0$.
- Correlation $\Gamma \equiv \exp(-\Delta T/\xi_T)$ between neighboring points
→ extra model parameter.

Polynomial-Hyperbolic Ansatz (PHA)

- Interpolates between [arXiv:1706.00455](#) and [arXiv:2201.02004](#)

$$V(\phi) = -12 \cosh(\gamma \phi) + b_2 \phi^2 + b_4 \phi^4 + b_6 \phi^6$$

$$f(\phi) = \frac{\operatorname{sech}(c_1 \phi + c_2 \phi^2 + c_3 \phi^3)}{1 + d_1} + \frac{d_1}{1 + d_1} \operatorname{sech}(d_2 \phi)$$

Parametric Ansatz (PA)

- Similar shapes, more interpretable parameters (\sim [arXiv:1706.02647](#))

$$V(\phi) = -12 \cosh \left[\left(\frac{\gamma_1 \Delta \phi_V^2 + \gamma_2 \phi^2}{\Delta \phi_V^2 + \phi^2} \right) \phi \right]$$

$$f(\phi) = 1 - (1 - A_1) \left[\frac{1}{2} + \frac{1}{2} \tanh \left(\frac{\phi - \phi_1}{\delta \phi_1} \right) \right] - A_1 \left[\frac{1}{2} + \frac{1}{2} \tanh \left(\frac{\phi - \phi_2}{\delta \phi_2} \right) \right]$$

Markov Chain Monte-Carlo (MCMC)

- Random evolution to sample from posterior.
- Transition probabilities such that \mathcal{P} is stationary limit.
- Differential evolution MCMC: suited for correlations.

C.J.F. Ter Braak, *Statistics and Computing* **16** (2006)

Differential evolution

- 1 Use other chains j, k to update chain $i \neq j \neq k$: $\theta_i \rightarrow \theta_i + \frac{b}{\sqrt{2d}}(\theta_j - \theta_k) + \xi_i$.
- 2 Compute \mathcal{P} from model EoS.
 - If $\mathcal{P}/\mathcal{P}_0 > 1$, transition to new parameters.
 - Otherwise, accept transition with probability $\mathcal{P}/\mathcal{P}_0$.
- 3 Repeat.

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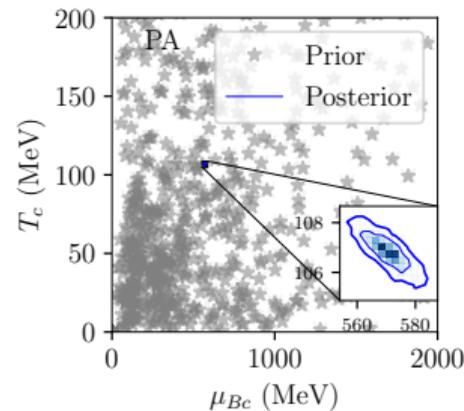
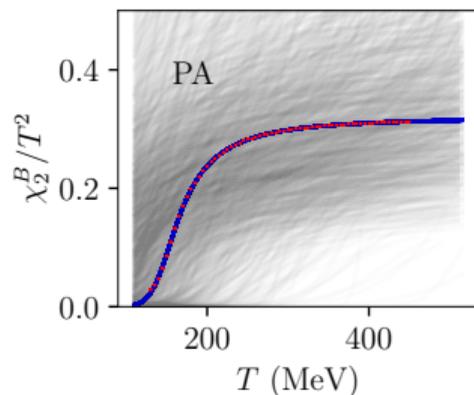
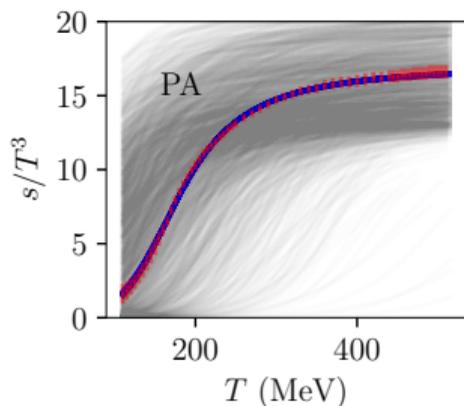
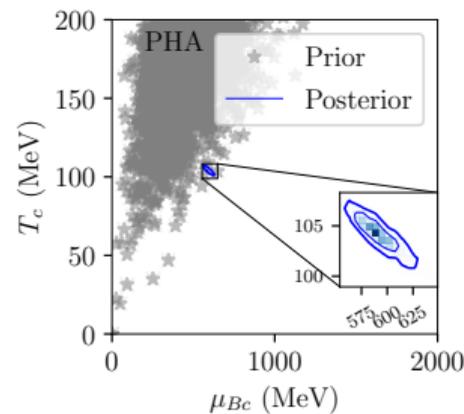
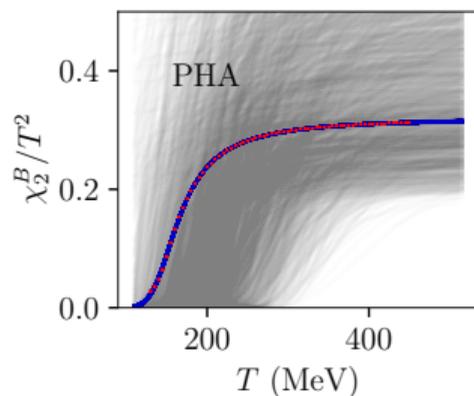
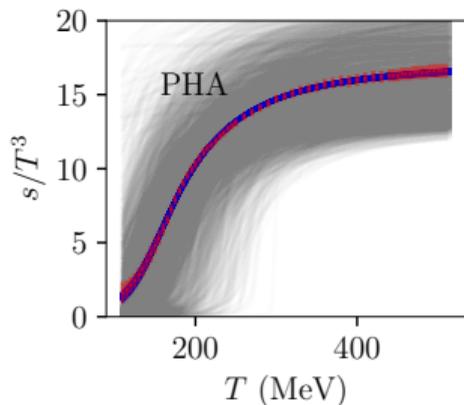
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Inputs: Baryon susceptibility and entropy density from the lattice.

S. Borsanyi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg and K. K. Szabo, *PRL* **730** (2014)
 Borsányi, Fodor, Guenther et al., *PRL* **126** (2021)

Results

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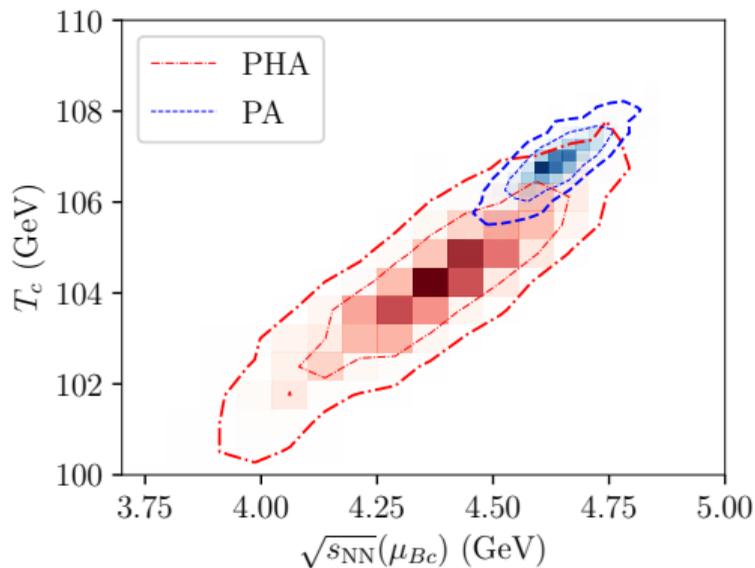
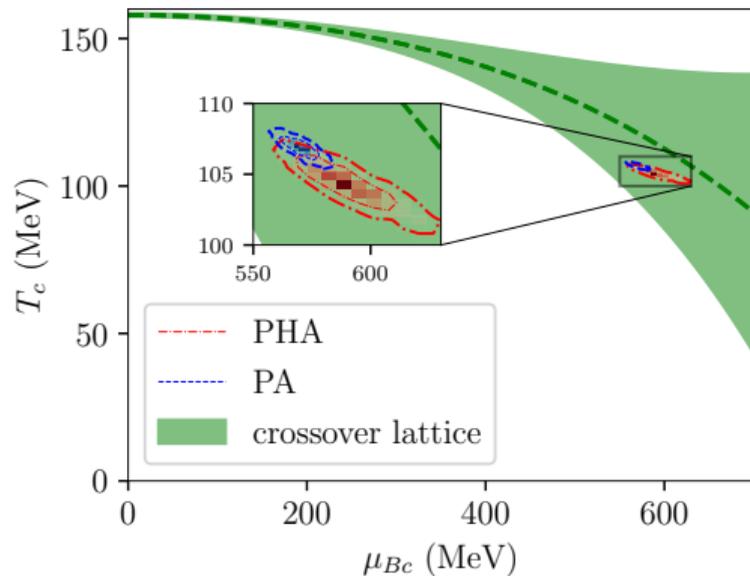


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Posterior critical points

$$(T_c, \mu_{Bc})_{PHA} = (104 \pm 3, 589^{+36}_{-26}) \text{ MeV},$$

$$(T_c, \mu_{Bc})_{PA} = (107 \pm 1, 571 \pm 11) \text{ MeV}.$$



- Both Ansätze overlap at 1σ . **Robust results!**

MH, J. Grefa, T.A. Manning, J. Noronha, J. Noronha-Hostler, I. Portillo, C. Ratti, R. Rougemont, M. Trujillo, [arXiv:2309.00579](https://arxiv.org/abs/2309.00579).

Conclusions

- 1 Powerful description of the QGP, matching *finite-density* lattice results.

J. Grefa, J. Noronha, J. Noronha-Hostler, I. Portillo, C. Ratti, R. Rougemont, PRD **104** (2021)
 J. Grefa, M. Hippert, J. Noronha, J. Noronha-Hostler, I. Portillo, C. Ratti and R. Rougemont, PRD **106** (2022)

J. Grefa — Wednesday 5:30pm

- 2 Bayesian black-hole engineering: systematic exploration of phase diagram, informed by lattice QCD.
- 3 Critical point at $\mu_c \approx 560 - 625$ MeV, corresponding to $\sqrt{s} \approx 4.0 - 4.8$ GeV.
- 4 Larger statistical preference for a critical point after constraints:
 PA model: $\sim 80\%$ of prior $\implies \sim 100\%$ of posterior.

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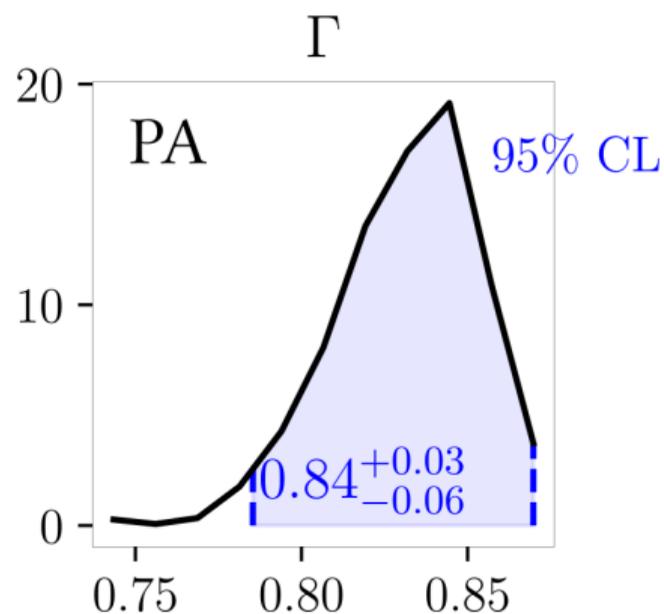
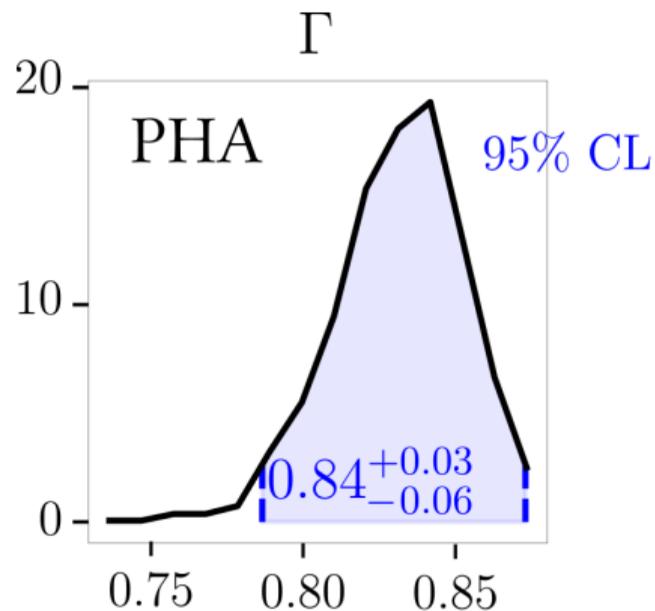
Backup slides...

New C++ code within MUSES Framework

- Development within the MUSES Framework: Multi-institutional collaboration for a unified solver for the equation of state, bridging models and applications.
- Support and advising by cyberinfrastructure and computer-science experts T. Andrew Manning and Roland Haas.
- Improved method to extract asymptotic UV scalings and thermodynamics.
- Large boost in performance and numerical stability.



Correlation strength



Equations of motion

$$\phi''(r) + \left[\frac{h'(r)}{h(r)} + 4A'(r) - B'(r) \right] \phi'(r) - \frac{e^{2B(r)}}{h(r)} \left[\frac{\partial V(\phi)}{\partial \phi} + \frac{e^{-2[A(r)+B(r)]} \Phi'(r)^2}{2} \frac{\partial f(\phi)}{\partial \phi} \right] = 0,$$

$$\Phi''(r) + \left[2A'(r) - B'(r) + \frac{d[\ln f(\phi)]}{d\phi} \phi'(r) \right] \Phi'(r) = 0,$$

$$A''(r) - A'(r)B'(r) + \frac{\phi'(r)^2}{6} = 0,$$

$$h''(r) + [4A'(r) - B'(r)]h'(r) - e^{-2A(r)}f(\phi)\Phi'(r)^2 = 0,$$

$$h(r)[24A'(r)^2 - \phi'(r)^2] + 6A'(r)h'(r) + 2e^{2B(r)}V(\phi) + e^{-2A(r)}f(\phi)\Phi'(r)^2 = 0,$$

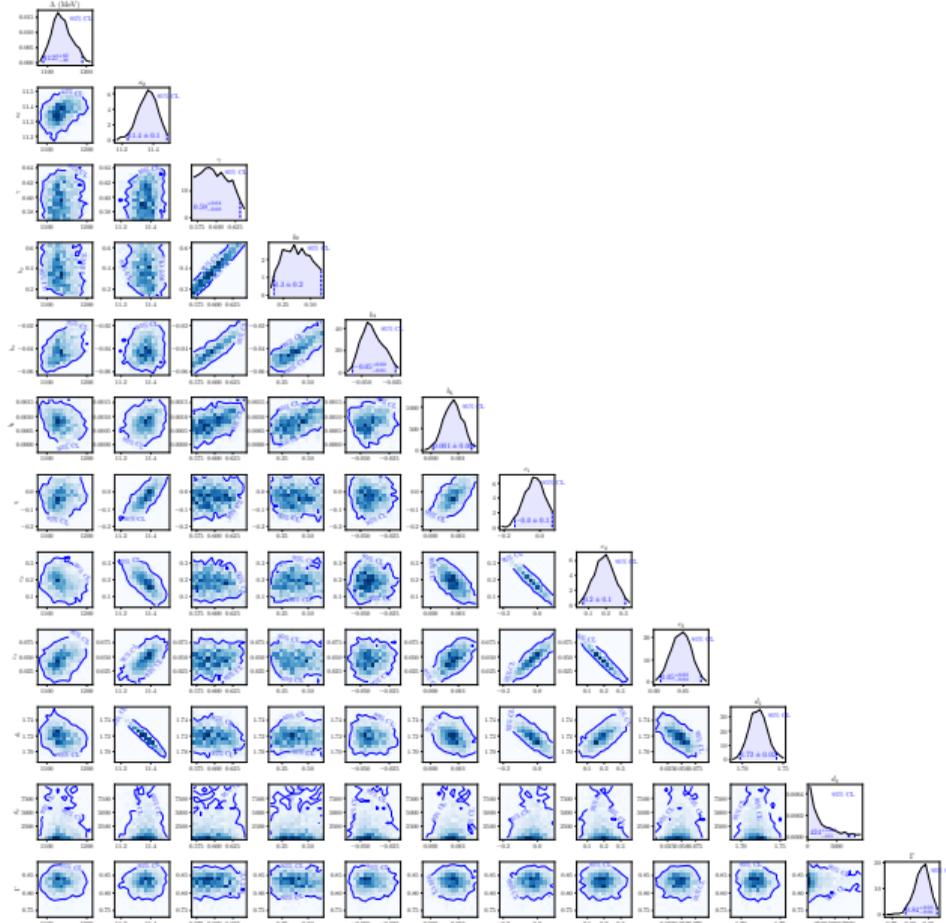
Extraction of Thermodynamics

- Thermodynamics extracted from scalings after conversion to physical units.
- Requires near-boundary scalings,

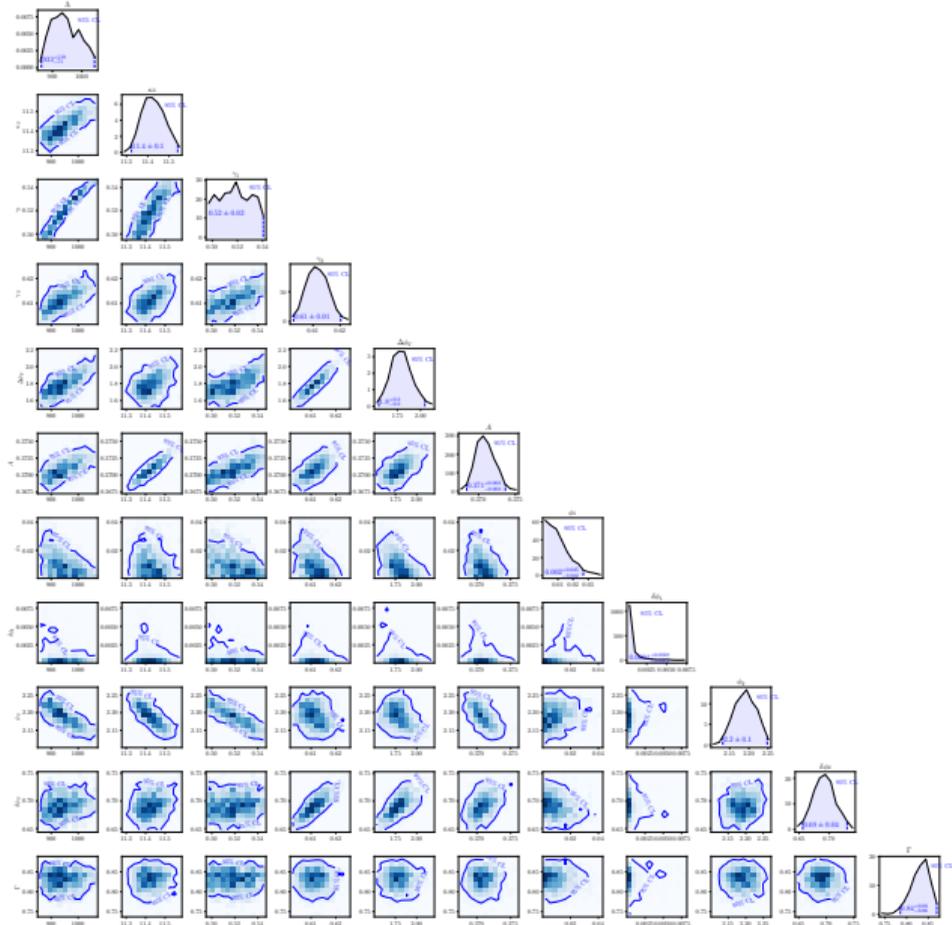
$$\phi \sim \phi_A e^{-\nu A(r)}, \quad \Phi \sim \Phi_0^{\text{far}} + \Phi_2^{\text{far}} e^{-2A(r)}, \quad A \sim A_{-1}^{\text{far}} r + A_0^{\text{far}}$$

- Inversion to find ϕ_A and Φ_2^{far} :
large coefficient \times tiny number = pure noise.

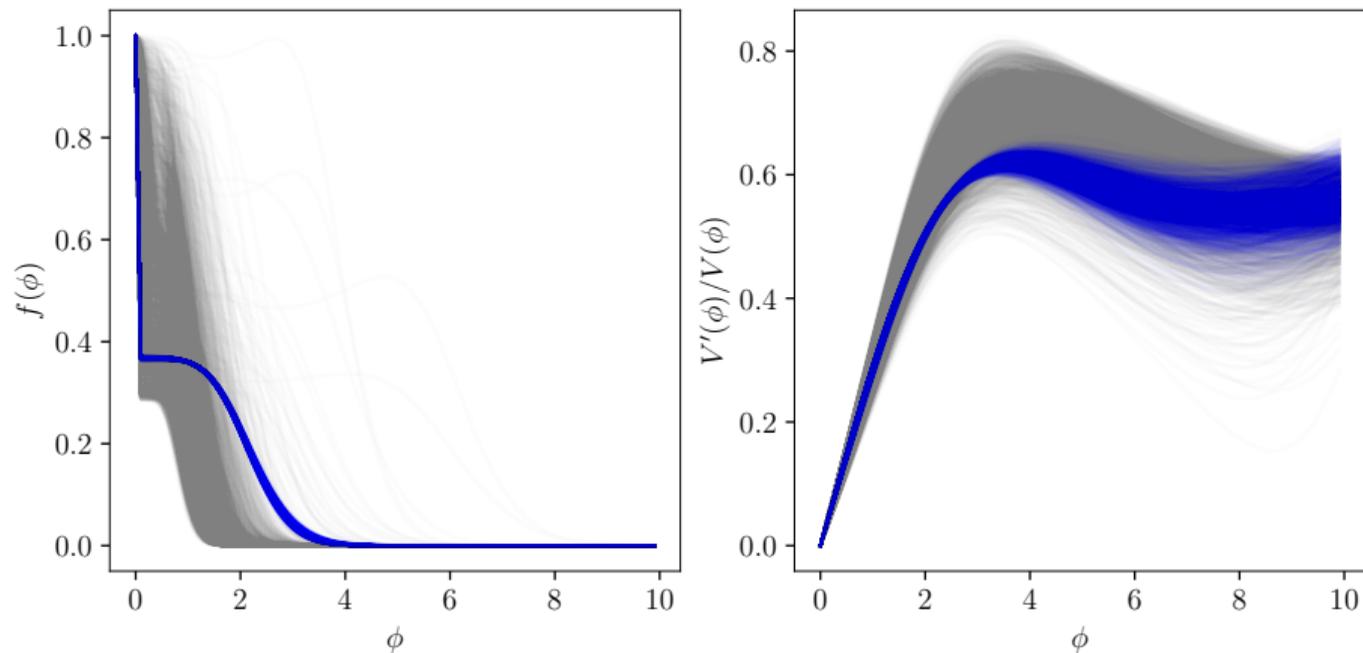
PHA model



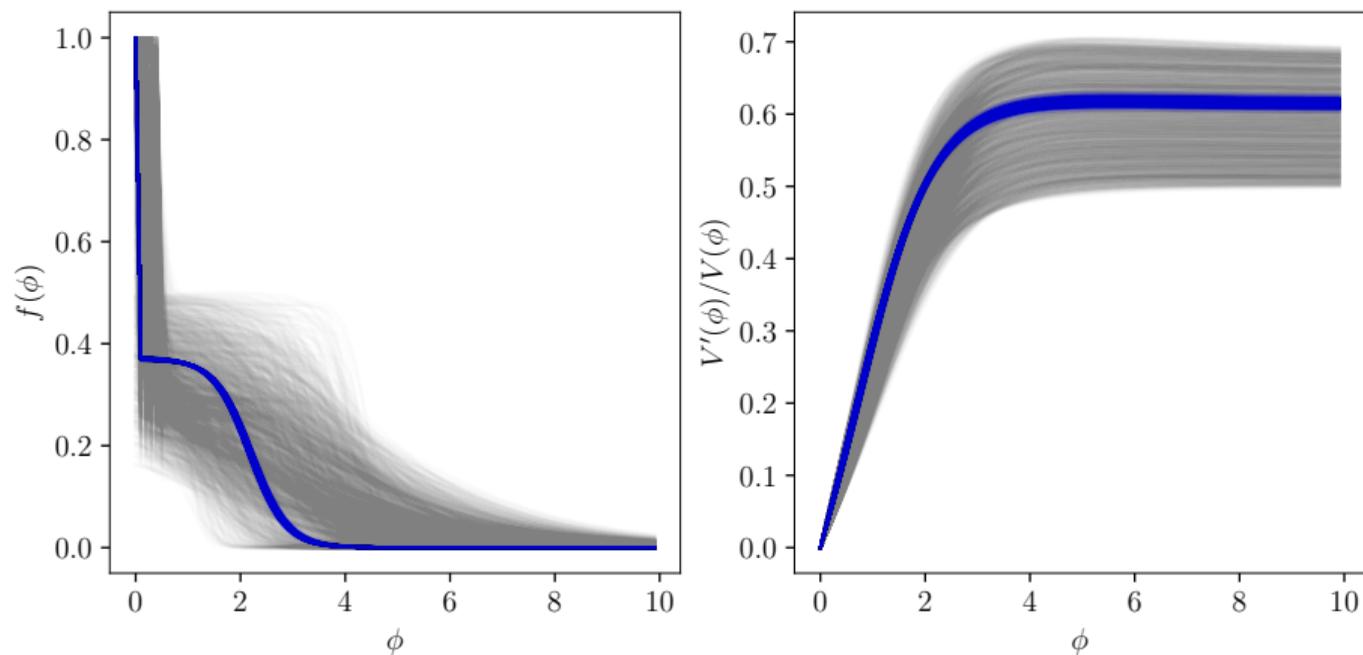
PA model



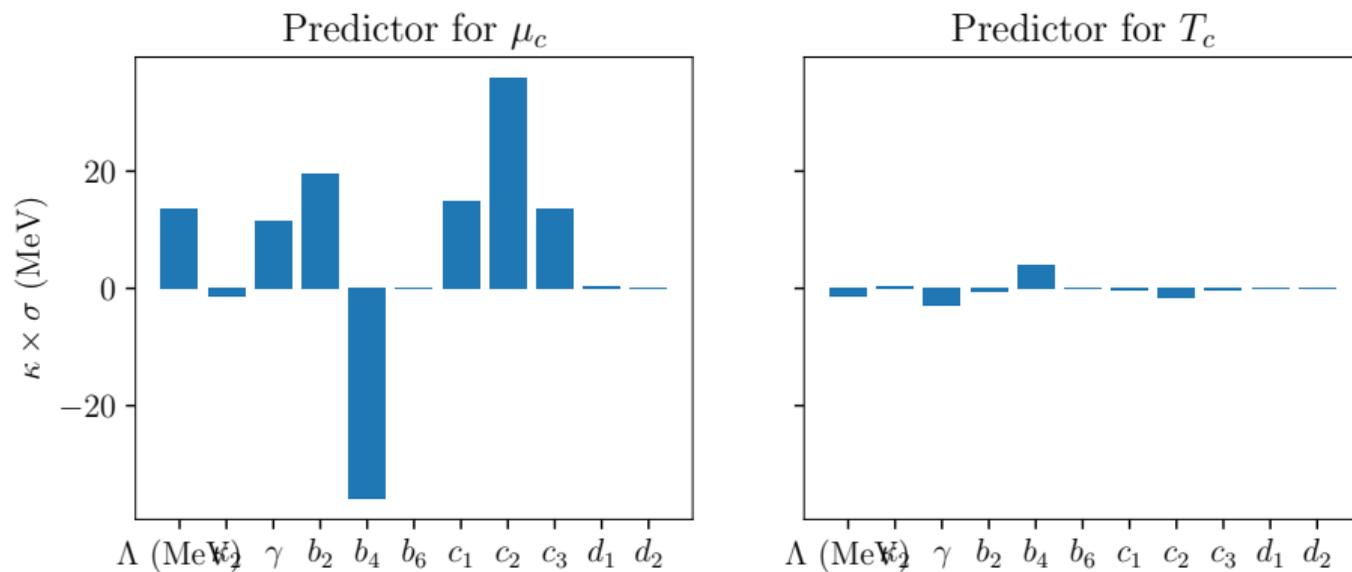
PHA potentials



PA potentials



PHA linear mapping



PA linear mapping

