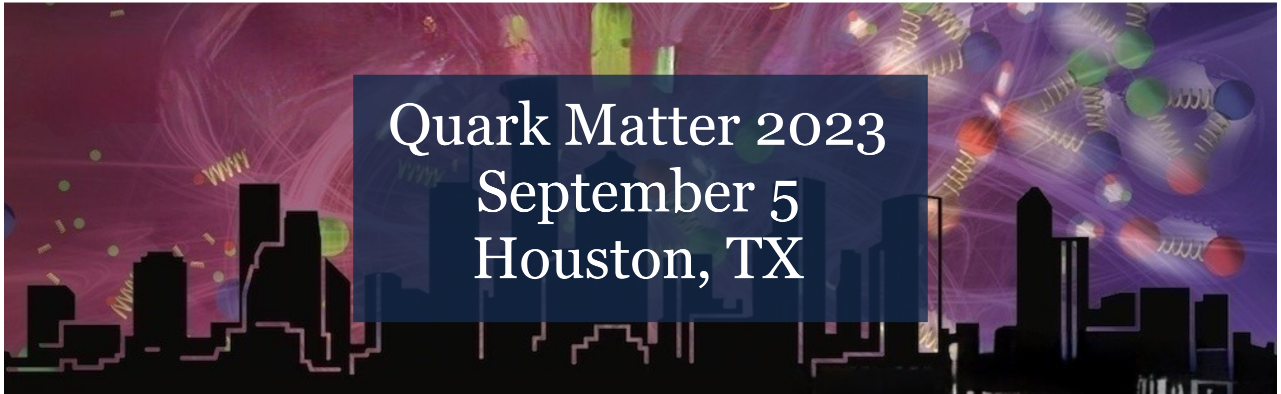
A new causal and stable theory of viscous chiral hydrodynamics

Nick Abboud

in collaboration with Enrico Speranza and Jorge Noronha

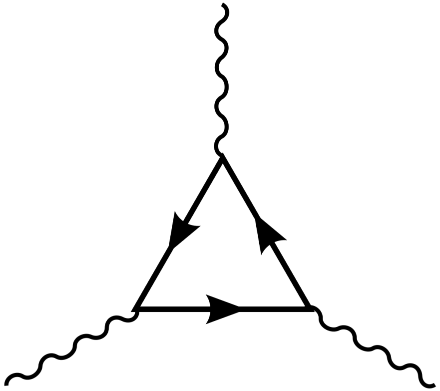
arXiv:2308.02928



Quantum anomalous hydrodynamics

How does the chiral anomaly show up in the macroscopic behavior of a relativistic fluid?

$+$

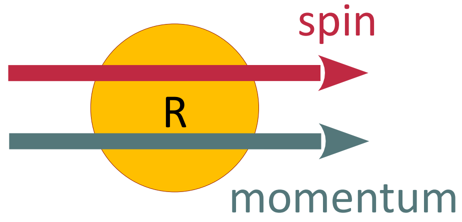


$= ?$

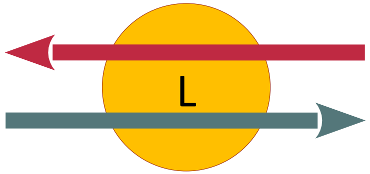
Chiral hydrodynamics

e.g. Kharzeev, Liao, Voloshin, Wang, Prog. Part. Nucl. Phys. 88, 1-28 (2016)

<u>chirality</u>

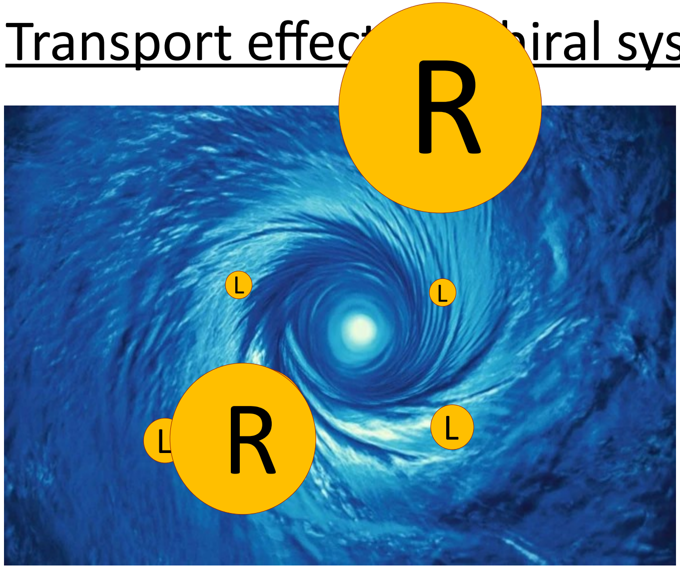


right-handed



left-handed

<u>Transport effec</u> ... <u>hiral systems</u>



- similar effects in $\vec{B}$-field
- transport coefficients largely determined by anomalies

$$U_{\text{int}} \sim -\vec{s} \cdot \vec{\omega}$$

spins align with fluid vorticity

$$\Downarrow$$

chiral transport effects

$$\vec{J} \propto \vec{\omega} \qquad \vec{J}_5 \propto \vec{\omega}$$

Son & Surowka, PRL 103, 191601 (2009)
Neiman & Oz, JHEP 03, 023 (2011)

Chiral hydrodynamics

chiral anomaly $+$ 2nd law of thermodynamics $\rightarrow$ new hydro terms

equations of motion

external electromagnetic fields

$$\nabla_\mu T^{\mu\nu} = \boxed{F^{\nu\lambda}} J_{5\lambda}$$
$$\nabla_\mu J_5^\mu = C\boxed{E^\mu B_\mu}$$

degrees of freedom

$$\varepsilon,\ n_5,\ u^\mu$$

energy, axial chg, velocity

In **Landau** hydrodynamic frame:

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu + (P - \zeta\partial_\alpha u^\alpha)\Delta^{\mu\nu} - 2\eta\sigma^{\mu\nu}$$
$$J_5^\mu = n_5 u^\mu + \sigma\left[E^\mu - T\Delta^{\mu\nu}\partial_\nu\left(\frac{\mu_5}{T}\right)\right] + \boxed{\xi_{J\omega}\omega^\mu + \xi_{JB}B^\mu}$$

demanded by thermodynamics!
determined by anomaly!

$$\xi_{J\omega} = C\mu_5^2 + \tilde{C}T^2$$
$$\xi_{JB} = C\mu_5$$

Chiral hydrodynamics

Son & Surowka, PRL 103, 191601 (2009)
Neiman & Oz, JHEP 03, 023 (2011)

chiral anomaly $+$ 2[nd] law of thermodynamics $\rightarrow$ new hydro terms

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$$\nabla_\mu J_5^\mu = C \boxed{E^\mu B_\mu}$$

degrees of freedom

$$\varepsilon,\ n_5,\ u^\mu$$

In **Eckart** hydrodynamic frame:

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu + (P - \zeta \partial_\alpha u^\alpha)\Delta^{\mu\nu} - 2\eta\sigma^{\mu\nu} + u^\mu \mathcal{Q}^\nu + u^\nu \mathcal{Q}^\mu$$

$$J_5^\mu = n_5 u^\mu$$

anomalous heat diffusion

$$\mathcal{Q}^\mu = -\sigma T\left[\frac{\Delta^{\mu\alpha}\partial_\alpha T}{T} + u^\alpha \partial_\alpha u^\mu\right] + \boxed{\xi_{T\omega}\omega^\mu + \xi_{TB} B^\mu}$$

vector + axial, other hydrodynamic frames, derivations from kinetic theory, Israel-Stewart-like theories, ...

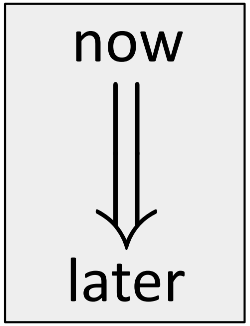
Isachenkov & Sadofyev PLB 697, 404 (2011)

Stephanov & Yee PRL 116, 122302 (2011)

Chen, Son, Stephanov PRL 115, 021601 (2015)
Yang, PRD 98, 076019 (2018)

Gorbar, Rybalka, Shovkovy, PRD 95, 096010 (2017)

Can these theories be solved?



Is chiral hydro **consistent™** with these principles?

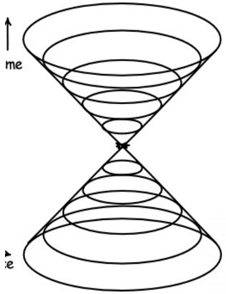
Local well-posedness

?

Unique solutions for arbitrary initial conditions

Causality

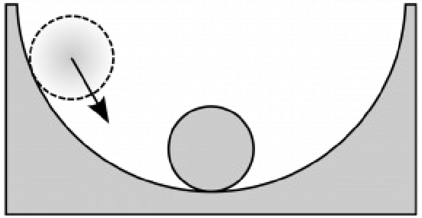
Information propagates subluminally



?

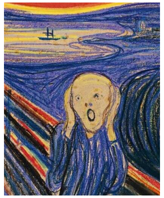
Stability

Deviations from equilibrium are bounded



?

Previous theories of viscous chiral hydrodynamics



fail to be consistent

OR

their consistent regime is ***not known***

In this talk...

the first consistent theory of viscous chiral hydrodynamics

NA, E. Speranza, J. Noronha, arXiv:2308.02928

BDNK approach to consistent 1st-order hydro

Bemfica, Disconzi, Noronha, PRD 98, 104064 (2018); PRD 100, 104020 (2019); PRX 12, 021044 (2022)
Kovtun, JHEP 10, 034 (2019); Hoult & Kovtun, JHEP 06, 067 (2020)

<u>degrees of freedom</u>

$$\varepsilon, \; n, \; u^\mu$$

<u>equations of motion</u>

$$\nabla_\mu T^{\mu\nu} = 0 \qquad \nabla_\mu J^\mu = 0$$

(vector current)

$$T^{\mu\nu} = (\varepsilon + \mathcal{A})u^\mu u^\nu + (P + \Pi)\Delta^{\mu\nu} + \mathcal{Q}^\mu u^\nu + \mathcal{Q}^\nu u^\mu + \mathcal{T}^{\mu\nu}$$

$$J^\mu = n u^\mu$$

Bemfica, Disconzi & Noronha, PRX 12, 021044 (2022)

$$\mathcal{A} = \tau_\varepsilon \left[u^\lambda \nabla_\lambda \varepsilon + (\varepsilon + P)\nabla_\lambda u^\lambda\right], \quad \mathcal{T}^{\mu\nu} = -2\eta\sigma^{\mu\nu}$$

$$\Pi = -\zeta \nabla_\lambda u^\lambda + \tau_P \left[u^\lambda \nabla_\lambda \varepsilon + (\varepsilon + P)\nabla_\lambda u^\lambda\right]$$

$$\mathcal{Q}^\nu = \tau_Q (\varepsilon + P) u^\lambda \nabla_\lambda u^\nu + \beta_\varepsilon \Delta^{\nu\lambda}\nabla_\lambda \varepsilon + \beta_n \Delta^{\nu\lambda}\nabla_\lambda n$$

has all leading dissipative contributions: shear/bulk viscosity & thermal conductivity

- extra transport parameters regulate inconsistent behavior
- proved locally well-posed, causal, and stable (and strongly hyperbolic)

Consistent 1st-order **chiral** hydro

NA, E. Speranza, J. Noronha
arXiv:2308.02928

degrees of freedom

$$\varepsilon,\ n_5,\ u^\mu$$

equations of motion

$$\nabla_\mu T^{\mu\nu} = F^{\nu\lambda} J_{5\lambda} \qquad \nabla_\mu J_5^\mu = C E^\mu B_\mu$$

$$T^{\mu\nu} = (\varepsilon + \mathcal{A}) u^\mu u^\nu + (P + \Pi)\Delta^{\mu\nu} + \mathcal{Q}^\mu u^\nu + \mathcal{Q}^\nu u^\mu + \mathcal{T}^{\mu\nu}$$

$$J_5^\mu = (n_5 + \mathcal{N}_5) u^\mu + \mathcal{J}_5^\mu$$

$$\mathcal{A} = \varepsilon_1 D\varepsilon + \varepsilon_2 \nabla_\lambda u^\lambda + \varepsilon_3 D n_5, \qquad \mathcal{T}^{\mu\nu} = -2\eta\sigma^{\mu\nu}$$

$$\Pi = \pi_1 D\varepsilon + \pi_2 \nabla_\lambda u^\lambda + \pi_3 D n_5$$

$$\mathcal{Q}^\nu = \theta_1 \nabla_\perp^\mu \varepsilon + \theta_2 D u^\mu + \theta_3 \nabla_\perp^\mu n_5 + \theta_E E^\mu + \boxed{\xi_{T\omega}\omega^\mu + \xi_{TB} B^\mu}$$

$$\mathcal{N}_5 = \nu_1 D\varepsilon + \nu_2 \nabla_\lambda u^\lambda + \nu_3 D n_5$$

$$\mathcal{J}_5^\mu = \gamma_1 \nabla_\perp^\mu \varepsilon + \gamma_2 D u^\mu + \gamma_3 \nabla_\perp^\mu n_5 + \gamma_E E^\mu + \boxed{\xi_{J\omega}\omega^\mu + \xi_{JB} B^\mu}$$

all possible first-order transport parameters are included

all chiral conductivities

Houston, we have a problem!

NA, E. Speranza, J. Noronha
arXiv:2308.02928

$$T^{\mu\nu} = (\varepsilon + \mathcal{A})u^\mu u^\nu + (P + \Pi)\Delta^{\mu\nu} + \mathcal{Q}^\mu u^\nu + \mathcal{Q}^\nu u^\mu + \mathcal{T}^{\mu\nu}$$

$$J_5^\mu = (n_5 + \mathcal{N}_5)u^\mu + \mathcal{J}_5^\mu$$

$$\mathcal{A} = \varepsilon_1 D\varepsilon + \varepsilon_2 \nabla_\lambda u^\lambda + \varepsilon_3 D n_5, \quad \mathcal{T}^{\mu\nu}$$

BAD

$$\Pi = \pi_1 D\varepsilon + \pi_2 \nabla_\lambda u^\lambda + \pi_3 D n_5$$

$$\mathcal{Q}^\nu = \theta_1 \nabla_\perp^\mu \varepsilon + \theta_2 D u^\mu + \theta_3 \nabla_\perp^\mu n_5 + \theta_E E^\mu + \boxed{\xi_{T\omega}\omega^\mu} + \xi_{TB} B^\mu$$

$$\mathcal{N}_5 = \nu_1 D\varepsilon + \nu_2 \nabla_\lambda u^\lambda + \nu_3 D n_5$$

$$\mathcal{J}_5^\mu = \gamma_1 \nabla_\perp^\mu \varepsilon + \gamma_2 D u^\mu + \gamma_3 \nabla_\perp^\mu n_5 + \gamma_E E^\mu + \xi_{J\omega}\omega^\mu + \xi_{JB} B^\mu$$

This term makes the theory acausal, unstable, and even ill-posed!

Why is the bad term bad?

We thank Lorenzo Gavassino for pointing out this argument

Strip away everything but the bad term

heat diffusion

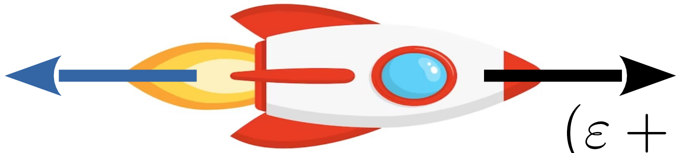
$$T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu + Pg^{\mu\nu} + Q^\mu u^\nu + Q^\nu u^\mu$$

Velocity perturbation atop static equilibrium:

$$u^\mu = (1, \delta\vec{u}) \quad \Longrightarrow \quad \delta T^{0j} = (\varepsilon + P)\delta u^j + \delta Q^j \not\propto \delta u^j$$

$\Longrightarrow$ unbounded acceleration by pushing heat in other direction

$\delta\vec{Q}$



$(\varepsilon + P)\delta\vec{u}$

this can be any function

$$\delta Q^j \sim \frac{1}{2}\xi_{T\omega}(\vec{\nabla} \times \delta\vec{u})^j \quad \Longrightarrow \quad \delta\vec{u} = \boxed{f(t)}(\sin(kz), \cos(kz), 0) \qquad k = \frac{2(\varepsilon + P)}{\xi_{T\omega}}$$

(bad term)

can increase without bound!

Why is the bad term bad?

We thank Lorenzo Gavassino for pointing out this argument

Strip away everything b

diffusion

$$T^{\mu\nu} = (\varepsilon + P \quad\quad\quad - Q^\nu u^\mu$$

unstable

and

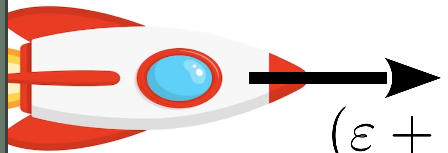
ill-posed

(non-deterministic)

Velocity perturbation a

$$u^\mu = (1, \vec{\delta u}) \quad\quad\quad u^j + \delta Q^j \not\propto \delta u^j$$

$\Longrightarrow$ unbounded acc
pushing heat in o



$(\varepsilon + P)\vec{\delta u}$

this can be any function

$$\delta Q^j \sim \frac{1}{2}\xi_{T\omega}(\vec{\nabla} \times \vec{\delta u})^j \quad \Longrightarrow \quad \vec{\delta u} = \boxed{f(t)}(\sin(kz), \cos(kz), 0) \quad\quad k = \frac{2(\varepsilon + P)}{\xi_{T\omega}}$$

(bad term)

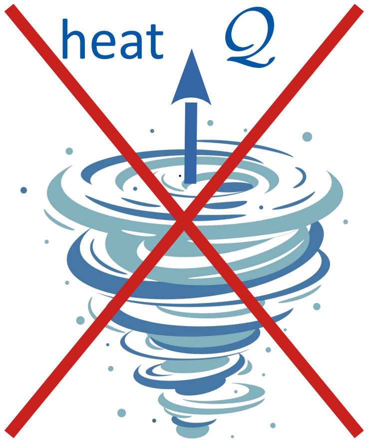
can increase without bound!

NA, E. Speranza, J. Noronha, arXiv:2308.02928

First-order viscous chiral hydro is ill-posed *unless*

$$\boxed{\xi_{T\omega} = 0}$$

no vorticity-induced heat flux!



the same happens in **ideal** chiral hydro

Speranza, Bemfica, Disconzi, Noronha, PRD 107, 054029 (2023)

Fixing the bad term

NA, E. Speranza, J. Noronha
arXiv:2308.02928

$$T^{\mu\nu} = (\varepsilon + \mathcal{A})u^\mu u^\nu + (P + \Pi)\Delta^{\mu\nu} + \mathcal{Q}^\mu u^\nu + \mathcal{Q}^\nu u^\mu + \mathcal{T}^{\mu\nu}$$

$$J_5^\mu = (n_5 + \mathcal{N}_5)u^\mu + \mathcal{J}_5^\mu$$

$$\ldots$$

BAD

$$\mathcal{Q}^\nu = \theta_1 \nabla_\perp^\mu \varepsilon + \theta_2 D u^\mu + \theta_3 \nabla_\perp^\mu n_5 + \theta_E E^\mu + \boxed{\xi_{T\omega}\omega^\mu} + \xi_{TB} B^\mu$$

hydrodynamic frame
=
definition of
hydrodynamic fields

$$\varepsilon,\ n_5,\ u^\alpha$$

frame transformation

$$u^\mu \to u^\mu - \frac{\xi_{T\omega}}{\varepsilon + P}\omega^\mu$$

Fixing the bad term

NA, E. Speranza, J. Noronha
arXiv:2308.02928

$$T^{\mu\nu} = (\varepsilon + \mathcal{A})u^\mu u^\nu + (P + \Pi)\Delta^{\mu\nu} + \mathcal{Q}^\mu u^\nu + \mathcal{Q}^\nu u^\mu + \mathcal{T}^{\mu\nu}$$

$$J_5^\mu = (n_5 + \mathcal{N}_5)u^\mu + \mathcal{J}_5^\mu$$

OK

$$\ldots$$

$$\mathcal{Q}^\nu = \theta_1 \nabla_\perp^\mu \varepsilon + \theta_2 D u^\mu + \theta_3 \nabla_\perp^\mu n_5 + \theta_E E^\mu +$$  $$+ \xi_{TB} B^\mu$$

frame transformation

$$u^\mu \to u^\mu - \frac{\xi_{T\omega}}{\varepsilon + P}\omega^\mu$$

requires departure from the
“thermodynamic frame”
of Jensen et al., PRL 109, 101601 (2012)

hydrodynamic frame

=

definition of
hydrodynamic fields

$$\varepsilon,\ n_5,\ u^\alpha$$

When is 1st-order chiral hydro consistent?

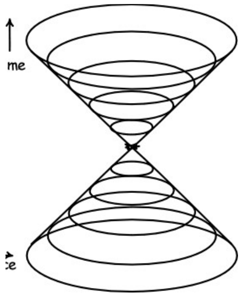
Local well-posedness

now

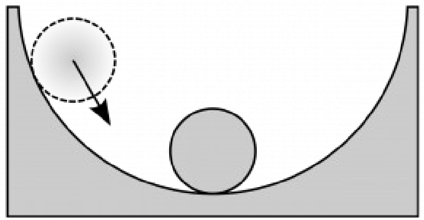
$\Downarrow$

later

Causality



Stability

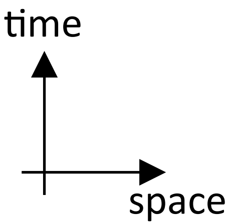


$\Updownarrow$

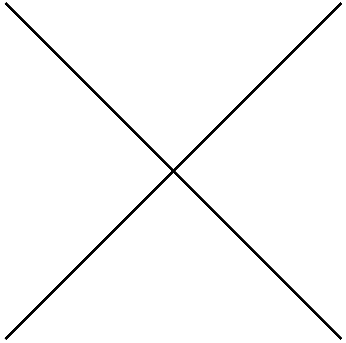
inequalities among transport coefficients

Nonlinear causality

e.g. Courant and Hilbert, *Methods of Mathematical Physics* (1989)
Choquet-Bruhat, *General Relativity and the Einstein Equations* (2009)

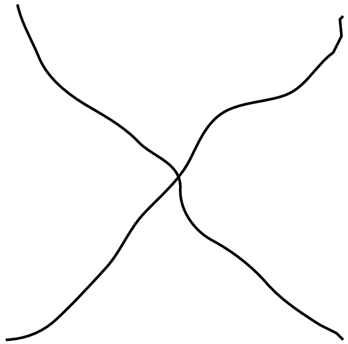


wave equation



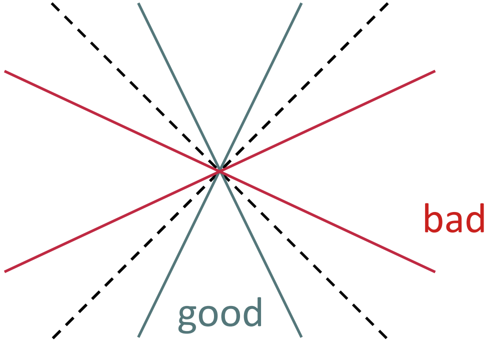
disturbances propagate along light cones

arbitrary PDE



disturbances propagate along **characteristic surfaces**

our theory



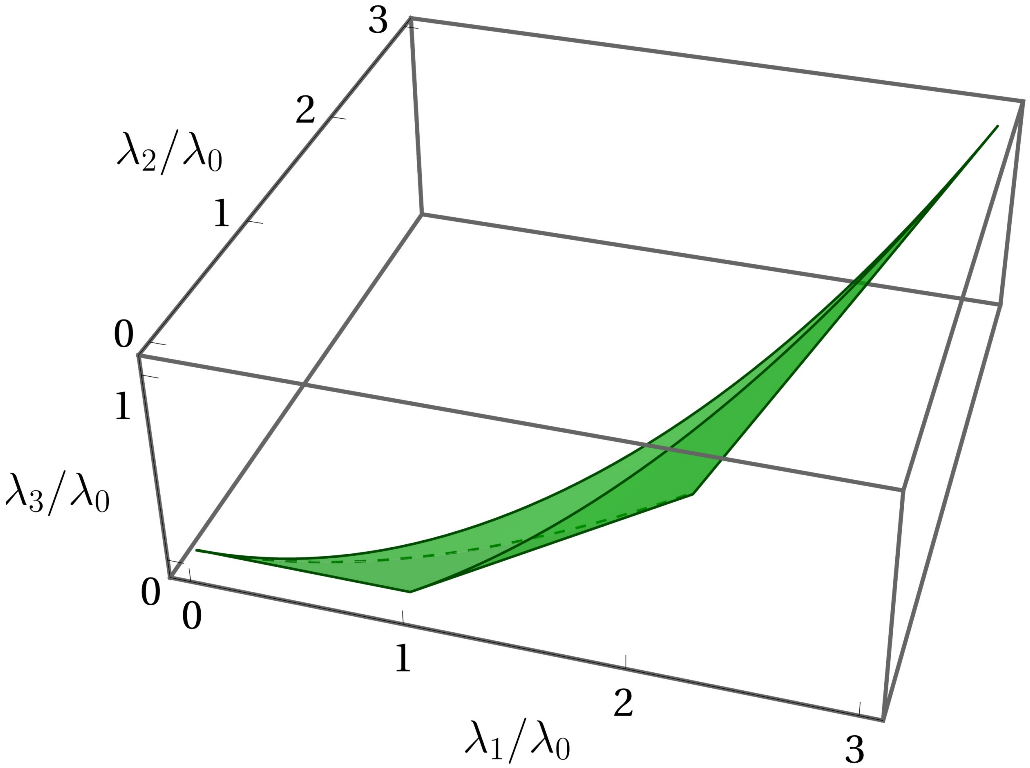
demand non-spacelike characteristic surfaces

$\Longleftrightarrow$ inequalities among transport parameters

Causality conditions

NA, E. Speranza, J. Noronha, arXiv:2308.02928

Causal if, and only if, $0 \leq \eta/\theta_2 \leq 1$ and



- The λ_i are combinations of the transport parameters
- Causality only depends on three combinations!
- These conditions imply local well-posedness in a restricted class of function spaces (Gevrey)

Lorentz-covariant linear stability

stability of **all** homogeneous equilibria

Fourier decomposition

$$\delta\varepsilon, \delta n_5, \delta u^\alpha \sim e^{i(-\Omega t + \vec{k}\cdot\vec{x})}$$

$$\mathrm{Im}[\Omega(\vec{k})] \leq 0 \quad \forall \vec{k}$$

(for all modes)

$\Longleftrightarrow$ inequalities among transport parameters

Causality is **necessary** for covariant stability!

Gavassino, PRX 12, 041001 (2022)
Bemfica, Disconzi & Noronha, PRX 12, 021044 (2022)

Stability conditions

NA, E. Speranza, J. Noronha, arXiv:2308.02928

$\theta_2 > 0 \quad \text{and} \quad \eta \geq 0,$

$\overline{\lambda}_0 > 0 \quad \text{and} \quad \overline{A} > 0 \quad \text{and} \quad \overline{F} > 0 \quad \text{and} \quad \overline{\lambda}_3 > 0,$

$\Delta_{(1,0)} \geq 0$ a

$(\Delta_{(2,4)}, \Delta_{(2,2)}$

$(\Delta_{(3,8)}, \Delta_{(3,6)}$

$(\Delta_{(4,12)}, \Delta_{(4,1}$

$S_2 = \{(a,b,c) \in \mathbb{R}^3 | a \geq 0, c \geq 0, b \geq -2\sqrt{ac}\}.$

S_3 :
- $a \geq 0$ and $b \geq 0$ and $c \geq 0$ and $d \geq 0,$
- $a > 0$ and $d > 0$ and $\mathrm{Disc}_3(a,b,c,d) \leq 0,$

$\chi_3 > 0$

≤ 0

The neccessary & sufficient conditions are complex

- Can be straightforwardly checked numerically
- Simplifications available in limiting cases (see paper)

$\Delta_{(1,0)} = \overline{A}\,\overline{B} - \overline{\lambda}_0,$

$\Delta_{(1,2)} = \overline{\lambda}_1 \overline{A} - \overline{\lambda}_0 \overline{C},$

$\Delta_{(2,0)} = \Delta_{(1,0)},$

$\Delta_{(2,2)} = \overline{C}\Delta_{(1,0)} + \Delta_{(1,2)} - \overline{A}$

$\Delta_{(2,4)} = \overline{C}\Delta_{(1,2)} - \overline{A}(\overline{\lambda}_2 \overline{A} - \overline{\lambda}$

$\Delta_{(3,2)} = (\overline{D} - \overline{B}c_s^2)\Delta_{(1,0)},$

$\Delta_{(3,4)} = (\overline{A}\,\overline{F} - \overline{B}\,\overline{E} - c_s^2 \overline{\lambda}_1 +$

$\Delta_{(3,6)} = (\overline{\lambda}_3 \overline{A} - \overline{\lambda}_1 \overline{E})\Delta_{(1,0)} +$

$\quad + \overline{\lambda}_0[\overline{E}(\overline{A}\,\overline{D} - \overline{\lambda}_0 c_s^2$

$\Delta_{(3,8)} = (\overline{\lambda}_3 \overline{A} - \overline{\lambda}_1 \overline{E})\Delta_{(1,2)} +$

$\Delta_{(4,4)} = -\overline{F}\Delta_{(1,0)} + c_s^2 \Delta_{(3,2)},$

$\Delta_{(4,6)} = [\overline{F}(\overline{A}c_s^2 - \overline{C}) - \overline{\lambda}_3]\Delta_{(1,0)} - \overline{F}\Delta_{(2,2)} + \overline{E}\Delta_{(3,2)} + c_s^2 \Delta_{(3,4)},$

$\Delta_{(4,8)} = [\overline{\lambda}_3(\overline{A}c_s^2 - \overline{C}) + \overline{A}\,\overline{E}\,\overline{F}]\Delta_{(1,0)} + \overline{A}\,\overline{F}c_s^2 \Delta_{(1,2)} - (\overline{\lambda}_3 + \overline{C}\,\overline{F})\Delta_{(2,2)} - \overline{F}\Delta_{(2,4)},$

$\quad + \overline{E}\Delta_{(3,4)} + c_s^2 \Delta_{(3,6)} - \overline{A}^3 \overline{F}^2,$

$\Delta_{(4,10)} = \overline{\lambda}_3 \overline{A}\,\overline{E}\Delta_{(1,0)} + \overline{A}(\overline{E}\,\overline{F} + \overline{\lambda}_3 c_s^2)\Delta_{(1,2)} - \overline{\lambda}_3 \overline{C}\Delta_{(2,2)} - (\overline{\lambda}_3 + \overline{C}\,\overline{F})\Delta_{(2,4)}$

$\quad + \overline{E}\Delta_{(3,6)} + c_s^2 \Delta_{(3,8)} - 2\overline{\lambda}_3 \overline{A}^3 \overline{F},$

$\Delta_{(4,12)} = \overline{\lambda}_3 \overline{A}\,\overline{E}\Delta_{(1,2)} - \overline{\lambda}_3 \overline{C}\Delta_{(2,4)} + \overline{E}\Delta_{(3,8)} - \overline{\lambda}_3^2 \overline{A}^3.$

$+ 4\sqrt{\chi_2} - 2).$

$C = \left(\frac{4}{3}\eta - \pi_2\right)(\tilde{\varepsilon}_1 + \tilde{\nu}_3) + (\varepsilon_2 + \theta_2)(\tilde{\pi}_1 + \tilde{\theta}_1 - \langle\tilde{\nu}\rangle) + (\nu_2 + \gamma_2)(\tilde{\pi}_3 + \tilde{\theta}_3 + \langle\tilde{\varepsilon}\rangle)$

$\quad + w(\{\tilde{\pi}, \tilde{\nu}\} + \{\tilde{\gamma}, \tilde{\varepsilon}\}) + n\{\tilde{\varepsilon}, \tilde{\pi} + \tilde{\theta}\} - \theta_2(\tilde{\theta}_1 + \tilde{\gamma}_3),$

$D = \frac{4}{3}\eta - \pi_2 + w(\tilde{\pi}_1 - \tilde{\gamma}_3 - \langle\tilde{\nu}\rangle) + n(\tilde{\pi}_3 + \tilde{\theta}_3 + \langle\tilde{\varepsilon}\rangle) + \left(\frac{\partial P}{\partial \varepsilon}\right)_n (\varepsilon_2 + \theta_2) + \left(\frac{\partial P}{\partial n}\right)_\varepsilon (\nu_2 + \gamma_2),$

$E = -\left(\frac{4}{3}\eta - \pi_2\right)(\tilde{\theta}_1 + \tilde{\gamma}_3) + w\{\tilde{\gamma}, \tilde{\pi}\} + n\{\tilde{\pi}, \tilde{\theta}\} + (\varepsilon_2 + \theta_2)\langle\tilde{\gamma}\rangle - (\nu_2 + \gamma_2)\langle\tilde{\theta}\rangle,$

$F = w\langle\tilde{\gamma}\rangle - n\langle\tilde{\theta}\rangle.$

Conclusions & Outlook

NA, E. Speranza, J. Noronha, arXiv:2308.02928

- New comprehensive formulation of first-order chiral hydrodynamics
- First theory of viscous chiral hydrodynamics proven to be causal and stable
- Suitable for numerical simulation of chiral effects throughout the hydro evolution
- Future work: chirality in consistent second-order theories, rotating equilibria, ...