

Limiting attractors in heavy-ion collisions — the interplay between bottom-up and hydrodynamical attractors

based on work in preparation with K. Boguslavski, A. Kurkela, T. Lappi, J. Peuron
see also arXiv:2303.12520 and arXiv:2303.12595

Florian Lindenbauer

TU Wien

Sept 5, 2023, Quark Matter 2023



TECHNISCHE
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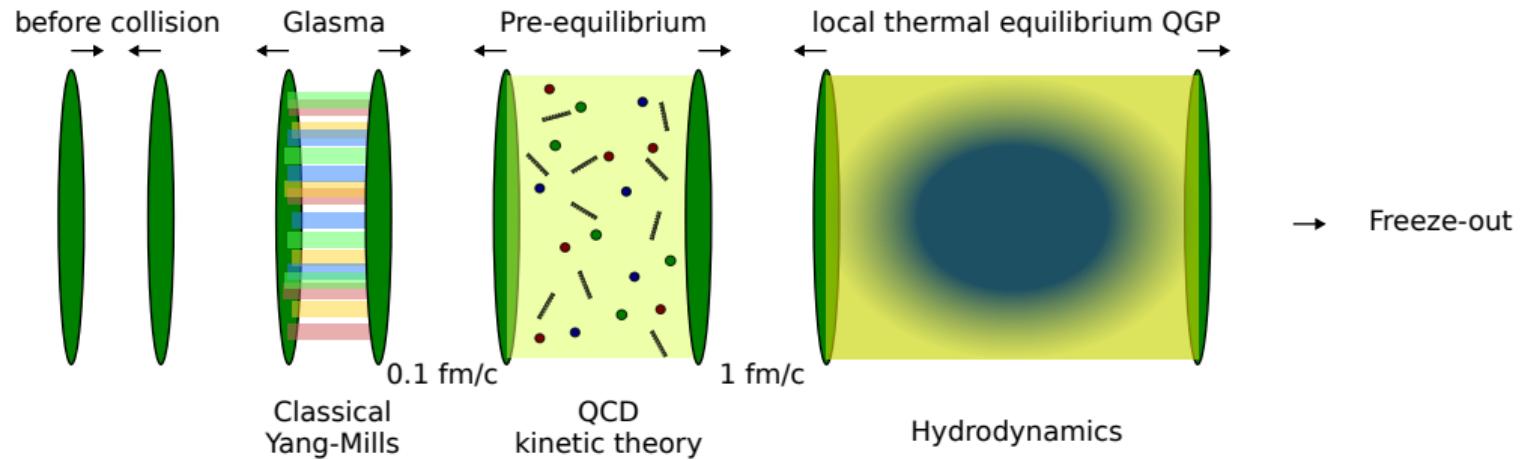
FWF Österreichischer
Wissenschaftsfonds

$\int dk \Pi$
Doktoratskolleg
Particles and Interactions

Outline

- 1 Introduction
- 2 Kinetic theory and bottom-up thermalization
- 3 Hydrodynamics and hydrodynamic attractors
- 4 Obtaining transport coefficients during bottom-up thermalization
- 5 Conclusion

Time-evolution of the quark-gluon plasma



Consider thermalization stage (kinetic theory)

Effective kinetic theory description of the QGP

■ Microscopic description:

Quasi-particles with **distribution function** $f(t, \mathbf{p})$

■ Time evolution described by **Boltzmann equation** at leading-order¹

$$(\partial_t + \mathbf{v} \cdot \nabla) f = \underbrace{\left| \begin{array}{c} \text{---} \\ \text{---} \\ | \quad | \\ \text{---} \end{array} \right|^2 + \left| \begin{array}{c} \text{---} \\ \text{---} \\ | \quad | \\ \text{---} \end{array} \right|^2}_{\text{Collision term}}$$

$$-\frac{\partial f(\mathbf{p}, \tau)}{\partial \tau} + \frac{p_z}{\tau} \frac{\partial f(\mathbf{p}, \tau)}{\partial p_z} = \mathcal{C}^{2 \leftrightarrow 2}[f] + \mathcal{C}^{'1 \leftrightarrow 2'}[f]$$

¹[JHEP 01 (2003) [Arnold, Moore, Yaffe], Int.J.Mod.Phys.E 16 (2007) [Arnold]]

Effective kinetic theory description of the QGP

- **Microscopic description:**

Quasi-particles with **distribution function** $f(t, \mathbf{p})$

- Time evolution described by **Boltzmann equation** at leading-order¹

$$(\partial_t + \mathbf{v} \cdot \nabla) f = \underbrace{\left| \begin{array}{c} \text{Diagram of two gluons interacting via a central collision} \\ | \\ \text{Diagram of a gluon interacting with a quark-gluon plasma (QGP) medium} \end{array} \right|^2}_{\text{Collision term}} + \dots$$

- Pure gluons, cylindrical symmetry around beam axis \hat{z} ,
Bjorken expansion, $x - y$ homogeneous

¹[JHEP 01 (2003) [Arnold, Moore, Yaffe], Int.J.Mod.Phys.E 16 (2007) [Arnold]]

Observables in EKT

- Fundamental quantity: Distribution function $f(\mathbf{k})$
- Energy-Momentum tensor:

$$T^{\mu\nu} = \nu_g \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{p^\mu p^\nu}{\rho} f(\mathbf{p})$$

- Longitudinal pressure $P_L = T_{zz}$
- Transverse pressure $P_T = T_{xx} = T_{yy}$
- Occupancy of the hard sector

$$\frac{\langle pf \rangle}{\langle p \rangle} = \frac{\int d^3\mathbf{p} p f(\mathbf{p})^2}{\int d^3\mathbf{p} p f(\mathbf{p})}$$

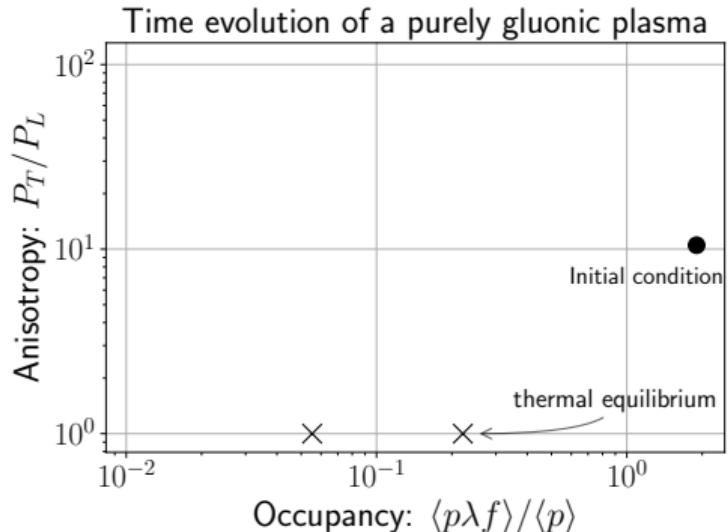
Bottom-up thermalization in heavy-ion collisions

- Initial condition², $\lambda = g^2 N_C = 4\pi N_C \alpha_s$

$$f_0(\mathbf{p}; \xi, \lambda) \sim \frac{1}{\lambda} \times \text{squeezed exponential}$$

$\xi \sim$ anisotropy,

$Q_s \sim$ saturation scale



²[Phys.Rev.Lett. 115 (2015) [Kurkela, Zhu]]

³[Phys.Lett.B 502 (2001) [Baier, Mueller, Schiff, Son]]

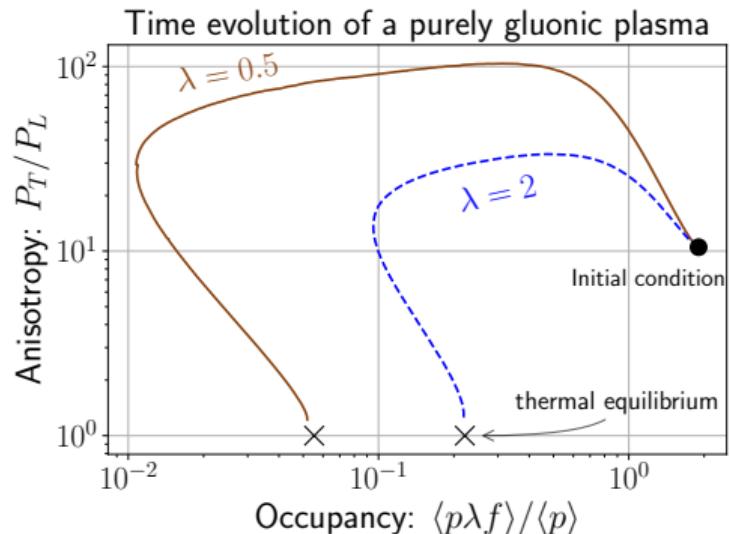
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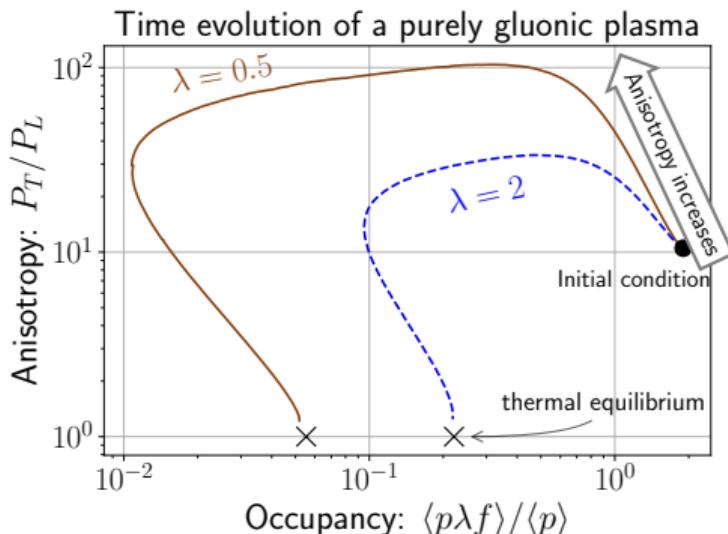
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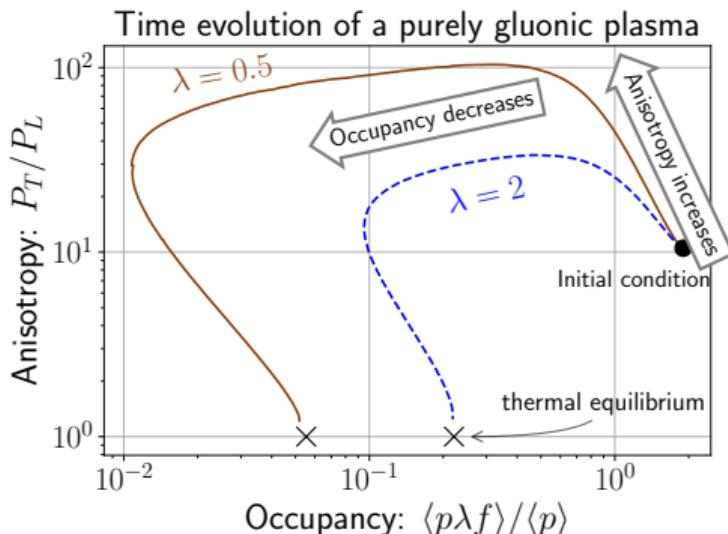
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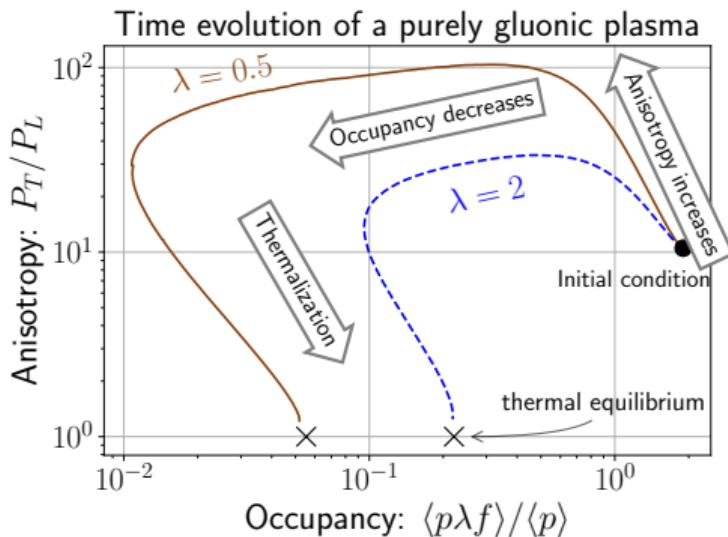
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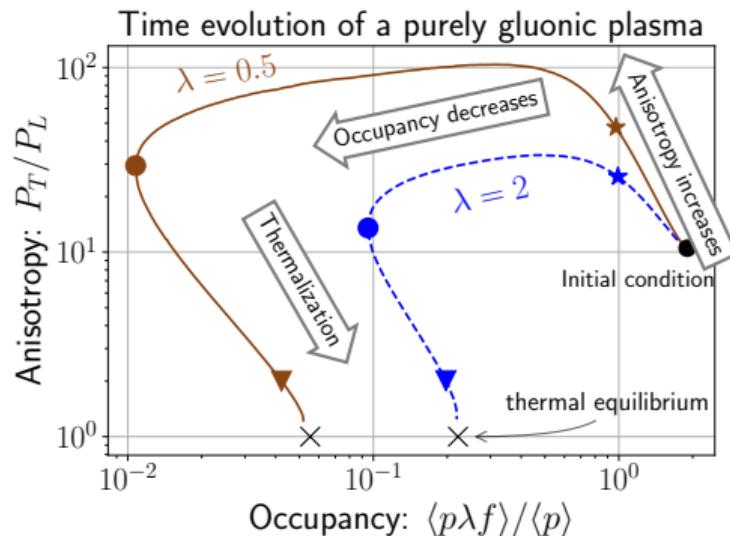
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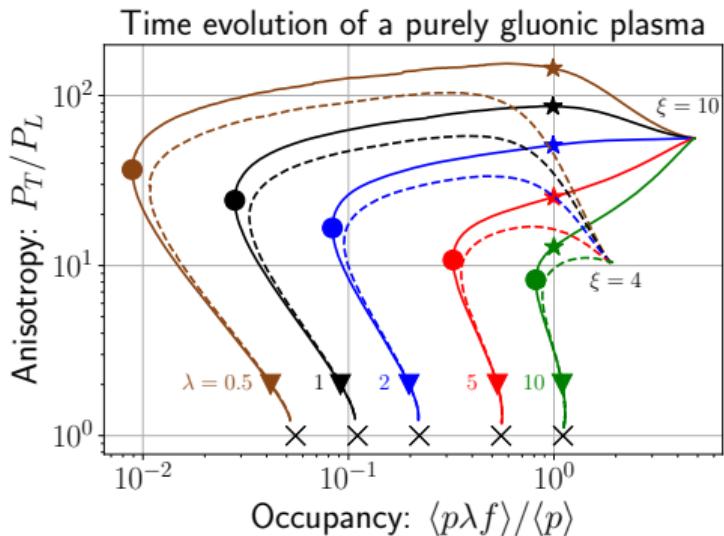
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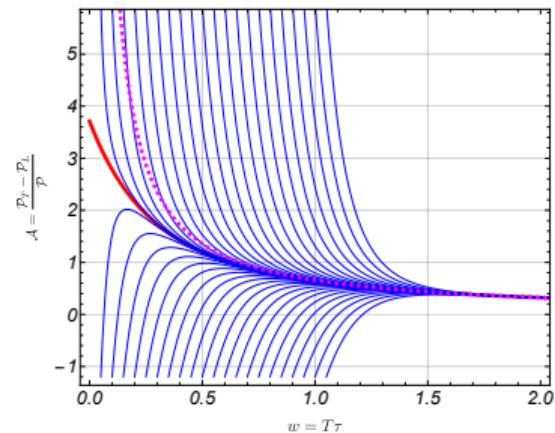
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Hydrodynamic attractors

- Hydrodynamics: **macroscopic description**
- Attractor found by solving hydro equations
- Complexity of initial state quickly reduced, rapid loss of information

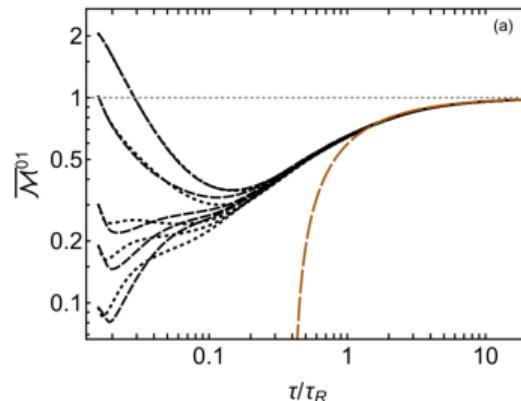


[Phys.Rev.Lett. 115 (2015) [Heller, Spalinski]]

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- Justifies applicability of fluid dynamics (even in early stages)



[Phys.Rev.Lett. 125 (2020) [Almaalol, Kurkela,

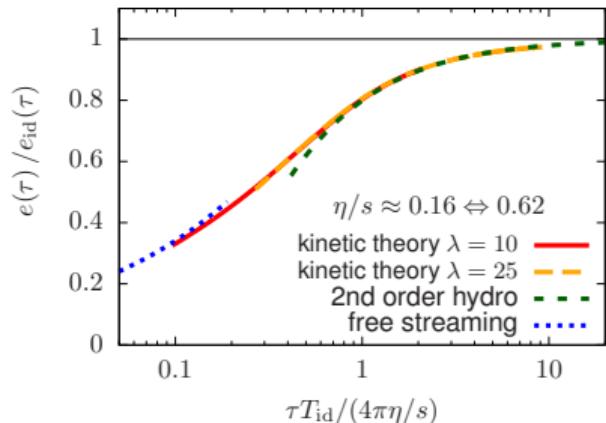
Strickland]]

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Hydrodynamic attractors

- Hydrodynamics: **macroscopic description**
- Attractor found by solving hydro equations
- Complexity of initial state quickly reduced, rapid loss of information
- Justifies applicability of fluid dynamics (even in early stages)
- Different couplings $\lambda \rightarrow$ same curve (when rescaled with η/s)
- Used to **connect initial state to hydro**

[Eur.Phys.J.C 82 (2022) [Soloviev], Prog.Part.Nucl.Phys. 132 (2023) [Jankowski, Spaliński]]



[Kurkela, Mazeliauskas, Paquet, Schlichting, Teaney,

Phys.Rev.Lett. 122 (2019)]

Relaxation time τ_R

- Conformal (first order) hydro:

$$\frac{P_L}{P_T} = 1 - 8 \underbrace{\frac{\eta/s}{\tau T}}_{\sim \tau_R/\tau}$$

[[Romatschke, Romatschke] (2019)]

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- Conformal: $P_L + 2P_T = \epsilon \rightarrow$ similar relation for $P_T/\epsilon, P_L/\epsilon, \dots$

[[Romatschke, Romatschke] (2019)]

Bottom-up vs. hydrodynamic attractor

- **Bottom-up** expects thermalization around $\tau_{\text{BMSS}} = \alpha_s^{-13/5} / Q_s$
- **Hydrodynamics** expects thermalization around $\tau_R = \frac{4\pi\eta/s}{T}$

How to reconcile these different time scales?

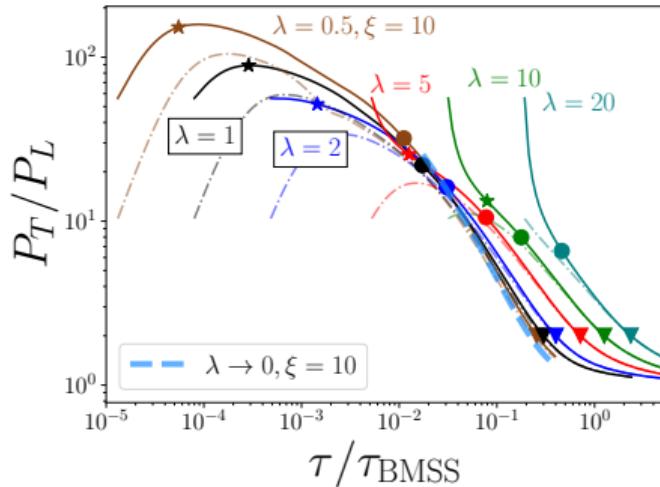
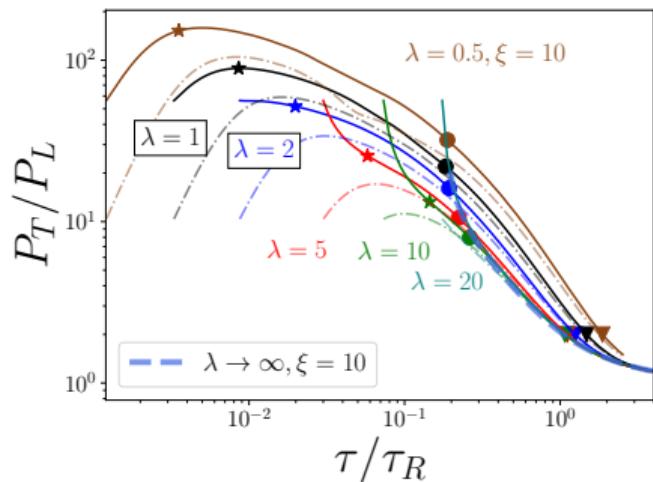
Method: Kinetic theory simulations for

- **different couplings:** $0.5 \leq \lambda \leq 20$
- **initial conditions:** vary anisotropy parameter ξ

Results: Pressure ratio

$$\tau_R = \frac{4\pi\eta/s}{T}, \tau_{\text{BMSS}} = \alpha_s^{-13/5} / Q_s$$

- Kinetic theory simulations for different couplings $0.5 \leq \lambda \leq 20$ and initial conditions.

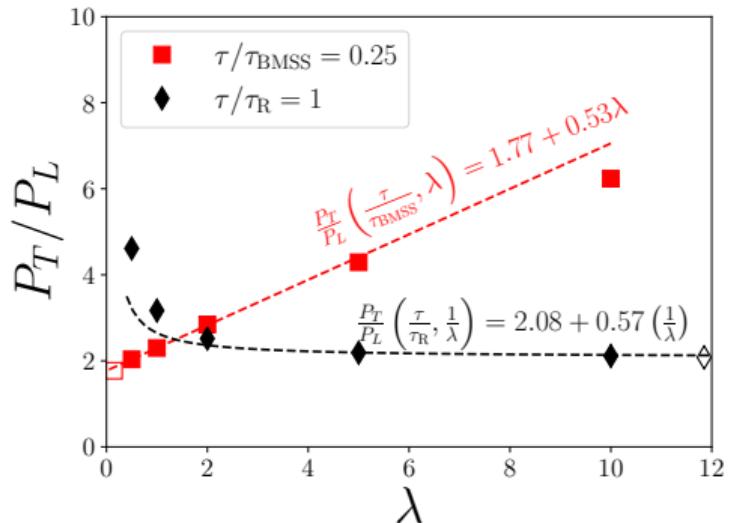


- Attractor for each λ (insensitive to IC)
- Curves approach limiting attractors after •

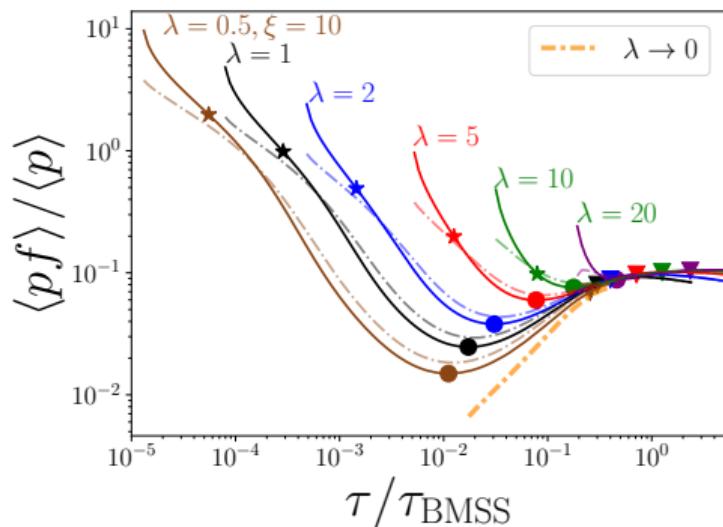
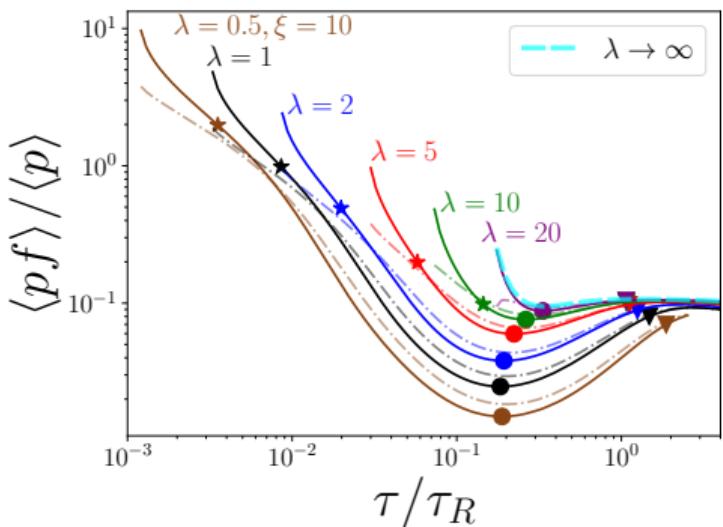
Extrapolation to limiting attractors

$$\tau_R = \frac{4\pi\eta/s}{T}$$
$$\tau_{\text{BMSS}} = \alpha_s^{-13/5} / Q_s$$

- Obtain limiting attractors by extrapolating at fixed τ/τ_R or τ/τ_{BMSS}
- **Bottom-up attractor:** Linear extrapolation to $\lambda \rightarrow 0$
- **Hydro attractor:** Linear extrapolation to $1/\lambda \rightarrow 0$



Occupancy



- Approach to weak coupling attractor even at moderate couplings

Transport coefficients

\hat{q} : F. Lindenbauer (poster session, 2303.12595)
 κ : J. Peuron (Tue 9:50 am, 2303.12520)

■ Jet quenching parameter:

Transverse momentum broadening of jets is quantified by $\hat{q} = \hat{q}^{yy} + \hat{q}^{zz}$,

$$\hat{q}^{ii} = \int d^2 q_\perp (q_\perp^i)^2 \frac{d\Gamma^{\text{el}}}{d^2 q_\perp} = \int d\Gamma (q^i)^2 |\mathcal{M}|^2 f(k)(1 + f(k'))$$

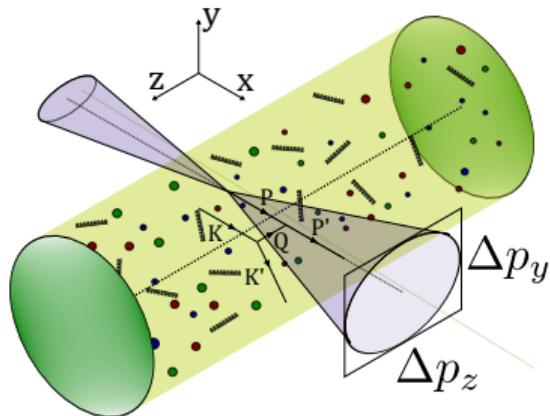


Figure: Schematic overview of jet momentum broadening

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■ Heavy-quark diffusion coefficient κ

$$\kappa^i = \int d\Gamma_\kappa (q^i)^2 |\mathcal{M}_\kappa|^2 f(k)(1 + f(k'))$$

measures momentum transfer to (infinitely) heavy quark

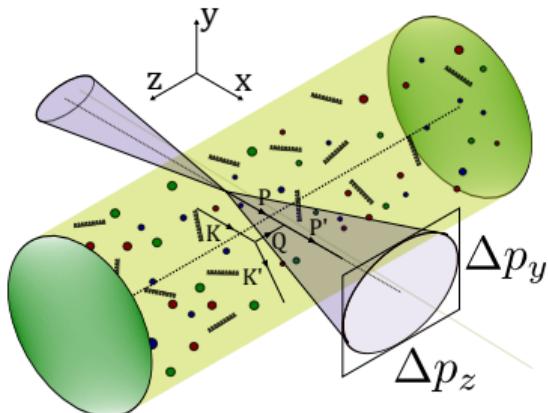
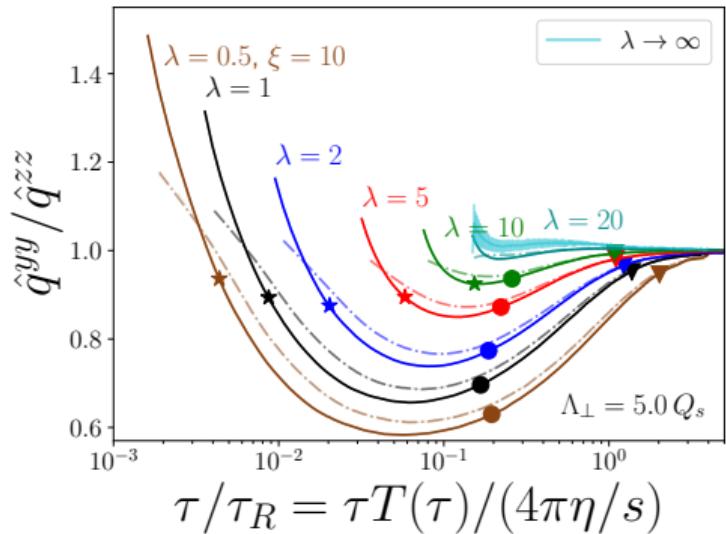


Figure: Schematic overview of jet momentum broadening

\hat{q} and the limiting attractors

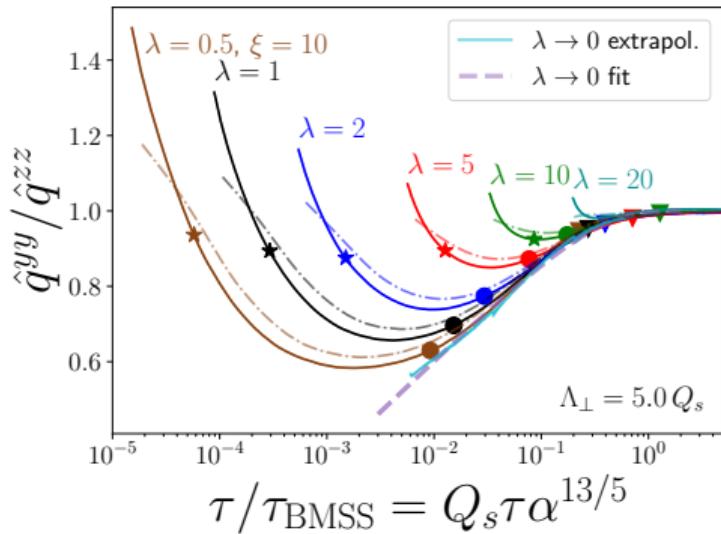
F. Lindenbauer (poster session)



$$\tau/\tau_R = \tau T(\tau)/(4\pi\eta/s)$$

- Approach to weak coupling attractor even at moderate couplings
- Fit for bottom-up attractor:

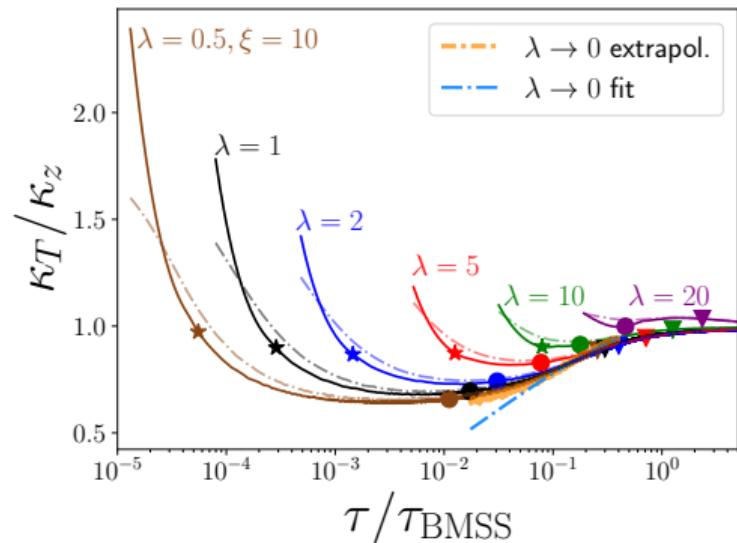
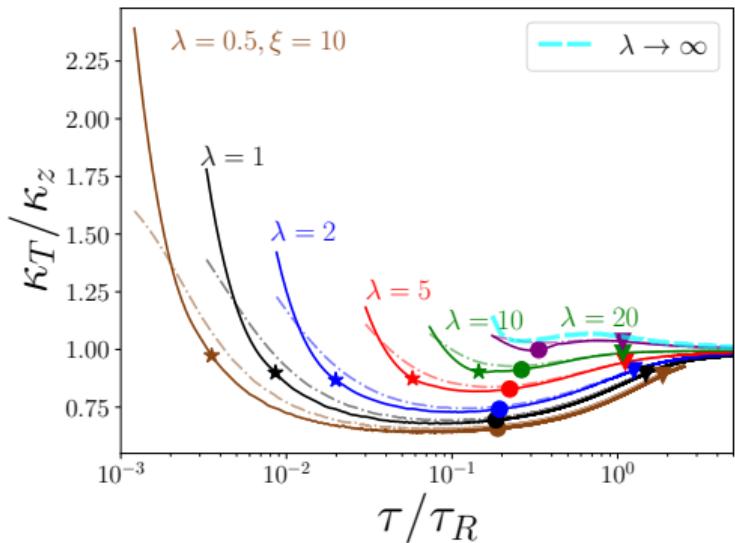
$$\hat{q}^{yy}/\hat{q}^{zz}(\tau) \approx 1 + c_1 \ln \left(1 - e^{-c_2 \tau/\tau_{\text{BMSS}}} \right) \text{ with } c_1 = 0.12, c_2 = 3.45.$$



$$\tau/\tau_{\text{BMSS}} = Q_s \tau \alpha^{13/5}$$

κ and the limiting attractors

J. Peuron (Tue 9:50 am)



- Similar to \hat{q} : Approach to weak coupling attractor even at moderate λ
- Fit for bottom-up attractor:

$$\kappa_T/\kappa_z(\tau) \approx 1 + c_1 \ln(1 - e^{-c_2 \tau/\tau_{\text{BMSS}}}) \quad \text{with } c_1 = 0.16, c_2 = 2.9.$$

Conclusions

- Performed kinetic theory simulations of bottom-up thermalization
- Limiting attractors emerge for P_T/P_L , occupancy, \hat{q} , κ
 - ◊ Strong coupling: $\lambda \rightarrow \infty$ Hydrodynamic attractor
 - ◊ Weak coupling: $\lambda \rightarrow 0$ Weak-coupling bottom-up attractor
- $\hat{q}^{yy}/\hat{q}^{zz}$, κ_T/κ_z , $\langle pf \rangle/\langle p \rangle$ even at moderate coupling follow **universal curves** in τ/τ_{BMSS}
 - ◊ **Previous conjecture** that hydro attractor governs late-time dynamics is **incomplete**
- ⇒ **Phenomenological** consequences for late-time observables in heavy-ion collisions (maybe important for small and medium-sized systems)?

Thank you very much for your attention!

Hydrodynamic description of the QGP

■ Macroscopic description

■ Expansion around local equilibrium

$$\langle T^{\mu\nu} \rangle = T_{(0)}^{\mu\nu} + \underbrace{T_{(1)}^{\mu\nu} + T_{(2)}^{\mu\nu} + \dots}_{\text{shear and bulk stress } \pi^{\mu\nu}, \Pi}$$

- Macroscopic properties (pressure, temperature, energy density, ...)
- Works **close to equilibrium** (small gradients)
- Approach to equilibrium governed by **transport coefficients** η, ζ, \dots

How to obtain the “temperature”?

- Out of equilibrium, no unambiguous definition of temperature
- Use **Landau matching**, i.e. at every timestep $\varepsilon^{\text{eq}}(T_\varepsilon) = \varepsilon^{\text{simulation}}$

$$T_\varepsilon(\tau) = \left(\frac{30\varepsilon(\tau)}{\pi^2 \times \#\text{d.o.f.}} \right)^{1/4}$$

This is the temperature of an equilibrium system with the same energy density.

- For heavy quark diffusion coefficient κ , this ϵ -matching works better than matching other quantities⁴

⁴[2303.12520 [Boguslavski, Kurkela, Lappi, FL, Peuron]]

Jet quenching parameter in kinetic theory

- Provided we know $f(\mathbf{k})$: Outgoing plasma particle

$$\hat{q}^{ij} = \int_{\substack{q_\perp < \Lambda \\ p \rightarrow \infty}} d\Gamma_{PS} q^i q^j |\mathcal{M}|^2 f(\mathbf{k}) (1 + f(\mathbf{k}'))$$

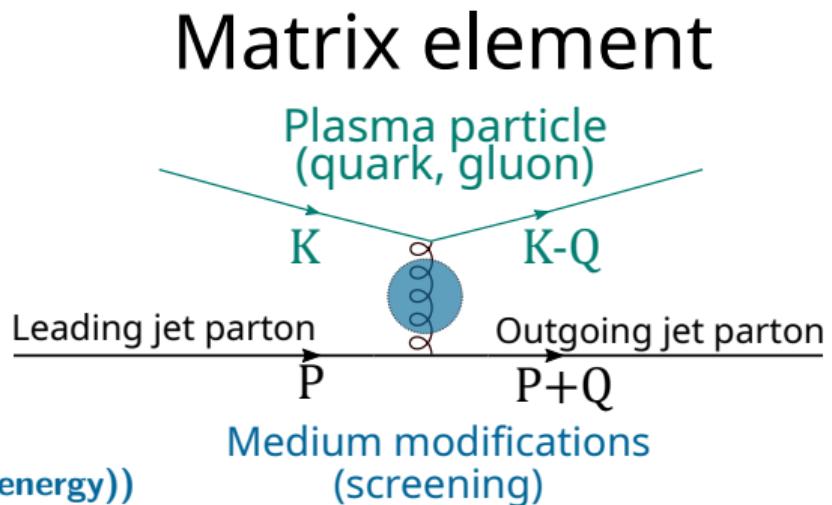
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Plasma particle (quark, gluon)

Incoming plasma particles with momentum \mathbf{k}

Matrix element with medium corrections (self-energy))

appropriate phase-space measure



Jet quenching parameter \hat{q}

- During initial collision, highly energetic particles are created
 - ◊ Move through Quark-Gluon plasma
 - ◊ Interact with it
 - ◊ Split into many particles
 - ◊ Measured as **Jets** in the detector

- \hat{q} is defined via

$$\hat{q} = \frac{d\langle p_\perp^2 \rangle}{dL} = \frac{d\langle p_\perp^2 \rangle}{dt} = \int d^2 q_\perp q_\perp^2 \frac{d\Gamma^{\text{el}}}{d^2 q_\perp}$$

- Quantifies **momentum broadening**
- \hat{z} : Beam direction, \hat{x} : jet direction

