Quarkonium transport in weakly and strongly coupled plasmas

30th International Conference on Ultra-relativistic Nucleus-Nucleus Collisions, Houston TX, United States of America
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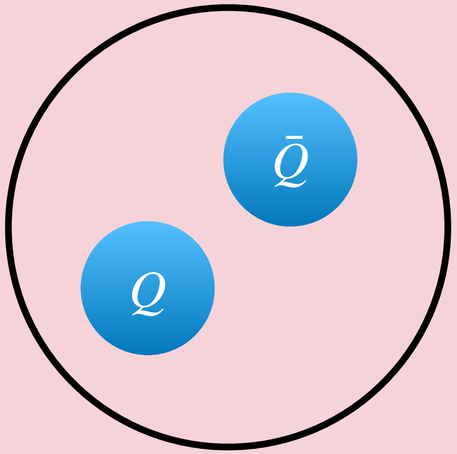
Bruno Scheihing-Hitschfeld (MIT)
based on 2107.03945, 2205.04477, 2304.03298, 2306.13127, 2309.XXXXX

Quarkonium in medium

$M \gg Mv \gg Mv^2$

M: heavy quark mass

v: typical relative speed



color singlet;
“bound” state

color octet;
“unbound” state

Q: c or b quark

$\bar{Q}$: $\bar{c}$ or $\bar{b}$ quark

$T > 0$

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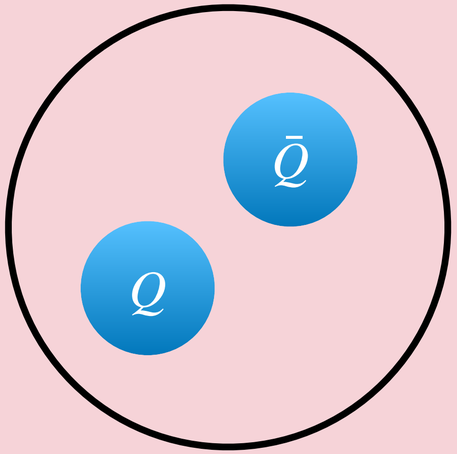

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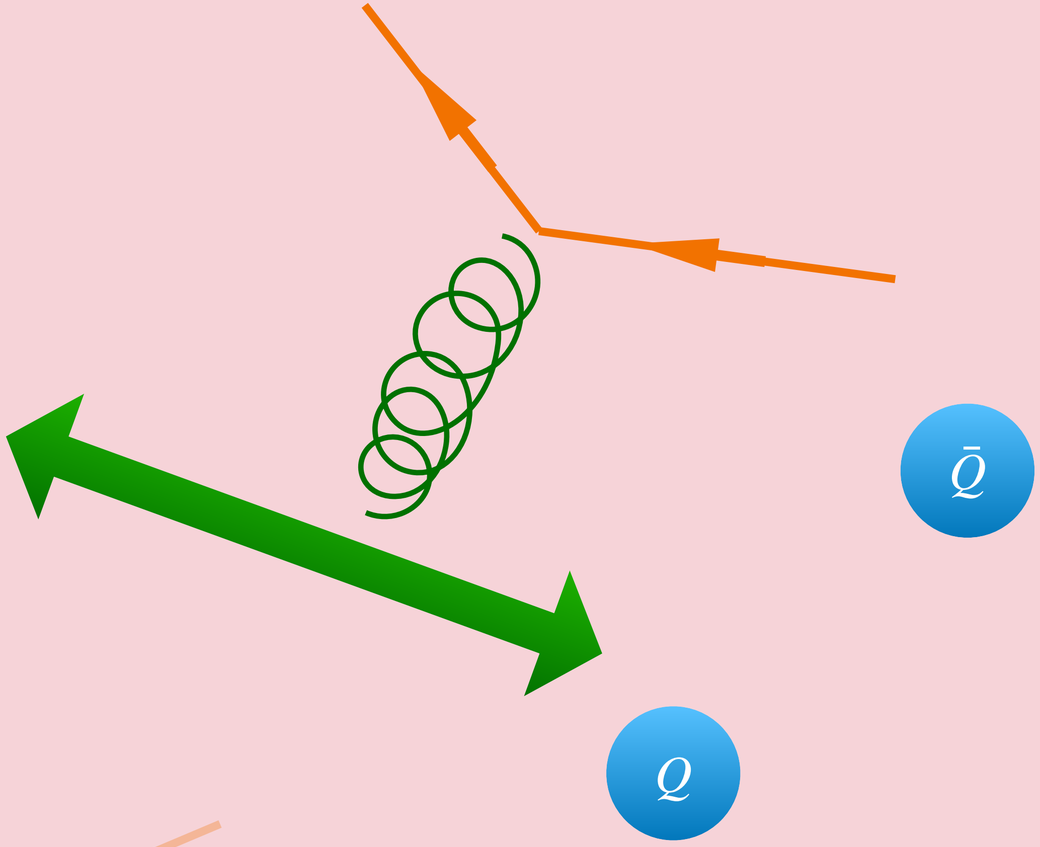

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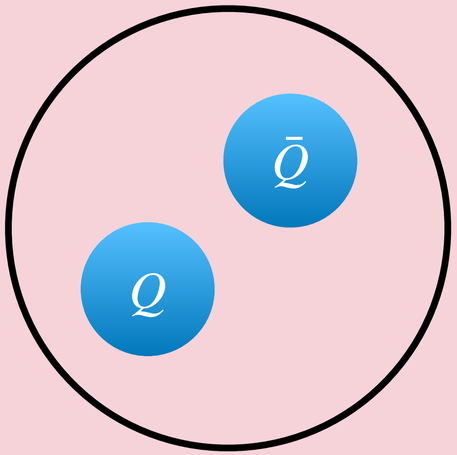

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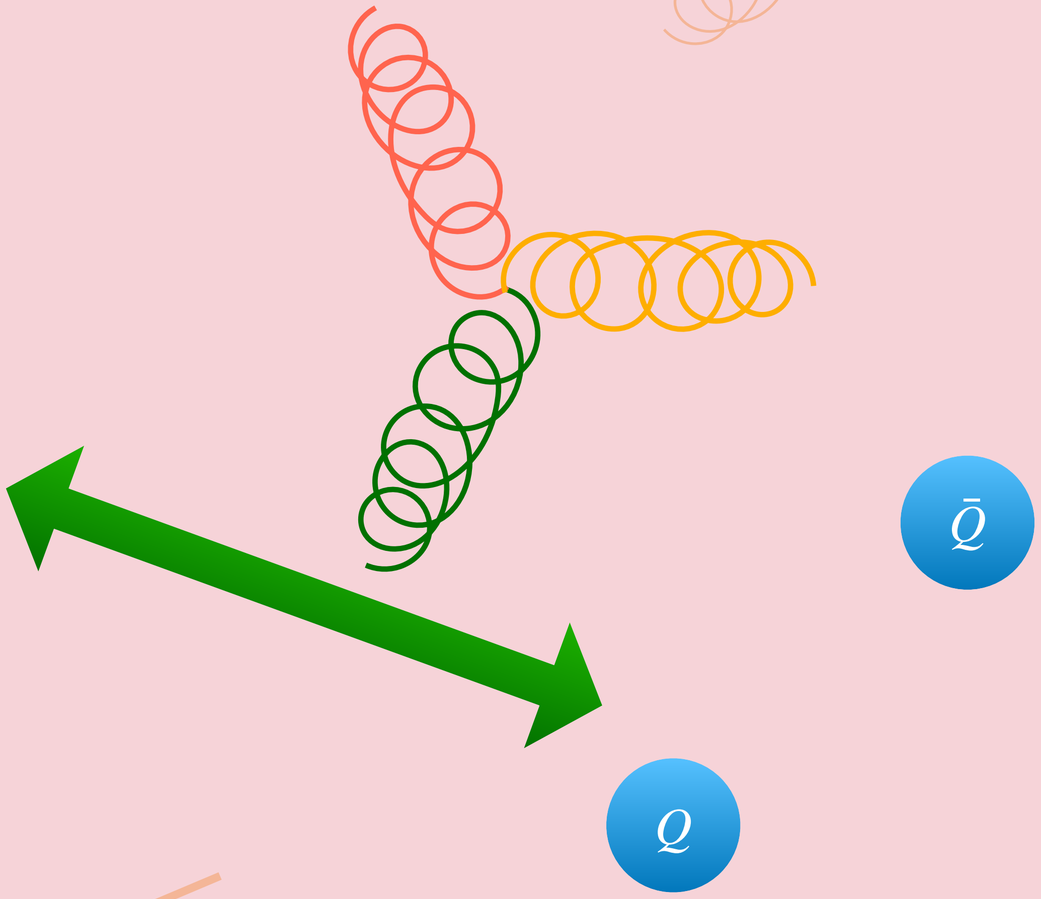

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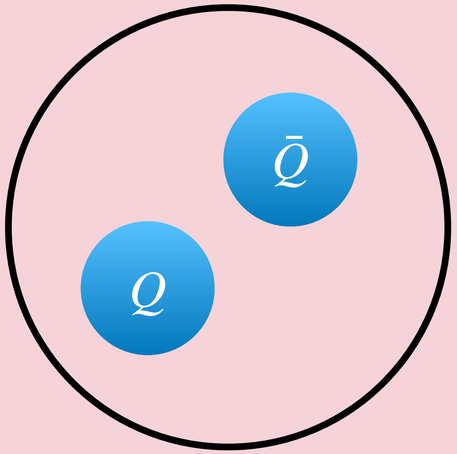

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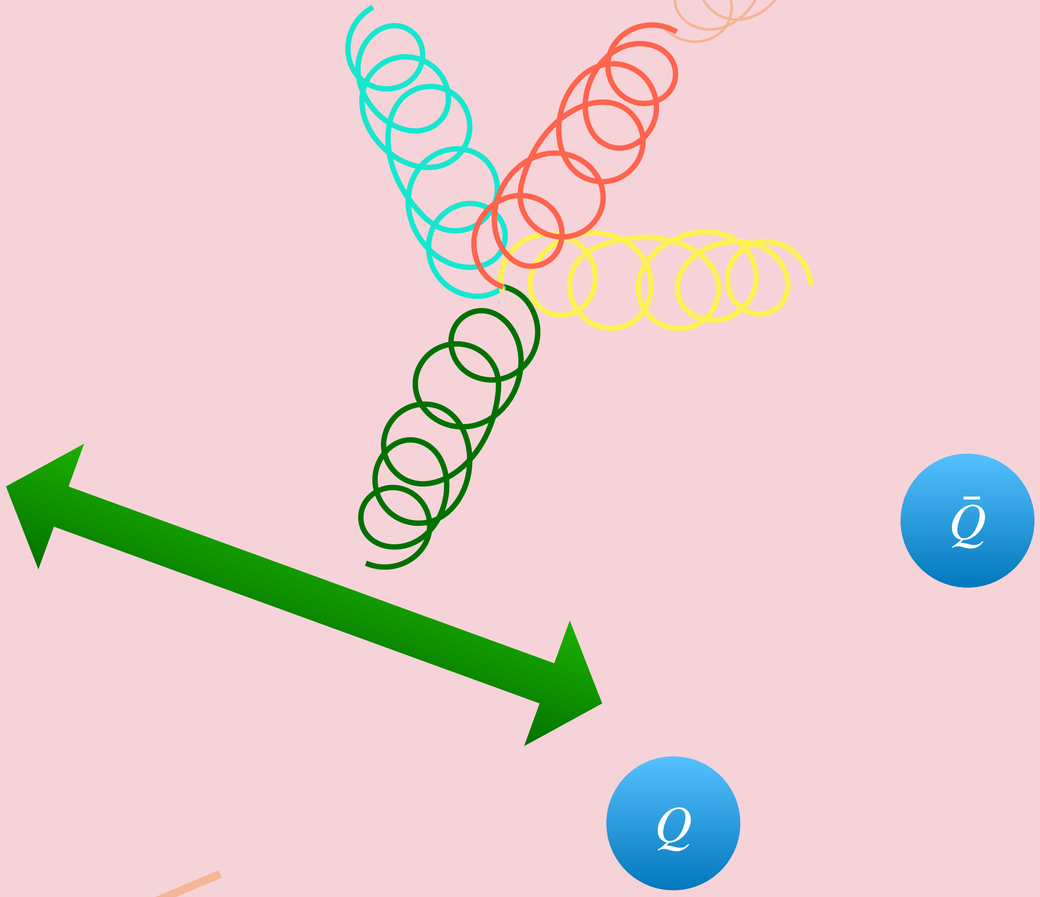

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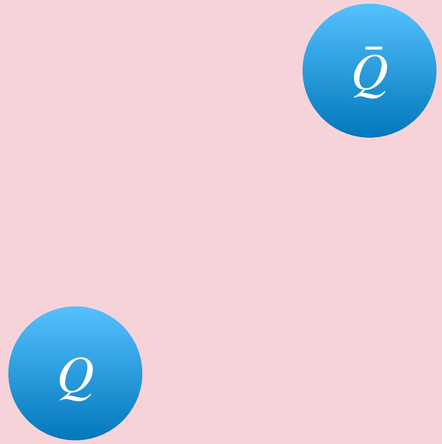
At high T, quarkonium "melts" because the medium screens the interactions between heavy quarks (Matsui & Satz 1986)

$Q\bar{Q}$ melts if $r \sim \dfrac{1}{Mv} \gg \dfrac{1}{T}$



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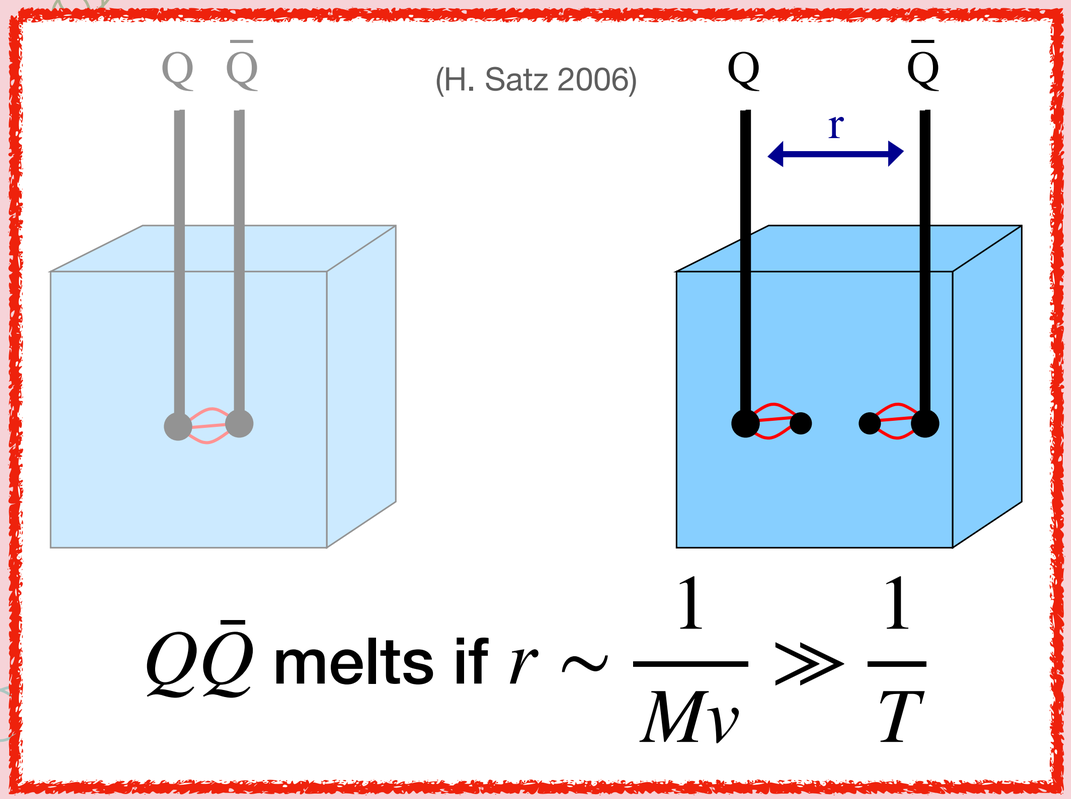

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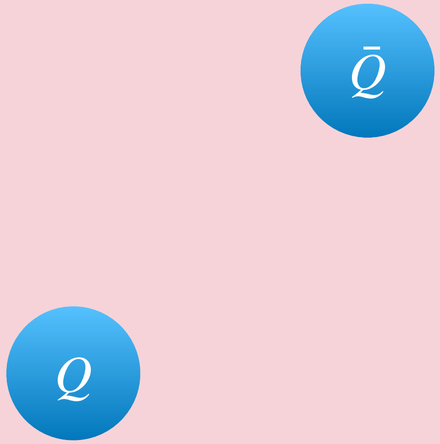

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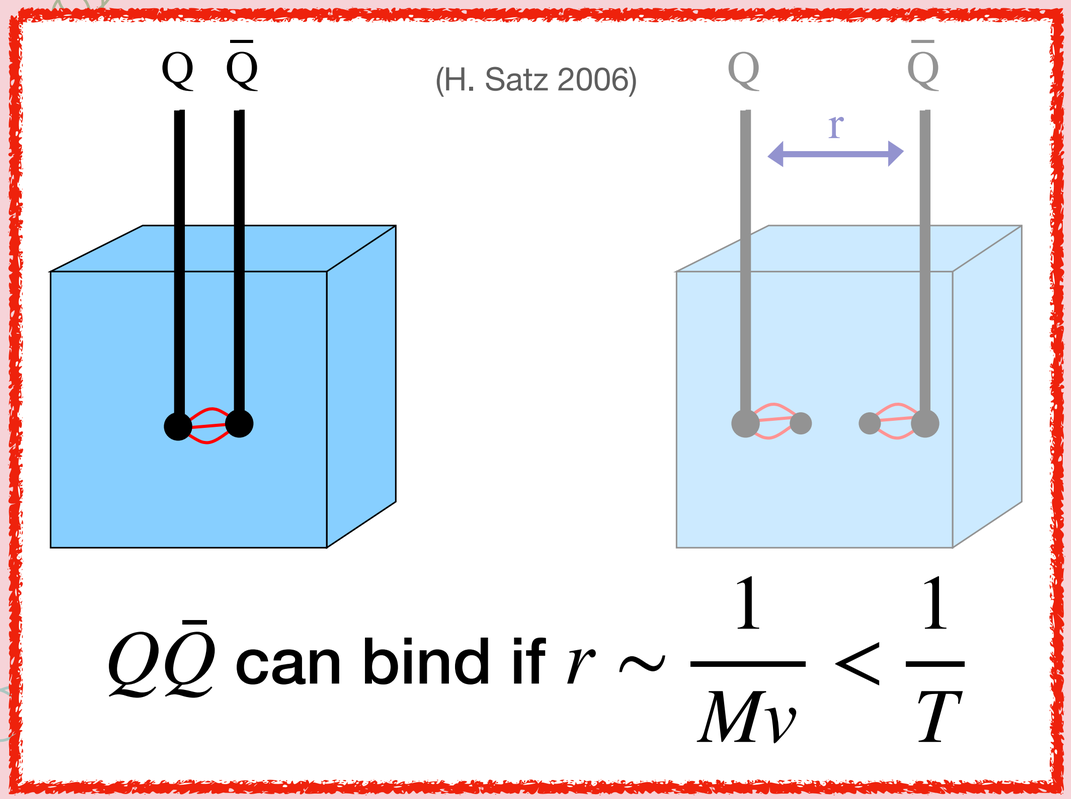

$Q\bar{Q}$ can bind if $r \sim \dfrac{1}{Mv} < \dfrac{1}{T}$

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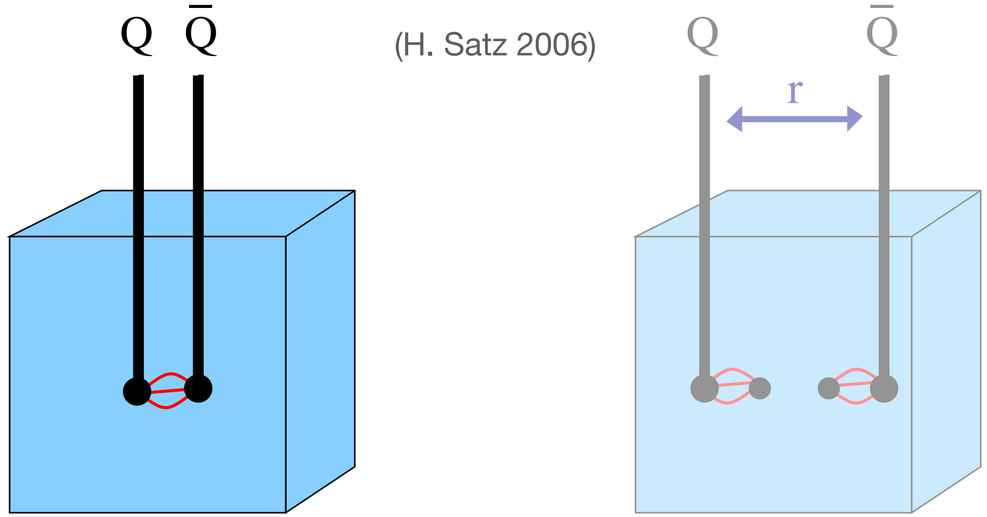

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$\Longrightarrow$ most of quarkonium starts to form when $Mv \gtrsim T$

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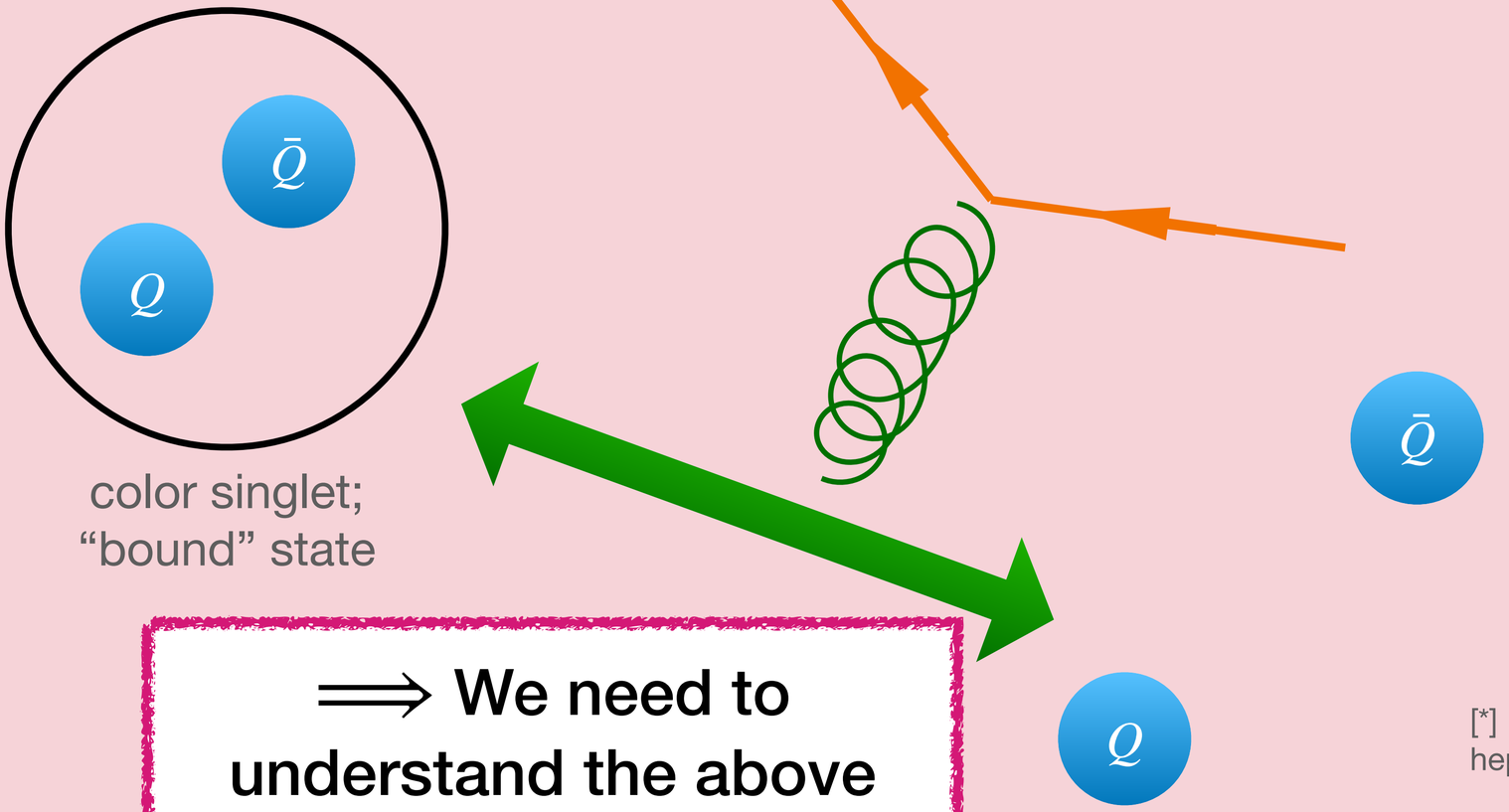

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$\Longrightarrow$ **We need to understand the above dynamics in the hierarchy**

$$Mv \gg T$$

$\Longrightarrow$ **pNRQCD [*]**

[*] N. Brambilla, A. Pineda, J. Soto. A. Vairo
hep-ph/9907240, hep-ph/0410047

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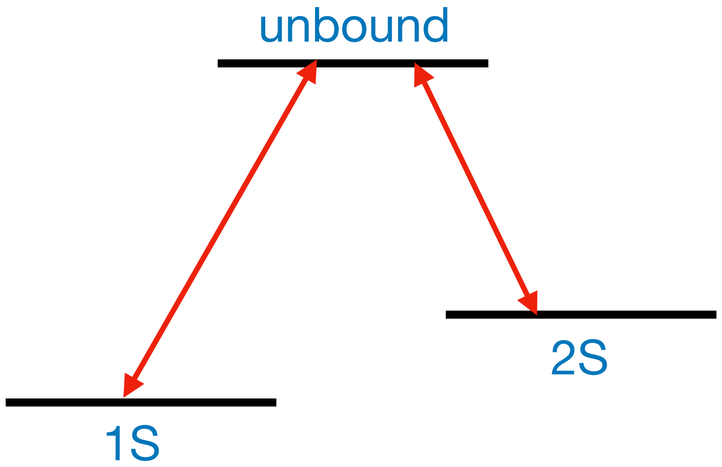
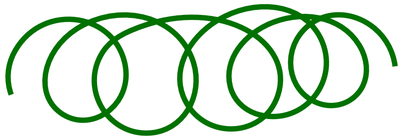
Quarkonium as an open quantum system

Transitions between quarkonium energy levels (the system)



$$\frac{1}{\tau_S} \sim \Delta E_n \sim Mv^2$$

Interaction with the environment



$$\frac{1}{\tau_I} \sim \frac{H_{\rm int}^2}{T} \sim T\frac{T^2}{(Mv)^2}$$

QGP (the environment)

$$\frac{1}{\tau_E} \sim T$$

$$\mathscr{L}_{\rm pNRQCD} = \mathscr{L}_{\rm light\,quarks} + \mathscr{L}_{\rm gluon} + \int d^3r \,{\rm Tr}_{\rm color}\Big[S^\dagger(i\partial_0 - H_s)S + O^\dagger(iD_0 - H_o)O$$

$$+ V_A\big(O^\dagger \mathbf{r}\cdot g\mathbf{E}S + {\rm h.c.}\big) + \frac{V_B}{2}O^\dagger\big\{\mathbf{r}\cdot g\mathbf{E}, O\big\} + \cdots\Big]$$

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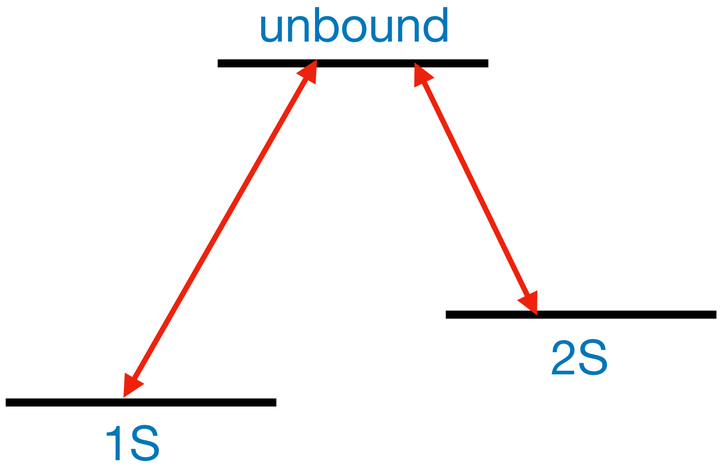

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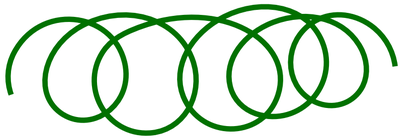

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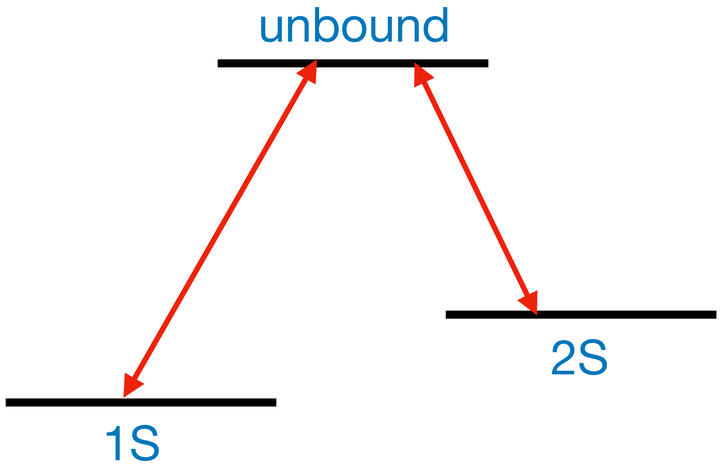

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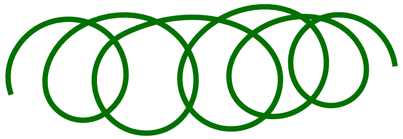

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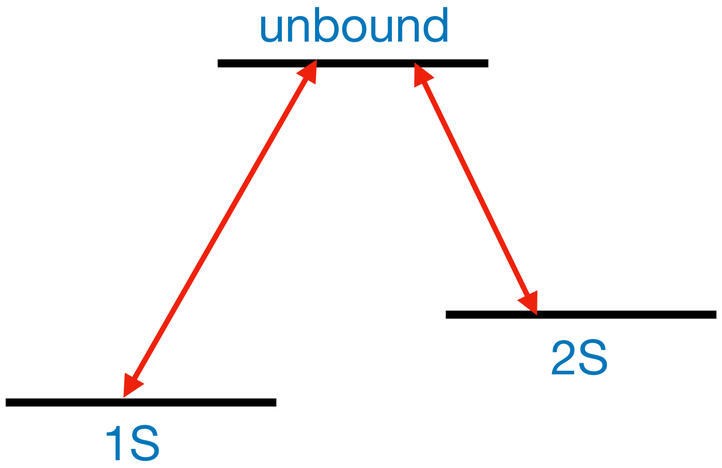

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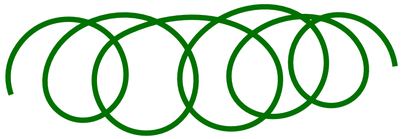

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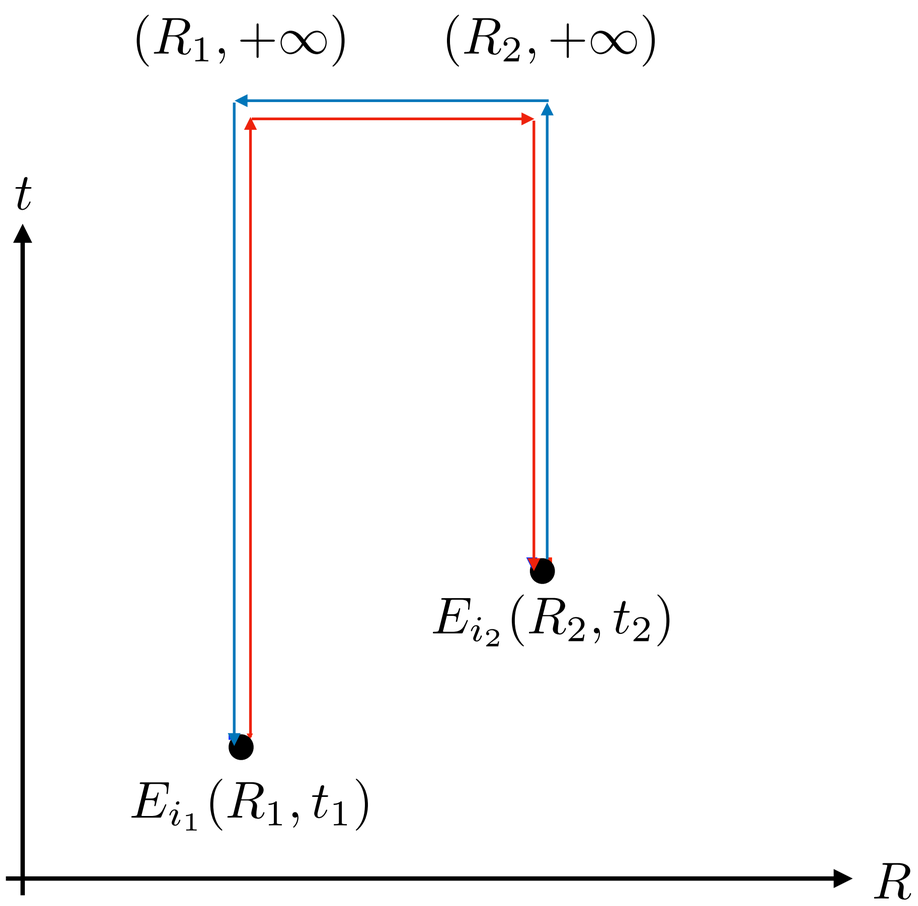
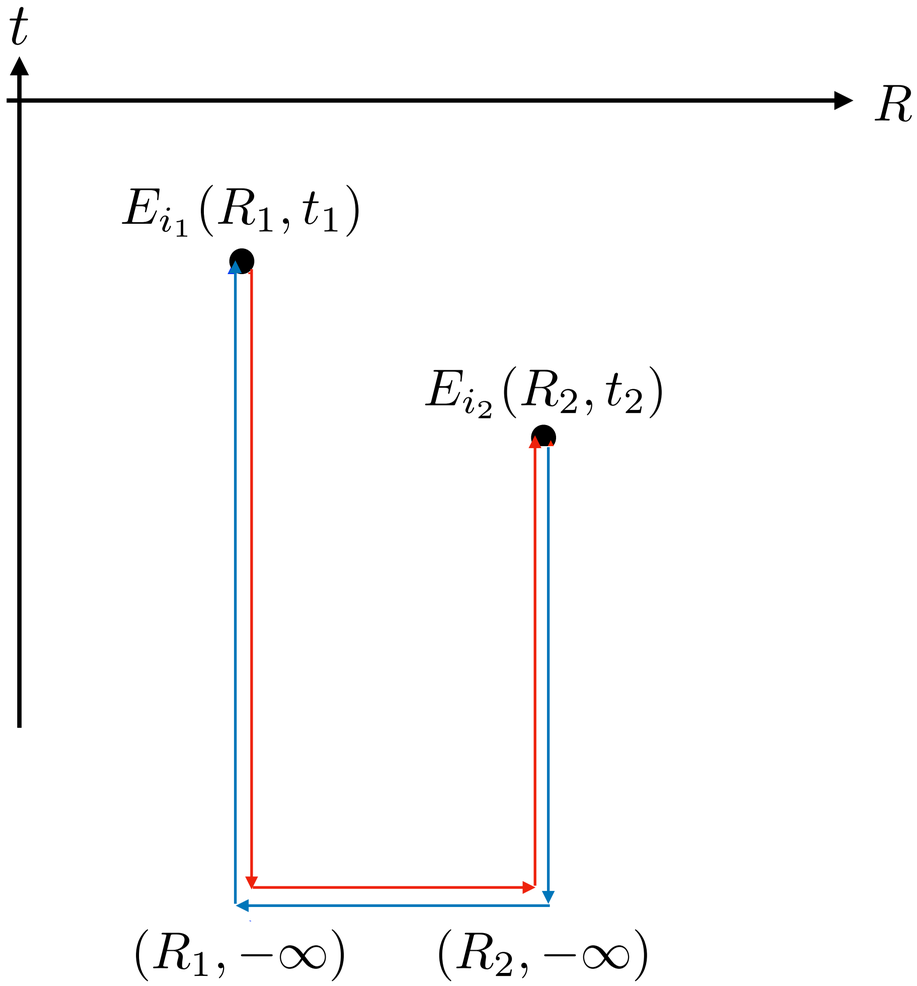
How does the QGP enter the dynamics?

QGP chromoelectric correlators

for quarkonia transport

$$[g_E^{--}]^{>}_{i_2 i_1}(t_2, t_1, \mathbf{R}_2, \mathbf{R}_1) = \left\langle \left(\mathscr{W}_{2'} E_{i_2}(\mathbf{R}_2, t_2)\right)^a \left(E_{i_1}(\mathbf{R}_1, t_1)\mathscr{W}_{1'}\right)^a \right\rangle_T$$





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See also: N. Brambilla et al. hep-ph/1612.07248, hep-ph/1711.04515, hep-ph/2205.10289

X. Yao and T. Mehen, hep-ph/2009.02408
T. Binder, K. Mukaida, B. Scheihing-Hitschfeld, X. Yao, hep-ph/2107.03945

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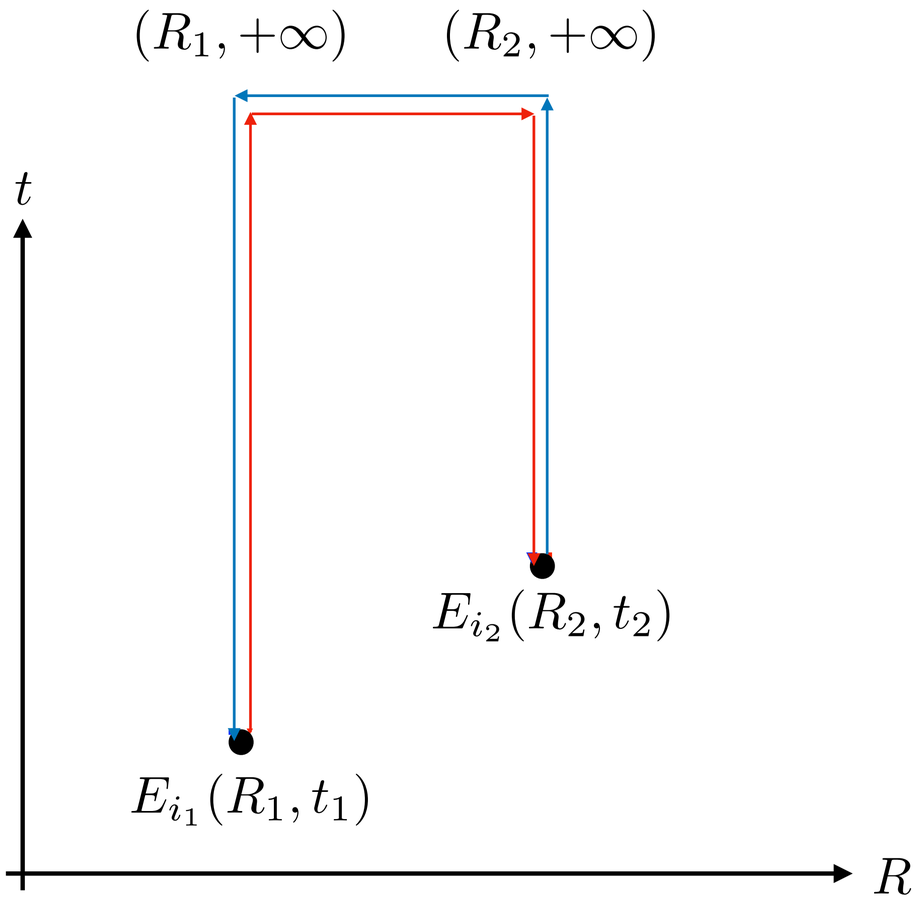

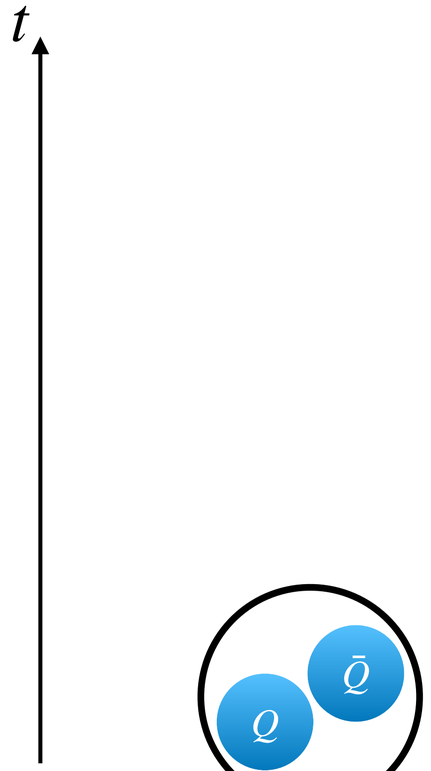

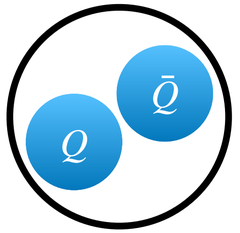

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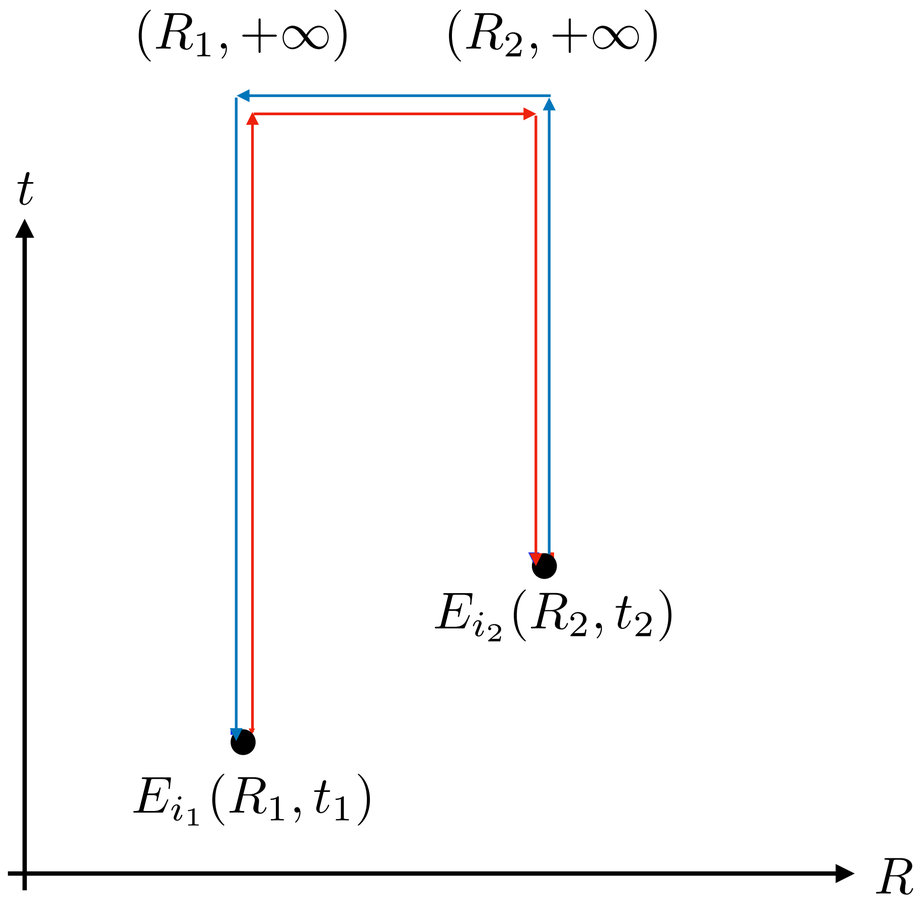

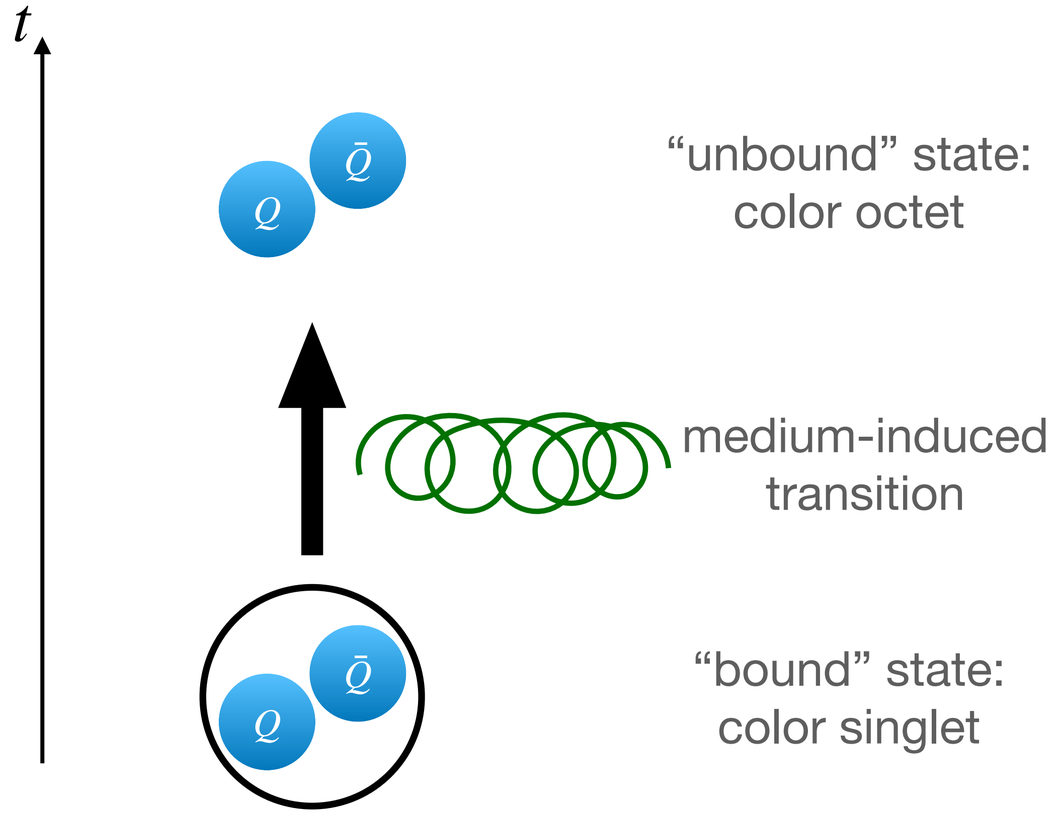

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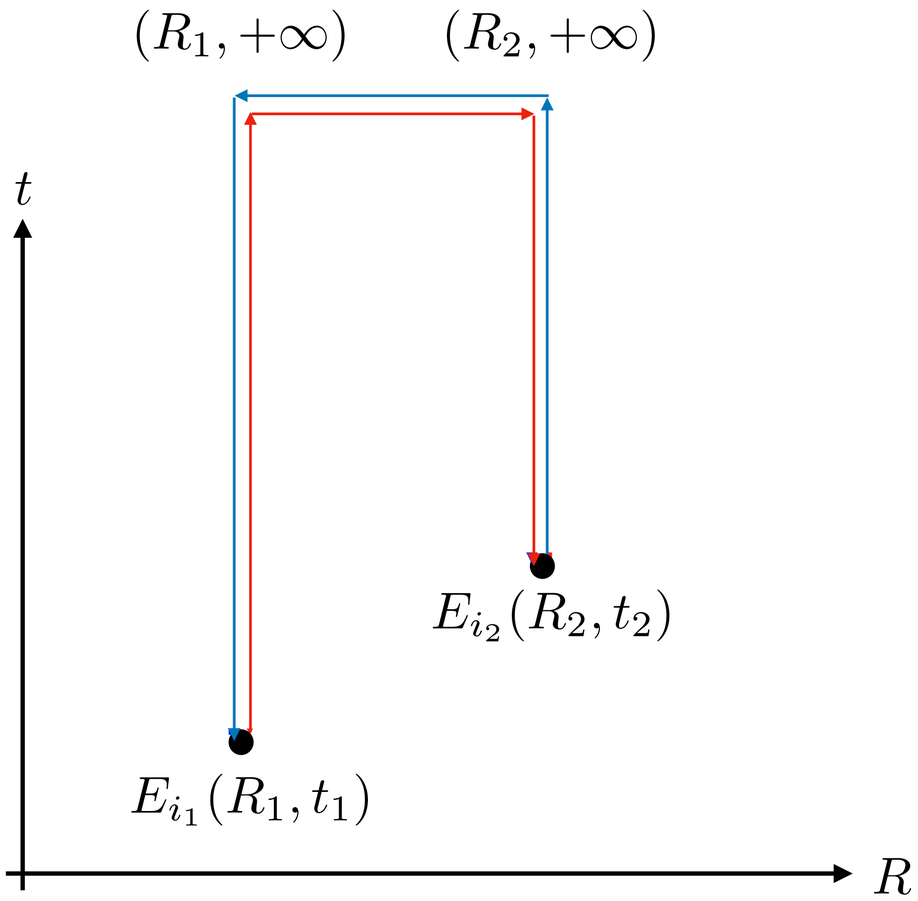

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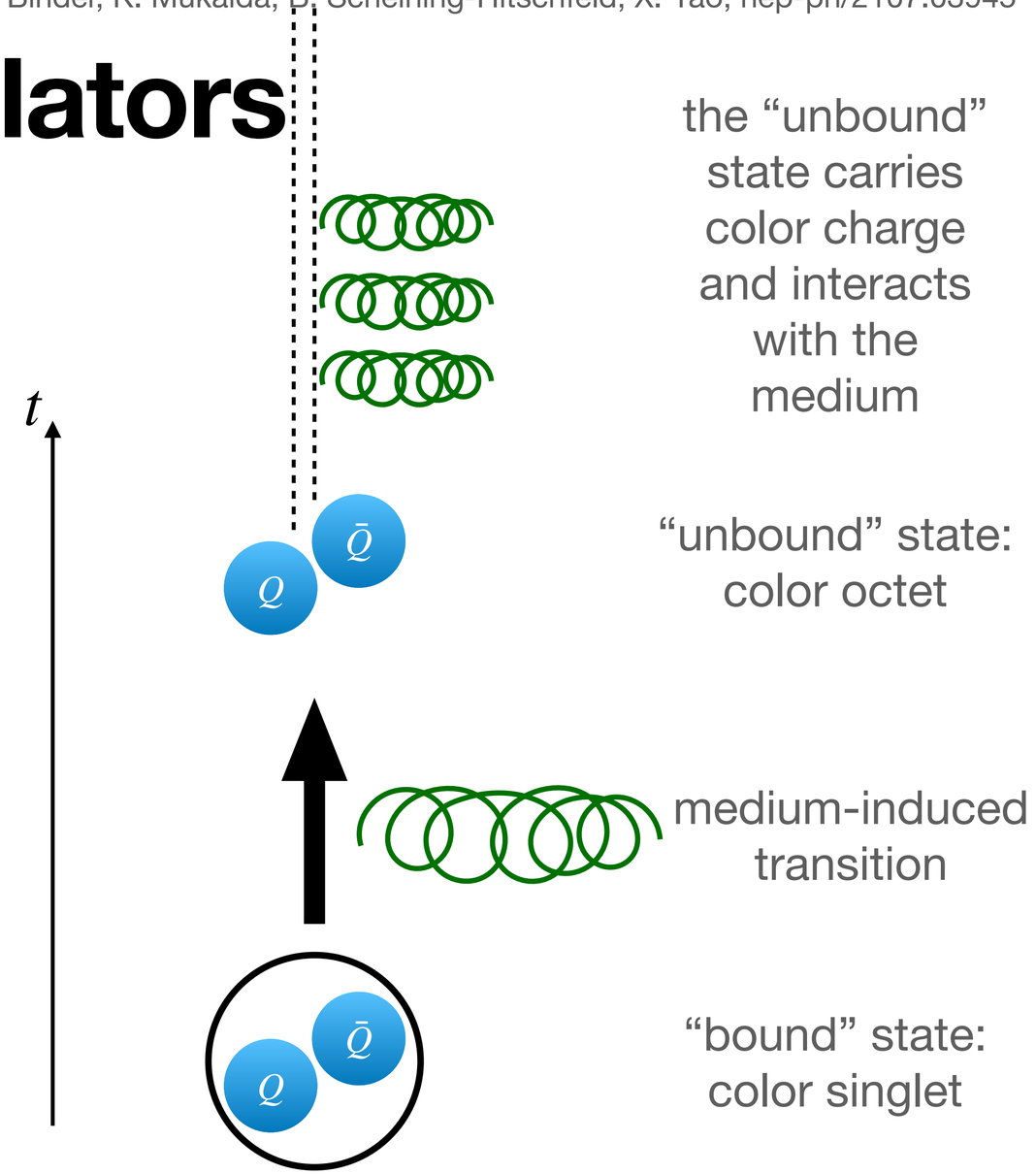

the “unbound” state carries color charge and interacts with the medium

“unbound” state: color octet

medium-induced transition

“bound” state: color singlet

See also: N. Brambilla et al. hep-ph/1612.07248, hep-ph/1711.04515, hep-ph/2205.10289

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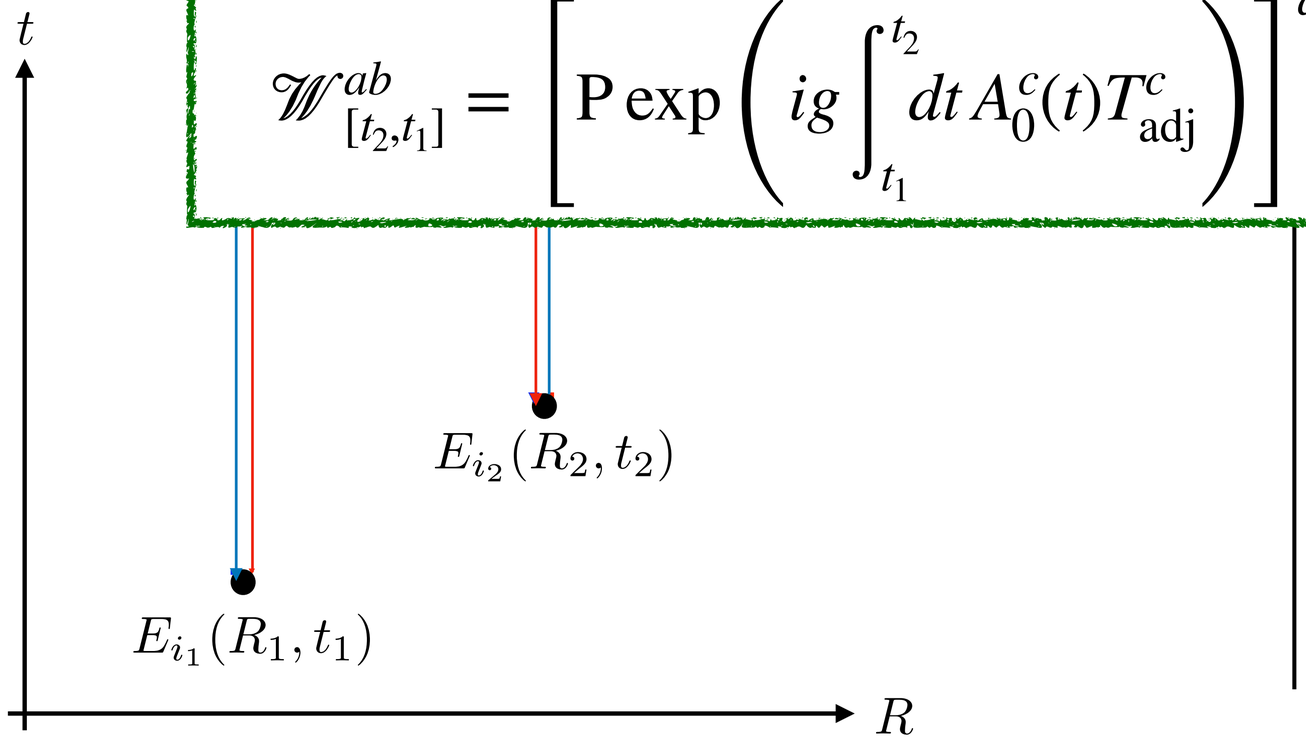
for quarkonia transport

$(R$

Summing the one-gluon insertions along the octet $Q\bar{Q}$ path generates a Wilson line:

$$\mathscr{W}^{ab}_{[t_2,t_1]} = \left[\mathrm{P} \exp\left(ig \int_{t_1}^{t_2} dt\, A_0^c(t) T^c_{\mathrm{adj}} \right) \right]^{ab}$$



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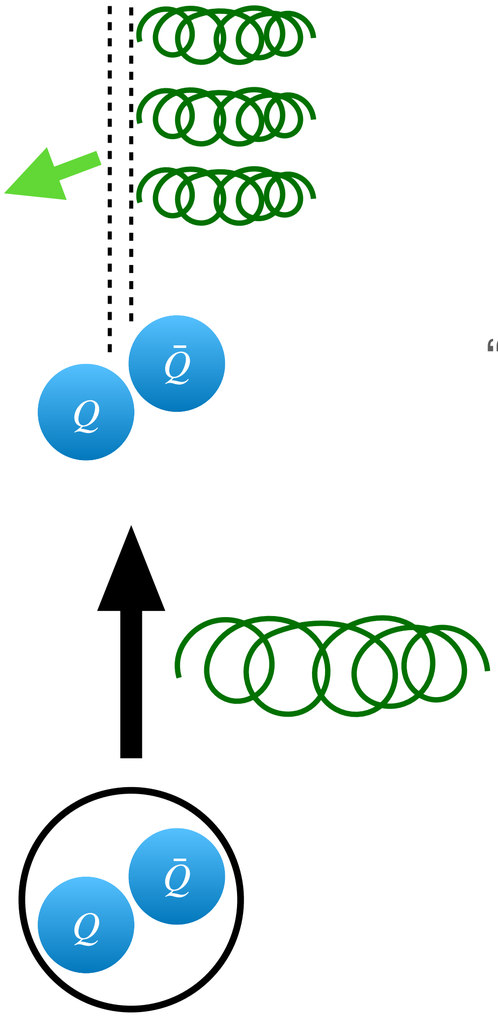

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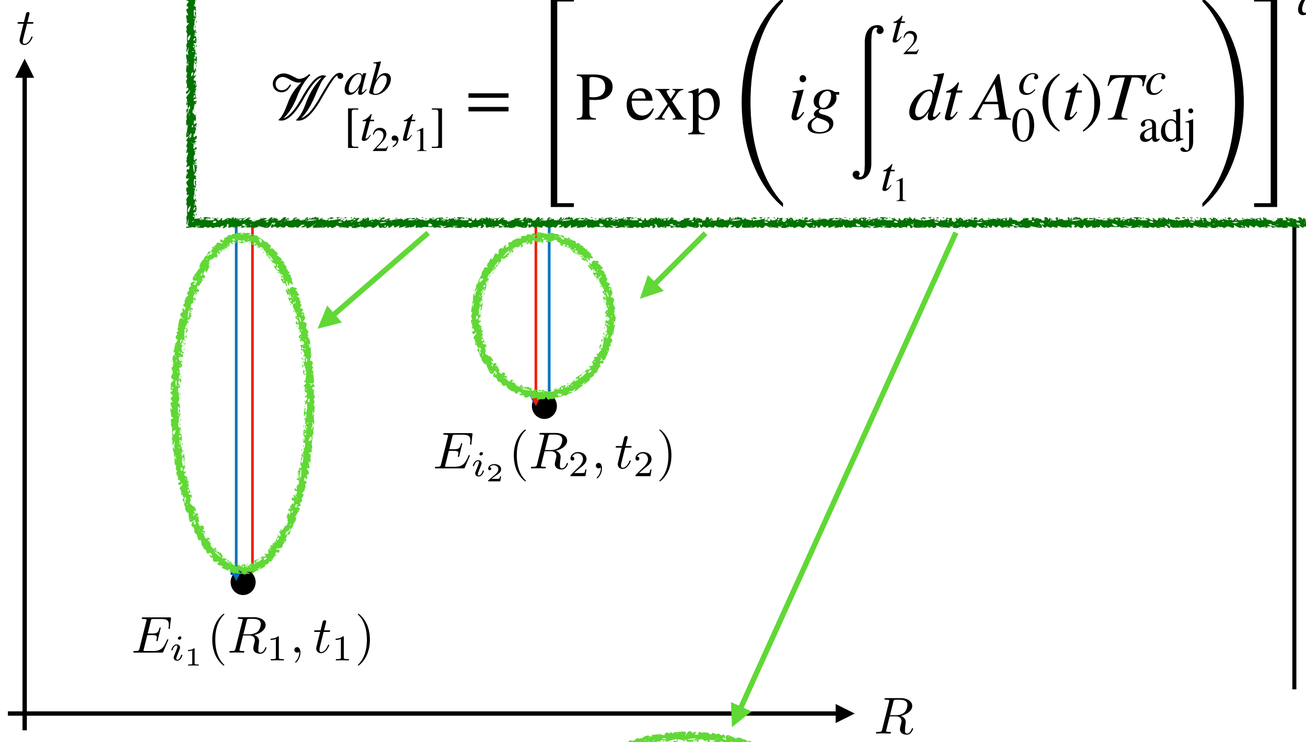
QGP chromoelectric correlators

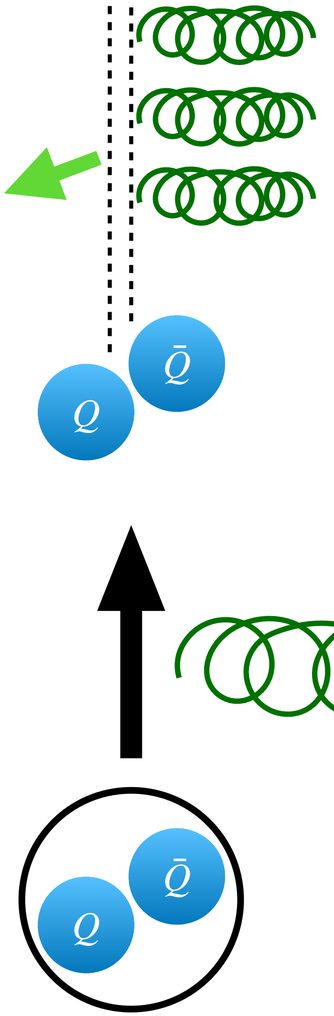
for quarkonia transport

Summing the one-gluon insertions along the octet $Q\bar{Q}$ path generates a Wilson line:

$$\mathscr{W}^{ab}_{[t_2,t_1]} = \left[\mathrm{P} \exp\left(ig \int_{t_1}^{t_2} dt\, A_0^c(t) T^c_{\mathrm{adj}} \right) \right]^{ab}$$



$$[g_E^{++}]^{>}_{i_2 i_1}(t_2, t_1, \mathbf{R}_2, \mathbf{R}_1) = \left\langle \left(E_{i_2}(\mathbf{R}_2, t_2)\mathscr{W}_2\right)^a \left(\mathscr{W}_1 E_{i_1}(\mathbf{R}_1, t_1)\right)^a \right\rangle_T$$



the “unbound” state carries color charge and interacts with the medium

“unbound” state: color octet

medium-induced transition

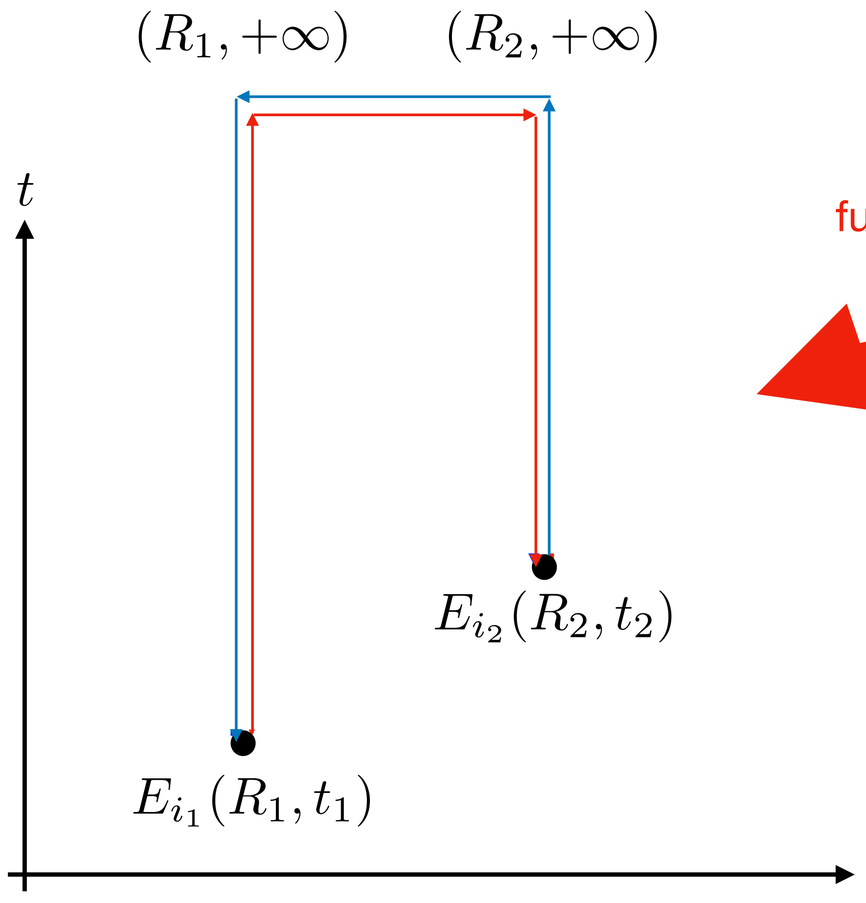
“bound” state: color singlet

See also: N. Brambilla et al. hep-ph/1612.07248, hep-ph/1711.04515, hep-ph/2205.10289

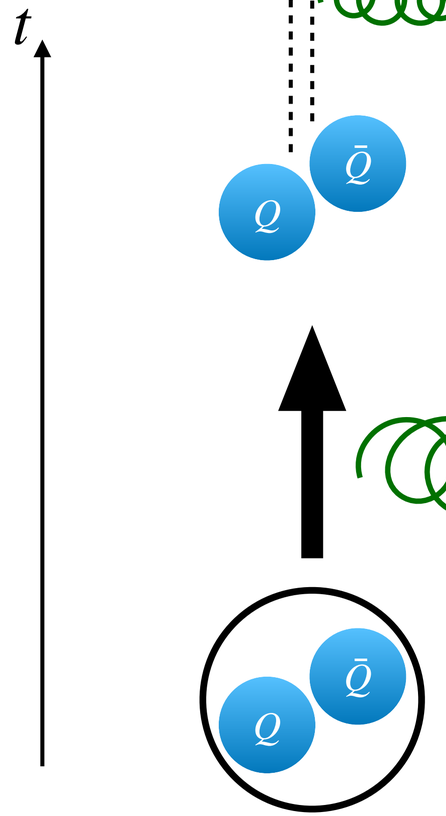
X. Yao and T. Mehen, hep-ph/2009.02408
T. Binder, K. Mukaida, B. Scheihing-Hitschfeld, X. Yao, hep-ph/2107.03945

QGP chromoelectric correlators

for quarkonia transport



the correlation function associated to this process



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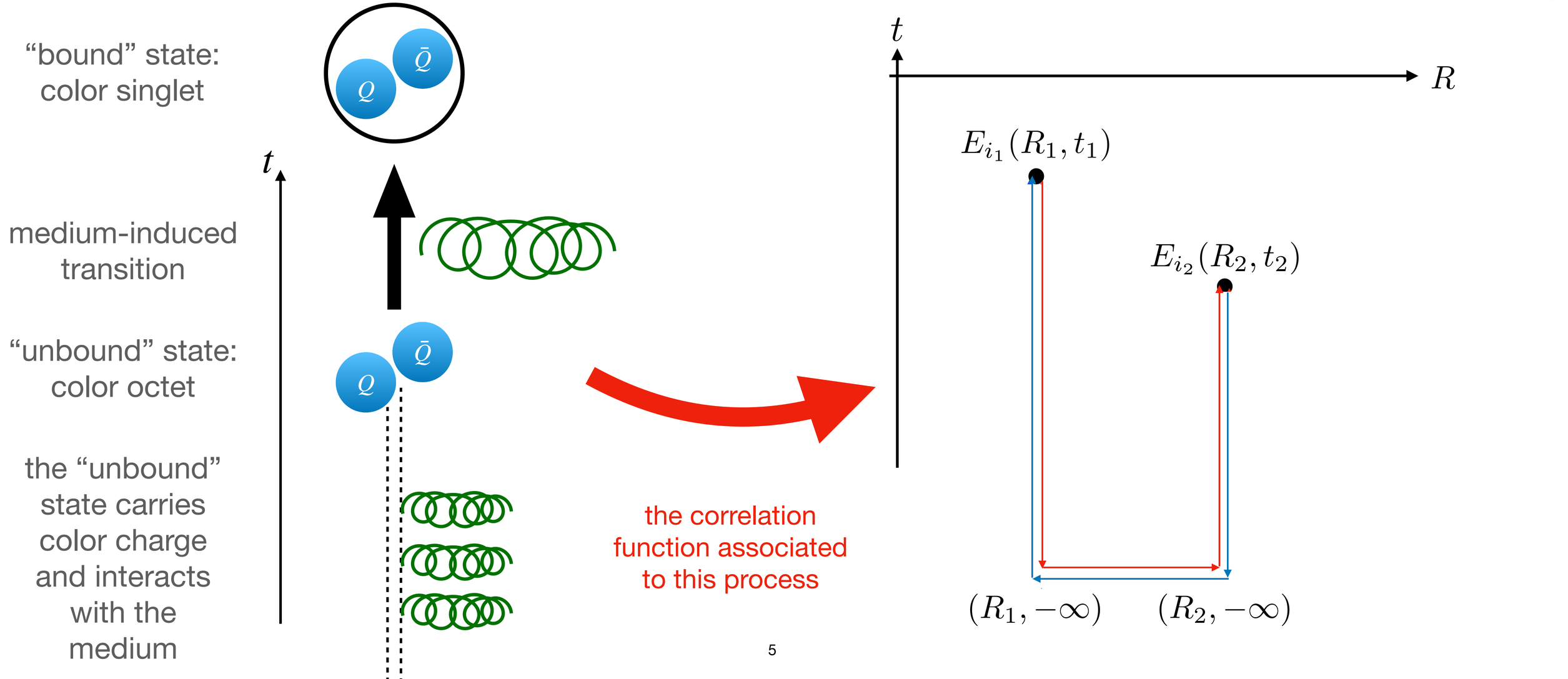
$$[g_E^{++}]^{>}_{i_2 i_1}(t_2, t_1, \mathbf{R}_2, \mathbf{R}_1) = \left\langle \left(E_{i_2}(\mathbf{R}_2, t_2)\mathscr{W}_2\right)^a \left(\mathscr{W}_1 E_{i_1}(\mathbf{R}_1, t_1)\right)^a_5 \right\rangle_T$$

QGP chromoelectric correlators

for quarkonia transport

$$[g_E^{--}]^{>}_{i_2 i_1}(t_2, t_1, \mathbf{R}_2, \mathbf{R}_1) = \left\langle \left(\mathscr{W}_{2'} E_{i_2}(\mathbf{R}_2, t_2)\right)^a \left(E_{i_1}(\mathbf{R}_1, t_1)\mathscr{W}_{1'}\right)^a \right\rangle_T$$



Why are these correlators interesting?

Quarkonium in the quantum brownian motion limit

$Mv \gg T \gg Mv^2$ **(Brambilla et al.)**

$$\frac{d\rho_S(t)}{dt} = -i\left[H_S + \Delta H_S, \rho_S(t)\right] + \kappa_{\rm adj}\left(L_{\alpha i}\rho_S(t)L_{\alpha i}^\dagger - \frac{1}{2}\left\{L_{\alpha i}^\dagger L_{\alpha i}, \rho_S(t)\right\}\right)$$

The correlators determine the transport coefficients:

$$\gamma_{\rm adj} \equiv \frac{g^2}{6N_c}{\rm Im}\int_{-\infty}^{\infty} ds\,\langle \mathcal{T}\, E^{a,i}(s,\mathbf{0})\mathscr{W}^{ab}[(s,\mathbf{0}),(0,\mathbf{0})]E^{b,i}(0,\mathbf{0})\rangle\,,$$

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Quarkonium in the quantum optical limit

Semiclassical approximation

+ $Mv \gg Mv^2, T$ **(Yao et al.)**

$$\frac{dn_b(t,\mathbf{x})}{dt} = -\Gamma^{\rm diss}\, n_b(t,\mathbf{x}) + R^{\rm form}(t,\mathbf{x})$$

These correlators determine the dissociation and formation rates of quarkonia:

$$\Gamma^{\rm diss} \propto \int \frac{{\rm d}^3\mathbf{p}_{\rm rel}}{(2\pi)^3}\frac{{\rm d}^3\mathbf{q}}{(2\pi)^3}\,|\langle\psi_{\mathscr{B}}|\,\mathbf{r}\,|\Psi_{\mathbf{p}_{\rm rel}}\rangle|^2[g_E^{++}]^{>}_{ii}\left(q^0 = E_{\mathscr{B}} - \frac{\mathbf{p}_{\rm rel}^2}{M}, \mathbf{q}\right),$$

$$R^{\rm form}(t,\mathbf{x}) \propto \int \frac{{\rm d}^3\mathbf{p}_{\rm cm}}{(2\pi)^3}\frac{{\rm d}^3\mathbf{p}_{\rm rel}}{(2\pi)^3}\frac{{\rm d}^3\mathbf{q}}{(2\pi)^3}\,|\langle\psi_{\mathscr{B}}|\,\mathbf{r}\,|\Psi_{\mathbf{p}_{\rm rel}}\rangle|^2[g_E^{--}]^{>}_{ii}\left(q^0 = \frac{\mathbf{p}_{\rm rel}^2}{M} - E_{\mathscr{B}}, \mathbf{q}\right)$$

$$\times f_{\mathscr{S}}(\mathbf{x}, \mathbf{p}_{\rm cm}, \mathbf{r}=0, \mathbf{p}_{\rm rel}, t)\,.$$

A comparison with heavy quark diffusion

Different physics with the same building blocks

Heavy quark diffusion

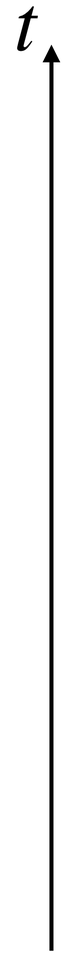
an analogous picture

J. Casalderrey-Solana and D. Teaney, hep-ph/0605199

- The heavy quark diffusion coefficient is also defined from a correlation of chromoelectric fields:

$$\langle \mathrm{Tr}\Big[(U_{[\infty,t]}E_i(t)U_{[t,-\infty]})^\dagger \times (U_{[\infty,0]}E_i(0)U_{[0,-\infty]})\Big]\rangle$$

- It reflects the typical momentum transfer $\langle p^2 \rangle$ received from "kicks" from the medium.



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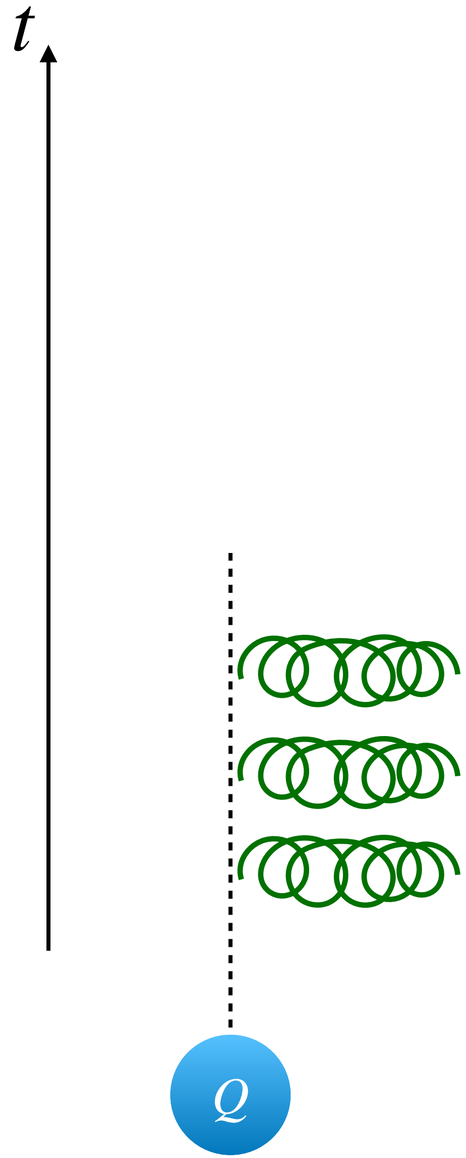

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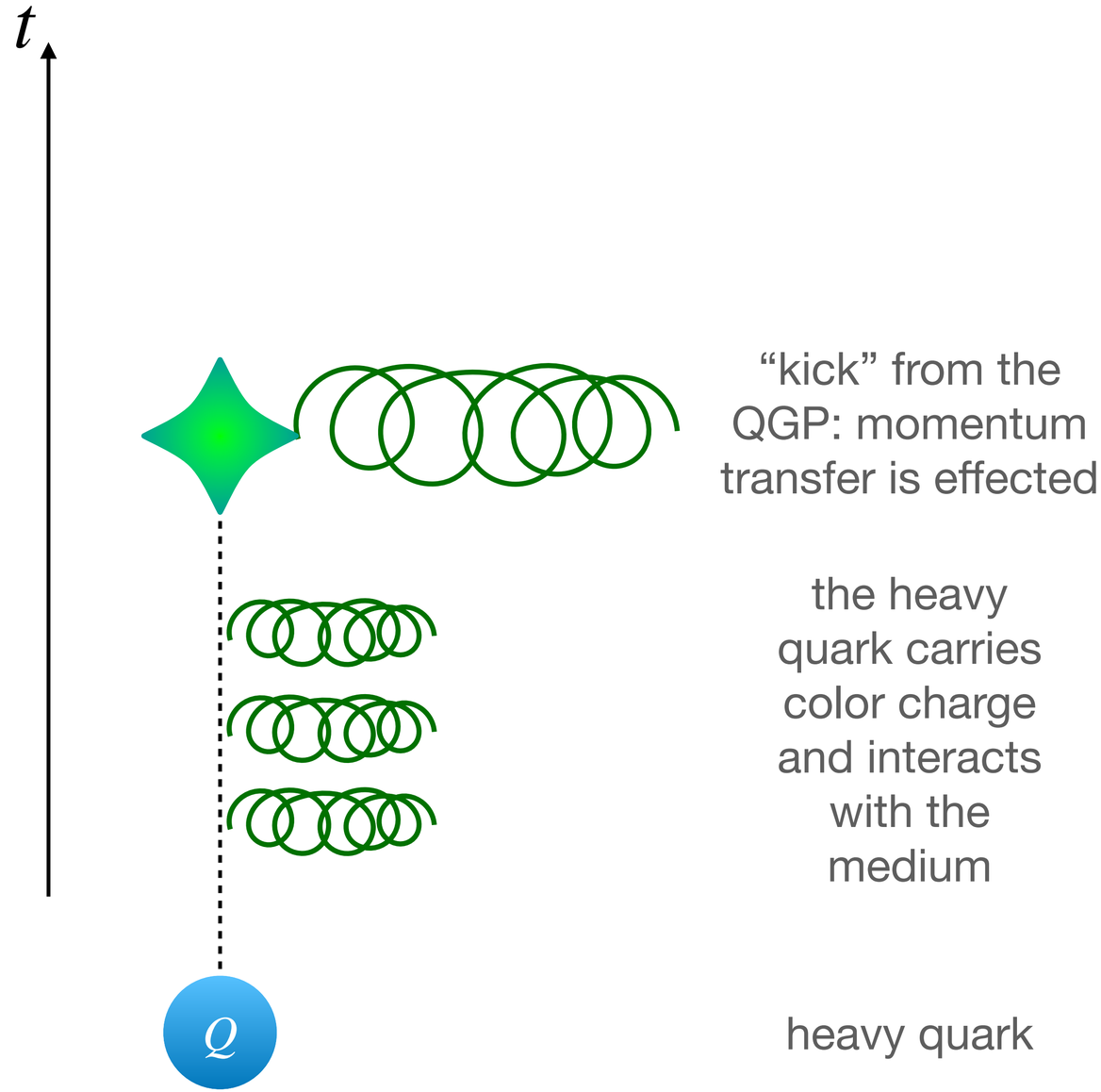

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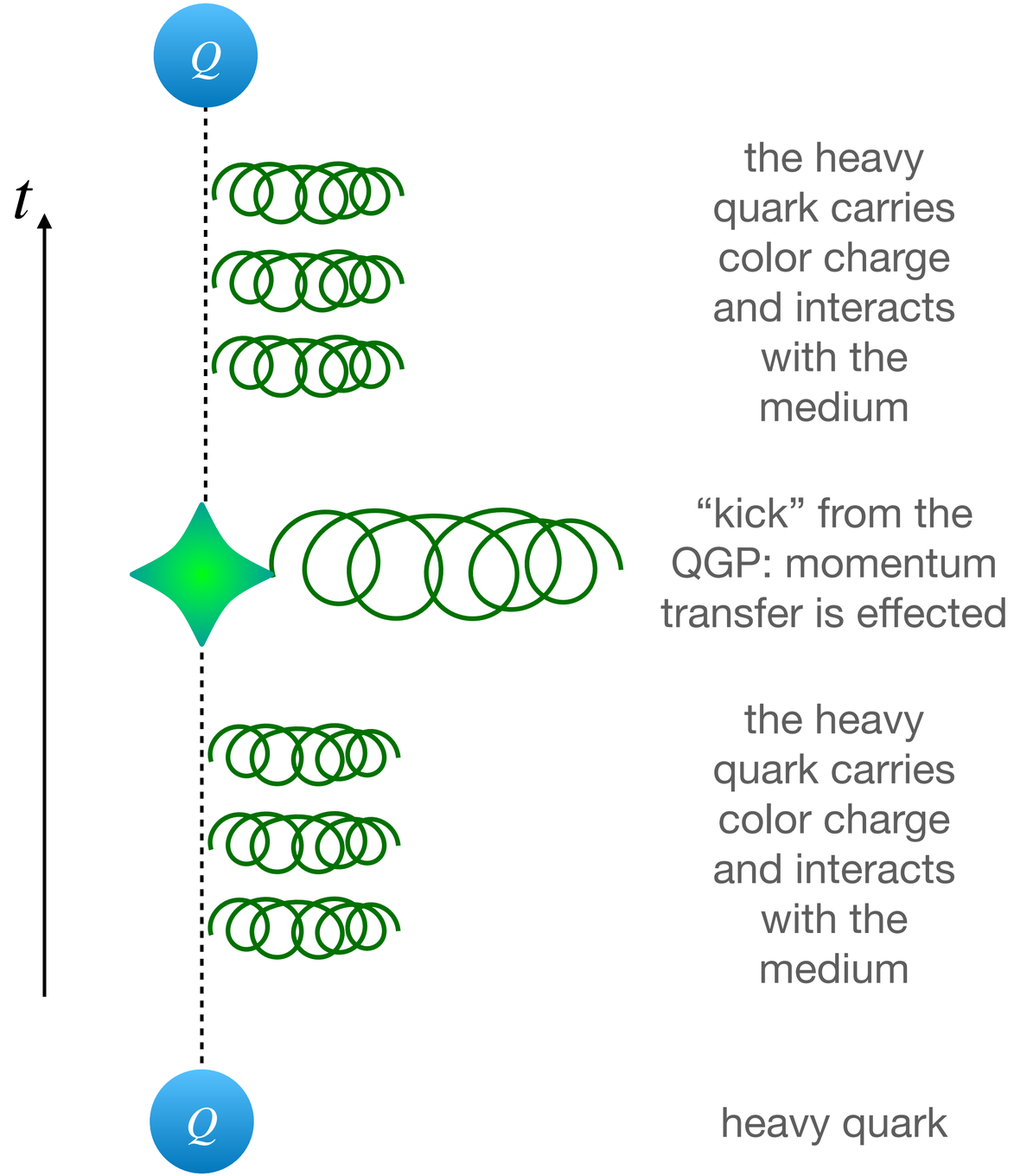


Heavy quark and quarkonia correlators

a small, yet consequential difference

The heavy quark diffusion coefficient can be defined from the real-time correlator J. Casalderrey-Solana and D. Teaney, hep-ph/0605199; see also A. M. Eller, J. Ghiglieri and G. D. Moore, hep-ph/1903.08064

$$\left\langle \mathrm{Tr}_{\mathrm{color}} \left[U(-\infty, t) E_i(t) U(t,0) E_i(0) U(0, -\infty) \right] \right\rangle_T ,$$

whereas for quarkonia the relevant quantity is ($\mathbf{R}_1 = \mathbf{R}_2$ in the preceding discussion)

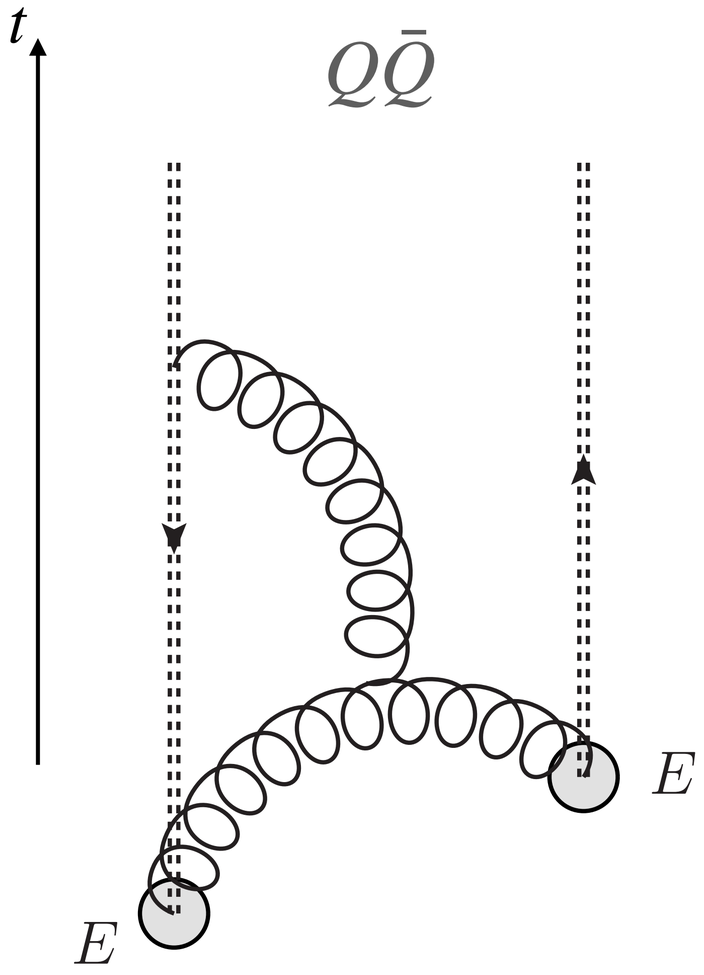
$$T_F \left\langle E_i^a(t) \mathscr{W}^{ab}(t,0) E_i^b(0) \right\rangle_T .$$

see also A. M. Eller, J. Ghiglieri and G. D. Moore, hep-ph/1903.08064

BS and X. Yao, hep-ph/2205.04477
Phys. Rev. Lett. 130, 052302

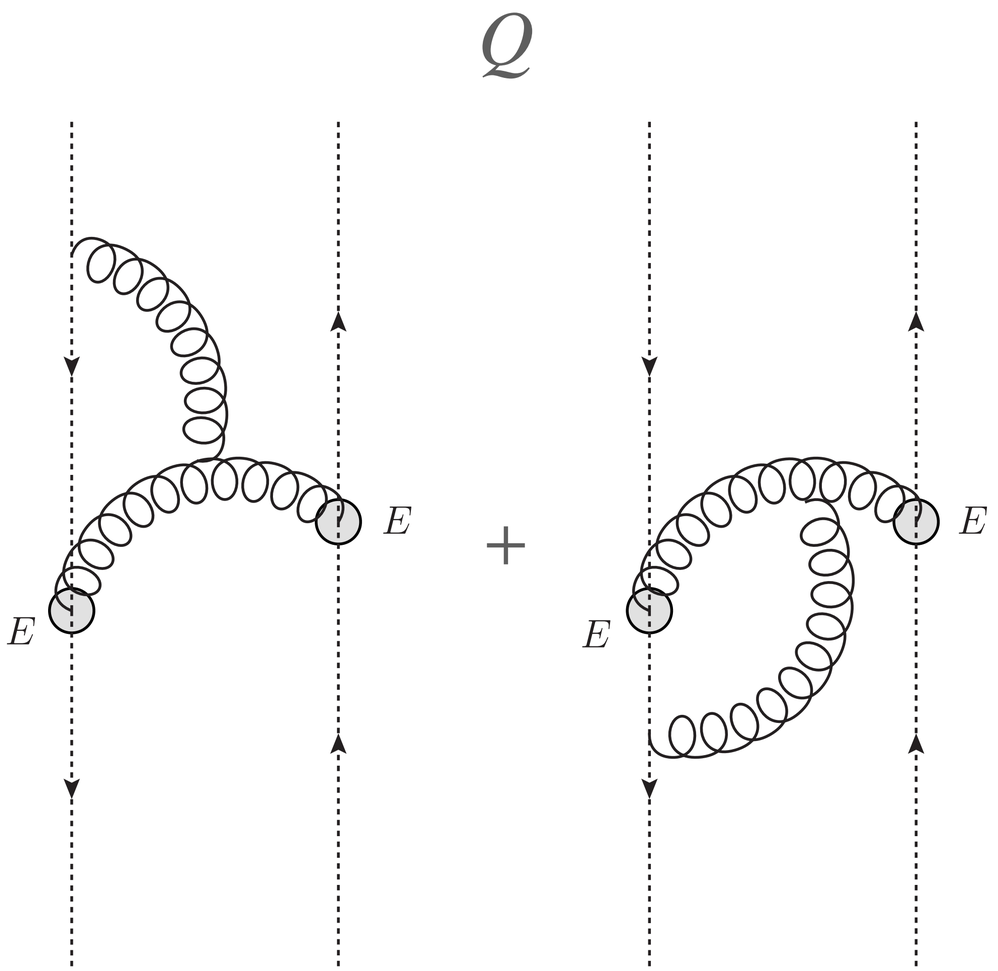
The difference in pQCD

operator ordering is crucial!



Perturbatively, one can isolate the difference between the correlators to these diagrams.

$$\Delta\rho(\omega) = \frac{g^4 N_c^2 C_F T_F}{4\pi} |\omega|^3$$



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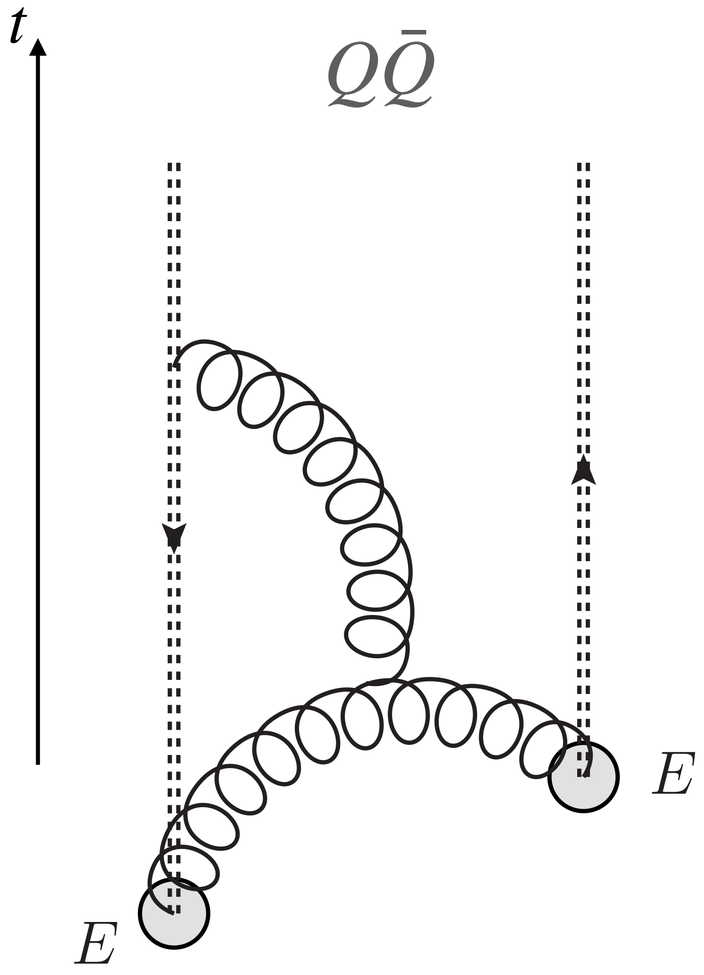

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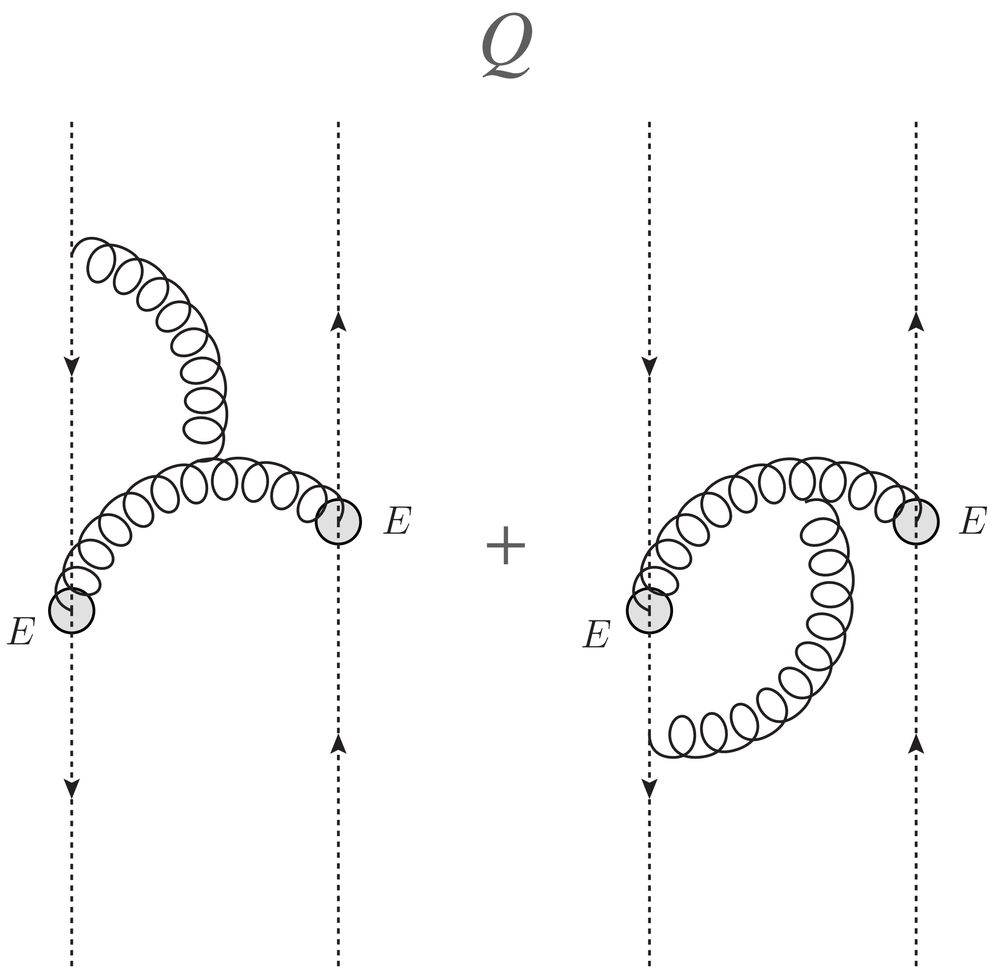

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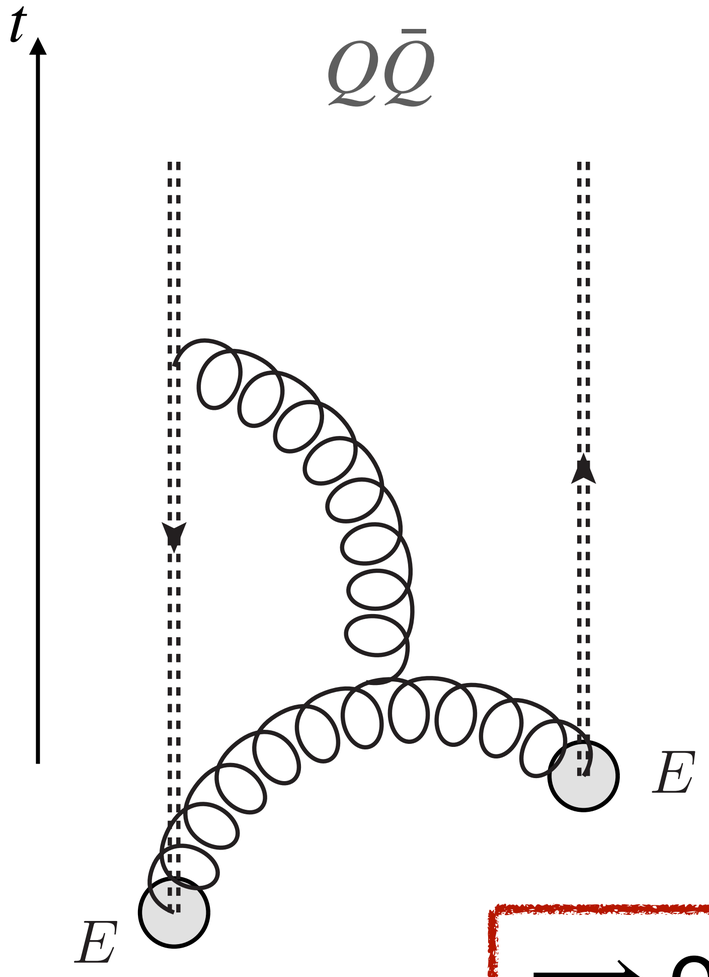

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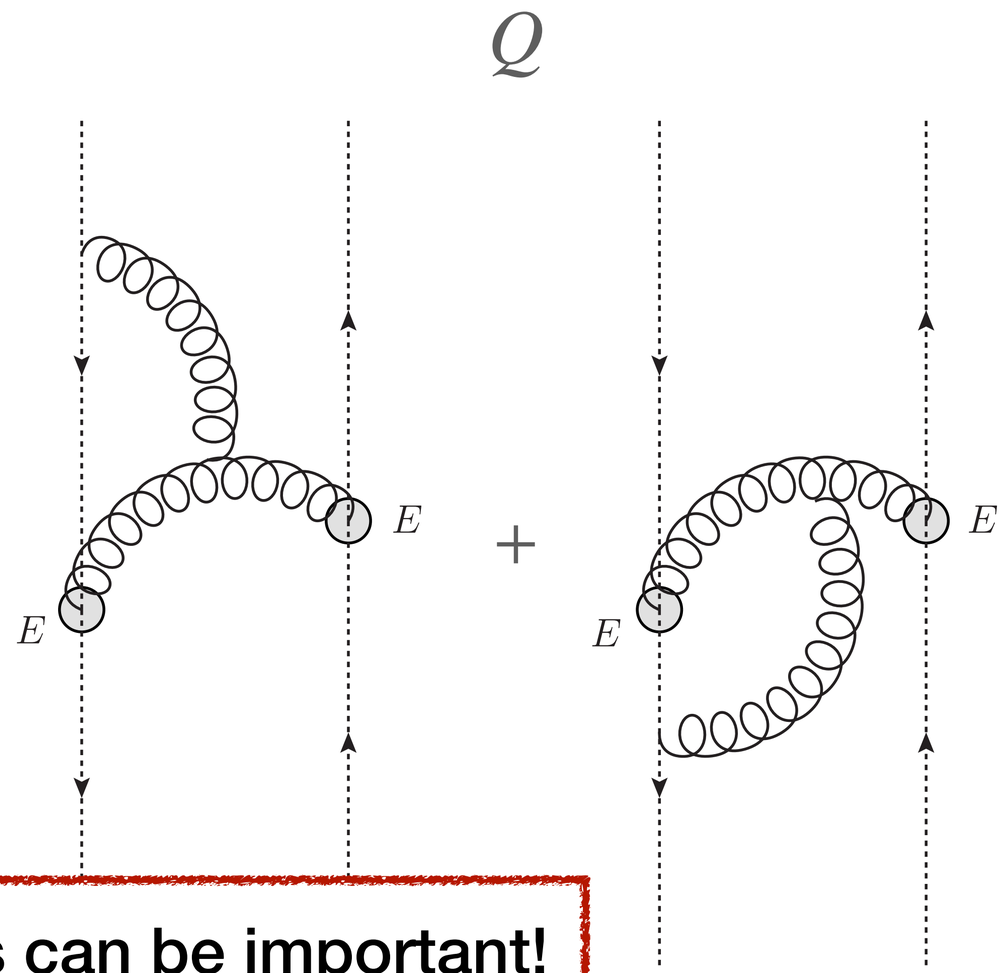
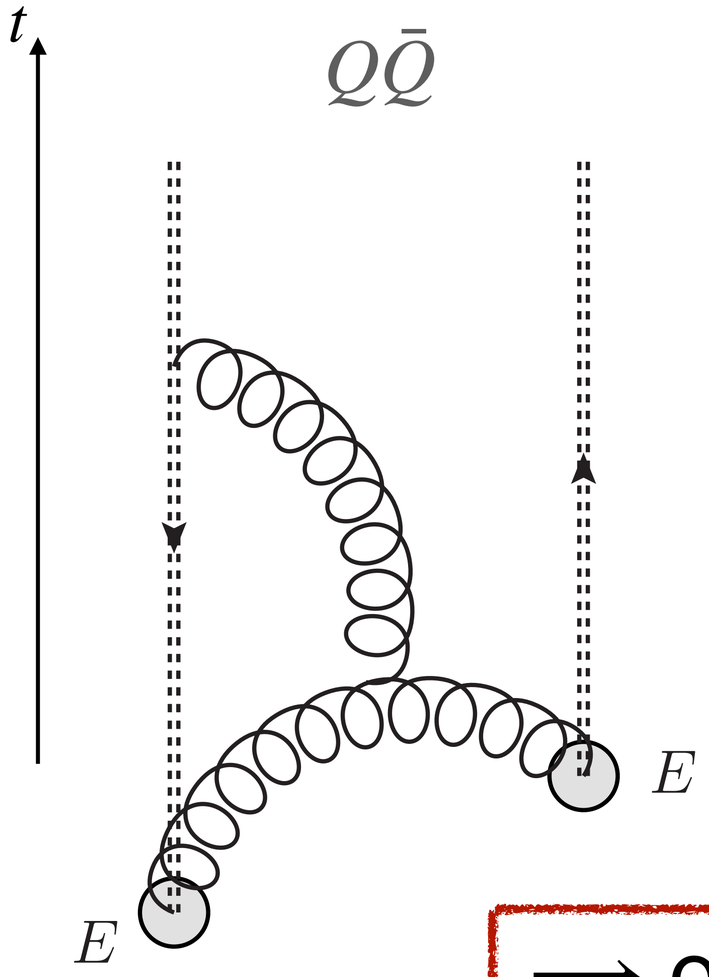


$\Longrightarrow$ **Quantum color correlations can be important!**

The difference in pQCD

Gauge invariant!

operator ordering is crucial!



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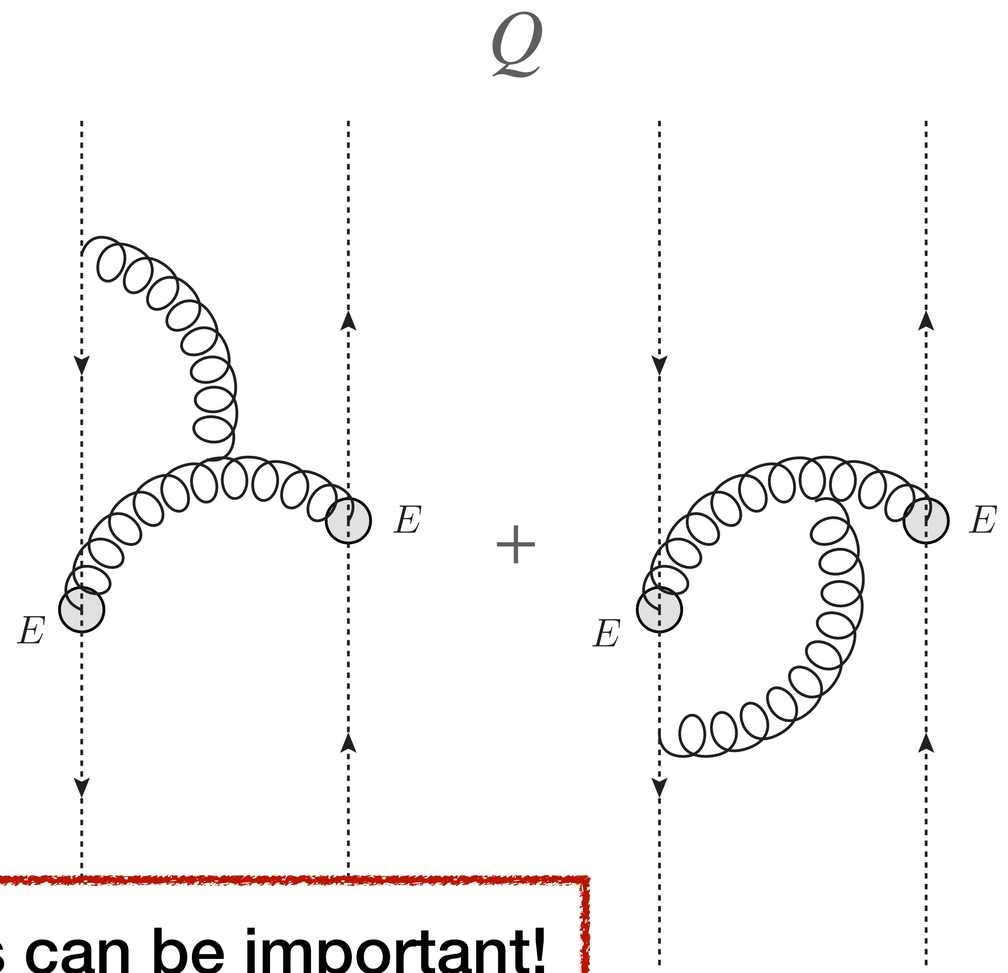


$\Longrightarrow$ **Quantum color correlations can be important!**

Can we calculate this difference non-perturbatively in QCD?

A Lattice QCD perspective

heavy quark diffusion

- The heavy quark diffusion coefficient has been studied by evaluating the following correlation function (e.g., Altenkort et al. 2009.13553, 2302.08501; Leino et al. 2212.10941):

$$G_{\rm fund}(\tau) = -\frac{1}{3}\frac{\left\langle {\rm ReTr}_c[U(\beta,\tau)\, gE_i(\tau)\, U(\tau,0)\, gE_i(0)]\right\rangle}{\left\langle {\rm ReTr}_c[U(\beta,0)]\right\rangle}\,.$$

- The heavy quark diffusion coefficient is extracted by reconstructing the corresponding spectral function (Caron-Huot et al. 0901.1195):

$$G_{\rm fund}(\tau) = \int_0^{+\infty}\frac{d\omega}{2\pi}\frac{\cosh\left(\omega(\tau-\frac{1}{2T})\right)}{\sinh\left(\frac{\omega}{2T}\right)}\rho_{\rm fund}(\omega)\,,\quad \kappa_{\rm fund} = \lim_{\omega\to 0}\frac{T}{\omega}\rho_{\rm fund}(\omega)\,.$$

A Lattice QCD perspective

quarkonium transport (hep-ph/2306.13127 w/ X. Yao)

- The quarkonium correlator in imaginary time has received less attention:

$$G_{\rm adj}(\tau) = \frac{T_F g^2}{3N_c} \left\langle E_i^a(\tau) \mathscr{W}^{ab}(\tau,0) E_i^b(0) \right\rangle .$$

- The transport coefficients can also be extracted by spectral reconstruction:

$$G_{\rm adj}(\tau) = \int_{-\infty}^{+\infty} \frac{{\rm d}\omega}{2\pi} \frac{\exp\left(\omega(\frac{1}{2T}-\tau)\right)}{2\sinh\left(\frac{\omega}{2T}\right)} \rho_{\rm adj}^{++}(\omega) , \quad \kappa_{\rm adj} = \lim_{\omega\to 0} \frac{T}{2\omega} \left[\rho_{\rm adj}^{++}(\omega) - \rho_{\rm adj}^{++}(-\omega)\right] .$$

- Main new ingredient: the spectral function $\rho_{\rm adj}^{++}(\omega)$ is not odd under $\omega \to -\omega$, because $G_{\rm adj}(\tau)$ is not invariant under $\tau \to 1/T - \tau$.

In summary, we have:

- ☑ Understood the weakly coupled limit in QCD, and 2107.03945, 2205.04477
- ☑ formulated how to extract the transport coefficients in lattice QCD. 2306.13127

However, the QGP is strongly coupled.

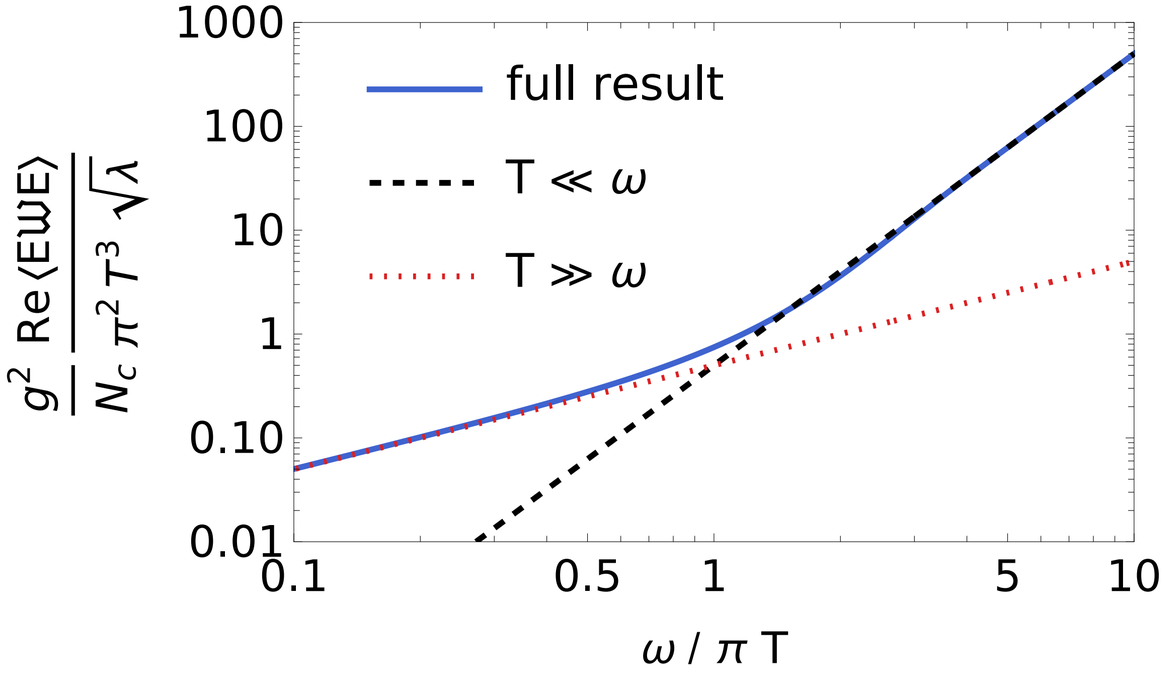
- Pending a lattice QCD determination, is there anything else we can learn at strong coupling?

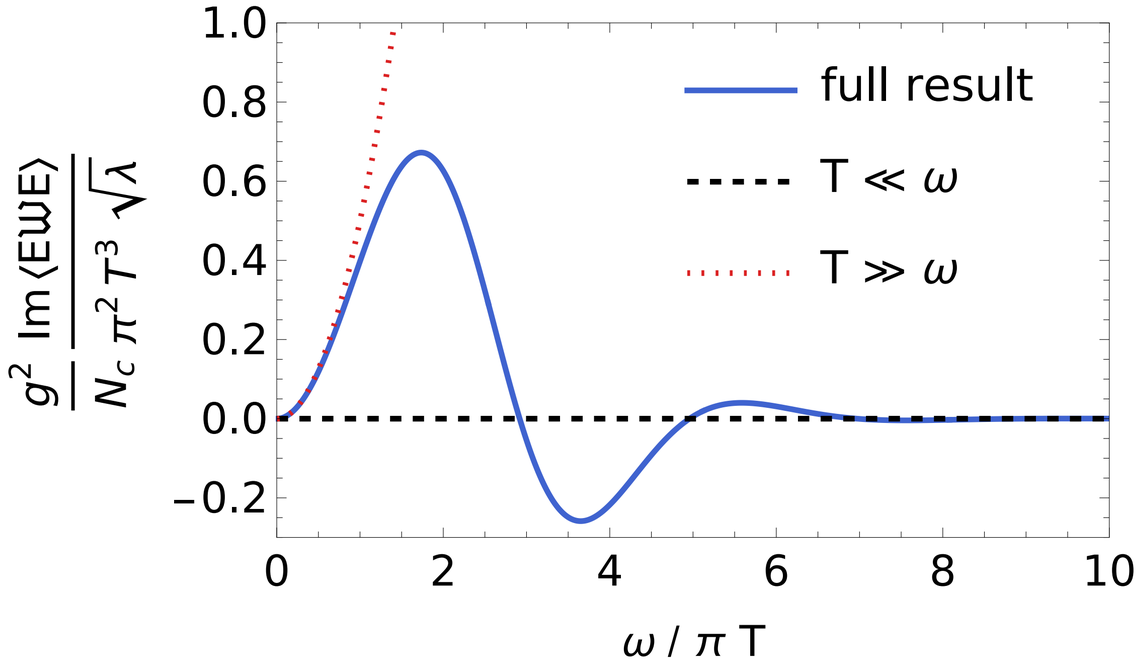
Yes! Using holography, we can:

- ☑ Understand the strong coupling in $\mathcal{N} = 4$ SYM, and 2304.03298
- ☑ calculate its velocity dependence. 2309.XXXXX

Results in $\mathcal{N} = 4$ SYM at strong coupling

novel calculation in 2304.03298 with G. Nijs and X. Yao





$$\kappa_{\rm adj}^{\mathcal{N}=4} + i\gamma_{\rm adj}^{\mathcal{N}=4} \equiv \frac{g^2}{6N_c}\int_{-\infty}^{\infty} ds\,\langle \mathcal{T}\, E^{a,i}(s)\mathscr{W}^{ab}_{[s,0]}E^{b,i}(0)\rangle = 0$$

* J. Casalderrey-Solana and D. Teaney, hep-ph/0605199

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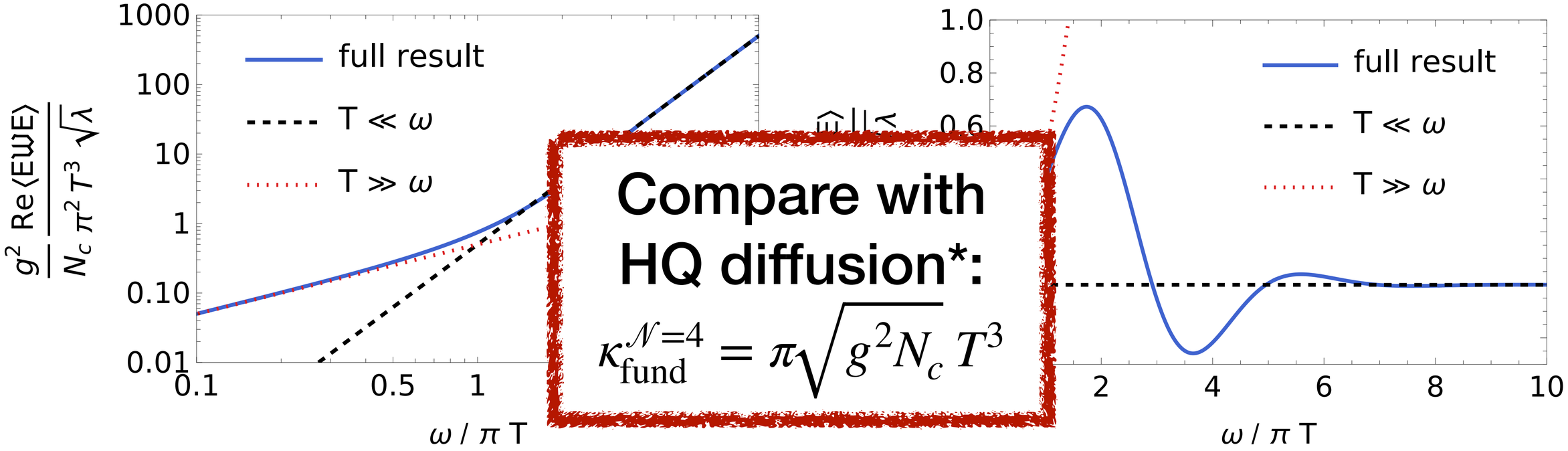
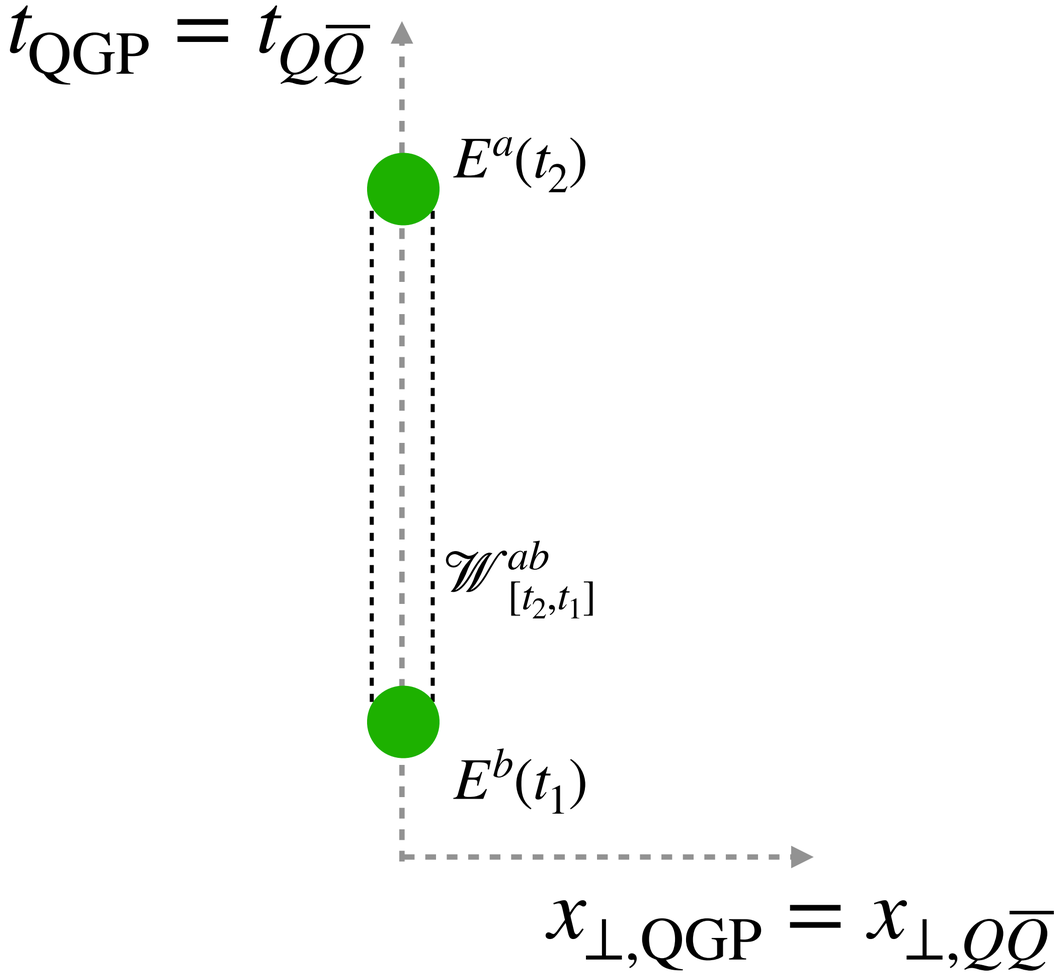


Compare with HQ diffusion*:

$$\kappa_{\mathrm{fund}}^{\mathcal{N}=4} = \pi\sqrt{g^2 N_c}\, T^3$$

$$\kappa_{\mathrm{adj}}^{\mathcal{N}=4} + i\gamma_{\mathrm{adj}}^{\mathcal{N}=4} \equiv \frac{g^2}{6N_c}\int_{-\infty}^{\infty} ds\, \langle \mathcal{T}\, E^{a,i}(s)\mathscr{W}_{[s,0]}^{ab} E^{b,i}(0)\rangle = 0$$

Velocity dependence of quarkonia transport rates in AdS/CFT 2309.XXXXX (w/ G. Nijs and X. Yao)



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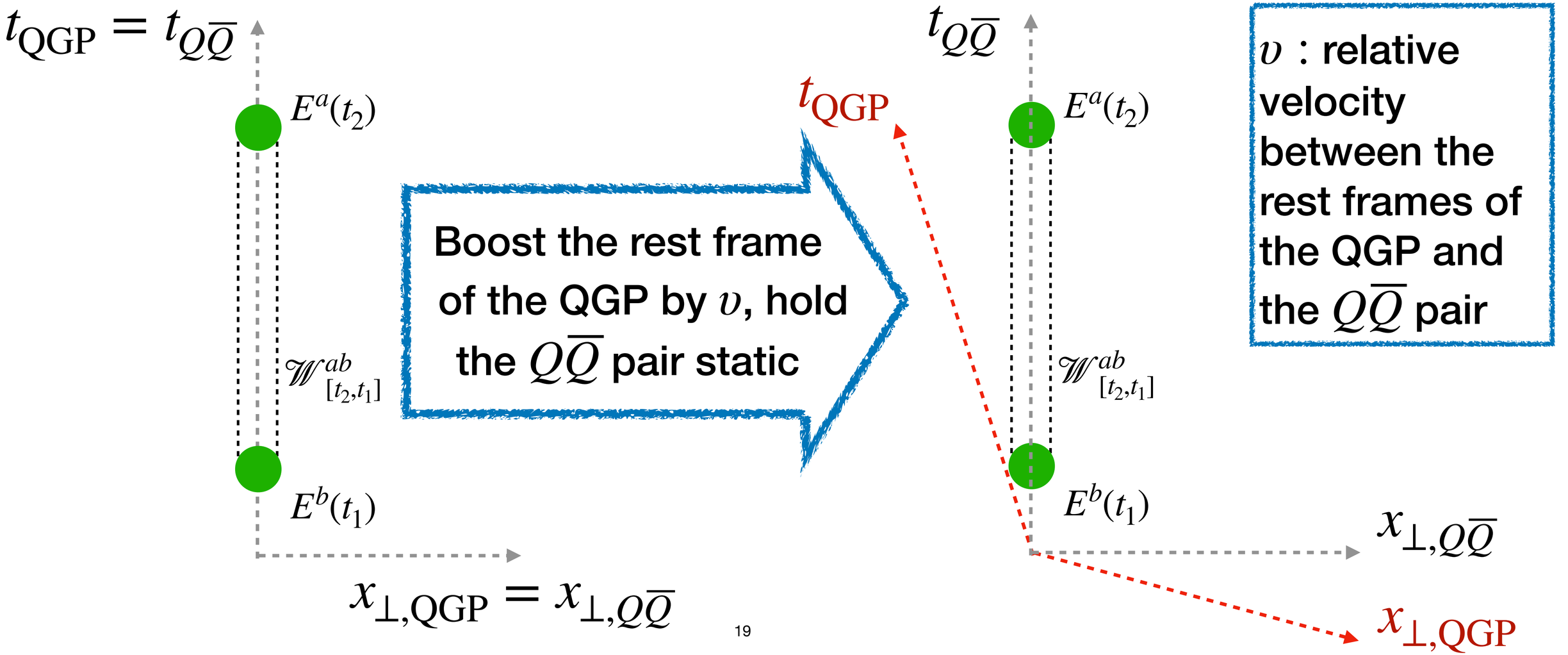


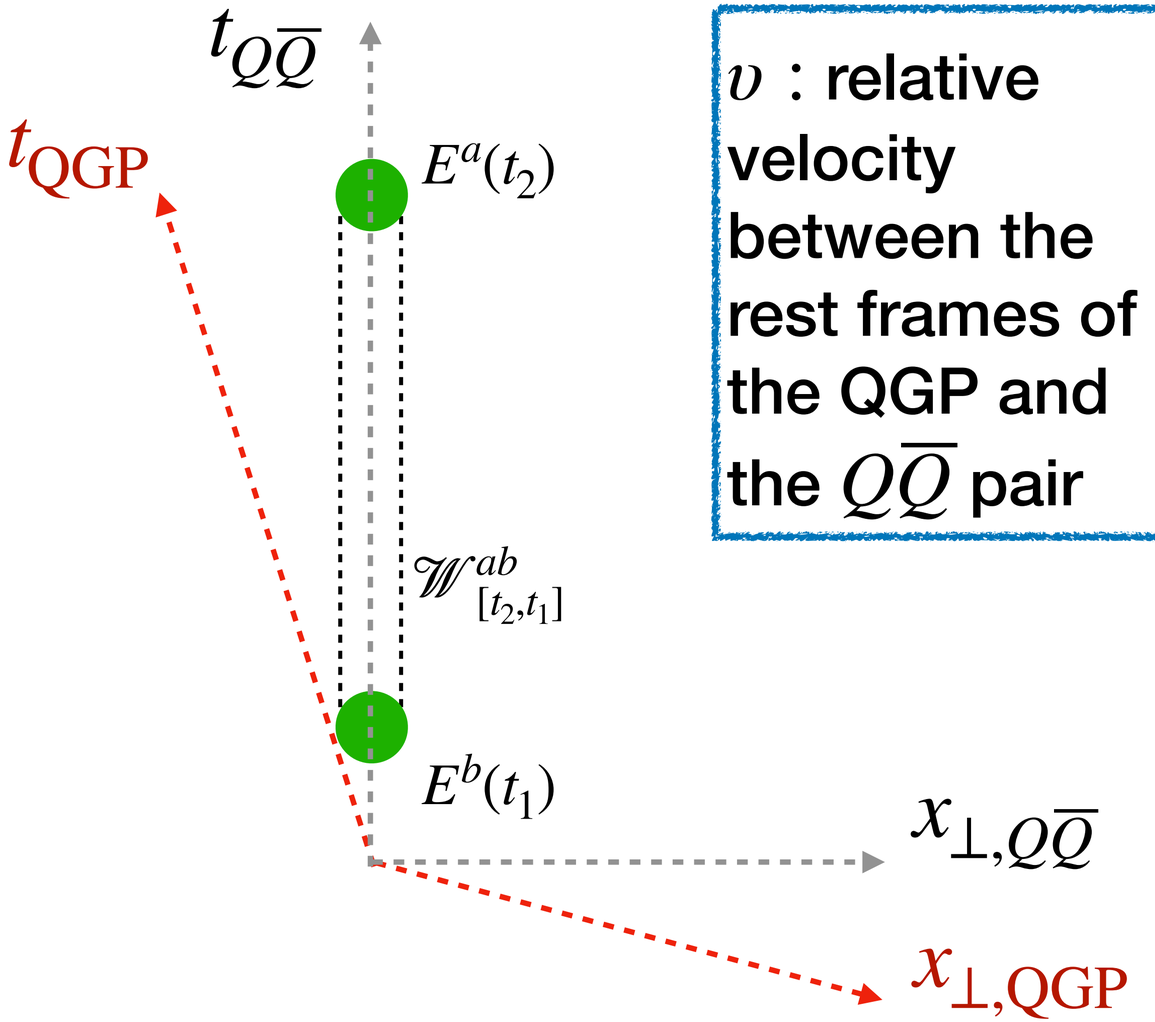
Velocity dependence of quarkonia transport rates in AdS/CFT 2309.XXXXX (w/ G. Nijs and X. Yao)

Result:

The correlator has the same form as in the $v = 0$ case, with

$$T_{\mathrm{eff},Q\bar{Q}} = \sqrt{\gamma} T_{\mathrm{QGP}} \,,$$

where $\gamma = 1/\sqrt{1 - v^2}$.



v : relative velocity between the rest frames of the QGP and the $Q\bar{Q}$ pair

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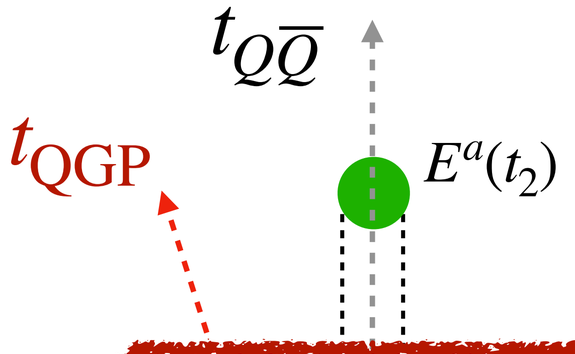


v : relative velocity between the rest frames of

Same temperature dependence as the $Q\bar{Q}$ potential in the same "hot wind" setup hep-ph/0612168

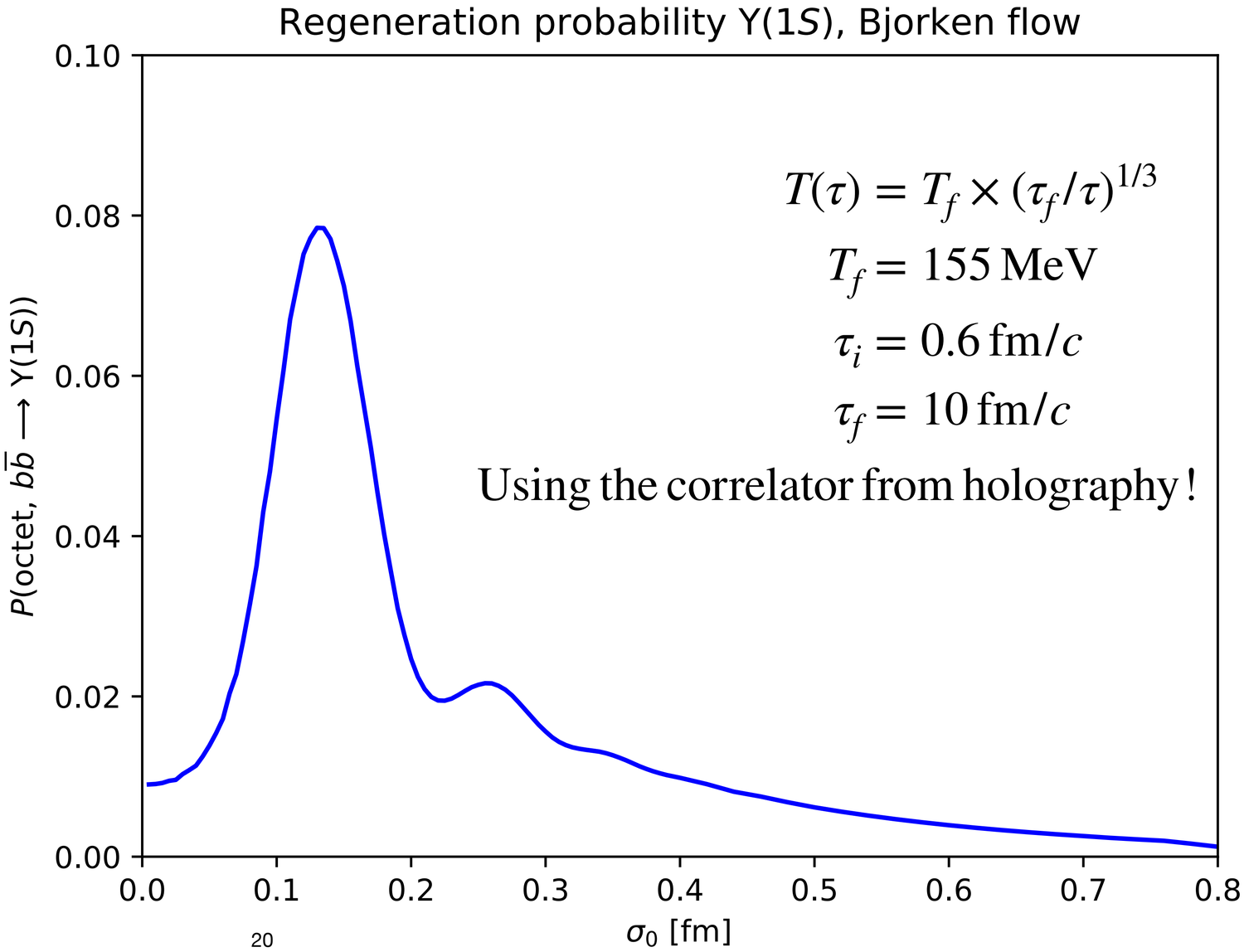
PRELIMINARY

A flavor of what we can do

23XX.XXXXX (w/ X. Yao)

We can now evolve a state of a heavy quark-antiquark pair, taking into account:

- ☑ Their wavefunction evolution using a potential model, allowing for different initial separations σ_0 between the pair.
- ☑ Their transition rates via the correlator we just discussed.



Summary and conclusions

- We have discussed how to calculate the chromoelectric correlators of the QGP that govern quarkonium transport.
  - Interesting prospects for interpolating between weak & strong coupling, and describing non-perturbative QGP physics.
- Next steps (as of Hard Probes 2023):
  - Generalize the calculations to include a boosted medium.
  - Calculate the chromo-magnetic correlators $\langle B^a(t)\mathscr{W}^{ab}_{[t,0]}B^b(0)\rangle_T$.
  - Use them as input for quarkonia transport codes.
- Stay tuned!

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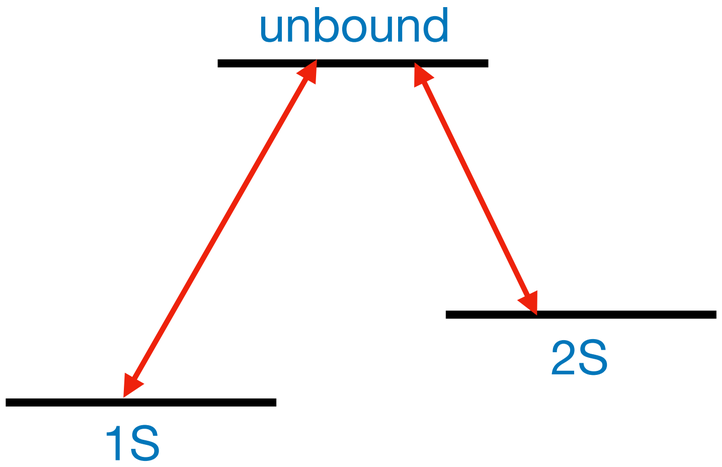
Thank you!

Extra slides

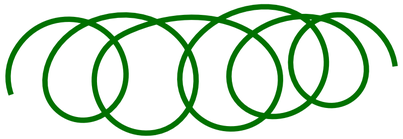
Time scales of quarkonia

Transitions between quarkonium energy levels (the system)



$$\frac{1}{\tau_S} \sim \Delta E_n \sim Mv^2$$

Interaction with the environment



$$\frac{1}{\tau_I} \sim \frac{H_{\rm int}^2}{T} \sim T\frac{T^2}{(Mv)^2}$$

QGP (the environment)

$$\frac{1}{\tau_E} \sim T$$

$$\mathscr{L}_{\rm pNRQCD} = \mathscr{L}_{\rm light\,quarks} + \mathscr{L}_{\rm gluon} + \int d^3r {\rm Tr}_{\rm color}\Big[S^\dagger(i\partial_0 - H_s)S + O^\dagger(iD_0 - H_o)O$$

$$+V_A\big(O^\dagger \mathbf{r}\cdot g\mathbf{E}S + {\rm h.c.}\big) + \frac{V_B}{2}O^\dagger\big\{\mathbf{r}\cdot g\mathbf{E}, O\big\} + \cdots\Big]$$

Open quantum systems

"tracing/integrating out" the QGP

- Given an initial density matrix $\rho_{\rm tot}(t=0)$, quarkonium coupled with the QGP evolves as

$$\rho_{\rm tot}(t) = U(t)\rho_{\rm tot}(t=0)U^\dagger(t).$$

- We will only be interested in describing the evolution of quarkonium and its final state abundances

$$\Longrightarrow \ \rho_S(t) = {\rm Tr}_{\rm QGP}\left[U(t)\rho_{\rm tot}(t=0)U^\dagger(t)\right].$$

- Then, one derives an evolution equation for $\rho_S(t)$, assuming that at the initial time we have $\rho_{\rm tot}(t=0) = \rho_S(t=0)\otimes e^{-H_{\rm QGP}/T}/\mathscr{Z}_{\rm QGP}$.

Open quantum systems

"tracing/integrating out" the QGP: semi-classic description

Unitary evolution of environment + subsystem

⇩ Trace out the environment degrees of freedom

OQS: ρ_S has non-unitary, time-irreversible evolution

⇩ Markovian approximation $\Longleftrightarrow$ weak coupling in H_I

OQS: Lindblad equation

⇩ Wigner transform: $f(\mathbf{x},\mathbf{k},t) \equiv \int_{k'} e^{i\mathbf{k}'\cdot\mathbf{x}} \left\langle \mathbf{k}+\frac{\mathbf{k}'}{2}\middle|\rho_S(t)\middle|\mathbf{k}-\frac{\mathbf{k}'}{2}\right\rangle$

Semi-classic subsystem: Boltzmann/Fokker-Planck equation

Lindblad equations for quarkonia at low $T \ll Mv$

quantum Brownian motion limit & quantum optical limit in pNRQCD

- After tracing out the QGP degrees of freedom, one gets a Lindblad-type equation:

$$\frac{\partial \rho}{\partial t} = -i[H_{\text{eff}}, \rho] + \sum_j \gamma_j \left(L_j \rho L_j^\dagger - \frac{1}{2} \left\{ L_j^\dagger L_j, \rho \right\} \right)$$

- This can be done in two different limits within pNRQCD:

Quantum Brownian Motion:

see works by Brambilla et al.

$$\tau_I \gg \tau_E$$
$$\tau_S \gg \tau_E$$

relevant for $Mv \gg T \gg Mv^2$

Quantum Optical:

$$\tau_I \gg \tau_E$$
$$\tau_I \gg \tau_S$$

see works by Yao et al.

relevant for $Mv \gg Mv^2, T$

Quantum Brownian Motion limit details

$$\frac{d\rho_S(t)}{dt} = -i\left[H_S + \Delta H_S, \rho_S(t)\right] + \kappa_{\rm adj}\left(L_{\alpha i}\rho_S(t)L_{\alpha i}^\dagger - \frac{1}{2}\left\{L_{\alpha i}^\dagger L_{\alpha i}, \rho_S(t)\right\}\right)$$

$$H_S = \frac{\mathbf{p}_{\rm rel}^2}{M} + \begin{pmatrix} -\frac{C_F\alpha_s}{r} & 0 \\ 0 & \frac{\alpha_s}{2N_c r} \end{pmatrix}, \qquad \Delta H_S = \frac{\gamma_{\rm adj}}{2} r^2 \begin{pmatrix} 1 & 0 \\ 0 & \frac{N_c^2-2}{2(N_c^2-1)} \end{pmatrix}$$

$$L_{1i} = \left(r_i + \frac{1}{2MT}\nabla_i - \frac{N_c}{8T}\frac{\alpha_s r_i}{r}\right)\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$L_{2i} = \sqrt{\frac{1}{N_c^2-1}}\left(r_i + \frac{1}{2MT}\nabla_i + \frac{N_c}{8T}\frac{\alpha_s r_i}{r}\right)\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$L_{3i} = \sqrt{\frac{N_c^2-4}{2(N_c^2-1)}}\left(r_i + \frac{1}{2MT}\nabla_i\right)\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

Heavy quark and quarkonia correlators

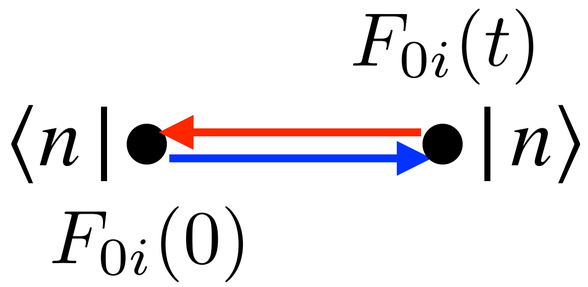
a small, yet consequential difference

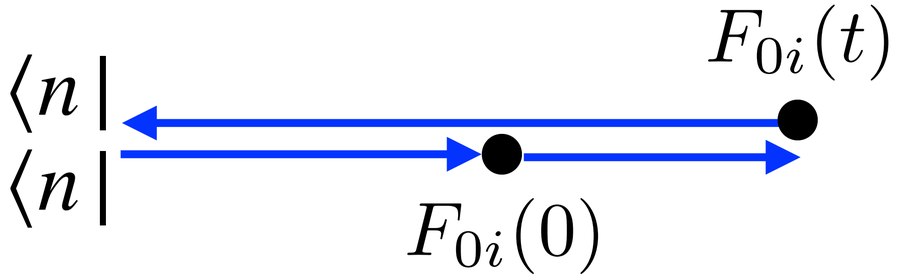
A. M. Eller, J. Ghiglieri and G. D. Moore, hep-ph/1903.08064

What we just found, and had been noticed even earlier by Eller, Ghiglieri and Moore, is simply stating that:

They compared M. Eidemuller and M. Jamin, hep-ph/9709419 with
Y. Burnier, M. Laine, J. Langelage and L. Mether, hep-ph/1006.0867

$$T_F \left\langle E_i^a(t) \mathscr{W}^{ab}(t,0) E_i^b(0) \right\rangle_T \neq \left\langle \mathrm{Tr}_{\mathrm{color}} \left[U(-\infty, t) E_i(t) U(t,0) E_i(0) U(0,-\infty) \right] \right\rangle_T$$





An axial gauge puzzle

an apparent (but not actual) inconsistency

- This finding presents a puzzle:
  - Let's say we were able to set axial gauge $A_0 = 0$.
  - Then, the two correlation functions would look the same:

$$T_F \left\langle E_i^a(t) E_i^a(0) \right\rangle_T = \left\langle \mathrm{Tr}_{\mathrm{color}} \left[E_i(t) E_i(0) \right] \right\rangle_T .$$

  - If true, this would imply that:
    A. one of the calculations is wrong, or
    B. one of the correlators is not gauge invariant.

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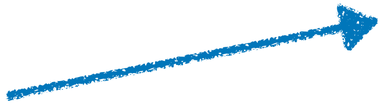

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BS and X. Yao, hep-ph/2205.04477

An axial gauge puzzle

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We verified that this difference between the correlators is gauge invariant using an interpolating gauge condition:

$$G_M^a[A] = \frac{1}{\lambda} A_0^a(x) + \partial^\mu A_\mu^a(x)$$

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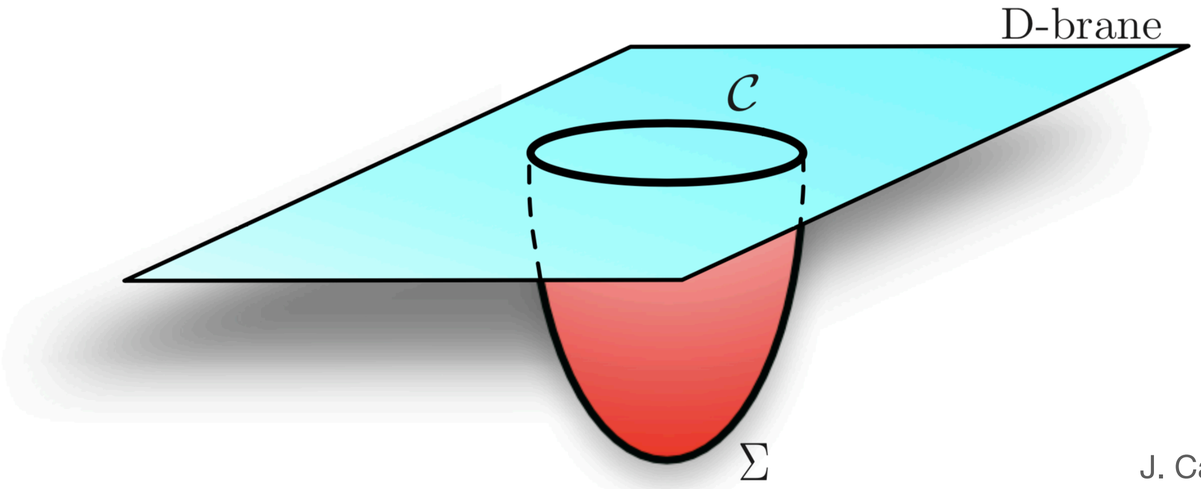
[**] J. Maldacena, hep-th/9711200

Wilson loops in AdS/CFT

setup

- The holographic duality provides a way to formulate the calculation of analogous correlators in strongly coupled theories. [**]
  - Wilson loops can be evaluated by solving classical equations of motion:

$$\langle W[\mathscr{C} = \partial\Sigma] \rangle_T = e^{iS_{\mathrm{NG}}[\Sigma]}$$



J. Casalderrey-Solana, H. Liu, D. Mateos, K. Rajagopal and U. A. Wiedemann, hep-ph/1101.0618

How do Wilson loops help?

setup — pure gauge theory

- Field strength insertions along a Wilson loop can be generated by taking variations of the path $\mathscr{C}$:

$$\frac{\delta}{\delta f^{\mu}(s_2)}\frac{\delta}{\delta f^{\nu}(s_1)}W[\mathscr{C}_f]\bigg|_{f=0} = (ig)^2\mathrm{Tr}_{\mathrm{color}}\Big[U_{[1,s_2]}F_{\mu\rho}(\gamma(s_2))\dot{\gamma}^{\rho}(s_2)U_{[s_2,s_1]}F_{\nu\sigma}(\gamma(s_1))\dot{\gamma}^{\sigma}(s_1)U_{[s_1,0]}\Big]$$

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- Same as the lattice calculation of the heavy quark diffusion coefficient:

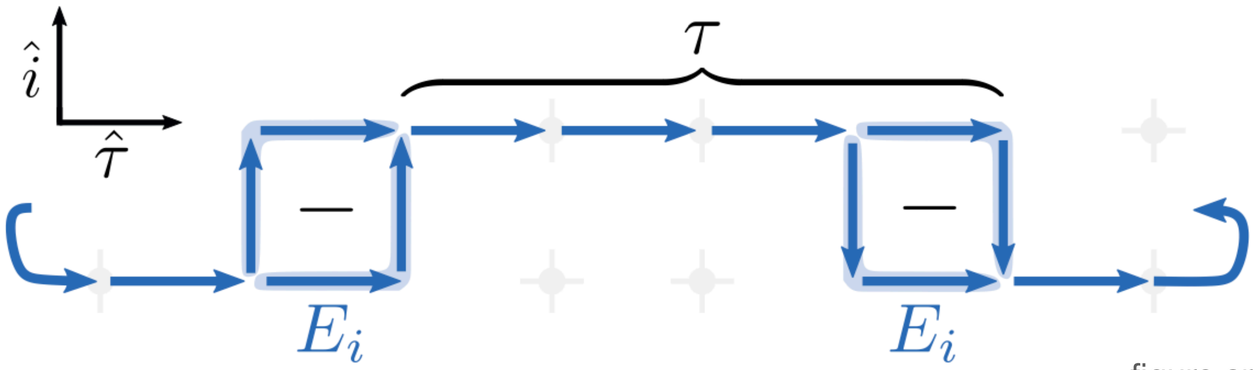

figure credit: L. Altenkort

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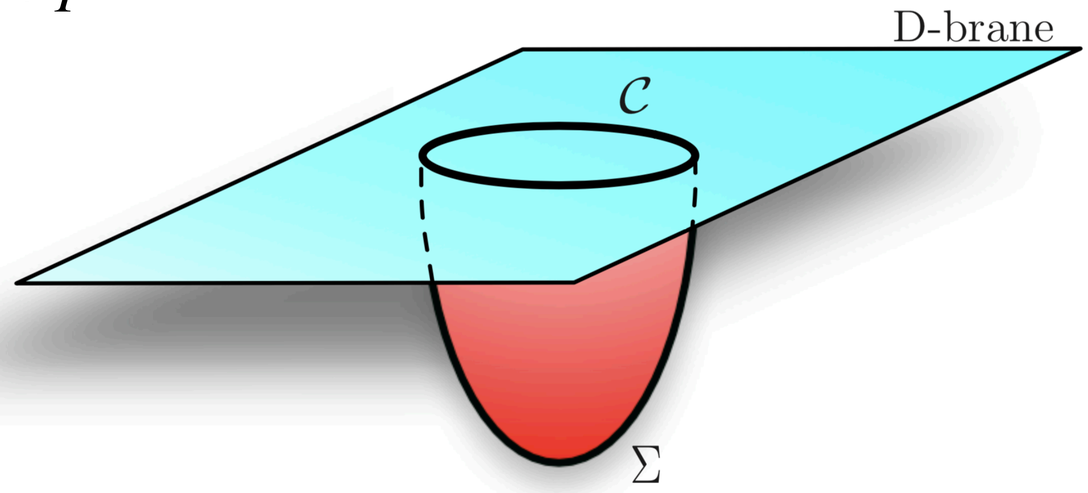
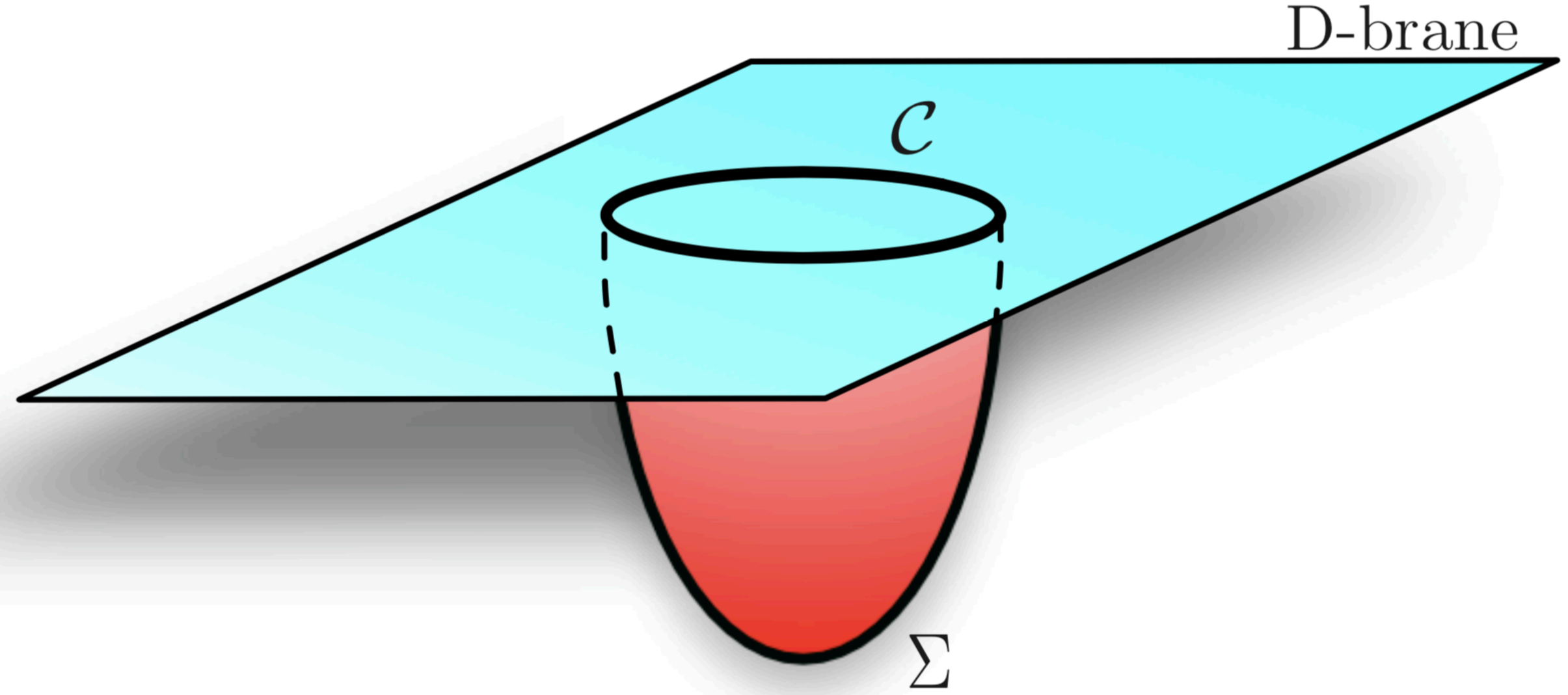


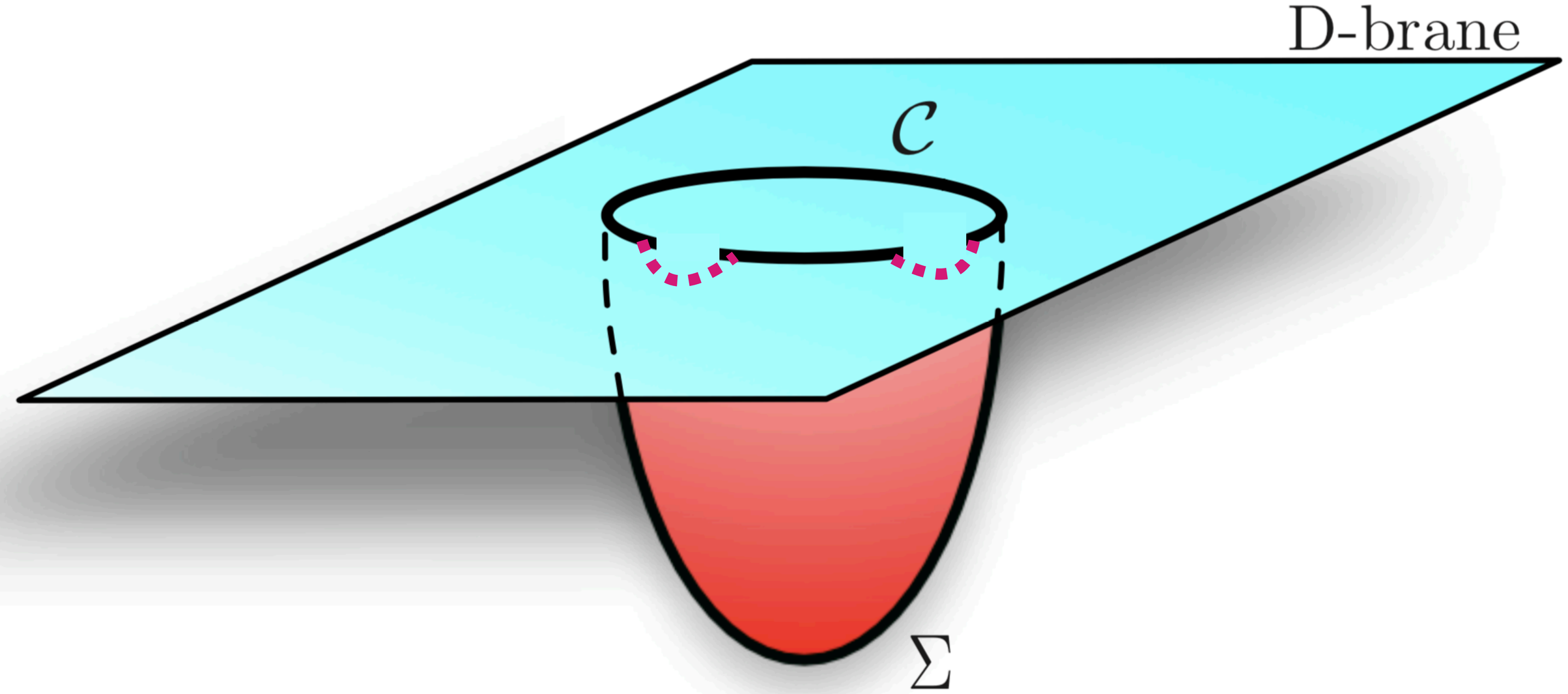
Metric of interest for finite T calculations:

$$ds^2 = \frac{R^2}{z^2}\left[-f(z)\,dt^2 + d\mathbf{x}^2 + \frac{1}{f(z)}dz^2 + z^2 d\Omega_5^2\right]$$

$$f(z) = 1 - (\pi T z)^4$$

J. Casalderrey-Solana, H. Liu, D. Mateos, K. Rajagopal and U. A. Wiedemann, hep-ph/1101.0618


D-brane
C
Σ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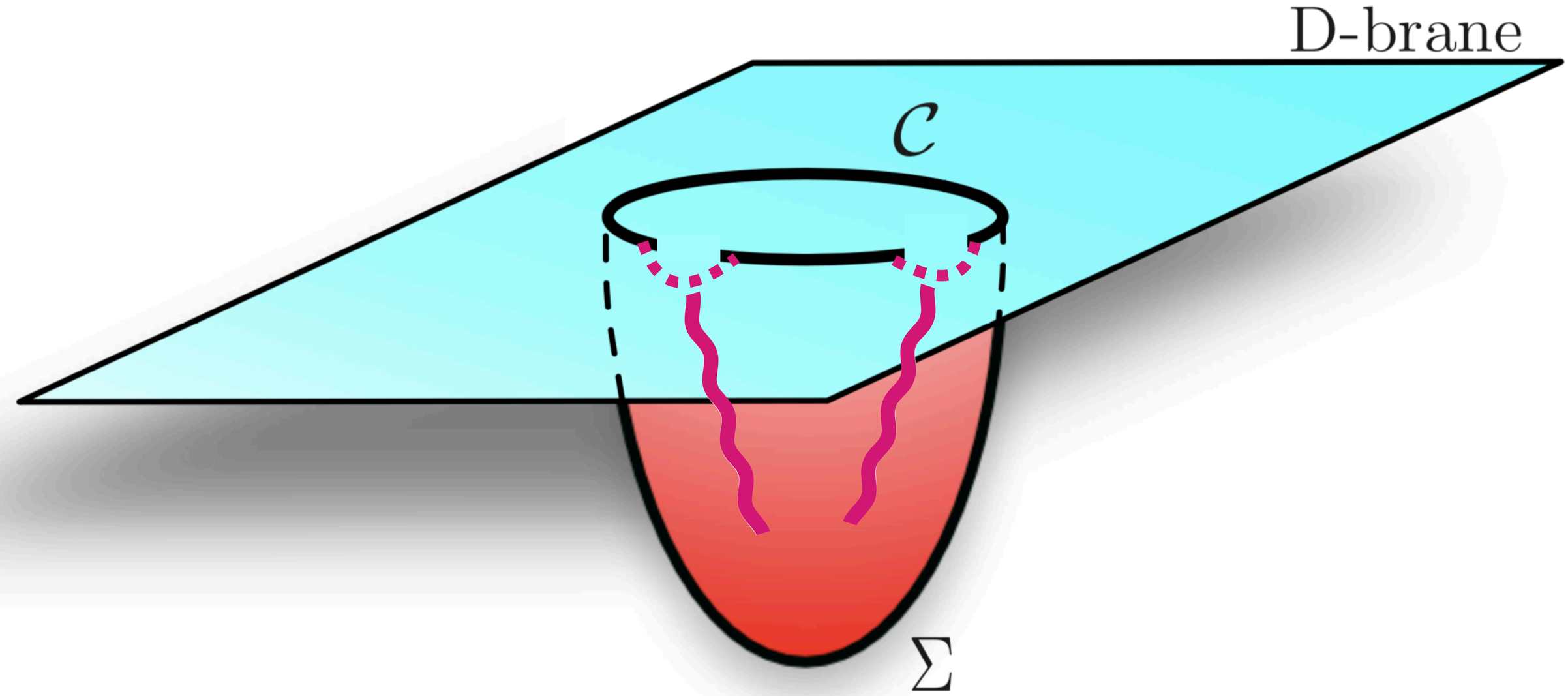
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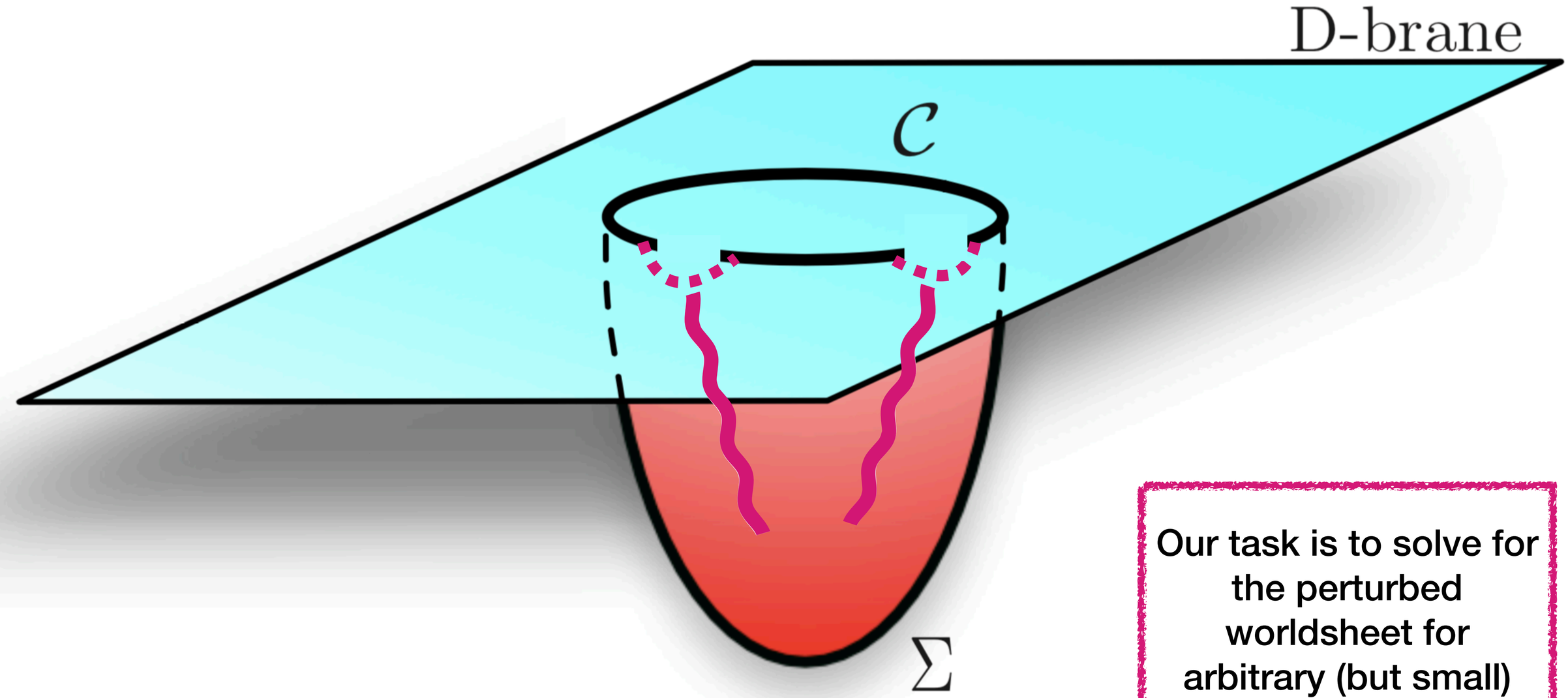

Our task is to solve for the perturbed worldsheet for arbitrary (but small) changes in the loop $\mathscr{C}$

Wilson loops in $\mathcal{N} = 4$ SYM

a slightly different observable

- A holographic dual in terms of an extremal surface exists for

$$W_{\text{BPS}}[\mathscr{C}; \hat{n}] = \frac{1}{N_c}\text{Tr}_{\text{color}}\left[\mathscr{P}\exp\left(ig\oint_{\mathscr{C}} ds\, T^a\left[A^a_\mu\, \dot{x}^\mu + \hat{n}(s)\cdot\vec{\phi}^{\,a}\sqrt{\dot{x}^2}\,\right]\right)\right],$$

which is *not* the standard Wilson loop.

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- $\mathcal{N} = 4$ SYM has 6 scalar fields $\vec{\phi}^{\,a}$, which enter the above Wilson loop through a direction $\hat{n} \in {\rm S}_5$. Also, its dual gravitational description is ${\rm AdS}_5 \times {\rm S}_5$.

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- What to do with this extra parameter? For a single heavy quark, just set $\hat{n} = \hat{n}_0$.

Choosing $\hat{n}$

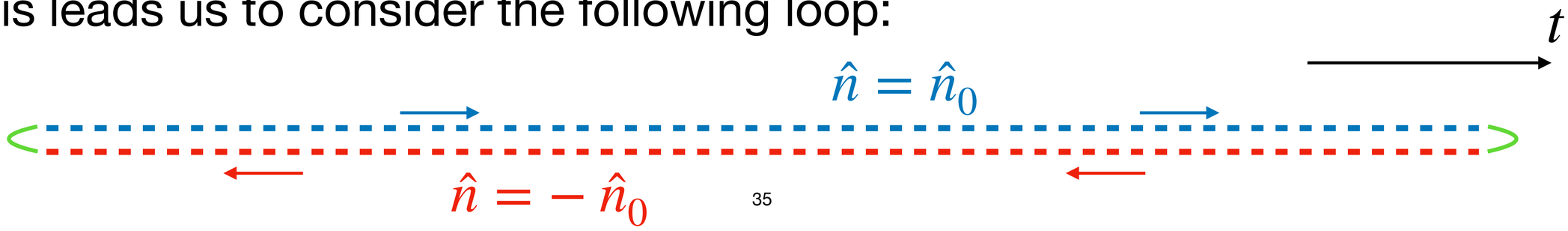
what is the best proxy for an adjoint Wilson line?

- A key property of the adjoint Wilson line is

$$\mathscr{W}^{ab}_{[t_2,t_1]} = \frac{1}{T_F}\mathrm{Tr}\left[\mathscr{T}\{T^a U_{[t_2,t_1]} T^b U^\dagger_{[t_2,t_1]}\}\right],$$

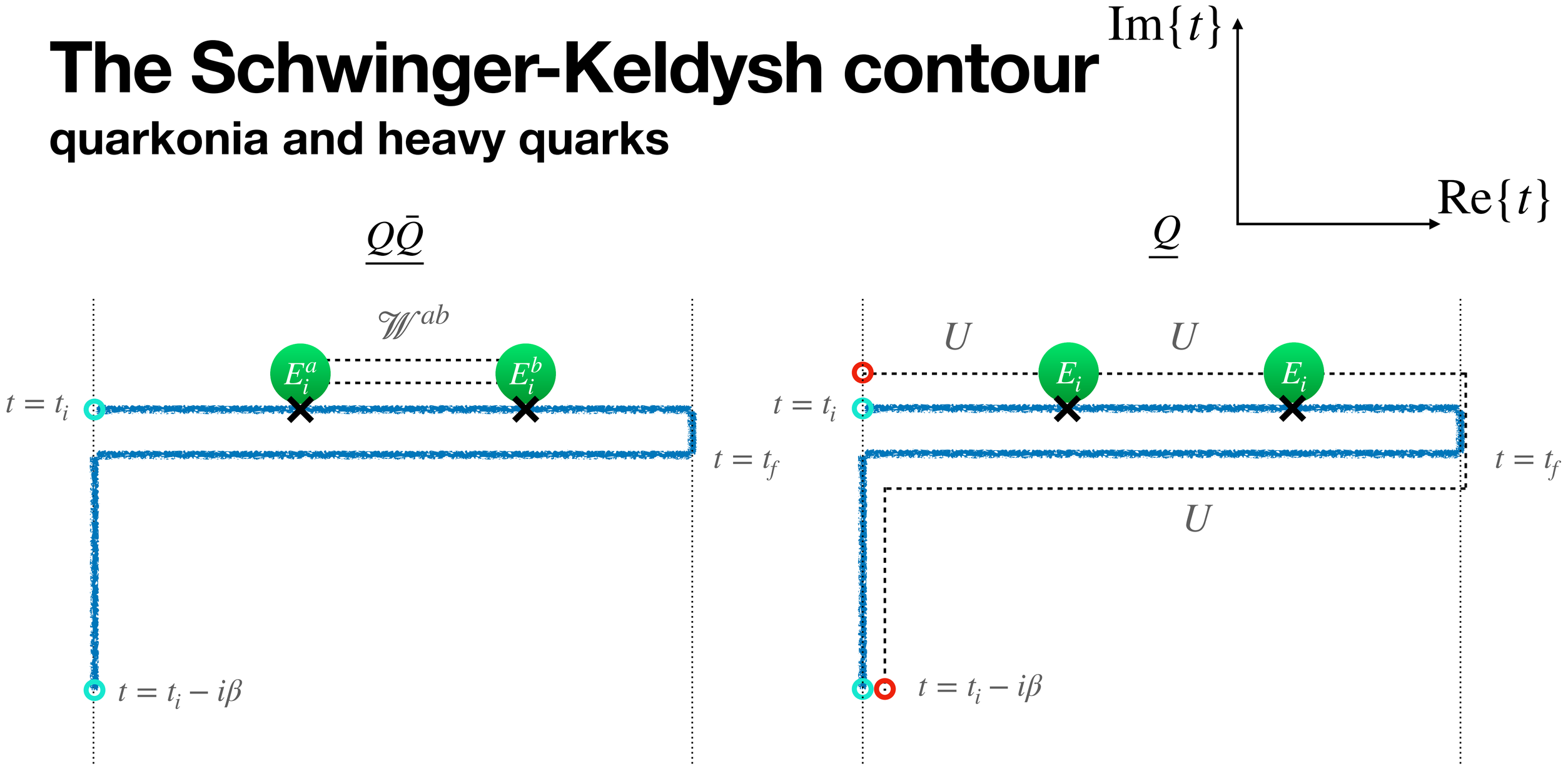
which means that we can obtain the correlator we want by studying deformations of a Wilson loop of the form $W = \frac{1}{N_c}\mathrm{Tr}\left[UU^\dagger\right] = 1.$

- This leads us to consider the following loop:



The Schwinger-Keldysh contour

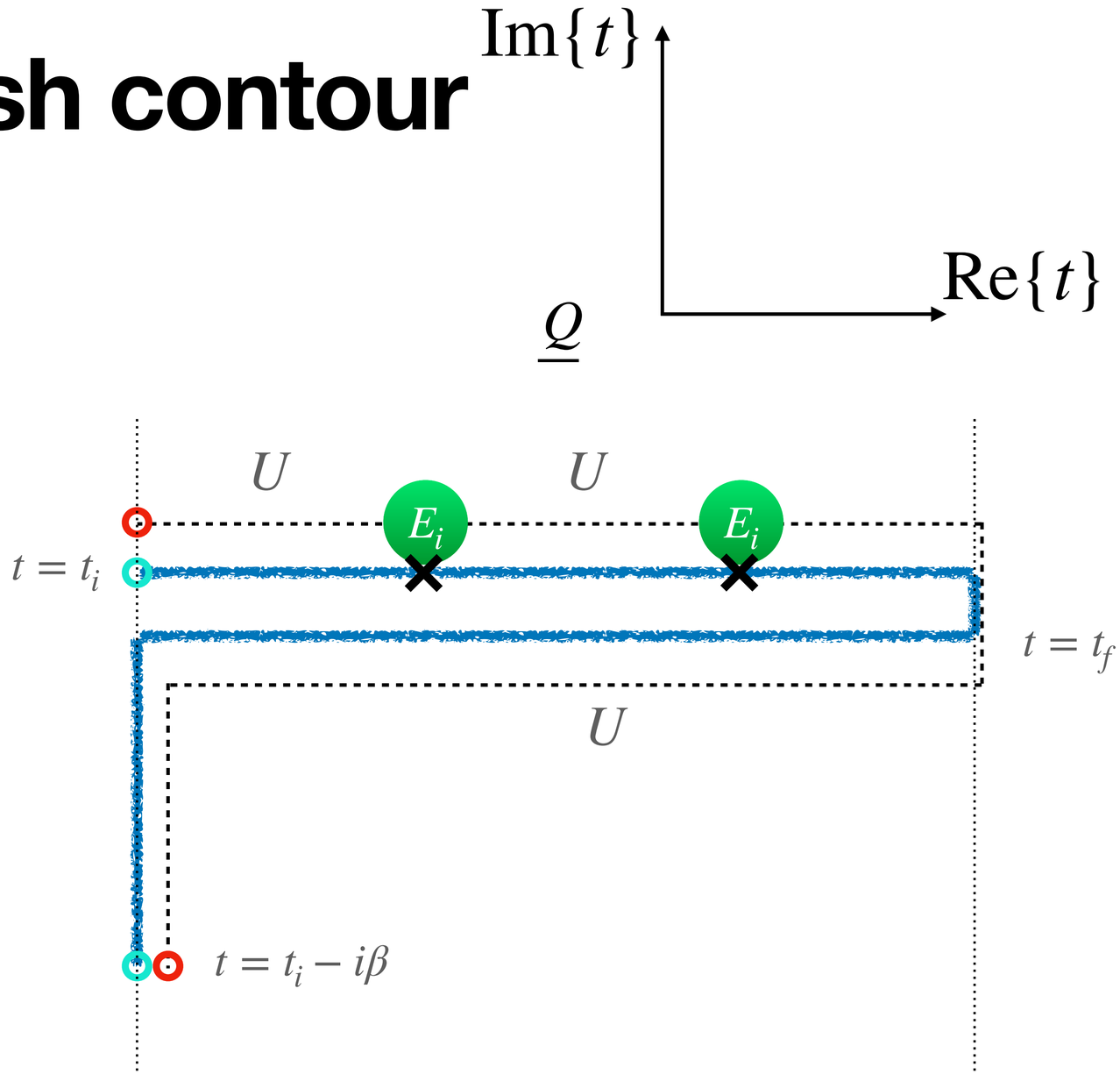
quarkonia and heavy quarks



The Schwinger-Keldysh contour

quarkonia and heavy quarks

- The heavy quark is present at all times:
  - It is part of the construction of the thermal state.
  - The Wilson line, which enforces the Gauss' law constraint due to the point charge, is also present on the Euclidean segment.



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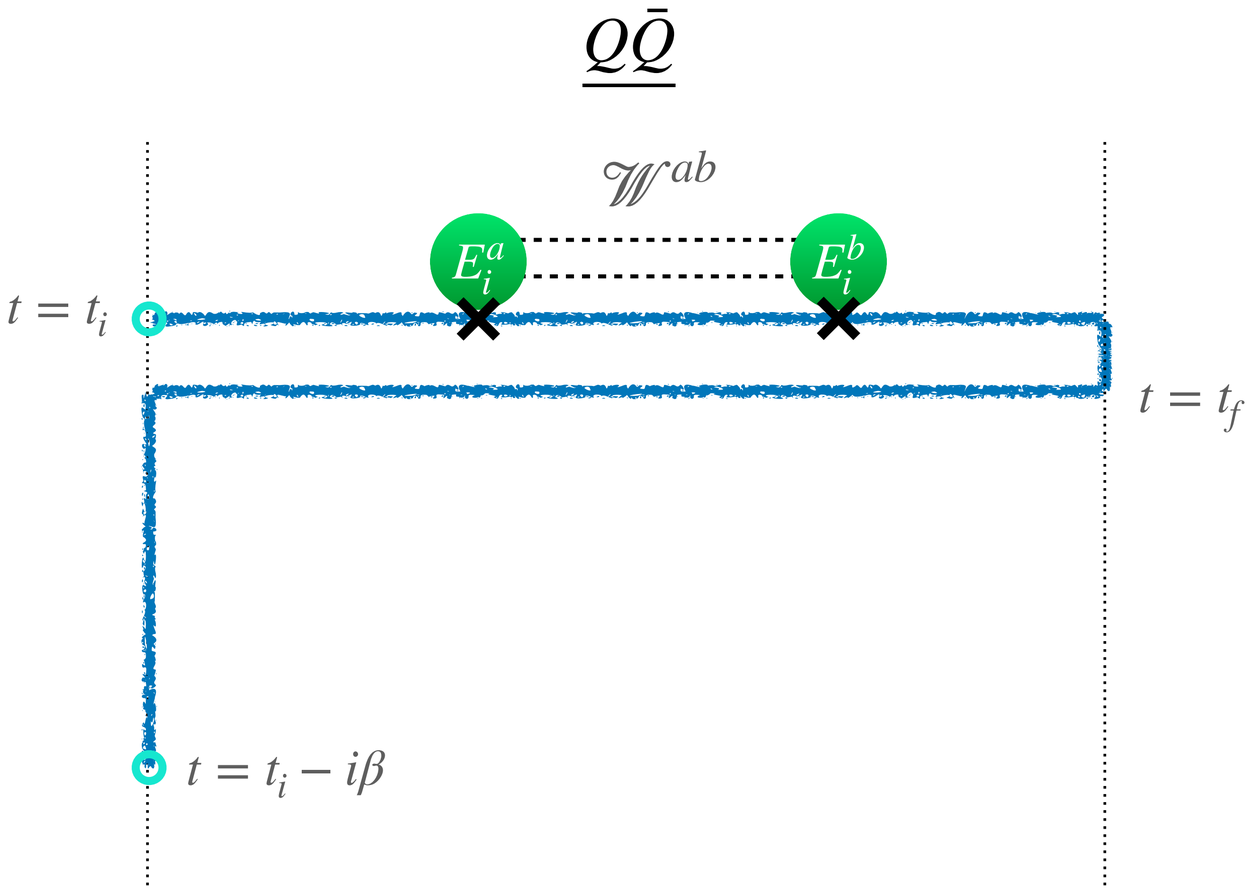


- In this correlator, the heavy quark pair is present at all times, but it is only color-charged for a finite time:
  - It is *not* part of the construction of the thermal state.
  - The adjoint Wilson line, representing the propagation of unbound quarkonium (in the adjoint representation), is only present on the real-time segment.

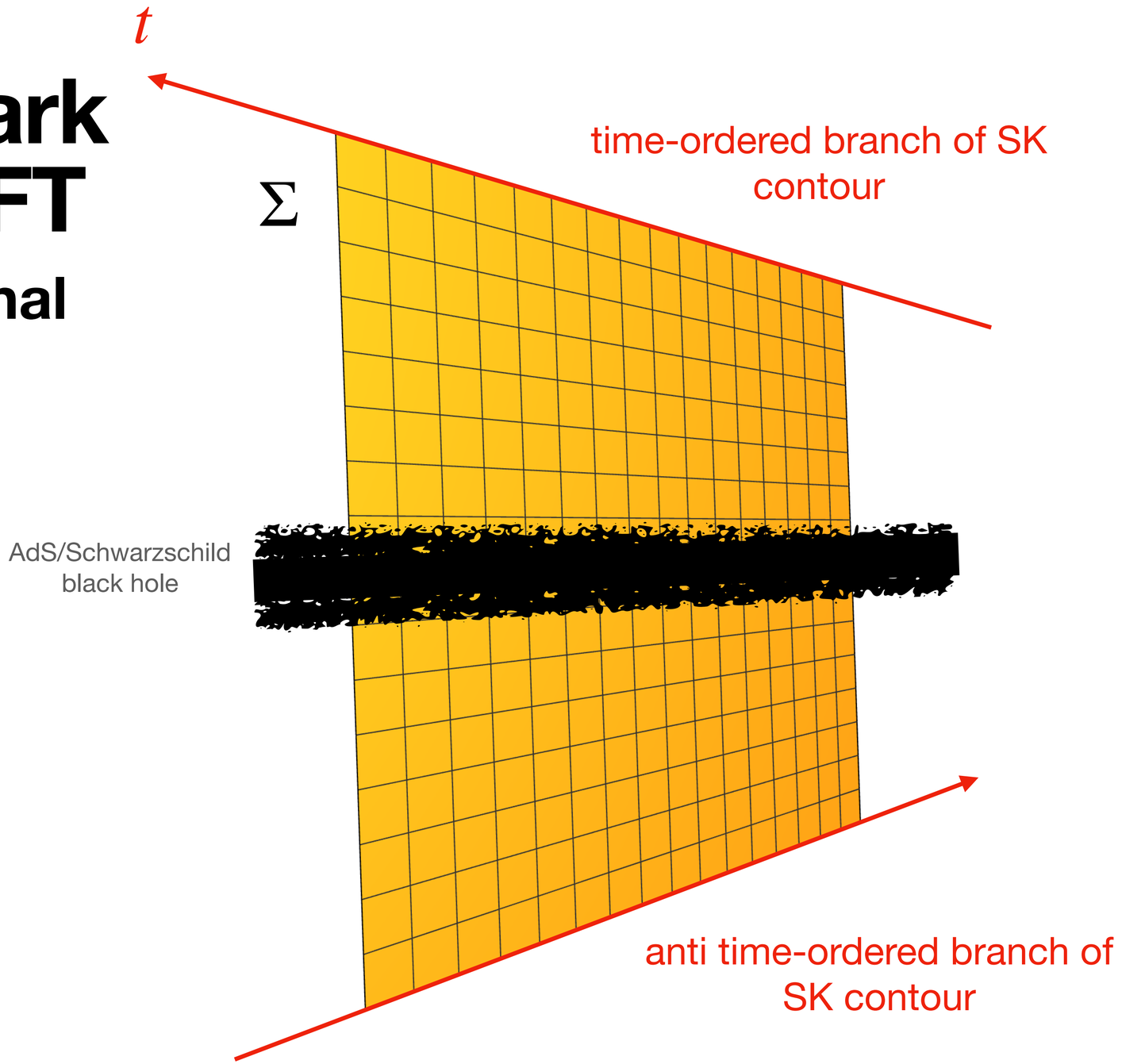
[***] J. Casalderrey-Solana and D. Teaney, hep-ph/0605199

Review: Heavy Quark Diffusion in AdS/CFT

using the same computational technique

Steps of the calculation:

1. Find the appropriate background solution



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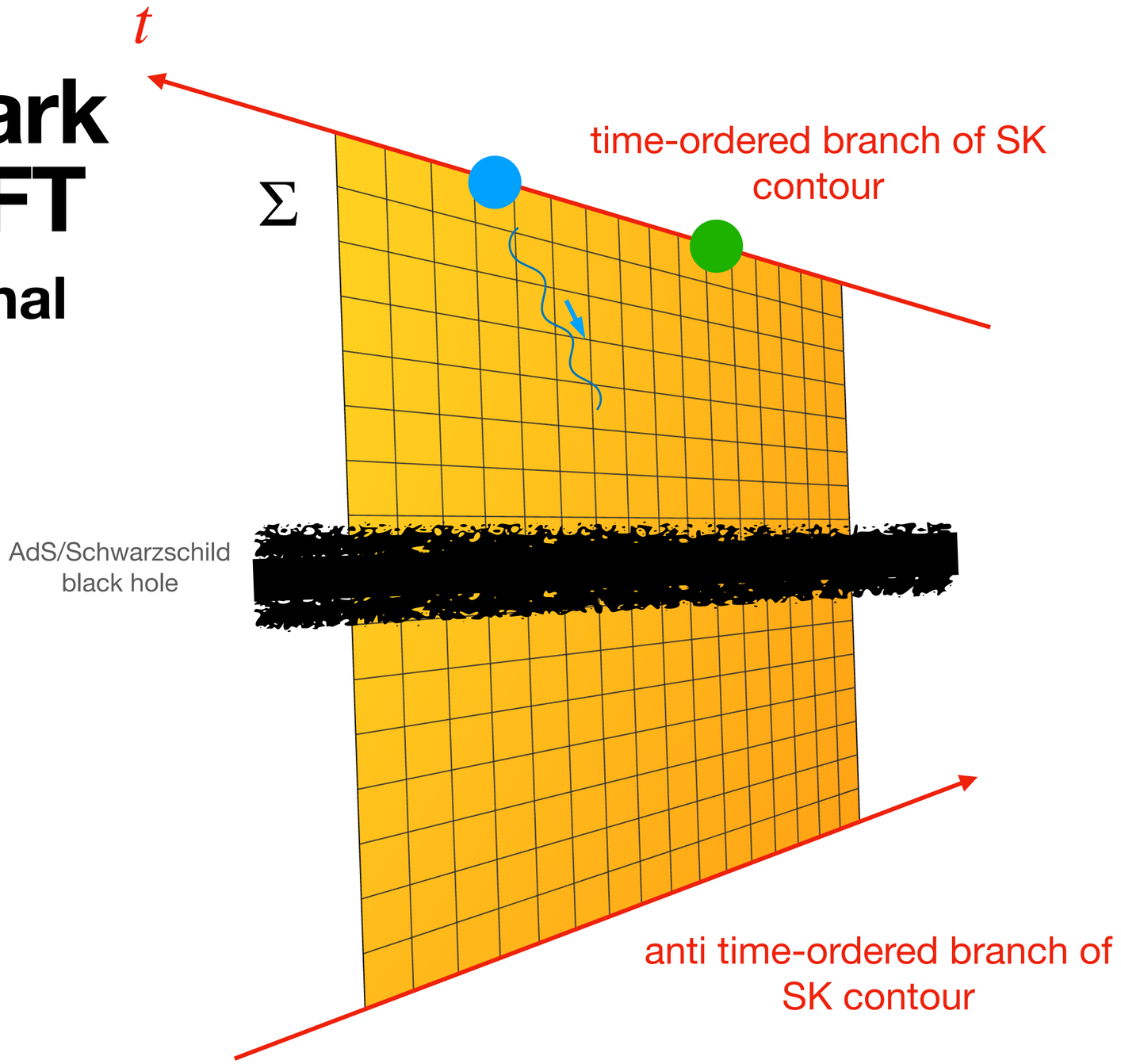
2. Introduce perturbations



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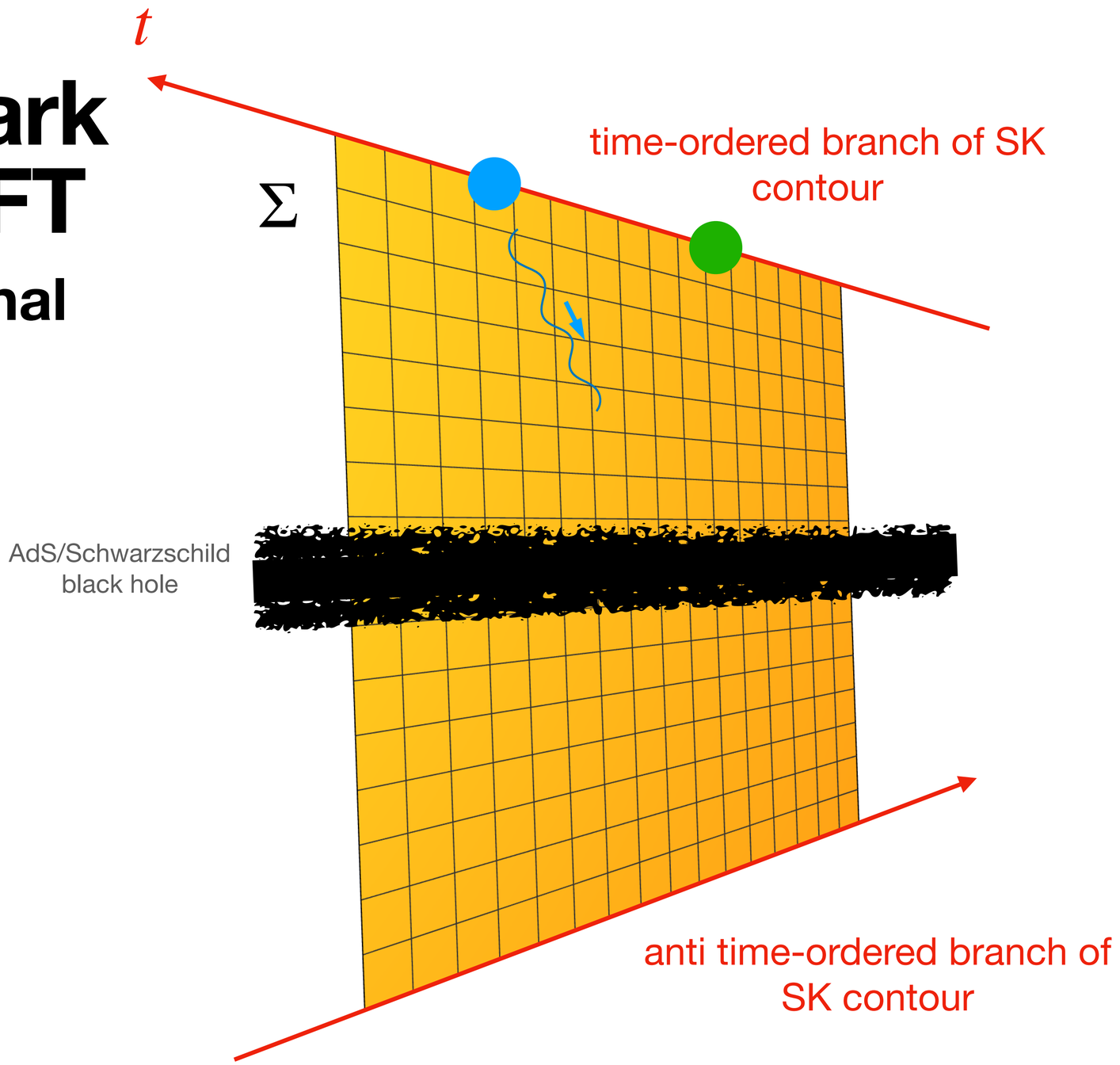

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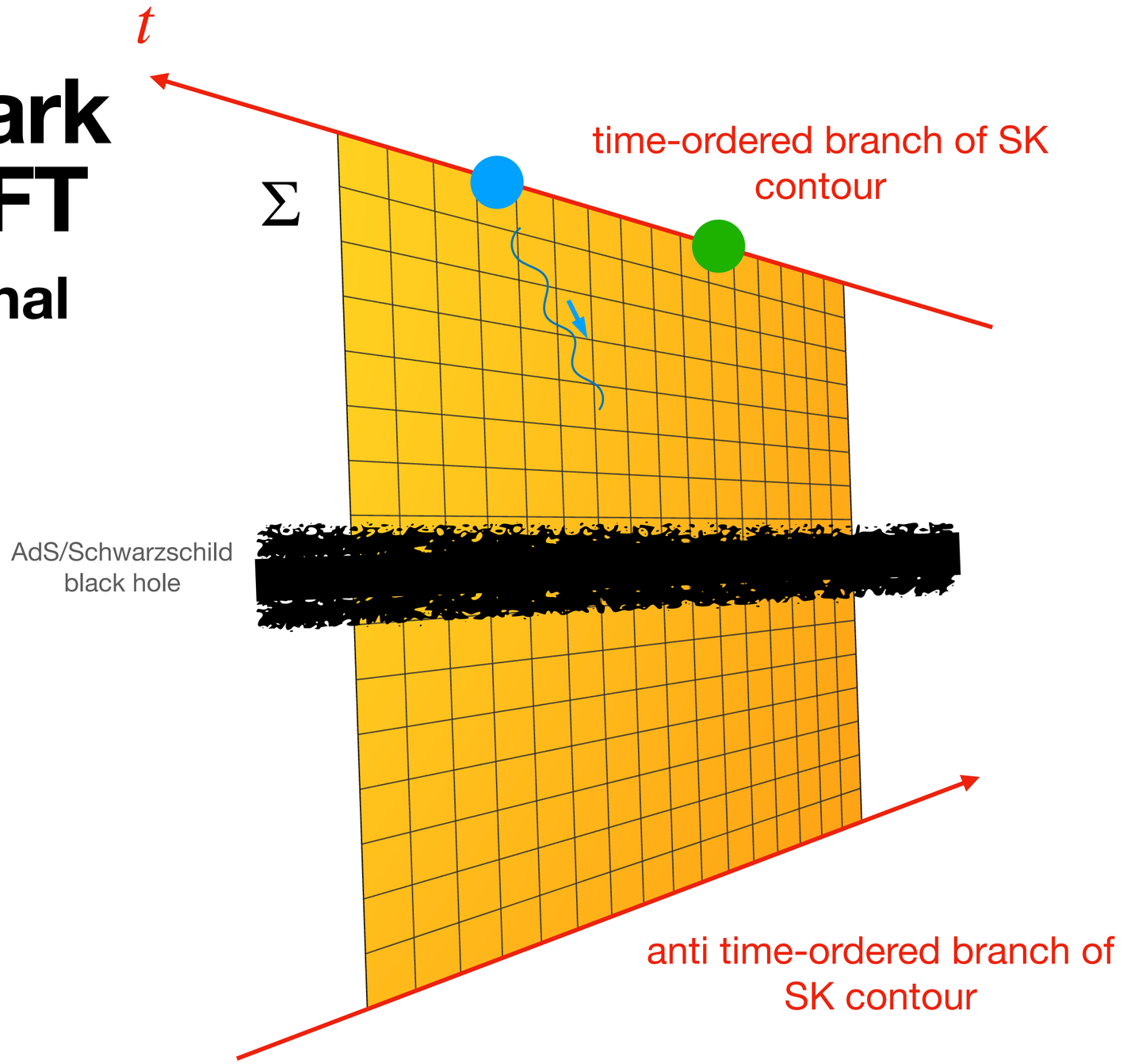
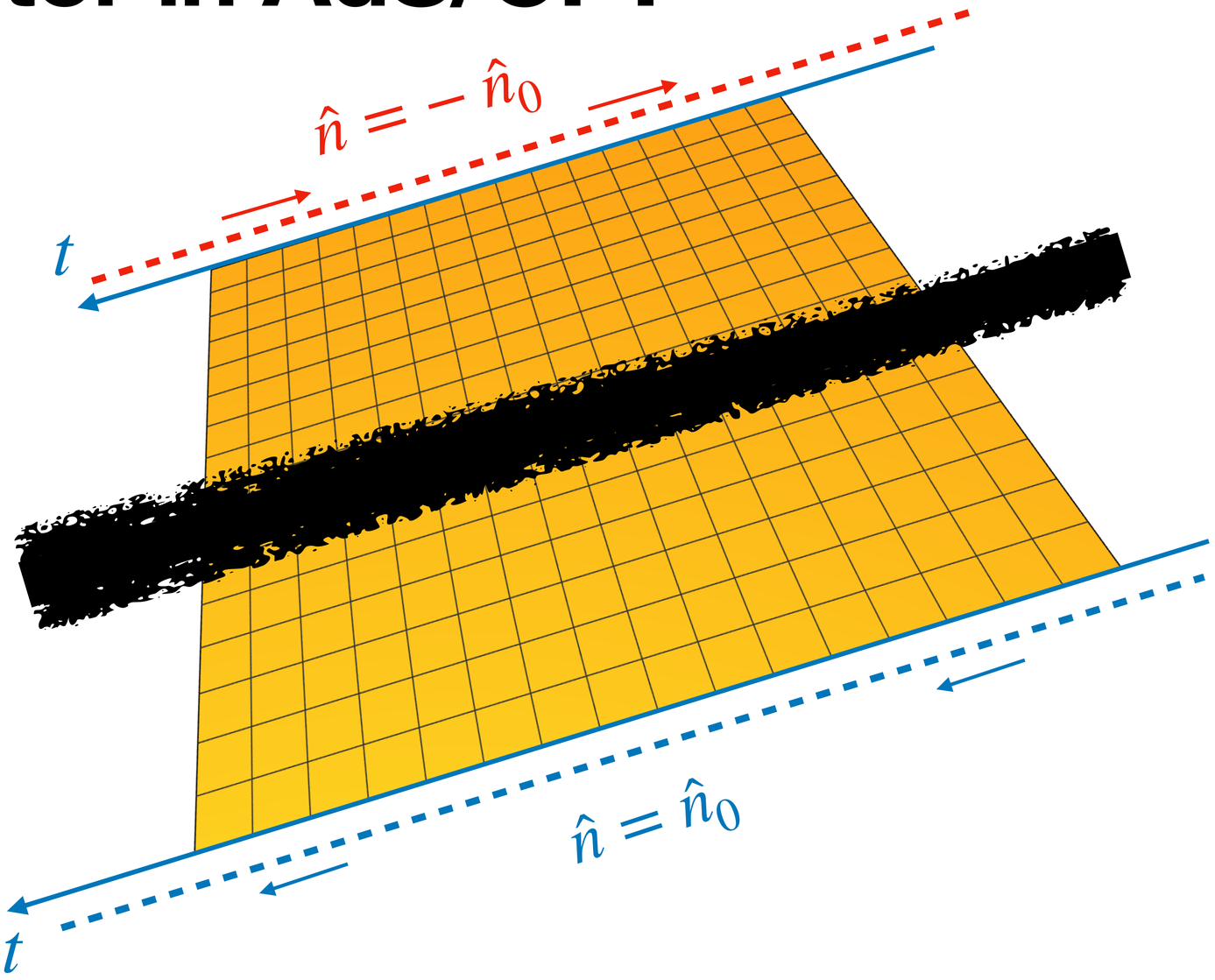
From here: $\kappa = \pi\sqrt{g^2 N_c}\, T^3$



Quarkonium correlator in AdS/CFT

a very similar picture

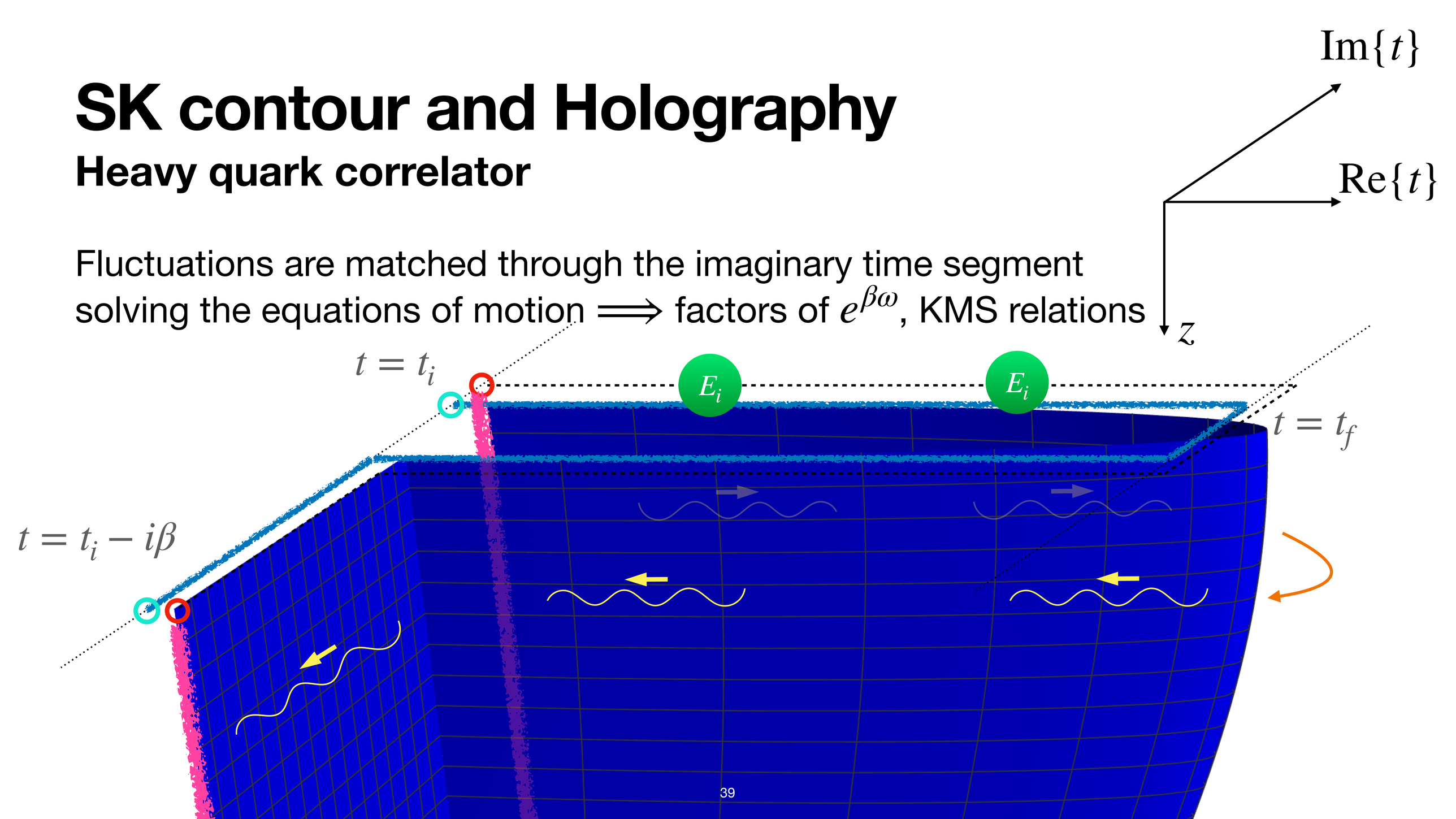
- Same steps as before:
  1. Find background solution
  2. Introduce perturbations
  3. Evaluate the derivatives
- Differences:
  - Boundary conditions
  - Time-ordered correlator; not retarded



SK contour and Holography

Heavy quark correlator

Fluctuations are matched through the imaginary time segment
solving the equations of motion $\Longrightarrow$ factors of $e^{\beta\omega}$, KMS relations



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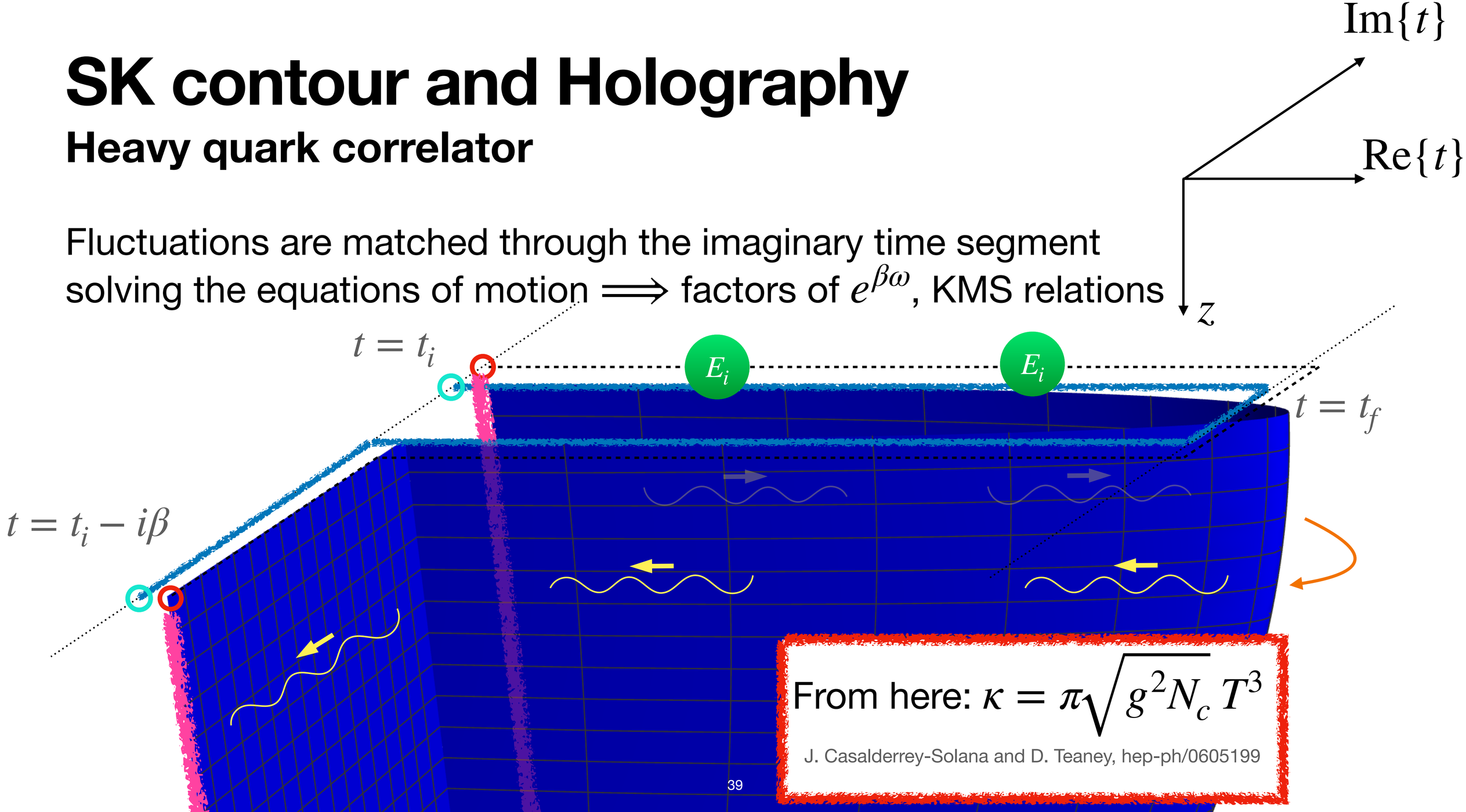


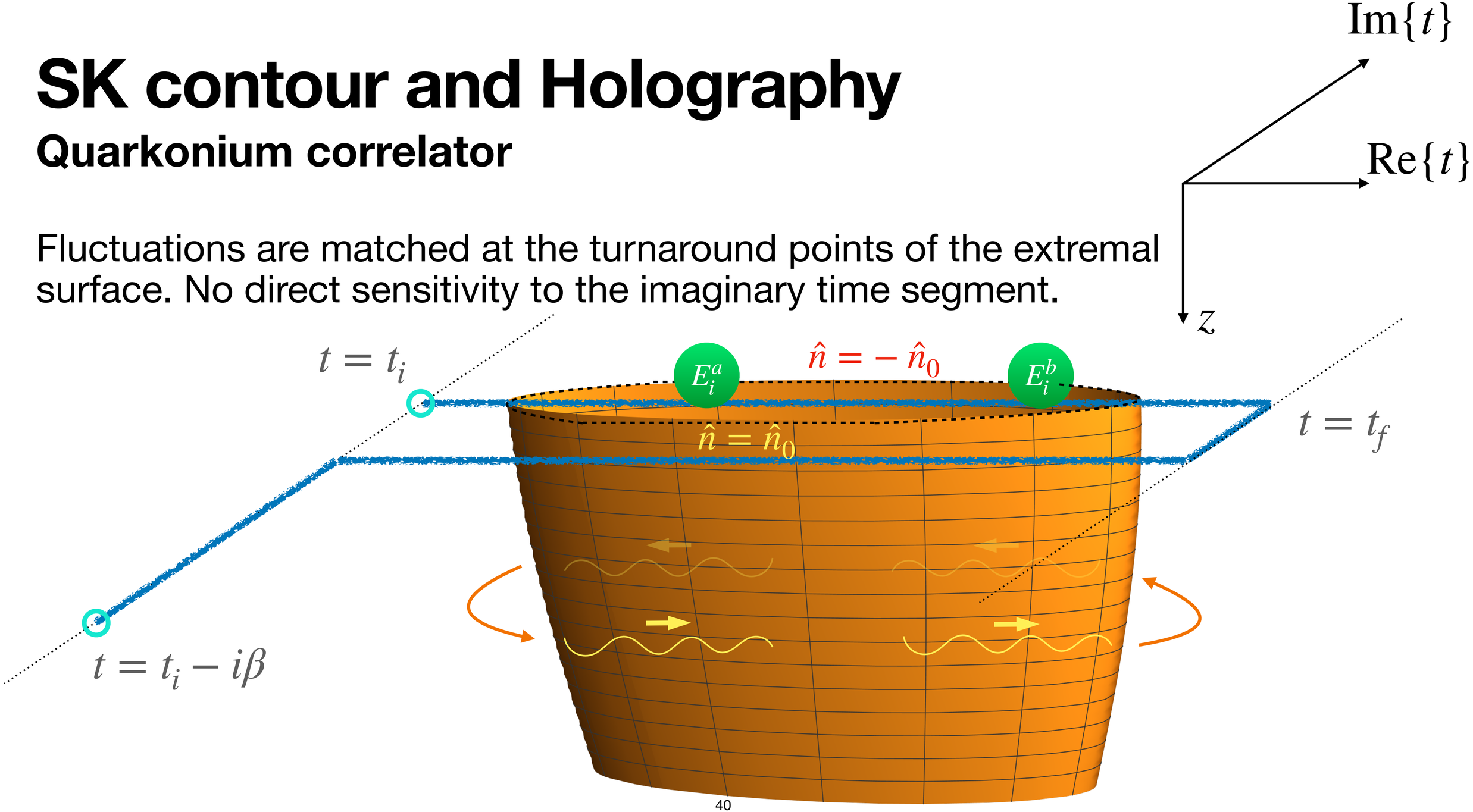
From here: $\kappa = \pi\sqrt{g^2 N_c}\, T^3$

J. Casalderrey-Solana and D. Teaney, hep-ph/0605199

SK contour and Holography

Quarkonium correlator

Fluctuations are matched at the turnaround points of the extremal surface. No direct sensitivity to the imaginary time segment.



How the calculation proceeds

what equations do we need to solve?

- The classical, unperturbed equations of motion from the Nambu-Goto action to determine Σ:

$$S_{\mathrm{NG}} = -\frac{1}{2\pi\alpha'}\int d\tau d\sigma \sqrt{-\det\left(g_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu\right)}\ .$$

- The classical, linearized equation of motion with perturbations in order to be able to calculate derivatives of $\langle W[\mathscr{C}_f]\rangle_T = e^{iS_{\mathrm{NG}}[\Sigma_f]}$:

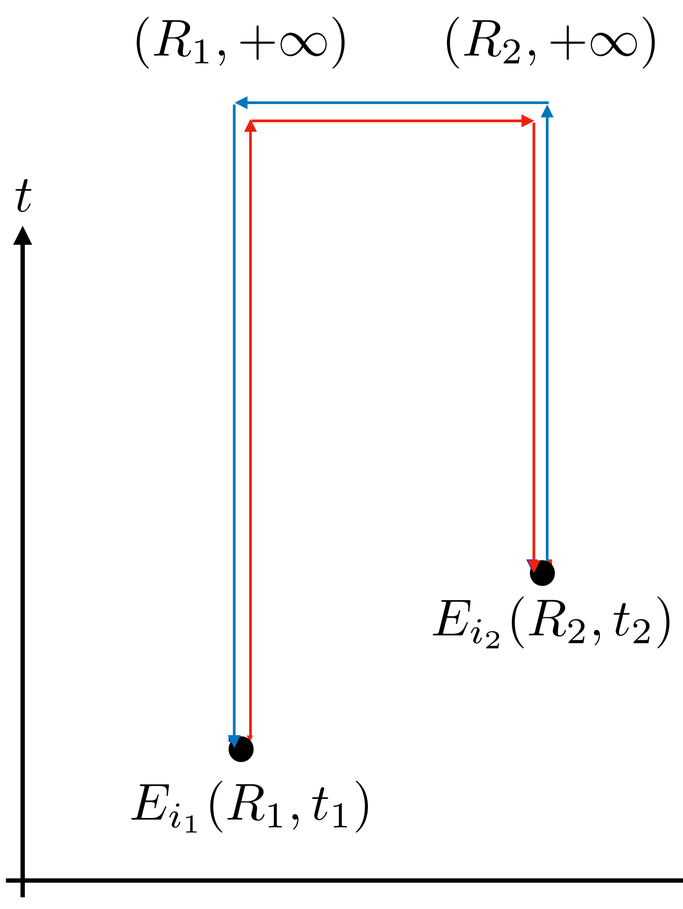
$$S_{\mathrm{NG}}[\Sigma_f] = S_{\mathrm{NG}}[\Sigma] + \int dt_1 dt_2 \left.\frac{\delta^2 S_{\mathrm{NG}}[\Sigma_f]}{\delta f(t_1)\delta f(t_2)}\right|_{f=0} f(t_1)f(t_2) + O(f^3)\ .$$

- In practice, the equations are only numerically stable in Euclidean signature, so we have to solve them and analytically continue back.

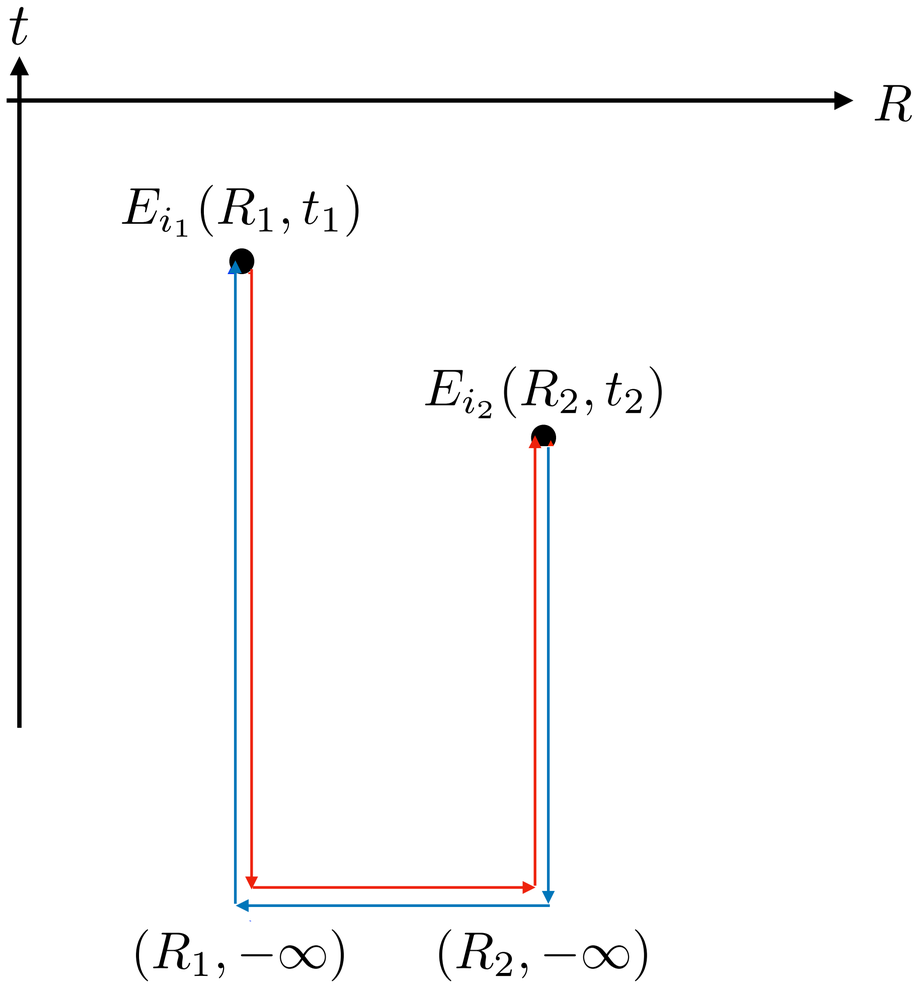
QGP chromoelectric correlators

for quarkonia transport

$$[g_E^{--}]^{>}_{i_2 i_1}(t_2, t_1, \mathbf{R}_2, \mathbf{R}_1) = \left\langle \left(\mathscr{W}_{2'} E_{i_2}(\mathbf{R}_2, t_2)\right)^a \left(E_{i_1}(\mathbf{R}_1, t_1)\mathscr{W}_{1'}\right)^a \right\rangle_T$$



Related by KMS conditions and parity + time reversal



$$[g_E^{++}]^{>}_{i_2 i_1}(t_2, t_1, \mathbf{R}_2, \mathbf{R}_1) = \left\langle \left(E_{i_2}(\mathbf{R}_2, t_2)\mathscr{W}_2\right)^a \left(\mathscr{W}_1 E_{i_1}(\mathbf{R}_1, t_1)\right)^a_{42} \right\rangle_T$$

The spectral function of quarkonia

symmetries and KMS relations

The KMS conjugates of the previous correlators are such that

$$[g_E^{++}]^{>}_{ji}(q) = e^{q^0/T}[g_E^{++}]^{<}_{ji}(q)\,, \quad [g_E^{--}]^{>}_{ji}(q) = e^{q^0/T}[g_E^{--}]^{<}_{ji}(q)\,,$$

and one can show that they are related by

$$[g_E^{++}]^{>}_{ji}(q) = [g_E^{--}]^{<}_{ji}(-q)\,, \quad [g_E^{--}]^{>}_{ji}(q) = [g_E^{++}]^{<}_{ji}(-q)\,.$$

The spectral functions $[\rho_E^{++/--}]_{ji}(q) = [g_E^{++/--}]^{>}_{ji}(q) - [g_E^{++/--}]^{<}_{ji}(q)$ are not necessarily odd under $q \leftrightarrow -q$. However, they do satisfy:

$$[\rho_E^{++}]_{ji}(q) = -\,[\rho_E^{--}]_{ji}(-q)\,.$$