Illinois Center for Advanced Studies of the Universe

UNIVERSITY OF ILLINOIS URBANA-CHAMPAIGN

The Grainger College of Engineering
Physics

A new approach to stochastic relativistic fluid dynamics from information flow

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arXiv:2306.08635, arXiv:2309.00512

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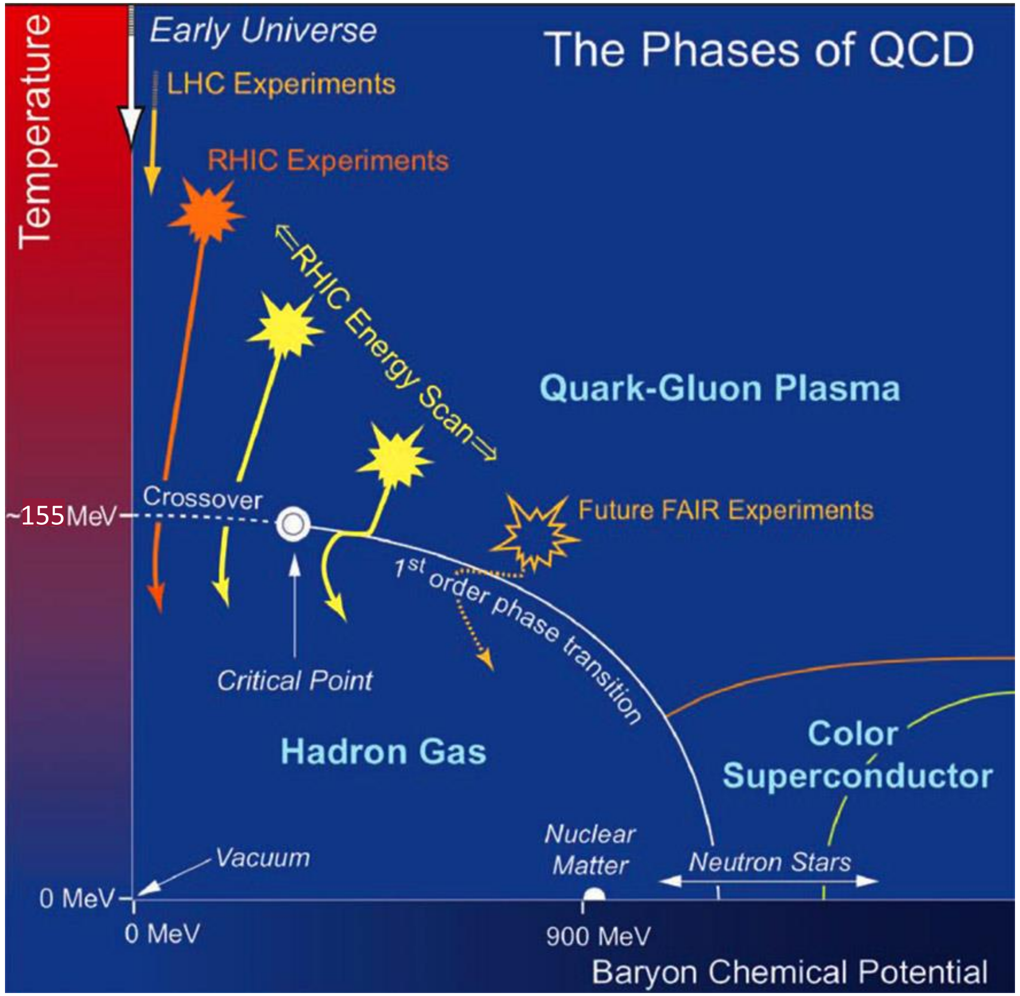
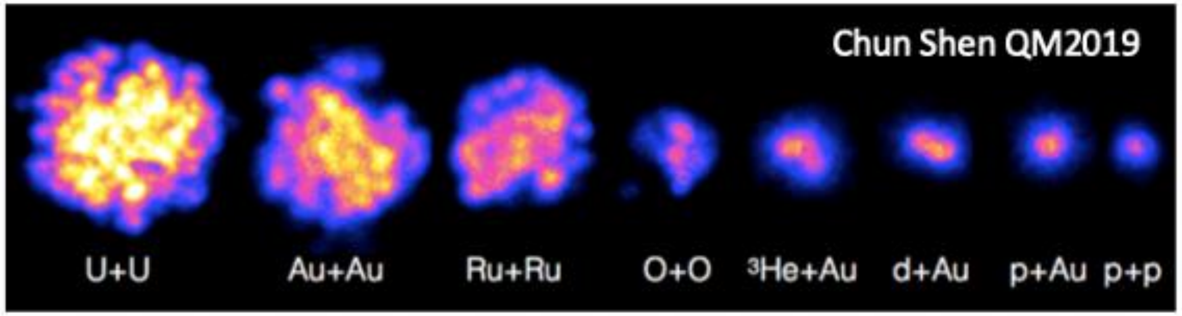
Stochastic fluctuations in the QGP



- Fluctuation-dissipation theorem implies thermal fluctuations in dissipative hydro.
- Correlation length of fluctuations diverges near critical point.
- Thermal fluctuations should become more important for small systems.



Approaches to fluctuating dynamics

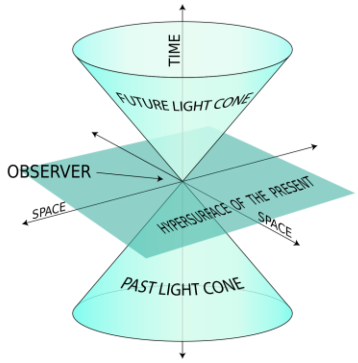
- Many different approaches for including fluctuations, e.g.:
- Obtain noise correlators from Green's functions.

  H.B. Callen, T. A. Welton PR 83 (1951); R. Kubo, J. Phys. Soc. Jpn. 12 (1957); P. Kovtun, J. Phys. A 45 (2012)
- Hydro-kinetics: derive equations of motion for correlation functions.

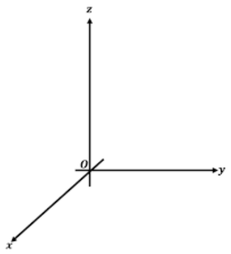
  Y. Akamatsu, A. Mazeliauskas, D. Teaney PRC 95 (2017); M. Martinez, T. Schafer PRC 99 (2019); X. An, G. Basar, M. Stephanov, H.U. Yee, PRC 100 (2019)
- Schwinger-Keldysh: construct effective field theory, use KMS symmetry to fix noise in action.

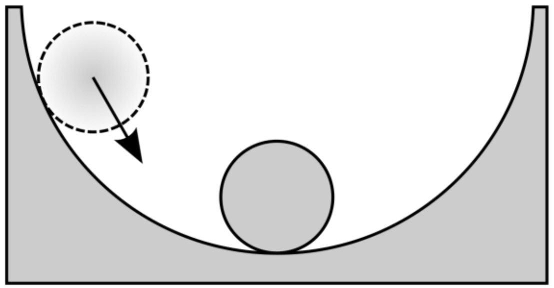
  L. M. Sieberer, A. Chiocchetta, A. Gambassi, U. C. Tauber, S. Diehl, PRB 92 (2015); M. Crossley, P. Glorioso, H. Liu, JHEP 09 (2017); H. Liu, P. Glorioso, TASI (2017)
- What is missing?

The missing piece



- Causality: information should remain within light-cone.
- Stability: fluctuations around equilibrium should not grow out of control.
- Causality is necessary for covariant stability.
- Previous formulations do not enforce these in a covariant manner.





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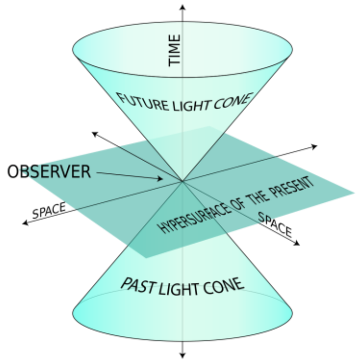

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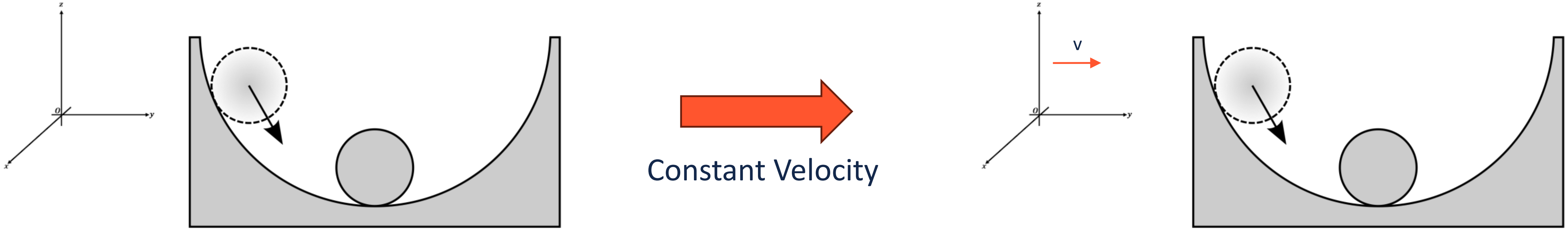


The information current

- Covariant fluctuations from maximum entropy principle.

L. D. Landau, E. M. Lifshitz, Statistical Physics (1980)

- Encoded in information current

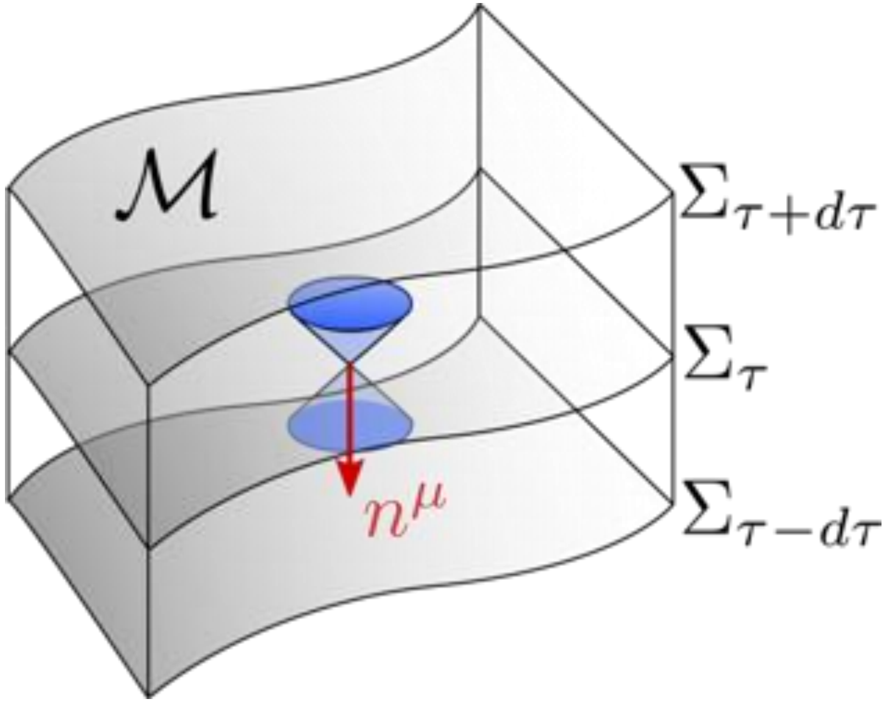
L. Gavassino, M. Antonelli, B. Haskell, PRL 128 (2022)

$$E^\mu = -\delta s^\mu - \beta_\nu \delta T^{\mu\nu} - \alpha_i \delta J_i^\mu$$

- Related to free energy by

$$\frac{\Omega}{T} = \int d\Sigma n_\mu E^\mu$$

$$\to p[\delta\phi] \sim e^{-\int d\Sigma n_\mu E^\mu}$$



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The information current

L. Gavassino, M. Antonelli, B. Haskell, PRL 128 (2022)

- System is causal and stable against fluctuations if

1. $n_\mu E^\mu \geq 0$ for all past-directed, timelike n^μ. Equilibrium minimizes the free energy.
2. $n_\mu E^\mu = 0$ iff the system is in equilibrium. Equilibrium is unique.
3. $\partial_\mu E^\mu \leq 0$, which is the second law of thermodynamics.

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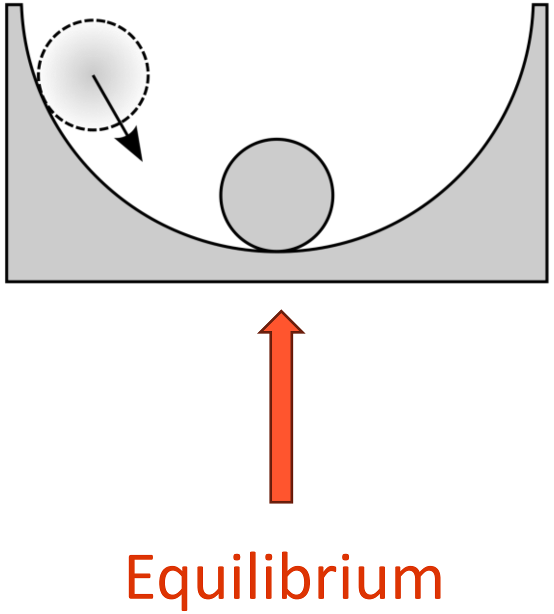

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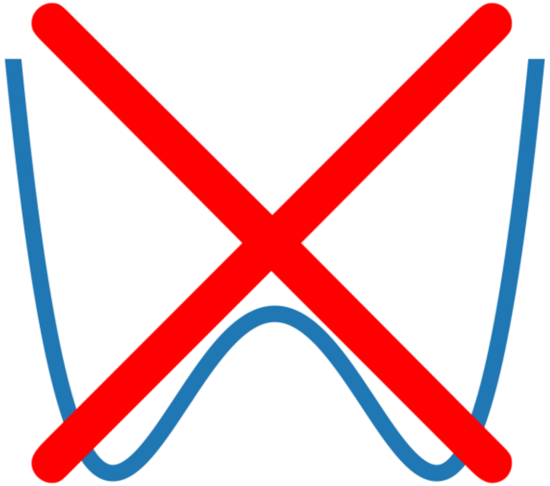

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Information current determines fluctuations

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- Using the information current, we constructed a new theory of fluctuations.

Construct EoM from information current.

$$\left(E^{\mu}_{AB}\partial_{\mu} + \sigma_{AB}\right)\delta\phi^{B} = \xi_{A} \qquad E^{\mu} = \frac{1}{2}\delta\phi^{A}E^{\mu}_{AB}\delta\phi^{B}$$

Write probability distribution using EoM

$$p[\delta\phi] \sim \exp\left[-\frac{1}{2}\int_{\Sigma} d\Sigma_1 d\Sigma_2 \delta\phi_A(x_1)\left(\langle\delta\phi^{A}(x_1)\delta\phi_B(x_2)\rangle\right)^{-1}\delta\phi^{B}(x_2)\right]$$

Compare to thermodynamic probability distribution.

$$p[\delta\phi] \sim e^{-\int_{\Sigma} d\Sigma n_{\mu} E^{\mu}}$$

Solve for noise correlator, note that foliation dependence drops out.

$$\langle\xi_A(x)\xi_B(x')\rangle = 2\sigma_{AB}\delta^{(4)}(x - x')$$

An example: diffusion

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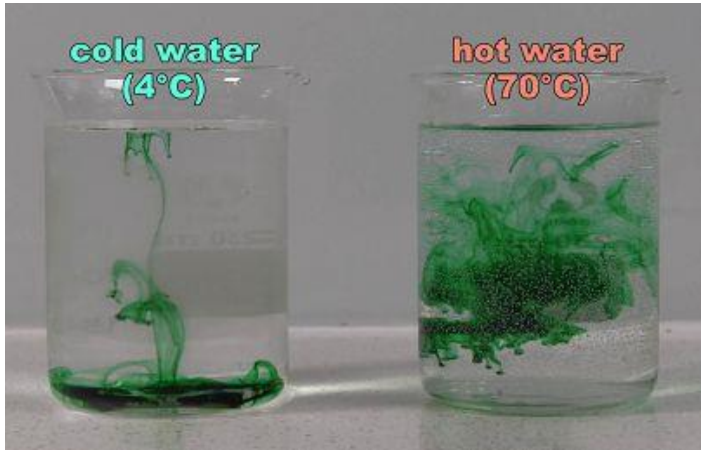

- Non-relativistic diffusion defined by

$$\partial_t \delta n - D\nabla^2 \delta n = \xi$$

- Maxwell-Cattaneo model (compatible with relativity)

$$\tau \partial_t^2 \delta n + \partial_t \delta n - D\nabla^2 \delta n = \xi$$

- Perform order reduction by introducing new vector degree of freedom

$$\frac{1}{\chi T}\left(\partial_t \delta n + \partial_i \delta q^i\right) = \xi_n$$

$$\frac{1}{\chi T}\partial_i \delta n + \frac{\beta_q}{T}\partial^t \delta q_i + \frac{1}{\kappa T}\delta q^i = \xi_q^i$$

$$\Rightarrow$$

$$J^\nu = n u^\nu + q^\nu$$

$$\frac{q^\nu}{\kappa T} = -\frac{\beta_J}{T} u^\lambda \partial_\lambda q^\nu - \Delta^{\lambda\nu}\partial_\lambda(\mu/T)$$

Spacelike projector

An example: diffusion

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- Information current and entropy production are

$$E^{\mu}_{AB} = \frac{1}{\chi T}\begin{pmatrix} u^{\mu} & \Delta^{\mu}{}_{\nu} \\ \Delta^{\mu\rho} & \beta_J \chi u^{\mu} \Delta^{\rho}{}_{\nu} \end{pmatrix} \qquad \sigma_{AB} = \frac{1}{\kappa T}\begin{pmatrix} 0 & 0 \\ 0 & \Delta^{\rho}{}_{\nu} \end{pmatrix}$$

- Fluctuation-dissipation theorem gives

$$\langle \xi_n(x)\xi_n(x')\rangle = 0, \quad \langle \xi^{\mu}_q(x)\xi^{\nu}_q(x')\rangle = \frac{1}{\kappa T}\Delta^{\mu\nu}\delta^{(4)}(x - x')$$

- Construction in terms of on-shell objects.

- Symmetrized Green's function obtained from EoM.

Effective Action Approach

The Martin-Siggia-Rose (MSR) action

- Action approaches useful for e.g., calculating Green's functions and non-linear extensions.
- MSR approach allows stochastic differential equation to be written as path integral over an effective action.

P. C. Martin, E. D. Siggia, H. A. Rose, PRA 8 (1973)

- Working from the information current, we find

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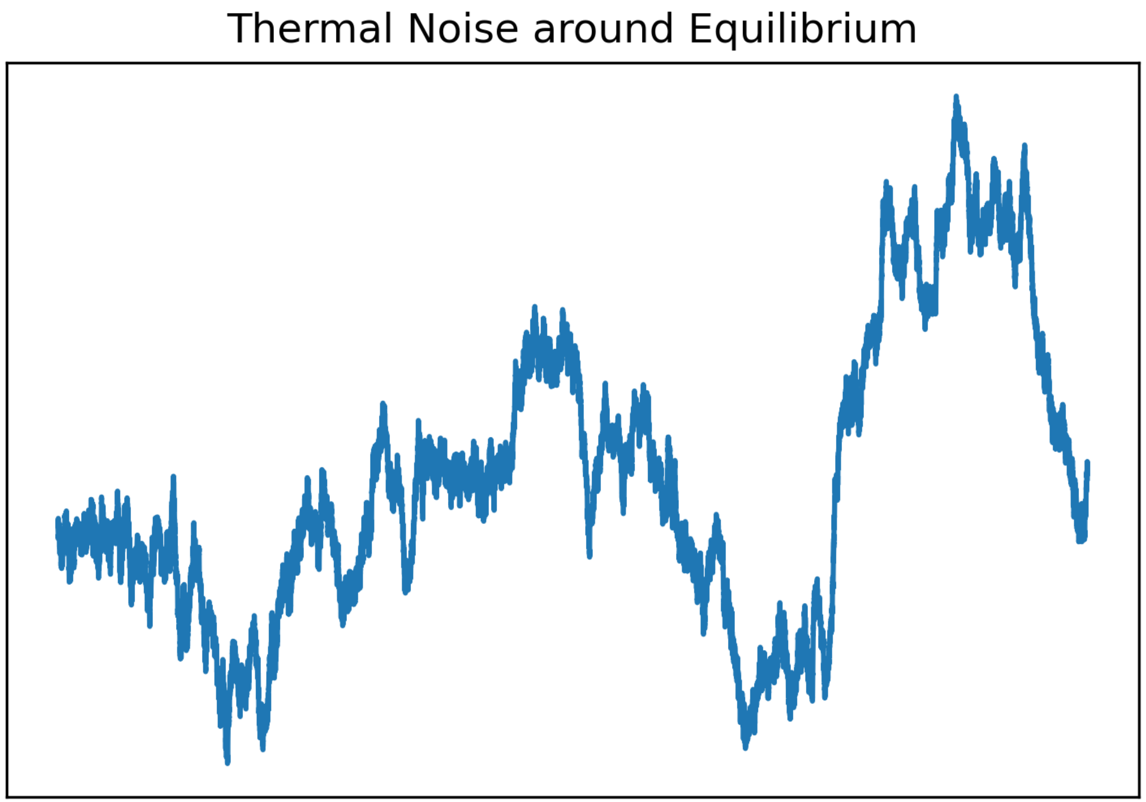
$$\mathcal{L}_{\mathrm{MSR}} = -\delta\bar{\phi}^A(x)\left(E^{\mu}_{AB}\partial_\mu + \sigma_{AB}\right)\delta\phi^B(x) + \frac{i}{2}\int d^4x'\,\delta\bar{\phi}^A(x)\langle\xi_A(x)\xi_B(x')\rangle\delta\bar{\phi}^B(x')$$

- Does this have a symmetry that implements FDT to all orders?

Detailed balance

J. Guo, P. Glorioso, A. Lucas, PRL 129 (2022); G. Torrieri, JHEP 02 (2022)



- Is this going forwards in time, or backwards?

New symmetry from detailed balance

J. Guo, P. Glorioso, A. Lucas, PRL 129 (2022); G. Torrieri, JHEP 02 (2022)

- Detailed balance allows comparison to other approaches

$$P[\delta\phi_f^A(x\in\Sigma_f)|\delta\phi_0^A(x\in\Sigma_0)]\,w[\delta\phi_0^A] = P[\Theta\delta\phi_0^A(x\in\Sigma_0)|\Theta\delta\phi_f^A(x\in\Sigma_f)]\,w[\delta\phi_f^A]$$

- Writing probability distributions as path integrals over the action gives,

$$\mathcal{L}_\theta = \mathcal{L} + i\delta\phi^A E^\mu_{AB}\partial_\mu\delta\phi^B$$

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- Used to derive a symmetry of the effective action with the form

$$\phi^A \to \Theta\phi^A,\;\; \bar{\phi}^A \to -\Theta\delta\bar{\phi}^A - i\Theta\delta\phi^A$$

Symmetry can be mapped to KMS symmetry.

- Detailed balance holds nonlinearly.

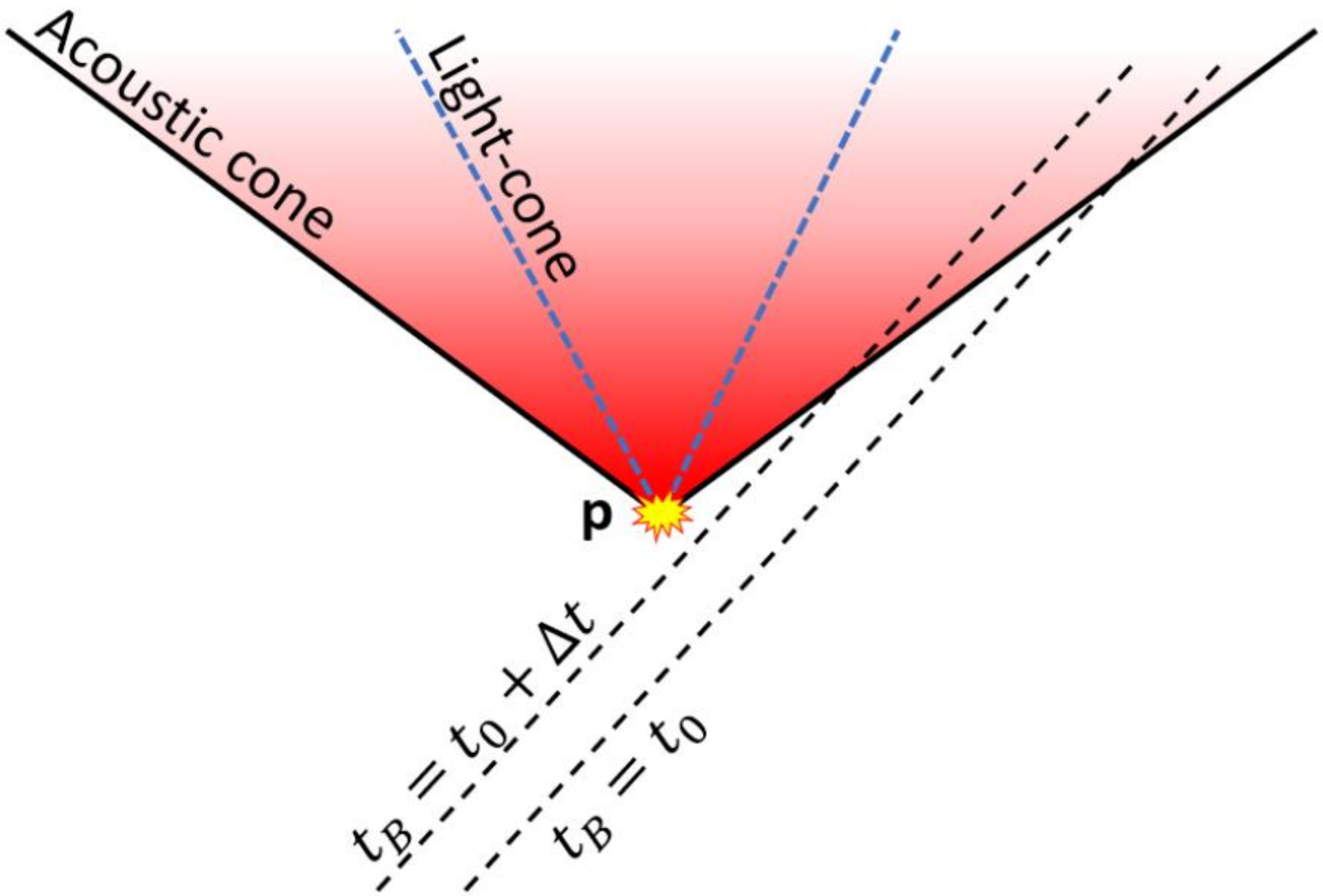
Conclusions

- Causality and stability are essential for fluctuations in relativistic fluids.
- New covariant approach for causal and stable stochastic hydrodynamics.
- Action formulation based on new (modified KMS) symmetry implementing detailed balance/FDT to all orders.
- Outlook: critical dynamics, nonlinear/non-Gaussian fluctuations.

Bonus Slides

Causality is necessary for covariant stability

L. Gavassino PRX 12 (2022)



The Problem with Fluctuating Relativistic Navier-Stokes

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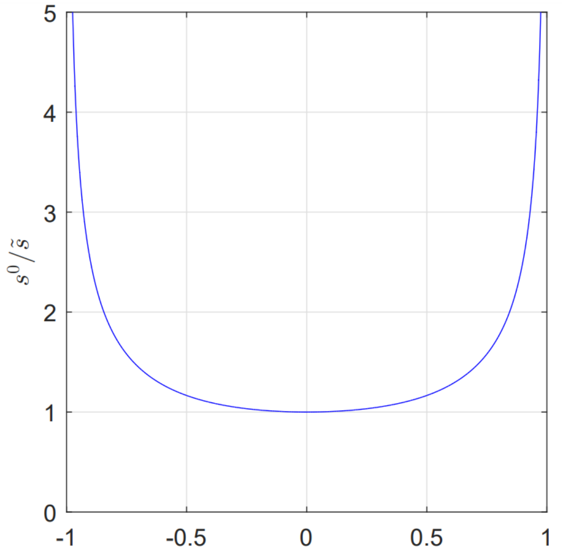
- Information current is given by

$$E^{\mu} \sim A u^{\mu} \delta u^{2} + B \Delta^{\mu\nu}_{\alpha\beta} \delta u_{\nu} \partial^{\alpha} \delta u^{\beta}$$

- Demanding that $n_{\mu} E^{\mu} = 0$ yields a PDE with homogenous solutions, this violated stability and causality conditions.

- Working in local rest frame hides the issue, doesn't fix issue.

- Most probable state have large gradients.



L. Gavassino, M. Antonelli, B. Haskell, PRD 102 (2020)

The Problem with Fluctuating BDNK theory

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F. Bemfica, M. Disconzi, J. Noronha, PRD 98 (2018); F. Bemfica, M. Disconzi, J. Noronha, PRD 100 2019; P. Kovtun, JHEP 10 (2019)

- Consider BDNK theory for conserved current, EoM is

$$T u^\nu u^\alpha \partial_\nu \left(\lambda \partial_\alpha (\mu/T)\right) + T\chi u^\nu \partial_\nu (\mu/T) - T\Delta^{\nu\alpha}\partial_\nu \left(\kappa \partial_\alpha (\mu/T)\right) = 0$$

- Causality and stability require $0 < \lambda/\kappa \leq 1$, parameters must have same sign.

- Schwinger-Keldysh effective action is given by

$$\mathcal{L}_{\mathrm{SK}} = J^\mu B_{a\mu} + i\,T\left(\kappa\Delta^{\mu\nu} - \lambda u^\mu u^\nu\right) B_{a\mu} B_{a\nu}$$

- Path integral diverges in stable and causal regime, inherent in first-order theories.

Fluctuations in Israel-Stewart

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- Consider conformal Israel-Stewart in a general hydrodynamic frame

J. Noronha, M. Spalinski, E. Speranza, PRL 128 (2022)

- Information current:

$$
\begin{aligned}
E^\mu &= \frac{c_s^2 u^\mu}{2(\epsilon+P)T}\delta\epsilon^2 + \frac{(\epsilon+P)u^\mu}{2T}\delta u^\nu \delta u_\nu + \frac{\delta P \delta u^\mu}{T} + \frac{\delta\mathcal{A}\delta u^\mu}{3T} + \frac{u^\mu}{T}\delta u^\nu \delta Q_\nu + \\
&+ \frac{\delta u_\nu \delta\pi^{\mu\nu}}{T} + \frac{u^\mu}{T^2}\delta\mathcal{A}\delta T + \frac{\delta T \delta Q^\mu}{T^2} + \frac{u^\mu}{2T}\left[\frac{\tau_A}{4\epsilon\tau_\phi}\delta\mathcal{A}^2 + \frac{\tau_Q}{4\epsilon\tau_\psi}\delta Q^2 + \frac{\tau_\pi}{\eta}\delta\pi^2\right]
\end{aligned}
$$

- Entropy production:

$$
\sigma = \frac{1}{2T}\left(\frac{\delta\mathcal{A}^2}{4\epsilon\tau_\phi} + \frac{\delta Q^2}{4\epsilon\tau_\psi} + \frac{\delta\pi^2}{\eta}\right)
$$

Fluctuations in Israel-Stewart

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- Energy-momentum correlator can be decomposed as P. K. Kovtun, A. O. Starinets, PRD 72 (2005)

$$\langle T^{\mu\nu} T^{\alpha\beta} \rangle = G_1(k) S^{\mu\nu\alpha\beta} + G_2(k) Q^{\mu\nu\alpha\beta} + G_3(k) L^{\mu\nu\alpha\beta}$$

- In Landau frame, there is only one structure $\Delta^{\mu\nu\alpha\beta}$.

- General frame is distinct from Landau frame, important for describing the most general possible fluctuations.

Action for Diffusion

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- For the example of diffusion, considered, the MSR action is

$$\mathcal{L}_{\rm MSR} = -\frac{\delta\bar{n}}{\chi T}\left(u^\mu\partial_\mu\delta n + \partial_\mu\delta q^\mu\right) - \frac{\delta\bar{q}^\mu}{\chi T}\left(\partial_\mu\delta n + \beta_J\chi u^\nu\partial_\nu\delta q_\mu + \frac{\chi}{\kappa}\delta q_\mu\right) + \frac{i}{\kappa T}\delta\bar{q}^2$$

- This couples to a source through terms of the form

$$\mathcal{L}_{\rm source} = \bar{A}_\mu J^\mu - A_\mu \mathcal{L}_\beta \bar{J}^\mu$$

- Allows for advanced, retarded, and symmetrized Green's functions to be obtained by evaluating the path integral.

Comparison to Schwinger-Keldysh approach

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- In linear limit, new effective symmetry derived from detailed balance

$$\phi^A \to \Theta\phi^A, \quad \bar{\phi}^A \to -\Theta\delta\bar{\phi}^A - i\Theta\delta\phi^A$$

- Can compare to KMS symmetry of SK theory.

$$\phi_r \to \Theta\phi_r, \quad \phi_a \to \Theta\phi_a + i\Theta\mathcal{L}_\beta\phi_r$$

- Provides mapping between variables

$$\phi_r = \delta\phi, \quad \phi_a = \mathcal{L}_\beta\delta\bar{\phi}$$

- Can use SK procedure to compute Green's functions.