

Fluctuations near the liquid-gas and chiral phase transitions

V. Koch, M. Marczenko, K. Redlich, C. Sasaki

Incubator of Scientific Excellence – Centre for Simulations of Superdense Fluids
University of Wrocław

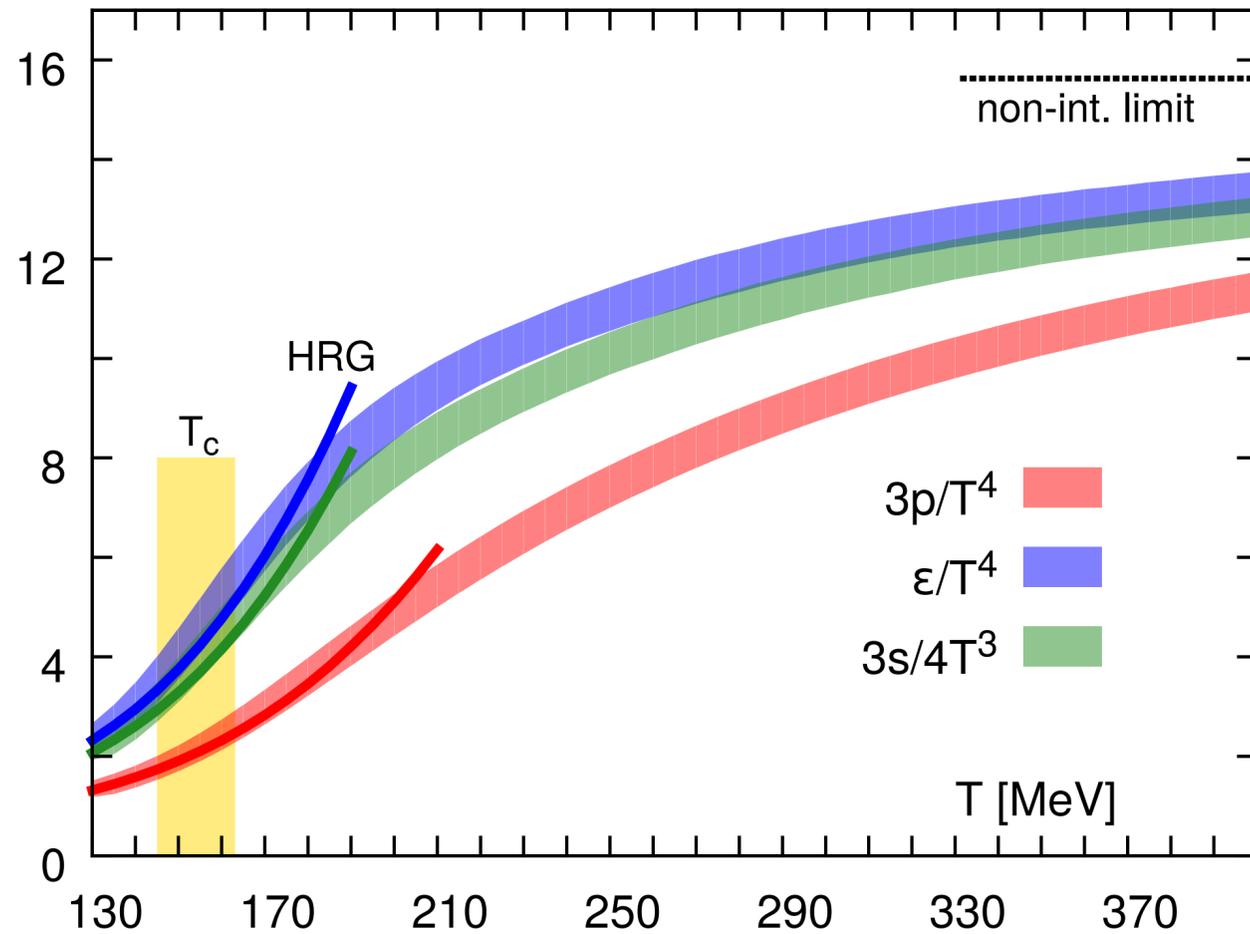
Phys. Rev. D 107 (2023) 5, 054046
arXiv:2308.15794

QUARK MATTER 2023,
3-9 SEPTEMBER, HOUSTON, TX, USA



Lattice QCD vs Hadron Resonance Gas

HotQCD, 2014

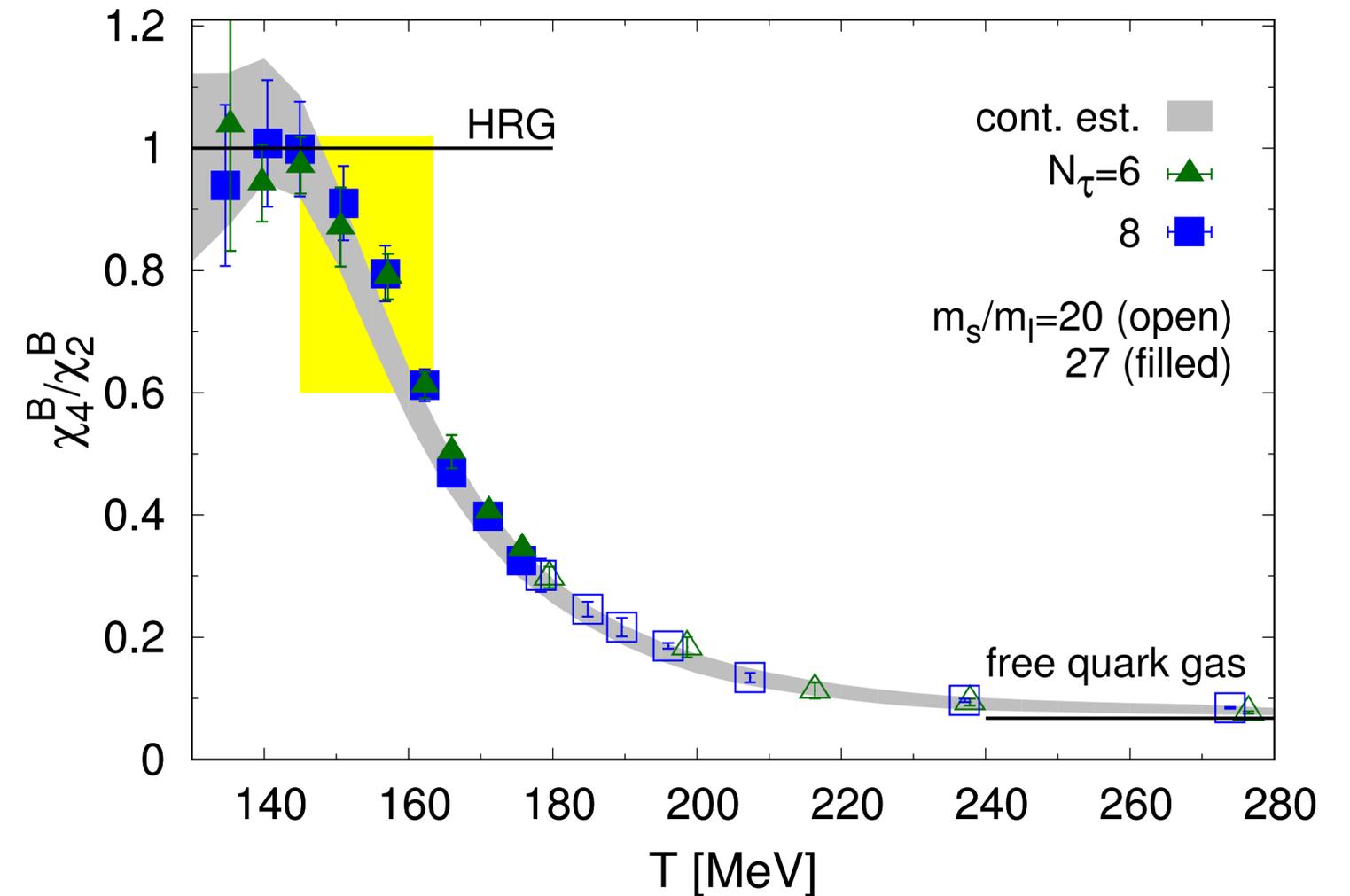


Pressure in the HRG model

$$P^{\text{HRG}} = \sum_{i \in \text{had}} P^{\text{id}}(T, \mu_i; m_i)$$

Agreement with LQCD EoS up to $\simeq T_c$

HotQCD, 2017

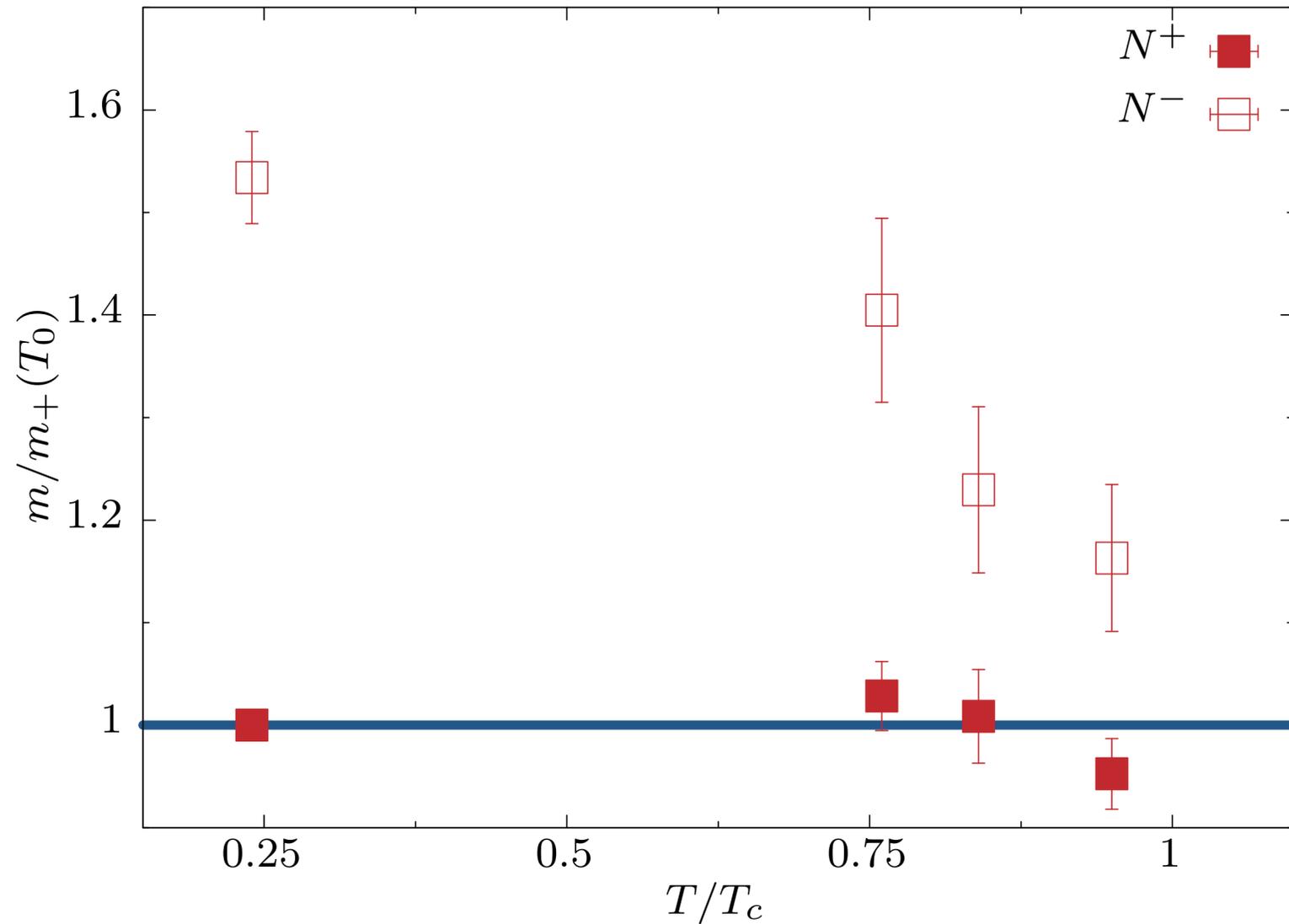


Taylor expansion of LQCD EoS

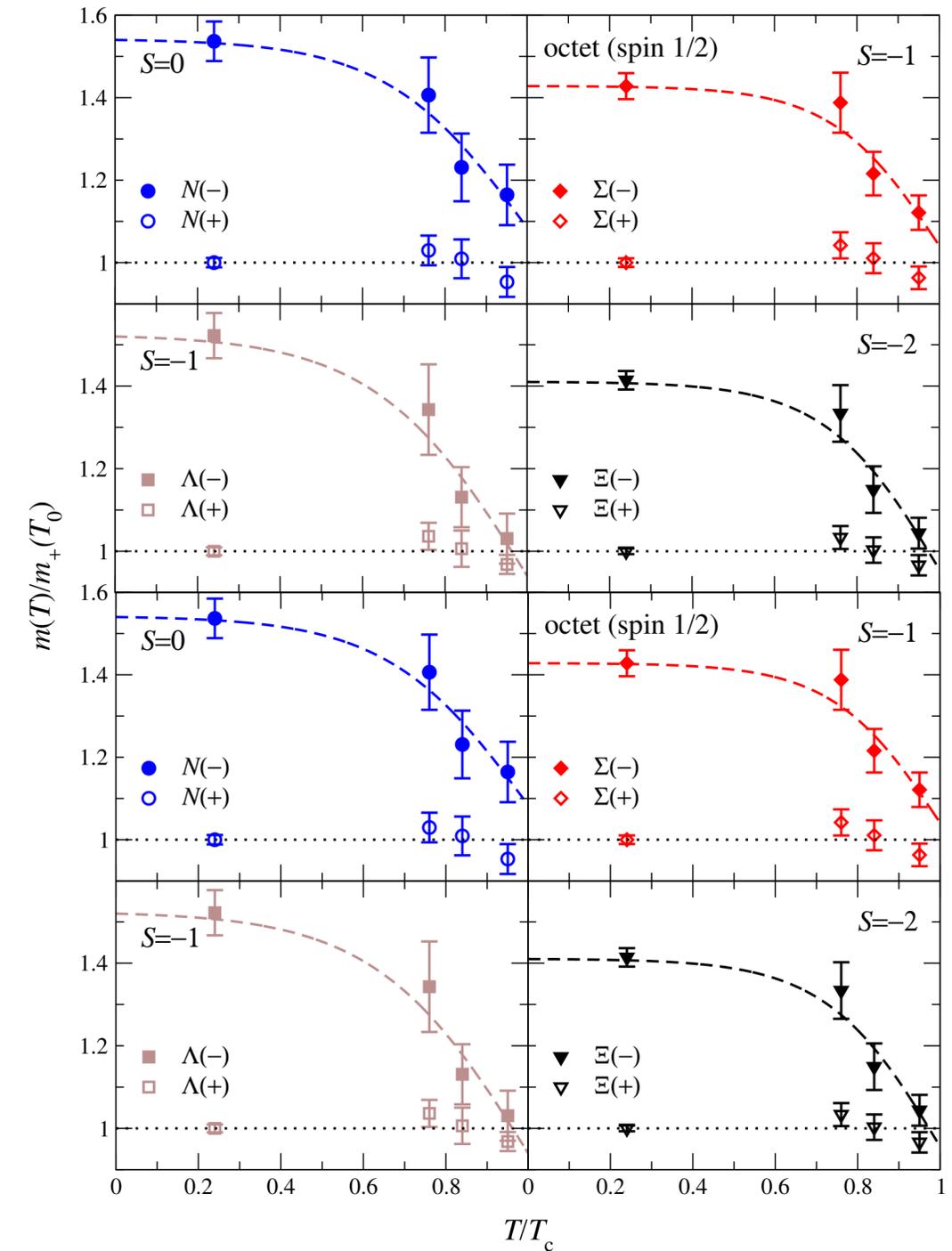
$$\frac{P}{T^4} = \sum_{k=0}^{\infty} \left(\frac{\mu_B}{T} \right)^k \frac{\chi_k^B}{k!}, \text{ where } \chi_k^B = \frac{\partial^k P/T^4}{\partial (\mu_B/T)^k}$$

Kurtosis: $\frac{\chi_4^B}{\chi_2^B} \sim B^2$: breakdown $\sim T_c$: changeover to QGP

Parity Doubling in Lattice QCD Aarts et al, 2017, 2019



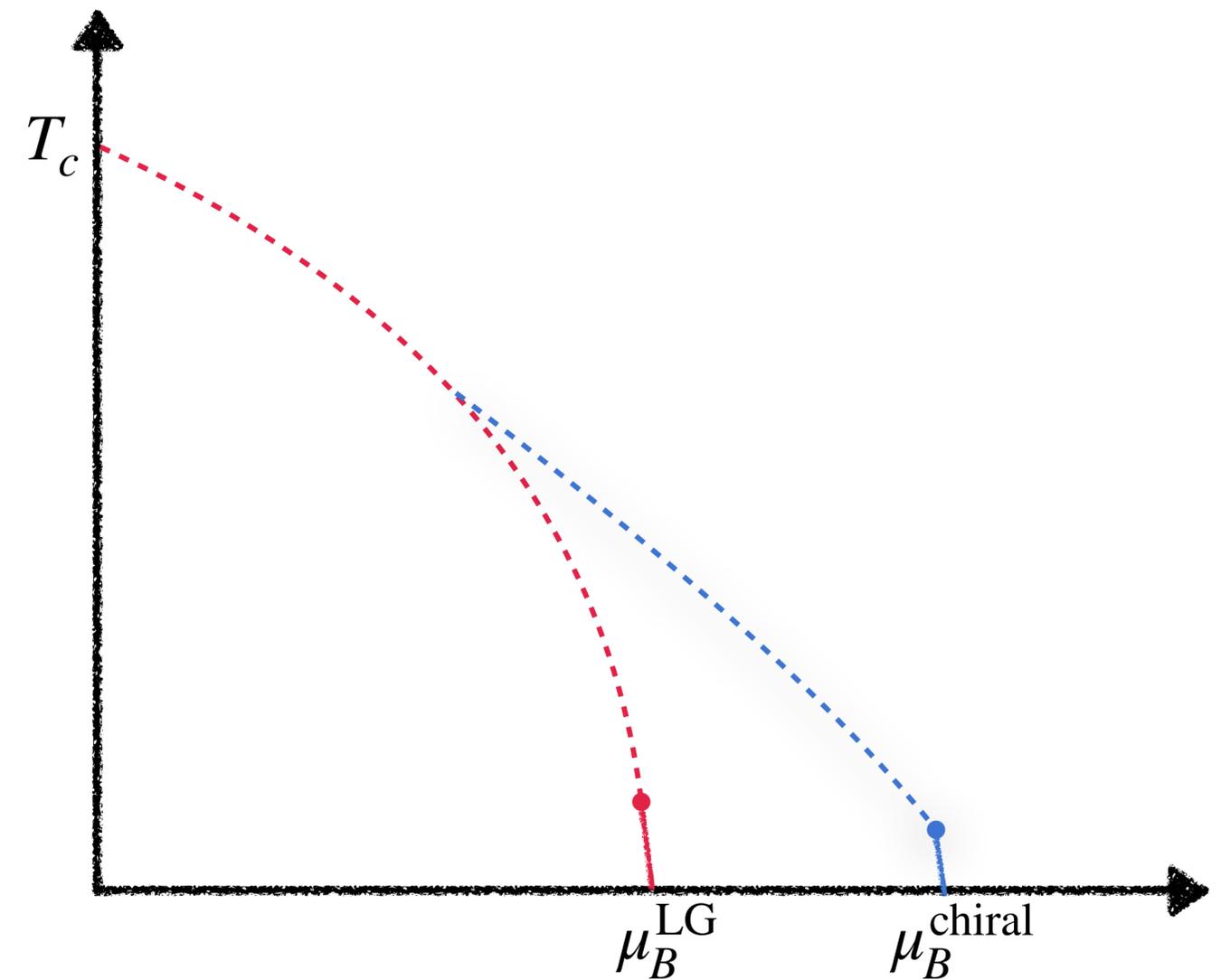
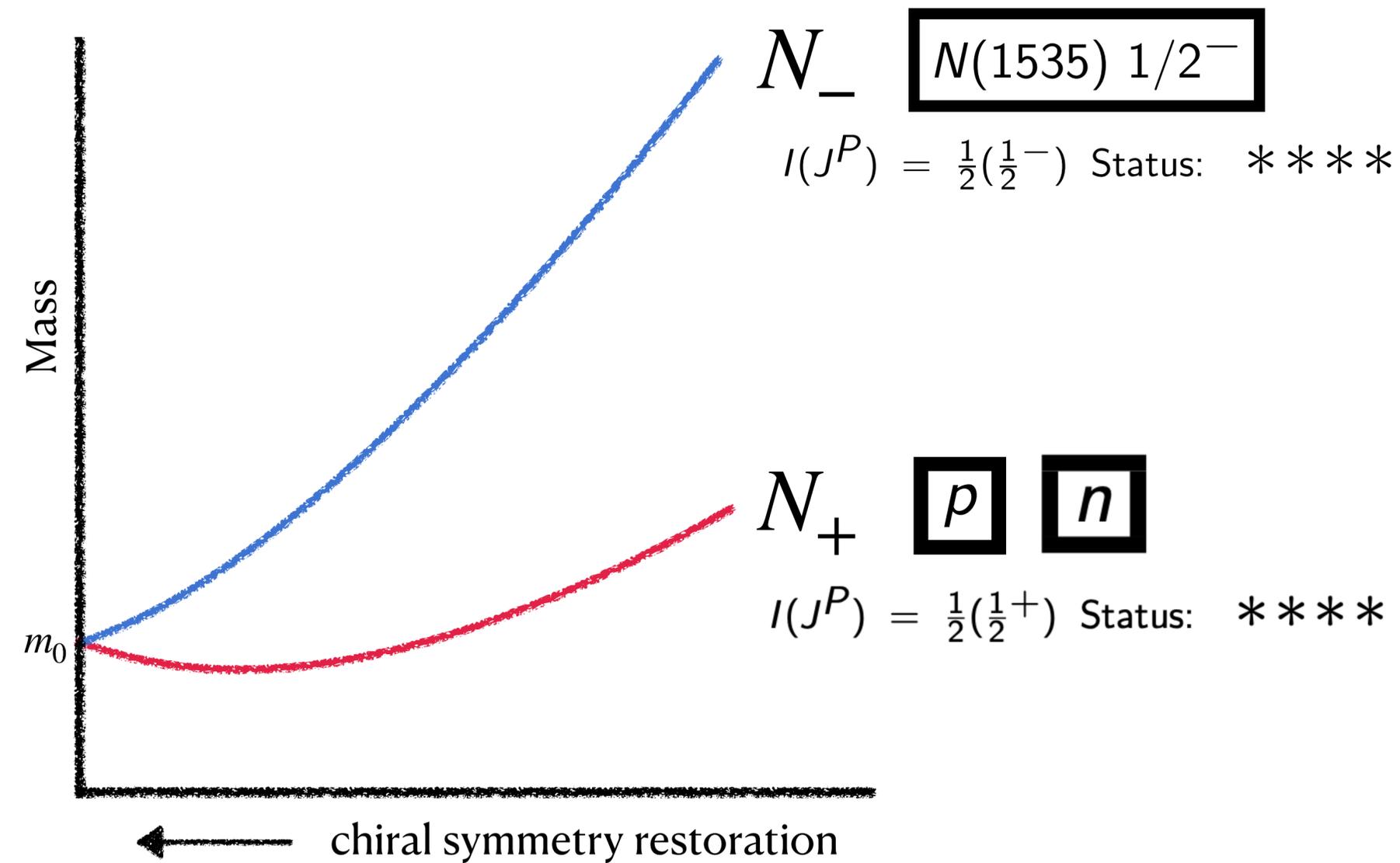
General tendency: $N^+ \sim \text{const}$; N^- - drastic drop toward T_c
 Chiral partners N^\pm degenerate but massive around T_c



Imprint of chiral symmetry restoration in the baryonic sector

Model a'la DeTar, Kunihiro 1989 $\longrightarrow \mathcal{L}_{\text{mass}} \sim m_0 (\bar{\psi}_1 \gamma_5 \gamma_2 + \bar{\gamma}_1 \gamma_5 \psi_2)$

$$M_{\pm} = \sqrt{4m_0^2 + \alpha^2 \sigma^2 \mp \beta \sigma} \xrightarrow{\sigma \rightarrow 0} m_0$$



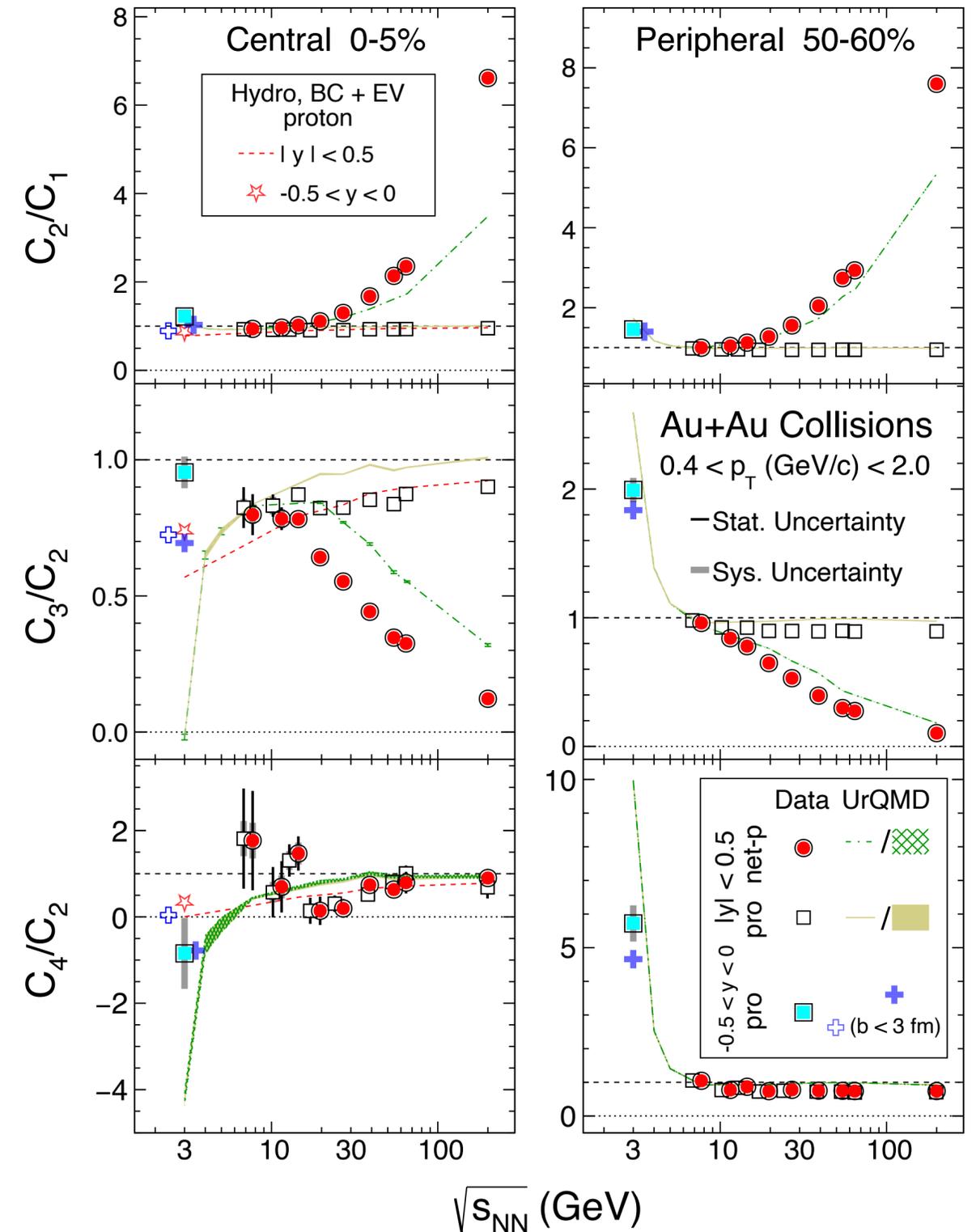
Cumulants vs Susceptibilities

STAR, 2023

Mean: M	$\langle N_B \rangle$	C_1
Variance: σ^2	$\langle (\delta N_B)^2 \rangle$	C_2
Skewness: S	$\langle (\delta N_B)^3 \rangle / \sigma^3$	$C_3 / C_2^{3/2}$
Kurtosis: K	$\langle (\delta N_B)^4 \rangle / \sigma^3 - 3$	C_4 / C_2^2

$$C_n \equiv VT^3 \frac{d^n P / T^4}{d(\mu_B / T)^n} \Bigg|_T \longleftrightarrow \chi_n^B \equiv \frac{d^n P / T^4}{d(\mu_B / T)^n} \Bigg|_T$$

$$C_n = VT^3 \chi_n^B$$



For multiplicity $N_B = N_+ + N_-$

$$\langle N_B \rangle = \langle N_+ \rangle + \langle N_- \rangle$$

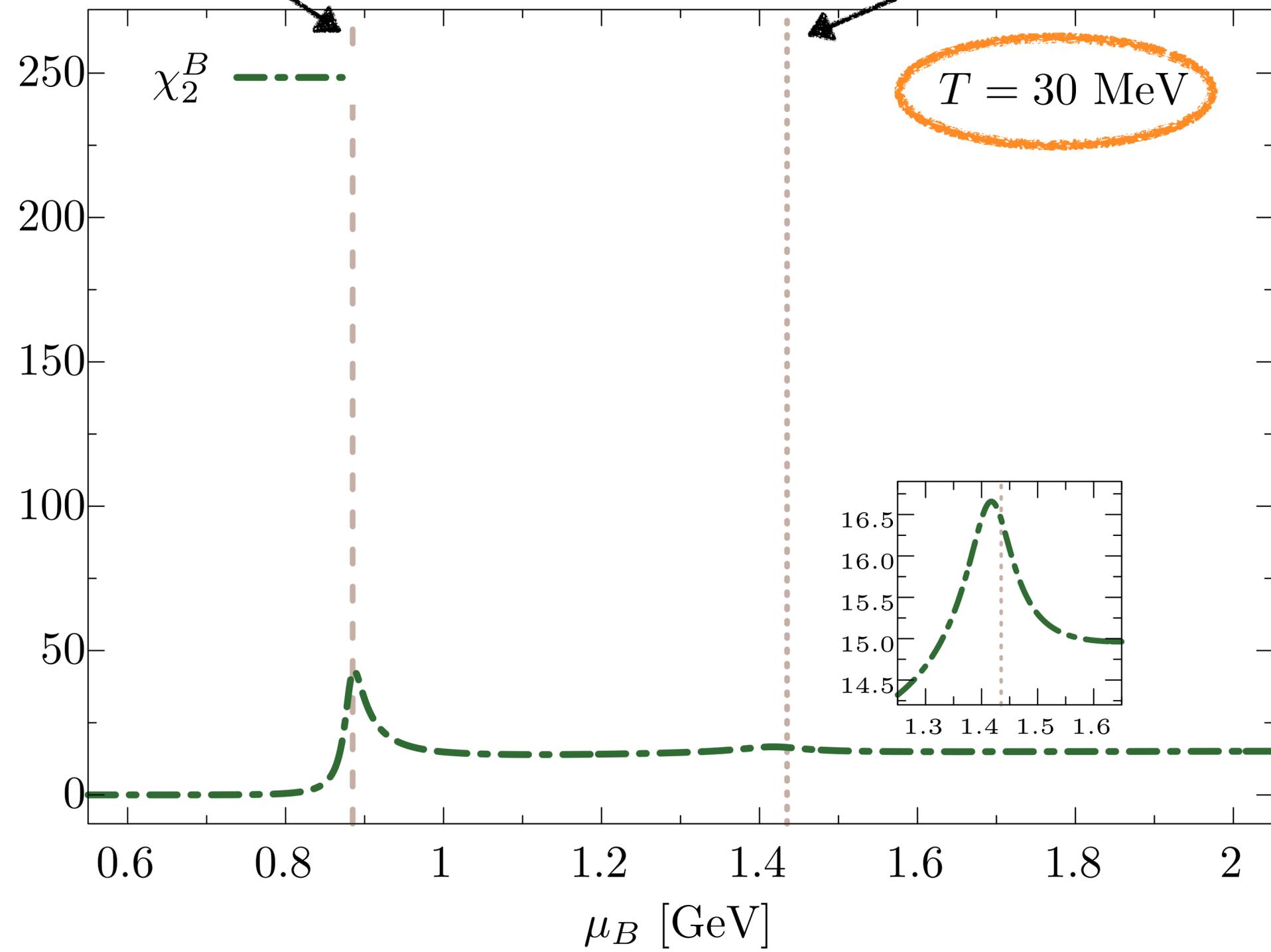
$$\langle \delta N_B \delta N_B \rangle = \langle (\delta N_+)^2 \rangle + \langle (\delta N_-)^2 \rangle + 2 \langle \delta N_+ \delta N_- \rangle$$

$$\langle \delta N_\alpha \delta N_\beta \rangle = VT^3 \chi_n^{\alpha\beta} \quad \longleftrightarrow \quad \chi_2^{\alpha\beta} = \frac{d^2 P / T^4}{d(\mu_\alpha / T) d(\mu_\beta / T)}$$

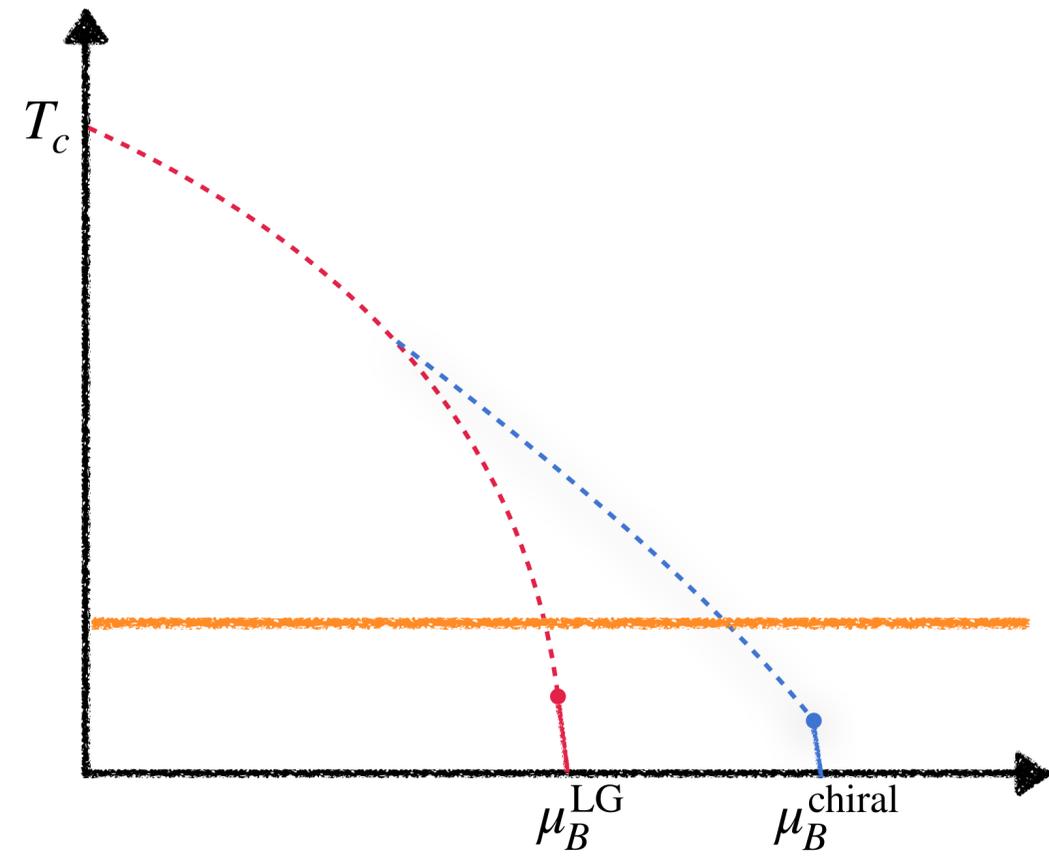
$$\chi_2^B = \chi_2^{++} + \chi_2^{--} + 2\chi_2^{+-}$$

Liquid-Gas

Chiral

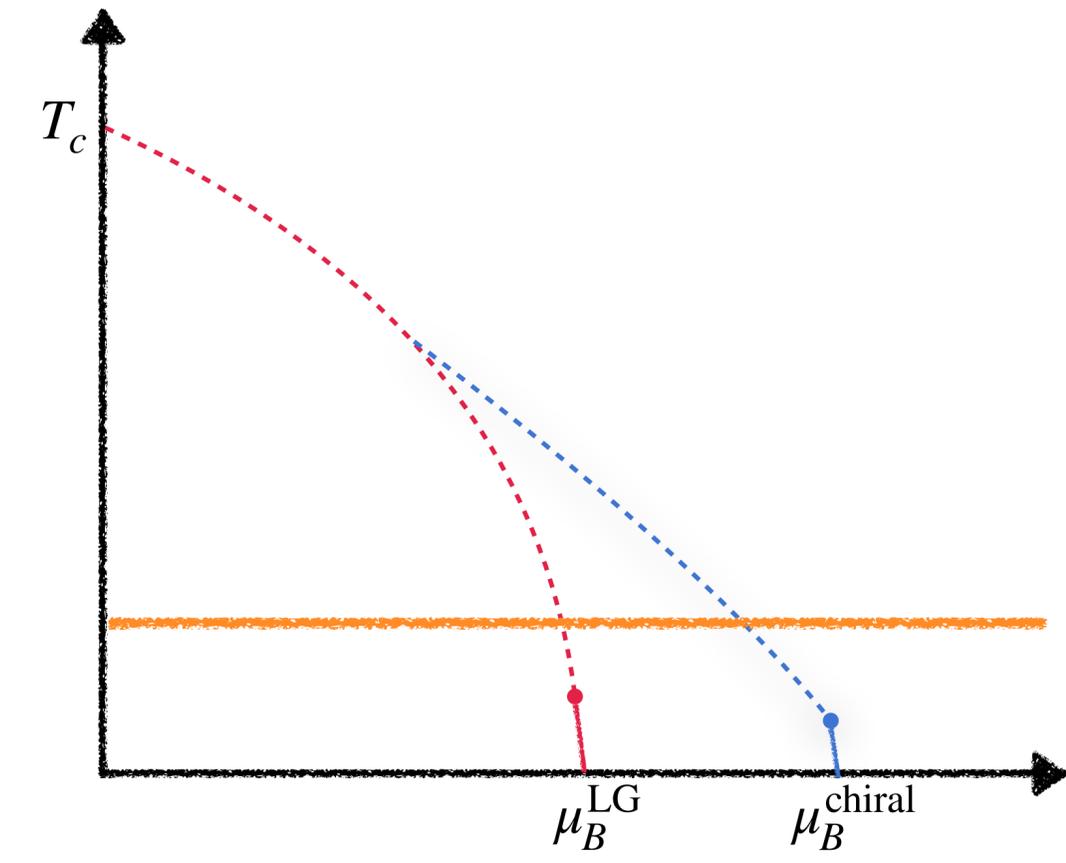
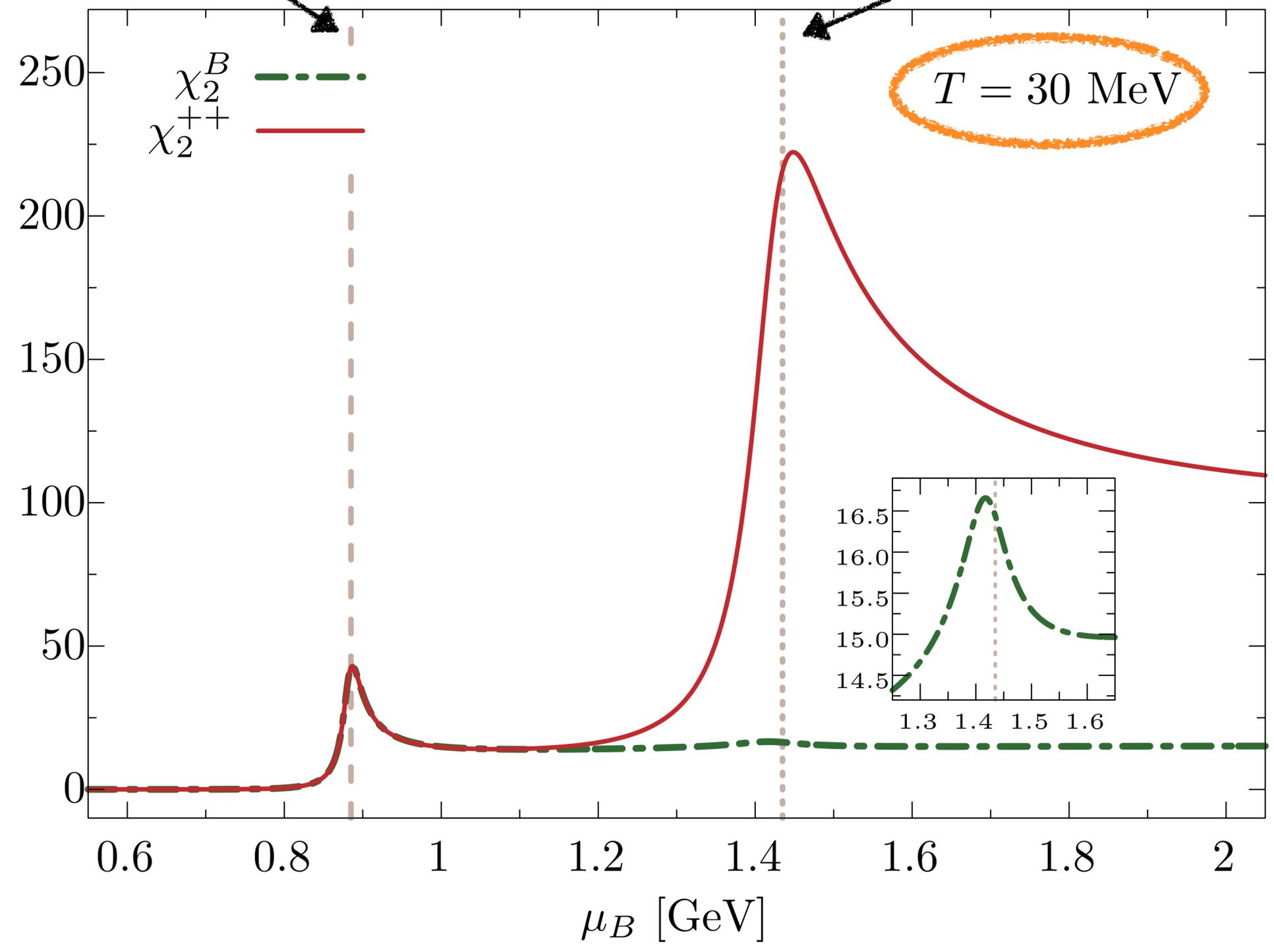


χ_2^B



Liquid-Gas

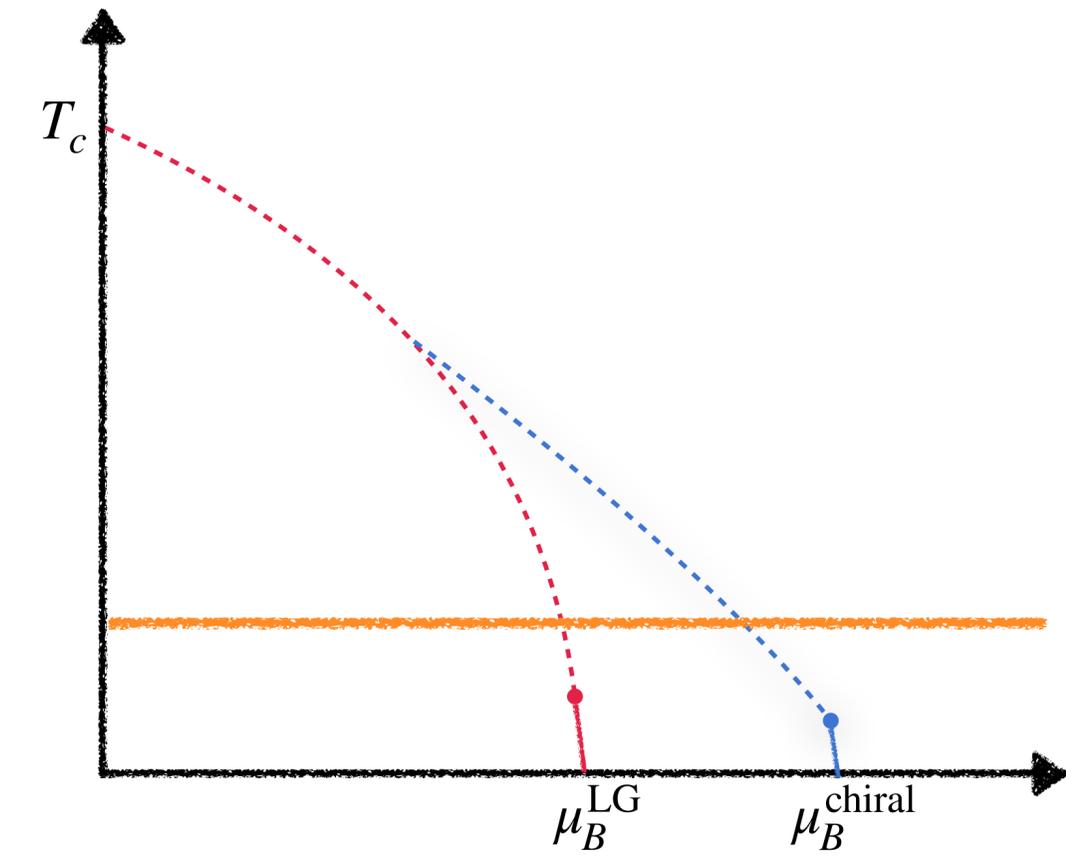
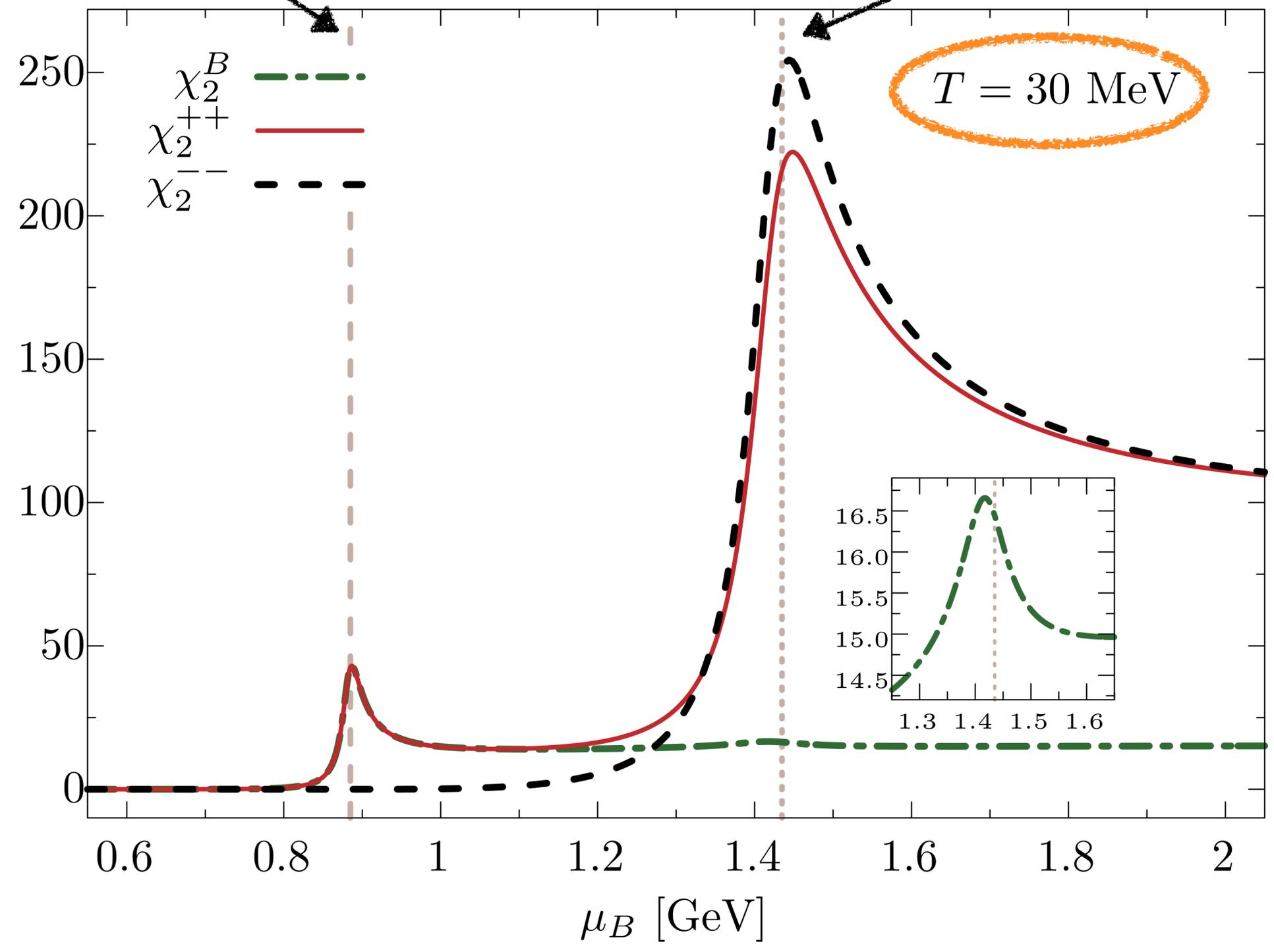
Chiral



$$\chi_2^B = \chi_2^{++}$$

Liquid-Gas

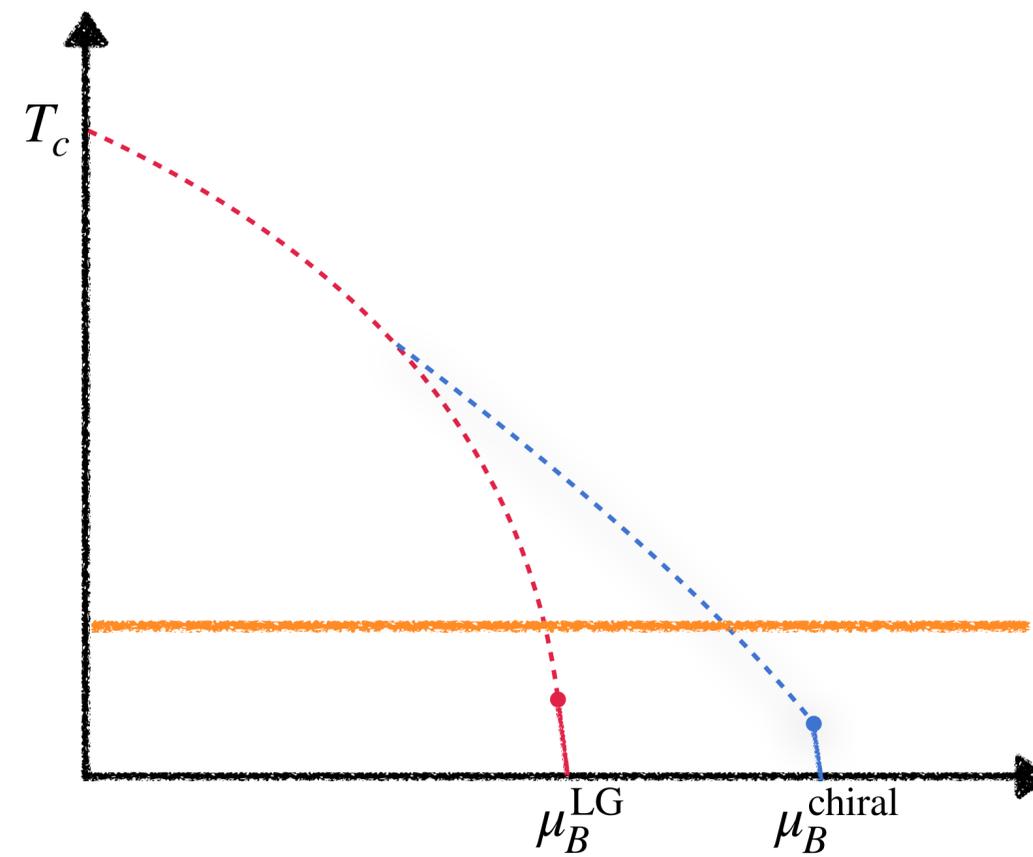
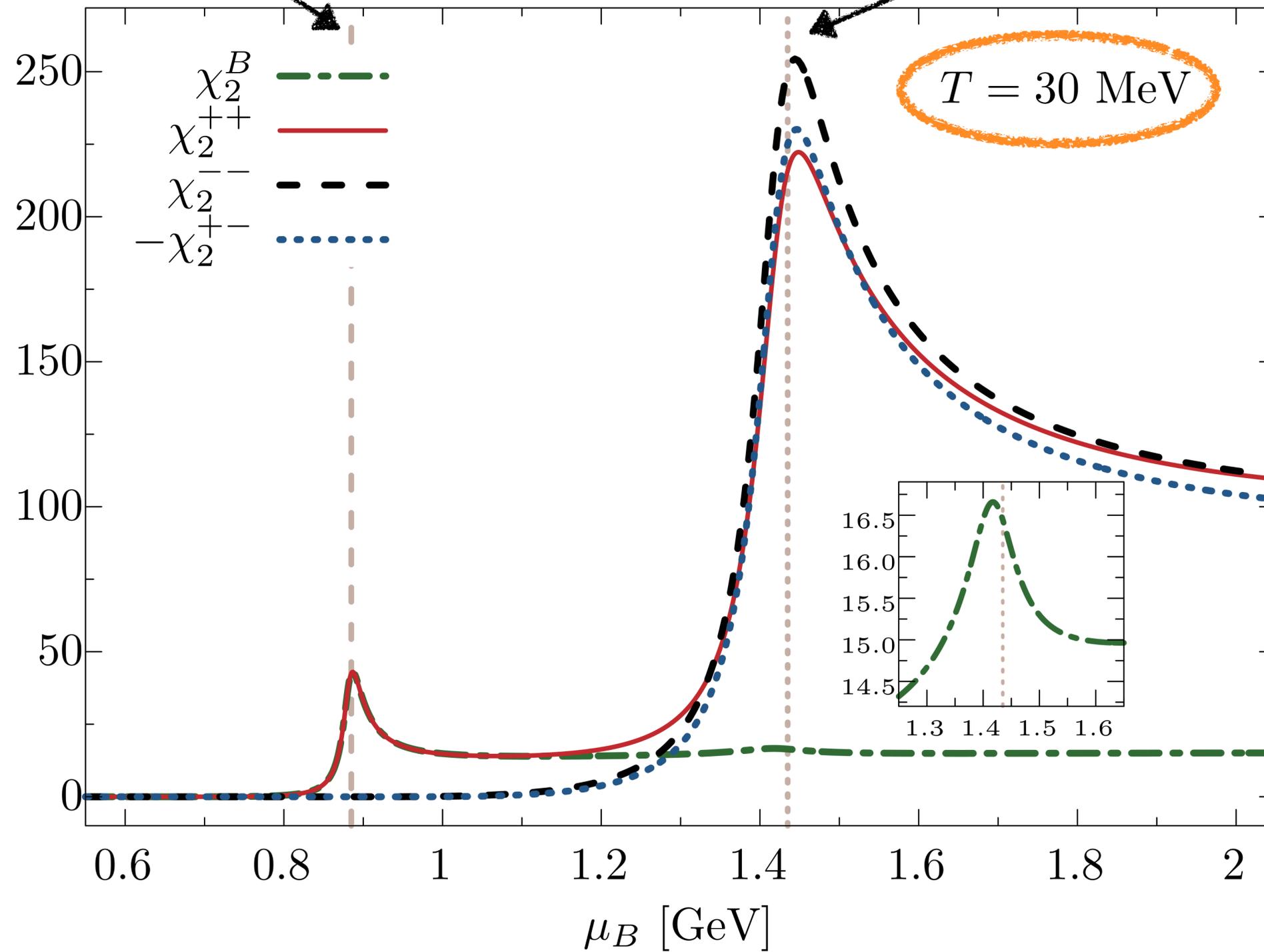
Chiral



$$\chi_2^B = \chi_2^{++} + \chi_2^{--}$$

Liquid-Gas

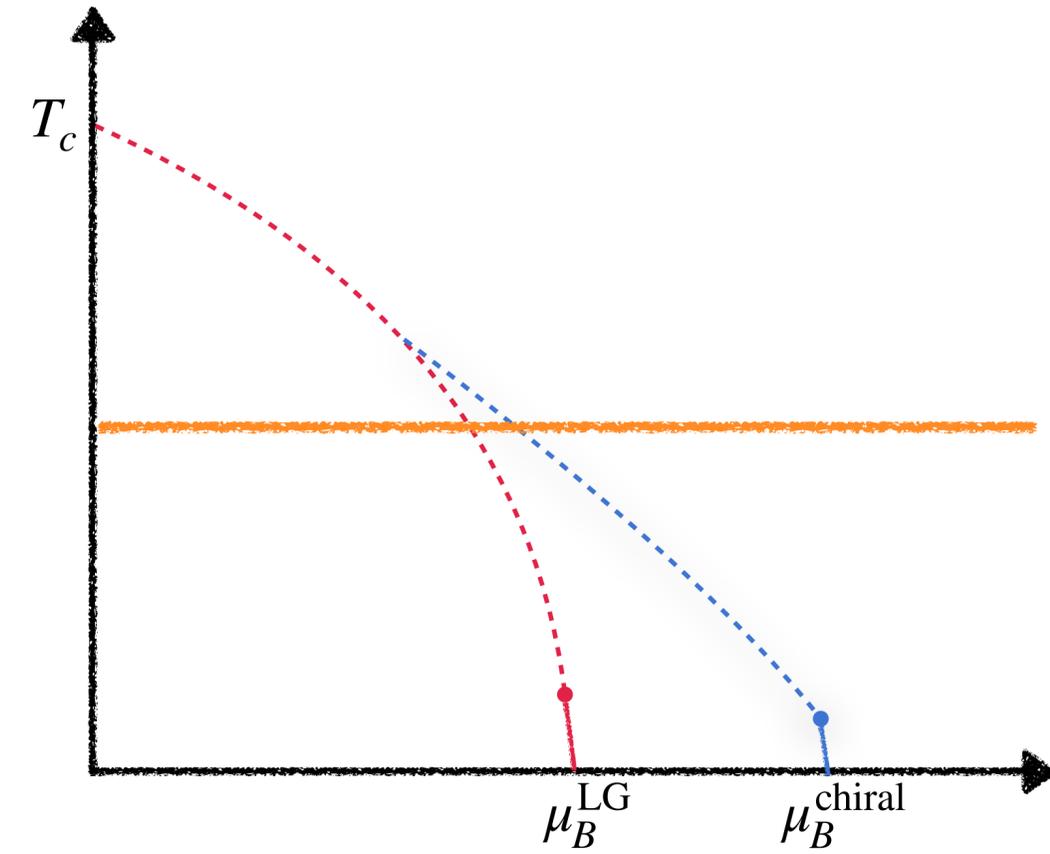
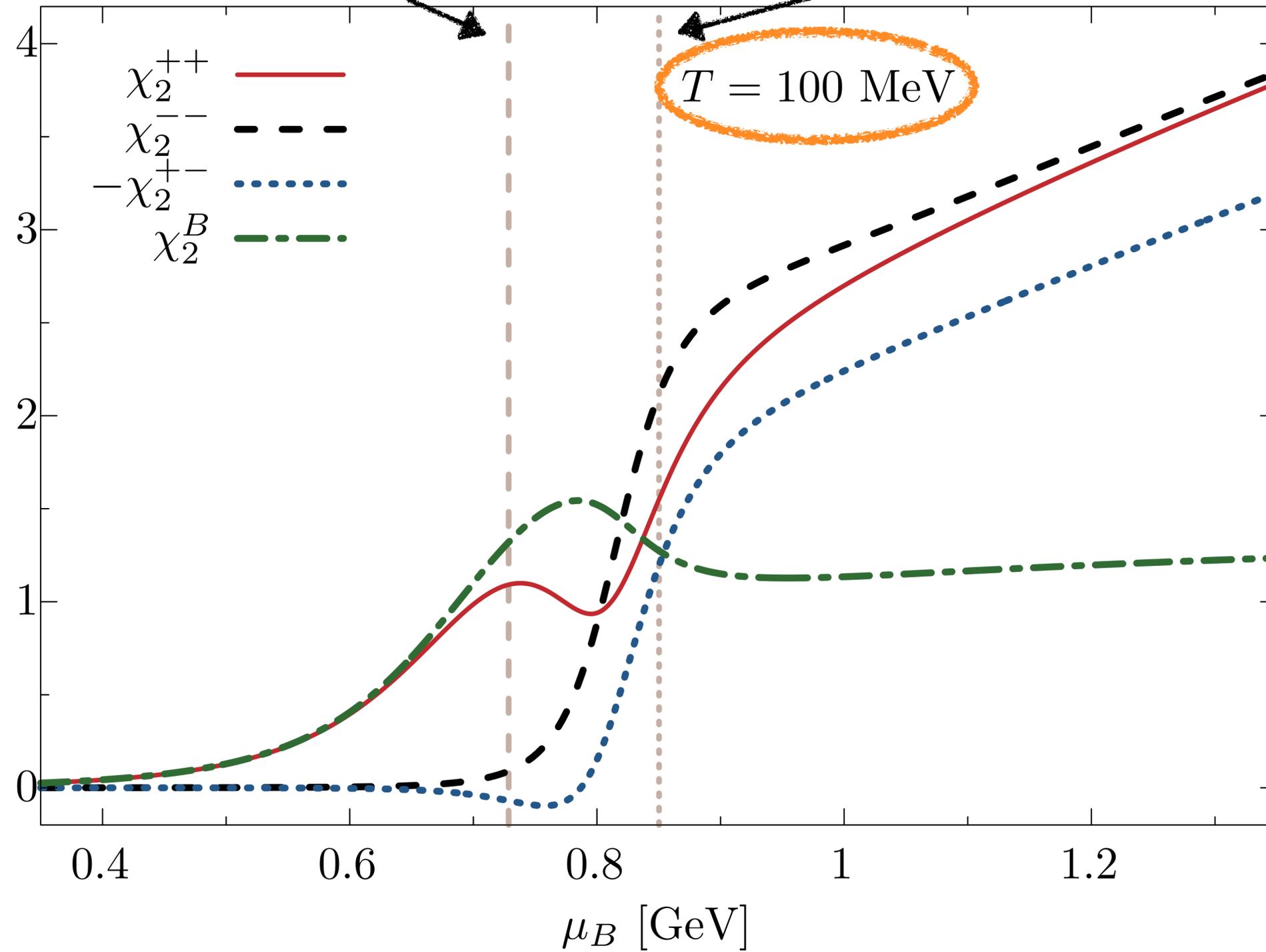
Chiral



$$\chi_2^B = \chi_2^{++} + \chi_2^{--} + 2\chi_2^{+-}$$

Liquid-Gas

Chiral



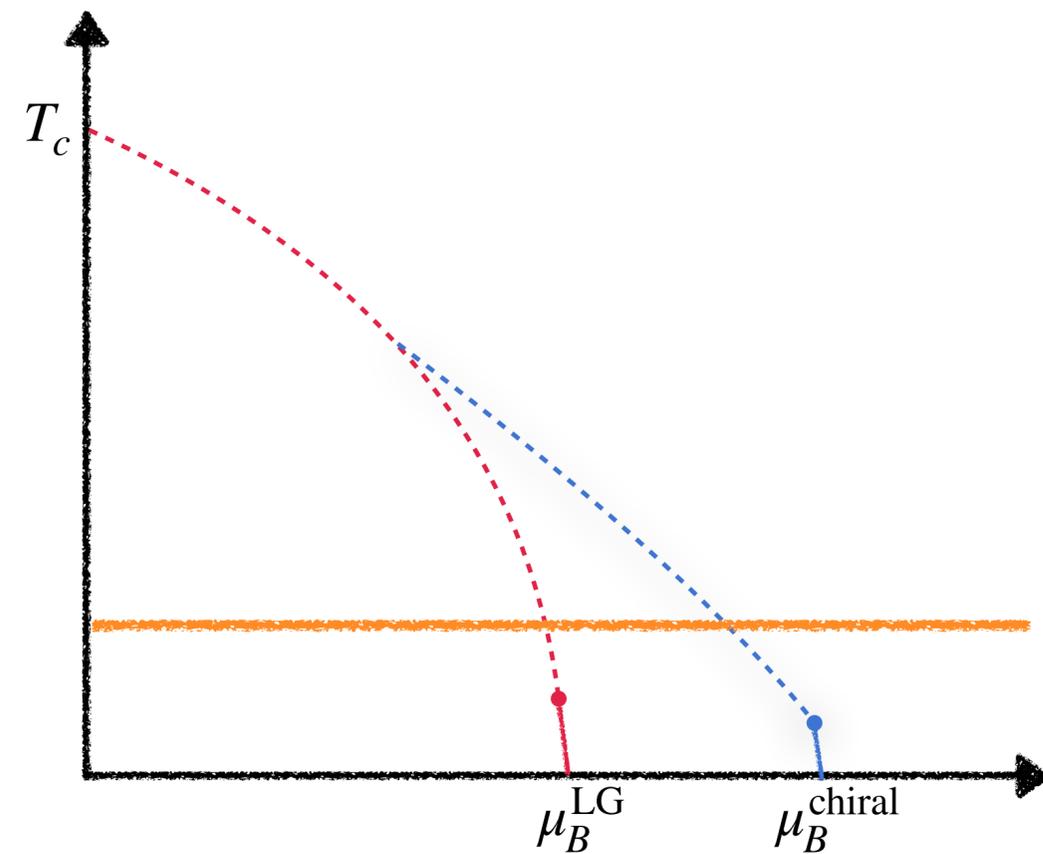
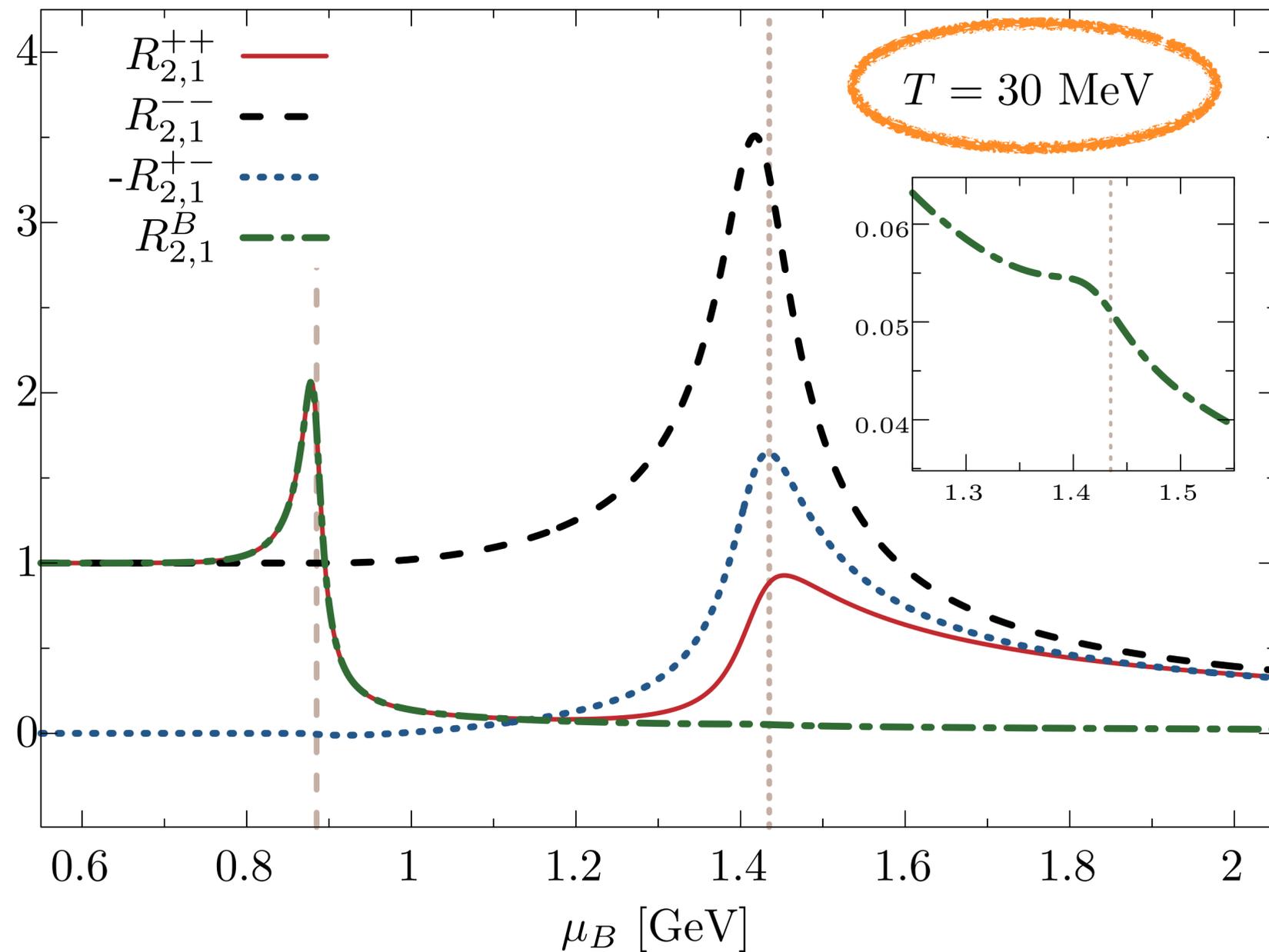
$$\chi_2^B = \chi_2^{++} + \chi_2^{--} + 2\chi_2^{+-}$$

Cumulants $C_n \sim V$

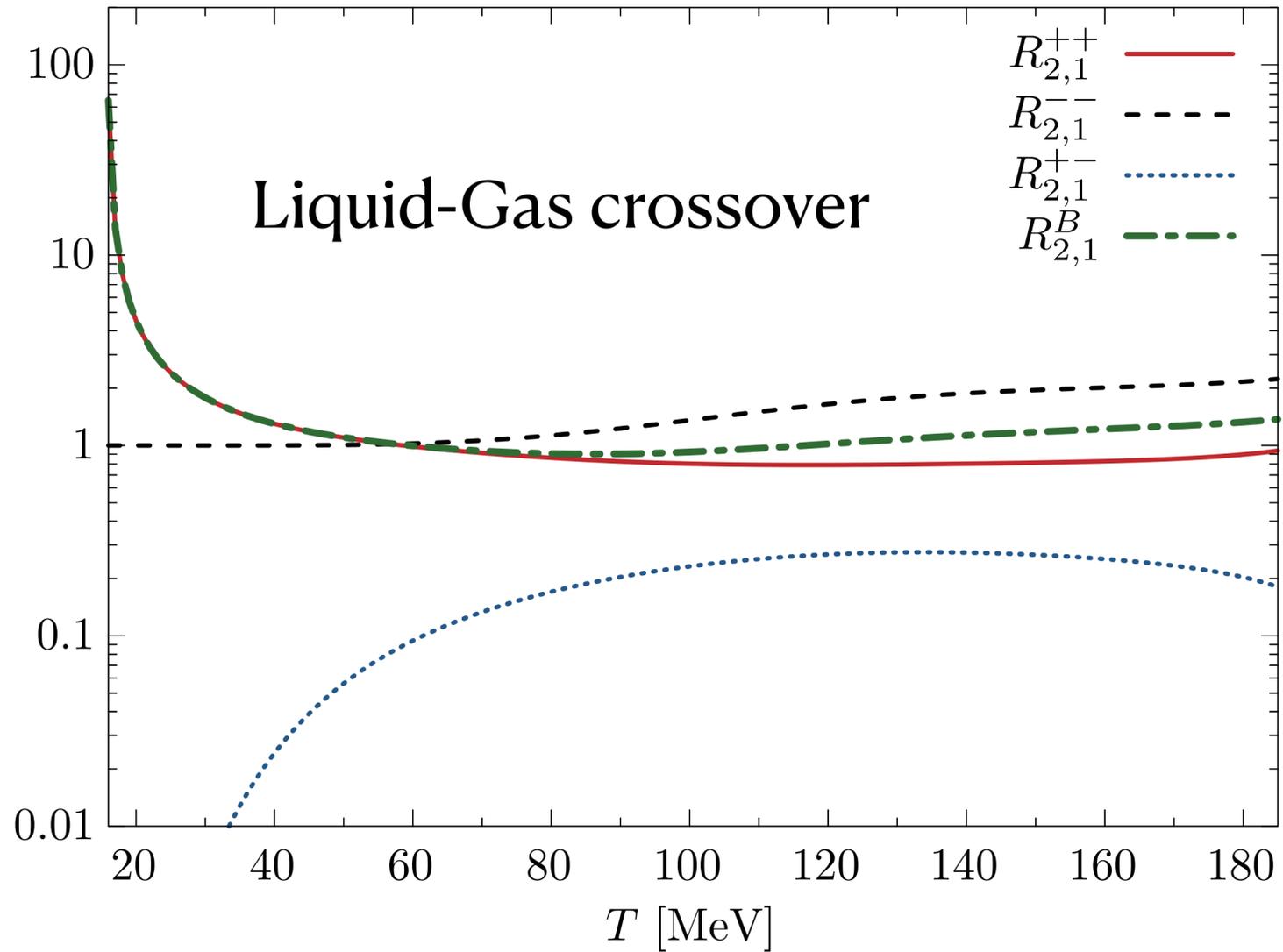


volume cancels in ratios

$$R_{2,1}^{\alpha\beta} \equiv \frac{C_2^{\alpha\beta}}{\sqrt{C_1^\alpha C_1^\beta}} = \frac{\chi_2^{\alpha\beta}}{\sqrt{\chi_1^\alpha \chi_1^\beta}} = \frac{\sigma^2}{M}$$



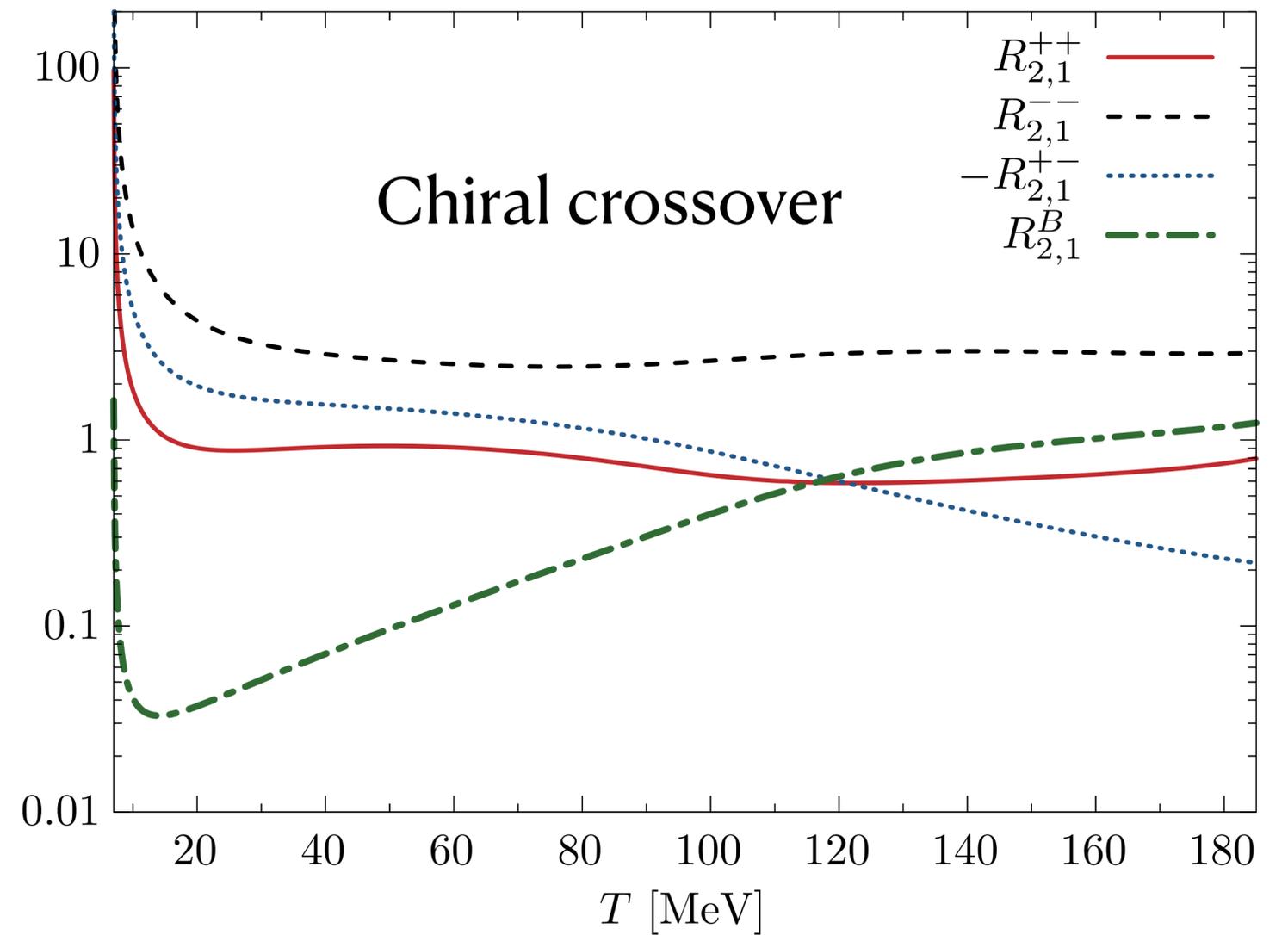
Critical Point \rightarrow enhanced fluctuations & non-monotonicity



Fluctuations dominated by **positive parity**



Net-nucleon \sim **net-baryon**

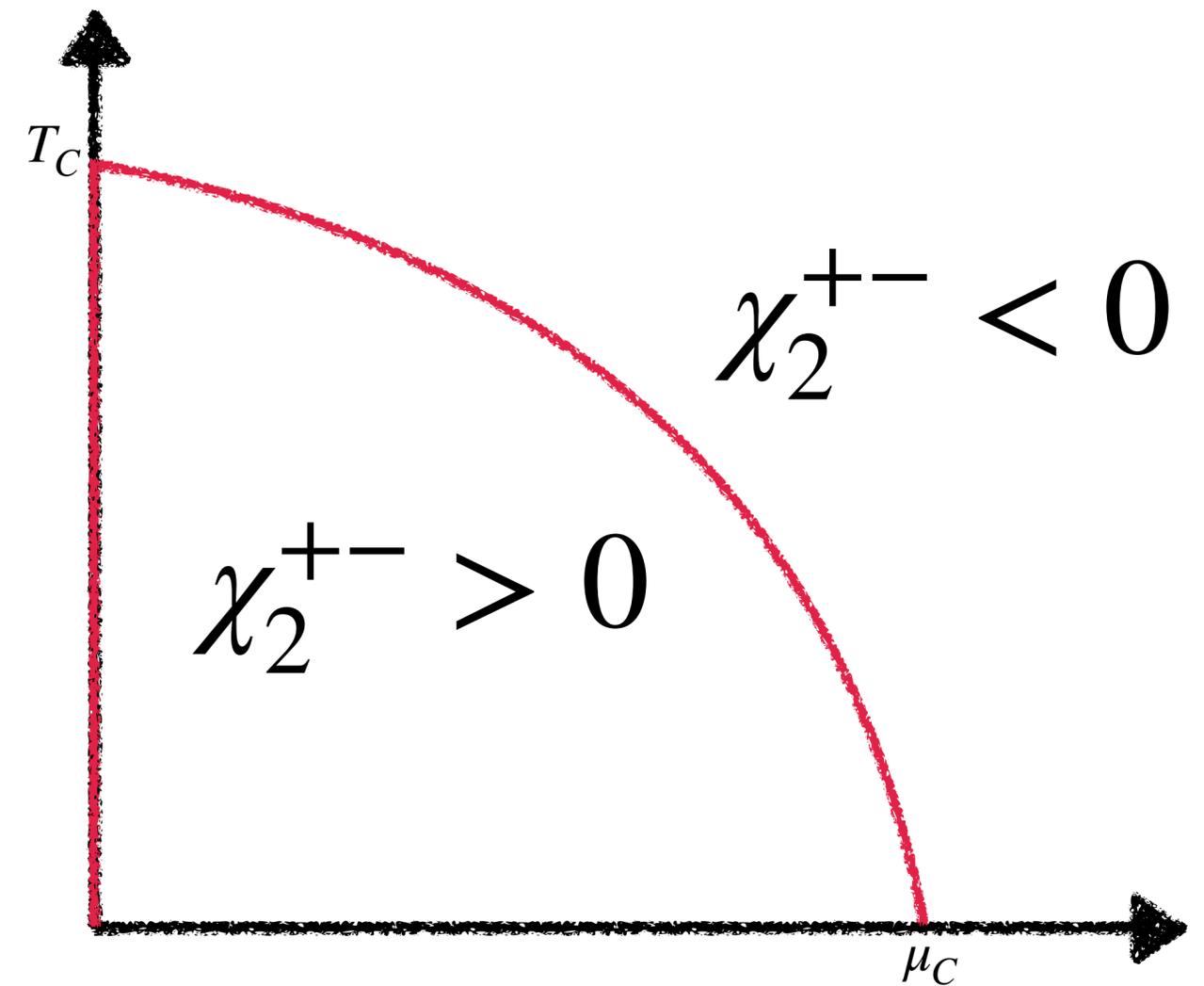
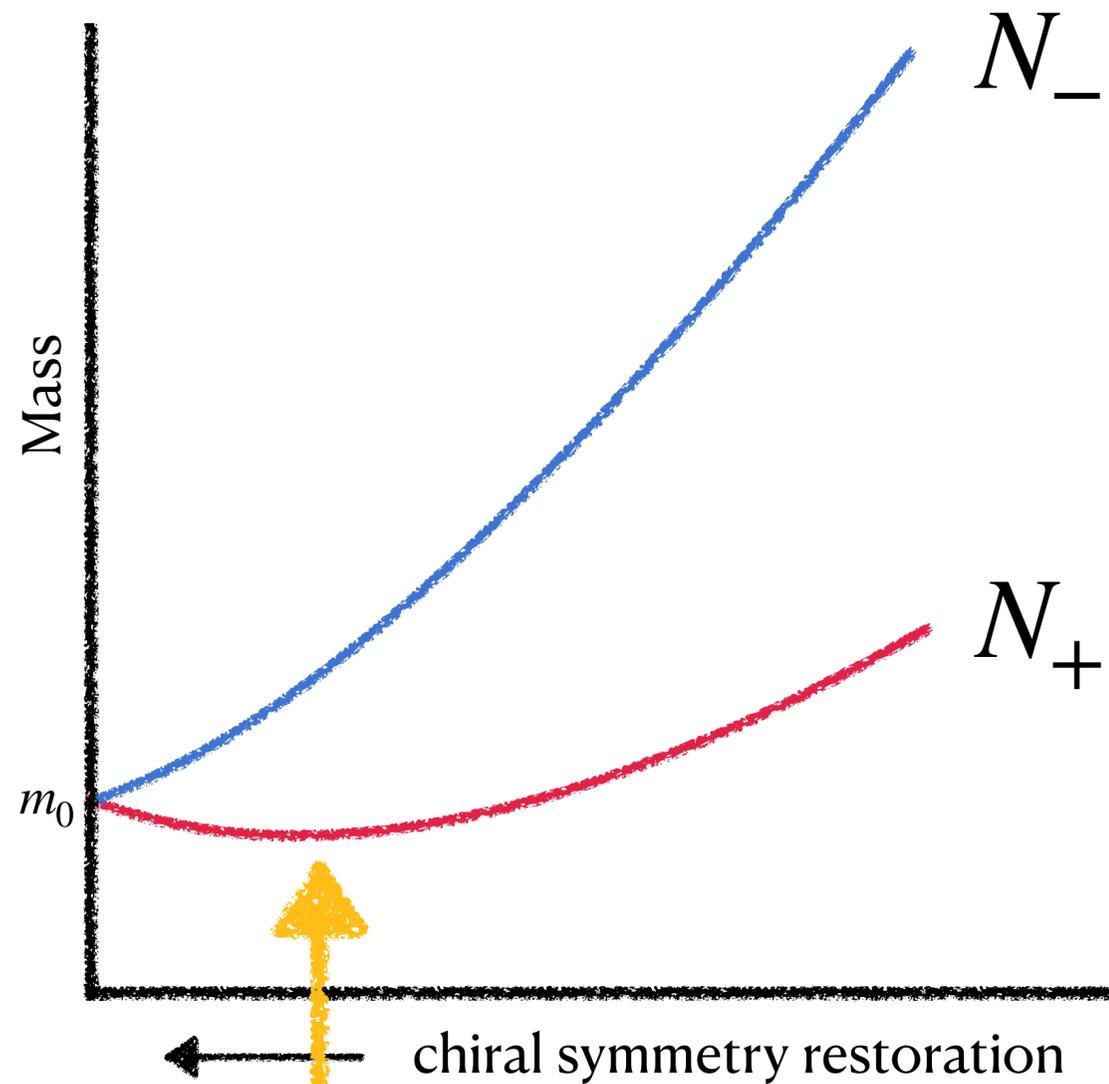


Presence of chiral partners + **correlations**

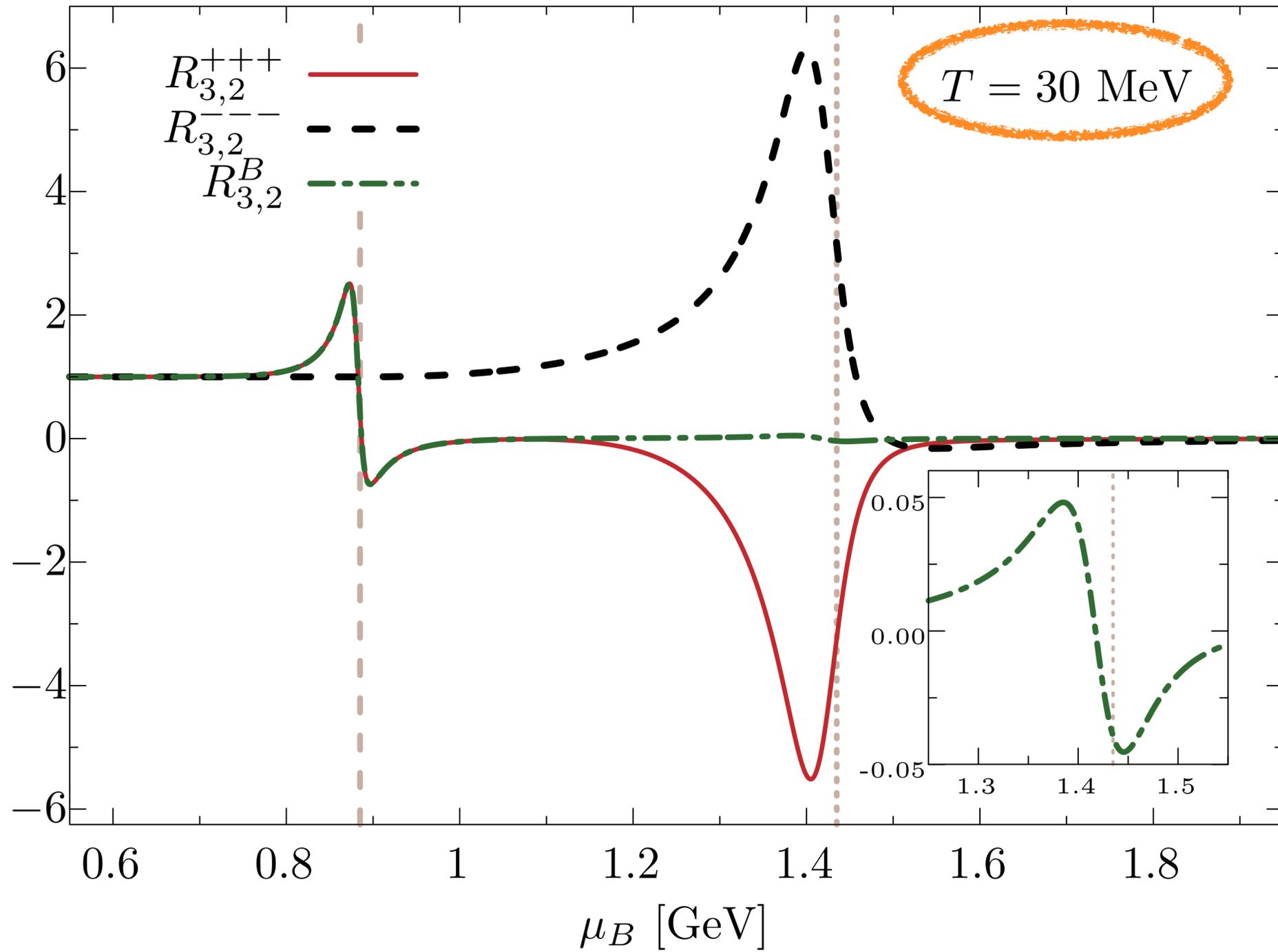


Net-baryon suppressed

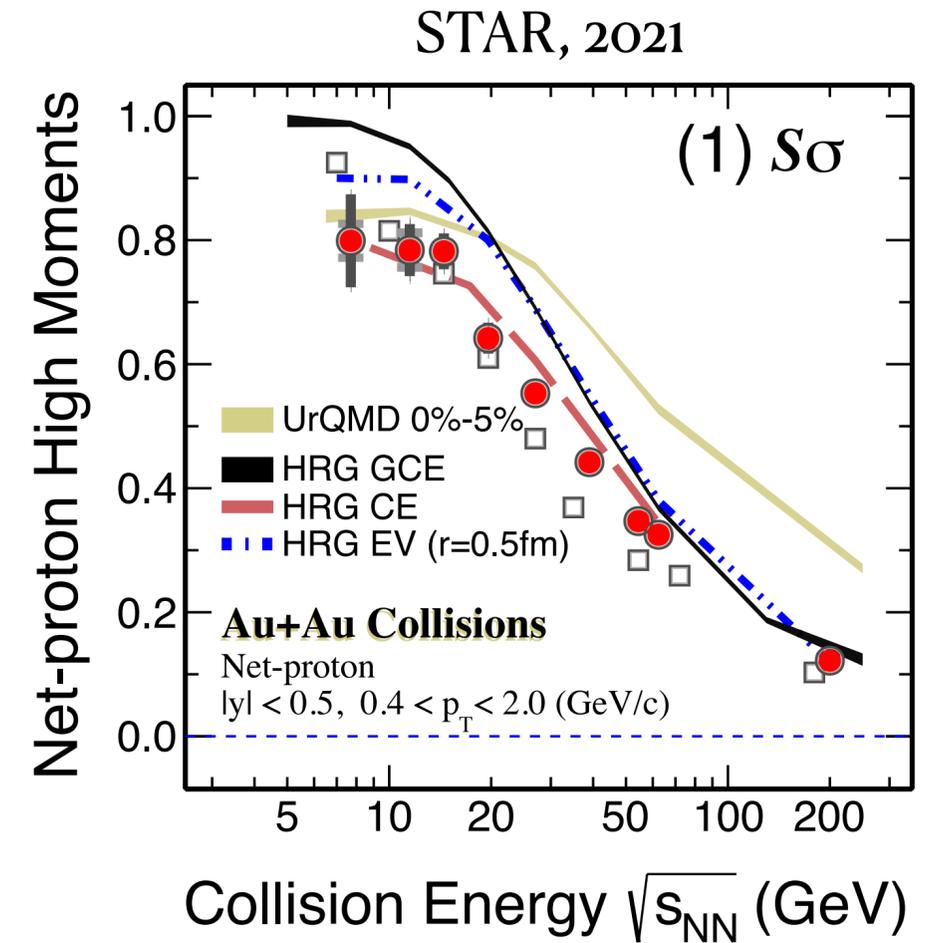
Idealized behavior of the correlator \longrightarrow no repulsive forces



Min of M_+ $\chi_2^{\alpha\beta} \sim \frac{\partial m_\alpha}{\partial \sigma} \frac{\partial m_\beta}{\partial \sigma}$ \longrightarrow but also repulsion



$$R_{3,2}^{\alpha\alpha\alpha} \equiv \frac{C_3^{\alpha\alpha}}{C_2^{\alpha\alpha}} = \frac{\chi_3^{\alpha\alpha}}{\chi_2^{\alpha\alpha}} = S\sigma$$



Individual fluctuations **do not qualitatively** reflect the total net-baryon number fluctuations

Summary

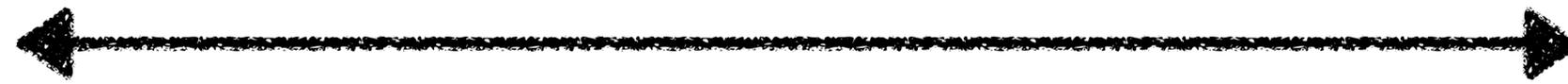
Individual fluctuations **may not** reflect the total net-baryon fluctuations



Chiral symmetry restoration and correlations between chiral partners



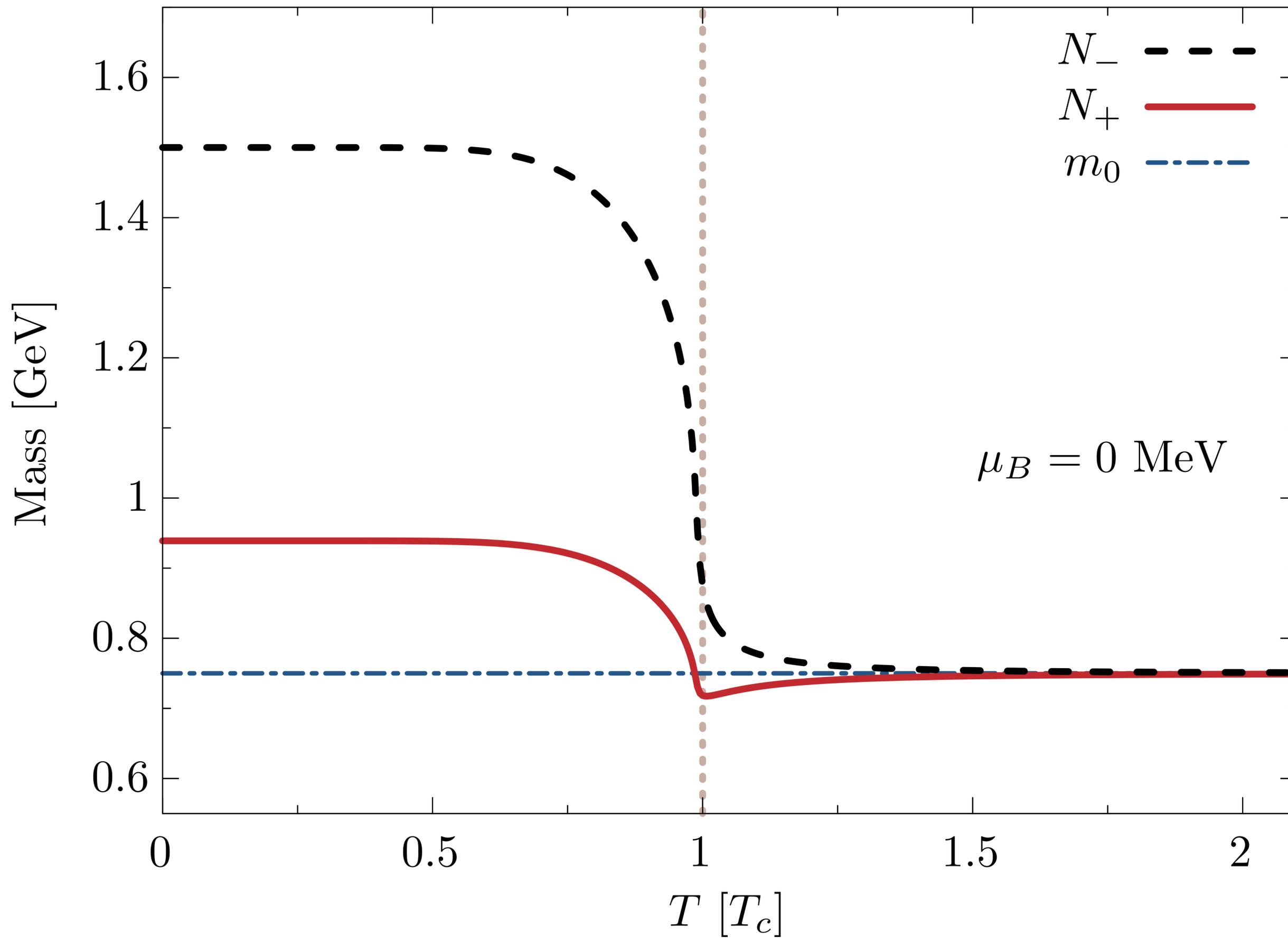
Net-proton

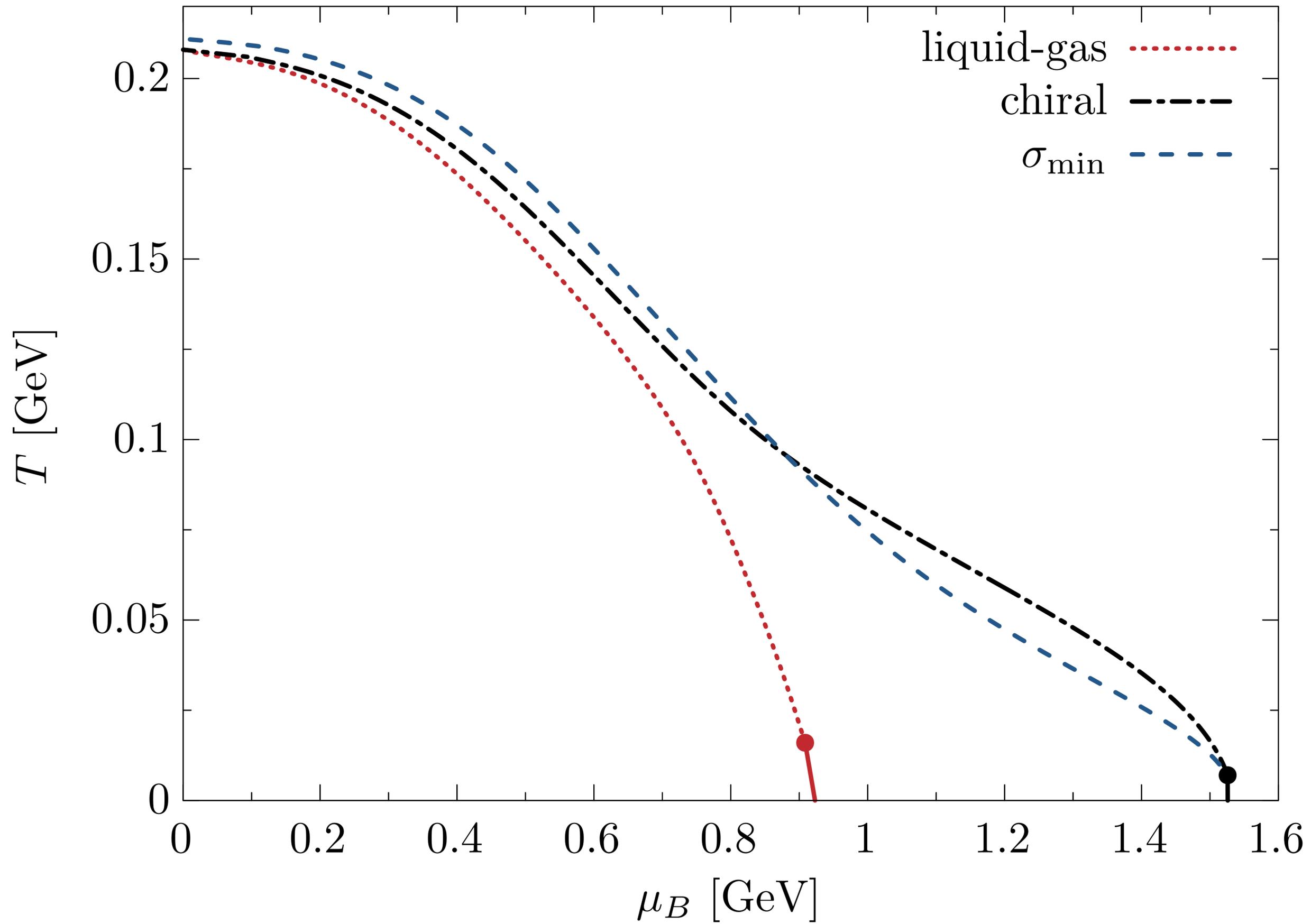


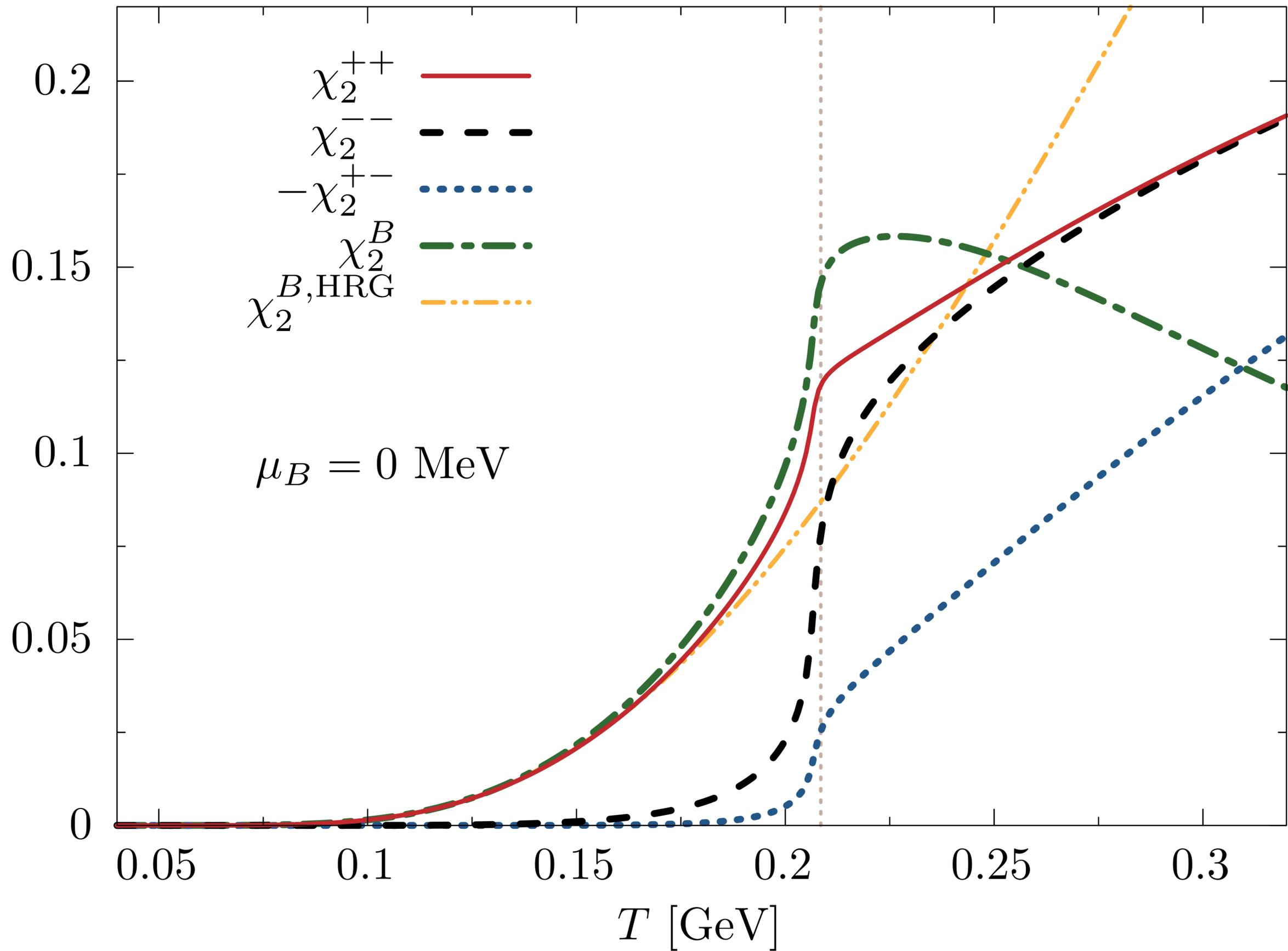
Net-baryon

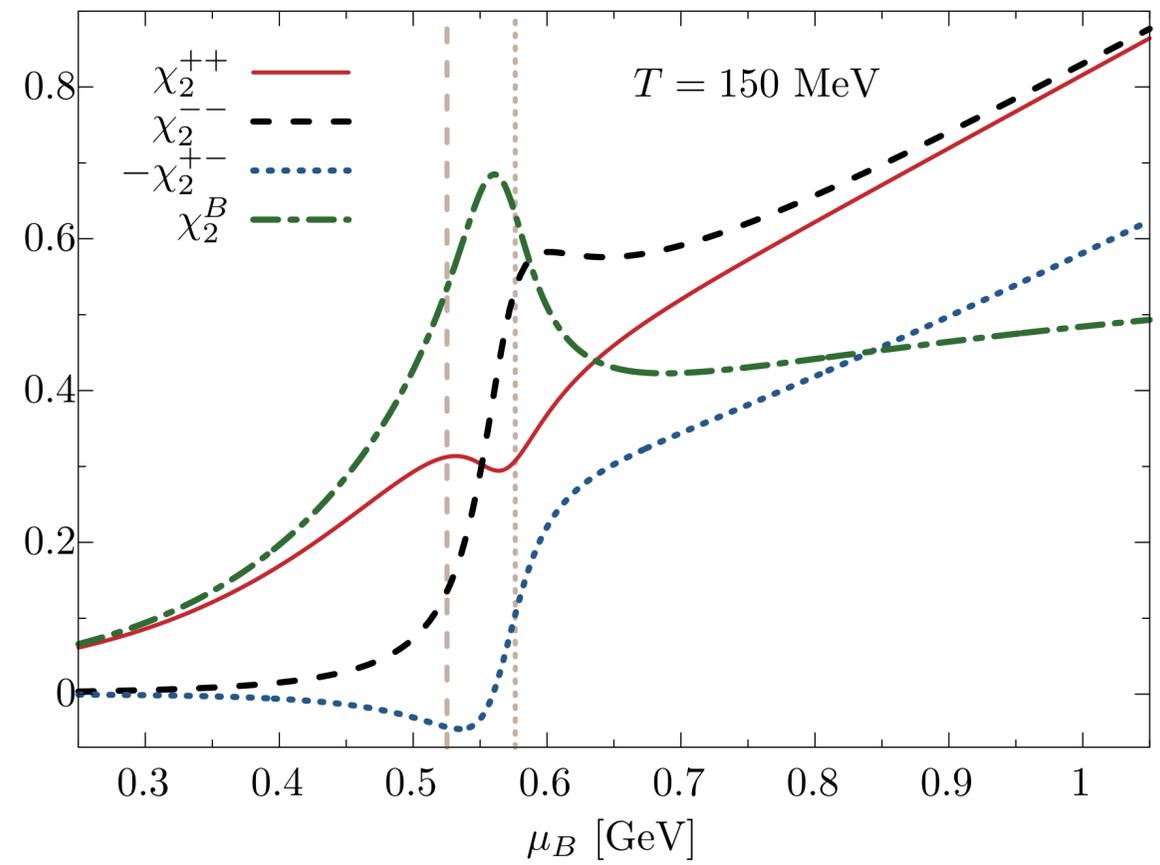
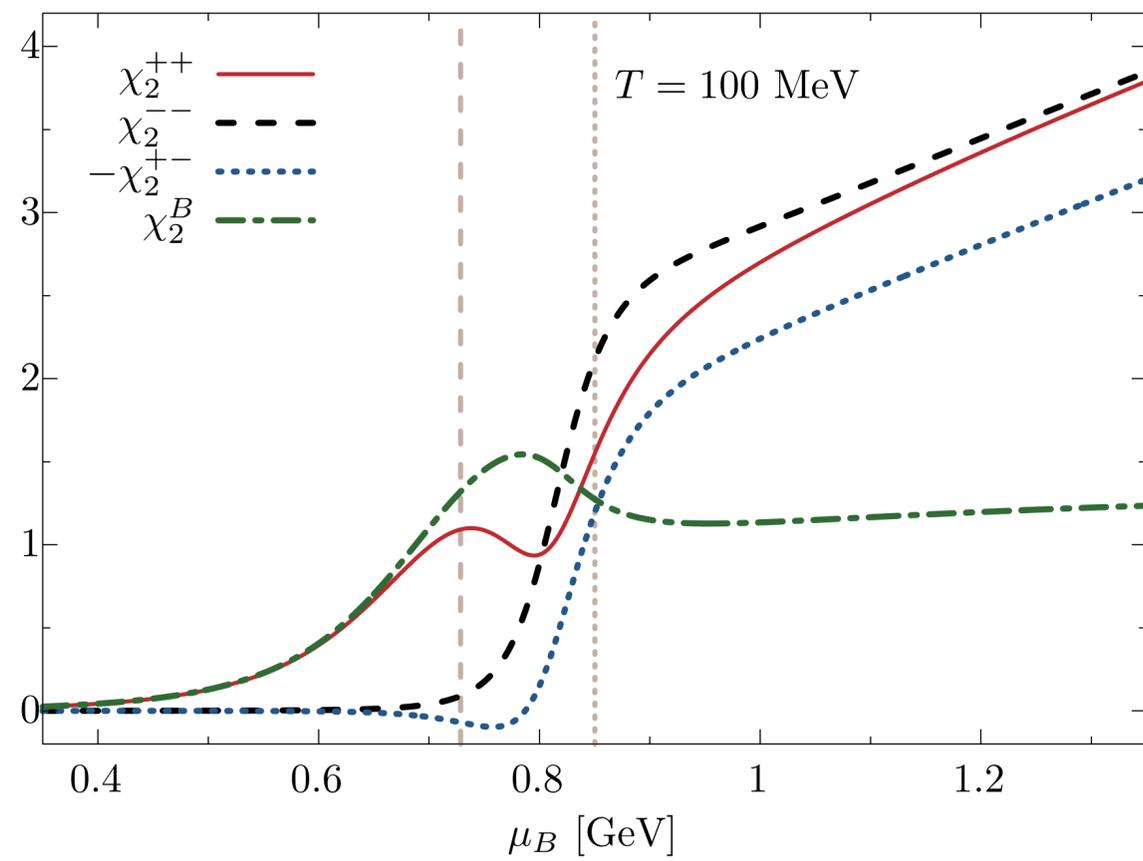
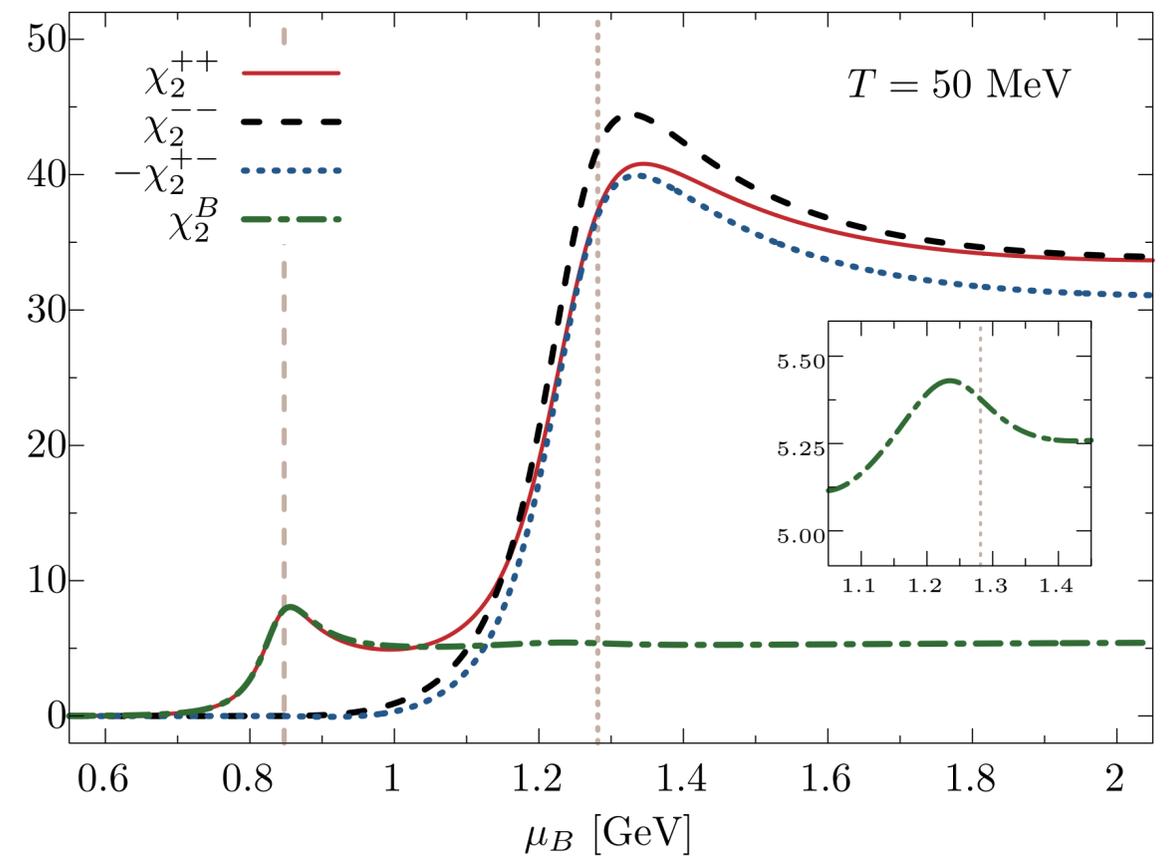
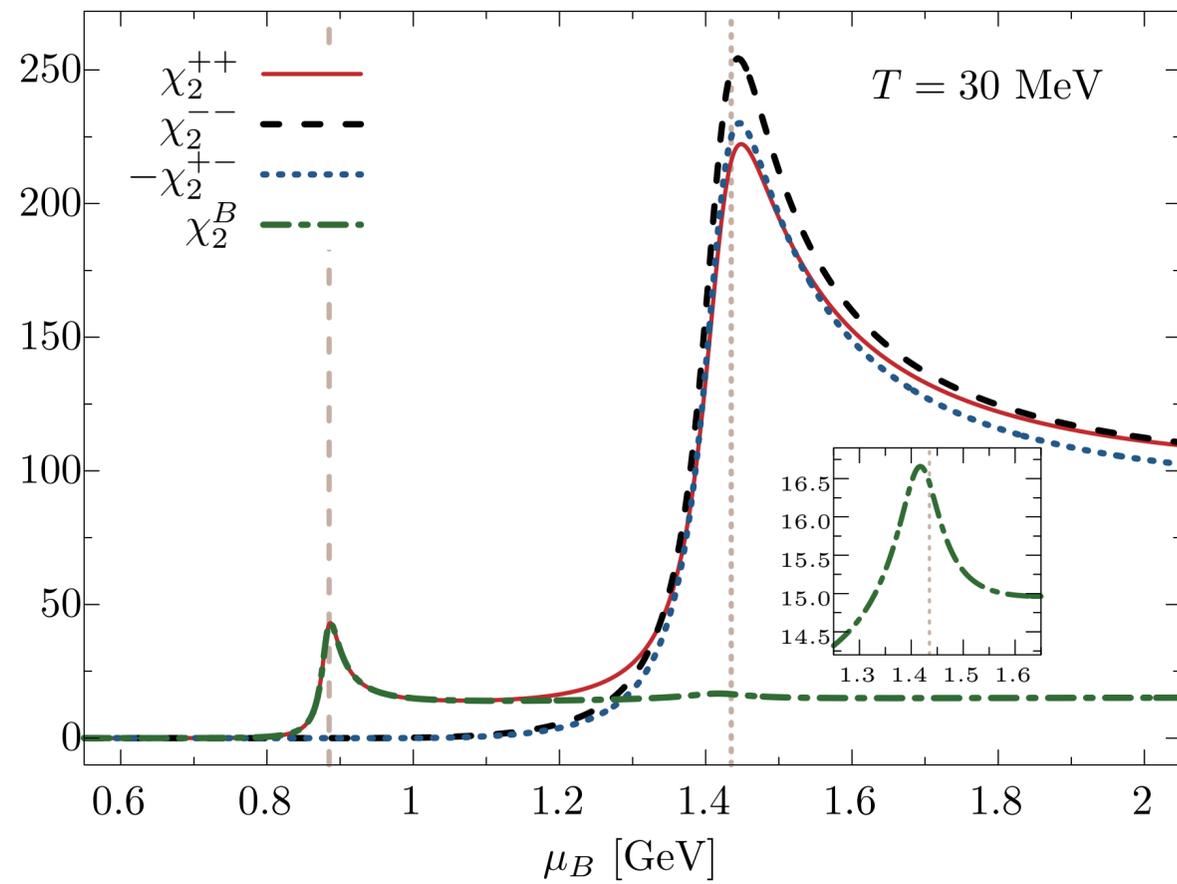
How N_- contributes to net-proton?

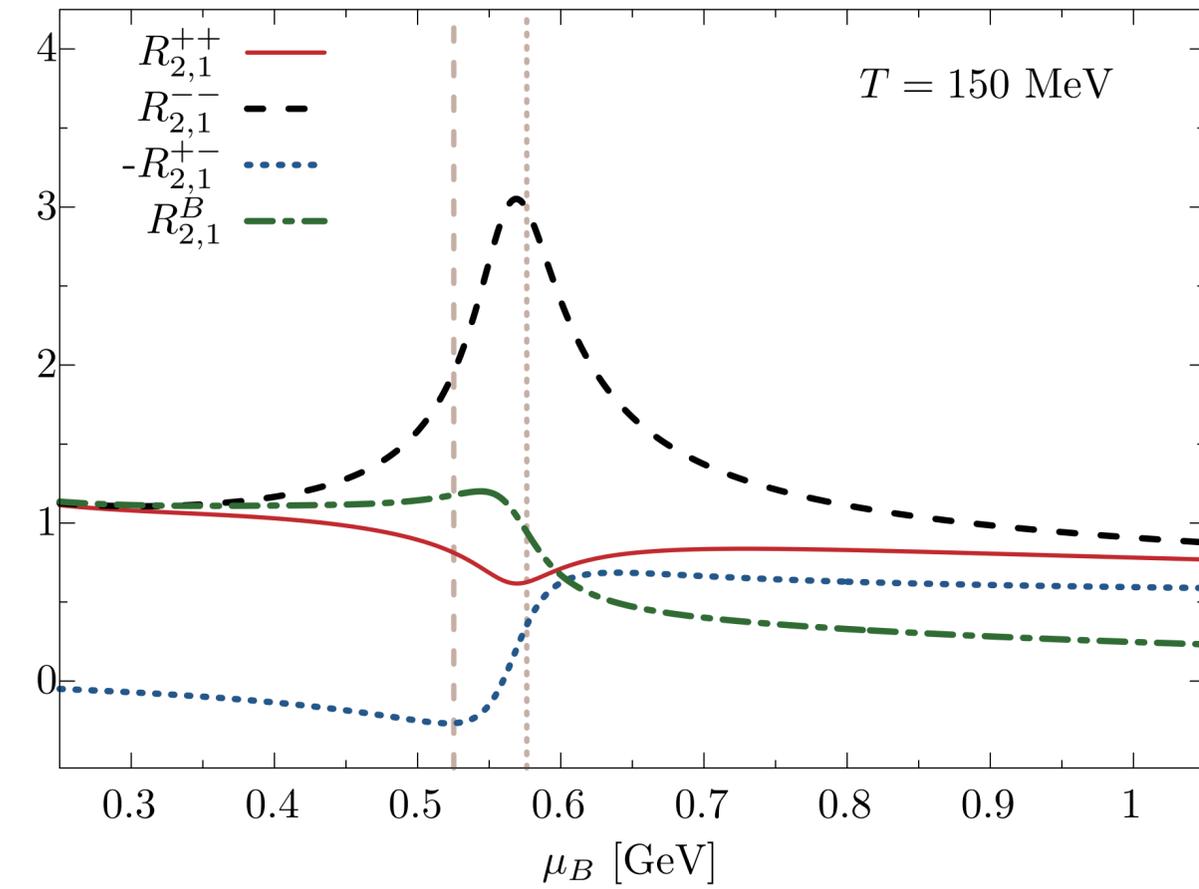
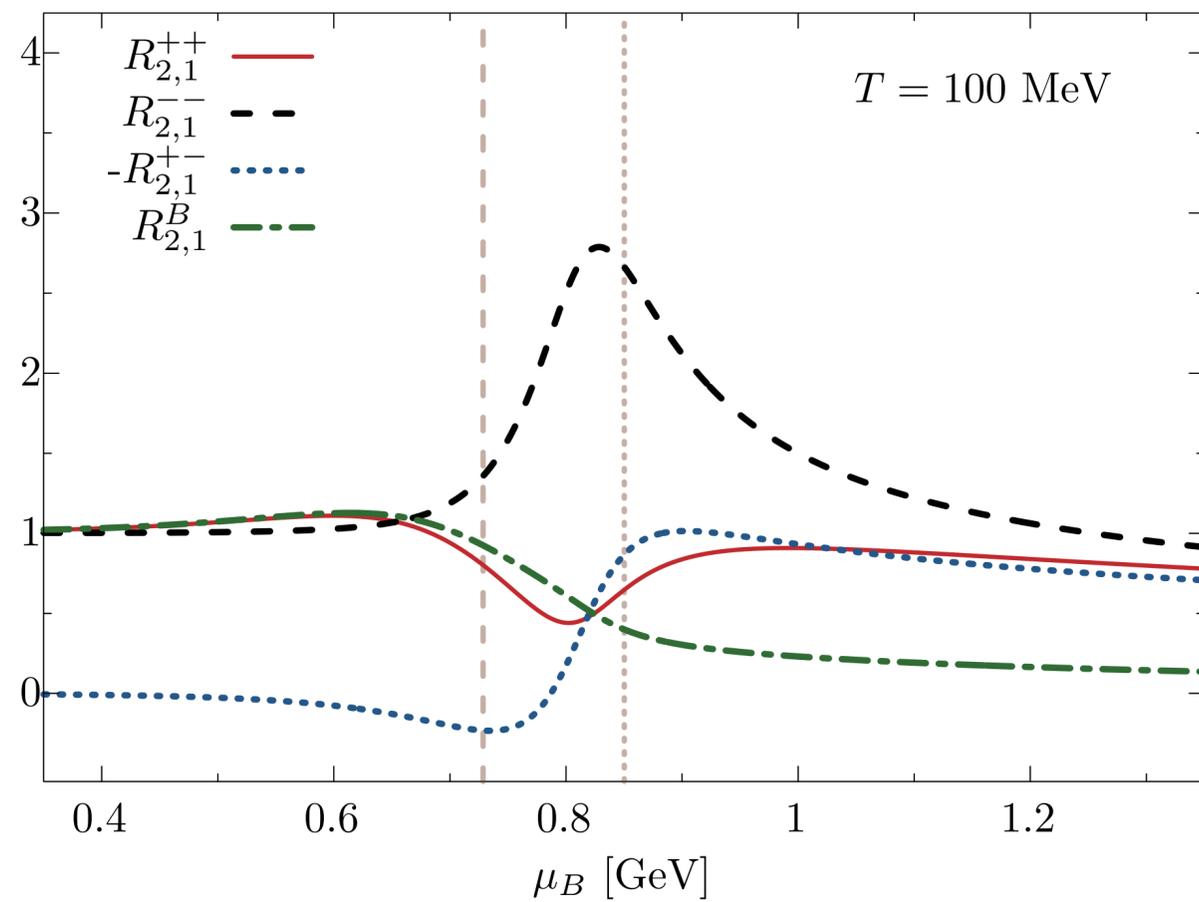
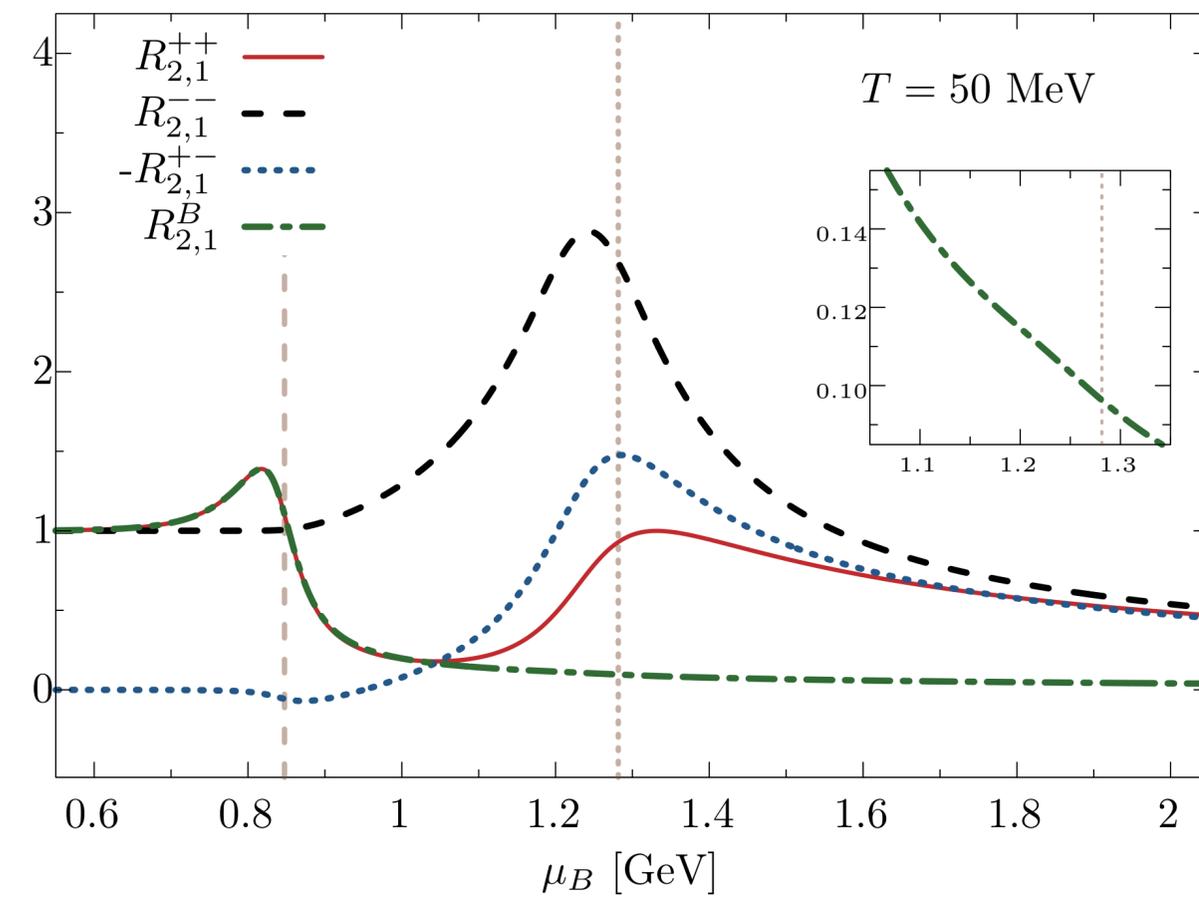
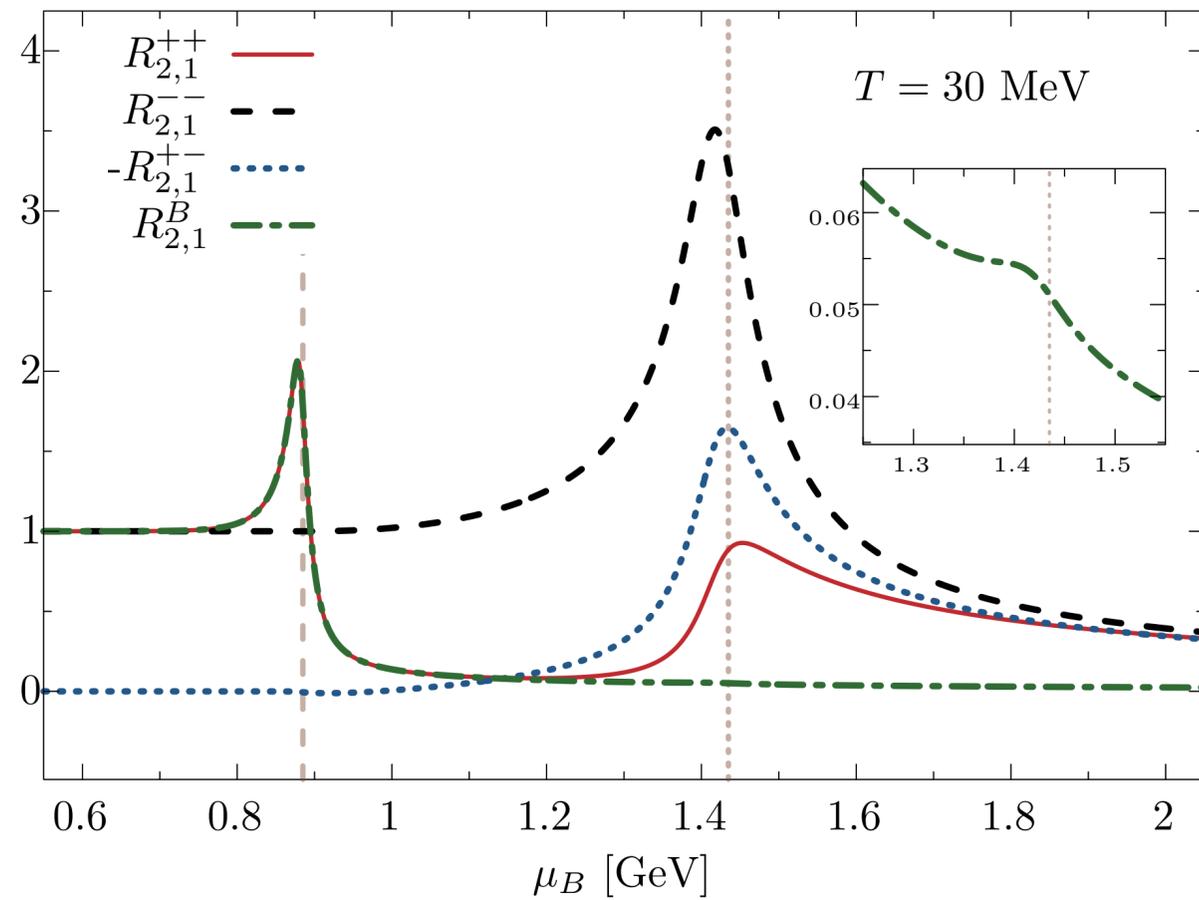
Thank You

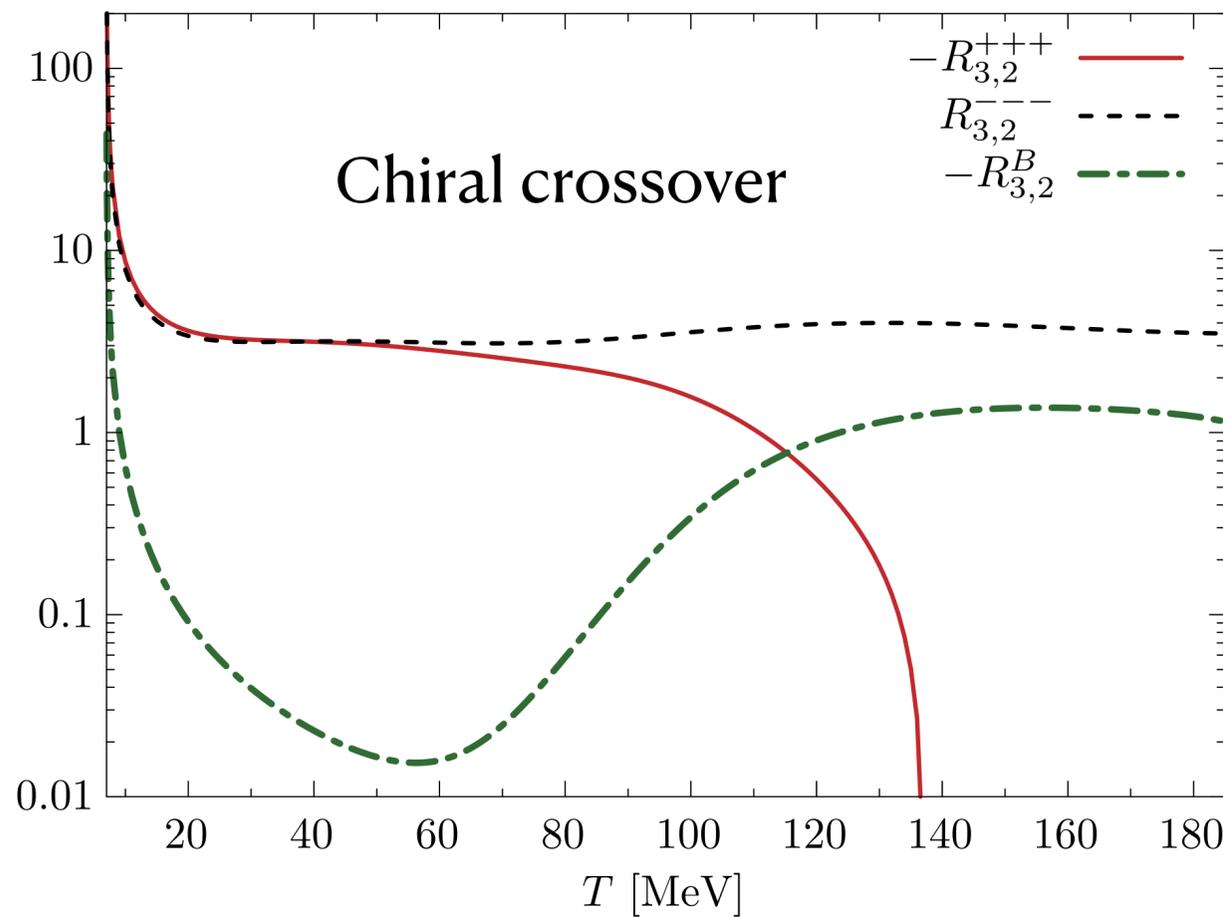
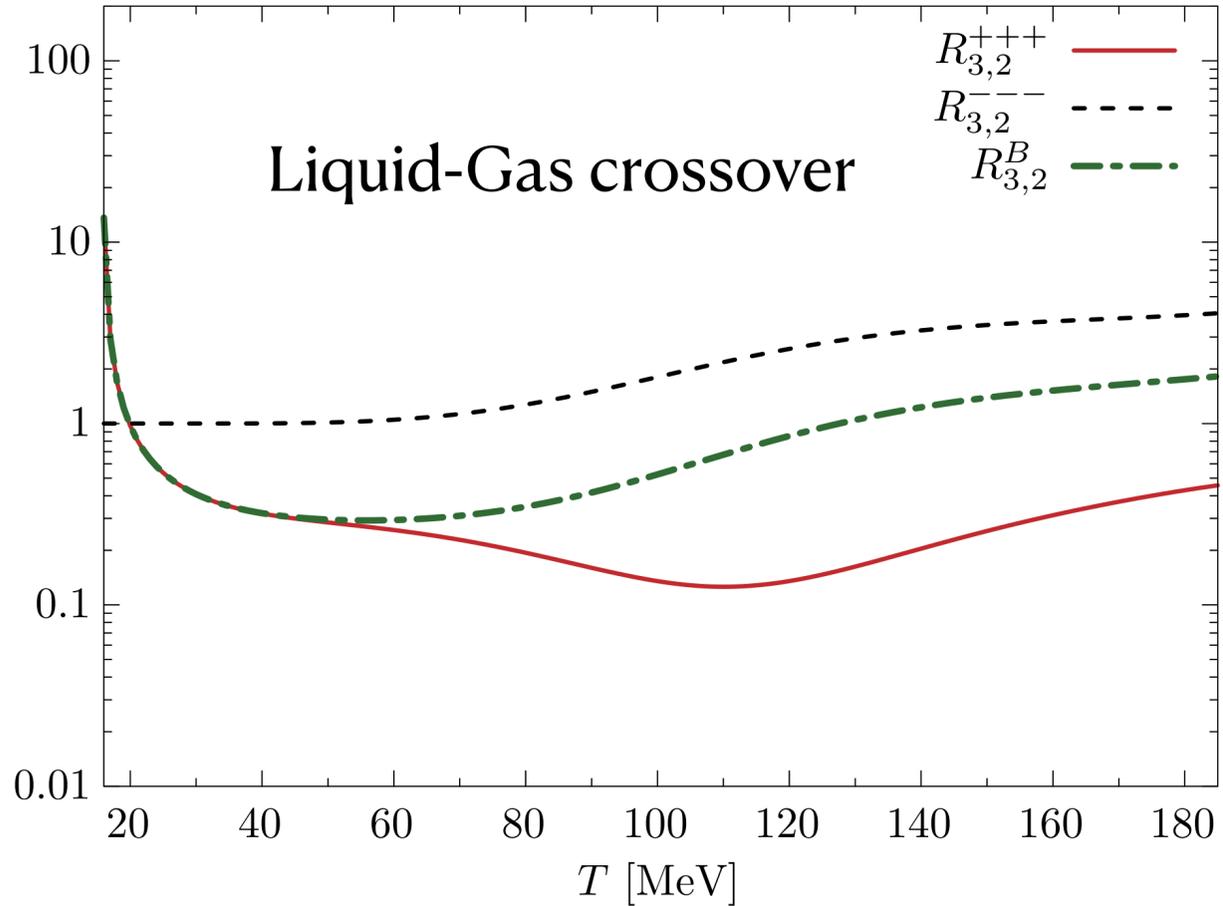












$$R_{3,2}^{\alpha\alpha\alpha} \equiv \frac{C_3^{\alpha\alpha}}{C_2^{\alpha\alpha}} = \frac{\chi_3^{\alpha\alpha}}{\chi_2^{\alpha\alpha}} = S\sigma$$

