

Spinodal enhancement of fluctuations in nucleus-nucleus collisions

Roman Poberezhnyuk

Quark Matter 2023

In collaborations with: Volodymyr Vovchenko, Volodymyr Kuznietsov, Oleh Savchuk, Mark Gorenstein, Jan Steinheimer, Horst Stoecker, and others



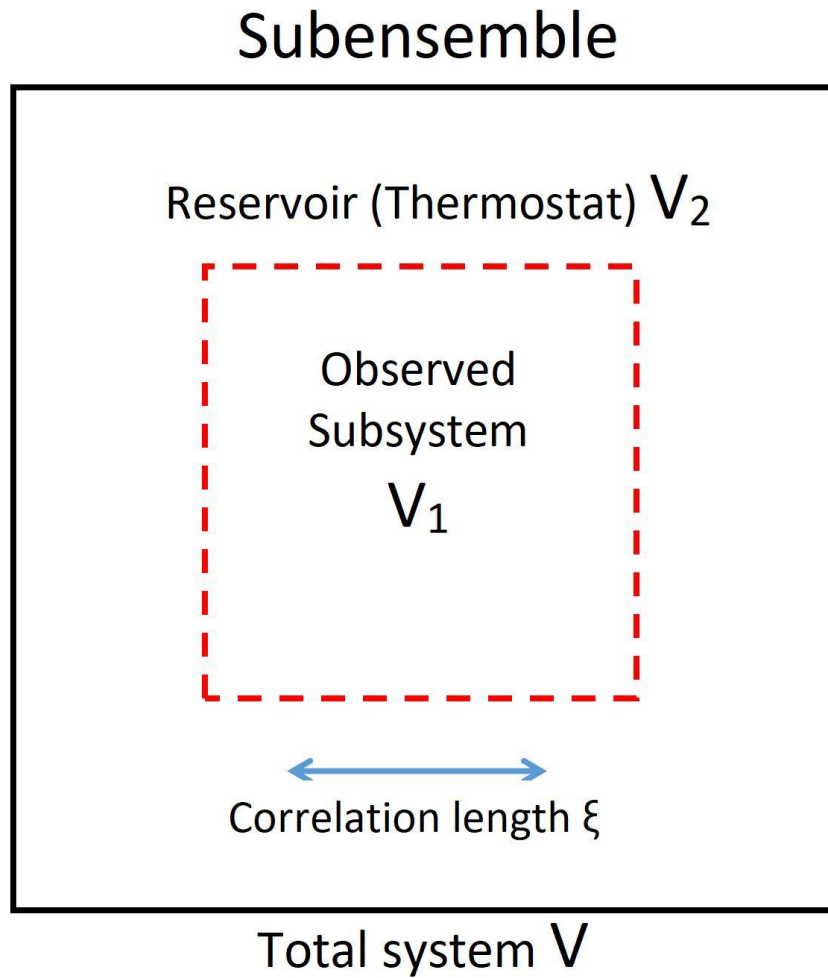
FIAS Frankfurt Institute
for Advanced Studies



Outline

- Brief overview of the Subensemble method for fluctuations in the presence of conservation laws
- Application to the fluctuations inside mixed phase of the FOPT
- Results for mixed phase fluctuations in the context of HIC
- HADES data on proton number fluctuations at $\sqrt{s_{NN}} = 2.4$ GeV

In HIC fluctuations are measured within Subensemble

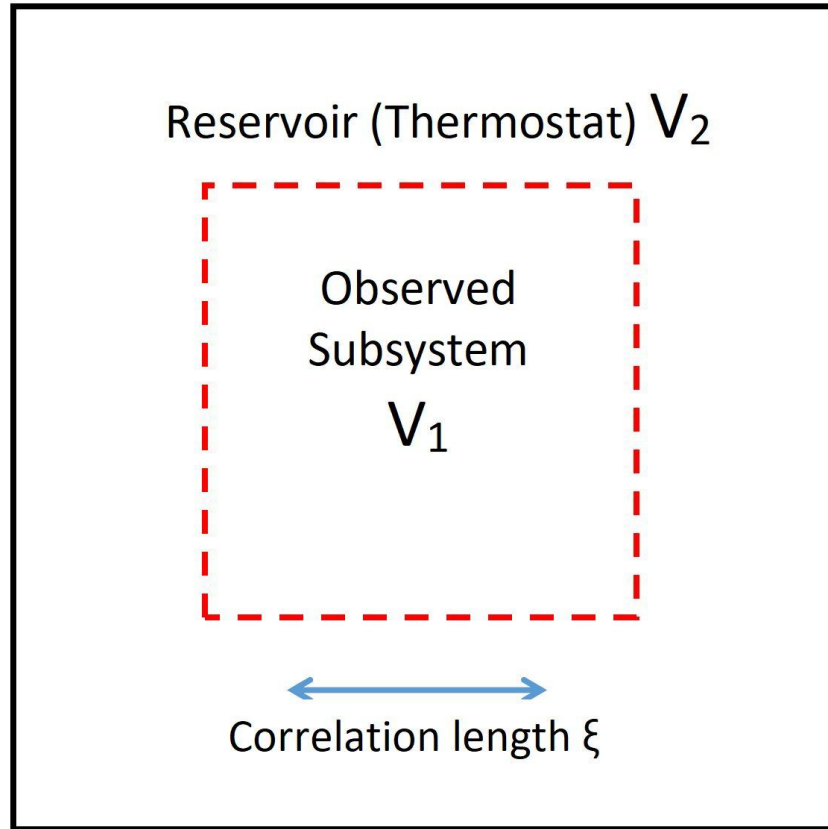


$$\xi \sim V_1 \sim V_2$$

Heavy Ion Collisions, Microscopic simulations

In HIC fluctuations are measured within Subensemble

Subensemble

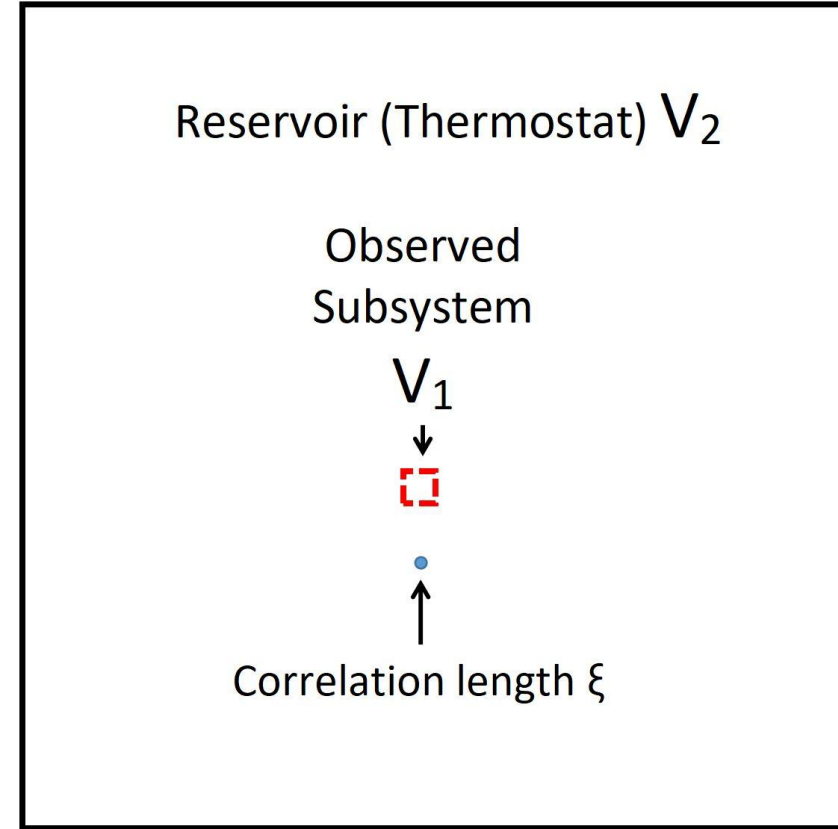


Total system V

$$\xi \sim V_1 \sim V_2$$

Heavy Ion Collisions, Microscopic simulations

Grand Canonical Ensemble



$$\xi \ll V_1 \ll V_2$$

Lattice QCD, Effective models of QCD

Ensembles are not equivalents w.r.t. fluctuations

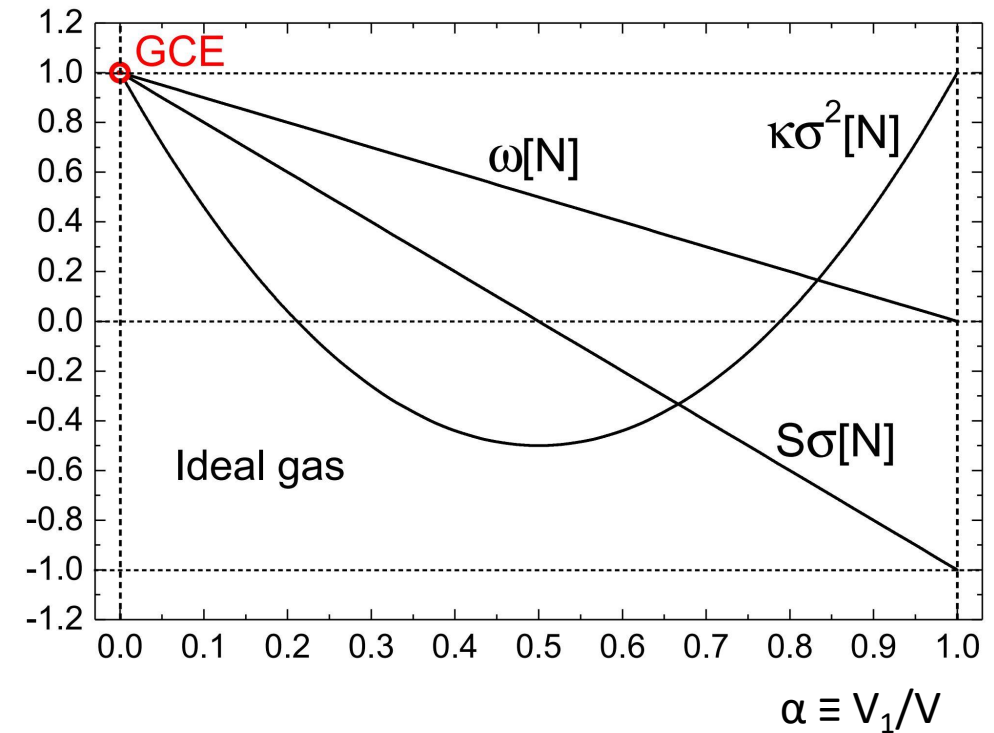
In HIC the system is too small to satisfy both GCE requirements

$$\xi \ll V_1 \ll V$$

- 1) $\xi \ll V_1$ (Thermodynamic limit)
ratios of extensive quantities become V -independent
- 2) $V_1 \sim V$; $\alpha \equiv V_1/V$ ($0 \ll \alpha \ll 1$)

Solution:

There is a simple connection between GCE and Subensemble cumulants in thermodynamic limit



Subensemble acceptance: Full result up to κ_6

$$\kappa_1[B_1] = \alpha VT^3 \chi_1^B \quad P(B_1) \propto Z^{\text{ce}}(T, \alpha V, B_1) Z^{\text{ce}}(T, (1 - \alpha)V, B - B_1), \quad \alpha \equiv V_1/V$$

$$\kappa_2[B_1] = \alpha VT^3 \beta \chi_2^B$$

Model-independent

$$\kappa_3[B_1] = \alpha VT^3 \beta (1 - 2\alpha) \chi_3^B$$

$$\kappa_4[B_1] = \alpha VT^3 \beta \left[\chi_4^B - 3\alpha\beta \frac{(\chi_3^B)^2 + \chi_2^B \chi_4^B}{\chi_2^B} \right]$$

$$\kappa_5[B_1] = \alpha VT^3 \beta (1 - 2\alpha) \left\{ [1 - 2\beta\alpha] \chi_5^B - 10\alpha\beta \frac{\chi_3^B \chi_4^B}{\chi_2^B} \right\}$$

$$\kappa_6[B_1] = \alpha VT^3 \beta [1 - 5\alpha\beta(1 - \alpha\beta)] \chi_6^B + 5 VT^3 \alpha^2 \beta^2 \left\{ 9\alpha\beta \frac{(\chi_3^B)^2 \chi_4^B}{(\chi_2^B)^2} - 3\alpha\beta \frac{(\chi_3^B)^4}{(\chi_2^B)^3} \right. \\ \left. - 2(1 - 2\alpha)^2 \frac{(\chi_4^B)^2}{\chi_2^B} - 3[1 - 3\beta\alpha] \frac{\chi_3^B \chi_5^B}{\chi_2^B} \right\}$$

$$\chi_n^B = \frac{\partial^n (p/T^4)}{\partial (\mu_B/T)^n} \quad \text{-- grand-canonical susceptibilities} \quad \beta = 1 - \alpha$$

Subensemble acceptance: Cumulant ratios

Some common cumulant ratios:

$$\langle B \rangle = \kappa_1, \quad \omega = \frac{\kappa_2}{\kappa_1}, \quad S\sigma = \frac{\kappa_3}{\kappa_2}, \quad \kappa\sigma^2 = \frac{\kappa_4}{\kappa_2}.$$

scaled variance

$$\frac{\kappa_2[B_1]}{\kappa_1[B_1]} = (1 - \alpha) \frac{\chi_2^B}{\chi_1^B},$$

skewness

$$\frac{\kappa_3[B_1]}{\kappa_2[B_1]} = (1 - 2\alpha) \frac{\chi_3^B}{\chi_2^B},$$

kurtosis

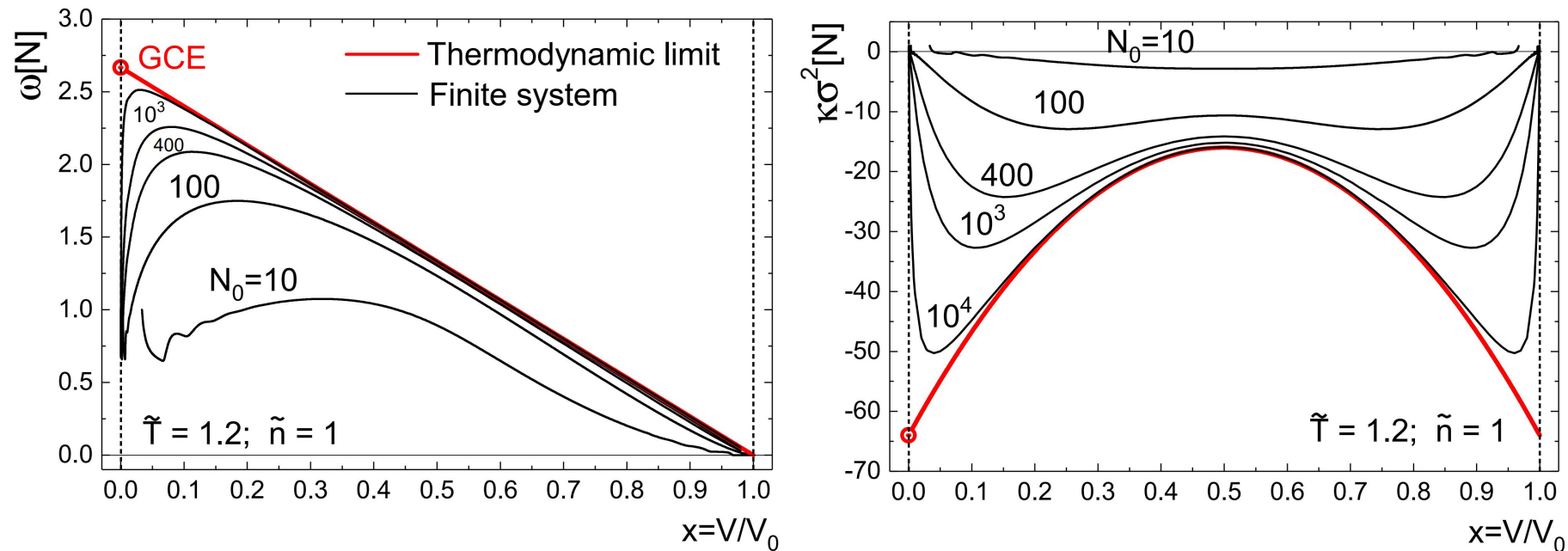
$$\frac{\kappa_4[B_1]}{\kappa_2[B_1]} = (1 - 3\alpha\beta) \frac{\chi_4^B}{\chi_2^B} - 3\alpha\beta \left(\frac{\chi_3^B}{\chi_2^B} \right)^2.$$

Subensemble acceptance: van der Waals fluid

Calculate cumulants $\kappa_n[N]$ in a subvolume directly from the partition function

$$P(N) \propto Z_{\text{vdW}}^{\text{ce}}(T, xV_0, N) Z_{\text{vdW}}^{\text{ce}}(T, (1-x)V_0, N_0 - N)$$

and compare with the subensemble acceptance results



Results agree with subensemble acceptance in thermodynamic limit ($N_0 \rightarrow \infty$)

Finite size effects are strong near the critical point: a consequence of large correlation length ξ

[R.P., Savchuk, Gorenstein, Vovchenko, Taradiy, Begun, Satarov, Steinheimer, Stoecker, *PRC*, 20]

[Kuznietsov, Savchuk, Gorenstein, Koch, Vovchenko, *PRC*, 22]

Further developments of the Subensemble Acceptance Method

- Multiple conserved charges (e.g., B,Q,S) [Vovchenko, R.P., Koch, *JHEP*, 20]
- Factorial cumulants [Barej, Bzdak, *PRC*, 22]
- NLO corrections [Barej, Bzdak, *PRC*, 23]
- Mixed phase of FOPT???

Mixed phase GCE fluctuations in homogeneous equilibrium

Model independent

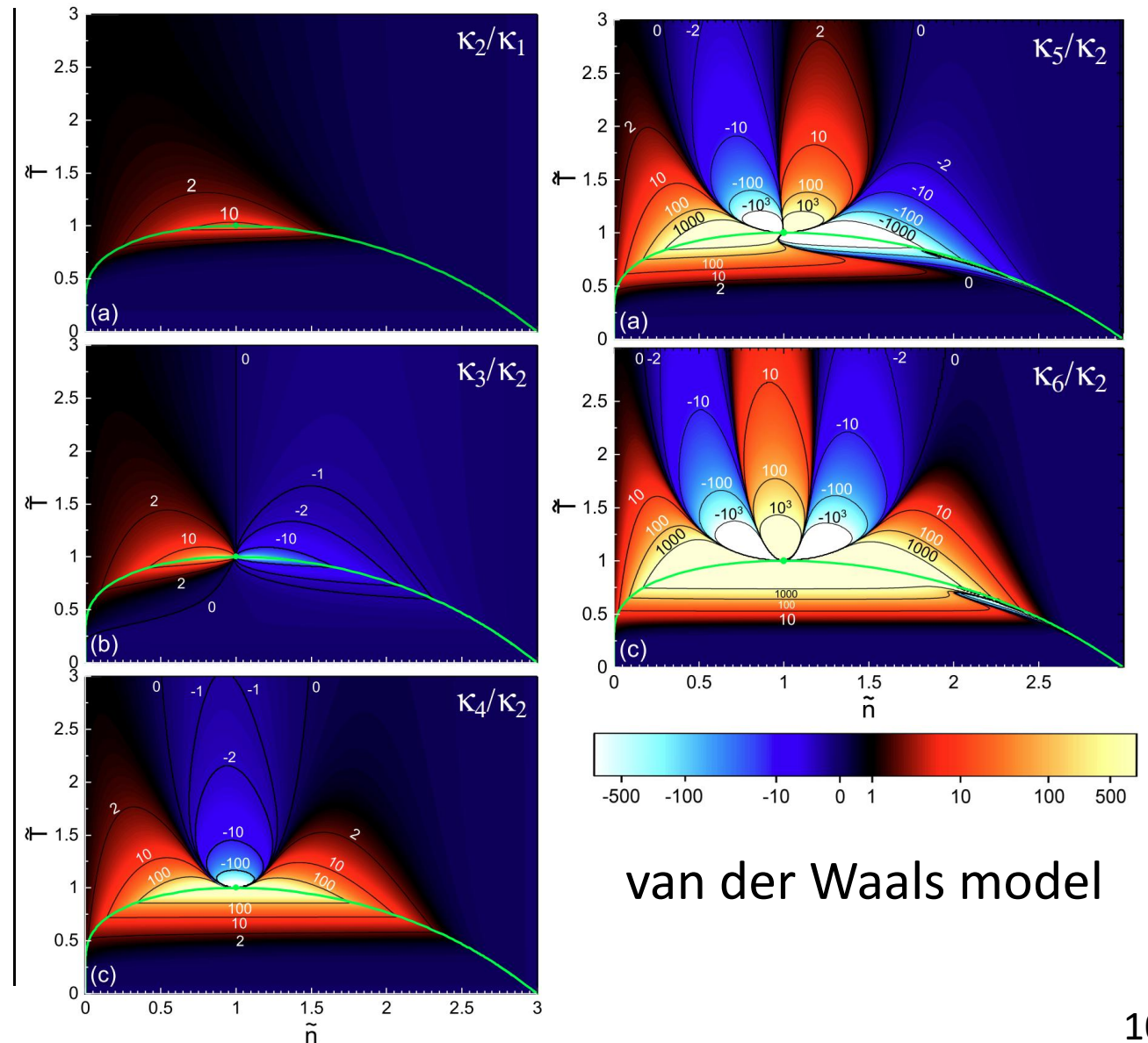
$$V_{liquid} \equiv V_1, \quad V_{gas} \equiv V_2, \quad x \equiv V_1/V, \quad y \equiv 1 - x$$

$$\langle B^r \rangle = \langle (B_1 + B_2)^r \rangle = V^r \langle (x\rho_1 + y\rho_2)^r \rangle$$

$$\langle \rho_1^l \rho_2^m x^n \rangle = \langle \rho_1^l \rangle \langle \rho_2^m \rangle \langle x^n \rangle$$

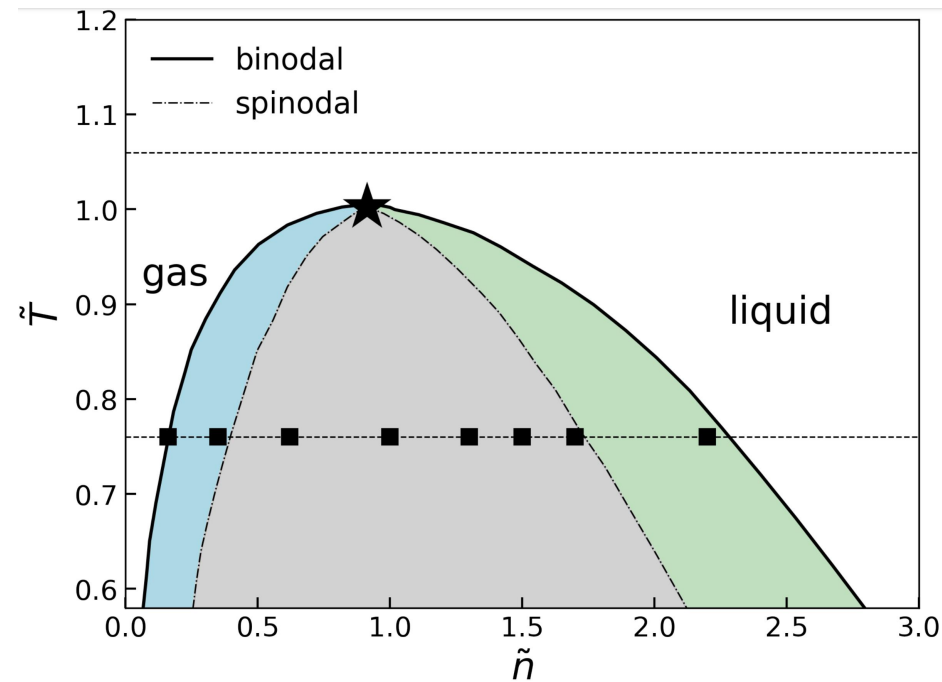
$$\kappa_1 = \kappa_{1,1} + \kappa_{1,2} ,$$

$$\kappa_j = \kappa_{j,1} + \kappa_{j,2} + [(n_1 - n_2)V]^j \kappa_{j,x} , \quad j \geq 2.$$



Mixed phase at finite size and time

$$\tilde{p} = p^*/p_c^*, \quad \tilde{n} = n^*/n_c^*, \quad \tilde{T} = T^*/T_c^*.$$

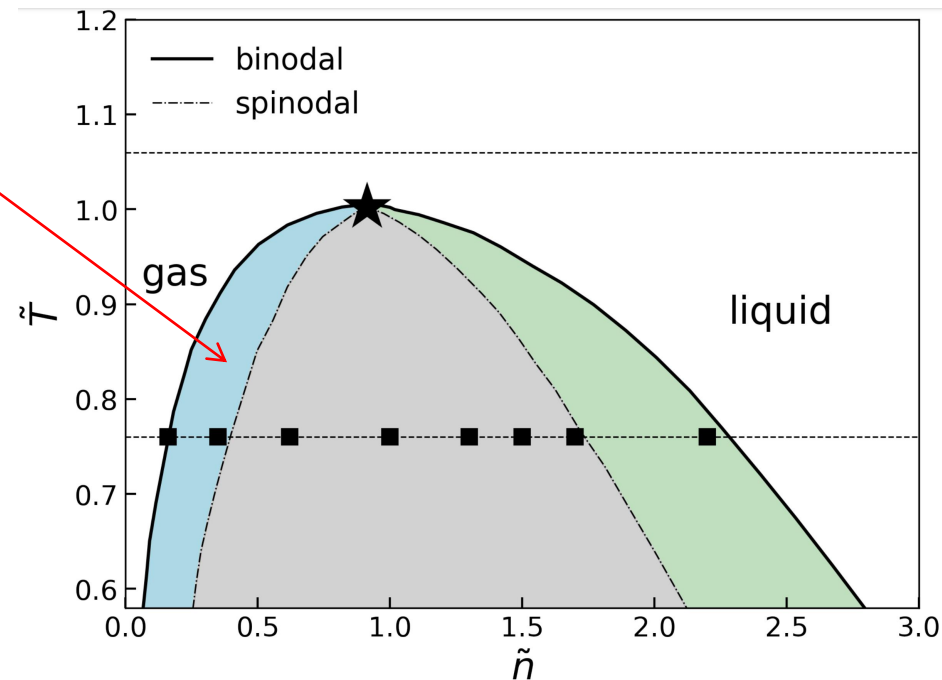
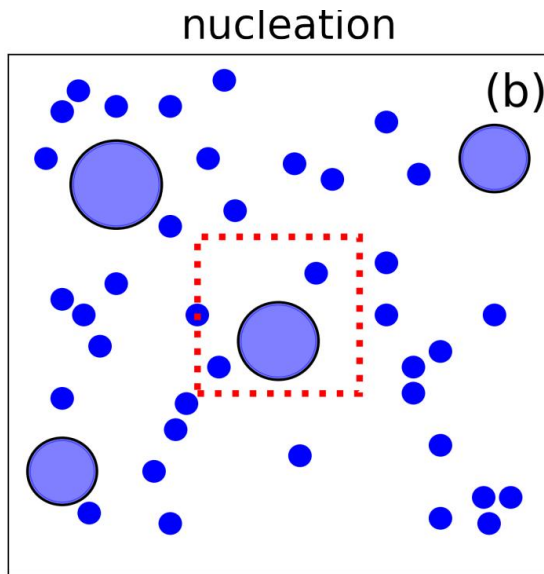


[Evans, da Gamma, *Molecular Phys.*, 79]

[Kuznietsov, Savchuk, **R.P.**, Vovchenko, Gorenstein, Stoecker, PRC, 23]

Mixed phase at finite size and time

$$\tilde{p} = p^*/p_c^*, \quad \tilde{n} = n^*/n_c^*, \quad \tilde{T} = T^*/T_c^*.$$

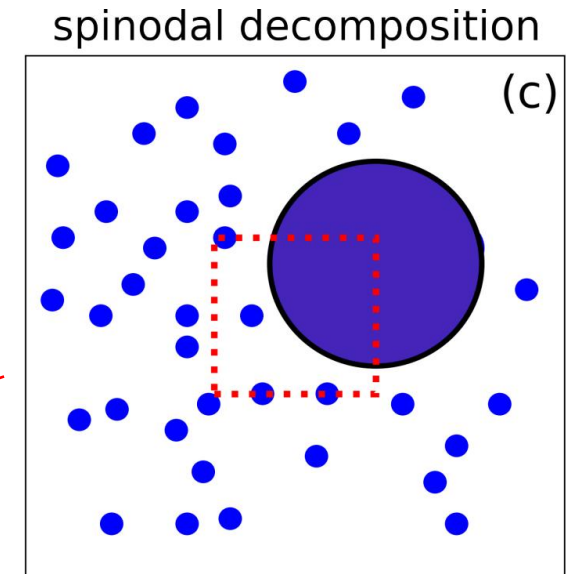
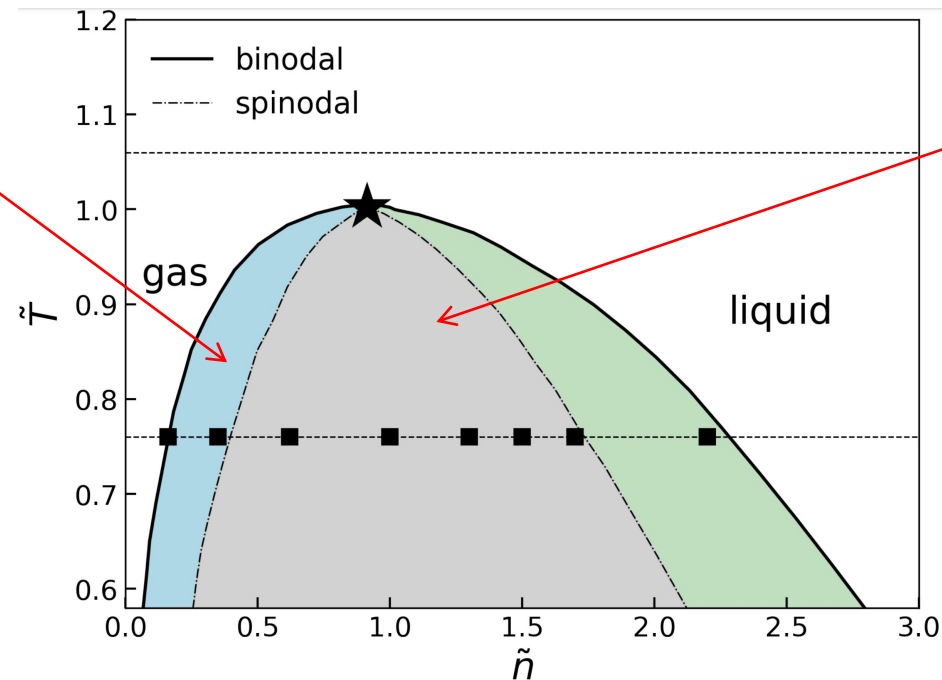
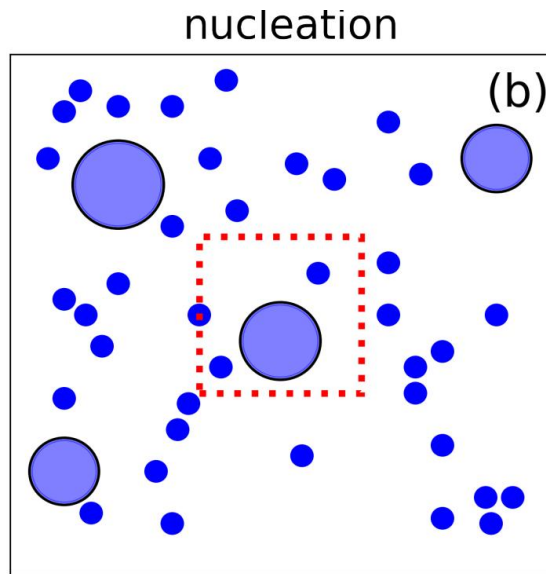


[Evans, da Gamma, *Molecular Phys.*, 79]

[Kuznietsov, Savchuk, **R.P.**, Vovchenko, Gorenstein, Stoecker, PRC, 23]

Mixed phase at finite size and time

$$\tilde{p} = p^*/p_c^*, \quad \tilde{n} = n^*/n_c^*, \quad \tilde{T} = T^*/T_c^*.$$

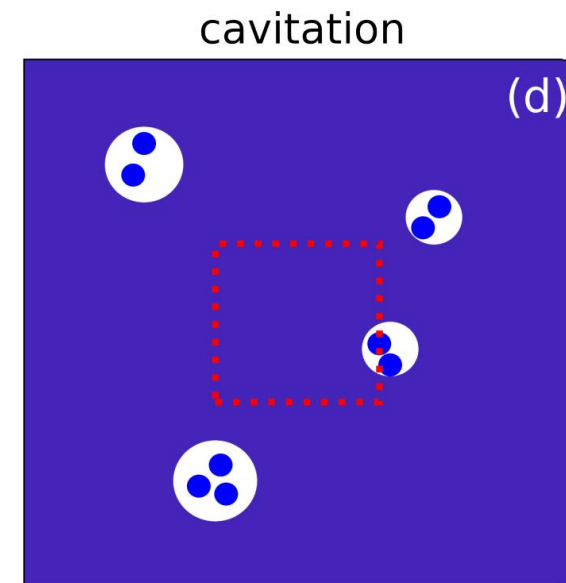
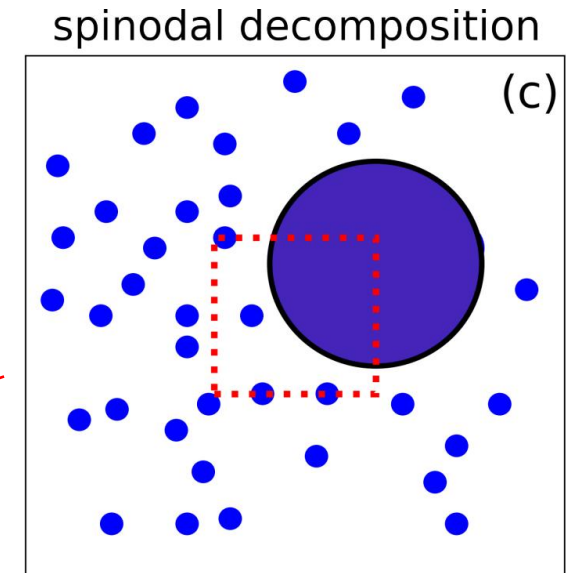
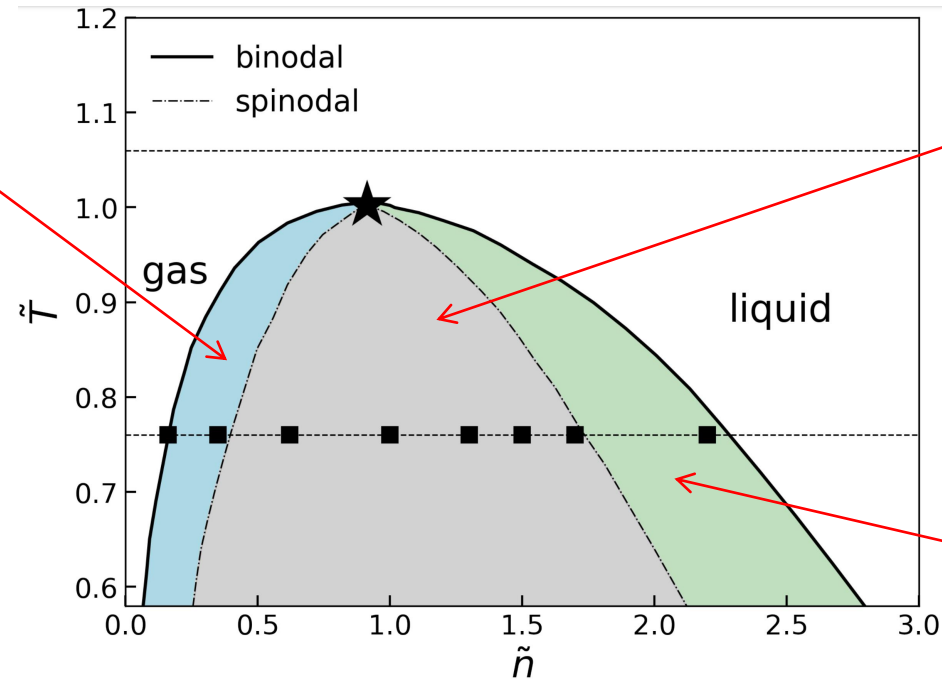
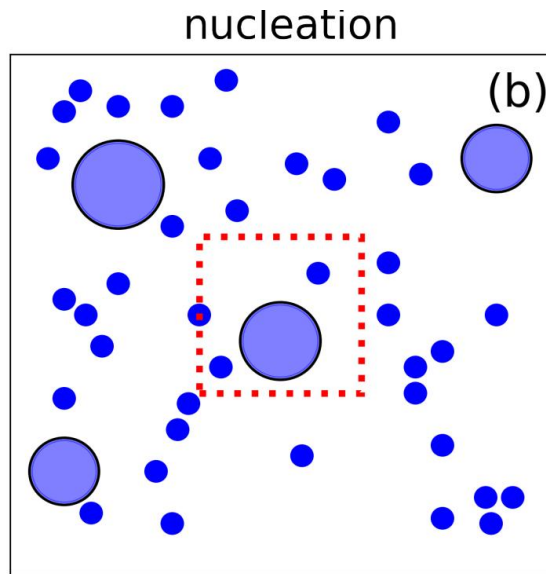


[Evans, da Gamma, *Molecular Phys.*, 79]

[Kuznietsov, Savchuk, **R.P.**, Vovchenko, Gorenstein, Stoecker, *PRC*, 23]

Mixed phase at finite size and time

$$\tilde{p} = p^*/p_c^*, \quad \tilde{n} = n^*/n_c^*, \quad \tilde{T} = T^*/T_c^*.$$

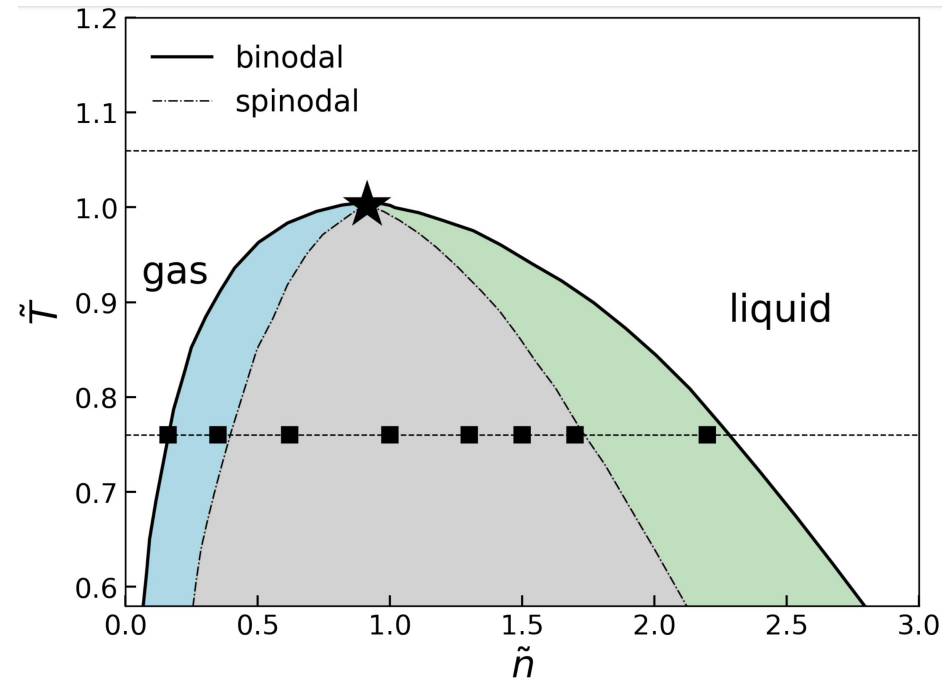


[Evans, da Gamma, *Molecular Phys.*, 79]

[Kuznietsov, Savchuk, **R.P.**, Vovchenko, Gorenstein, Stoecker, *PRC*, 23]

Lennard-Jones fluid simulations

$$\text{Potential: } V_{\text{LJ}} = 4\epsilon \left[\left(\frac{\sigma}{r} \right)^6 - \left(\frac{\sigma}{r} \right)^{12} \right]$$
$$\alpha = 0.2, N = 400 \quad \tilde{\omega} = \omega / (1 - \alpha)$$



Lennard-Jones fluid simulations

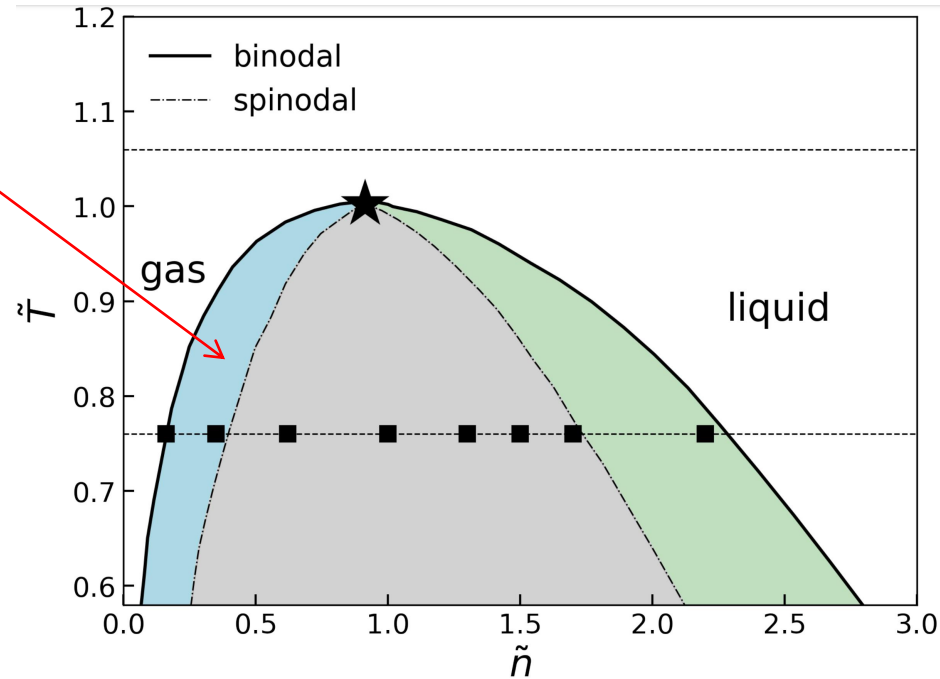
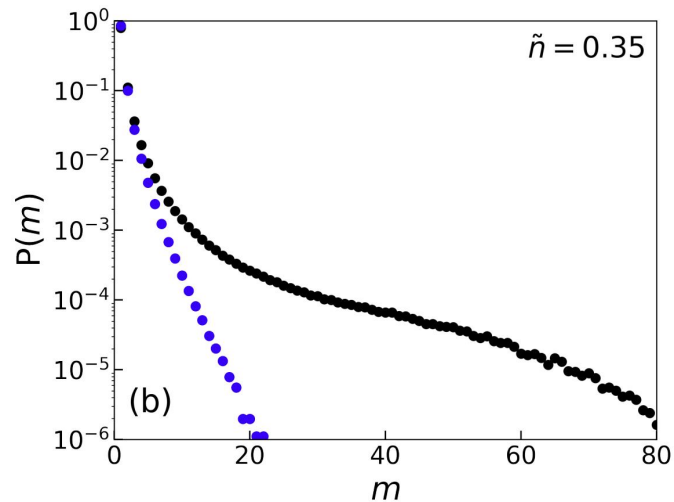
$$\text{Potential: } V_{\text{LJ}} = 4\epsilon \left[\left(\frac{\sigma}{r} \right)^6 - \left(\frac{\sigma}{r} \right)^{12} \right]$$

$$\alpha = 0.2, N = 400 \quad \tilde{\omega} = \omega / (1 - \alpha)$$

Model of non-interacting clusters in GCE:

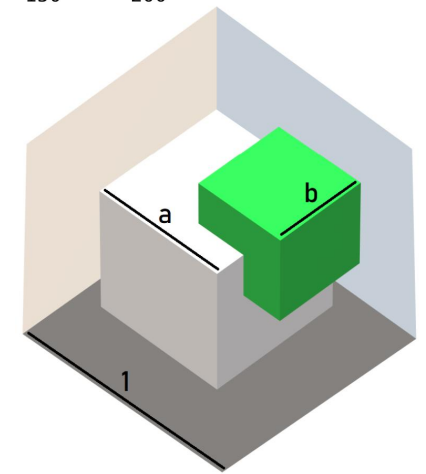
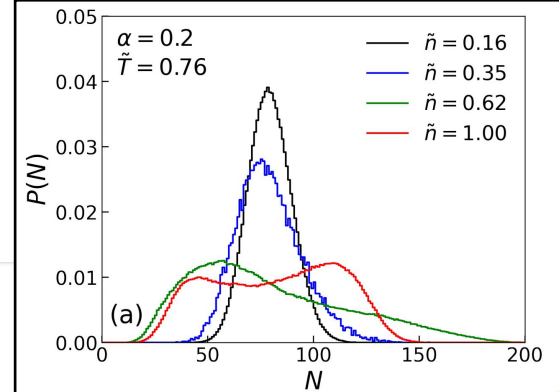
$$Z_{\text{GCE}} = \prod_{k \geq 1} \exp \left[V (2\pi k m T)^{3/2} g(k) \exp \left(\frac{\mu k}{T} \right) \right]$$

Distribution of clusters from LJ:



Lennard-Jones fluid simulations

Potential: $V_{\text{LJ}} = 4\epsilon \left[\left(\frac{\sigma}{r} \right)^6 - \left(\frac{\sigma}{r} \right)^{12} \right]$
 $\alpha = 0.2, N = 400 \quad \tilde{\omega} = \omega / (1 - \alpha)$



Minecraft Model

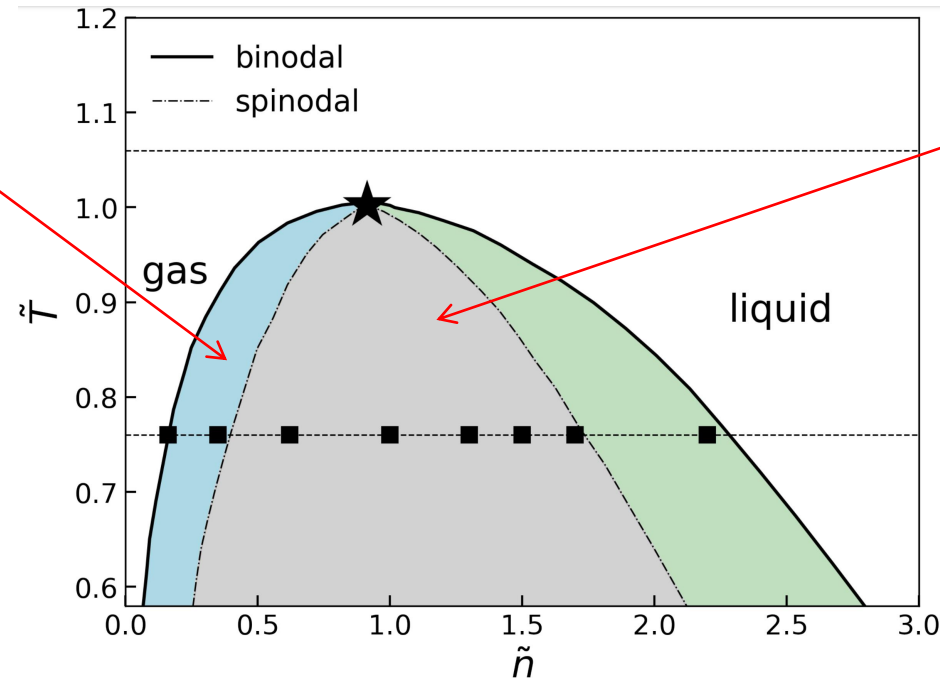
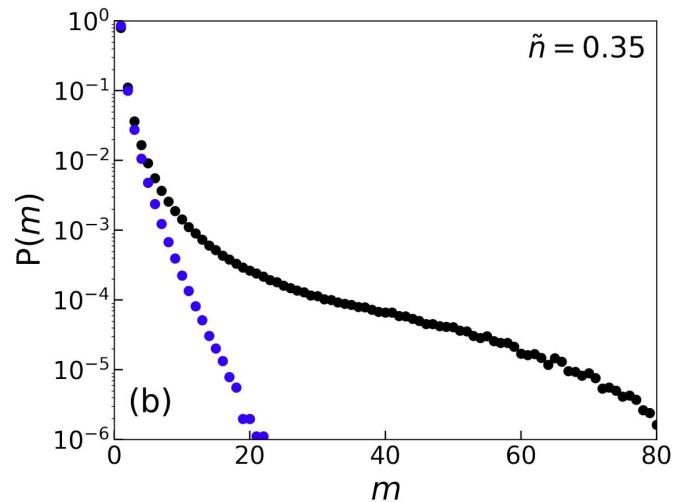
$$\tilde{\omega}[N] = \alpha(1 - \alpha)N_0 \frac{(n_l - n_g)^2}{n^2} \text{Var}[x].$$

$$\text{Var}[x] = -b^6 + \left[\frac{b^2(3a - b) + \Theta_{a+b-1}(a + b - 1)^3 + \Theta_{b-a}(b - a)^3}{3a^2} \right]^3$$

Model of non-interacting clusters in GCE:

$$Z_{\text{GCE}} = \prod_{k \geq 1} \exp \left[V(2\pi k m T)^{3/2} g(k) \exp \left(\frac{\mu k}{T} \right) \right]$$

Distribution of clusters from LJ:



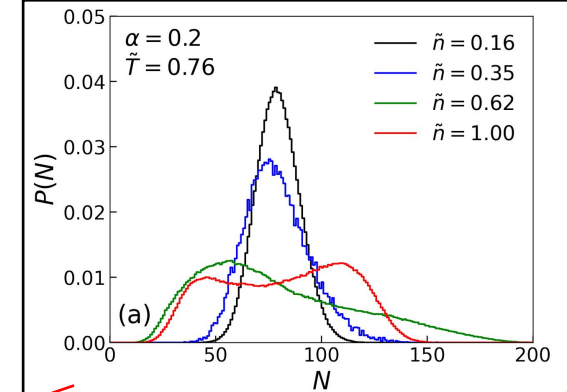
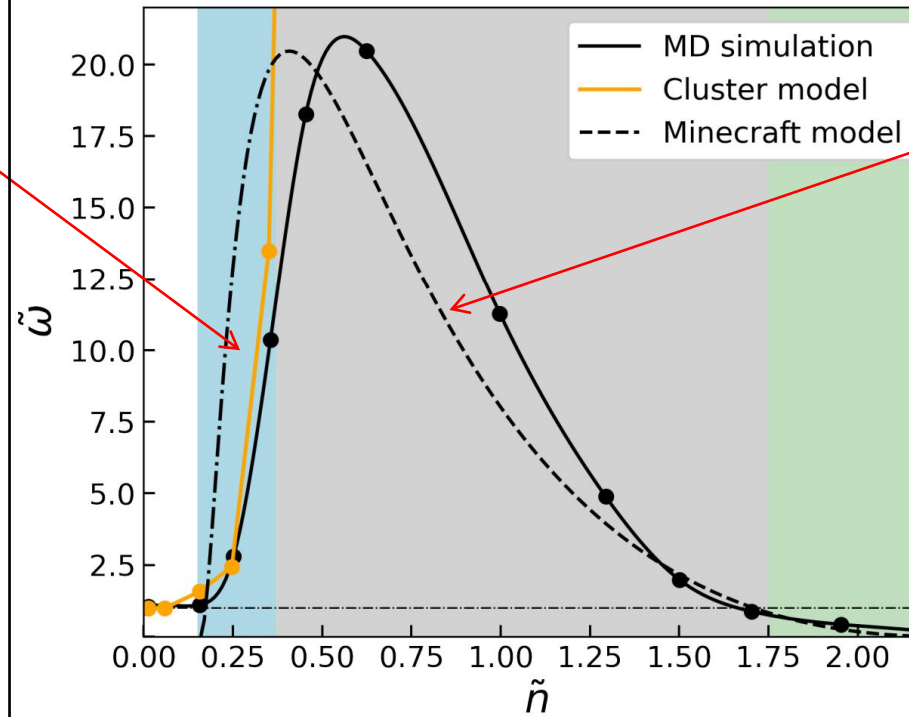
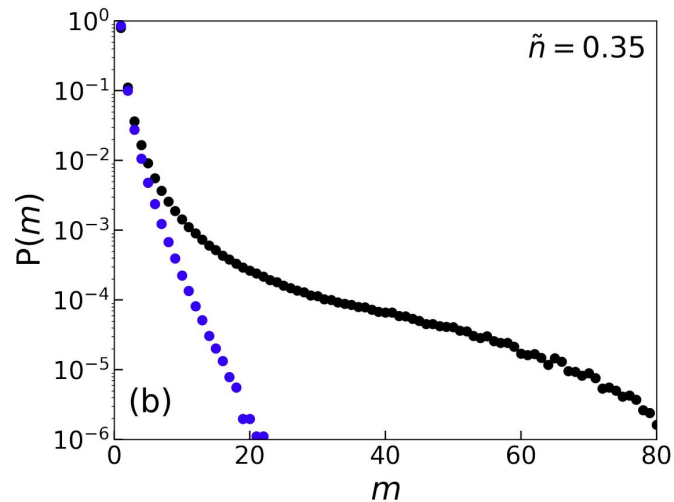
Lennard-Jones fluid simulations

Potential: $V_{LJ} = 4\epsilon \left[\left(\frac{\sigma}{r} \right)^6 - \left(\frac{\sigma}{r} \right)^{12} \right]$
 $\alpha = 0.2, N = 400 \quad \tilde{\omega} = \omega / (1 - \alpha)$

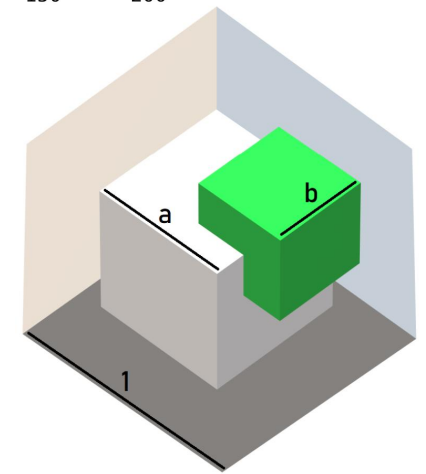
Model of non-interacting clusters in GCE:

$$Z_{GCE} = \prod_{k \geq 1} \exp \left[V (2\pi k m T)^{3/2} g(k) \exp \left(\frac{\mu k}{T} \right) \right]$$

Distribution of clusters from LJ:



Minecraft Model



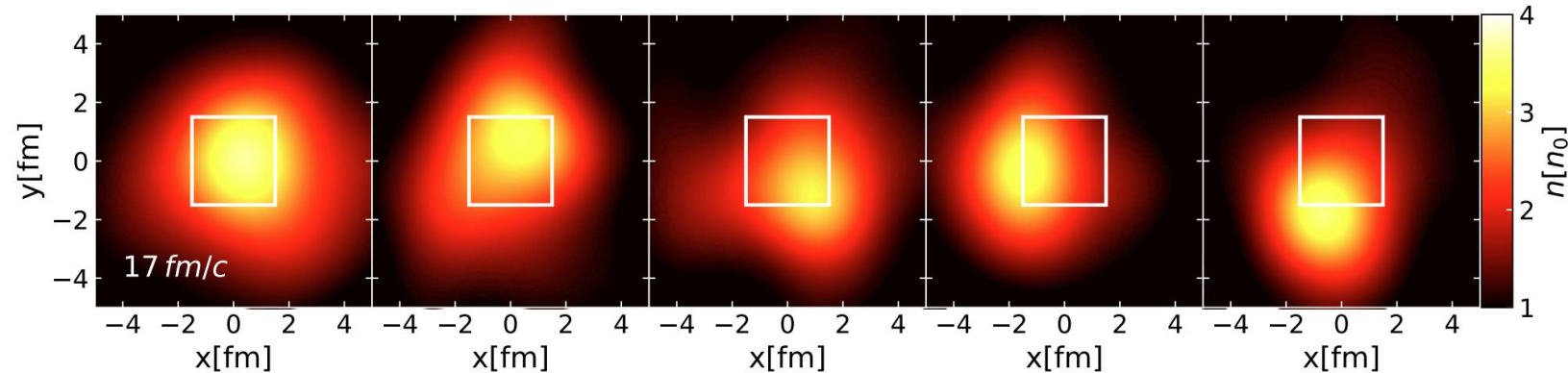
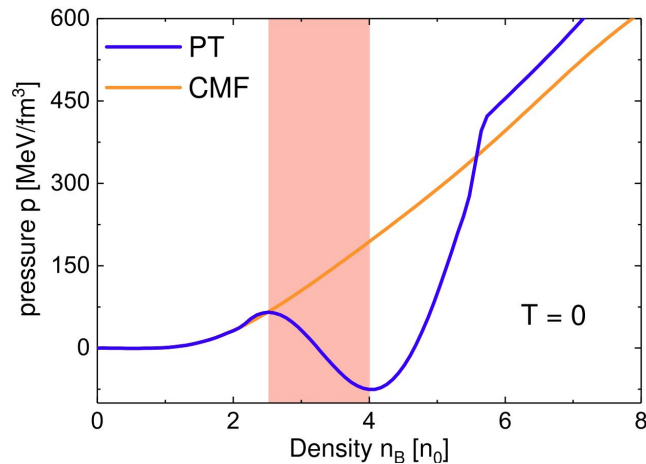
$$\tilde{\omega}[N] = \alpha(1 - \alpha) N_0 \frac{(n_l - n_g)^2}{n^2} \text{Var}[x].$$

$$\text{Var}[x] = -b^6 + \left[\frac{b^2(3a - b) + \Theta_{a+b-1}(a + b - 1)^3 + \Theta_{b-a}(b - a)^3}{3a^2} \right]^3$$

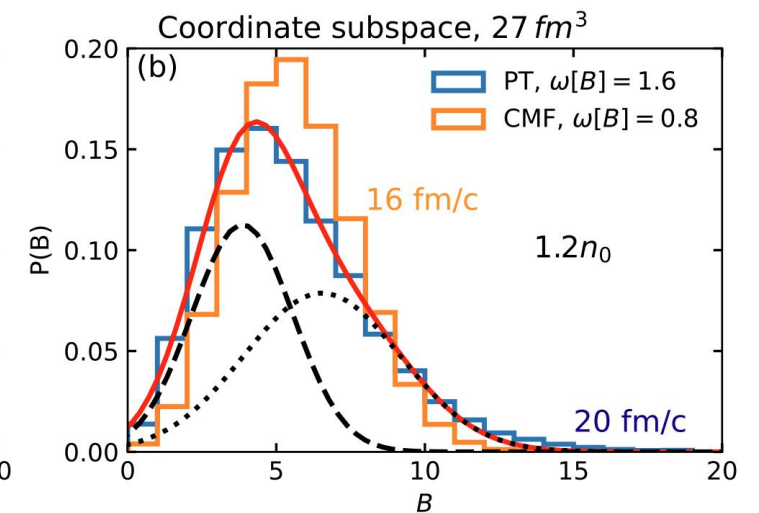
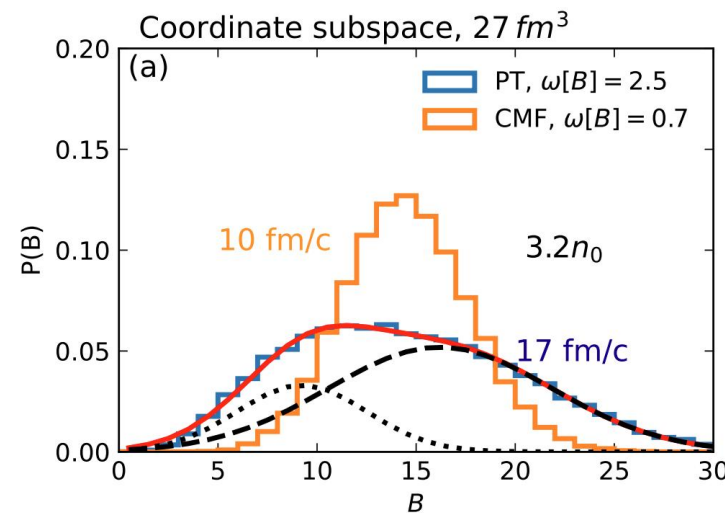
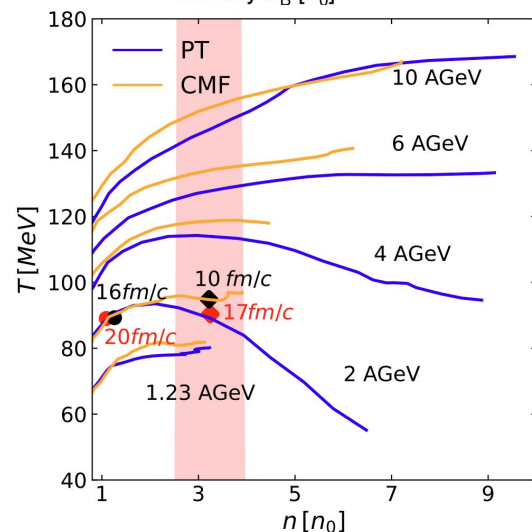
Application to HIC — UrQMD with Phase Transition

Using [\[Manjunath Omana Kuttan et al., *EPL*, 22\]](#)

UrQMD-3.5 with Chiral SU(3)-flavor parity-doublet Polyakov-loop quark-hadron mean-field model (**CMF**) potential

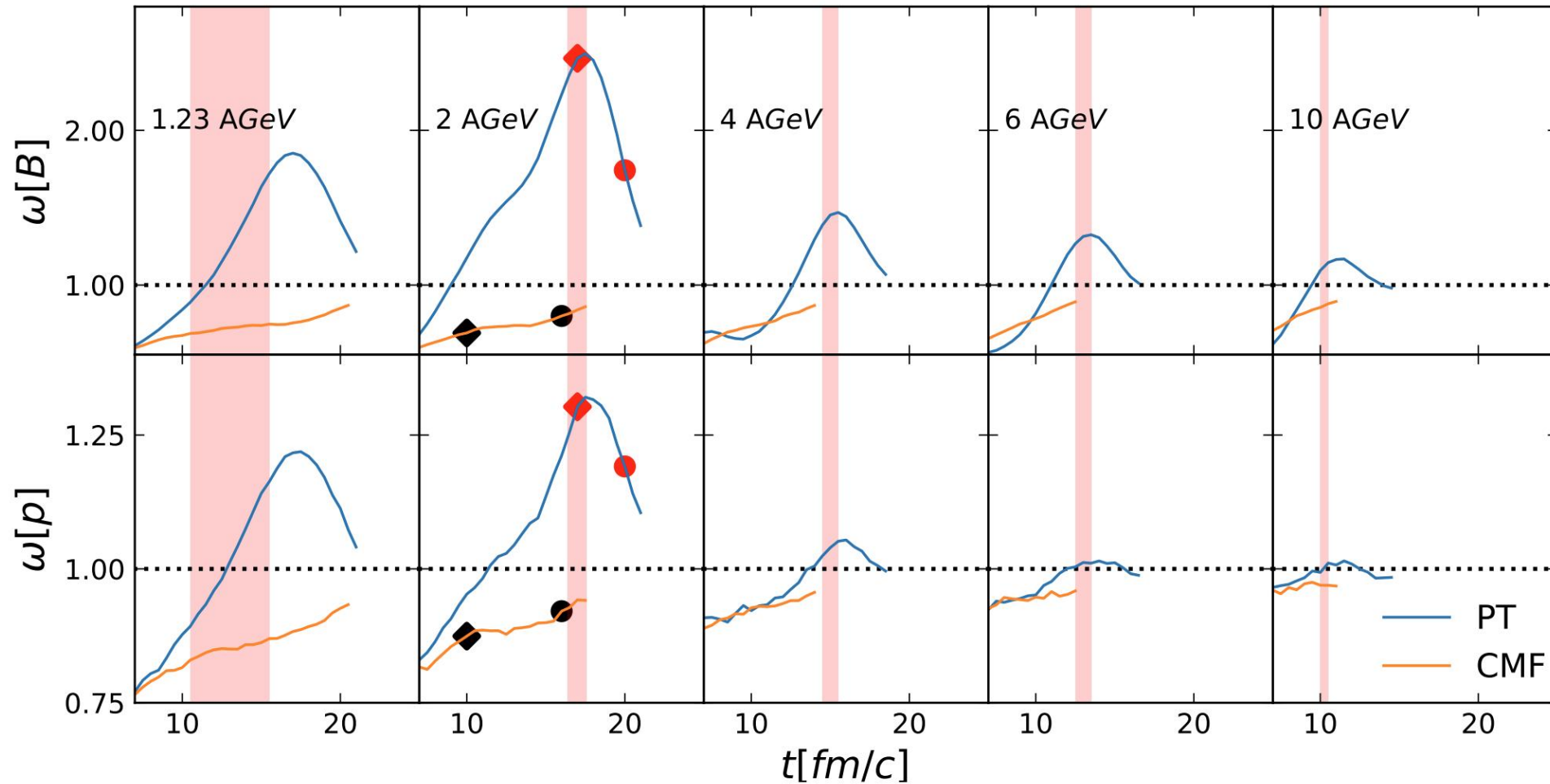


System
Trajectories

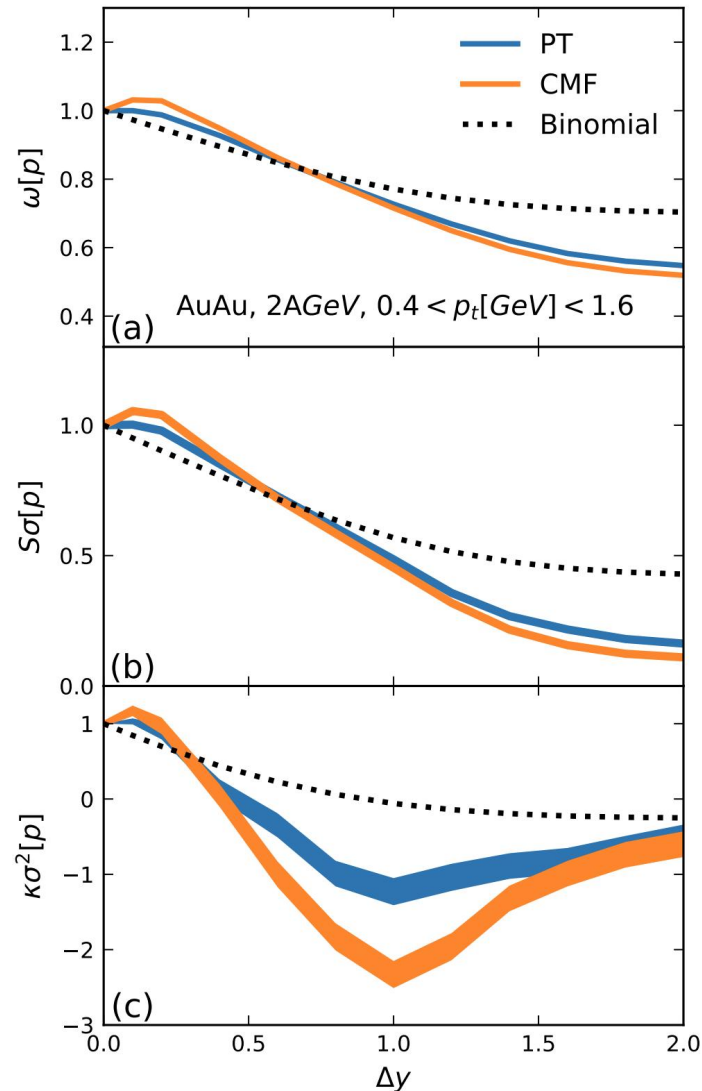


[\[Savchuk, R.P., Motornenko, Steinheimer, Gorenstein, Vovchenko, PRC, 23\]](#)

Application to HIC — Fluctuations in coordinate space

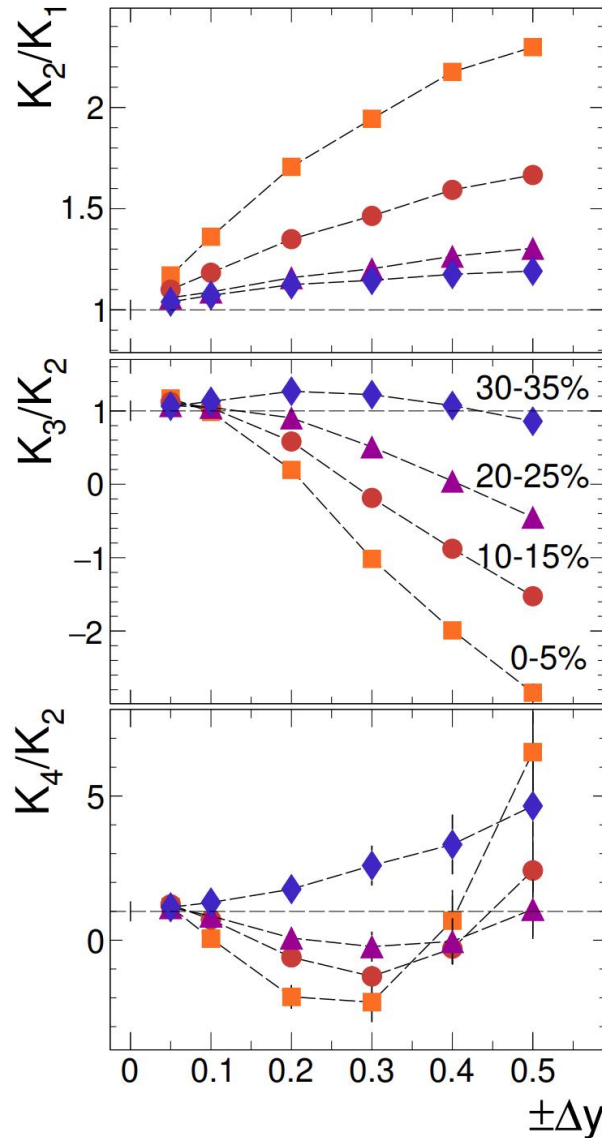


Application to HIC — Fluctuations in momentum space

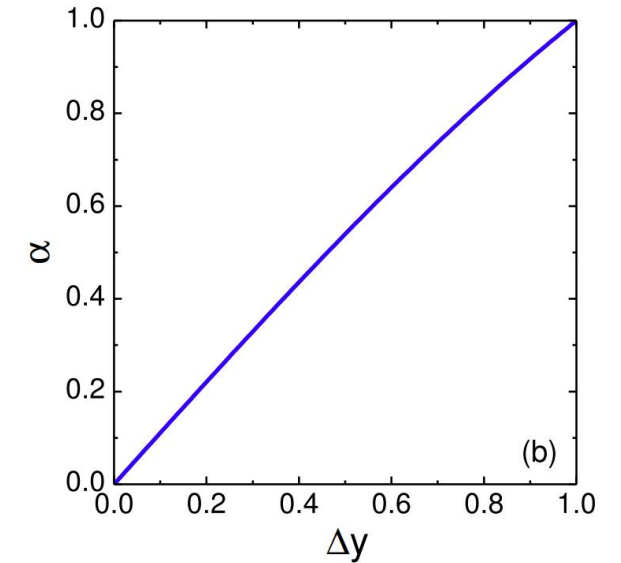
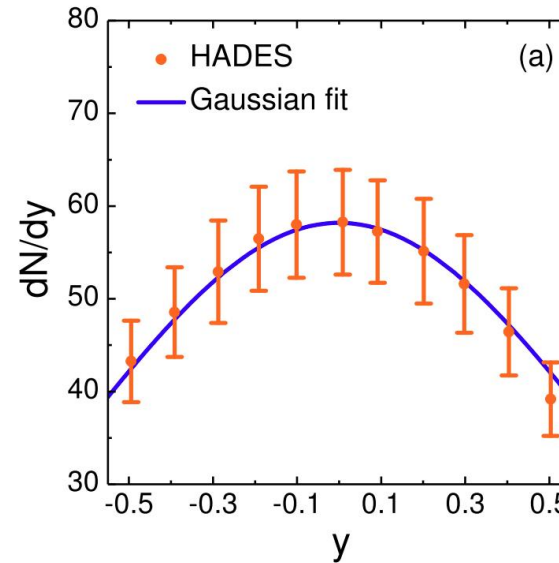


At low collision energy
space-momentum correlations are small,
thus the signal of PT in fluctuations is washed out

HADES data on proton number fluctuations at $\sqrt{s_{NN}} = 2.4$ GeV



[HADES, *PRC*, 20]



$$\frac{dN}{dy} = C \exp \left[-\frac{y^2}{2a^2} \right]$$

$$\alpha = \frac{\int_{-\Delta y/2}^{\Delta y/2} dy \, dN/dy}{\int_{-1/2}^{1/2} dy \, dN/dy}$$

[Savchuk, **R.P.**, Gorenstein, *PLB*, 22]

HADES data on proton number fluctuations at $\sqrt{s_{NN}} = 2.4$ GeV

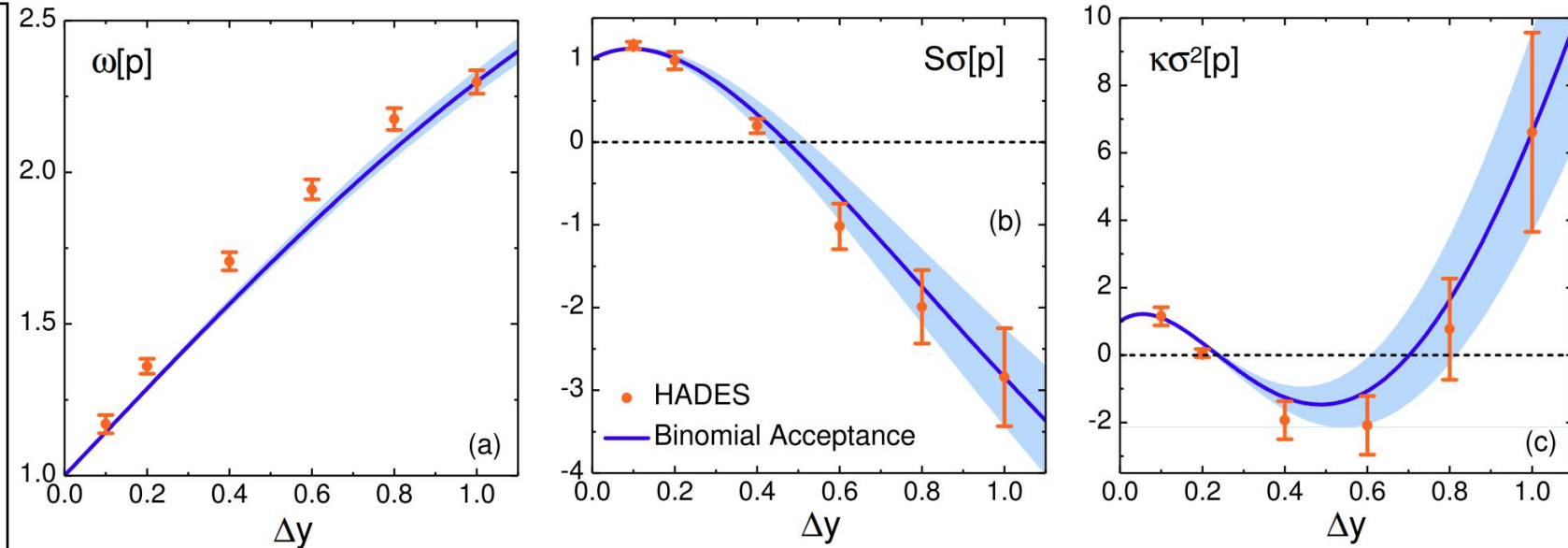
Binomial Acceptance formulas assume uncorrelated emission of protons:

$$p(n, \alpha) = \sum_{N=n}^{\infty} \frac{N!}{n!(N-n)!} \alpha^n (1-\alpha)^{N-n} P(N)$$

$$\omega_{\alpha}[n] \equiv \frac{\kappa_2[n|\alpha]}{\kappa_1[n|\alpha]} = 1 - \alpha + \alpha \omega[N] ,$$

$$S\sigma_{\alpha}[n] = \frac{\kappa_3[n|\alpha]}{\kappa_2[n|\alpha]} = \frac{\omega[N]}{\omega_{\alpha}[n]} \{ \alpha^2 S\sigma[N] + 3\alpha(1-\alpha) \} + \frac{1-\alpha}{\omega_{\alpha}[n]} (1-2\alpha) ,$$

$$\begin{aligned} \kappa\sigma_{\alpha}^2[n] &= \frac{\kappa_4[n|\alpha]}{\kappa_2[n|\alpha]} = \frac{\omega[N]}{\omega_{\alpha}[n]} \{ \alpha^3 \kappa\sigma^2[N] \} \\ &+ \frac{\omega[N]}{\omega_{\alpha}[n]} (1-\alpha) \{ 6\alpha^2 S\sigma[N] + \alpha(7-11\alpha) \} \\ &+ \frac{1-\alpha}{\omega_{\alpha}[n]} \{ 1 - 6\alpha(1-\alpha) \} , \end{aligned}$$



Consequence of picture with uncorrelated protons:

$$\rho(n_1, n_2) \equiv \langle N \rangle \frac{\langle n_1 n_2 \rangle - \langle n_1 \rangle \langle n_2 \rangle}{\langle n_1 \rangle \langle n_2 \rangle} = \omega[N] - 1$$

Summary

- Subensemble acceptance formulas allow to compare fluctuations measured in different subensembles and grand canonical calculations. The method requires a sufficiently large system such as created in central HIC and sufficient space-momentum correlations. It is applicable in the metastable region of FOPT.
- Directly in the vicinity of the CP or in the spinodal region the system is “never large enough”. Here different approximations should be used that account for finite size effects.
- The expanding system created in HIC exhibits large fluctuations when crossing the spinodal region of FOPT. This signal survives till the later stages of a collision via memory effect.
- However, at low energies the space-momentum correlation is small and this signal is not transferred to second and third order cumulants measured in momentum subspace.
- This agrees with HADES data on proton number fluctuations at $\sqrt{s_{NN}}=2.4\text{GeV}$ which are consistent with binomial baseline of non-interacting hadrons.

Summary

- Subensemble acceptance formulas allow to compare fluctuations measured in different subensembles and grand canonical calculations. The method requires a sufficiently large system such as created in central HIC and sufficient space-momentum correlations. It is applicable in the metastable region of FOPT.
- Directly in the vicinity of the CP or in the spinodal region the system is “never large enough”. Here different approximations should be used that account for finite size effects.
- The expanding system created in HIC exhibits large fluctuations when crossing the spinodal region of FOPT. This signal survives till the later stages of a collision via memory effect.
- However, at low energies the space-momentum correlation is small and this signal is not transferred to second and third order cumulants measured in momentum subspace.
- This agrees with HADES data on proton number fluctuations at $\sqrt{s_{NN}}=2.4\text{GeV}$ which are consistent with binomial baseline of non-interacting hadrons.

Thank you for attention!