



Far-from-equilibrium transport coefficients in neutron star mergers

Yumu Yang

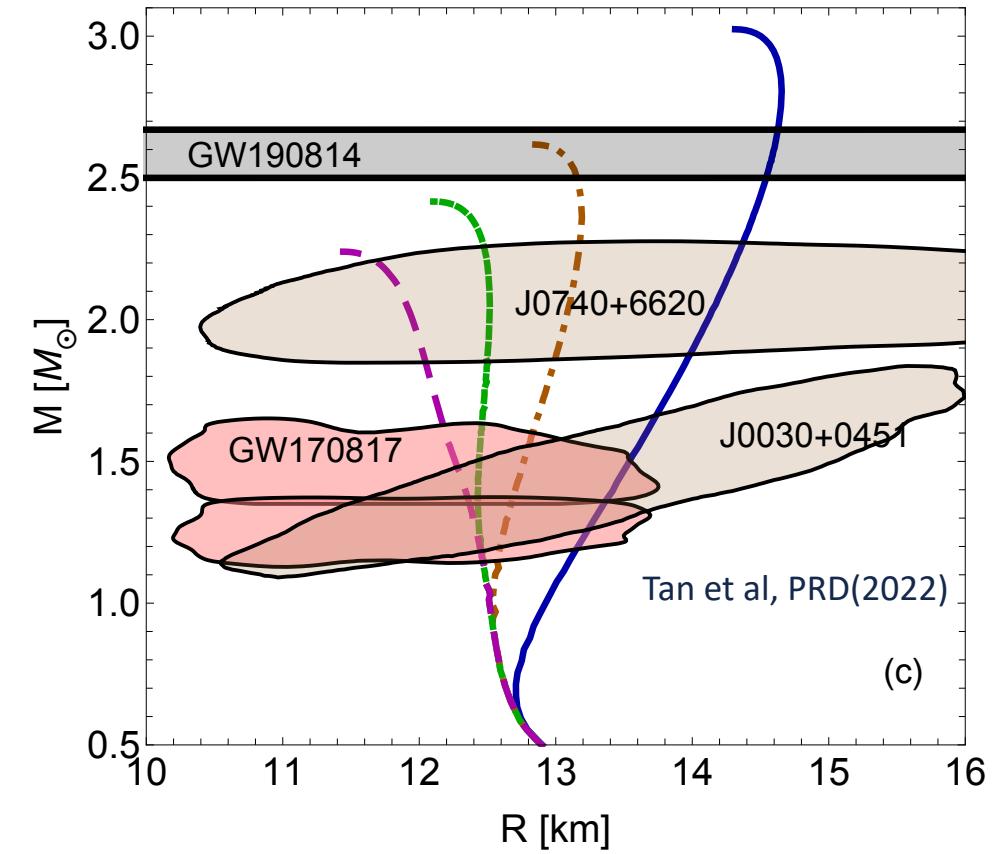
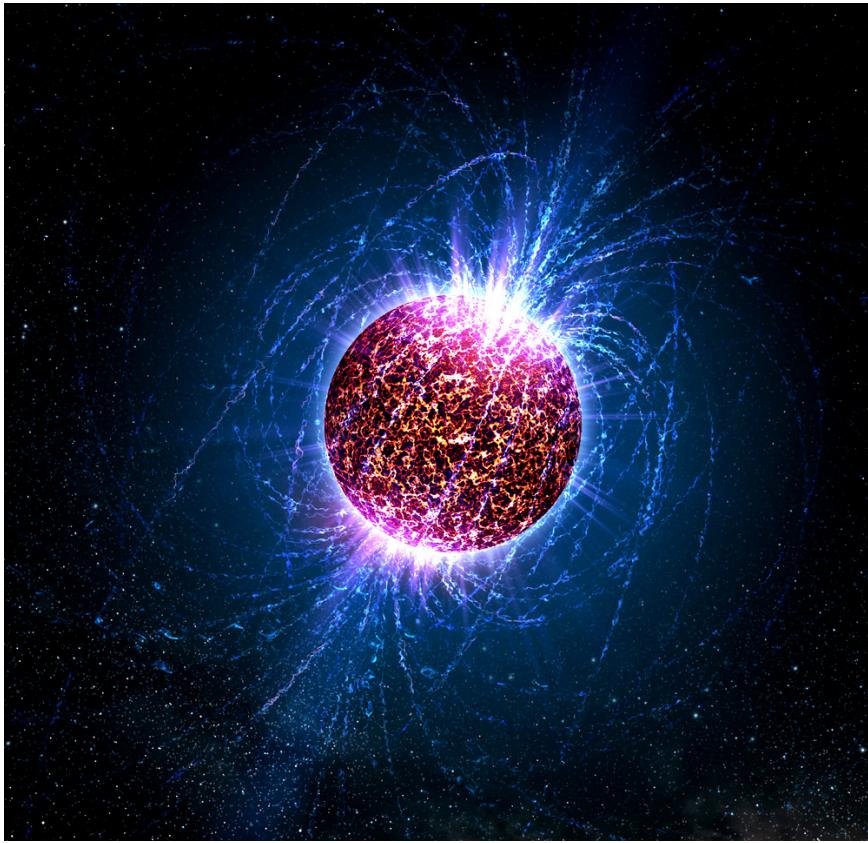
arXiv: [2309.01864](https://arxiv.org/abs/2309.01864)

Collaboration with: Mauricio Hippert, Enrico Speranza, Jorge Noronha



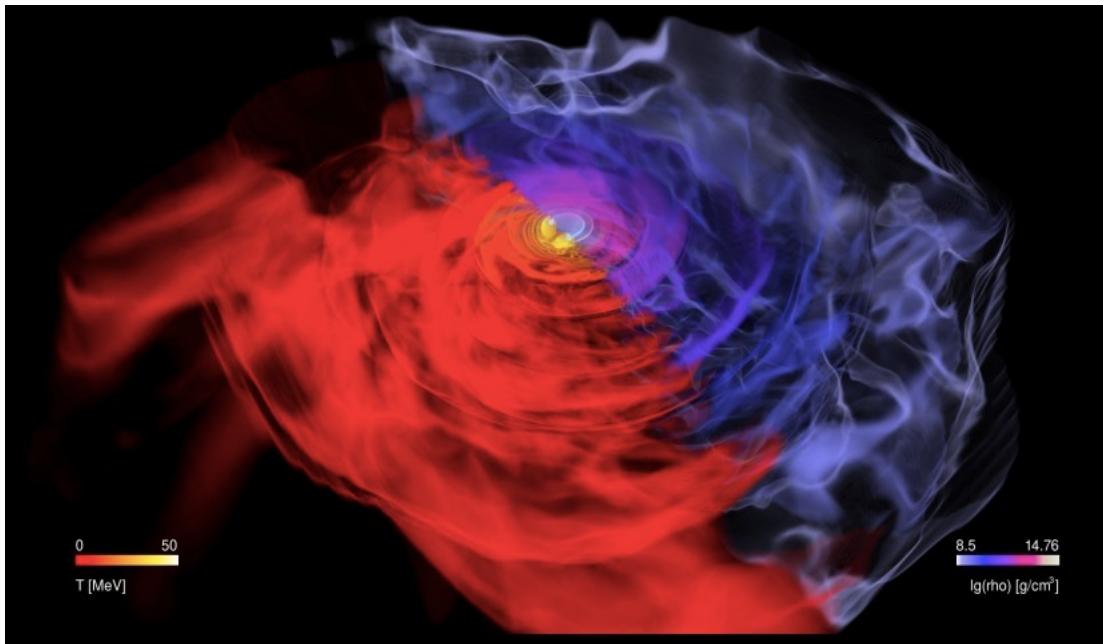
Equation of state of neutron stars

- Observations constrain EoS, $p = p(\varepsilon, n_B)$



How about neutron star mergers?

In order to accurately simulate neutron star mergers



Most et al., PRL (2019)

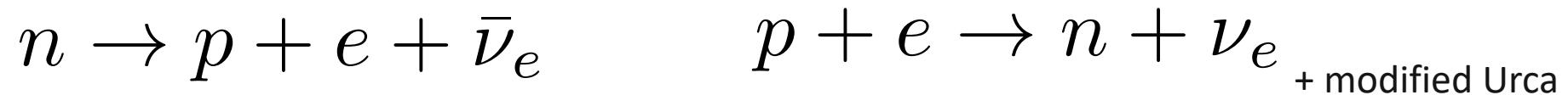
- Need more than just the EoS
- Solve fluid dynamics coupled to GR
- Dissipative processes (neutrino physics)
- Deviations from beta equilibrium

How does it get out of equilibrium?

How does it get out of equilibrium?

Sawyer, PRD (1989)

- Consider neutrino-transparent npe matter: neutron, proton, and electron
- When nuclear matter is compressed, electron fraction Y_e wants to change



Out of equilibrium physics from chemical imbalance

- Beta equilibrium: $\mu_n = \mu_p + \mu_e \rightarrow$ Equilibrium electron fraction: Y_e^{eq}

How does it get out of equilibrium?

- Change in electron fraction is related to the Urca rates

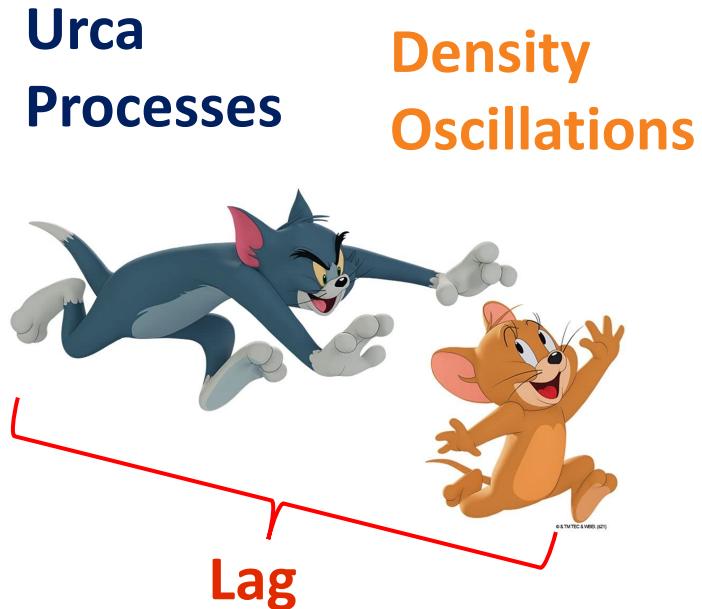
$$\Gamma_{\bar{\nu}} - \Gamma_{\nu} = n_B \frac{d(\delta Y_e)}{dt} = \Gamma_e$$

- Urca processes & density oscillations: same timescale



Deviation from beta equilibrium

- Energy loss from compression/expansion: bulk viscosity?

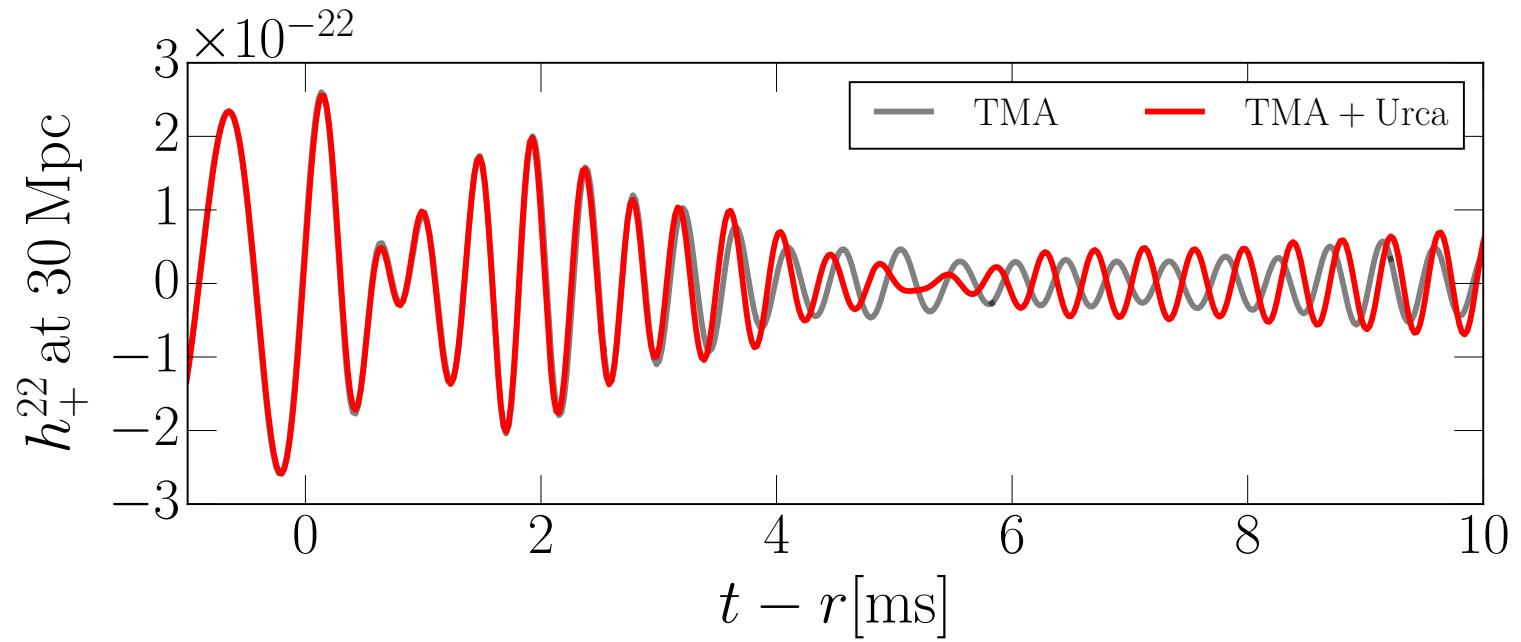
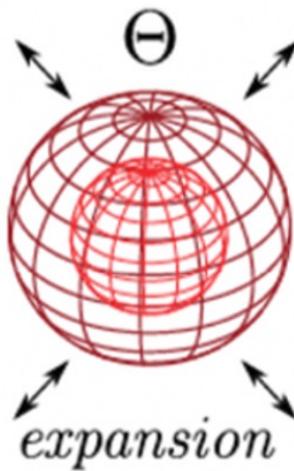


Bulk viscosity in neutron star mergers?

Alford, Bovard, Hanuske, Rezzolla, Schwenzer, PRL (2018)

- Density oscillations + weak interactions: deviations from beta equilibrium
→ realistic simulations with URCA rates

Bulk viscosity ζ



Most et al, arXiv:2207.00442

What do some state-of-the-art codes actually solve?

Reactive fluid

(coupled with Einstein's equations)

- Baryon conservation

$$\nabla_\mu(n_B u^\mu) = 0$$

- Energy-momentum conservation

$$\nabla_\mu [(\varepsilon + P)u^\mu u^\nu - Pg^{\mu\nu}] = 0$$

- Non-conserved electron current

$$\nabla_\mu(n_e u^\mu) = \boxed{\Gamma_e}$$

Reaction rates
(weak interactions)

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Reaction rates
(weak interactions)



Exact Duality!

no approximations

Bulk-viscous fluid

- Total pressure

$$P = P_{eq} + \Pi$$

bulk scalar

Israel-Stewart Equation!

$$\tau_\Pi u^\mu \nabla_\mu \Pi + \Pi = -\zeta \theta$$

Israel, Stewart, Annals of Physics (1979)

Resummed bulk-viscous transport coefficients

$$\zeta(\varepsilon, n_B, \Pi), \tau_\Pi(\varepsilon, n_B, \Pi)$$

Gavassino, Noronha, arXiv:2305.04119

Reactive fluid

(coupled with Einstein's equations)

Bulk-viscous fluid



Exact Duality!

Neutron star mergers are bulk-viscous systems

What are the transport coefficients for realistic EoSs?

YY, Hippert, Speranza, Noronha, arXiv: [2309.01864](https://arxiv.org/abs/2309.01864)

Equations of state

YY, Hippert, Speranza, Noronha, arXiv: [2309.01864](https://arxiv.org/abs/2309.01864)

- Walecka-like relativistic mean-field model (mesons + baryons)

$$\mathcal{L} = \mathcal{L}_\sigma + \mathcal{L}_\omega + \mathcal{L}_\rho + \mathcal{L}_e + \mathcal{L}_{baryon} + \mathcal{L}_{interactions}$$

- No deconfined matter, superfluidity, etc.
- EoS characterized by **symmetry energy J** and its **slope L**

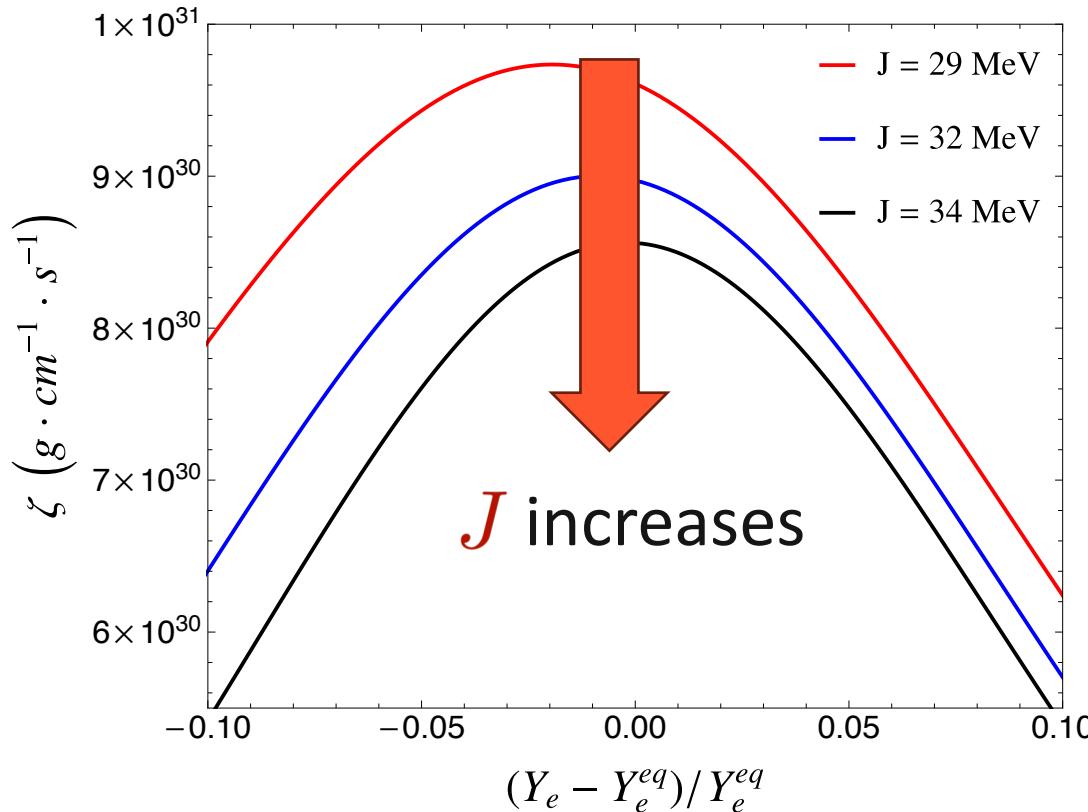
$$E_{sym}(n_B) = \boxed{J} + \boxed{\frac{L}{3}} \left(\frac{n_B}{n_{sat}} - 1 \right) + \mathcal{O} \left[\left(1 - \frac{n_B}{n_{sat}} \right)^2 \right]$$

Resummed bulk viscosity

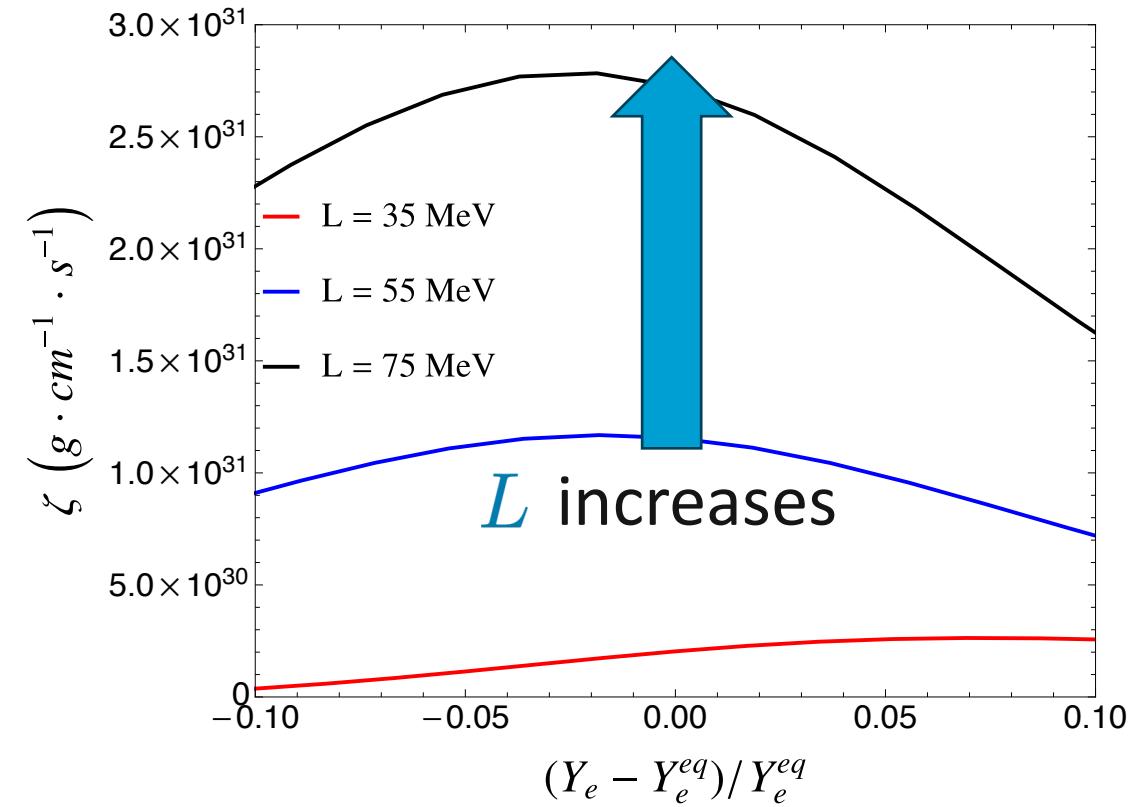
YY, Hippert, Speranza, Noronha,
arXiv: [2309.01864](https://arxiv.org/abs/2309.01864)

Bulk viscosity sensitive to **symmetry energy J** and **slope L !**

Fix L ε at n_{sat} , T at 2 MeV, $L = 51$ MeV



Fix J ε at n_{sat} , T at 2 MeV, $J = 30.7$ MeV

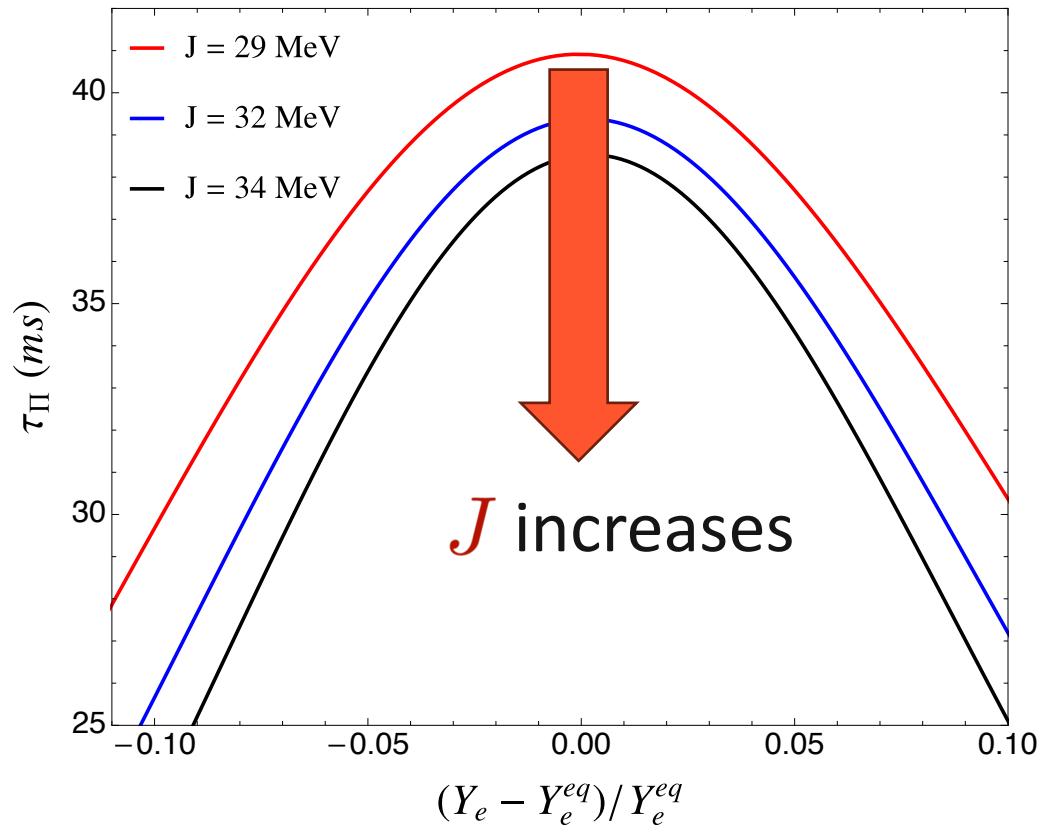


Resummed relaxation time

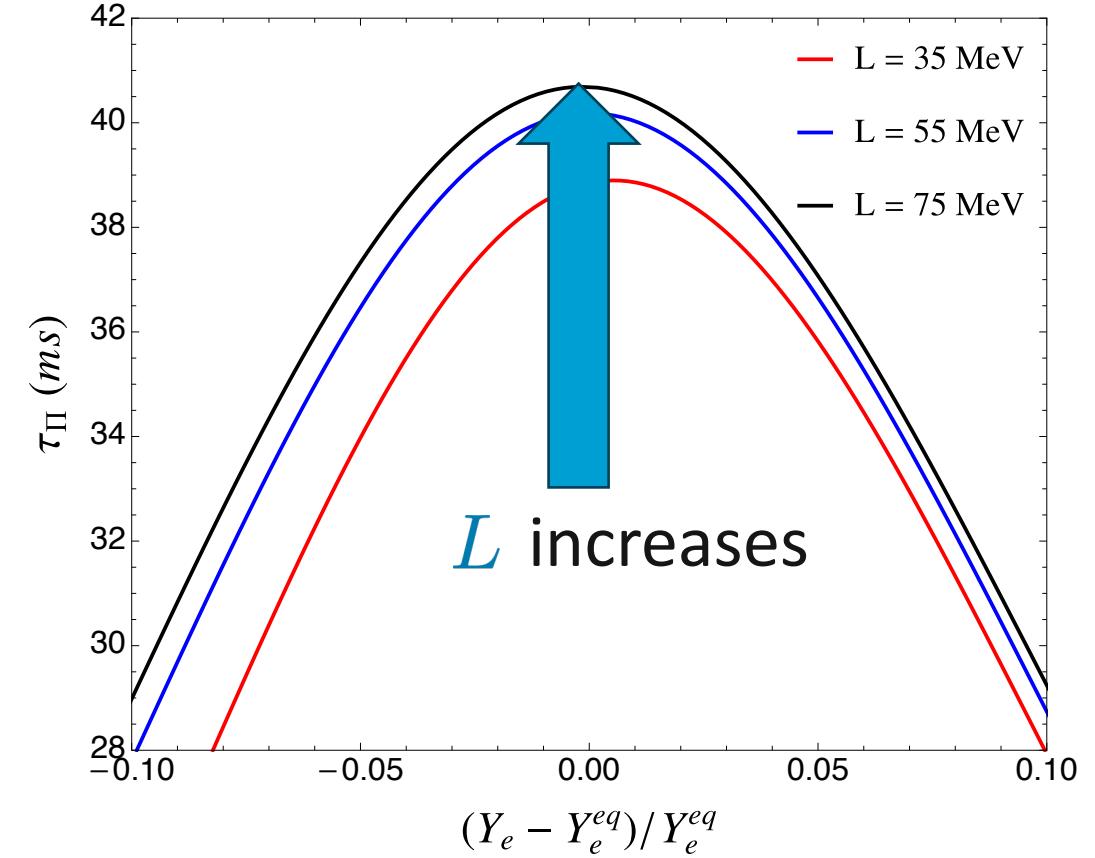
YY, Hippert, Speranza, Noronha,
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Relaxation time sensitive to **symmetry energy J** and **slope L !**

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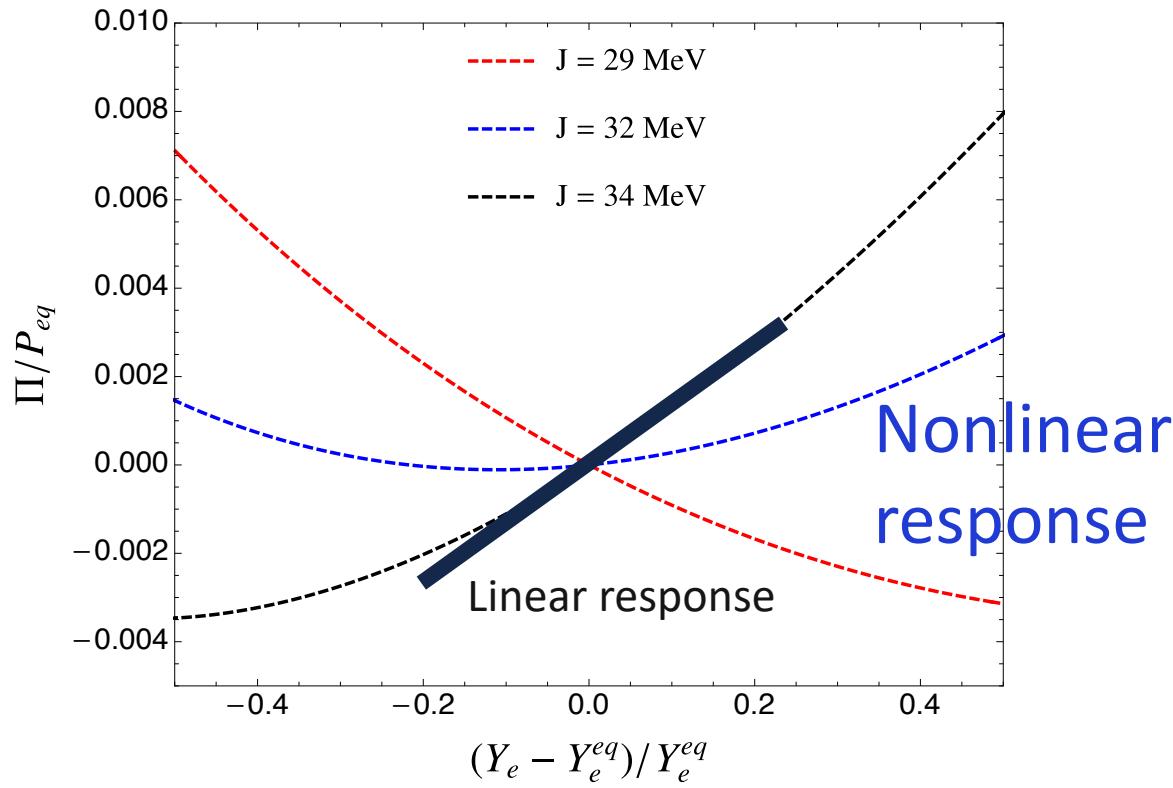
Fix J ε at n_{sat} , T at 2 MeV, J = 30.7 MeV



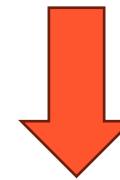
Linear vs. nonlinear response

YY, Hippert, Speranza, Noronha, arXiv: [2309.01864](https://arxiv.org/abs/2309.01864)

ε at $2 n_{sat}$, T at 2 MeV, L = 51 MeV



- For realistic EoSs, the near beta equilibrium response can be nonlinear
- Invalidates linear response approaches to bulk viscosity



Need a resummed bulk-viscous approach!

Regime of applicability of bulk description

YY, Hippert, Speranza, Noronha, arXiv: [2309.01864](https://arxiv.org/abs/2309.01864)

- The resummed $\zeta \tau_\Pi$ are functions of Y_e
- Is it always true that $\Pi(Y_e) \xrightarrow{?} Y_e(\Pi)$
- Minimum pressure/Quadratic behavior of Π against Y_e : two values of Y_e for one value of Π

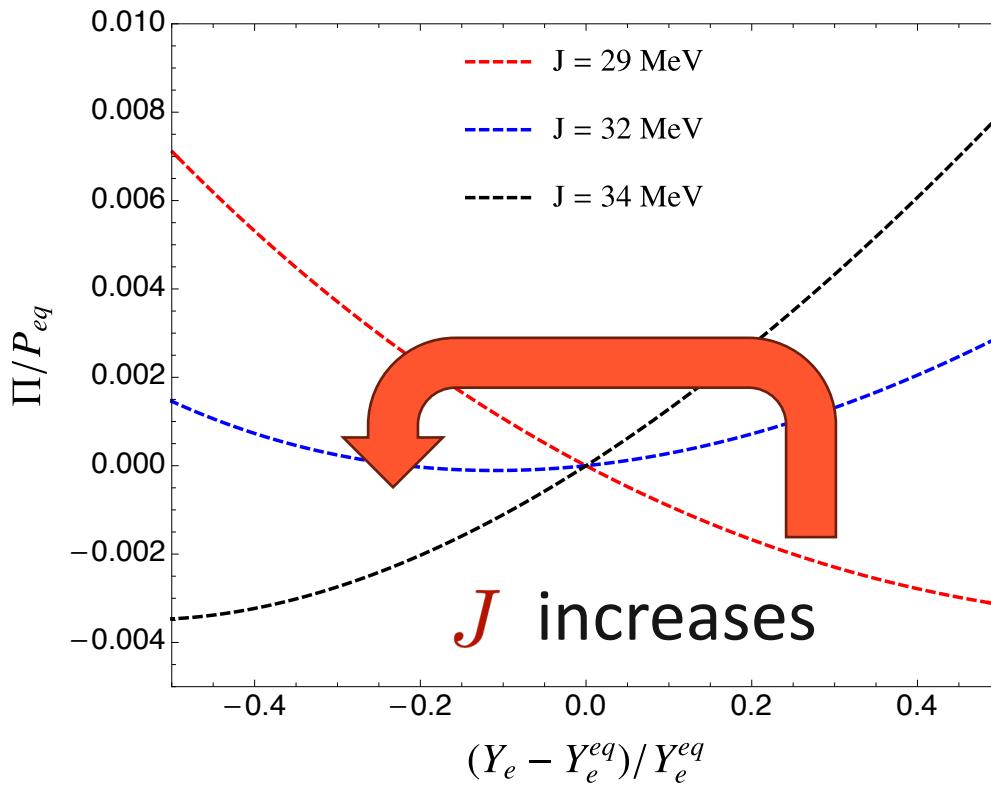
A standard bulk-viscous description does not make sense beyond the minimum pressure point

Regime of validity

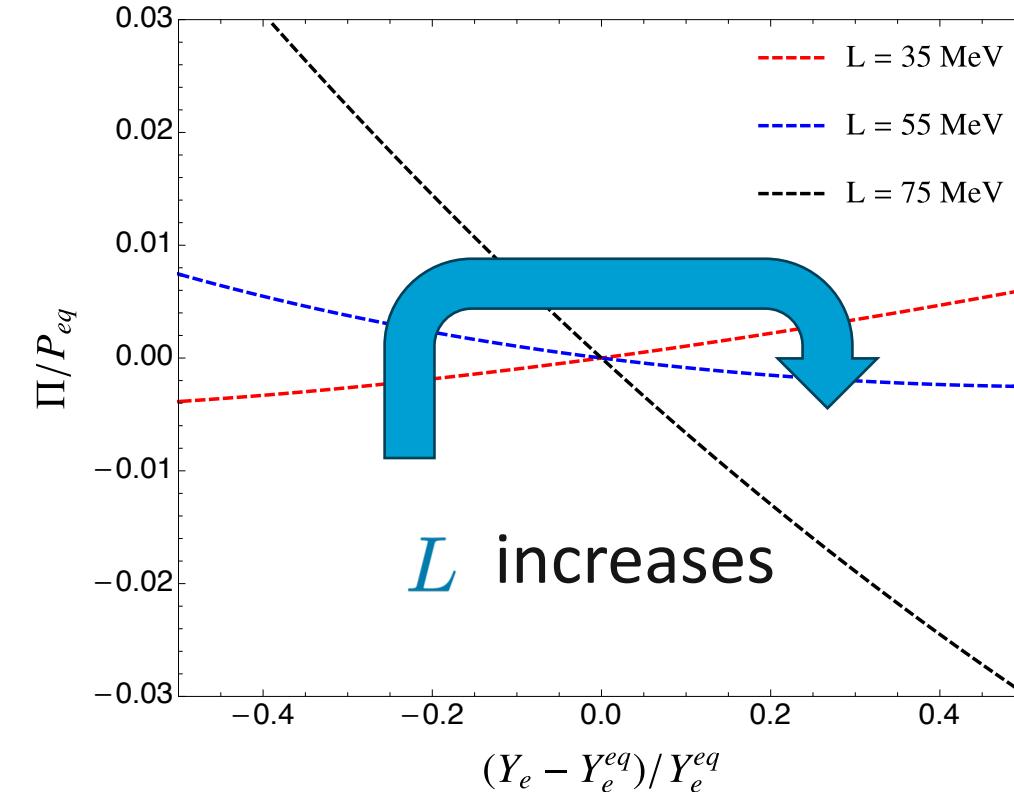
YY, Hippert, Speranza, Noronha, arXiv: [2309.01864](https://arxiv.org/abs/2309.01864)

Minimum pressure point sensitive to **symmetry energy J** and its **slope L !**

Fix L ε at $2 n_{sat}$, T at 2 MeV, $L = 51$ MeV



Fix J ε at $2 n_{sat}$, T at 2 MeV, $J = 30.7$ MeV



Conclusions

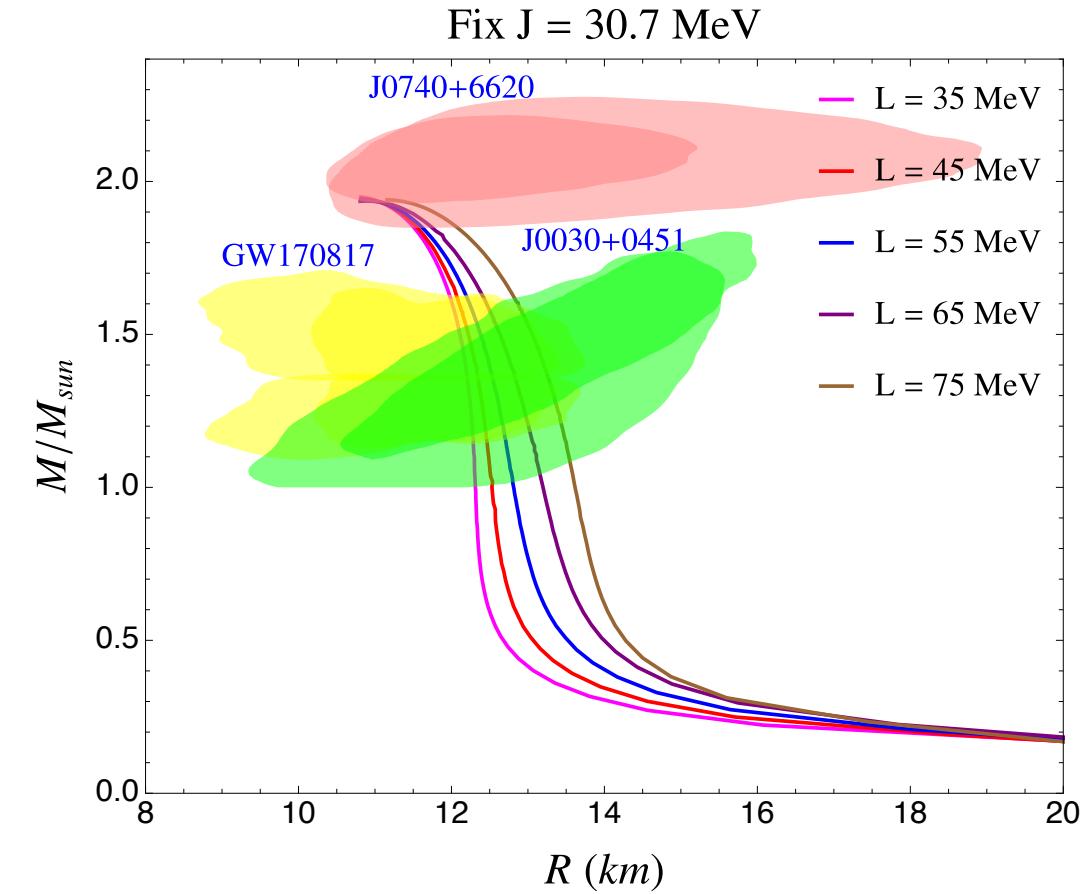
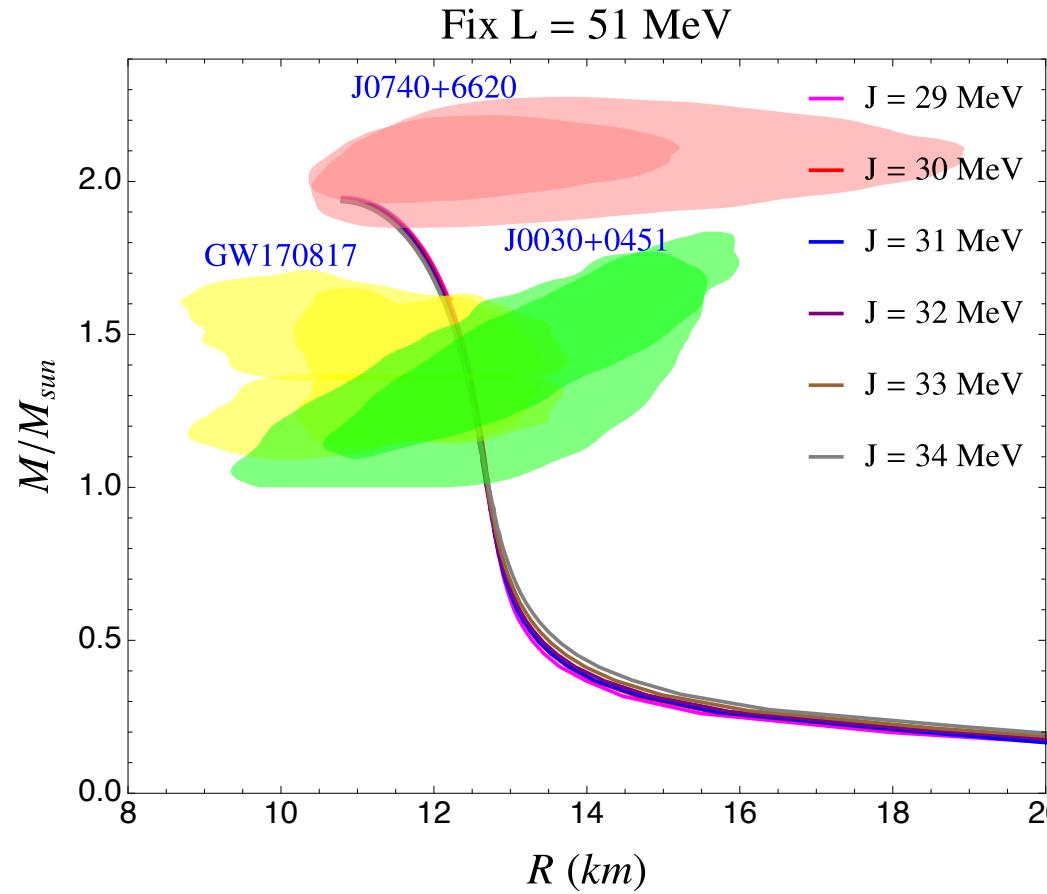
YY, Hippert, Speranza, Noronha, arXiv: [2309.01864](https://arxiv.org/abs/2309.01864)

- Neutron star mergers probe ultradense matter out of beta equilibrium
- Neutrino transparent npe matter  resummed bulk-viscous Israel-Stewart
- First calculation of resummed transport coefficients under merger conditions
- Transport coefficients sensitive to symmetry energy and its slope (EoS)

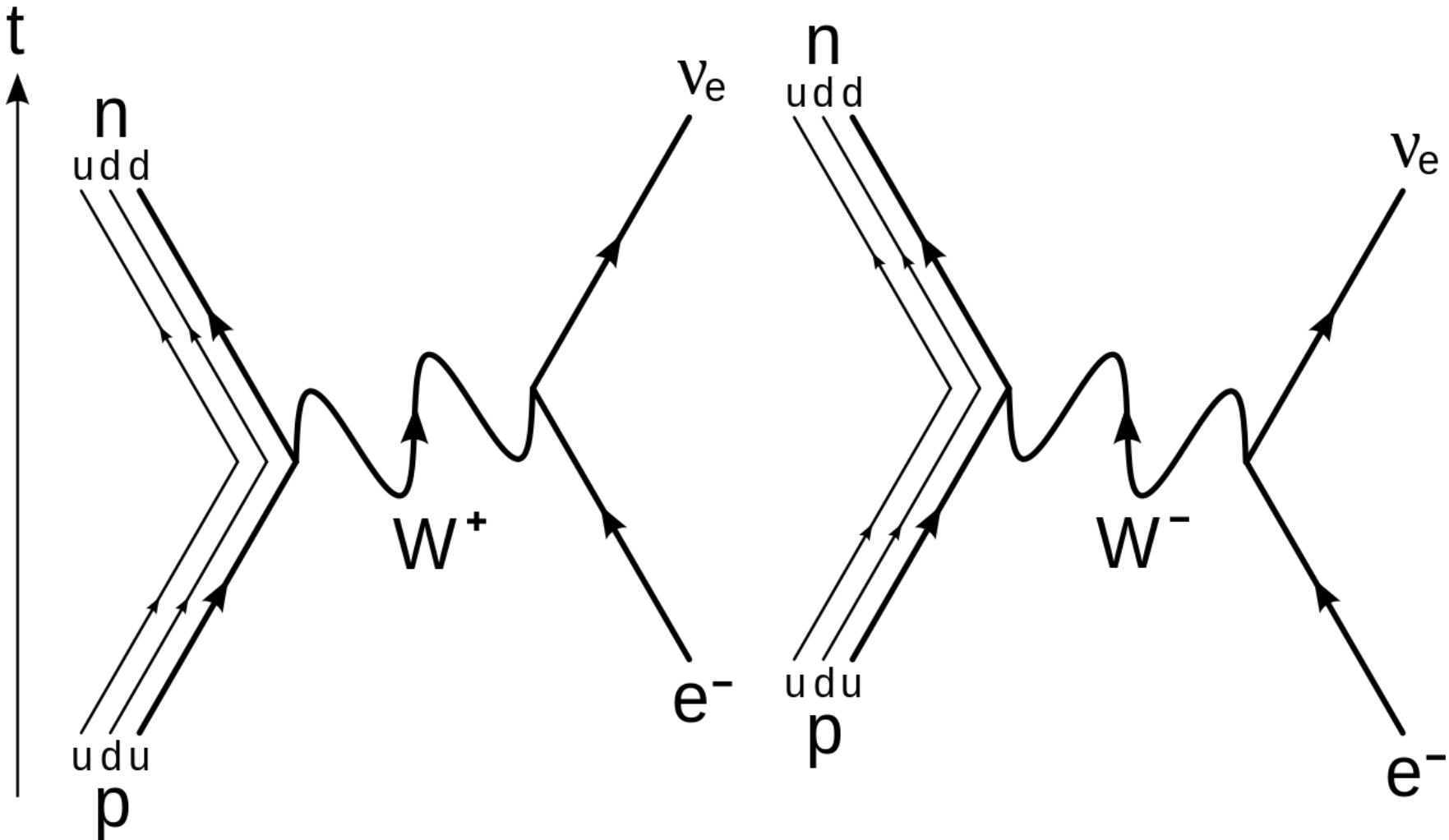
EXTRA SLIDES

Mass vs. radius curves

- EoSs consistent with observations



Direct Urca



Modified Urca

- Chemical reactions with a spectator X



Uncertainties of symmetry energy and its slope

MUSES, arXiv:2303.17021

J (MeV)	31.6 ± 2.7	Li et al., EPJ (2019)
L (MeV)	58.16 ± 16	Li et al., EPJ (2019)
	50 ± 15.5	Fan et al., PRC (2014)
	54 ± 8	Reinhard et al. PRL (2019)
	106 ± 37	Reed et al., PRL (2021)

Constraints on bulk viscosity transport coefficients

$$\left[\frac{\zeta}{\tau_{\Pi}} + n_B \left. \frac{\partial P}{\partial n_B} \right|_{\varepsilon, Y_e = Y_e^{eq}} \right] \frac{1}{\varepsilon + P} \leq 1 - \left. \frac{\partial P}{\partial \varepsilon} \right|_{n_B, Y_e = Y_e^{eq}}$$

Bemfica, Disconzi, Noronha, PRL (2019)

Effect of equilibration timescale

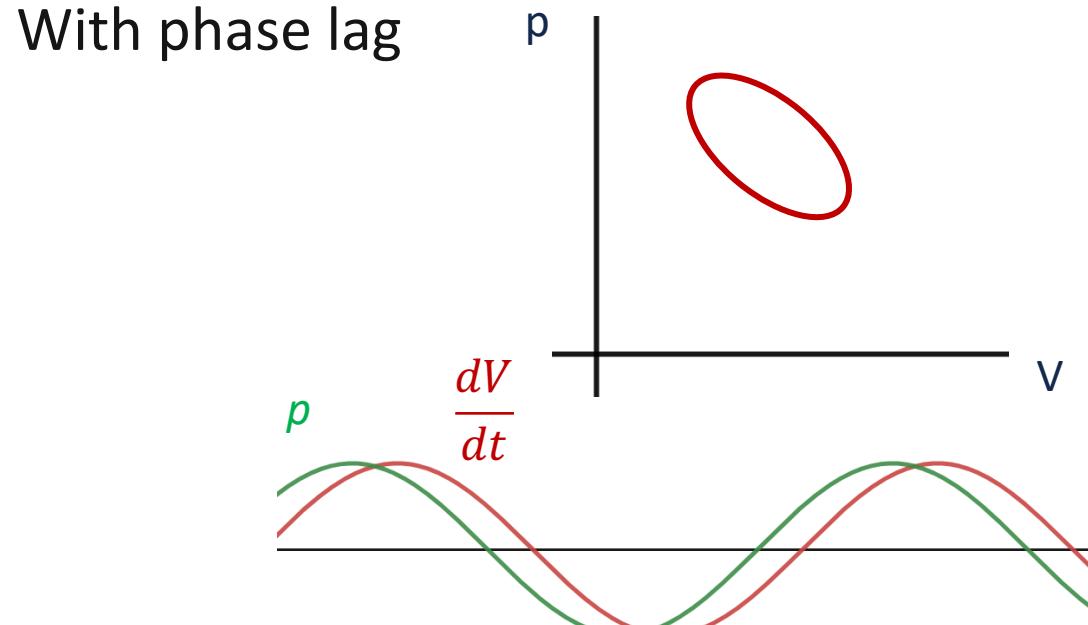
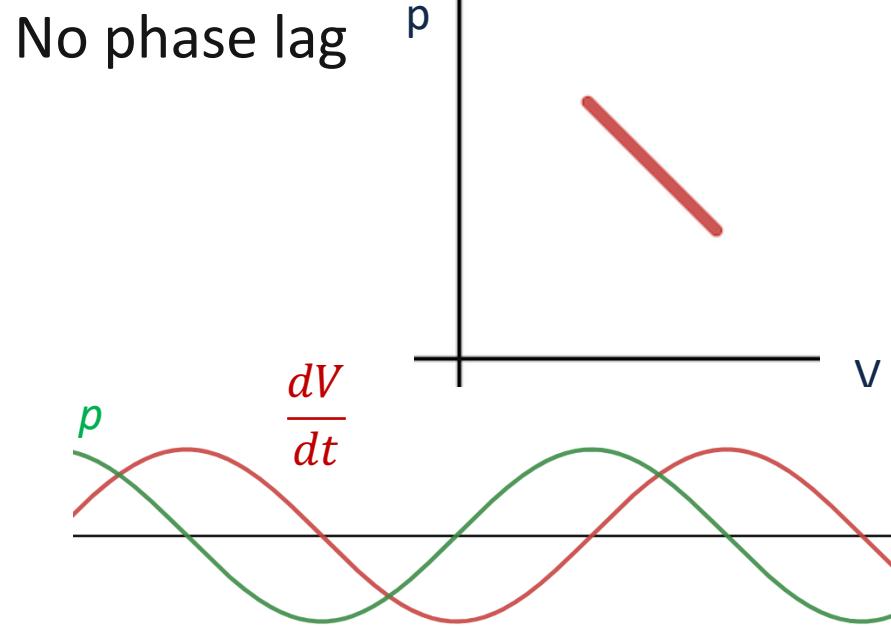
Alford, Bovard, Hanuske, Rezzolla, Schwenzer, PRL (2018)

- Slow equilibration: composition is fixed, and the process is reversible
- Fast equilibration: mixture is instantaneously equilibrated
- Same order as the pressure: **Phase Lag -> Bulk Viscosity!**

Pressure and volume becomes out of phase

Sawyer, PRD (1989)

$$\text{Dissipation} = - \int p dV = - \int p \frac{dV}{dt} dt$$



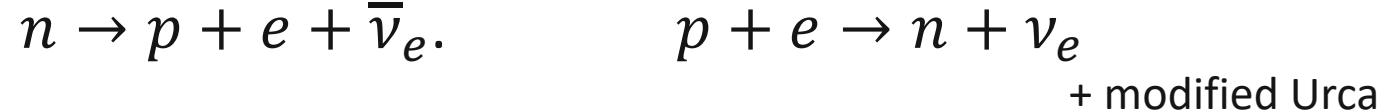
Equations of State (Lagrangian)

$$\begin{aligned}\mathcal{L} = & \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{b}{3}m_B(g_\sigma\sigma)^3 - \frac{c}{4}(g_\sigma\sigma)^4 + \frac{1}{2}m_\omega^2\omega^\mu\omega_\mu - \frac{1}{4}\omega^{\mu\nu}\omega_{\mu\nu} \\ & + \frac{1}{2}m_\rho^2\vec{\rho}^\mu\vec{\rho}_\mu - \frac{1}{4}\vec{\rho}_{\mu\nu}\vec{\rho}^{\mu\nu} + G_{\omega\rho}\omega^\mu\omega_\mu\vec{\rho}^\mu\vec{\rho}_\mu + \bar{\psi}_e(i\gamma_\mu\partial^\mu - m_e)\psi \\ & + \bar{\psi}\left[i\gamma_\mu\partial^\mu - m_B + \gamma^0\mu_B + \gamma_0\frac{\tau_3}{2}\mu_I - g_\omega\omega^\mu\gamma_\mu - g_\rho\gamma^\mu\vec{\rho}_\mu \cdot \frac{\vec{\tau}}{2} + g_\sigma\sigma\right]\psi\end{aligned}$$

Hippert et al, PRD(2022)

Expansion around beta equilibrium

- Consider neutrino-transparent npe matter with Urca processes



- Beta equilibrium: $\mu_n = \mu_p + \mu_e$

Deviation: $\delta\mu = \mu_n - \mu_p - \mu_e$

- Nonzero $\delta\mu$: $\Gamma_{\bar{\nu}} - \Gamma_\nu = n_B \frac{d(\delta Y_e)}{dt} = \Gamma_e$
Small deviations: $\Gamma_e = -\gamma\delta\mu$, first order in $\delta\mu$

Israel-Stewart Equation

- Small deviations

$$u^\nu \nabla_\nu \delta\mu = -\gamma \mathcal{A} \delta\mu + \mathcal{B} \nabla_\nu u^\nu$$

- Pressure can also be expanded around beta equilibrium

$$P = P|_{\delta\mu=0} + \Pi, \quad \Pi = P_1 \delta\mu, \quad P_1 = \frac{\partial P}{\partial \delta\mu} \Big|_{\delta\mu=0}$$

Gavassino, Antonelli, Haskell, CQG (2021)

Celora et al., PRD (2022)

Bulk scalar equation of motion is the same equation as in heavy ions!

Israel, Stewart, Annals of Physics (1979)
Denicol et al., PRD (2012)

$$\tau_\Pi u^\mu \nabla_\mu \Pi + \delta_{\Pi\Pi} \Pi \nabla_\mu u^\mu + \Pi = -\zeta \nabla_\mu u^\mu$$

Bulk viscosity from metric perturbation

- Assume periodic perturbation of the metric tensor

$$g^{\mu\nu} = \eta^{\mu\nu} + \delta g^{\mu\nu} \quad \delta g^{\mu\nu} \propto e^{i\omega t}$$

- Variation of the energy-momentum tensor from the variation of the metric tensor

$$\delta T^{\mu\nu} \sim G_R \delta g^{\mu\nu} \quad \nabla_\nu u^\nu = \partial_\nu u^\nu + \Gamma_{\nu\rho}^\nu u^\rho$$

AC bulk viscosity

$$\zeta_{AC}(\omega) = n_B P_1 \frac{\frac{\gamma}{n_B} \left. \frac{\partial \delta \mu}{\partial Y_e} \right|_{n_B} \left. \frac{\partial \delta \mu}{\partial n_B} \right|_{Y_e}}{\omega^2 + \left(\frac{\gamma}{n_B} \left. \frac{\partial \delta \mu}{\partial Y_e} \right|_{n_B} \right)^2}$$

Bulk viscosity from the phase lag

Alford et al, JPGNPP (2010)

Sa'd, Schaffner-Bielich, arXiv 0908.4190 (2010)

- Assume

$$n_i = n_i(\mu_i)$$

$$Y_e = Y_{e,0} + Re(\delta Y_{e,0} e^{i\omega t})$$

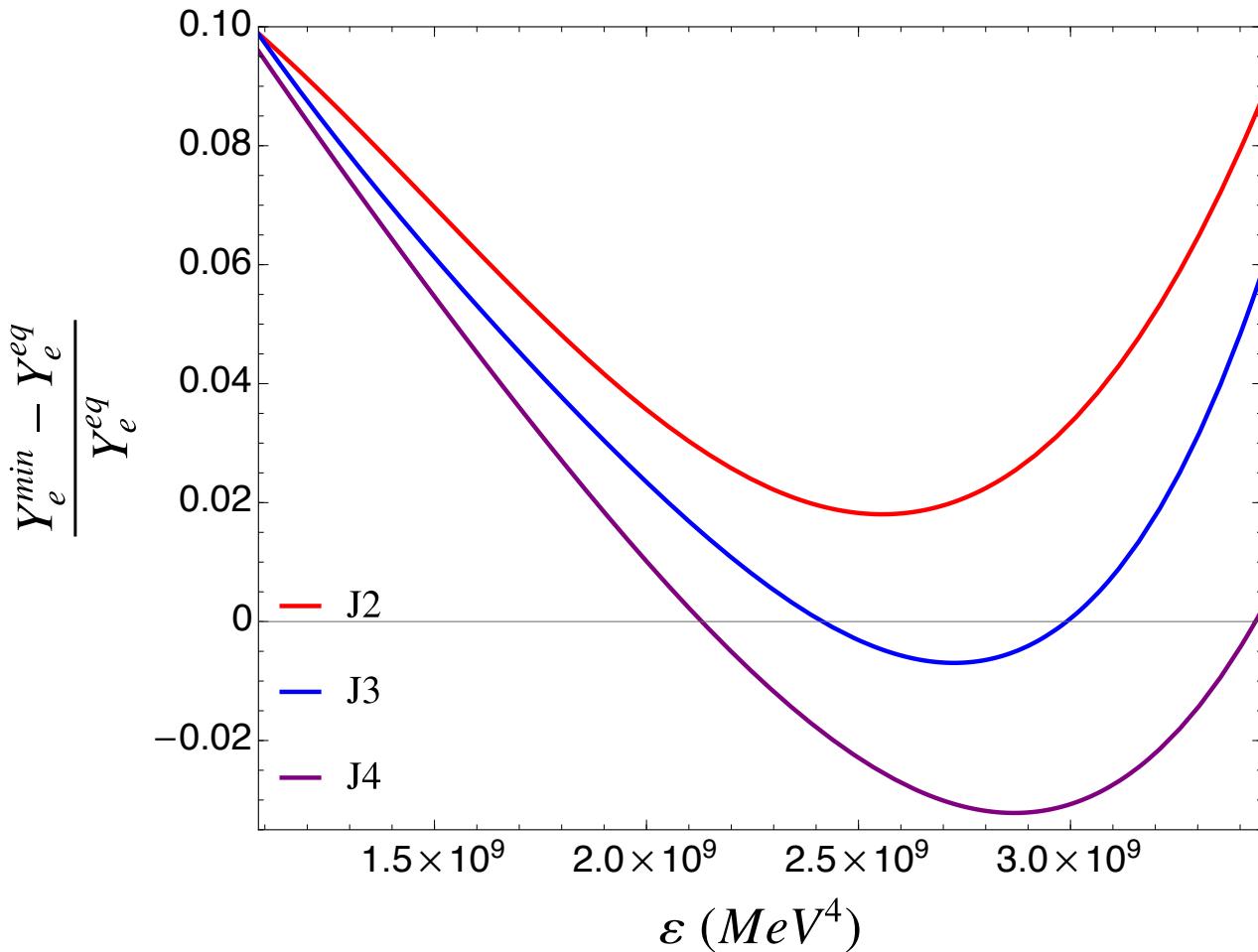
- Energy density dissipation

$$\langle \dot{\mathcal{E}}_{diss} \rangle = -\frac{\zeta}{\tau} \int_0^\tau dt (\nabla \cdot \vec{v})^2 = \frac{n_{B,0}}{\tau} \int_0^\tau (P + \delta P) \frac{d}{dt} (V + \delta V) dt$$

Bulk viscosity

$$\zeta = \frac{-\gamma C^2}{\omega^2 + (\gamma B/n_B)^2}$$

Regime of validity



Energy density moves the pressure minimum.

Symmetry energy shifts the curve.

Israel-Stewart Equation

- Total pressure is the sum of equilibrium pressure and out-of-equilibrium correction

$$P = P_{eq} + \Pi$$

Gavassino, Noronha, 2305.04119 (2023)

- Evolution of Π can be described by three variables

$$u^\mu \nabla_\mu \Pi = \frac{\partial \Pi}{\partial \varepsilon} \Bigg|_{n_B, Y_e} u^\mu \nabla_\mu \varepsilon + \frac{\partial \Pi}{\partial n_B} \Bigg|_{\epsilon, Y_e} u^\mu \nabla_\mu n_B + \frac{\partial \Pi}{\partial Y_e} \Bigg|_{n_B, \varepsilon} u^\mu \nabla_\mu Y_e$$

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