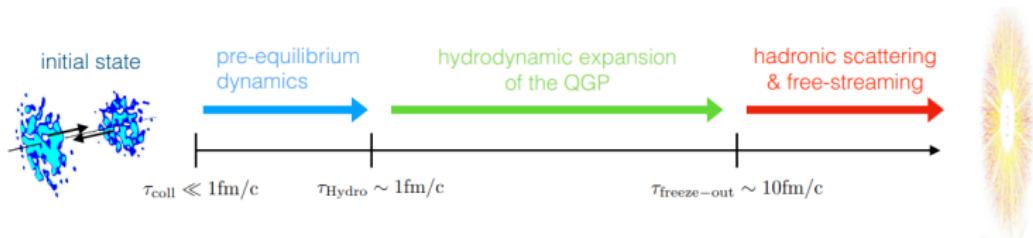


Establishing the Range of Applicability of Hydrodynamics in High-Energy Collisions

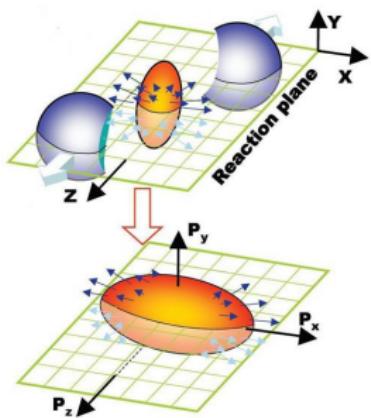
Clemens Werthmann
in Collaboration with Victor Ambruś and Sören Schlichting
based on PRD 107, 094013 and PRL 130, 152301

University of Wrocław

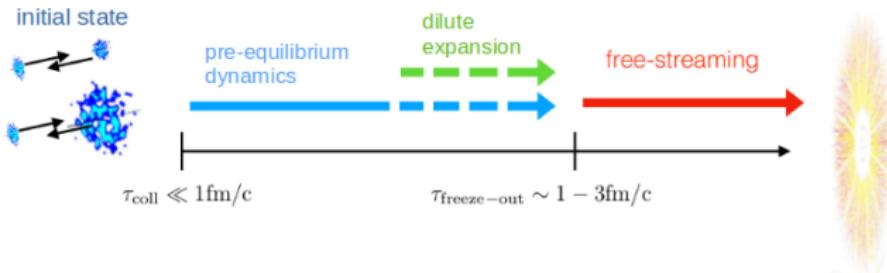




- ▶ early stage requires non-equilibrium description, but system quickly equilibrates
- ▶ strongly interacting QGP leaves imprints of thermalization and collectivity in final state observables:
 v_n , $\langle p_T \rangle$, particle yields, ...



Hiroshi Masui (2008)



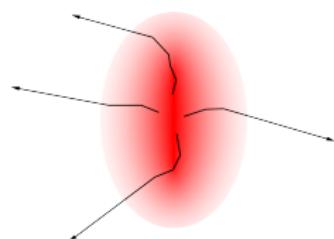
Very dilute, hydrodynamics not necessarily applicable

- ▶ still collective behaviour is observed!

Nagle, Zajc Ann.Rev.Nucl.Part. 68 (2018) 211

collectivity can also be explained in kinetic theory, a microscopic description which does not rely on equilibration

- ▶ interpolate between free streaming at small opacities and hydrodynamics at large opacities!



Aim

Case study in simplified kinetic theory description on full range from small to large system size with comparison to hydrodynamics for transverse flow observables

Model and Setup

- ▶ microscopic description in terms of averaged on-shell phase-space distribution of massless bosons:

$$f(\tau, \mathbf{x}_\perp, \eta, \mathbf{p}_\perp, y) = \frac{(2\pi)^3}{\nu_{\text{eff}}} \frac{dN}{d^3x d^3p}(\tau, \mathbf{x}_\perp, \eta, \mathbf{p}_\perp, y)$$

- boost invariance
- initialized with vanishing longitudinal pressure and no transverse momentum anisotropies
- ▶ time evolution: Boltzmann equation in conformal relaxation time approximation

$$p^\mu \partial_\mu f = C_{\text{RTA}}[f] = -\frac{p^\mu u_\mu}{\tau_R} (f - f_{\text{eq}}) , \quad \tau_R = 5 \frac{\eta}{s} T^{-1}$$

results will depend only on initial state and opacity

- dimensionless parameter: opacity \sim “total interaction rate”

Kurkela, Wiedemann, Wu EPJC 79 (2019) 965

$$\hat{\gamma} = \left(5 \frac{\eta}{s} \right)^{-1} \left(\frac{1}{a\pi} R \frac{dE_{\perp}^{(0)}}{d\eta} \right)^{1/4}$$

- encodes dependencies on viscosity, transverse size and energy scale

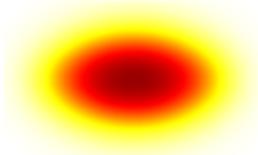
- our initial condition:

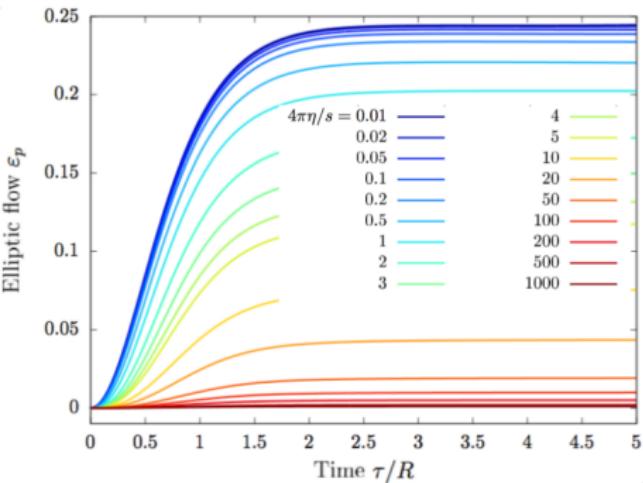
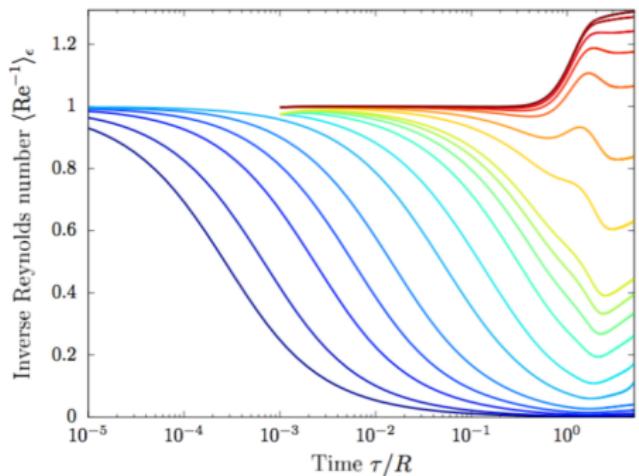
average profiles for centrality classes of Pb+Pb at 5.02 TeV

Borghini, Borrell, Feld, Roch, Schlichting, Werthmann PRC 107 (2023), 034905

- for fixed profile, vary $\hat{\gamma}$ via η/s : $\hat{\gamma} \approx 11 \cdot (4\pi\eta/s)^{-1}$

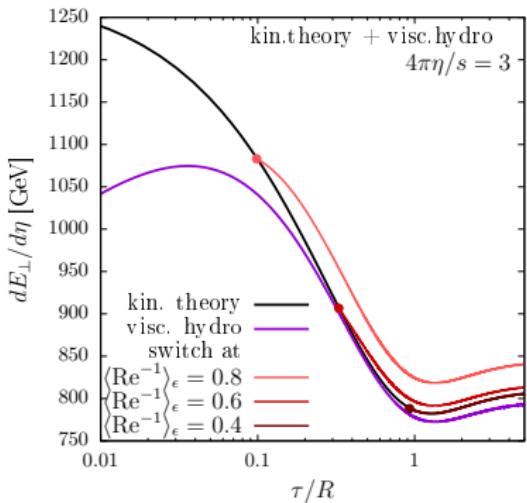
30-40%



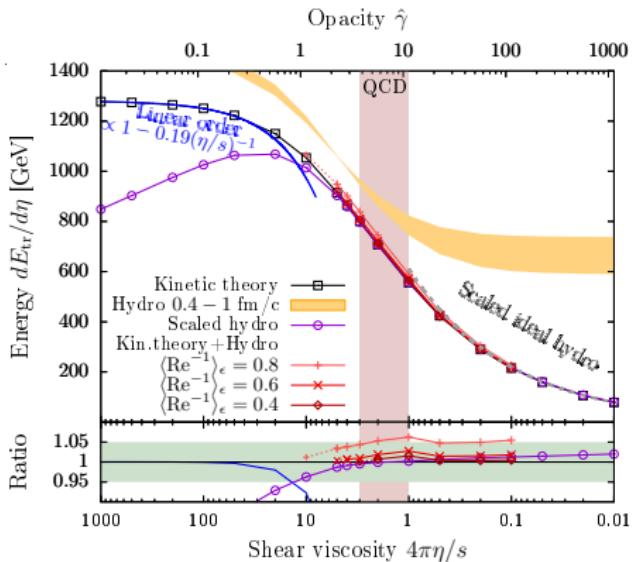
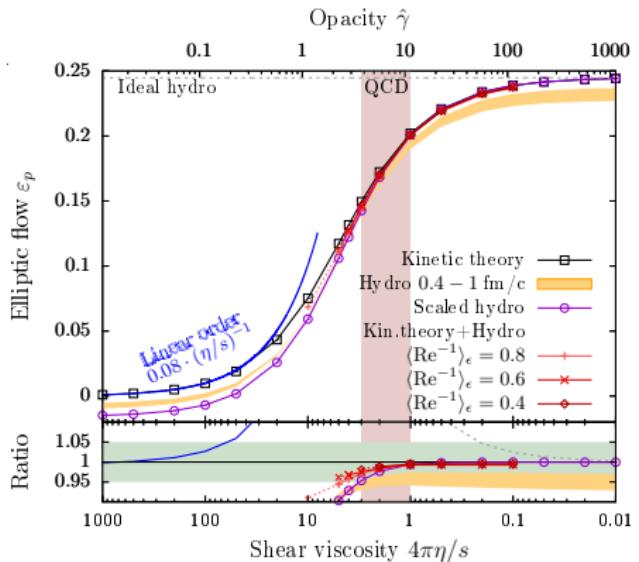


- ▶ $\text{Re}^{-1} = \left(\frac{6\pi^{\mu\nu}\pi_{\mu\nu}}{e^2} \right)^{1/2}$ measures relative size of non-equilibrium effects
 - equilibration timescale strongly depends on opacity; smaller systems take longer to equilibrate

- ▶ elliptic flow on similar timescales; continuously varying strength of response



- ▶ caveat: even at large opacities, naive hydrodynamics does not accurately describe pre-equilibrium
- ▶ can be counteracted in hybrid simulations: switching from kinetic theory to hydrodynamics after Re^{-1} has dropped to a specific value
 - later switch \Rightarrow more accurate results

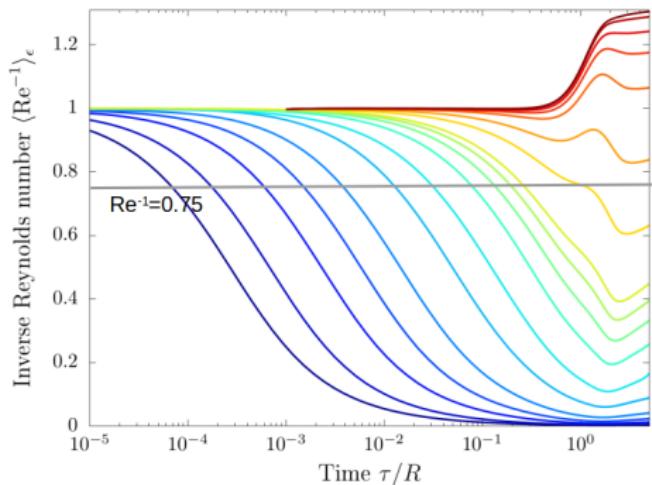


- ▶ naive hydro is off; scaled hydro accurate if $\hat{\gamma} \gtrsim 4$
- ▶ Hybrid kin. theory scheme can improve on scaled hydro at intermediate opacities
- ▶ later switching improves agreement: accurate on 5% level if $\text{Re}^{-1} < \text{Re}_c^{-1} \sim 0.75$

Characterize dynamics by timescales of:
transition to hydrodynamic behaviour

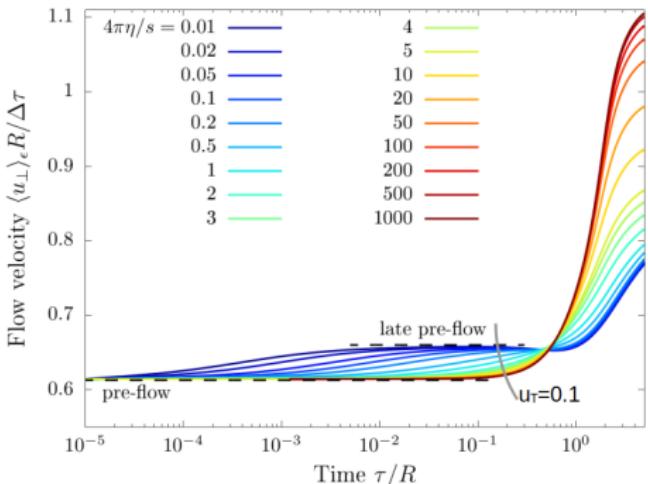
and onset of transverse expansion

- ▶ hydro applicable for $\text{Re}^{-1} \lesssim 0.75$



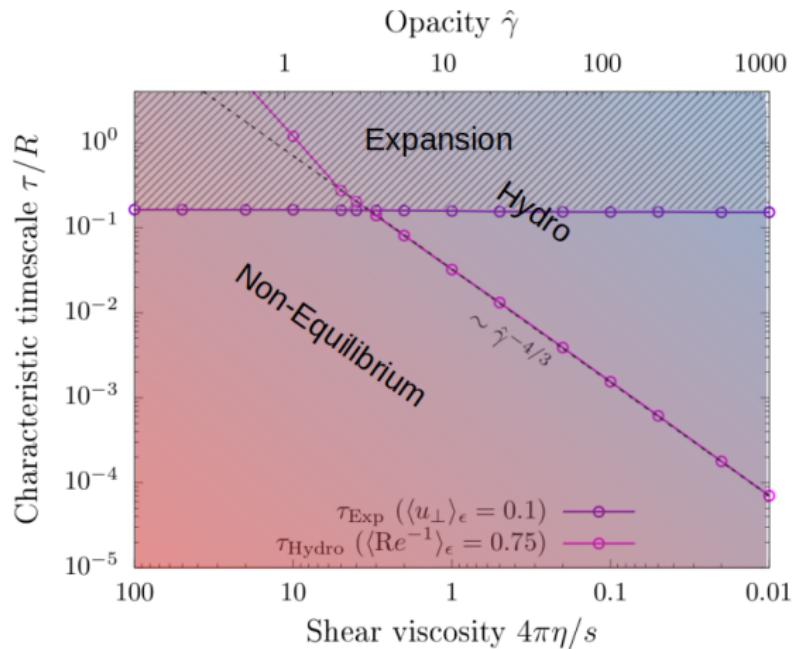
- takes longer for smaller systems; some systems never equilibrate enough!

- ▶ transverse expansion sets in: $u_\perp \sim 0.1$



- almost independent of opacity:
 $\tau_{\text{Exp}} \sim 0.2R$

Regime of applicability of hydrodynamics



- ▶ upper area: regime of transverse expansion
- ▶ Hydro applicable in top right corner
- ▶ $\hat{\gamma} \lesssim 3$: hydro does not become applicable before onset of transverse expansion!

Taking the criterion of $\hat{\gamma} \gtrsim 3$ seriously, what does this mean for the applicability of hydrodynamics to “real-life” collisions?

$$\text{Pb + Pb : } \hat{\gamma} \sim 5.7 \left(\frac{\eta/s}{0.16} \right)^{-1} \left(\frac{R}{2.78 \text{ fm}} \right)^{1/4} \left(\frac{dE_{\perp}^{(0)}/d\eta}{1280 \text{ GeV}} \right)^{1/4} \sim \frac{70-80\%}{2.7} - \frac{0-5\%}{9.0}$$

hydrodynamic behaviour in all but peripheral collisions

$$\text{p + Pb : } \hat{\gamma} \stackrel{\text{min. bias}}{\sim} 1.5 \left(\frac{\eta/s}{0.16} \right)^{-1} \left(\frac{R}{0.81 \text{ fm}} \right)^{1/4} \left(\frac{dE_{\perp}^{(0)}/d\eta}{24 \text{ GeV}} \right)^{1/4} \stackrel{\text{high mult.}}{\lesssim} 2.7$$

very high multiplicity events approach regime of applicability, but do not reach it

$$\text{p + p : } \hat{\gamma} \stackrel{\text{min. bias}}{\sim} 0.7 \left(\frac{\eta/s}{0.16} \right)^{-1} \left(\frac{R}{0.12 \text{ fm}} \right)^{1/4} \left(\frac{dE_{\perp}^{(0)}/d\eta}{7.1 \text{ GeV}} \right)^{1/4}$$

far from hydrodynamic behaviour

Taking the criterion of $\hat{\gamma} \gtrsim 3$ seriously, what does this mean for the applicability of hydrodynamics to "real-life" collisions?

$$\text{Pb + Pb : } \hat{\gamma} \sim 5.7 \left(\frac{\eta/s}{0.16} \right)^{-1} \left(\frac{R}{2.78 \text{ fm}} \right)^{1/4} \left(\frac{dE_{\perp}^{(0)}/d\eta}{1280 \text{ GeV}} \right)^{1/4} \sim \begin{array}{c} 70-80\% \\ 2.7 \end{array} - \begin{array}{c} 0-5\% \\ 9.0 \end{array}$$

hydrodynamic behaviour in all but peripheral collisions

$$\text{O + O : } \hat{\gamma} \sim 2.2 \left(\frac{\eta/s}{0.16} \right)^{-1} \left(\frac{R}{1.13 \text{ fm}} \right)^{1/4} \left(\frac{dE_{\perp}^{(0)}/d\eta}{55 \text{ GeV}} \right)^{1/4} \sim \begin{array}{c} 70-80\% \\ 1.4 \end{array} - \begin{array}{c} 0-5\% \\ 3.1 \end{array}$$

probes transition region to hydrodynamic behaviour

$$\text{p + Pb : } \overset{\text{min.bias}}{\hat{\gamma}} \sim 1.5 \left(\frac{\eta/s}{0.16} \right)^{-1} \left(\frac{R}{0.81 \text{ fm}} \right)^{1/4} \left(\frac{dE_{\perp}^{(0)}/d\eta}{24 \text{ GeV}} \right)^{1/4} \overset{\text{high mult.}}{\lesssim} 2.7$$

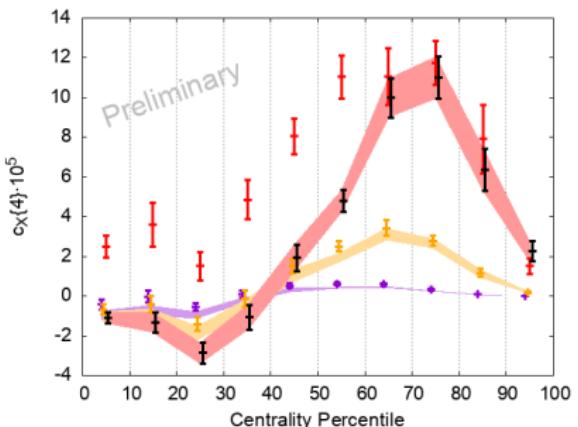
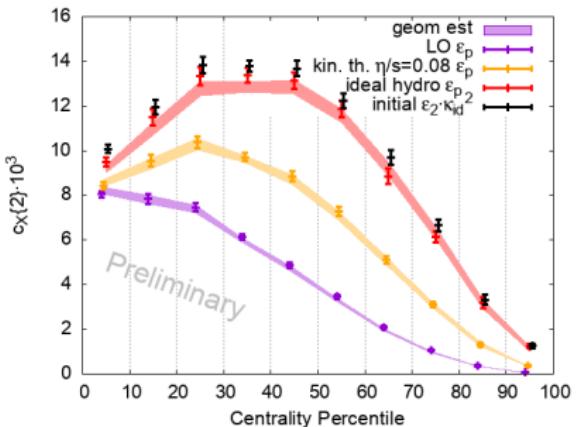
very high multiplicity events approach regime of applicability, but do not reach it

$$\text{p + p : } \overset{\text{min.bias}}{\hat{\gamma}} \sim 0.7 \left(\frac{\eta/s}{0.16} \right)^{-1} \left(\frac{R}{0.12 \text{ fm}} \right)^{1/4} \left(\frac{dE_{\perp}^{(0)}/d\eta}{7.1 \text{ GeV}} \right)^{1/4}$$

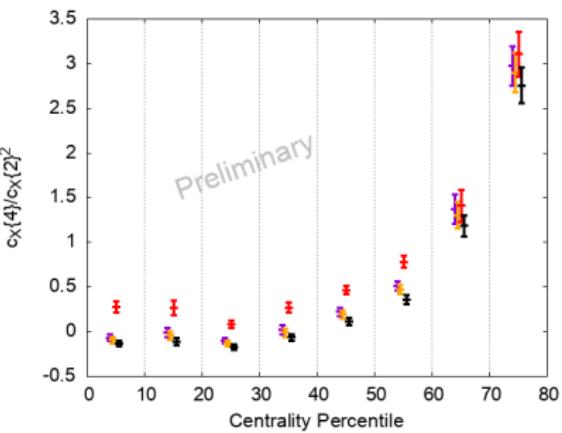
far from hydrodynamic behaviour

- ▶ kinetic theory description of transverse flow
on whole range in system size
- ▶ comparison to hydrodynamics:
accurate at 5% level if $\text{Re}^{-1} \lesssim 0.75$
- ▶ small systems ($p+p$, $p+\text{Pb}$):
transverse expansion faster than equilibration
 \Rightarrow hydro not applicable!
 - O+O covers transition regime to hydro behaviour

Teaser: flow cumulants in O+O



- $\langle (\epsilon_p)^n \rangle = \langle (\kappa \epsilon_2)^n \rangle = \bar{\kappa}^n \langle (\epsilon_2)^n \rangle + \dots$
- ▶ flow fluctuations dominated by avg. response to geometry fluctuations
- ▶ no $\hat{\gamma}$ -dependence in ideal hydro \Rightarrow same fluctuation curve
- ▶ ratio eliminates avg. response



Backup

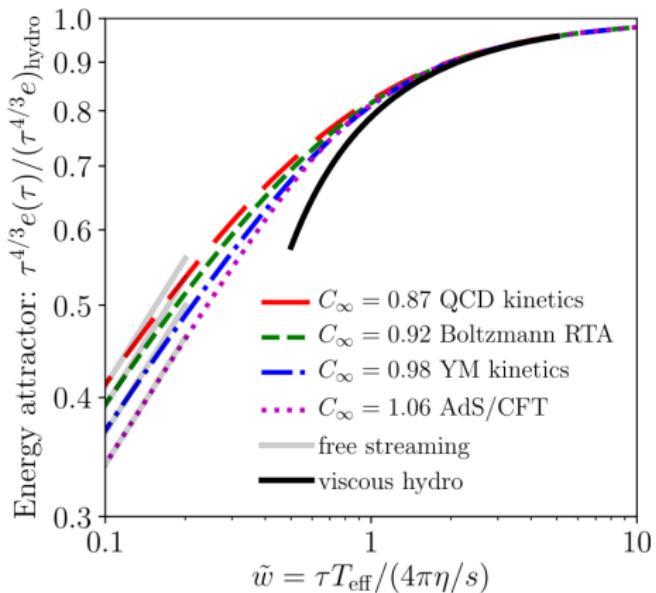
In theoretical descriptions:

$$v_n = \kappa_{n,n} \cdot \epsilon_n$$

- ▶ Flow can be compared to experiment
- ▶ Response depends on the dynamical model
- ▶ Initial state geometry is poorly constrained in small systems

Varying initial condition in order to fit flow measurements will mask inaccuracies in the description of the dynamical response!

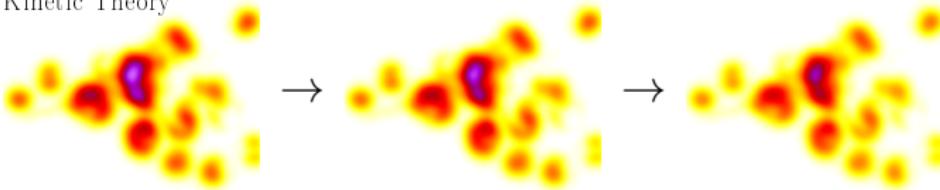
- ▶ more complex kernels will introduce further parameter dependence, but opacity dependence might still be "leading order approximation"
- ▶ in Bjorken flow, equilibration happens in very similar ways across different model descriptions:



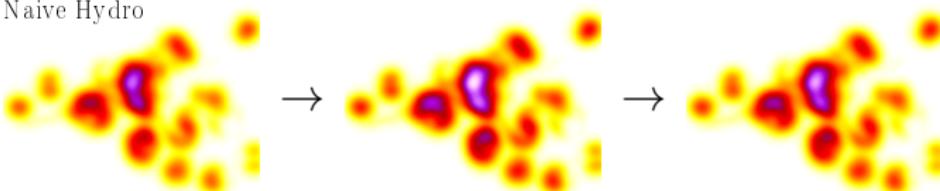
Giacalone, Mazeliauskas, Schlichting, PRL 123 (2019) 262301

evolution of τe :

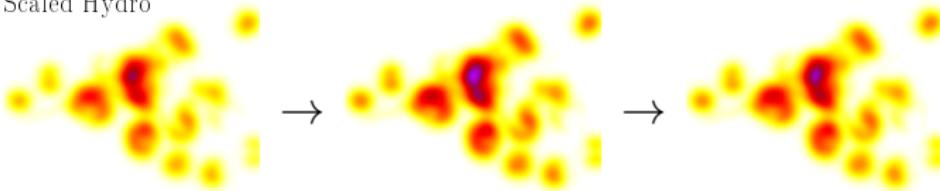
Kinetic Theory



Naive Hydro



Scaled Hydro



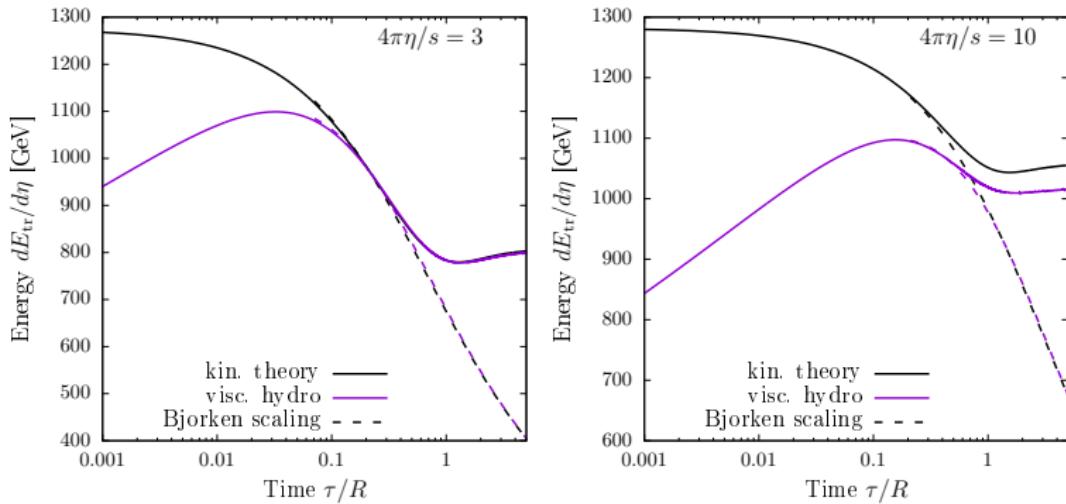
$$\tau = 3 \cdot 10^{-6} \text{ fm}$$

$$\tau = 8 \cdot 10^{-4} \text{ fm}$$

(times for $4\pi\eta/s = 0.05$)

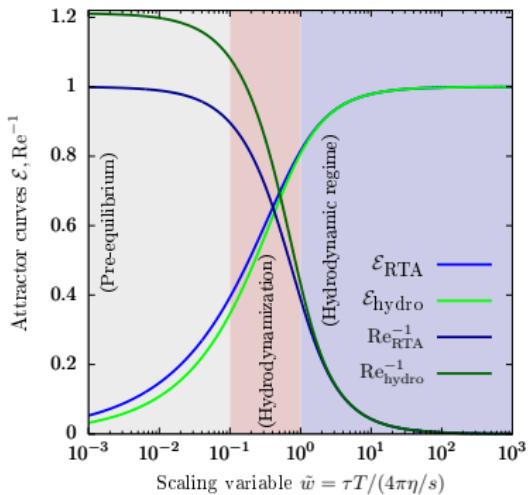
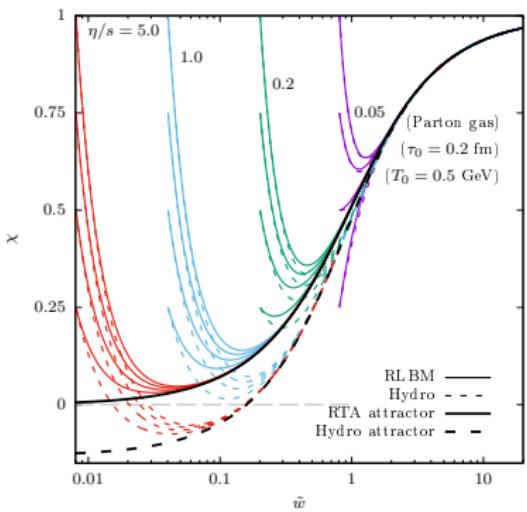
$$\tau = 3 \cdot 10^{-3} \text{ fm}$$

- accuracy depends on timescale separation of pre-equilibrium and transv. expansion



- ▶ longitudinal boost-invariant Bjorken flow exhibits universal behaviour
- ▶ time evolution curves converge to an attractor curve when expressed via the scaling variable $\tilde{w} = \frac{T\tau}{4\pi\eta/s}$
 \Rightarrow expressed via universal scaling functions
 $\chi(\tilde{w}) = p_L/p_T, \quad \mathcal{E}(\tilde{w}) \propto \tau^{4/3} e, \quad f_{E_\perp}(\tilde{w}) \propto \tau^{1/3} \frac{dE_\perp}{dy}, \dots$

Giacalone, Mazeliauskas, Schlichting, PRL 123 (2019) 262301



Ambruş, Bazzanini, Gabbana, Simeoni, Succi, Tripicione, arXiv:2201.09277

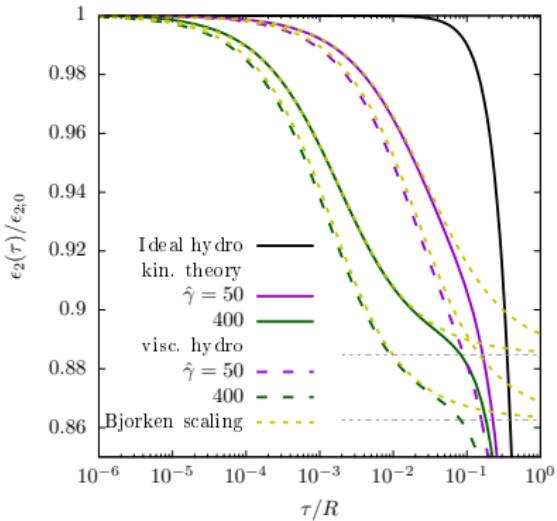
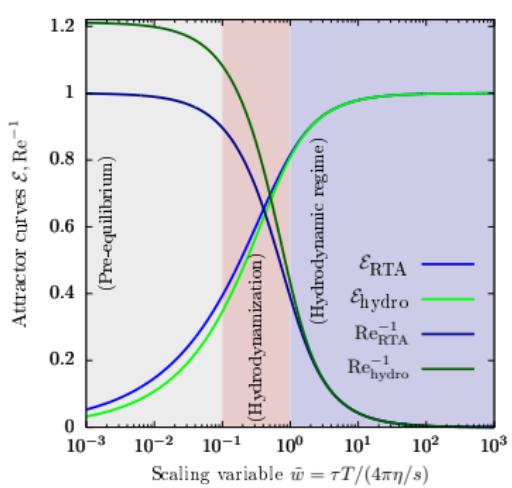
- $\tau \ll R$: no transverse expansion, system locally behaves like 0+1D Bjorken flow

- universal attractor curve scaling in the variable $\tilde{w}(\tau, \mathbf{x}_\perp) = \frac{T(\tau, \mathbf{x}_\perp)\tau}{4\pi\eta/s}$

Giacalone, Mazeliauskas, Schlichting, PRL 123 (2019) 262301

- $\tilde{w} \gg 1$: $\tau^{4/3}e = \text{const.}$, $\tau^{1/3}\frac{dE_\perp}{dy} = \text{const.}$

- $\tilde{w} \ll 1$: model dependent power law $\tau^{4/3}e \sim \tilde{w}^\gamma$



- inhomogeneous cooling changes energy density profile

Early Time Bjorken Scaling

Bjorken flow universal attractor curve in scaling variable $\tilde{w}(\tau, \mathbf{x}_\perp) = \frac{T(\tau, \mathbf{x}_\perp)\tau}{4\pi\eta/s}$:

$$\epsilon(\tau)\tau^{4/3} = (4\pi\eta/s)^{4/9} a^{1/9} (\epsilon\tau)_0^{8/9} C_\infty \mathcal{E}(\tilde{w}),$$

$$\tau^{1/3} \frac{dE_\perp}{d^2\mathbf{x}_\perp d\eta} = (4\pi\eta/s)^{4/9} a^{1/9} (\epsilon\tau)_0^{8/9} C_\infty f_{E_\perp}(\tilde{w})$$

- ▶ using $\epsilon = aT^4$, recast first eq. into self consistency eq. for \tilde{w}
- ▶ use together with initial cond. for $\epsilon\tau$ to relate differentials of $d\tilde{w}$ and $d\mathbf{x}_\perp$
- ▶ integrate second equation to find scaling of $dE_\perp/d\eta$
- ▶ use $\frac{(4\pi\eta/s)^4 a}{dE_\perp^0/d\eta R} = \frac{1}{\pi} \left(\frac{4\pi}{5\hat{\gamma}} \right)^4$ to identify $\hat{\gamma}$

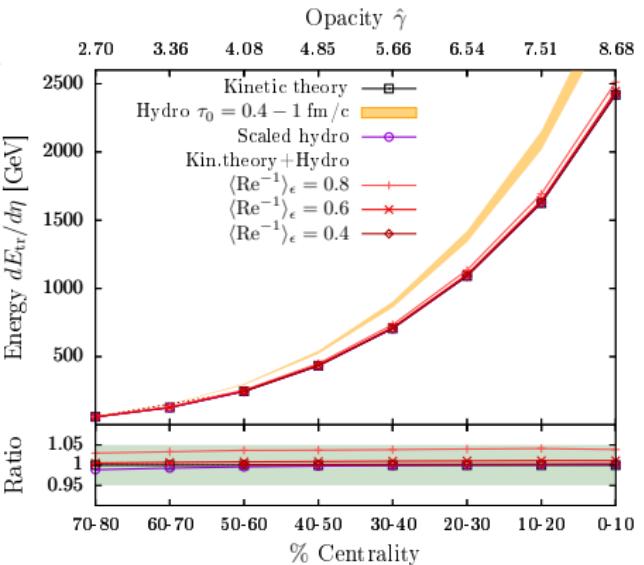
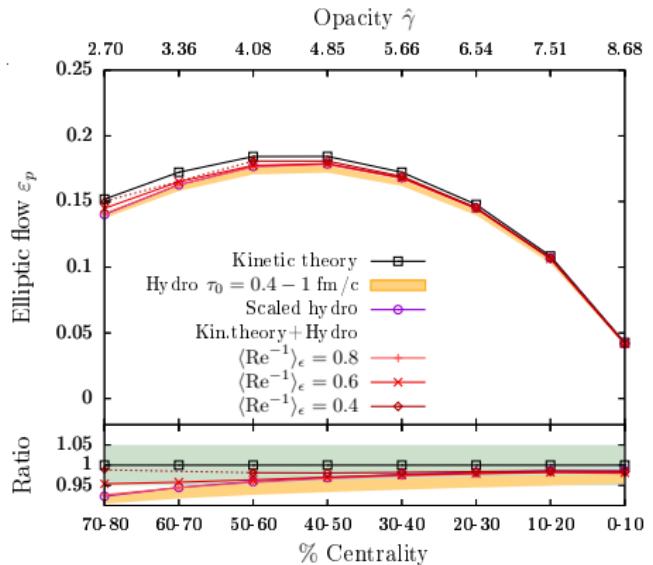
$$\frac{dE_\perp/d\eta}{dE_\perp^0/d\eta} = \frac{9}{2} \left(\frac{4\pi}{5\hat{\gamma}} \right)^4 \left(\frac{R}{\tau} \right)^3 \int_0^{\tilde{w}(\tau, \mathbf{x}_\perp=0)} \frac{\tilde{w}^3 d\tilde{w}}{\mathcal{E}(\tilde{w})} \left[1 - \frac{\tilde{w}}{4} \frac{\mathcal{E}'(\tilde{w})}{\mathcal{E}(\tilde{w})} \right] f_{E_\perp}(\tilde{w}),$$

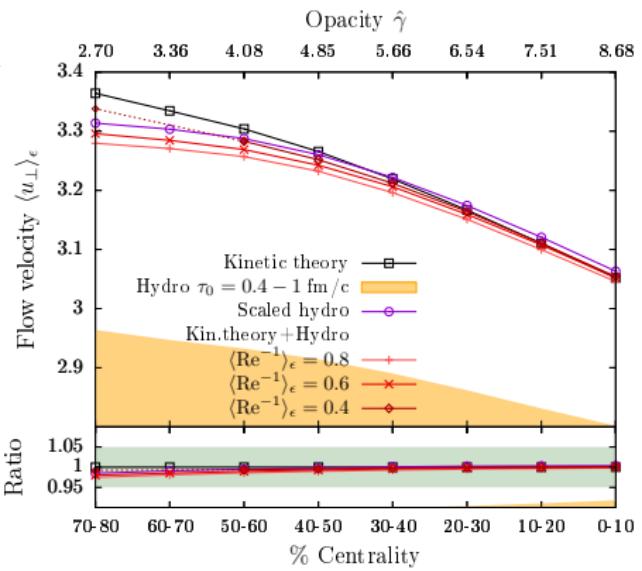
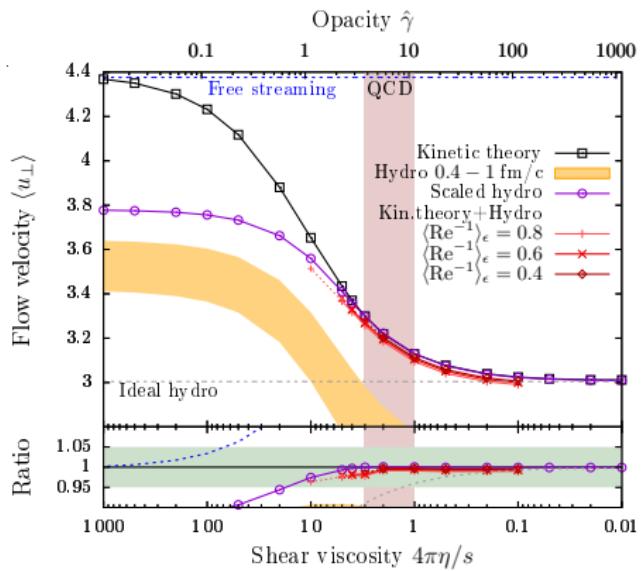
$$\tilde{w}(\tau, \mathbf{x}_\perp = 0) = \left(\frac{5\hat{\gamma}}{4\pi} \right)^{8/9} \left(\frac{\tau}{R} \right)^{2/3} [C_\infty \mathcal{E}(\tilde{w})]^{1/4}$$

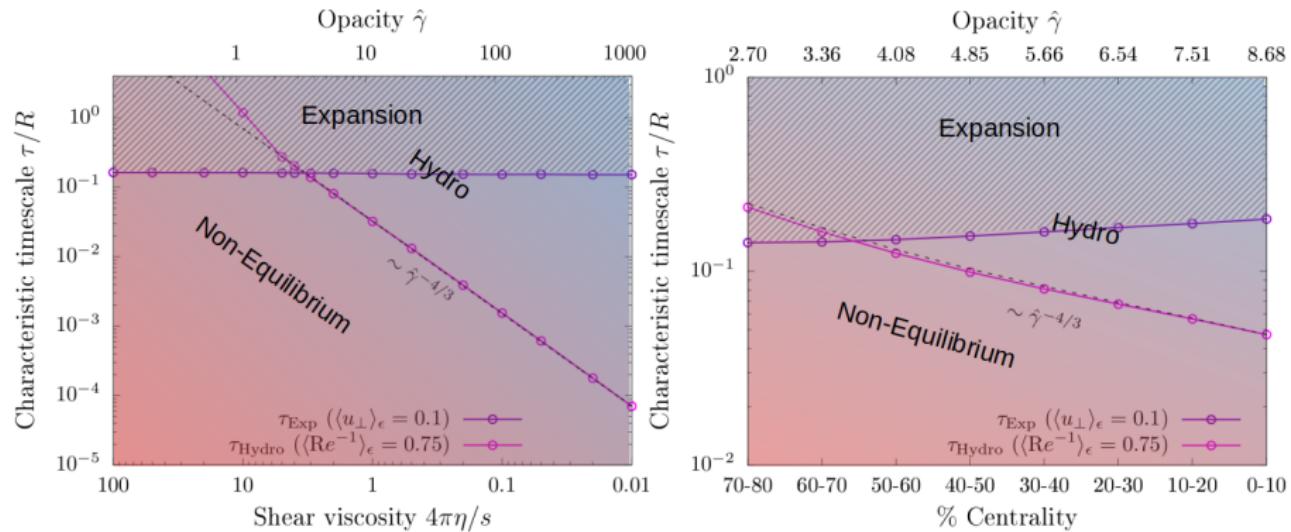
Limits of this scaling law:

- ▶ $\hat{\gamma} \left(\frac{\tau}{R} \right)^{3/4} \ll 1 \Rightarrow \tilde{w} \ll 1 \Rightarrow \mathcal{E}(\tilde{w}) \approx f_{E_\perp}(\tilde{w}) \approx C_\infty^{-1} \tilde{w}^{4/9} \Rightarrow \frac{dE_\perp/d\eta}{dE_\perp^0/d\eta} = 1$
- ▶ $\hat{\gamma}^{3/4} \left(\frac{\tau}{R} \right) \gg 1 \Rightarrow \tilde{w} \gg 1 \Rightarrow \mathcal{E}(\tilde{w}) \approx 1, f_{E_\perp} \approx \frac{\pi}{4}$
 $\Rightarrow \frac{dE_\perp/d\eta}{dE_\perp^0/d\eta} = \frac{9\pi}{32} \left(\frac{4\pi}{5\hat{\gamma}} \right)^{4/9} \left(\frac{R}{\tau} \right)^{1/3} C_\infty$

Centrality dependence







- ▶ transverse expansion sets in at $\tau_\perp \sim 0.2R$, independent of opacity
- ▶ Hydro applicable when $\text{Re}^{-1} < \text{Re}_c^{-1} \sim 0.75$ after timescale

$$\tau_{\text{Hydro}}/R \approx 1.53 \hat{\gamma}^{-4/3} \left[(\text{Re}_c^{-1})^{-3/2} - 1.21 (\text{Re}_c^{-1})^{0.7} \right]$$

- ▶ hydrodynamization before transv. Expansion for $\hat{\gamma} \gtrsim 3$