Far-off-equilibrium early-stage dynamics in high-energy nuclear collisions

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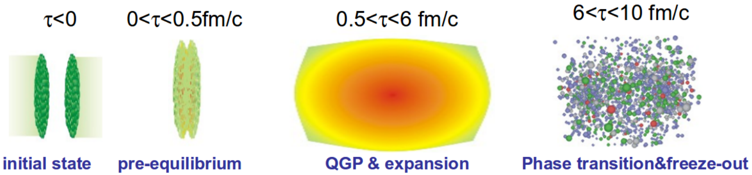
Introduction



(Figure by Steffen A. Bass)

- Traditional hydro: Description using macroscopic variables (T, μ, u^μ) and their gradients accompanied by transport coefficients (η, ζ, σ). Should be distinguished from Israel-Stewart type hydro (ISH) [Muller '67, Israel, Stewart '76] where the dissipative fluxes are promoted to independent dynamical degrees of freedom.
- ISH is remarkably successful in describing intermediate stages of heavy-ion collisions [Heinz et al., Romatschke et al., Dusling and Teaney, Song et al., and several others].
- ISH derived from kinetic theory works even when a fluid is not close to equilibrium [Heller et al., Romatschke, Strickland, Noronha, and others]. Can ISH serve as a proxy for kinetic theory in the far-from-equilibrium early stages of heavy-ion collisions?

Setup: Non-conformal quark-gluon gas at finite μ_B

- Consider a weakly interacting gas of quarks, anti-quarks, and gluons.
- Assume a kinetic description in terms of single-particle distribution functions, $f^i(x,p)$; 'i' denotes species.
- Evolution of $f^i(x,p)$ governed by Boltzmann equation:

$$p_i^\mu\,\partial_\mu f^i = \mathcal{C}[f^i]$$

- Approximate collisional kernel of the relaxation type [Andersen & Witting '74]:

$$\mathcal{C}[f^i] \approx -\frac{u\cdot p_i}{\tau_R}\,\left(f^i - f^i_{eq}\right),$$

τ_R is relaxation time for local equilibration, $u^\mu(x)$ is local fluid velocity.

- f_{eq} are given by Fermi-Dirac (for quarks, anti-quarks) or Bose-Einstein (for gluons) distributions in fluid rest frame. E.g.,

$$f^q_{eq} = \left[\exp\left(\beta(u\cdot p) - \alpha\right)\right]^{-1}, \,\text{where}\, \beta = 1/T, \alpha = \mu/T.$$

Hydro from kinetic theory [C.C., Heinz, Schäfer, PRC 107 (2023) 4, 044905]

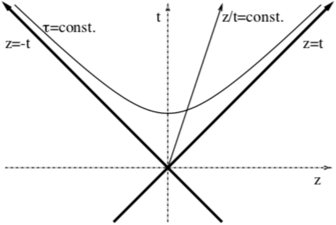
- Using $f(x,p)$ one obtains conserved currents of hydro:
$$T^{\mu\nu}(x) = \sum_i \int dP_i\, p_i^\mu\, p_i^\nu\, f^i = e\, u^\mu\, u^\nu - (P + \Pi)\, \Delta^{\mu\nu} + \pi^{\mu\nu},$$
$$N^\mu(x) = \int dP_q\, p_q^\mu\, \left(f^q - f^{\bar{q}}\right) = n\, u^\mu + n^\mu$$
The viscous stresses stem from $\delta f^i \equiv f^i - f^i_{eq}$, and satisfy relaxation-type equations [Denicol et al. '12]. For example,
$$\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_R} = \Delta^{\mu\nu}_{\alpha\beta} \sum_{i=1}^{3} \int dP_i\, p_i^\alpha\, p_i^\beta\, \frac{1}{u\cdot p_i} p_i^\gamma\, \nabla_i\, f^i.$$
- We obtained f^i up to 2nd-order in velocity gradients (ignoring n^μ) by solving RTA BE perturbatively in the Knudsen number ($Kn \sim \tau_R\, |\nabla^\mu u^\nu| \ll 1$):
$$\dot{\Pi} + \frac{\Pi}{\tau_R} = -\beta_\Pi\, \theta - \delta_{\Pi\Pi}\, \Pi\, \theta + \lambda_{\Pi\pi}\, \pi^{\mu\nu}\, \sigma_{\mu\nu},$$
$$\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_R} = 2\, \beta_\pi\, \sigma^{\mu\nu} + 2\, \pi_\gamma^{\langle\mu}\, \omega^{\nu\rangle\gamma} - \tau_{\pi\pi}\, \pi_\gamma^{\langle\mu}\, \sigma^{\nu\rangle\gamma} - \delta_{\pi\pi}\, \pi^{\mu\nu}\, \theta + \lambda_{\pi\Pi}\, \Pi\, \sigma^{\mu\nu},$$
where $(\beta_\Pi, \delta_{\Pi\Pi}, \cdots)$ are functions of (T, μ). Standard definitions: $\theta = \partial_\mu u^\mu$, velocity stress-tensor $\sigma^{\mu\nu} = \nabla^{\langle\alpha} u^{\beta\rangle}$, vorticity $\omega^{\mu\nu} = (\nabla^\mu u^\nu - \nabla^\nu u^\mu)/2$, $A^{\langle\mu\nu\rangle} = \Delta^{\mu\nu}_{\alpha\beta}\, A^{\alpha\beta}$, where $\Delta^{\mu\nu}_{\alpha\beta}$ is a double-symmetric traceless projector orthogonal to u^μ.

Application: Bjorken flow [J.D. Bjorken, PRD, 27, 140 (1983)]

- Bjorken flow is valid during the early stages of ultra-relativistic heavy-ion collisions. The fluid is assumed to be homogeneous in (x, y)-plane.
- The medium expands boost-invariantly along the beam $(z-)$ direction: $v^z = z/t$. Best described in Milne (expanding) coordinates $\tau \equiv \sqrt{t^2 - z^2}$, and $\eta_s \equiv \tanh^{-1}(z/t)$.

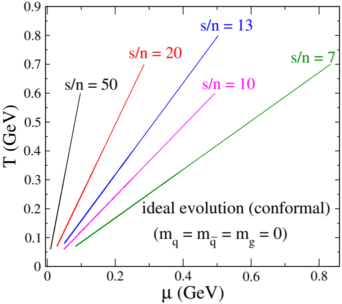
- Fluid expansion rate, $\theta = 1/\tau$, $\pi^{\mu\nu} \to \mathrm{diag}(0, \pi/2, \pi/2, -\pi/\tau^2)$, baryon diffusion vanishes, and everythings depends only on τ.
- Hydro quantities evolve as,

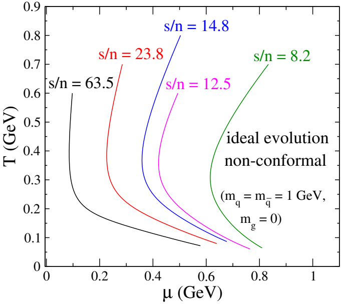
$$
\begin{aligned}
\frac{de}{d\tau} &= -\frac{1}{\tau}\left(e + P + \Pi - \pi\right), \\
\frac{dn}{d\tau} &= -\frac{n}{\tau}, \\
\frac{d\Pi}{d\tau} + \frac{\Pi}{\tau_R} &= -\frac{\beta_\Pi}{\tau} + \cdots, \\
\frac{d\pi}{d\tau} + \frac{\pi}{\tau_R} &= \frac{4}{3}\frac{\beta_\pi}{\tau} + \cdots .
\end{aligned}
$$



Case I: Ideal hydrodynamics

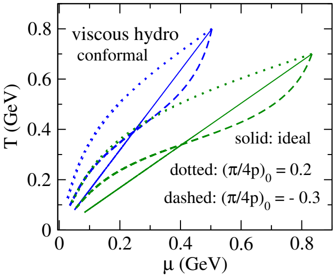
- Isentropic evolution $\implies s \propto 1/\tau$ where $s = (e + P - \mu n)/T$, $n \propto 1/\tau$ such that entropy per baryon s/n is constant.

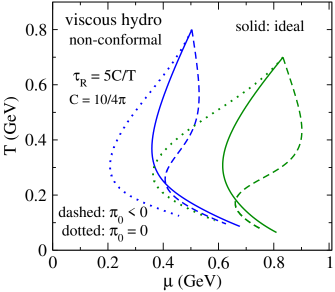




- For conformal gas, fixed s/n implies fixed μ/T.
- For non-conformal gas ($m_q = 1$ GeV):
  - At high T, EoS dominated by quarks, anti-quarks, and gluons. At low T, EoS dominated by quarks.
  - As $T \to 0$, Fermi statistics of quarks imply $\mu \to m_q$.
- s/n increases from right to left in the phase diagram.

Case II: Viscous hydrodynamics





- Dissipation substantially shuffles around phase trajectories.
- Trajectories with non-negative π_0 lie to left of ideal trajectory. Expected: dissipation produces entropy. ✓
- However, trajectories with negative π_0 move to the right for some time. But entropy should not decrease! Similar behavior first observed by Travis Dore et al. [PRD 102 (2020), 074017, PRD 106 (2022) 9, 094024] in a more complex setting: second-order hydro with Lattice QCD based EoS used to study critical dynamics.
- Is hydro breaking down for these far-off-equilibrium initial conditions?

Resolution

- Statement of the second law: $\partial_\mu S^\mu \geq 0$.
- Thus far we have assumed $S^\mu = s_{eq}\, u^\mu$ with $s_{eq} = (e + P - \mu n)/T$.
- But is it justified when the system deviates substantially from local equilibrium?
- Need an expression for non-equilibrium entropy.

Non-equilibrium entropy current

- Start from Boltzmann's H-function,

$$S^\mu = -\sum_i g_i \int dP_i \, p_i^\mu \, \phi_i[f^i],$$

The functions $\phi_i[f^i]$ are given as,

$$\phi_i[f^i] = f^i \ln f^i - \frac{1 + a_i f^i}{a_i} \ln(1 + a_i f^i),$$

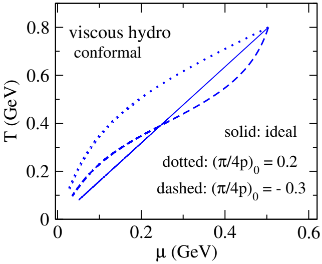
with $a_1 = a_2 = -1$ (Fermi-Dirac) and $a_3 = 1$ (Bose-Einstein).

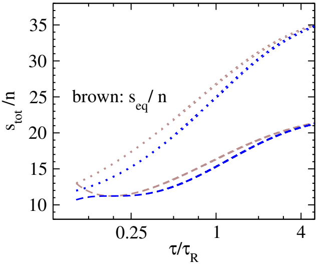
- Writing $f^i = f^i_{eq} + \delta f^i$, and expand to second-order in δf^i:

$$\boxed{S^\mu = s_{\rm eq} \, u^\mu - \alpha \, n^\mu - \frac{\beta}{4\beta_\pi} \, u^\mu \, \pi^{\alpha\beta} \, \pi_{\alpha\beta} + c_{nn} \, u^\mu \, n^\alpha \, n_\alpha + c_{n\pi} \, \pi^{\mu\alpha} \, n_\alpha .}$$

The coefficients $(\beta_\pi, c_{nn}, c_{n\pi})$ are derived for a massless QG-gas in PRC 107 (2023) 4, 044905 [C.C., Heinz, Schäfer].

Second law in conformal hydrodynamics [C.C., Heinz, Schäfer, '23]





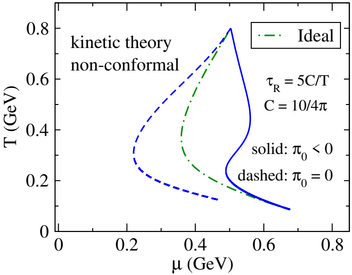
- Once second-order corrections are included, trajectories with same (T_0, μ_0) but different π_0, start with different $s_{\rm tot}/n < s_{\rm eq}/n$:

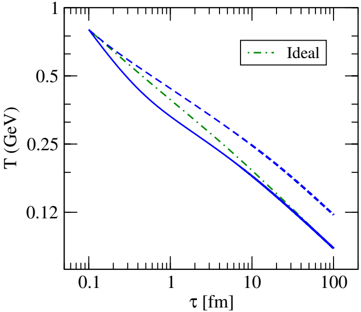
$$s_{\rm tot} = s_{\rm eq} - \frac{3\beta}{8\beta\pi}\pi^2.$$

- For $\pi_0 > 0$, both the ideal and total entropy per baryon increase.
- For trajectory that moves initially to the right, the ideal entropy per baryon decreases initially. However, this curve starts from a much lower total s/n.
- The total entropy per baryon never decreases. Hydro trajectories are consistent with the second law!

Viscous cooling! [C.C., Heinz, Schäfer, PRC 107 (2023) 4, 044905]

- Usually dissipative fluxes causes viscous heating $\implies$ Temperature falls slower than in ideal evolution.



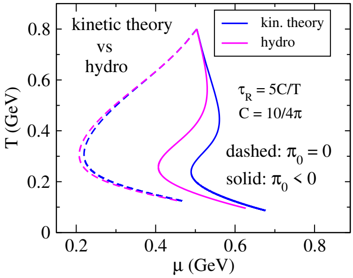


- However, for trajectories where the ideal entropy per baryon decreases, the temperature drops faster than even the ideal case.
- This happens when the effective longitudinal pressure $P_L > P \implies \pi - \Pi < 0$ such that,

$$\frac{d(s_{eq}/n)}{d\tau} = \frac{\pi - \Pi}{\tau_0\, n_0\, T} < 0.$$

Hydro vs kinetic theory

- Plenty of room for improvement...



- Substantial difference between second-order hydro and kinetic theory for non-equilibrium initial conditions.
- Perhaps the Knudsen number (perturbative) expansion is not well-suited for these scenarios.
- Need for a 'non-perturbative' framework that models both far-off-equilibrium and hydro regimes with more accuracy.

Far-off-equilibrium theory using maximum-entropy principle

- Recall, conserved curents $(T^{\mu\nu}, N^{\mu})$ are moments of $f(x,p)$. For example,

$$T^{\mu\nu}(x) \equiv \int dP\, p^{\mu}\, p^{\nu}\, f(x,p) = e\, u^{\mu} u^{\nu} - (P + \Pi)\,\Delta^{\mu\nu} + \pi^{\mu\nu}.$$

For RTA Boltzmann eq., the viscous stresses satisfy relaxation-type evolution equations. For eg., the exact evolution of Π is [Denicol et al. '12],

$$\dot{\Pi} + \frac{\Pi}{\tau_R} = -\alpha_1\,\theta + \alpha_2\,\Pi\,\theta + \alpha_3\,\pi^{\mu\nu}\,\sigma_{\mu\nu} + \frac{m^2}{3}\rho^{\mu\nu}_{(-2)}\,\sigma_{\mu\nu}$$
$$+ \frac{m^2}{3}\nabla_{\mu}\,\rho^{\mu}_{(-1)} + \frac{m^4}{9}\rho_{(-2)}\,\theta.$$

However, the eq. is **not closed** due to couplings to ρ-tensors.

- The ρ-tensors are non-hydrodynamic moments of $\delta f = f - f_{\rm eq}$:

$$\rho^{\mu}_{(-1)} \equiv \Delta^{\mu}_{\alpha} \int dP\,(u\cdot p)^{-1}\; p^{\alpha}\;\delta f,$$
$$\rho^{\mu\nu}_{(-2)} \equiv \int dP\,(u\cdot p)^{-2}\; p^{\langle\mu}\,p^{\nu\rangle}\;\delta f.$$

- Similar feature exists for shear stress evolution equation. Needs **truncation**, i.e., to express δf in terms of quantities appearing in $T^{\mu\nu}$.

A new truncation scheme

- Standard truncation schemes: Grad's 14-moment approximation [Dusling, Teaney '08], Chapman-Enskog approximation [Bhalerao, Jaiswal et al. '14], anisotropic hydro using Romatschke-Strickland ansatz [Romatschke and Strickland '03], $f_{\mathrm{RS}} \sim \exp\left(-\sqrt{p_T^2 + (1+\xi)p_z^2}/\Lambda\right)$.
- Grad assumes δf to be quadratic in momenta (ad-hoc); Chapman-Enskog δf should not be valid far-from-equilibrium. Both become negative (unphysical) at large momenta. Resulting hydrodynamics breaks down in certain flow profiles.
- The aHydro ansatz does not become negative and can handle large shear deformations at early stages of heavy-ion collisions.
    - But: its form is ad-hoc and custom-built for Bjorken flow (not general 3-d flow).
- We want to implement a truncation scheme that (i) leads to a framework which may work both near and far from local equilibrium, ii) does not invoke uncontrolled assumptions about the microscopic physics, and iii) does not restrict the expansion geometry.

The 'least-biased' distribution [E. Jaynes, Phys. Rev. 106, 620 (1957)]

- To truncate the moment-hierarchy of BE, we want to re-construct an approximate $f(x,p)$ at each time step solely in terms of quantities appearing in $T^{\mu\nu}$.
- The '**least-biased**' distribution that uses **all of, and only** the information provided by $T^{\mu\nu}$ is one that maximizes the non-equilibrium entropy,

$$s[f] = -\int dP\ (u\cdot p)\,\Phi[f],\ \ \Phi[f] \equiv f\ln f - \frac{1+a\,f}{a}\ln(1+a\,f),$$

$a = (-1, 0, 1)$ for FD, MB, BE statistics,

- subject to constraints,

$$\int dP\,(u\cdot p)^2\ f = e,\ -\frac{1}{3}\int dP\,p_{\langle\mu\rangle}p^{\langle\mu\rangle}\ f = P + \Pi,$$

$$\int dP\,p^{\langle\mu}p^{\nu\rangle}\ f = \pi^{\mu\nu}.$$

- The solution of such an $f(x,p)$ is obtained using $\delta s[f]/\delta f = 0$, after including appropriate Lagrange multipliers in $s[f]$.

The maximum-entropy distribution

- The solution for maximum entropy distribution is,

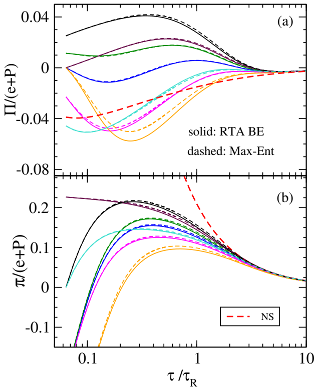
$$\boxed{f_{\rm ME}(x,p) = \left[\exp\left(\Lambda\left(u\cdot p\right) - \frac{\lambda_\Pi}{u\cdot p}p_{\langle\alpha\rangle}p^{\langle\alpha\rangle} + \frac{\gamma_{\langle\alpha\beta\rangle}}{u\cdot p}p^{\langle\alpha}p^{\beta\rangle}\right) - a\right]^{-1}}$$

where $(\Lambda, \lambda_\Pi, \gamma^{\langle\mu\nu\rangle})$ are Lagrange multipliers corresponding to the information (constraints) provided by hydrodynamics: $(e, \Pi, \pi^{\mu\nu})$

[Derek Everett, C.C., U. Heinz, PRC 103 (2021) 6, 064902].

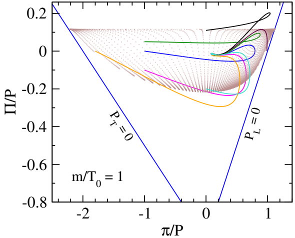
- Features of $f_{\rm ME}$:
  - Positive-definite for all momenta.
  - Non-linear dependence on $(\Pi, \pi^{\mu\nu})$; exact matching to $T^{\mu\nu}$ for large range of viscous stresses allowed by kinetic theory.
  - Reduces to linearized Chapman-Enskog δf of RTA BE for weak dissipative stresses; yields second-order hydro.
  - Resulting framework satisfies the second-law of thermodynamics.
- Maximum-entropy idea pursued before: in non-relativistic context by Levermore '96, in neutrino transport in astrophysical context by Murchikova et al. '17, by Calzetta et al. '19 for conformal fluids, and Pradeep & Stephanov '23 for freeze-out of critical fluctuations.

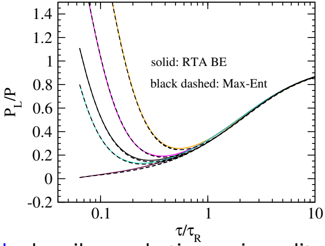
Application I: Bjorken flow [C.C., Heinz, Schäfer, arXiv:2307.10769]



- ME-hydro is in good agreement with KT throughout evolution. Nicely interpolates between early free-streaming and late Navier-Stokes dynamics.

- Generates dissipative stresses within domain allowed by KT.





Accurately describes early-time universality of P_L/P.

Application II: Gubser Flow [S.S. Gubser, PRD, 82, 085027 (2010)]

- Gubser flow is longitudinally boost-invariant: $v^z = z/t$, and has $u^\phi = 0$. But it has transverse dynamics: $u^r(x) \neq 0$.
- Re-scale metric, $ds^2 \to d\hat{s}^2 = ds^2/\tau^2$, followed by coordinate transform: $(\tau, r, \phi, \eta) \to (\rho, \theta, \phi, \eta)$,

$$\rho = -\sinh^{-1}\left(\frac{1 - q^2\tau^2 + q^2 r^2}{2q\tau}\right),\ \theta = \tan^{-1}\left(\frac{2qr}{1 + q^2\tau^2 - q^2 r^2}\right),$$

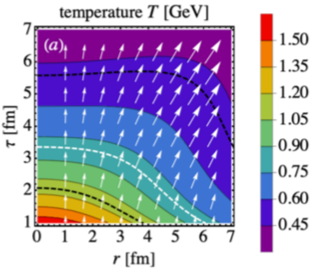
such that $\hat{u}^\mu = (1, 0, 0, 0)$.

Weyl rescaled unitless quantities,

$$e(\tau, r) = \frac{\hat{e}(\rho)}{\tau^4},$$

$$\pi_{\mu\nu}(\tau, r) = \frac{1}{\tau^2}\frac{\partial \hat{x}^\alpha}{\partial x^\mu}\frac{\partial \hat{x}^\beta}{\partial x^\nu}\hat{\pi}_{\alpha\beta}(\rho).$$

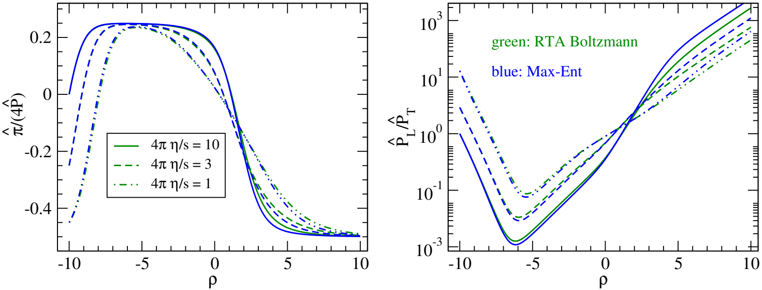
Also, $\hat{\pi}^{\mu\nu} = \mathrm{diag}(0, \hat{\pi}/2, \hat{\pi}/2, -\hat{\pi})$.



Du et al. [2019]

Results: ME-hydro [C.C., Heinz, Schäfer, arXiv:2307.10769]

- Evolution of shear inverse Reynolds number and pressure anisotropy using ME-hydro:



- Rapid transverse expansion in Gubser flow at late times (or large ρ) prevents system from thermalizing; fluid approaches transverse free-streaming: $\hat{P}_T \to 0$;
- ME-hydro correctly describes not only near-equilibrium dynamics but also longitudinal ($\hat{\bar{\pi}} \approx 0.25$) & transverse ($\hat{\bar{\pi}} \approx -0.5$) free-streaming domains.

Conclusions

- Derived second-order Israel-Stewart type hydro for a non-conformal quark-gluon gas at finite μ_B, undergoing Bjorken flow.
    - Certain trajectories seemed to violate the second-law of thermodynamics. Provided a microscopic explanation for this 'anomalous' behavior and established their thermodynamic consistency.
    - Pointed out a novel effect of viscous cooling associated with such trajectories.
- Derived far-from-equilibrium fluid dynamics from the Boltzmann equation using a maximum-entropy distribution: **ME-hydro**.
    - This scheme does not introduce ad-hoc assumptions about the microscopic physics or the flow profile being modeled; uses only information contained within hydro conserved currents.
    - This framework can be applied to systems of general microscopic dynamics as long as the constituents admit a description using distribution function.
    - ME-hydro accurately predicts the kinetic theory evolution of $T^{\mu\nu}$ in both free-streaming and hydrodynamic regimes for Bjorken and Gubser flows.
    - The description of $T^{\mu\nu}$ within this approach for flow profiles with less restrictive symmetries remains to be studied.

Extra slides

ME-hydro for Bjorken flow [C.C., Heinz, Schäfer, arXiv:2307.10769]

- Exact evolution of e, $P_L = (P + \Pi - \pi)$, and $P_T = (P + \Pi + \pi/2)$:

$$\frac{de}{d\tau} = -\frac{e + P_L}{\tau},$$

$$\frac{dP_L}{d\tau} = -\frac{P_L - P}{\tau_R} + \frac{\bar{\zeta}_z^L}{\tau}, \quad \frac{dP_T}{d\tau} = -\frac{P_T - P}{\tau_R} + \frac{\bar{\zeta}_z^\perp}{\tau}$$

- The couplings $\bar{\zeta}_z^L$ and $\bar{\zeta}_z^\perp$ involve non-hydro moments:

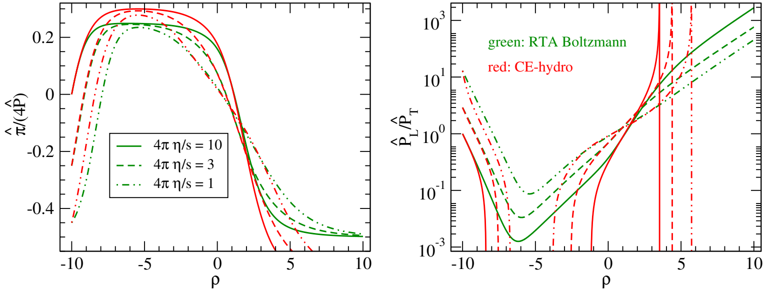
$$\bar{\zeta}_z^L = -3P_L + \int dP \, (p^\tau)^{-2} \, p_\eta^4 \, f,$$

$$\bar{\zeta}_z^\perp = -P_T + \frac{1}{2} \int dP \, (p^\tau)^{-2} \, p_\eta^2 \, p_T^2 \, f.$$

- To truncate, we set $f \to f_{\mathrm{ME}}$. This makes $\bar{\zeta}_z^L$ and $\bar{\zeta}_z^\perp$ functions of (e, P_L, P_T). Now, solve 3 equations for Lagrange multipliers; same complexity as hydro.

Breakdown of second-order hydro

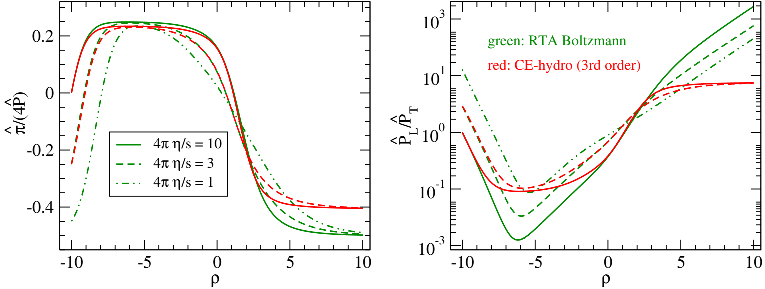
- Evolution of normalised shear and pressure anisotropy using second-order CE hydro (identical to Denicol et al. or DNMR):



- Rapid transverse expansion in Gubser flow at late times (or large ρ) prevents system from thermalizing; fluid approaches transverse free-streaming : $\hat{P}_T \to 0$; not described by second-order hydro.
- Second-order CE and DNMR yield negative $\hat{P}_L$ and $\hat{P}_T$.

Breakdown of third-order hydro

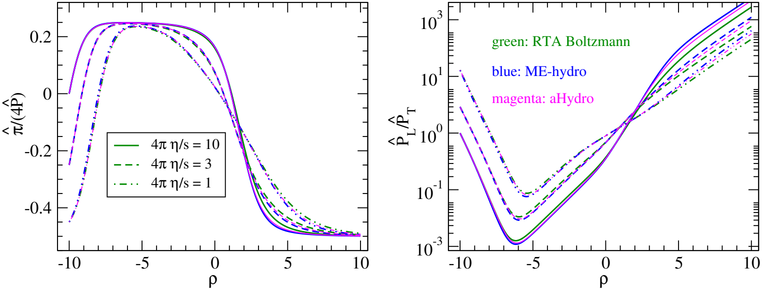
- Evolution of normalised shear and pressure anisotropy using third-order CE hydro [C.C., Heinz, et al. '18]:



- Third-order CE yields incorrect asymptotic value of $\hat{\pi}/(4\hat{P}) \approx -0.4$.
- For initialisations $\hat{\pi}/(4\hat{P}) \lesssim -0.4$, third-order CE equations become numerically unstable.

ME-hydro vs anisotropic hydro (in preparation)

- Comparison with aHydro, based on truncation distribution
  $$f_{\rm RS} = \exp\left(-\sqrt{\hat{p}_T^2 + (1+\hat{\xi})\hat{p}_z^2}/\hat{T}_{\rm RS}\right).$$



- aHydro is slightly better than ME-hydro. Need testing in more complicated flow profiles to distinguish between their performances.

Second law in non-conformal kinetic theory [C.C., Heinz, Schäfer, '23]

- Solved RTA Boltzmann equation exactly for QG-gas with non-zero quark masses.
- Although s_{eq}/n decreases, the total s/n computed using Boltzmann's H-function does not.
- 3 distinct regimes of s/n evolution:
  - early rapid increase of s/n: expansion driven isotropization,
  - intermediate plateau where $s/n \approx \text{const.}$ (free-streaming),
  - eventual merging with s_{eq}/n (interaction driven).