

Far-from-equilibrium slow modes and momentum anisotropy

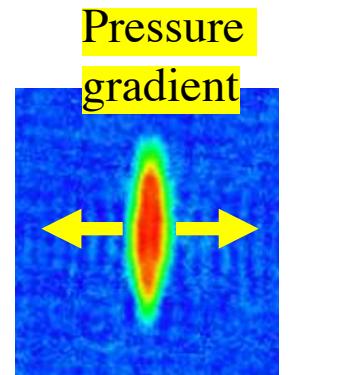
Jasmine Brewer



Based on [2212.00820] with Weiyao Ke, Li Yan, and Yi Yin

What is the origin of observed momentum anisotropies in small systems?

- Initial spatial ellipticity
- Initial momentum-space ellipticity

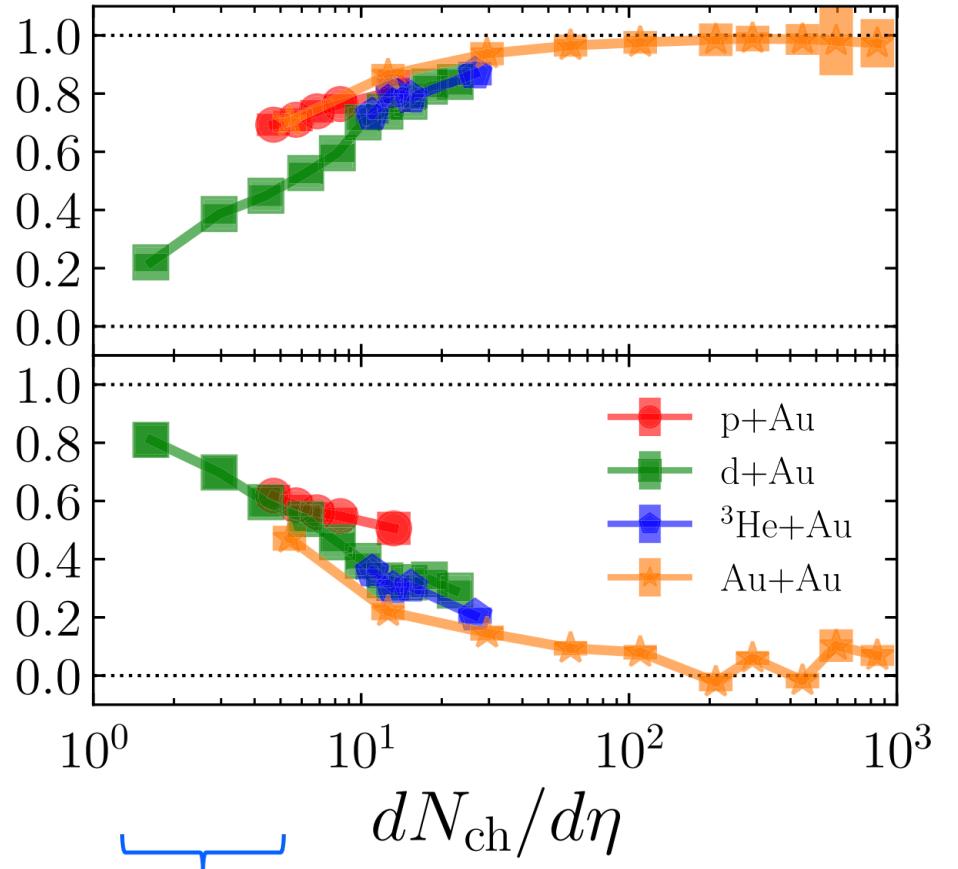


effect of hydrodynamics

residual momentum-space anisotropy
very important in small systems



correlation of v_2 with



Schenke, Shen, Tribedy [1908.06212]

but hydrodynamics is breaking down in the far-from-equilibrium regime –
how does momentum-space anisotropy evolve in kinetic theory?

Studying the evolution of momentum-space anisotropy in kinetic theory

Kinetic theory for a boost-invariant and *spatially-homogenous* system

$$\partial_\tau F - \frac{\hat{p}_z (1 - \hat{p}_z^2)}{\tau} \partial_{\hat{p}_z} F + \frac{4\hat{p}_z^2}{\tau} F = -C[f]$$



longitudinal expansion collisions

$F(\tau; \mathbf{x}, \hat{\mathbf{p}}) = \int p f$: integrated version of the single-particle distribution function contributing to the stress-energy tensor

Study the momentum-space anisotropies of $F(\tau, \hat{p})$ by decomposing in spherical harmonics

$$L_{lm} \equiv \int_{-1}^1 d\hat{p}_z \int \frac{d\phi}{4\pi} F(\tau, \hat{p}) \underbrace{(-1)^m P_l^m(\hat{p}_z)}_{\text{momentum distribution}} \underbrace{\cos(m\phi)}_{\text{spherical harmonics}}$$

$\tan \phi = \hat{p}_y / \hat{p}_x$

* No spatial anisotropy means all momentum anisotropies are driven by the initial state! *

Moments of F divide into “sectors” that evolve independently

$$L_{lm} \equiv \int_{-1}^1 d\hat{p}_z \int \frac{d\phi}{4\pi} F(\tau, \hat{p}) (-1)^m P_l^m(\hat{p}_z) \cos(m\phi)$$

moments with different m and parity in $p_z \rightarrow -p_z$ (+/-) evolve independently of one another

(0, +)

(1, +)

(0, -)

(2, +)

...

Hydrodynamic modes

$$L_{00} = \epsilon$$

$$L_{20} = \frac{1}{2}(3\epsilon - p_L)$$

$$L_{40}$$

⋮

⋮

⋮

$$L_{11} = T^{0x}$$

$$\begin{matrix} L_{31} \\ L_{51} \\ \vdots \\ \vdots \end{matrix}$$

$$L_{10} = \tau T^{0\eta}$$

$$\begin{matrix} L_{30} \\ L_{50} \\ \vdots \\ \vdots \end{matrix}$$

$$L_{22} = 3(T^{xx} - T^{yy})$$

$$L_{42}$$

$$L_{62}$$

⋮

⋮

⋮

Attractor for p_L/ϵ as a consequence of adiabatic evolution

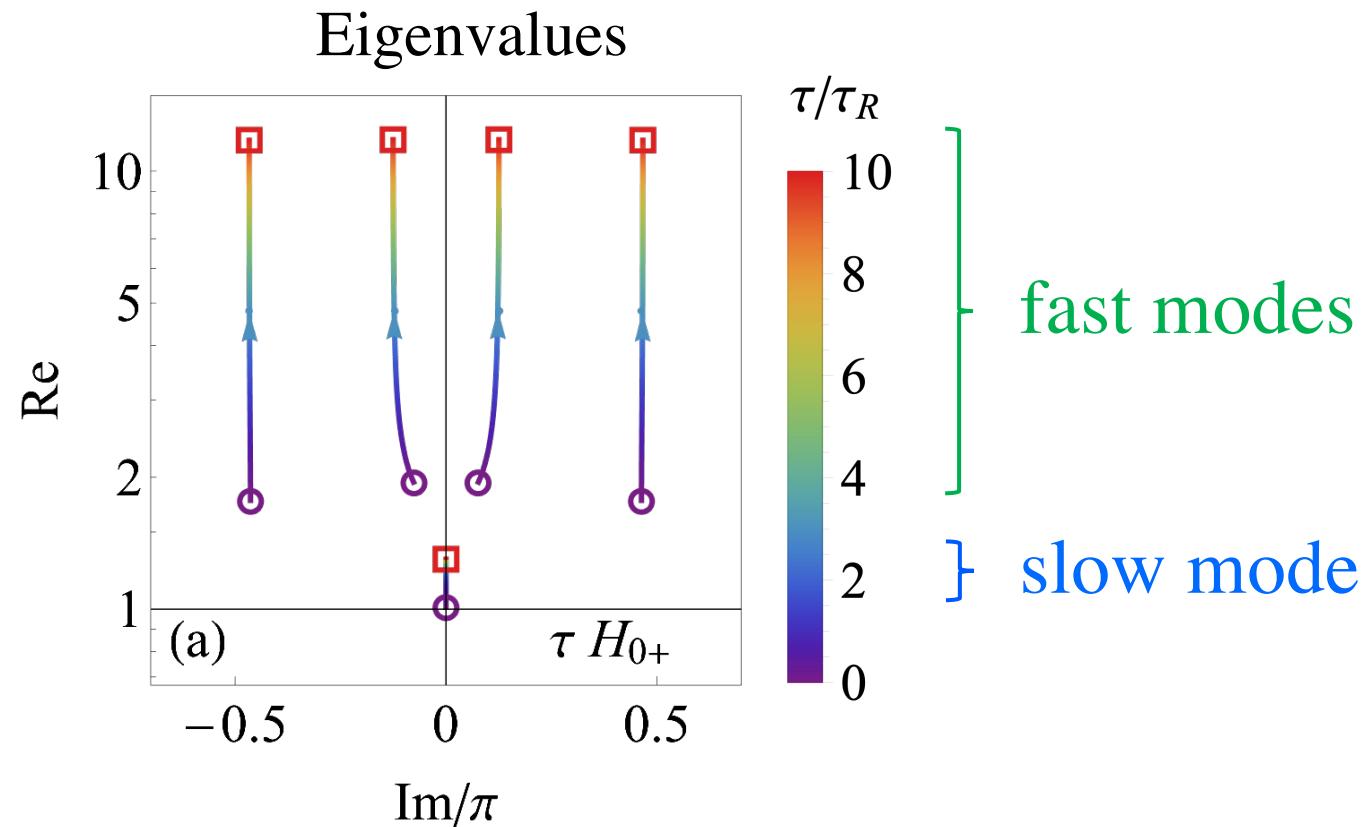
$(0, +)$

L_{00}

L_{20}

L_{40}

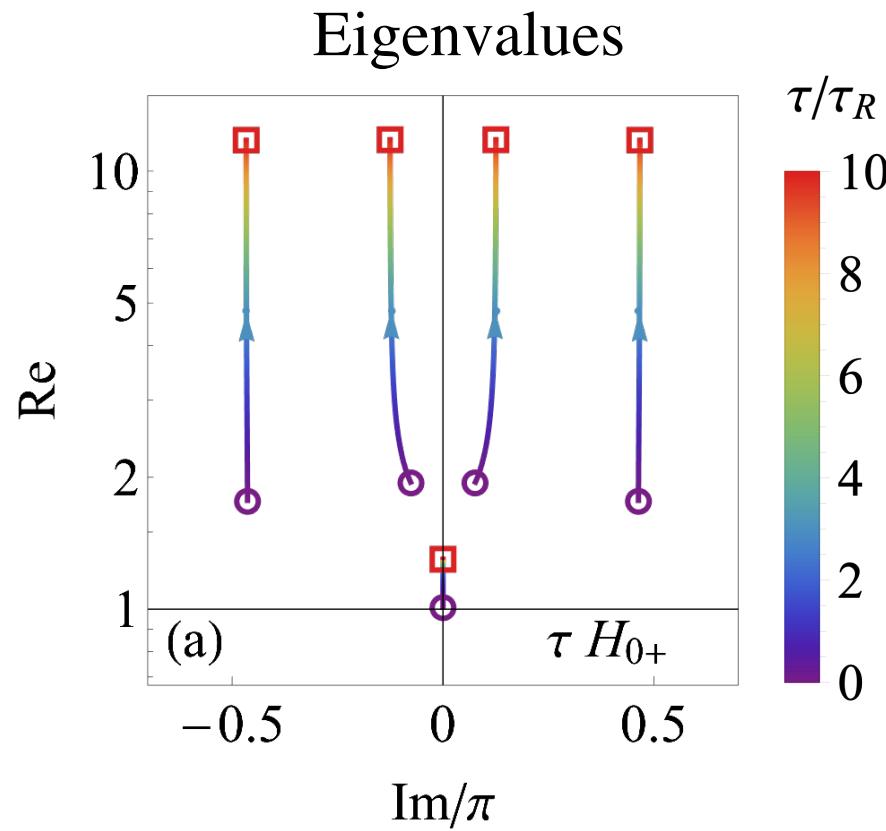
\vdots



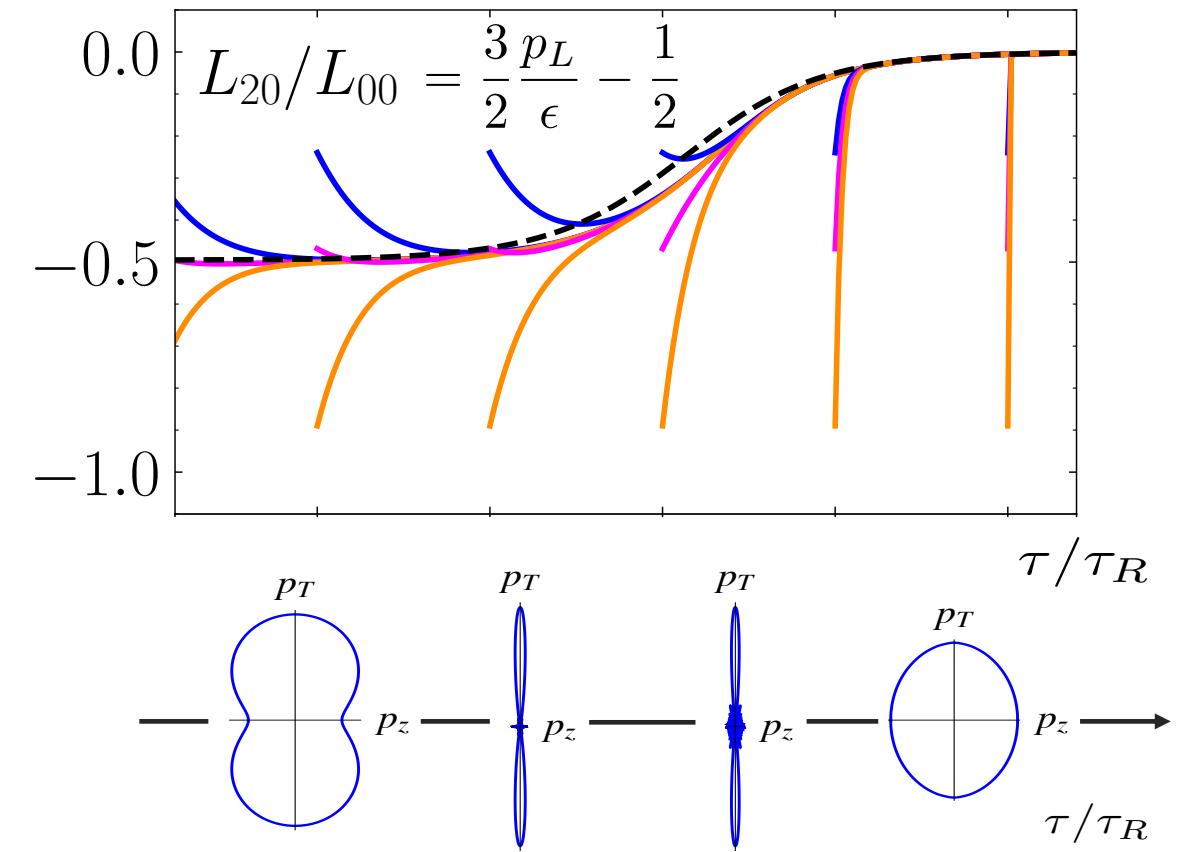
Attractor for p_L/ϵ as a consequence of adiabatic evolution

(0, +)

L_{00}
 L_{20}
 L_{40}
⋮
⋮



Extensive previous work; Heller and Spalinski; Romatschke; Almaalol, Kurkela, Strickland; Kurkela, van der Schee, Weidemann, Wu, many others



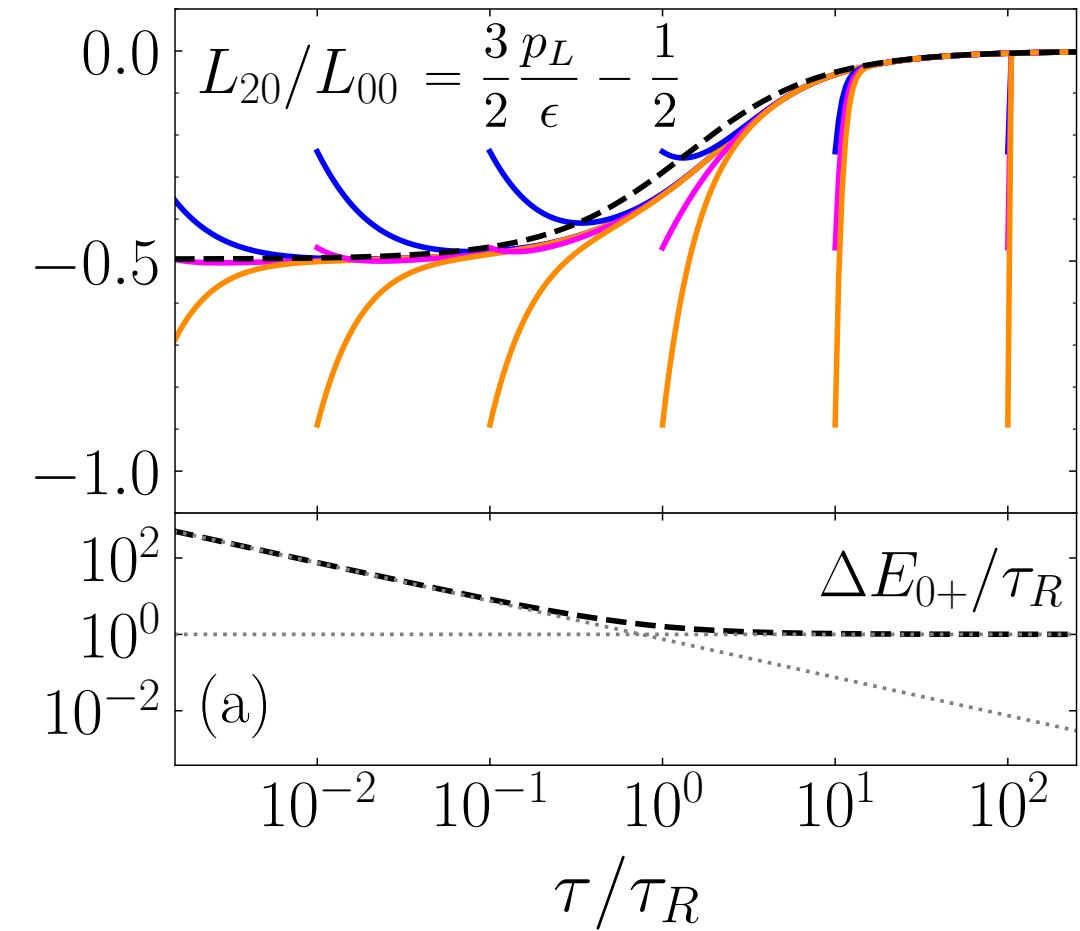
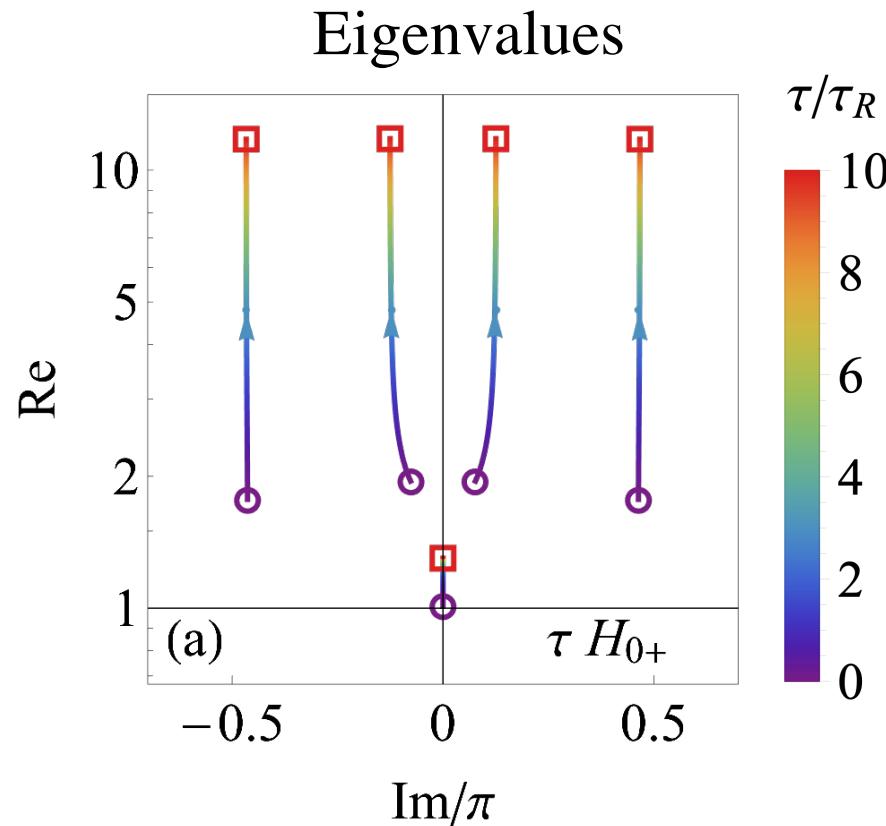
The evolution of moments is dominated by the evolution of the ground state

Attractor for p_L/ϵ as a consequence of adiabatic evolution

Extensive previous work; Heller and Spalinski; Romatschke; Almaalol, Kurkela, Strickland; Kurkela, van der Schee, Weidemann, Wu, many others

$(0, +)$

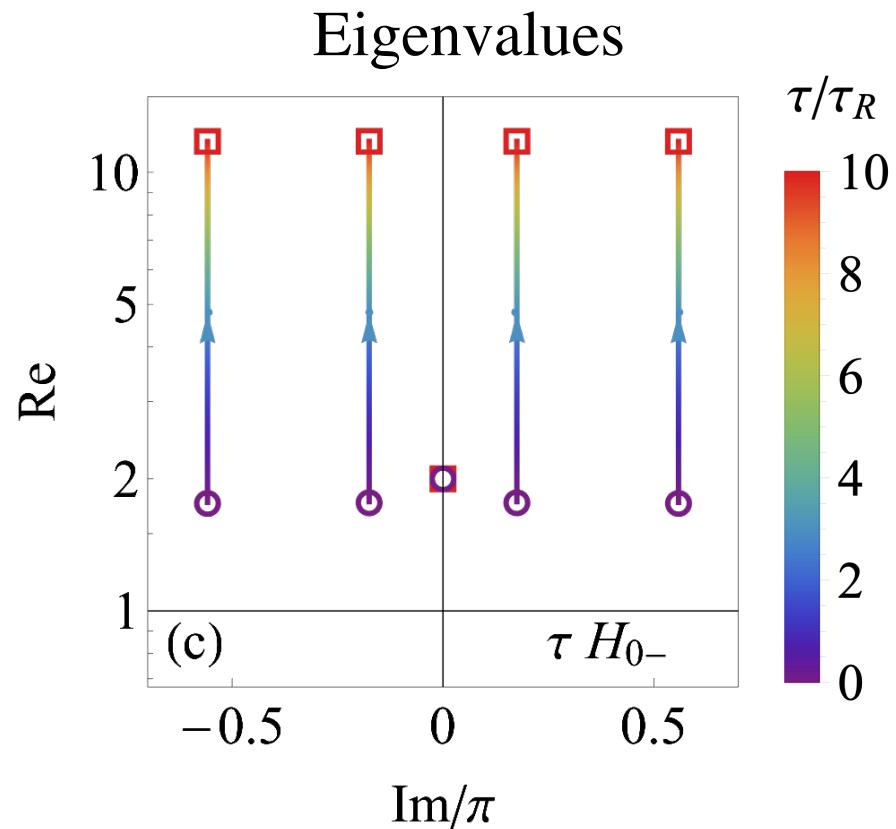
L_{00}
 L_{20}
 L_{40}
⋮
⋮



Decay of solutions to the ground state set by gap between ground and lowest excited states

Extending the concept of the attractor for other sectors of moments...

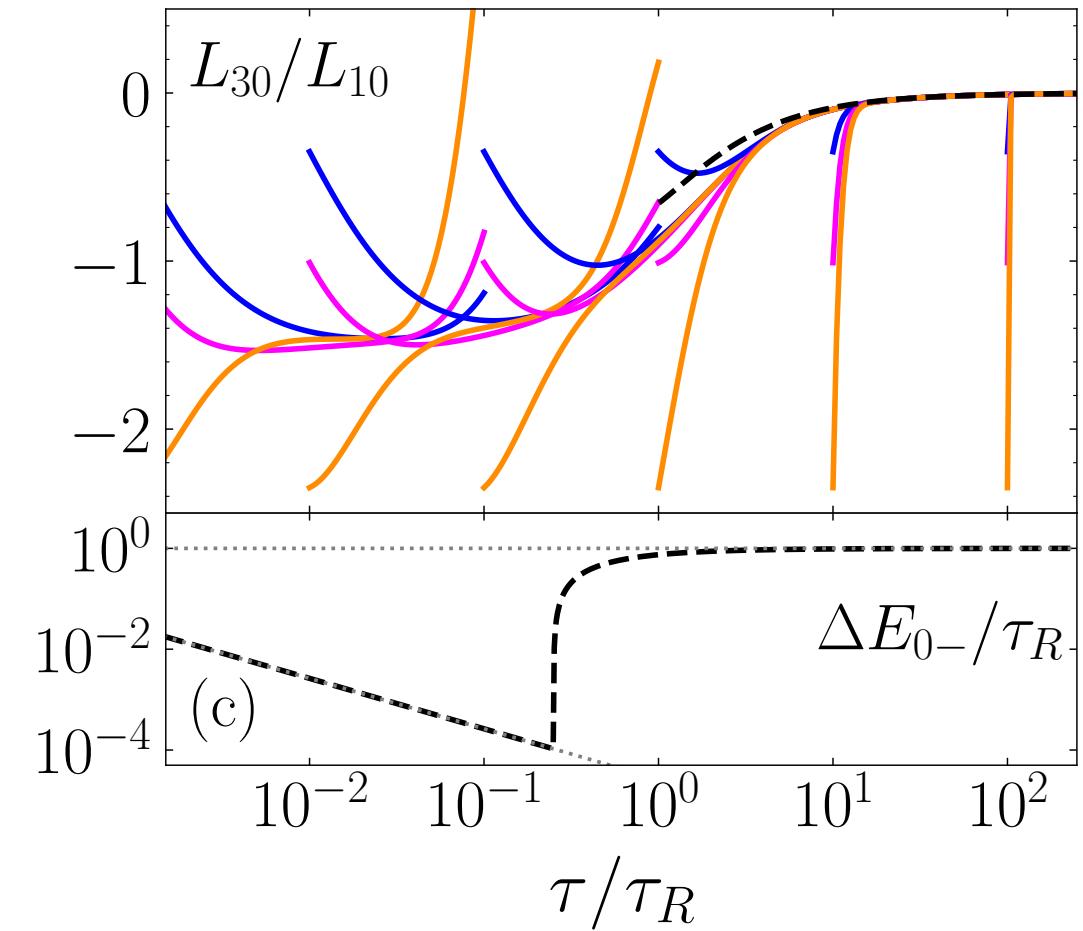
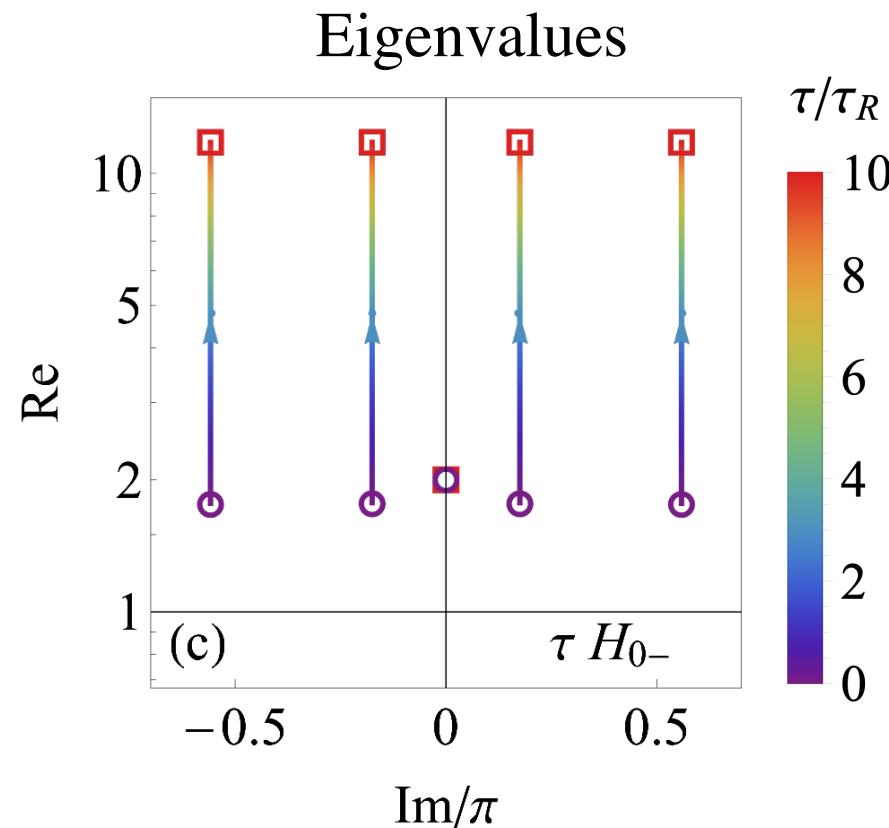
$(0, -)$



Extending the concept of the attractor for other sectors of moments...

$(0, -)$

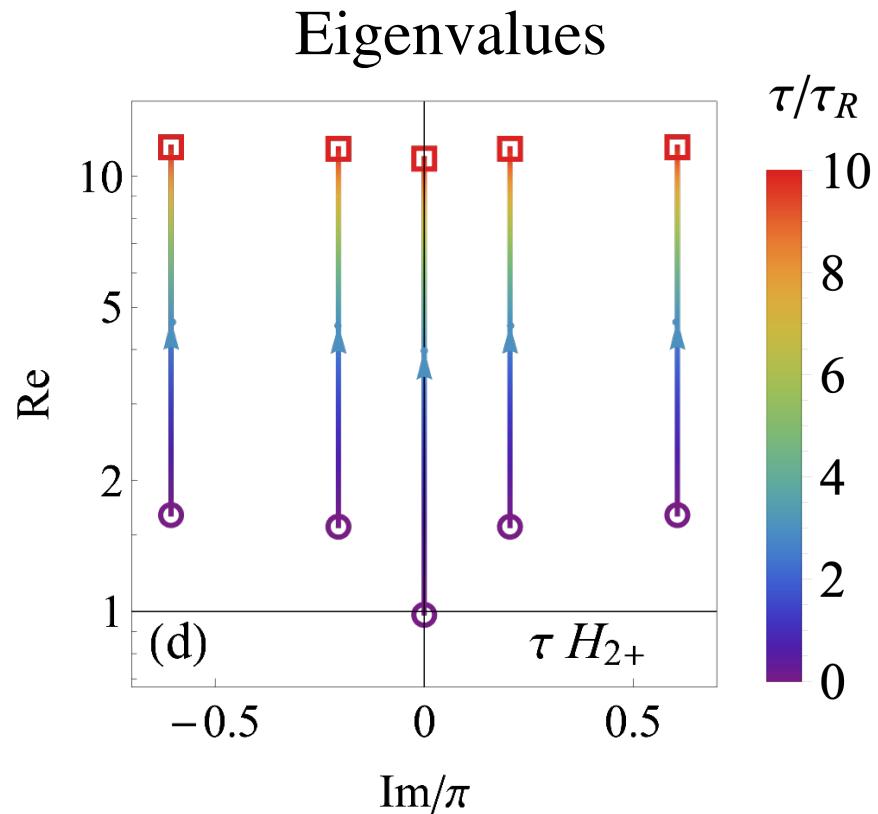
L_{10}
 L_{30}
 L_{50}
⋮
⋮



No or weak early-time attractor due to the decaying energy gap, even though there is a hydrodynamic mode

Extending the concept of the attractor for other sectors of moments...

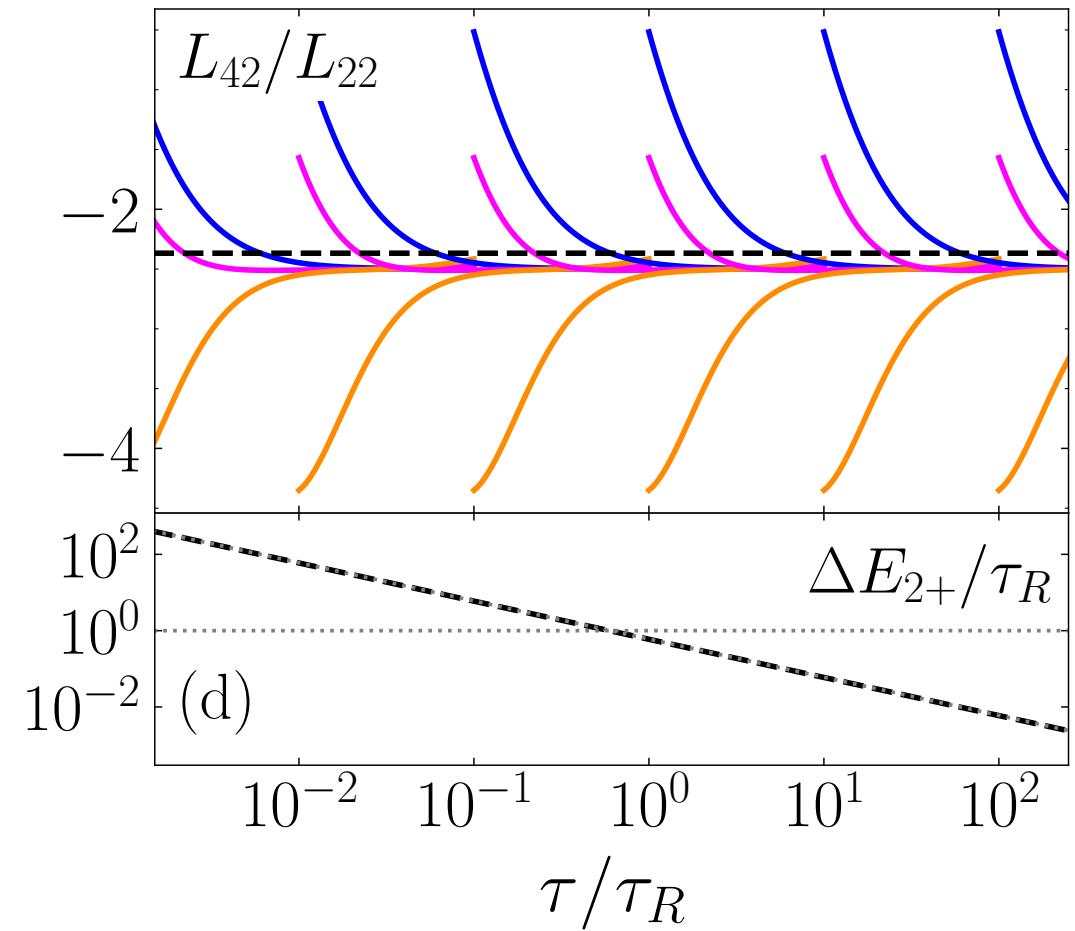
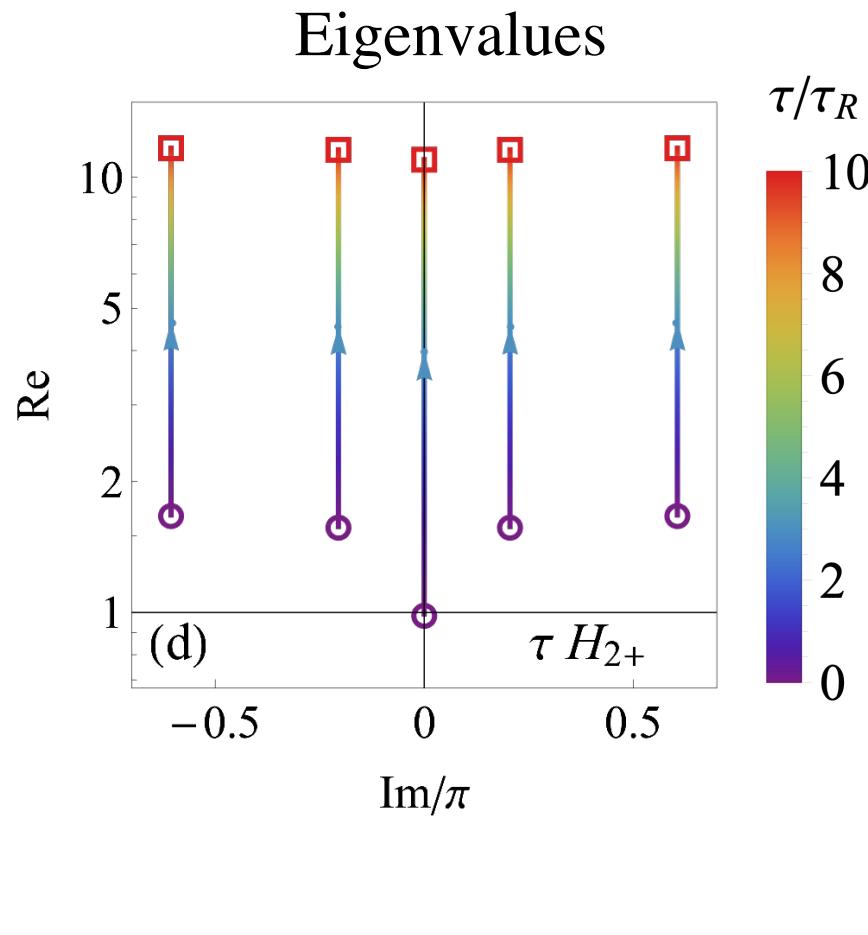
(2, +)



Extending the concept of the attractor for other sectors of moments...

$(2, +)$

L_{22}
 L_{42}
 L_{62}
 \vdots
 \vdots

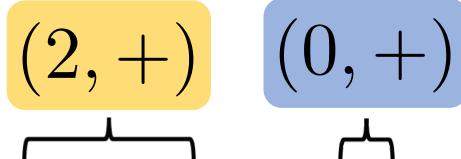


Power-law decay to the attractor for all times in sector not including a hydrodynamic mode

Implications for the decay of initial momentum anisotropy

Momentum anisotropies are related to moments in sectors with attractors

$$\mathcal{E}_{2p} \sim (T^{xx} - T^{yy}) / \epsilon$$

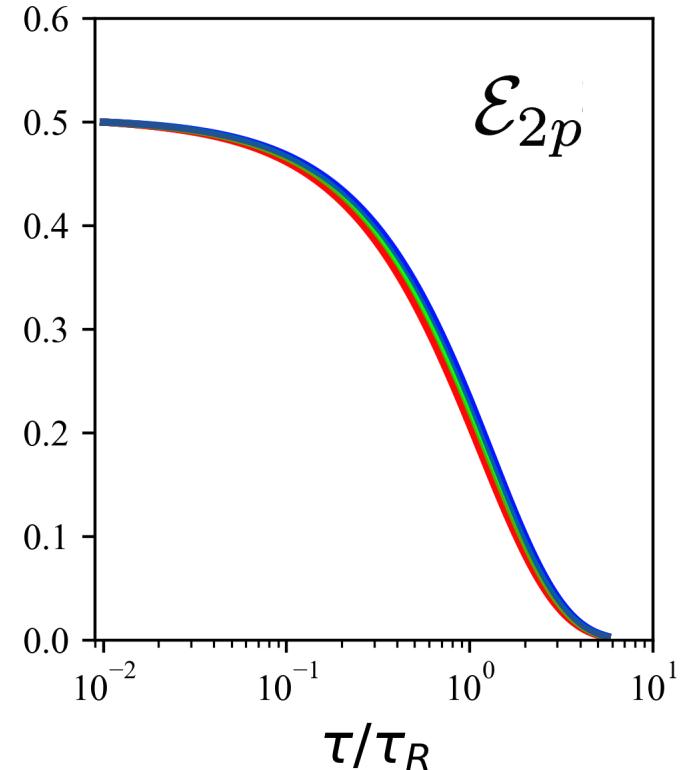


Their decay rate is set by difference in ground state eigenvalues for $(2,+)$ and $(0,+)$ sectors:

$$\partial_\tau \ln \mathcal{E}_{2p} \sim - (E_{2+}^G - E_{0+}^G)$$



zero at early times!



(different colors are different momentum weights)

Slow decay of momentum anisotropy far-from-equilibrium!

Summary: slow modes, attractors, and the momentum anisotropy

Diverse eigenspectrum for other moments of the distribution

- Early-time attractor only for sectors that have an energy gap at early times
- Slowest modes at early times do not necessarily become slowest modes at late times

Slow far-from-equilibrium evolution of momentum-space anisotropy

