# Effects of the First-Order Chiral Phase Transition in Relativistic Heavy-Ion Collisions

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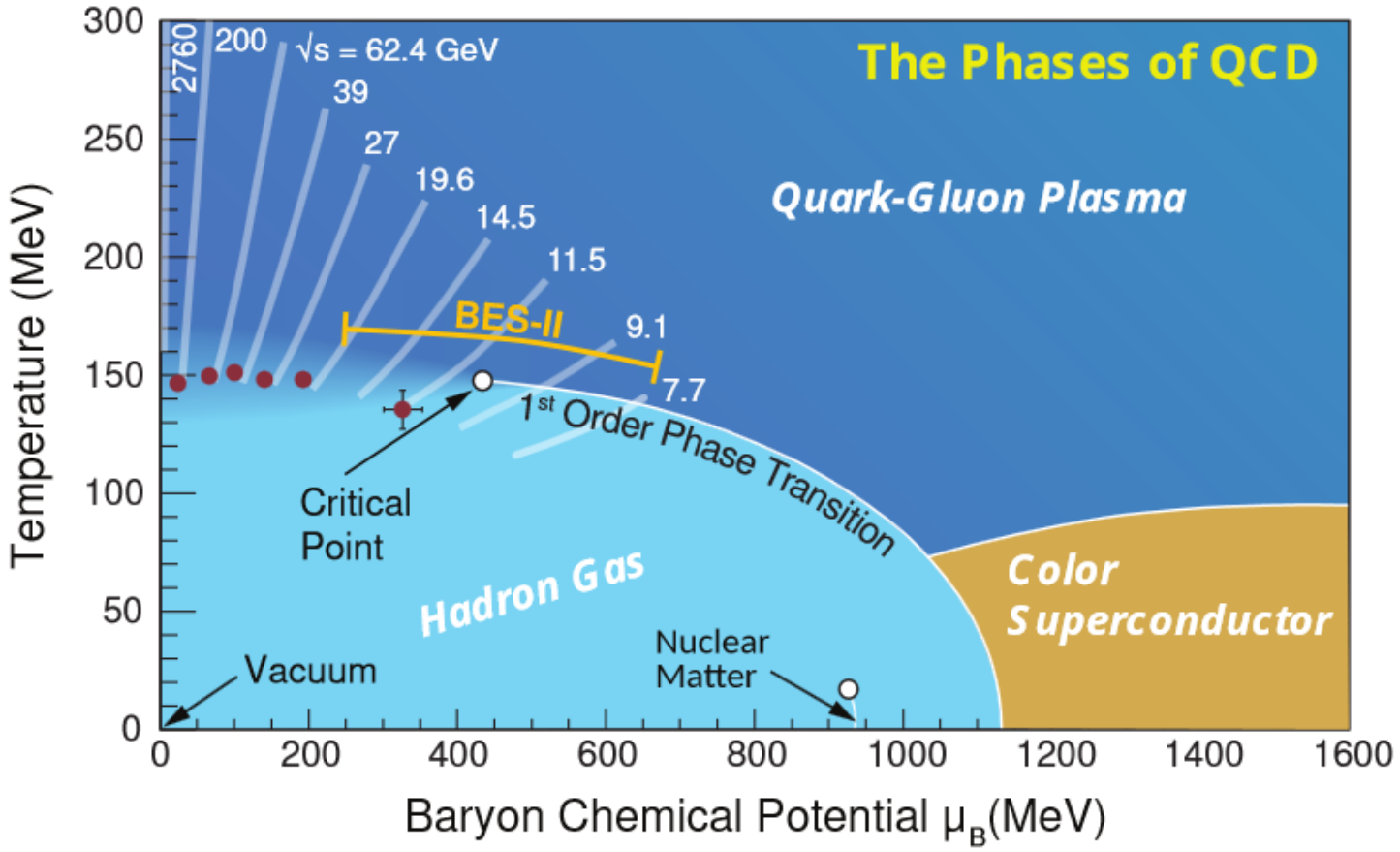
# Outline

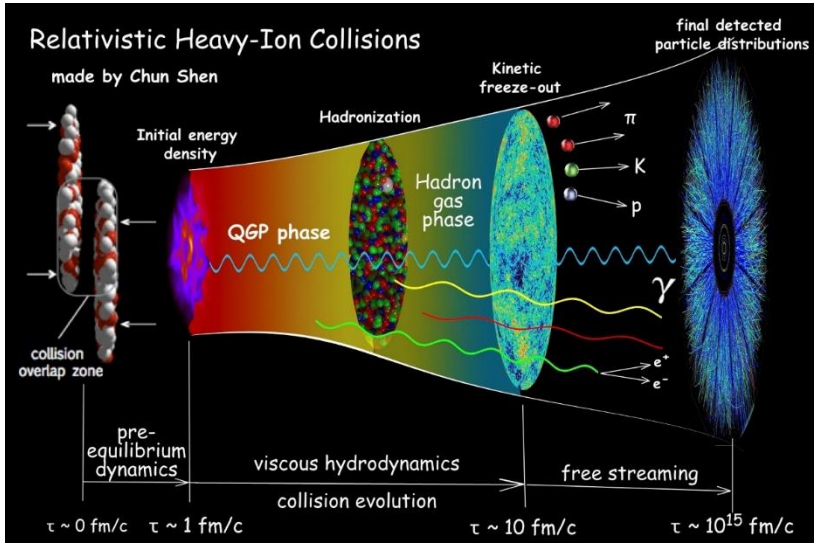
1. Motivation
2. Transport approach to the first-order chiral phase transition
3. Spinodal enhancement of $N_t N_p / N_d^2$
4. The triton 'puzzle' and its solution (RHIC and LHC)
5. Summary

Refs.: K. J. Sun, W. H. Zhou, L. W. Chen, C. M. Ko, and F. Li, R. Wang, and J. Xu, arXiv:2205.11010(2022)
K. J. Sun, R. Wang, C. M. Ko, Y. G. Ma, and C. Shen, 2207.12532(2022)

# 1. Motivation

# Hunt for the QCD critical point (1)




**Neutron star merger**

From Wiki.

**Critical Point: long-range correlation**

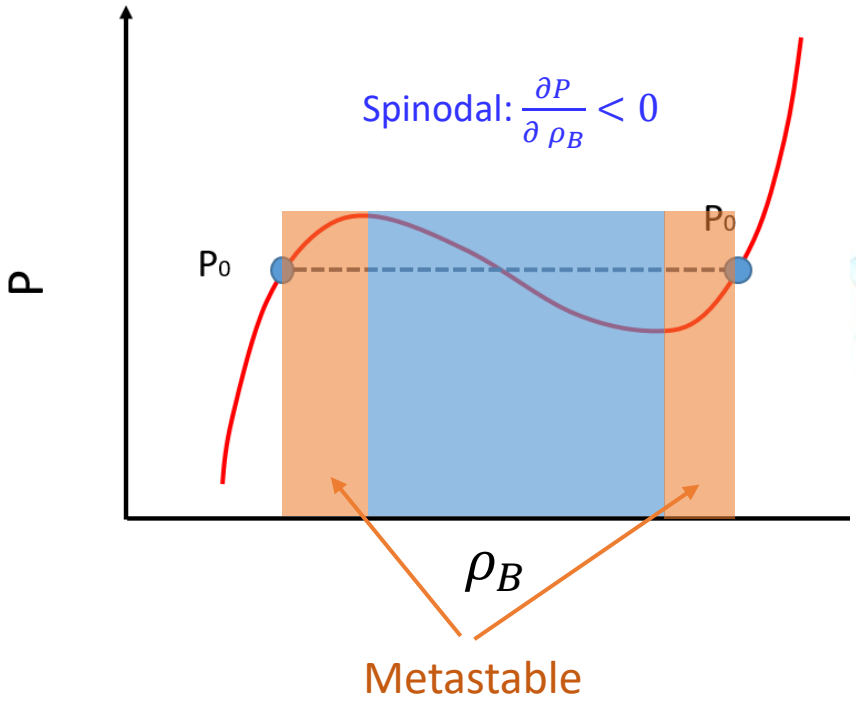
**First-order Phase Transition: Spinodal instability**

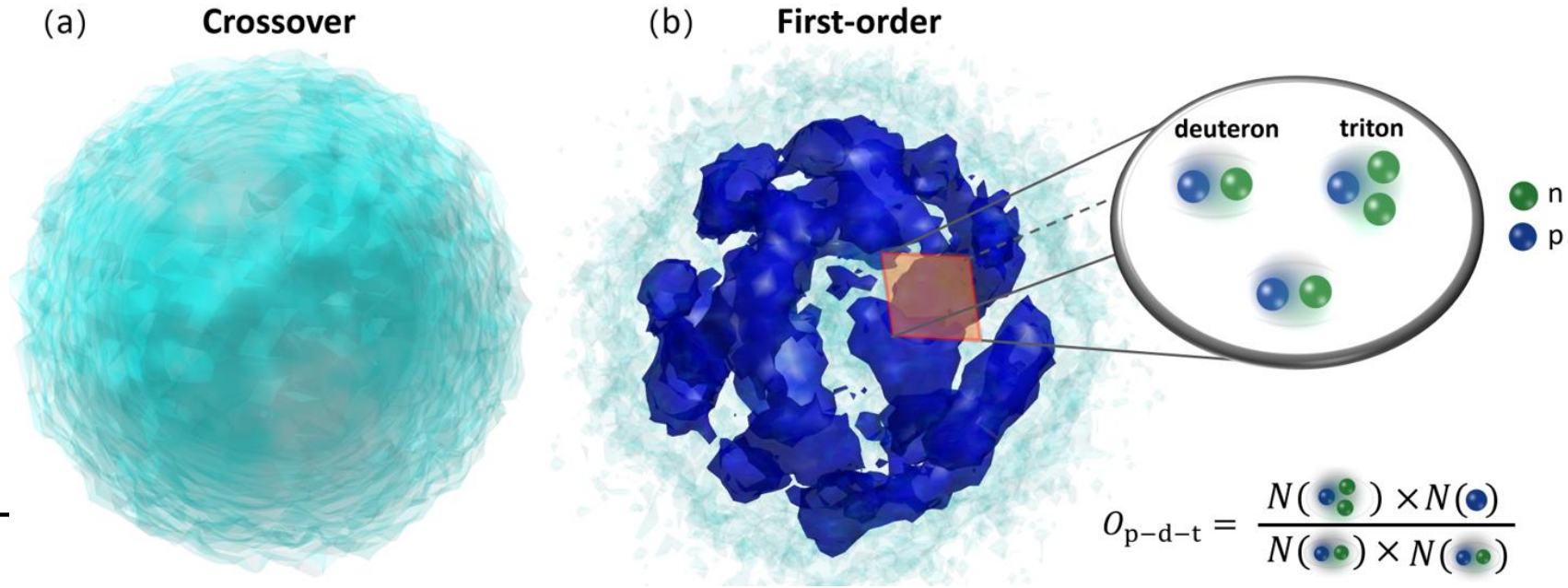
X. Luo and N. Xu, Nucl. Sci. Tech. 28, 112 (2017) A. Bzdak et al., Phys. Rept. 853, 1 (2020);
W. J. Fu, J. M. Pawlowski, F. Renneke, Phys. Rev. D101, 054032 (2020); LIGO & VIRGO, Phys. Rev. Lett. 119, 161101 (2017)

# Idea: Probing QCD phase transition with light nuclei(2)

C. M. Ko, NST 34, 80 (2023).

[Poster by Koichi Murase]



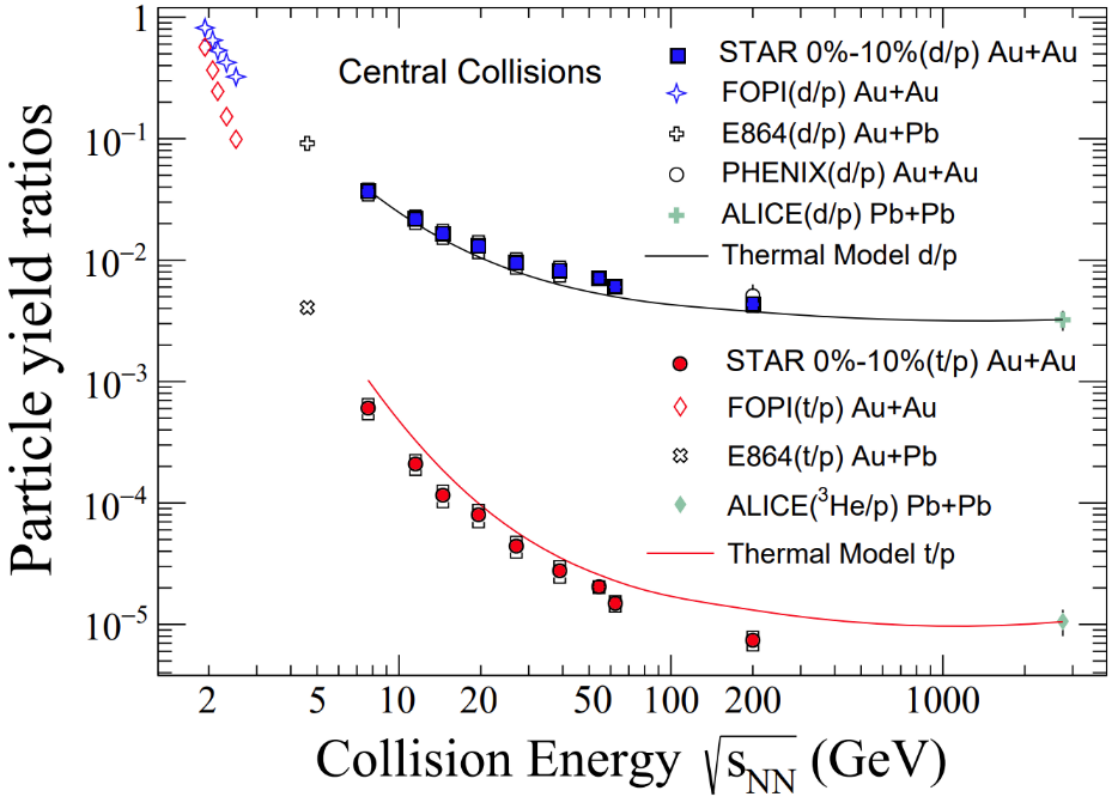

**Large density fluctuations lead to enhancements of** $N_t N_p / N_d^2 \approx \frac{1}{2\sqrt{3}}\left[1 + \Delta\rho_n + \frac{\lambda}{\sigma} G\left(\frac{\xi}{\sigma}\right)\right]$

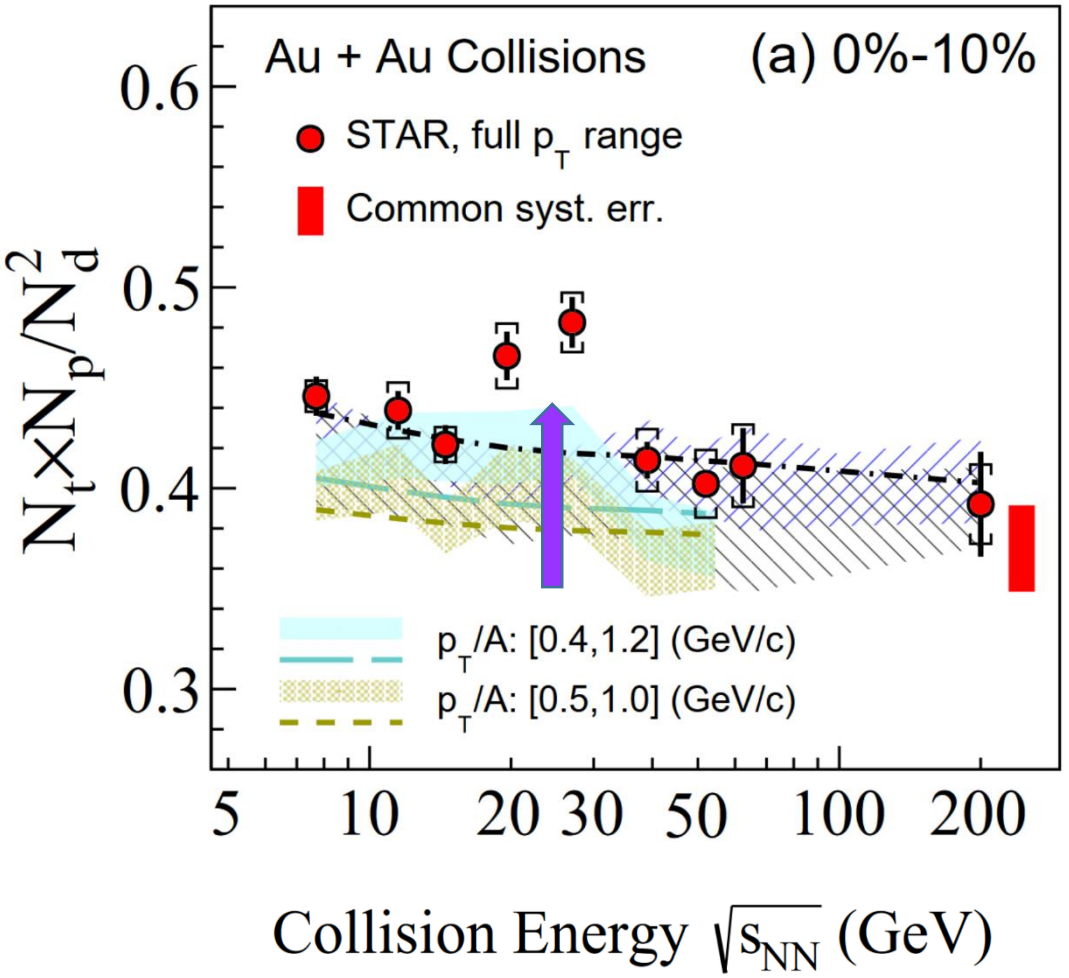
K. J. Sun, L. W. Chen, C. M. Ko, and Z. Xu, Phys. Lett. B 774, 103 (2017); K. J. Sun, C. M. Ko, and F. Li, PLB 816, 136258 (2021);
E. Shuryak, J.M.Torres-Rincon et al., PRC 100, 024903(2019); S. Wu et al., PRC 106,034905(2022).

# Recent STAR measurements (3)

STAR: PRL130, 202302 (2023)

[Poster by Yixuan Jin for STAR]



# 2. Transport approach to the first-order chiral phase transition

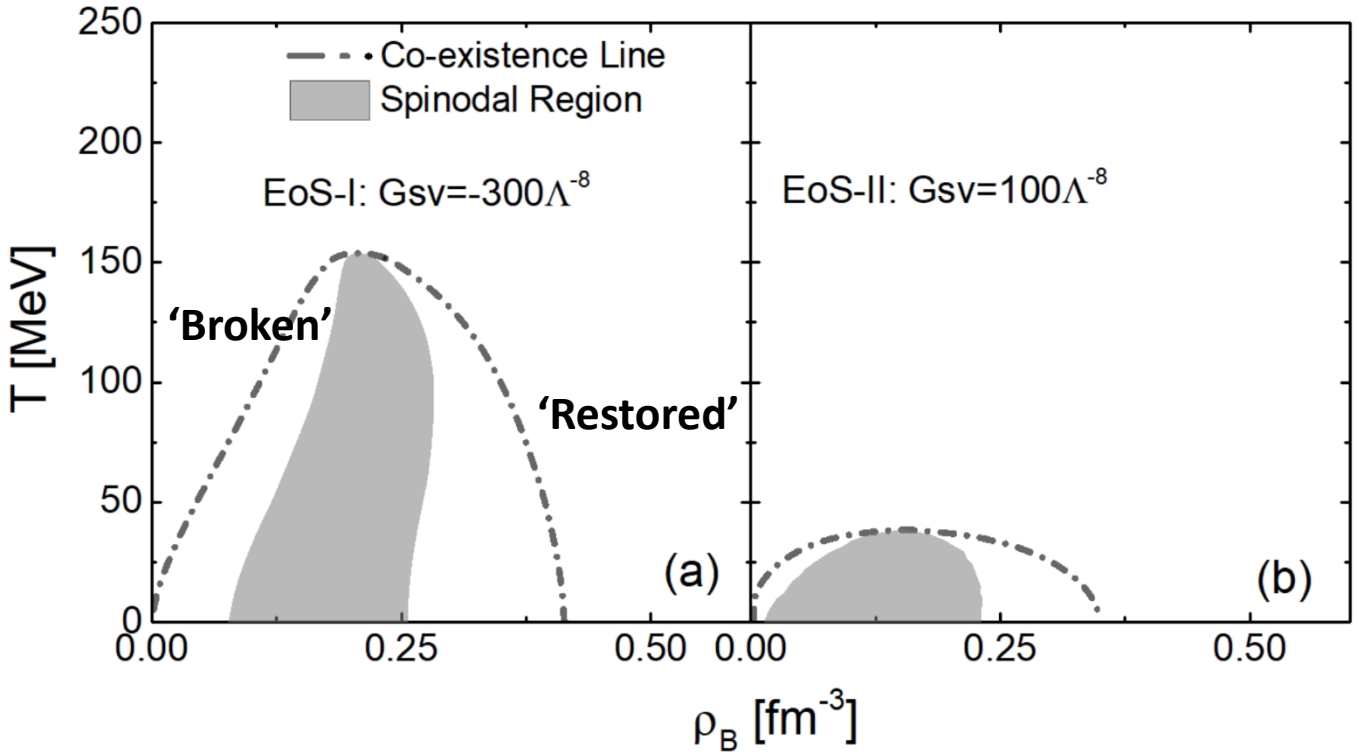
# Equation of state (extended NJL model) (4)

The eNJL provides a flexible equation of state (EoS) . The critical temperature can be easily changed by varying the strength of the scalar-vector interaction without affecting the vacuum properties.

Lagrangian density for an extended Nambu-Jona-Lasinio (eNJL) model

$$\begin{aligned}\mathcal{L} &= \bar{\psi}(i\gamma^\mu\partial_\mu - \hat{m})\psi + G_S\sum_{a=0}^{}[(\bar{\psi}\lambda^a\psi)^2 + (\bar{\psi}i\gamma_5\lambda^a\psi)^2] \\ &- K\{[\det[\bar{\psi}(1+\gamma_5)\psi] + \det[\bar{\psi}(1-\gamma_5)\psi]\} \\ &+ G_{SV}\left\{\sum_{a=1}^{3}[(\bar{\psi}\lambda^a\psi)^2 + (\bar{\psi}i\gamma_5\lambda^a\psi)^2]\right\} \\ &\times\left\{\sum_{a=1}^{3}[(\bar{\psi}\gamma^\mu\lambda^a\psi)^2 + (\bar{\psi}\gamma_5\gamma^\mu\lambda^a\psi)^2]\right\},\end{aligned}$$

| | | | |
|---|---|---|---|
| $\Lambda$ [MeV] | 602.3 | $M_{u,d}$ [MeV] | 367.7 |
| $G\Lambda^2$ | 1.835 | $M_s$ [MeV] | 549.5 |
| $K\Lambda^5$ | 12.36 | $(\langle\bar{u}u\rangle)^{1/3}$ [MeV] | -241.9 |
| $m_{u,d}$ [MeV] | 5.5 | $(\langle\bar{s}s\rangle)^{1/3}$ [MeV] | -257.7 |
| $m_s$ [MeV] | 140.7 | | |




M. Buballa, Phys. Rept. 407, 205 (2005)

K. J. Sun, C. M. Ko, S. Cao, and F. Li., Phys. Rev. D 103, 014006 (2021)

# Box simulation (5)

**[Spinodal instability: see talk by Roman Poberezhnyuk]**

*Test –particle method:* J. Xu, arXiv:1904.00131 (2019)

Effective mass:

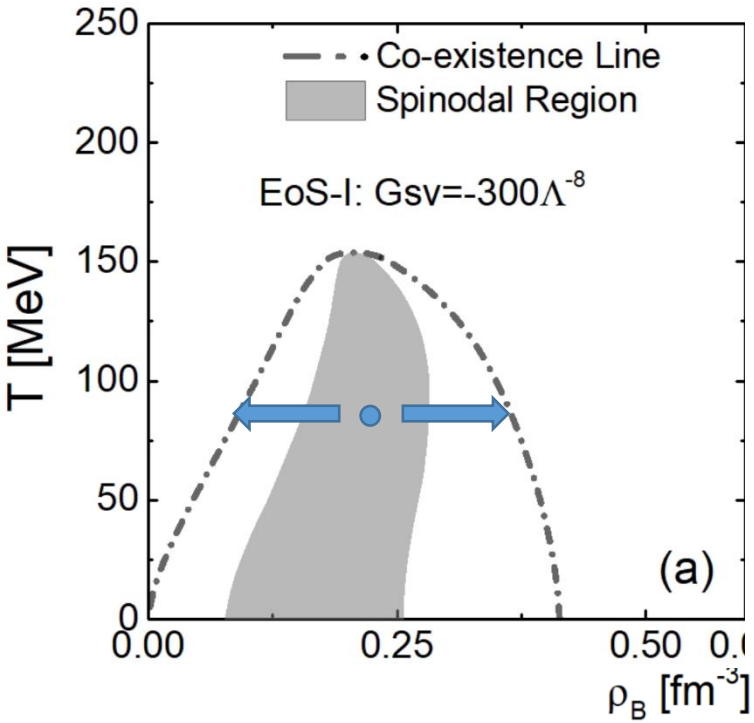
$$M_u = m_u - 4G_S\phi_u + 2K\phi_d\phi_s - 2G_{SV}(\rho_u+\rho_d)^2(\phi_u+\phi_d),$$
$$M_d = m_d - 4G_S\phi_d + 2K\phi_u\phi_s - 2G_{SV}(\rho_u+\rho_d)^2(\phi_u+\phi_d),$$
$$M_s = m_s - 4G_S\phi_s + 2K\phi_u\phi_d$$

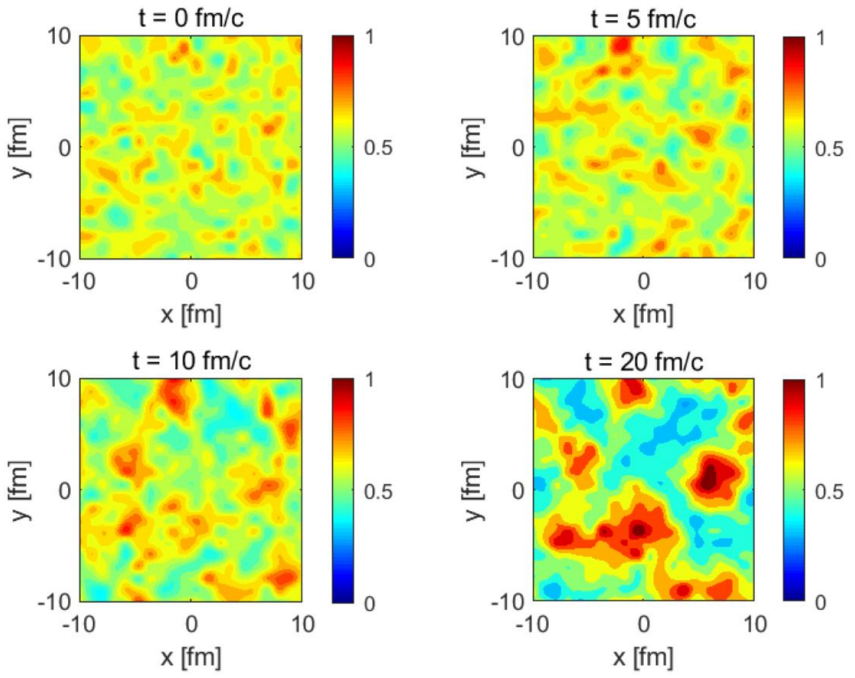
$$\phi_i = -2N_c\int_0^\Lambda \frac{\mathrm{d}^3p}{(2\pi\hbar)^3}\frac{M_i}{E_i}(1-f_i-\bar{f}_i)$$

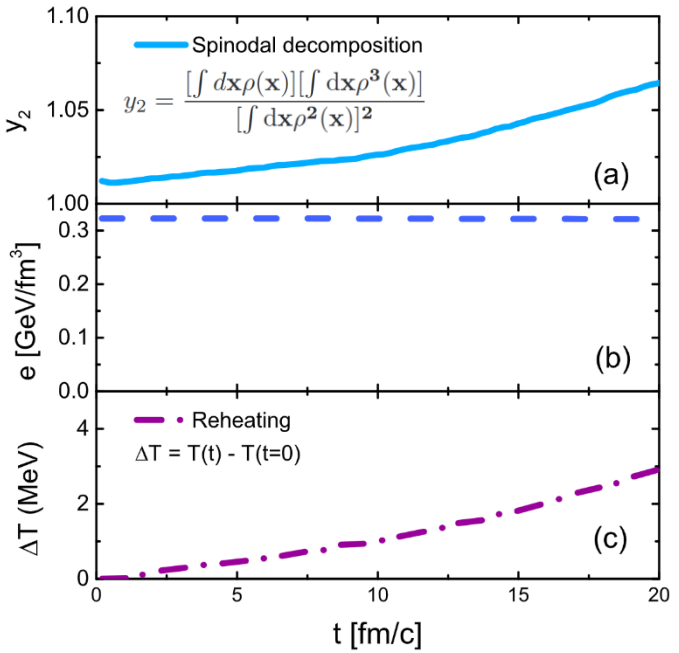
$$\rho_i = 2N_c\int_0^\Lambda \frac{\mathrm{d}^3p}{(2\pi\hbar)^3}(f_i-\bar{f}_i)$$

$$\frac{d\mathbf{r}}{dt} = \mathbf{v},$$
$$\frac{d\mathbf{p}}{dt} = -\frac{M}{E^*}\nabla_r M \pm \mathbf{E} \pm \mathbf{v}\times\mathbf{B}$$

Strong EM fields




Small irregularities will grow exponentially and soon the evolution becomes 'chaotic'.

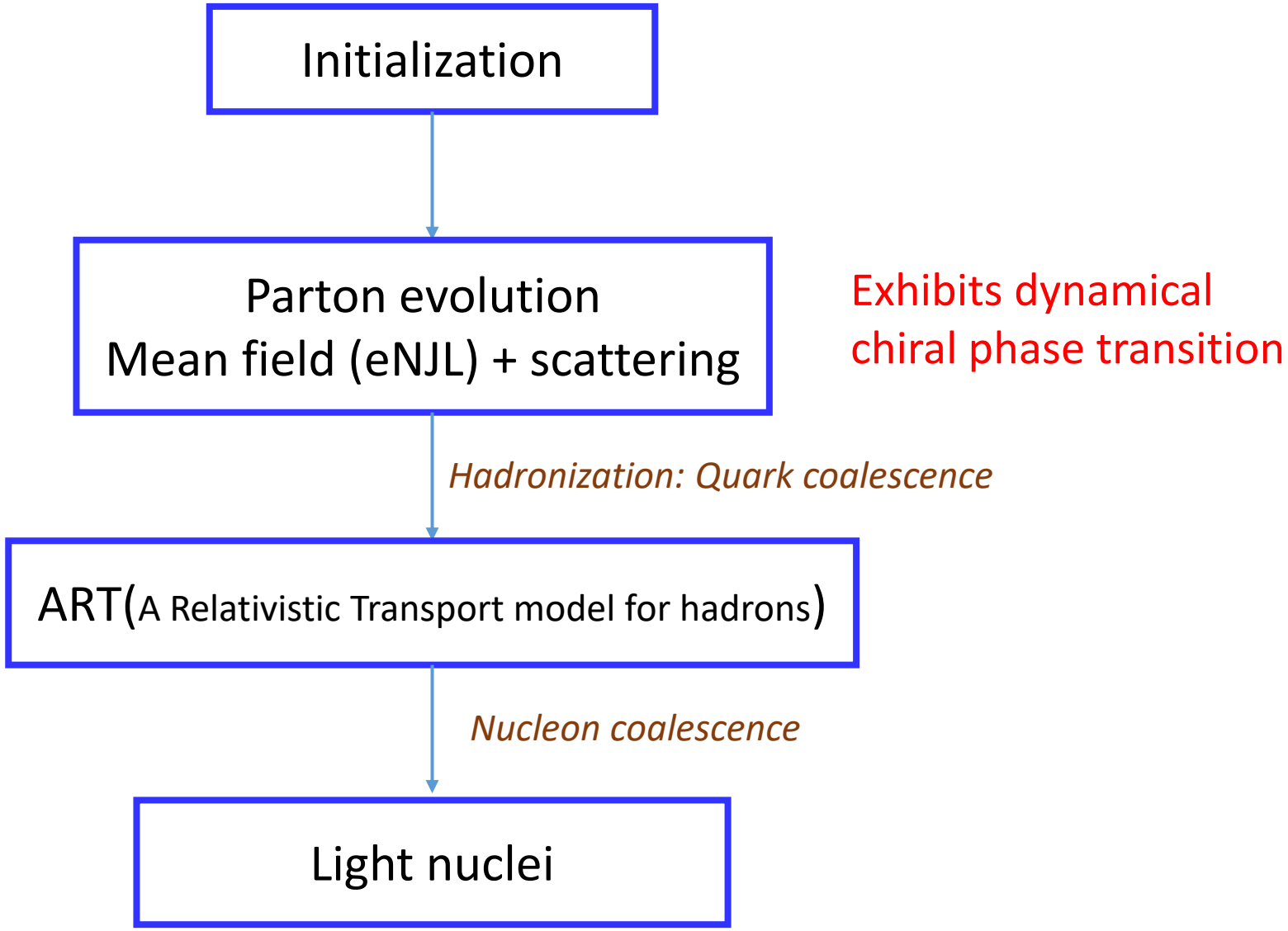
*For small k,* $\Gamma_k = -i\,vk$,

$v^2 = \frac{n}{\varepsilon+p}(\frac{\partial p}{\partial n})_S$ or $v^2 = \frac{n}{\varepsilon+p}(\frac{\partial p}{\partial n})_T < 0$

M. Buballa, Phys. Rept. 407, 205 (2005)

K. J. Sun, C. M. Ko, S. Cao, and F. Li., Phys. Rev. D 103, 014006 (2021)

# Relativistic heavy-ion collisions (6)

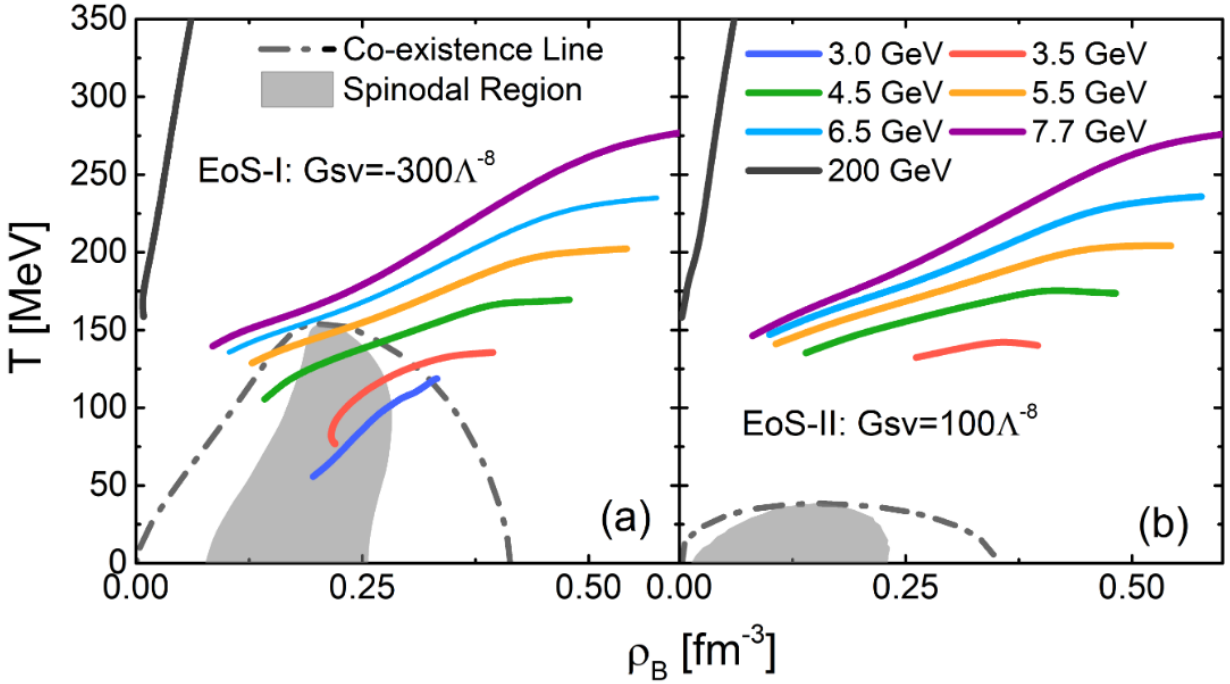


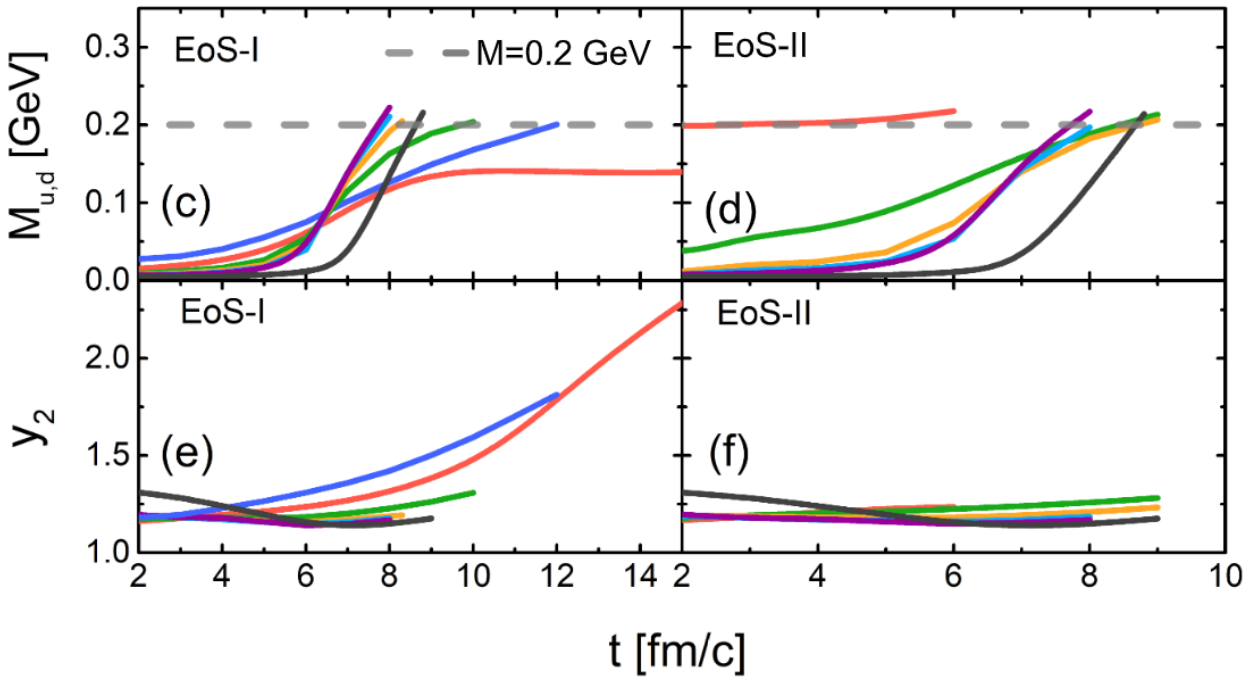
# Trajectories in the phase diagram



Phase trajectories of central cells in the phase diagram

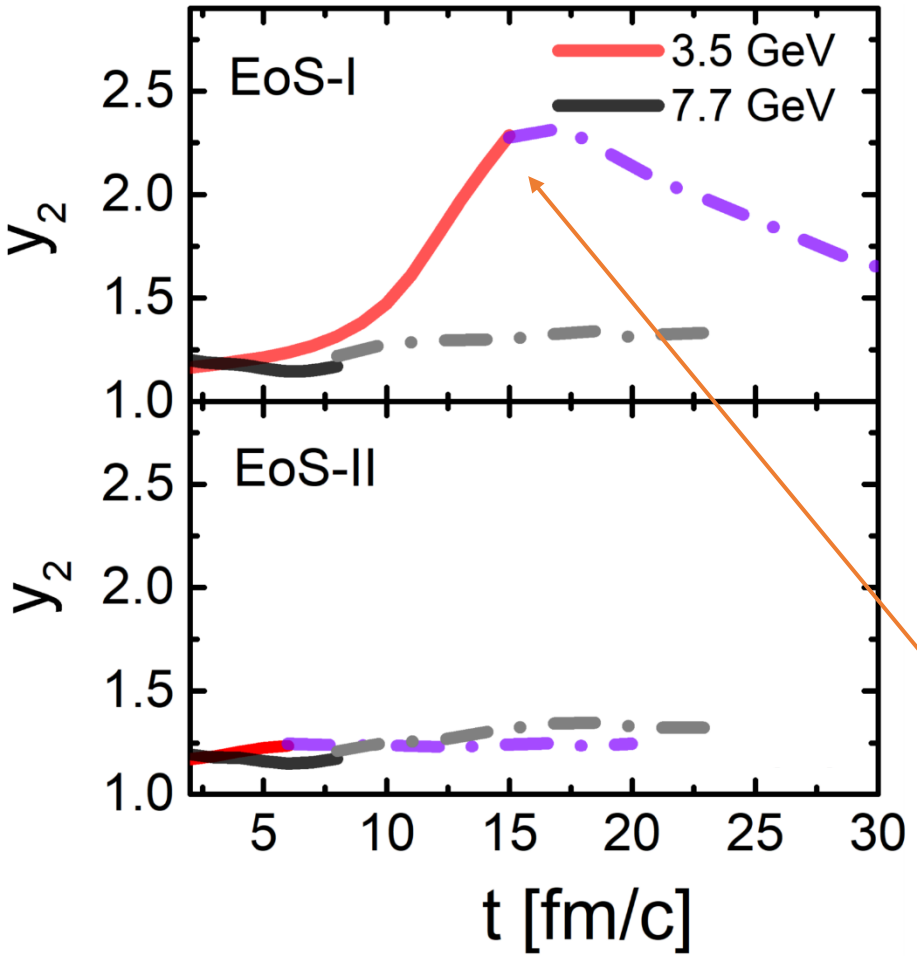
$$\overline{\rho^N} = \frac{\int d\mathbf{x}\rho^{(N+1)}(\mathbf{x})}{\int d\mathbf{x}\rho(\mathbf{x})} \qquad y_2 = \frac{[\int d\mathbf{x}\rho(\mathbf{x})][\int d\mathbf{x}\rho^3(\mathbf{x})]}{[\int d\mathbf{x}\rho^2(\mathbf{x})]^2}$$



# Survival of density fluctuation in an expanding fireball (8)

**Off-equilibrium effects**




Density moment:

$$\overline{\rho^N} = \frac{\int d\mathbf{x}\rho^{(N+1)}(\mathbf{x})}{\int d\mathbf{x}\rho(\mathbf{x})}$$

$$y_2 = \frac{[\int d\mathbf{x}\rho(\mathbf{x})][\int d\mathbf{x}\rho^{\mathbf{3}}(\mathbf{x})]}{[\int d\mathbf{x}\rho^{\mathbf{2}}(\mathbf{x})]^{\mathbf{2}}}$$

If the expansion is self-similar or scale invariant

$$\rho(\lambda(t)x, t) = \alpha(t)\rho(x, t_h)$$

then $y_2(t) = y_2(t_h)$, i.e., remains a constant

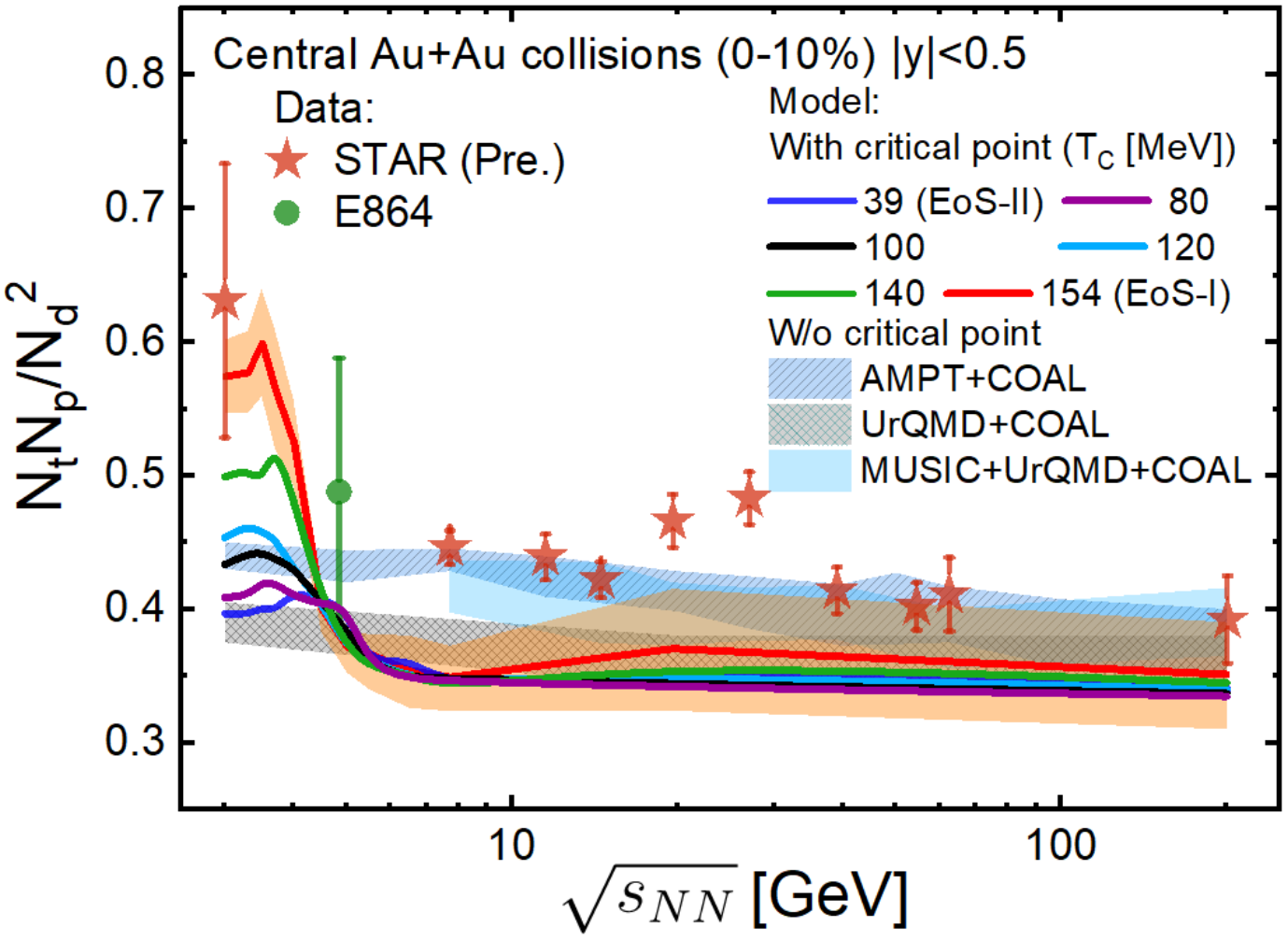
K.J.Sun et al., Eur.Phys.J.A 57 (2021) 11, 313

'Memory effects': Large density inhomogeneity survives to kinetic freezeout

# 3. Spinodal enhancement of $N_t N_p / N_d^2$

Ref.: K. J. Sun, W. H. Zhou, L. W. Chen, C. M. Ko, and F. Li, R. Wang, and J. Xu, arXiv:2205.11010(2022)

# Collision energy dependence (9)

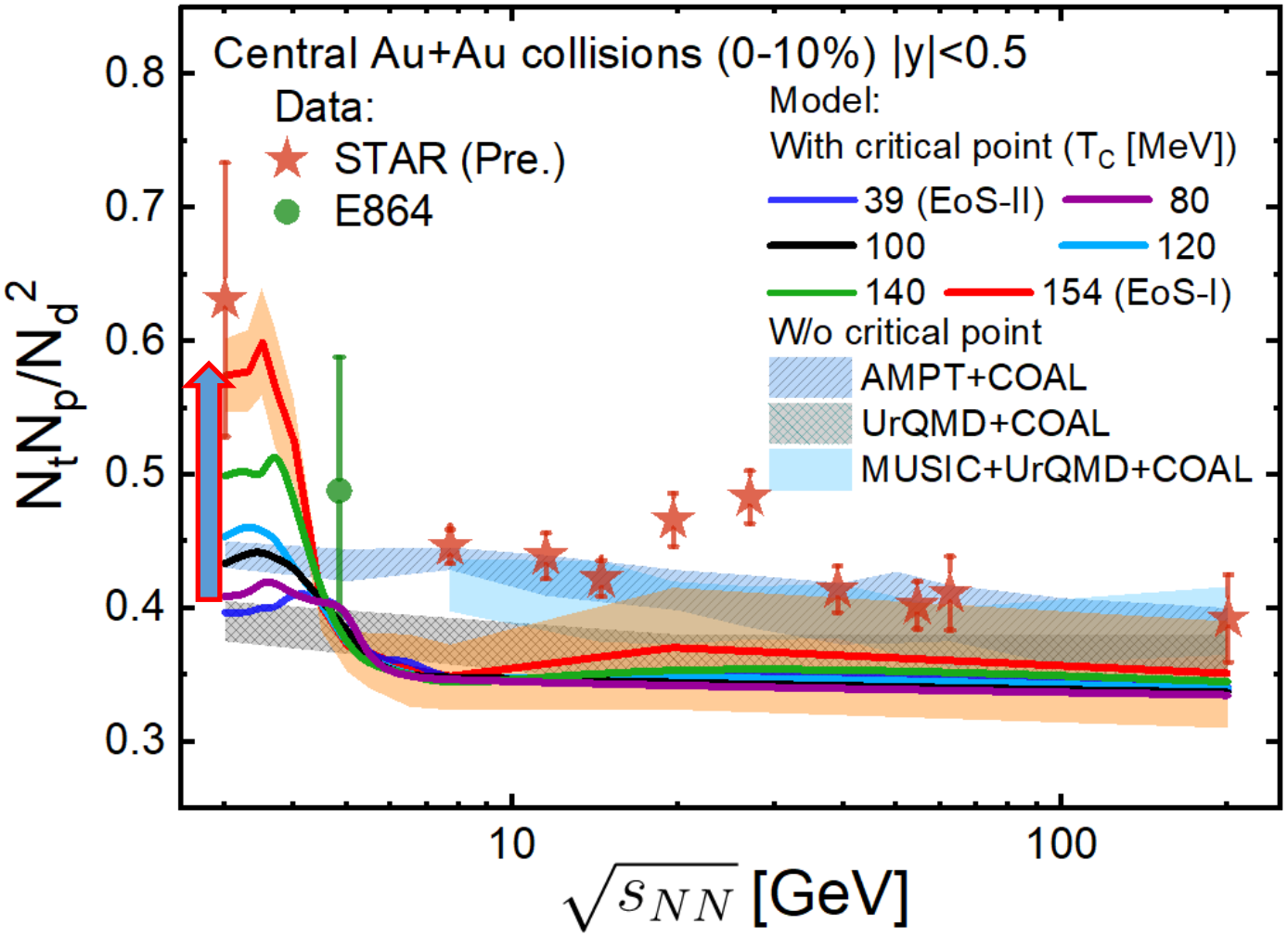



1. Without a critical point:
   The energy dependence of $tp/d^2$ is almost flat.

2. With a first-order phase transition:
   The spinodal instability induced enhancement of $tp/d^2$ during the first-order phase transition increases as increasing the critical temperature.

STAR, PRL130, 202302 (2023)
Hui Liu (STAR), QM2022
T. A. Armstrong et al. (E864), Phys. Rev. C 61, 064908 (2000).

# Collision energy dependence (10)

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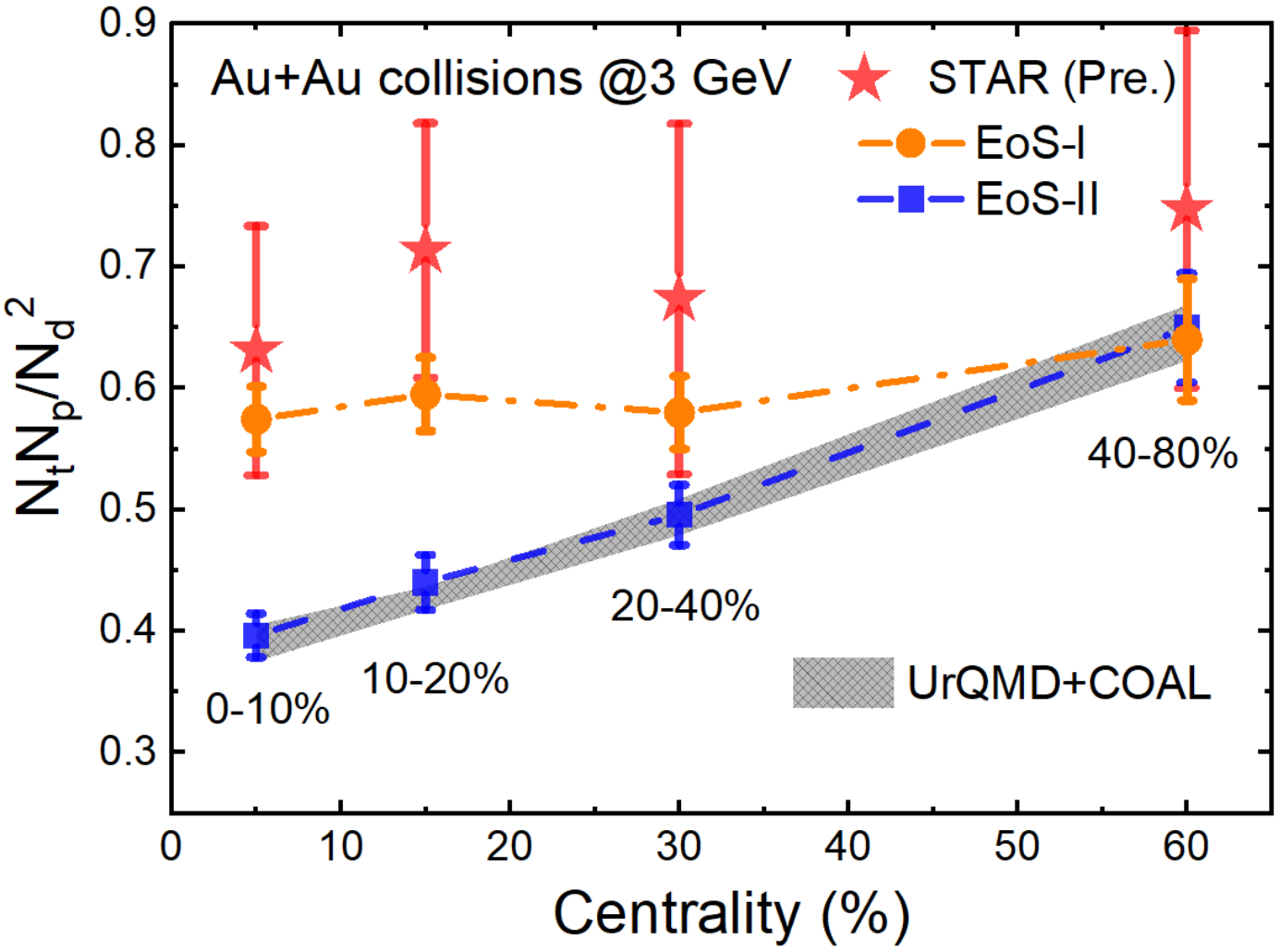
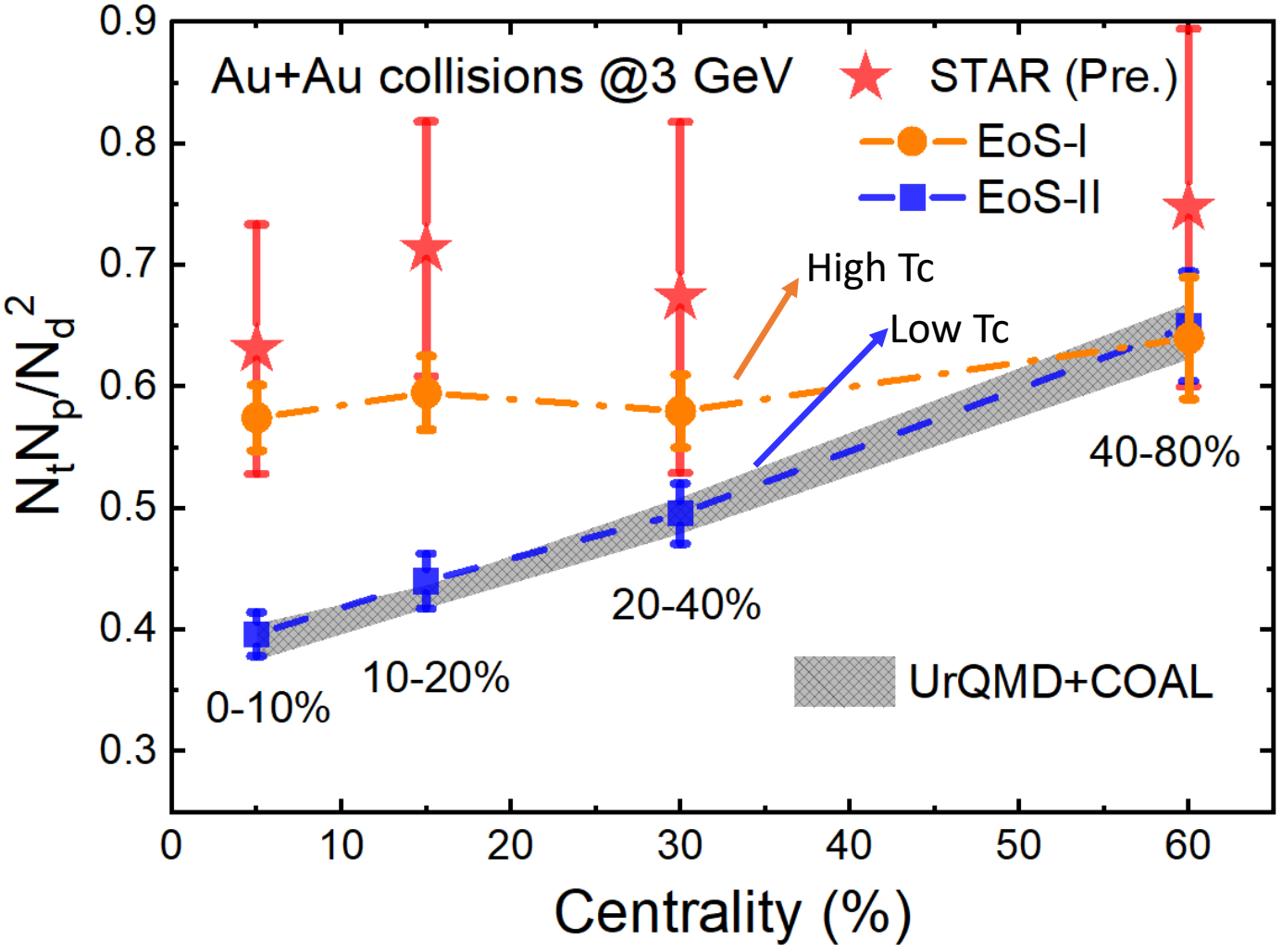
STAR, PRL130, 202302 (2023)
Hui Liu (STAR), QM2022
T. A. Armstrong et al. (E864), Phys. Rev. C 61, 064908 (2000).

# Centrality dependence





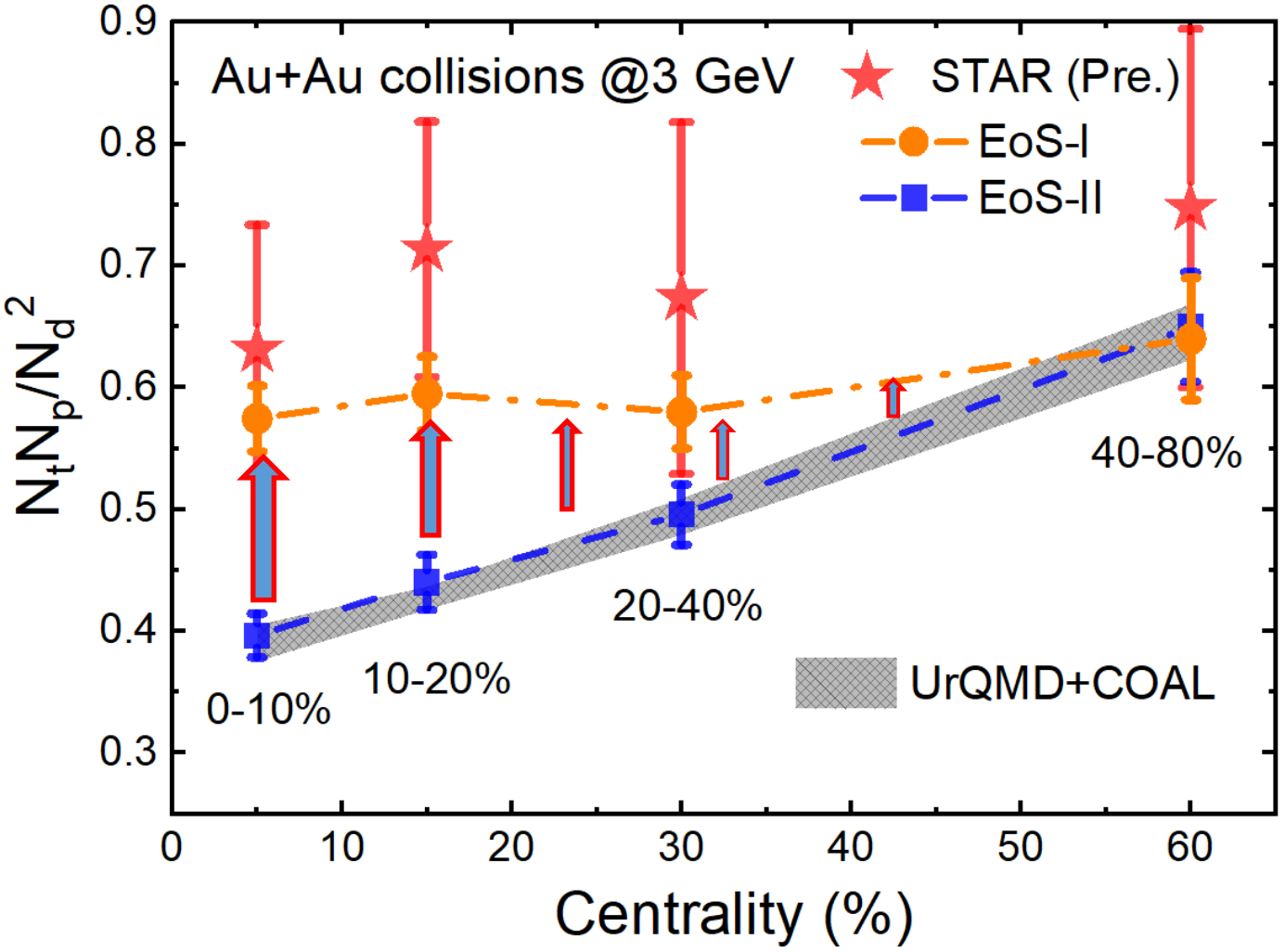
# Centrality dependence (12)




The spinodal enhancement of $tp/d^2$ subsides with increasing collision centrality because of smaller fireball lifetime in more peripheral collisions.

# Centrality dependence






The spinodal enhancement of $tp/d^2$ subsides with increasing collision centrality because of smaller fireball lifetime in more peripheral collisions.

The slope with EoS-I is 5 times smaller

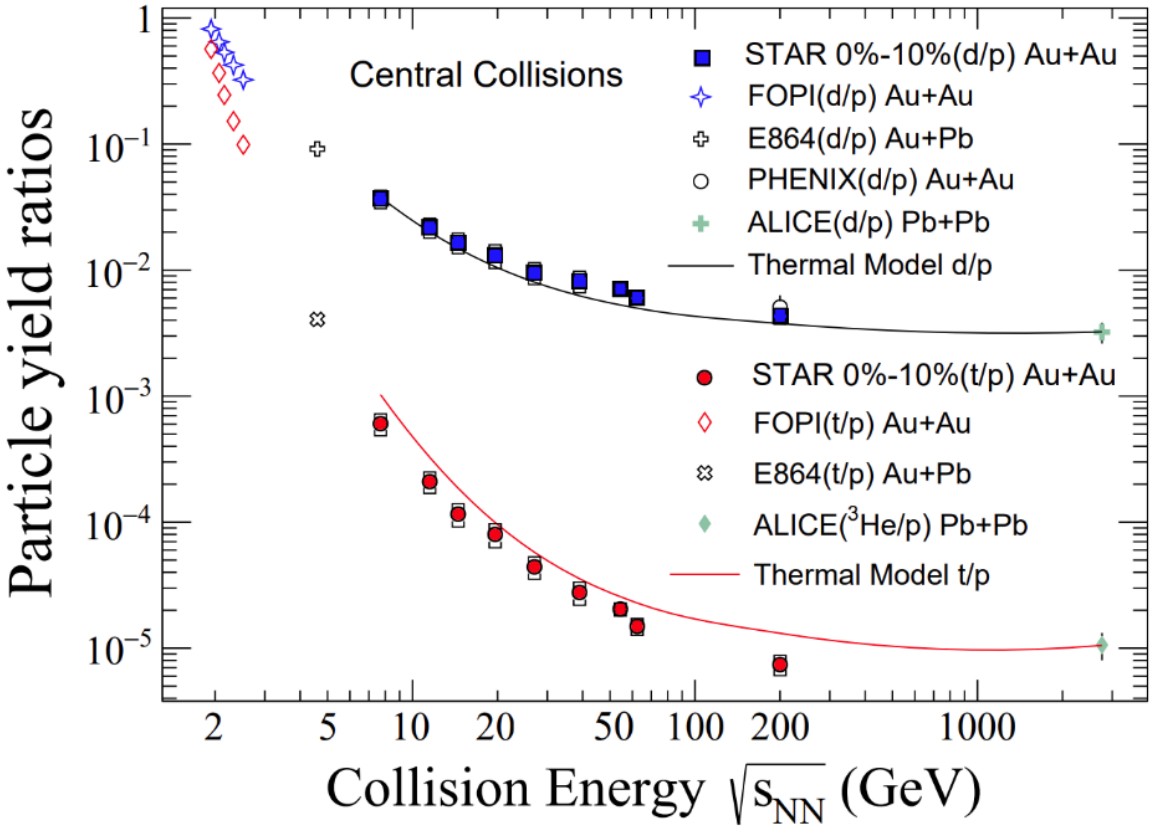
# 4. The triton ‘puzzle’ and its solution

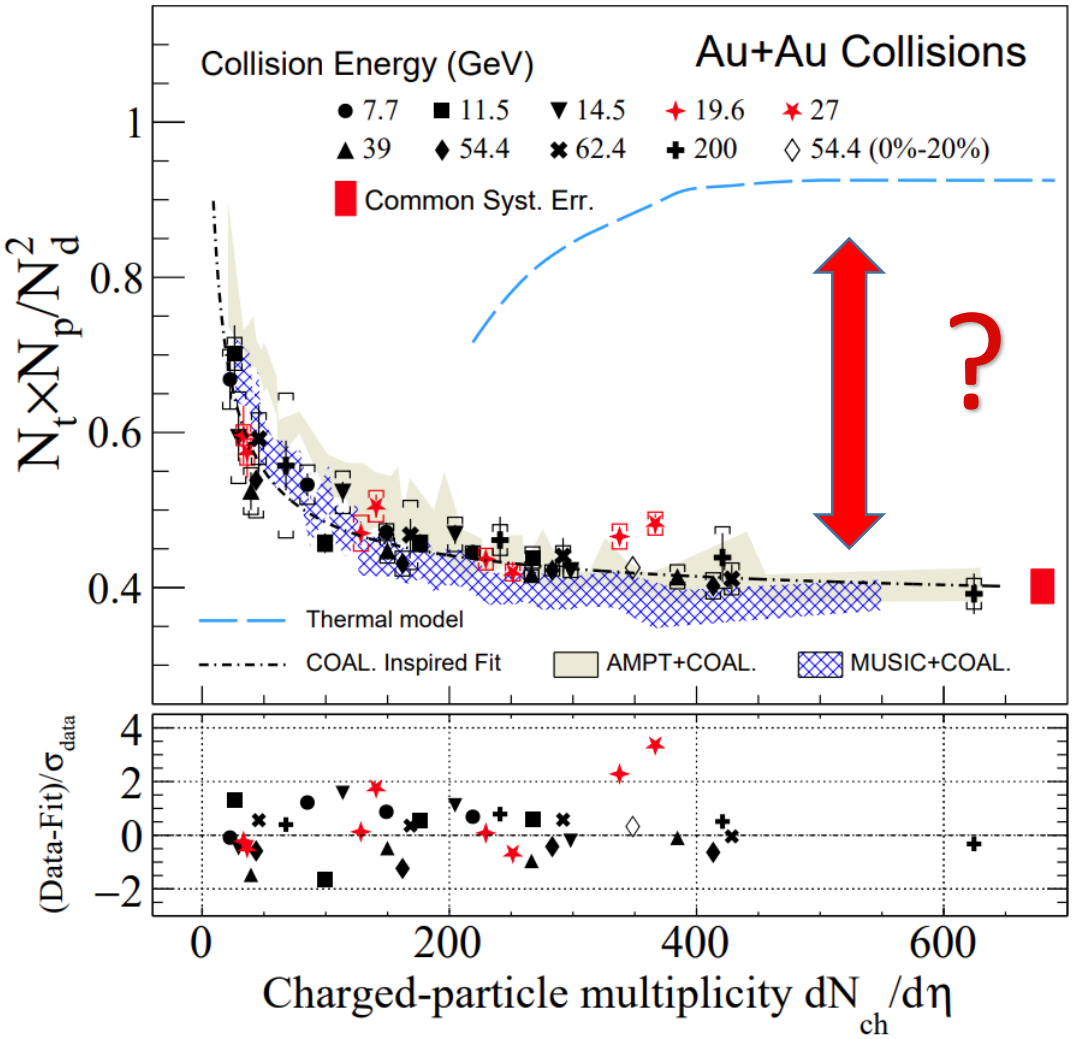
K. J. Sun, R. Wang, C. M. Ko, Y. G. Ma, and C. Shen, 2207.12532(2022)

# The Triton 'Puzzle' at RHIC



STAR, Phys.Rev.Lett. 130 (2023) 202301



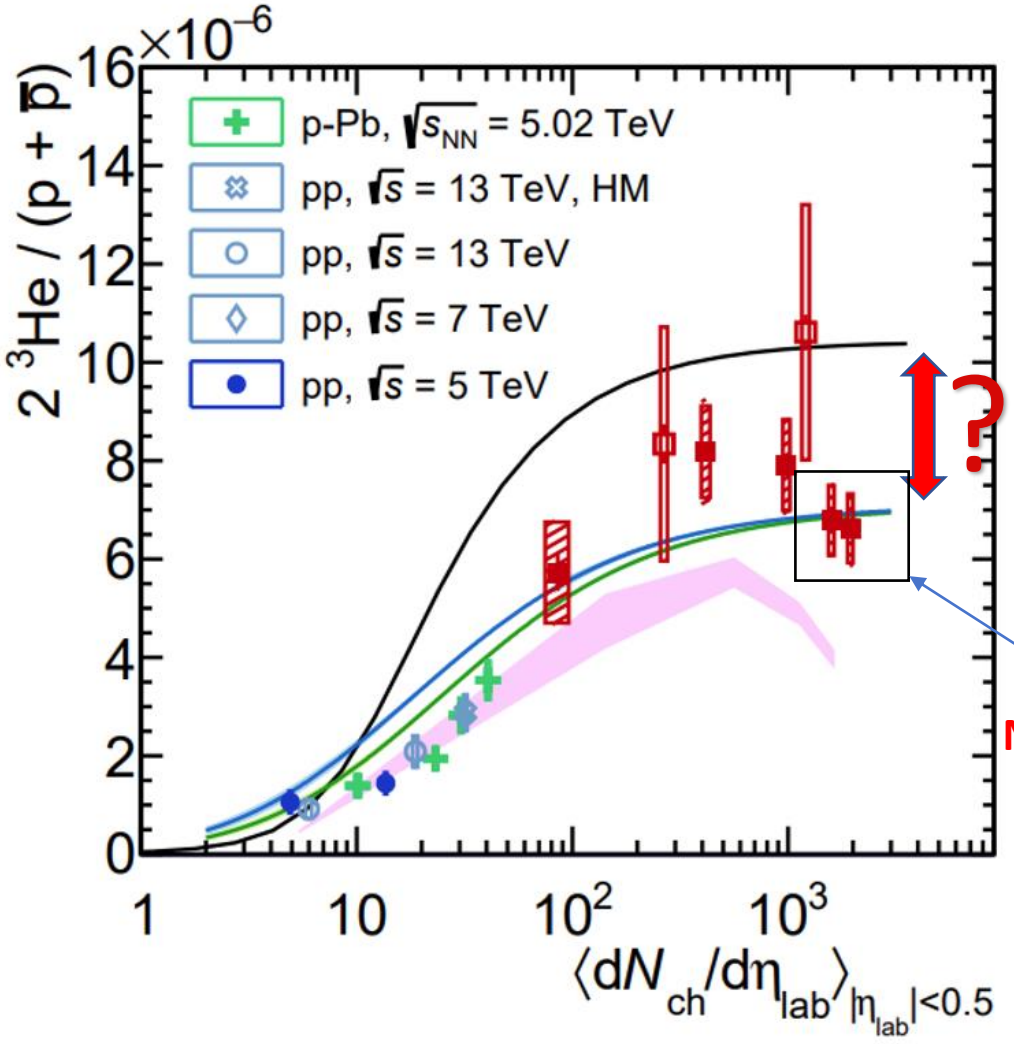




**Triton yields at RHIC are overestimated by the statistical hadronization model.**

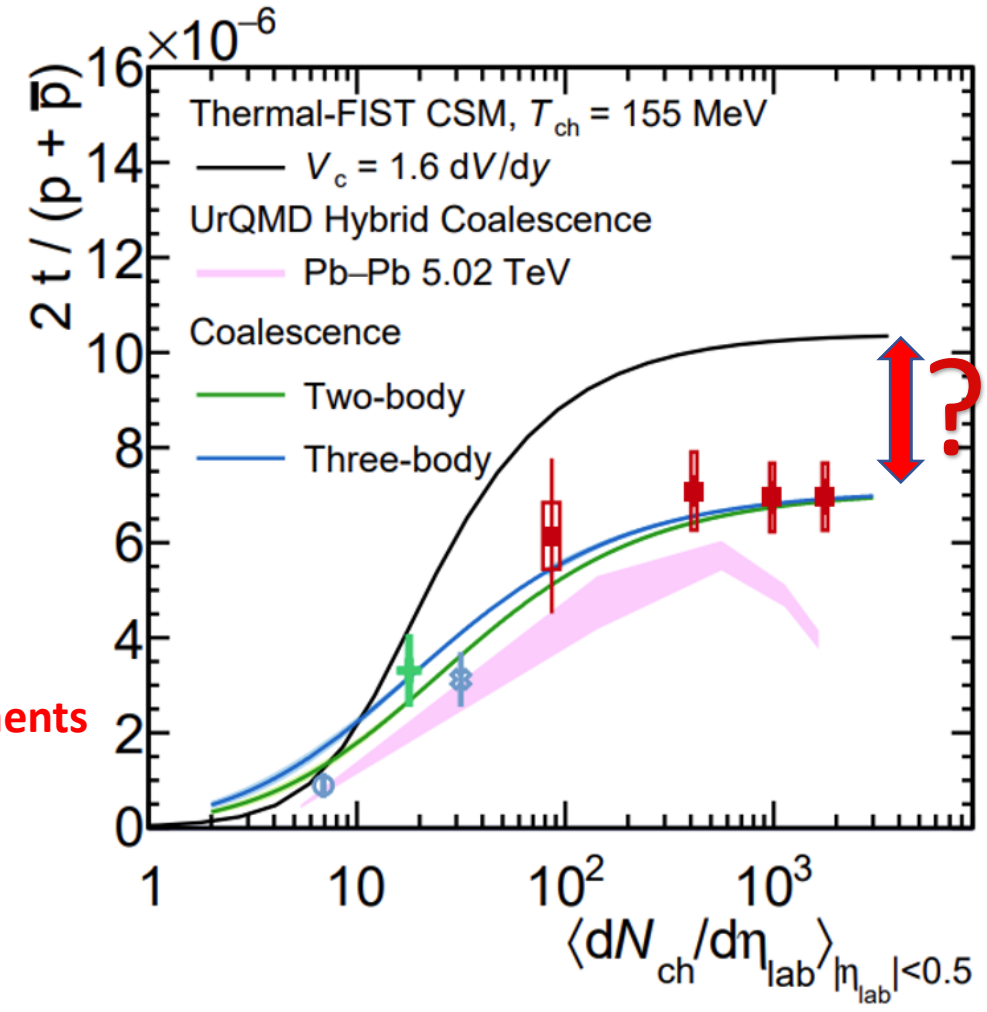
# The Triton 'Puzzle' at LHC



ALICE, Phys. Rev. C 107, 064904 (2023)




New measurements

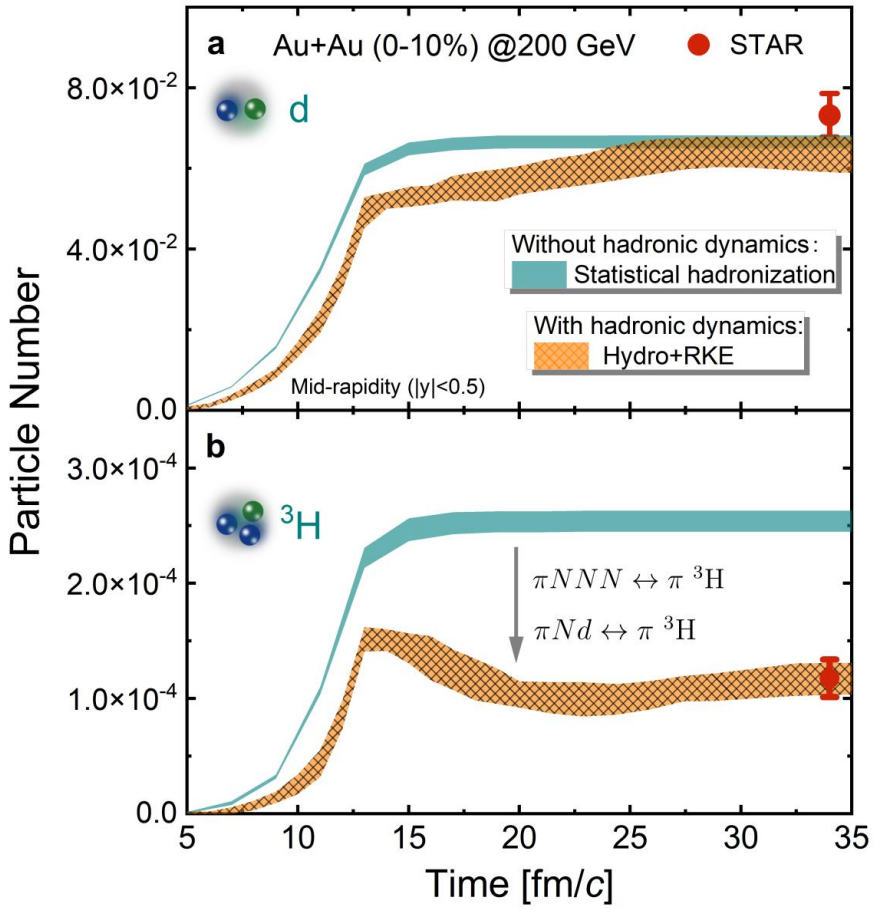


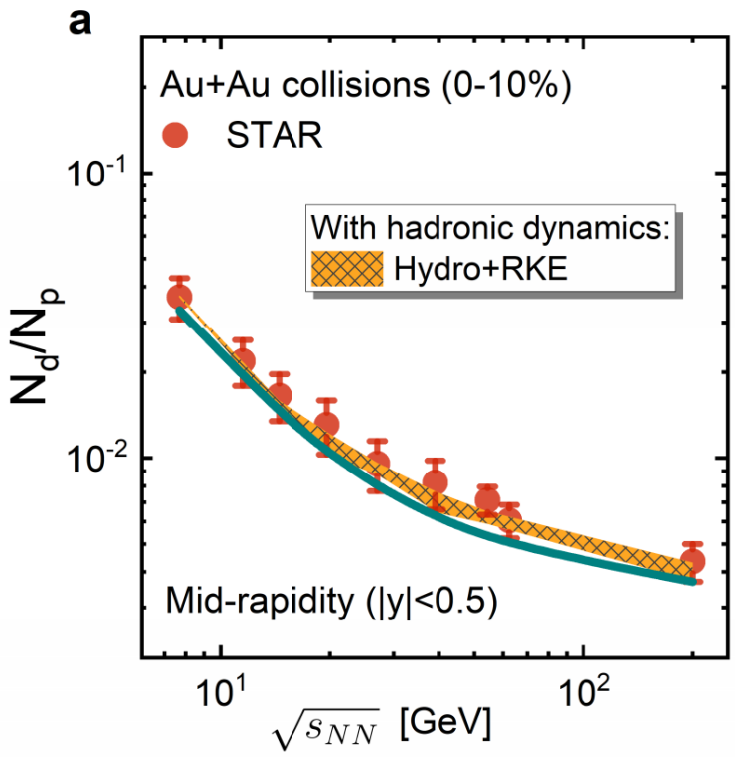

**Triton (helium-3) yields at LHC are overestimated by the statistical hadronization model.**

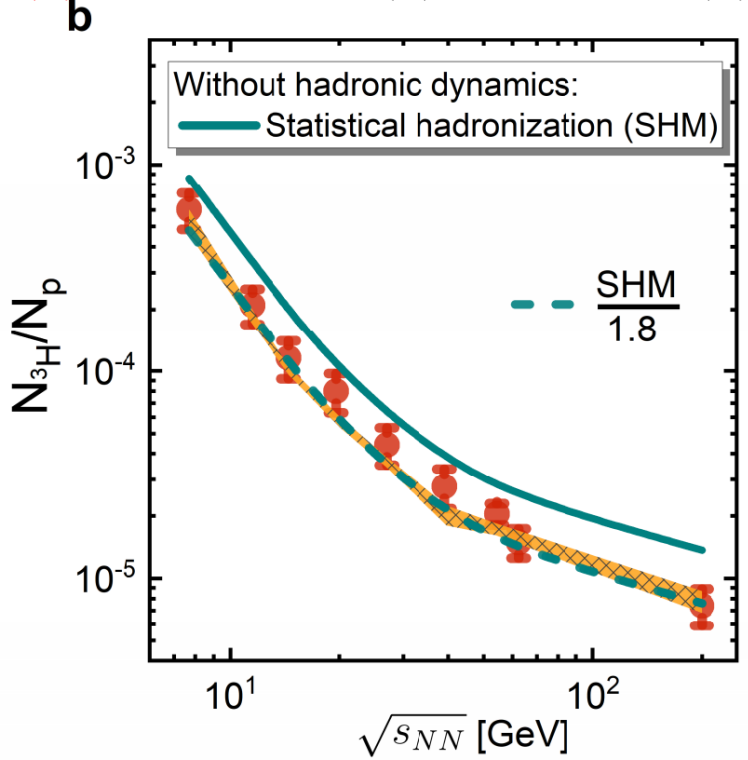
# Solution: Hadronic re-scatterings (16)

K. J. Sun, R. Wang, C. M. Ko, Y. G. Ma, and C. Shen, 2207.12532(2022) Data from STAR, PRL 130, 202301 (2023)

- A = 2 $\pi NN \leftrightarrow \pi d, NNN \leftrightarrow Nd$
- A = 3 $\pi NNN \leftrightarrow \pi t(h), \pi Nd \leftrightarrow \pi t(h), NNNN \leftrightarrow Nt(h), NNd \leftrightarrow Nt(h)$




Hadronic re-scatterings have small effects on the final deuteron yields (see also D. Oliinychenko et al. PRC 99, 044907 (2019)), but they reduce the triton yields by about a factor of 1.8. Similar strong hadronic effects occur at the LHC energies.

# Summary (17)

Main findings:

1. Using a novel transport model, we find that large density inhomogeneities generated by the **spinodal instability** during the first-order QCD phase transition can survive the fast expansion of the subsequent hadronic matter expansion and lead to an enhanced $N_t N_p / N_d^2$ in central Au+Au collisions at $\sqrt{s_{NN}}$ =3 – 5 GeV for $T_c \geq 80$ MeV, which is in accordance with the STAR measurements.

2. **Hadronic re-scatterings** are found to reduce the triton yields by about a factor of 2 during the post-hadronization stage, which resolves the overestimation of SHM on the triton production at both RHIC and LHC energies. (Hadronic re-scatterings also play an important role in explaining the production of hadronic resonances like $K^*$, $\Lambda(1520)$, and etc. [see talk by Sonali Padhan for ALICE])

# Backup

# Why $N_t N_p / N_d^2$ ? (1)

Density matrix formulation

$$N_d \propto Tr[\hat{\rho}_s \hat{\rho}_d]$$

Phys. Lett. B 774, 103 (2017);781, 499 (2018);816, 136258 (2021)

Encodes many-body density fluctuation/correlation

Phase-space representation:

$$N_d = \frac{3}{4}\int d\Gamma\, f_{pn}(\vec{p}_1, \vec{r}_1, \vec{p}_2, \vec{r}_2) \times W_d(\vec{r}, \vec{p})$$

$$W_d(\vec{r}, \vec{p}) = \frac{1}{\pi\hbar}\int d\vec{r}'\, \psi_d^*(\vec{r} + \vec{r}')\psi_d(\vec{r} - \vec{r}')e^{2i\vec{p}\cdot\vec{r}'}$$

Wigner function(Gaussian):

$$W_d(r, k) = 8\exp\left(-\frac{r^2}{\sigma_d^2} - \sigma_d^2 p^2\right) \qquad \sigma_d \approx 2.26 \text{ fm}$$

with density fluctuation and correlation:

$$f_{np}(x_1, p_1; x_2, p_2) = \rho_{np}(x_1, x_2)(2\pi mT)^{-3} e^{-\frac{p_1^2 + p_2^2}{2mT}}$$

$$\rho_{np}(x_1, x_2) = \rho_n(x_1)\rho_p(x_2) + C_2(x_1, x_2)$$

$$\rho_n(x) = <\rho_n> + \delta\rho_n(x) \qquad \rho_p(x) = <\rho_p> + \delta\rho_p(x)$$

$\delta\rho(x)$ denotes density fluctuation over space or inhomogeneity,

$$C_{np} = \langle\delta\rho_n(x)\delta\rho_p(x)\rangle / (\langle\rho_n\rangle\langle\rho_p\rangle)$$
$$\Delta\rho_n = \langle\delta\rho_n(x)^2\rangle / \langle\rho_n\rangle^2$$

$$<\cdots> \equiv \frac{1}{V}\int dx$$

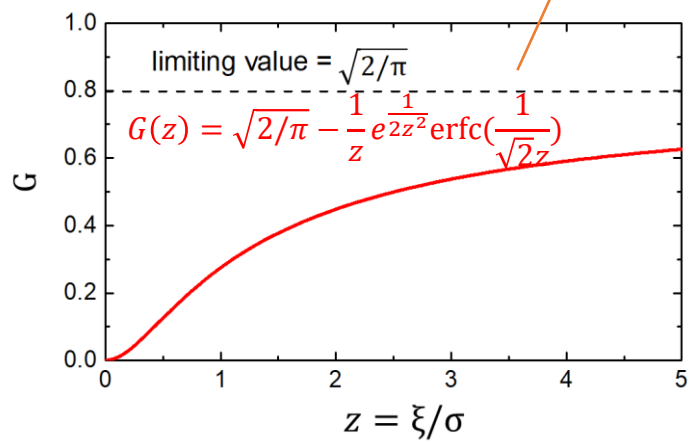
$$C_2(x_1 - x_2) \approx \lambda\langle\rho_n\rangle\langle\rho_p\rangle \frac{e^{-|x_1 - x_2|/\xi}}{|x_1 - x_2|^{1+\eta}} \quad (singular\ part\ only)$$

*with $\xi$ being the density − density correlation length*

$$0 < \langle\delta N^2\rangle \sim \int dx C_2(x) \sim \lambda\xi^2 \rightarrow \lambda > 0$$

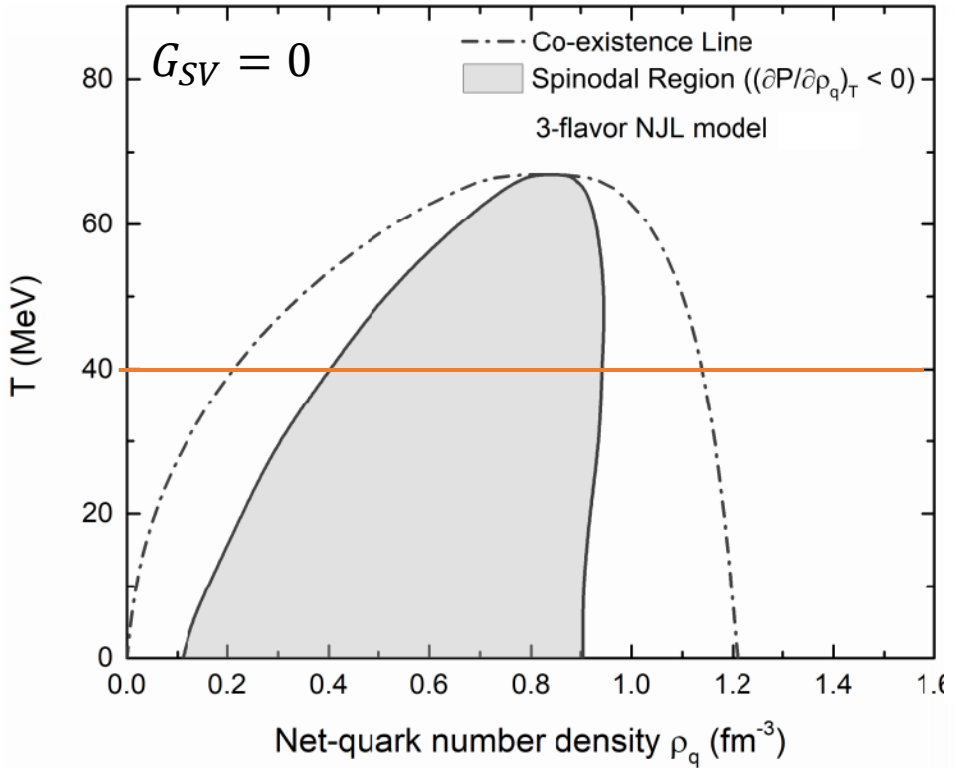
$$N_d \approx \frac{3}{\sqrt{2}}\left(\frac{2\pi}{mT}\right)^{\frac{3}{2}} N_p\langle\rho_n\rangle\left[1 + C_{np} + \frac{\lambda}{\sigma_d} G\left(\frac{\xi}{\sigma_d}\right)\right]$$

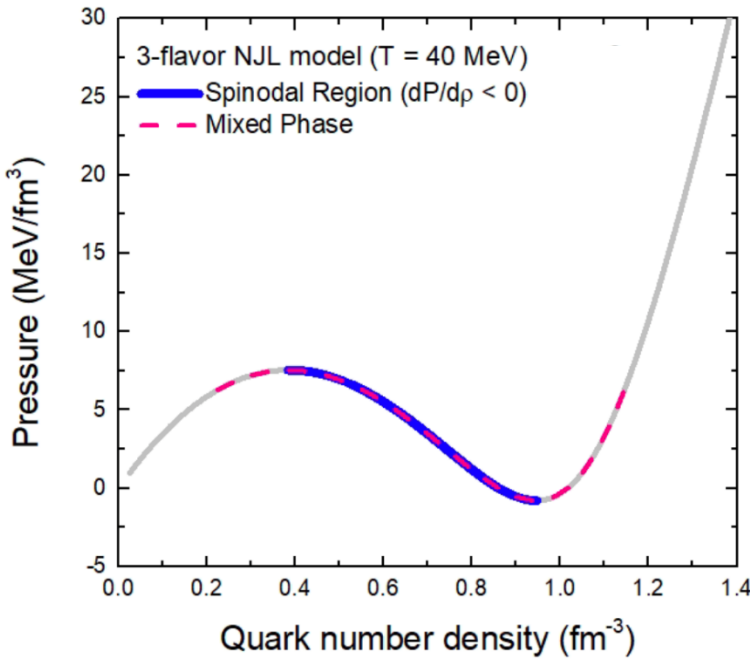
$$N_t = \frac{3^{3/2}}{4}\left(\frac{2\pi}{mT}\right)^3 N_p\langle\rho_n\rangle^2\left[1 + 2C_{np} + \Delta\rho_n + \frac{3\lambda}{\sigma_t} G\left(\frac{\xi}{\sigma_t}\right) + O(G^2)\right]$$

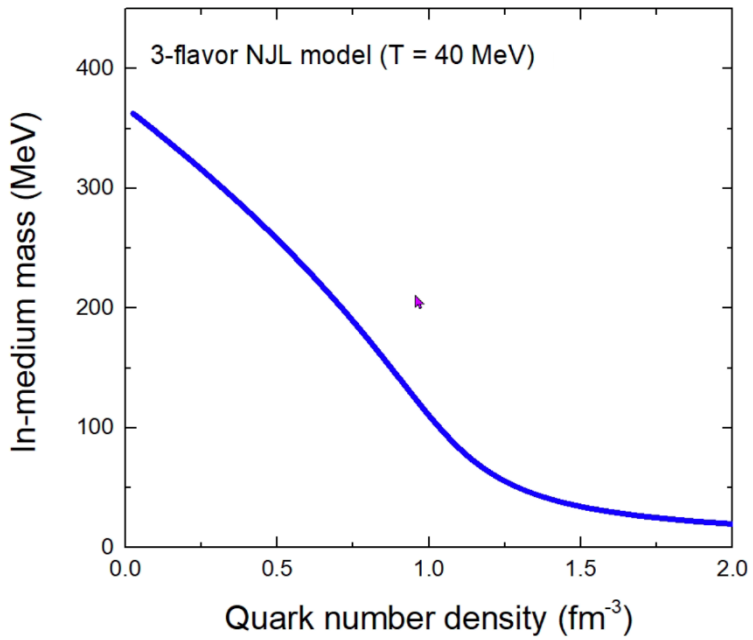



Ratio: $$\frac{N_t N_p}{N_d^2} \approx \frac{1}{2\sqrt{3}}\left[1 + \Delta\rho_n + \frac{\lambda}{\sigma} G\left(\frac{\xi}{\sigma}\right)\right]$$

# In-medium quark mass (2)




K. J. Sun, C. M. Ko, S. Cao, and F. Li., Phys. Rev. D 103, 014006 (2021)

# The Triton 'Puzzle'

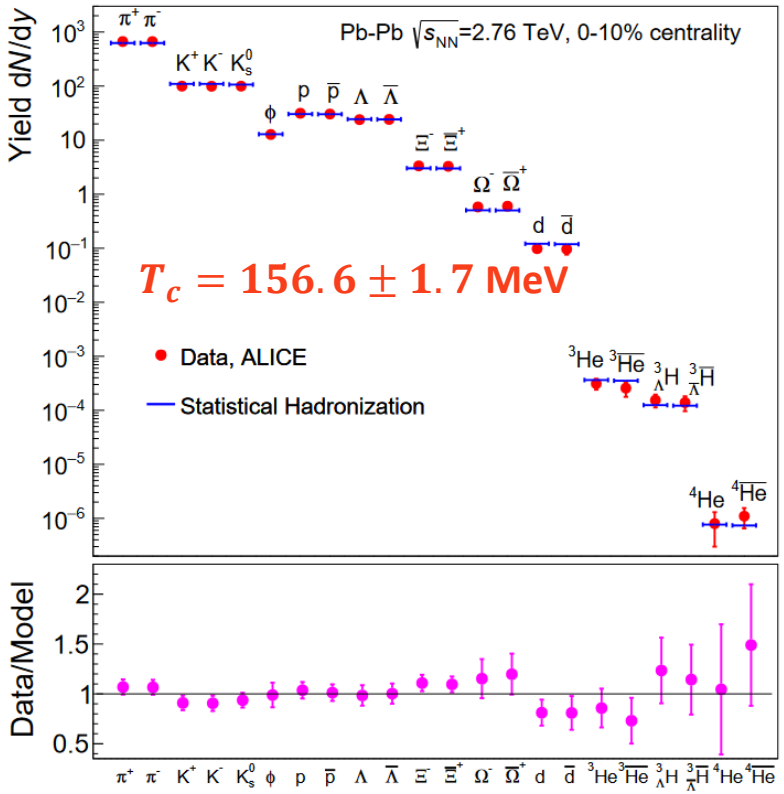
(3)

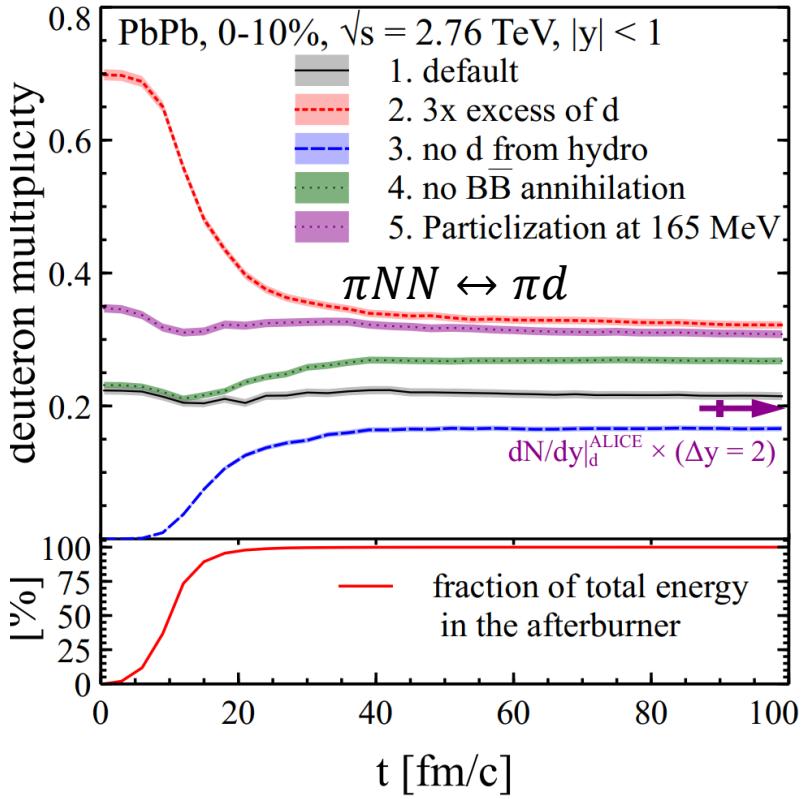
## Earlier theoretical results

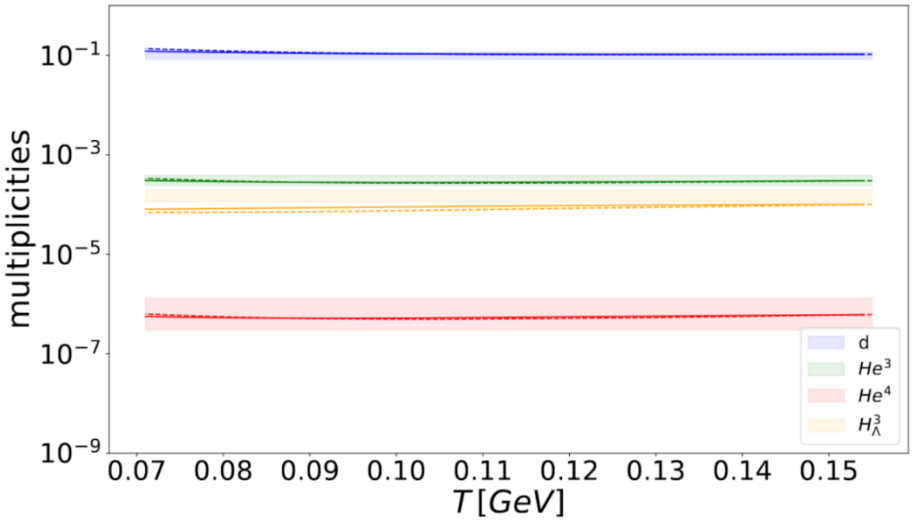
D. Oliinychenko, et al., PRC99, 044907 (2019)

V. Vovchenko, et al., PLB800, 135131 (2020)

T. Neidig, et al., PLB827, 136891 (2022)




**The obtained hadronic effects on light nuclei production are small!**

A. Andronic, P. Braun-Munzinger, J. Stachel, H. Stöcker, PLB 697, 203 (2011)

A. Andronic, P. Braun-Munzinger, K. Redlich, J. Stachel, Nature 561, 321 (2018)

# Relativistic Kinetic Approach to the Little-Bang Nucleosynthesis (4)

K. J. Sun, R. Wang, C. M. Ko, Y. G. Ma, and C. Shen, 2207.12532(2022)

Relativistic kinetic equation for $\pi NN \leftrightarrow \pi d$

$$\frac{\partial f_d}{\partial t} + \frac{\mathbf{P}}{E_d} \cdot \frac{\partial f_d}{\partial \mathbf{R}} = -\mathcal{K}^{>} f_d + \mathcal{K}^{<}(1 + f_d)$$

with collision integral:

$$\text{R.H.S.} = \frac{1}{2g_d E_d} \int \prod_{i=1'}^{3'} \frac{\mathrm{d}^3\mathbf{p}_i}{(2\pi)^3 2E_i} \frac{\mathrm{d}^3\mathbf{p}_\pi}{(2\pi)^3 2E_\pi} \frac{E_d \mathrm{d}^3\mathbf{r}}{m_d}$$

$$\times 2m_d W_d(\tilde{\mathbf{r}}, \tilde{\mathbf{p}}) \left(\overline{|\mathcal{M}_{\pi^+ n \to \pi^+ n}|^2} + n \leftrightarrow p\right)$$

$$\times \Big[ -\Big(\prod_{i=1'}^{3'} (1 \pm f_i)\Big) g_\pi f_\pi g_d f_d + \frac{3}{4}\Big(\prod_{i=1'}^{3'} g_i f_i\Big)$$

$$\times (1 + f_\pi)(1 + f_d)\Big] \times (2\pi)^4 \delta^4(p_{\text{in}} - p_{\text{out}})$$

Nonlocal collision integral to take into account the effects of finite nuclei sizes.
$W_d$ denotes deuteron Wigner function.

P. Danielewicz et al., NPA533, 712 (1991); PLB274, 268 (1992); Annals of Physics 152, 239(1984);

Impulse approximation (IA):

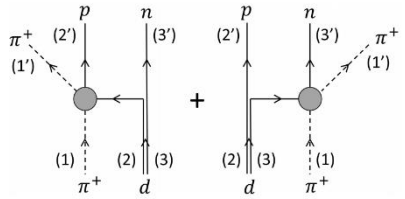



FIG. 1. Diagrams for the reaction $\pi^+ d \leftrightarrow \pi^+ np$ in the impulse approximation. The filled bubble indicates the intermediate states such as a $\Delta$ resonance.

Length/energy scale:

$$\lambda_{thermal} \sim 0.5\ fm \ll r_{np} \sim 4\ fm$$

**Solving kinetic equations with the stochastic method using test particles**

Probability for reaction $\pi d \leftrightarrow \pi NN$ to take place in volume $\Delta V$ and time interval $\Delta t$ are given by

$$P_{23}\big|_{\text{IA}} \approx F_d v_{\pi^+ p} \sigma_{\pi^+ p \to \pi^+ p} \frac{\Delta t}{N_{\text{test}} \Delta V} + (p \leftrightarrow n)$$

$$P_{32}\big|_{\text{IA}} \approx \frac{3}{4} F_d v_{\pi^+ p} \sigma_{\pi^+ p \to \pi^+ p} \frac{\Delta t W_d}{N_{\text{test}}^2 \Delta V} + (p \leftrightarrow n)$$

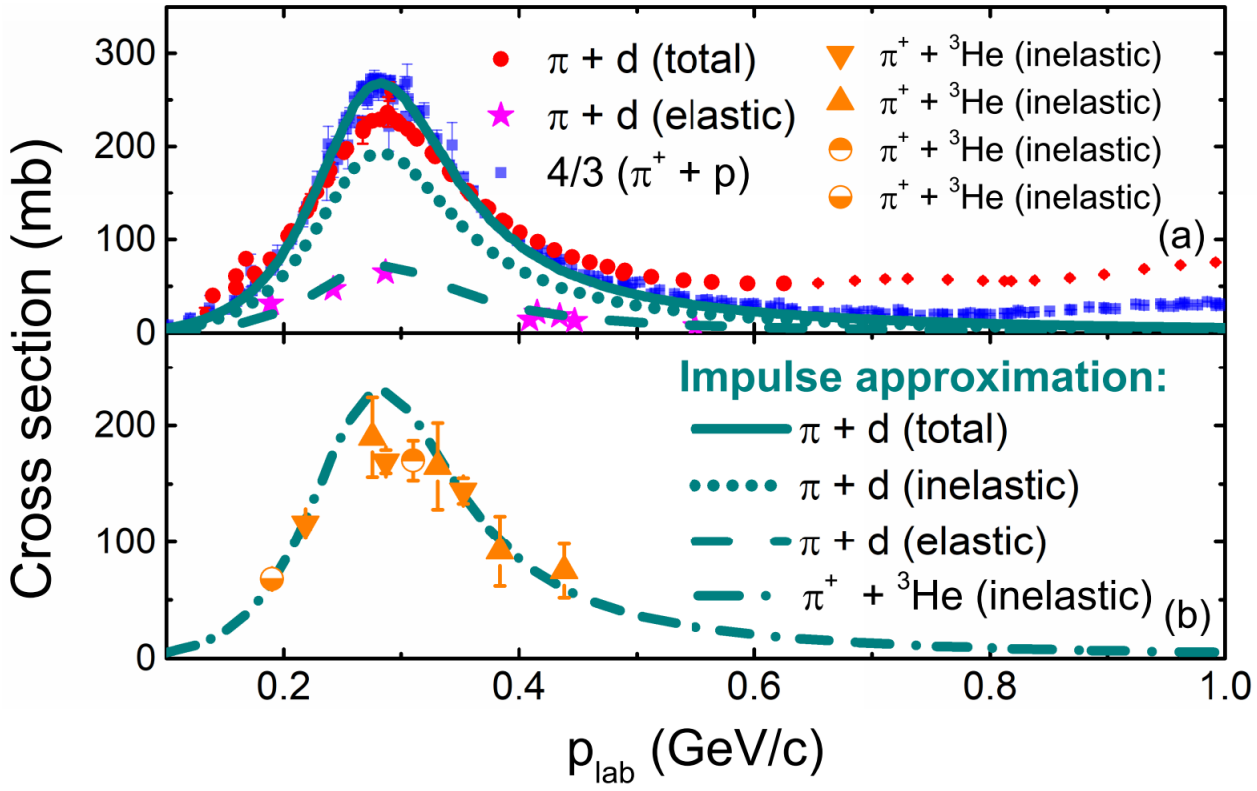
For triton or helium-3:

$$P_{42}\big|_{\text{IA}} \approx \frac{1}{4} F_t \frac{v_{\pi N} \sigma_{\pi N \to \pi N} \Delta t}{N_{\text{test}}^3 \Delta V} W_t$$

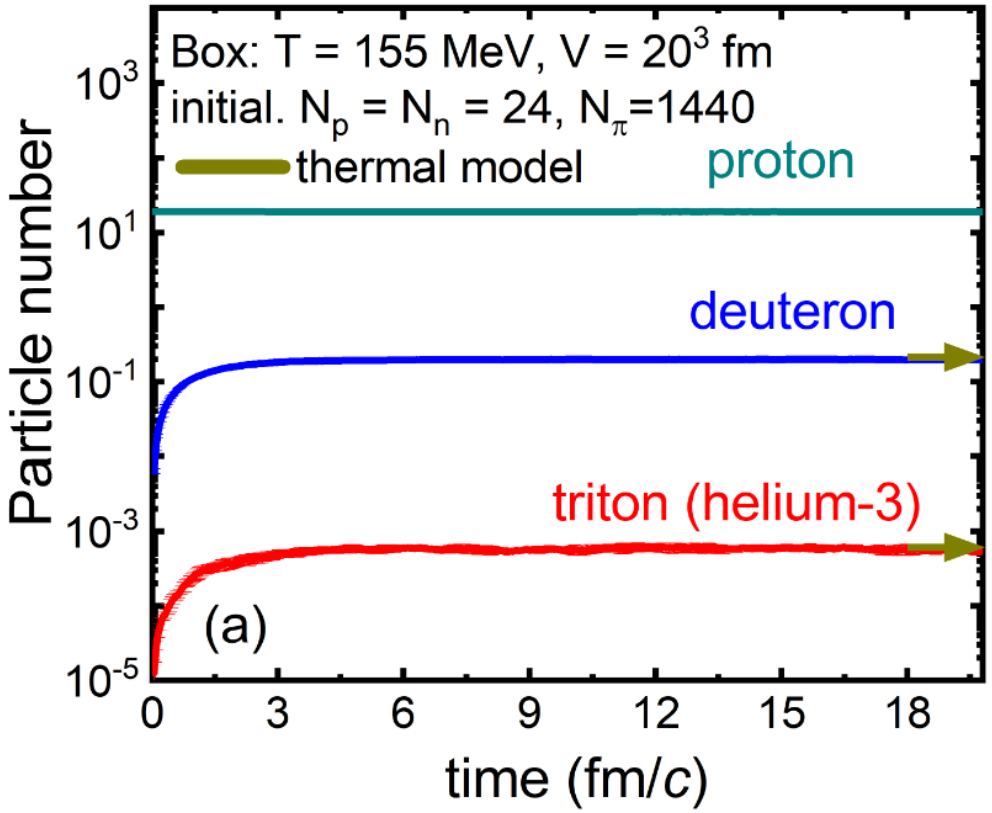
'renormalization' factor $F_d, F_t$ which can be fixed by $\pi d$ and $\pi t$ cross sections.

# Thermal Limit






**Periodic boundary condition**



# Relativistic Kinetic Approach versus Coalescence (6)

**Coalescence and flow in ultra-relativistic heavy ion collisions**

Rüdiger Scheibl and Ulrich Heinz
*Institut für Theoretische Physik, Universität Regensburg, D-93040 Regensburg, Germany*
(March 16, 1999 — published in Physical Review C 59 (1999), 1585-1602)

$$\frac{dN_{\rm d}}{d^3P_{\rm d}} = \frac{-3i}{(2\pi)^3}\int d^4r_{\rm d}\, d^3r \int \frac{d^4p_1}{(2\pi)^4}\,\frac{d^3p_2}{(2\pi)^3}$$
$$\times(2\pi)^4\delta^4(P_{\rm d}-p_1^*-p_2)\,\mathcal{D}\left(\boldsymbol{r}, \tfrac{\boldsymbol{p}_1-\boldsymbol{p}_2}{2}\right) \qquad (3.12)$$
$$\times\Big(\Sigma_{\rm p}^<(p_1^*,r_+)\,f_{\rm n}^{\rm W}(p_2,r_-) + \Sigma_{\rm n}^<(p_1^*,r_+)\,f_{\rm p}^{\rm W}(p_2,r_-)\Big),$$

$$-i\Sigma_{\rm N}^<(p^*,x) = \sum_j \int \frac{d^3q}{(2\pi)^3}\,\frac{d^3p'}{(2\pi)^3}\,\frac{d^3q'}{(2\pi)^3}$$
$$\times\,(2\pi)^4\delta^4(p^*+q-p'-q')\,|M_{{\rm N}j\to{\rm N}j}|^2$$
$$\times\,f_{\rm N}^{\rm W}(p',x)\,f_j^{\rm W}(q',x)\,\big(1\pm f_j^{\rm W}(q,x)\big)\,. \qquad (3.13)$$

$$-i\Sigma_{\rm N}^<(p^*,x) = \frac{f_{\rm N}(p^*,x)}{\tau_{\rm sca}^{\rm N}(\boldsymbol{p},x)}\,. \qquad (3.16)$$

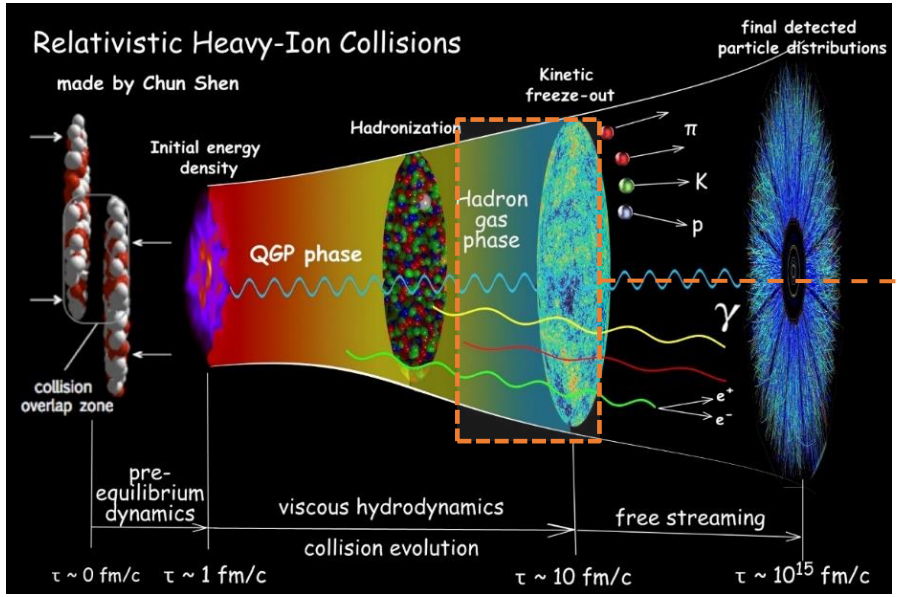
$$\frac{dN_{\rm d}}{d^3P_{\rm d}} = \frac{3}{(2\pi)^3}\int d^3r_{\rm d}\int \frac{d^3r\,d^3q}{(2\pi)^3}\,\mathcal{D}(\boldsymbol{r},\boldsymbol{q})$$
$$\times f_{\rm p}(q_+,r_+)\,f_{\rm n}(q_-^*;r_-)\,. \qquad (3.17)$$

# Final-state coalescence

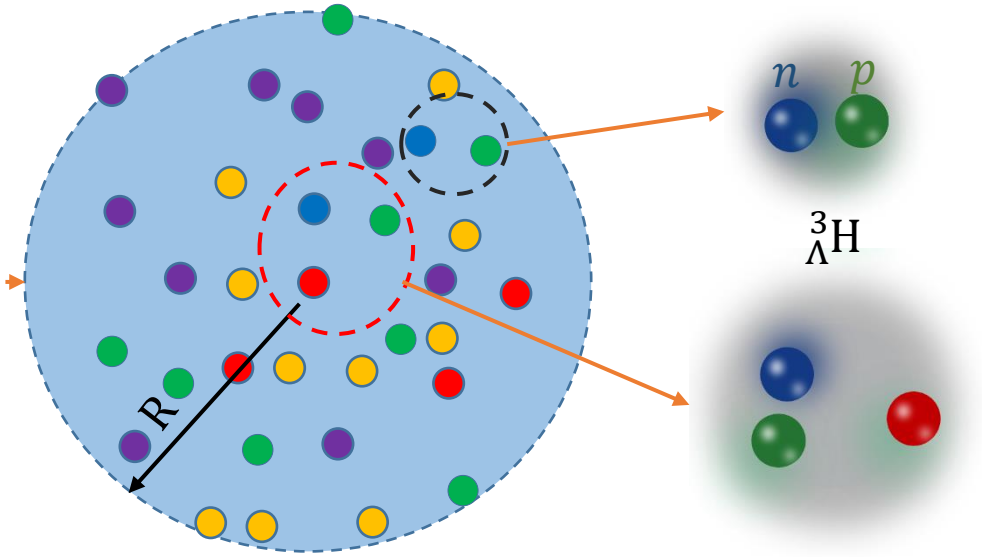


R. Scheibl and U. W. Heinz, PRC59. 1585(1999);

R. Scheibl and U. W. Heinz, PRC59. 1585(1999)




Coalescence Model




Deuteron




$^{3}_{\Lambda}\mathrm{H}$



Density Matrix Formulation
(sudden approximation)

Wigner function of light cluster

$$N_A = Tr(\hat{\rho}_s \hat{\rho}_A)$$
$$= g_c \int d\Gamma \, \rho_s(\{x_i, p_i\}) \times W_A(\{x_i, p_i\})$$

**Overlap between source distribution function and Wigner function of light nuclei**