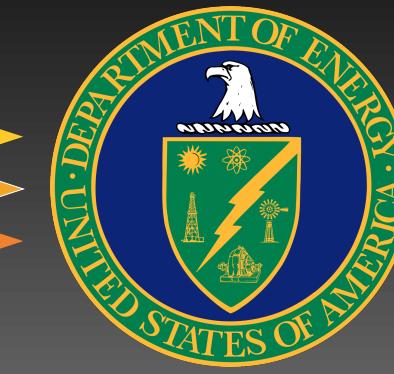




RBRC  
RIKEN BNL Research Center

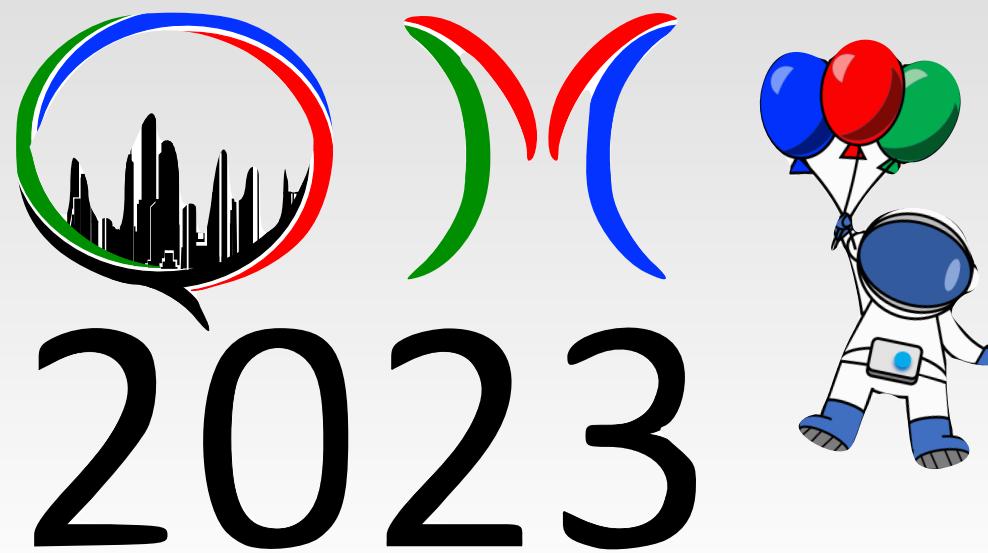


BEST  
COLLABORATION

# Bayesian Inference of QGP Properties and 3D Dynamics of Heavy-Ion Collisions in the RHIC Beam Energy Scan Program

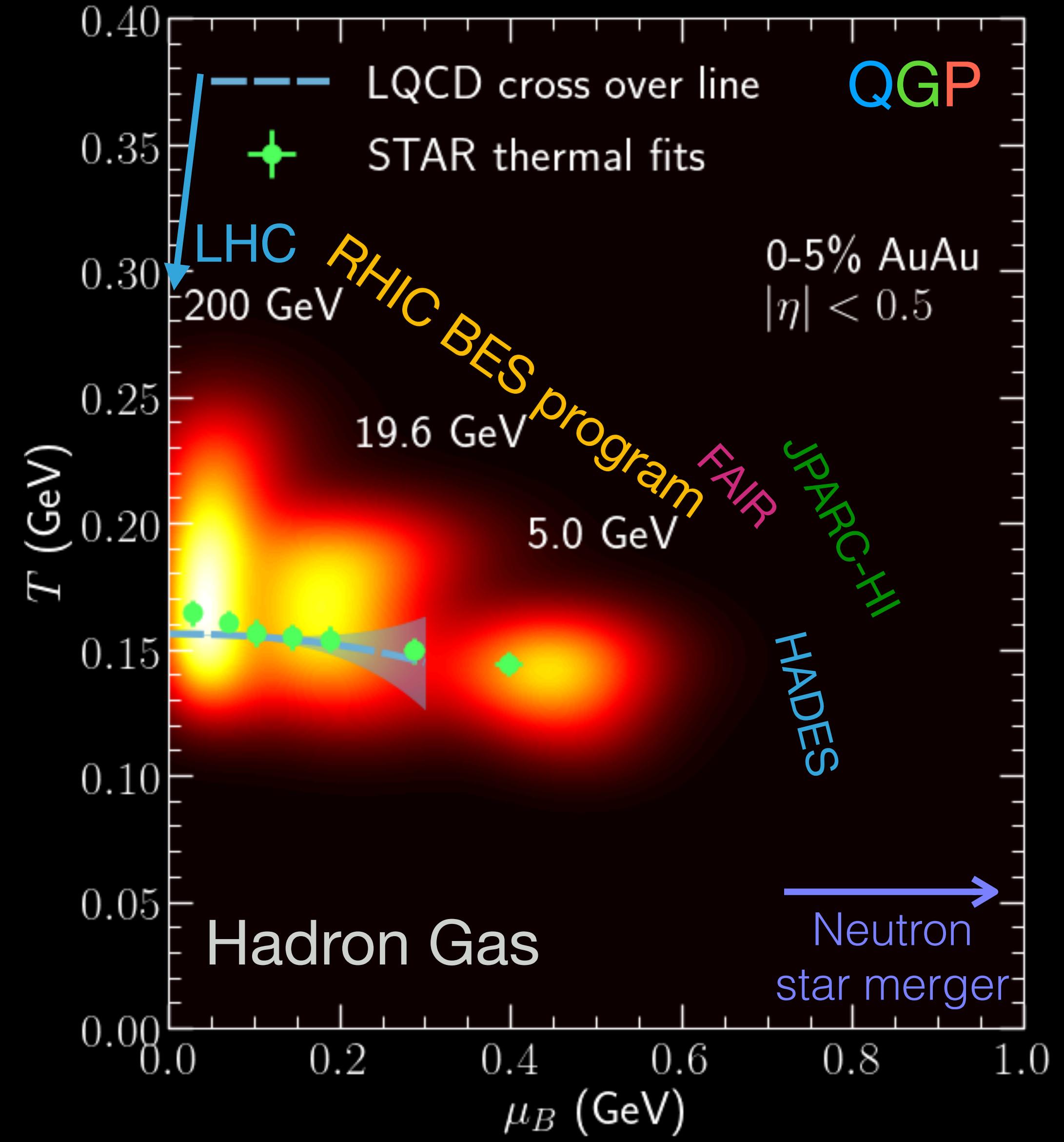
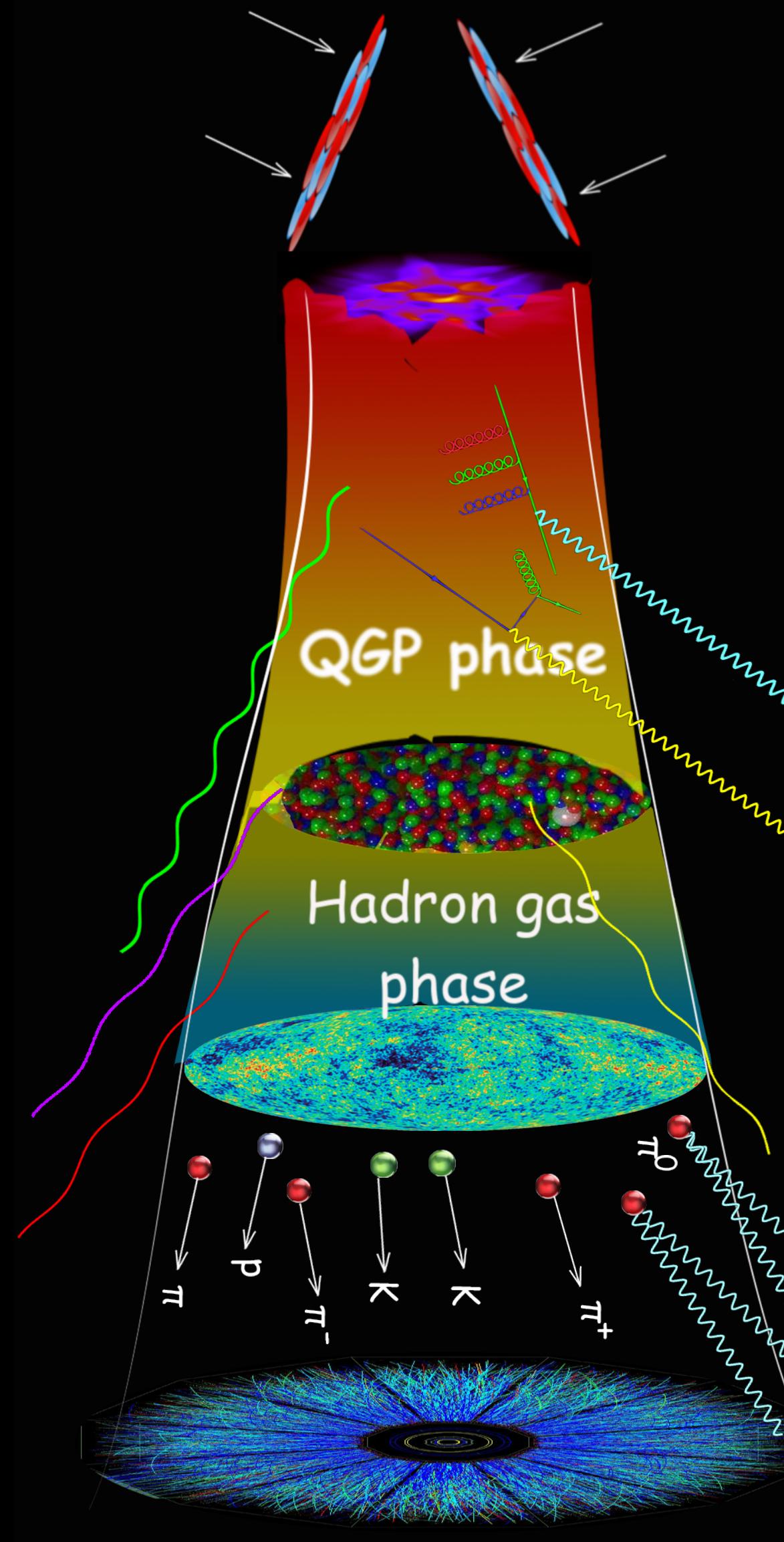
Chun Shen (Wayne State University)

In collaboration with Björn Schenke and Wenbin Zhao



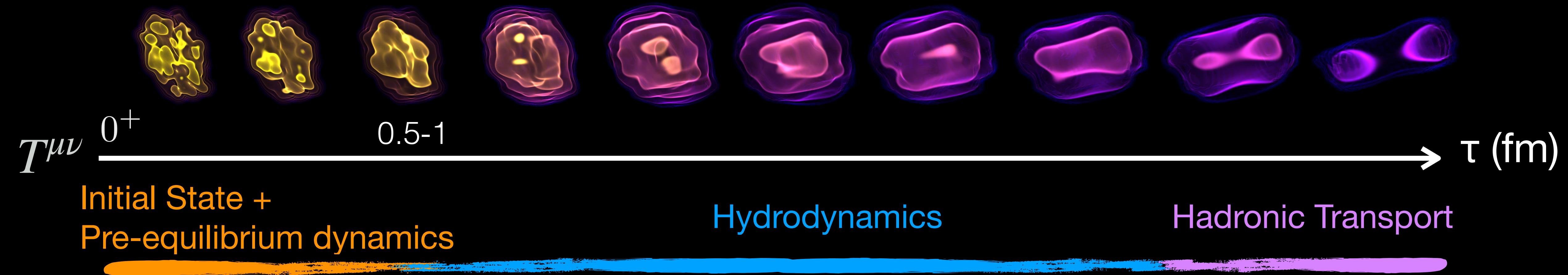
Sept. 5, 2023

# PROBING THE NUCLEAR MATTER PHASE DIAGRAM

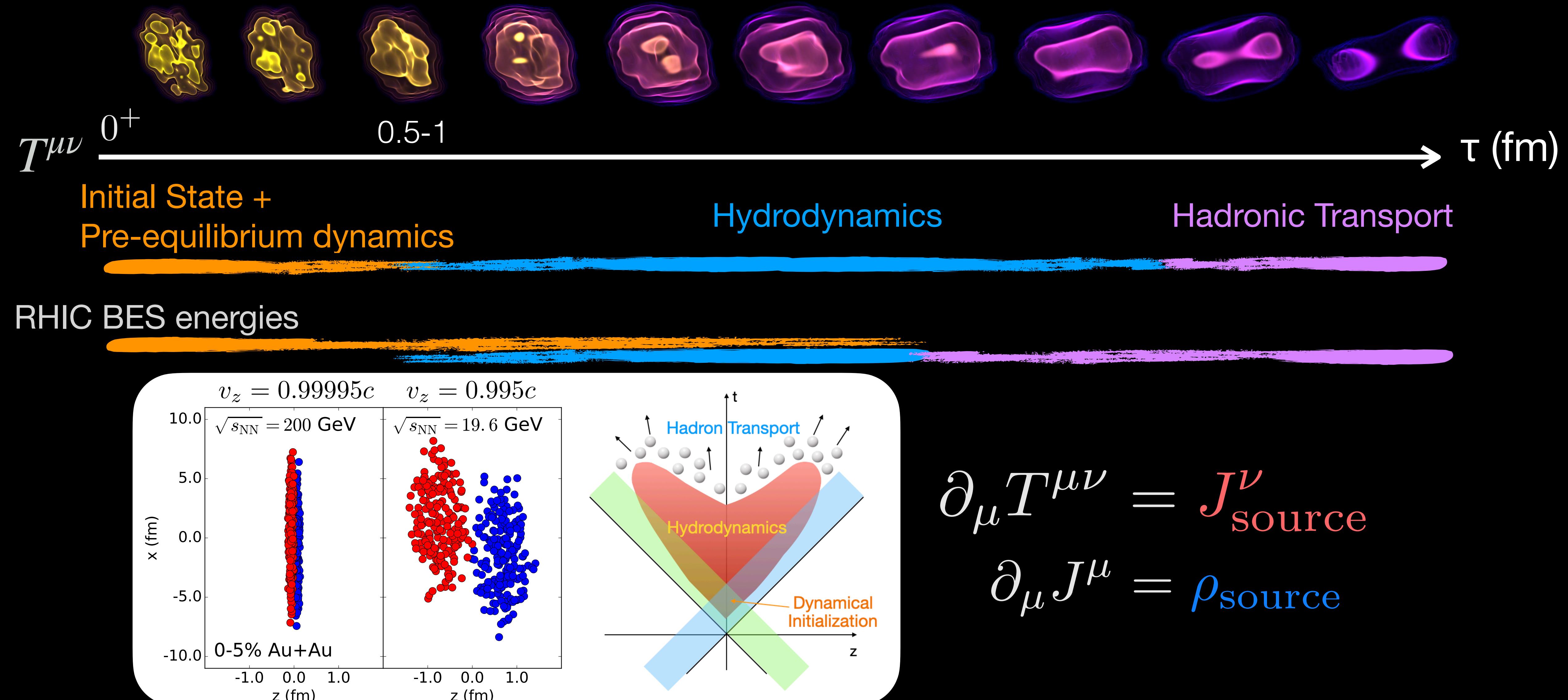


- Search for a critical point & 1st order phase transition  
 $c_s^2(T, \{\mu_q\})$
- How do the QGP transport properties change with baryon doping?  
 $(\eta/s)(T, \{\mu_q\}), (\zeta/s)(T, \{\mu_q\})$
- Access to new transport phenomena  
*Charge diffusion*

# THE MULTI-STAGE THEORETICAL FRAMEWORK



# THE MULTI-STAGE THEORETICAL FRAMEWORK

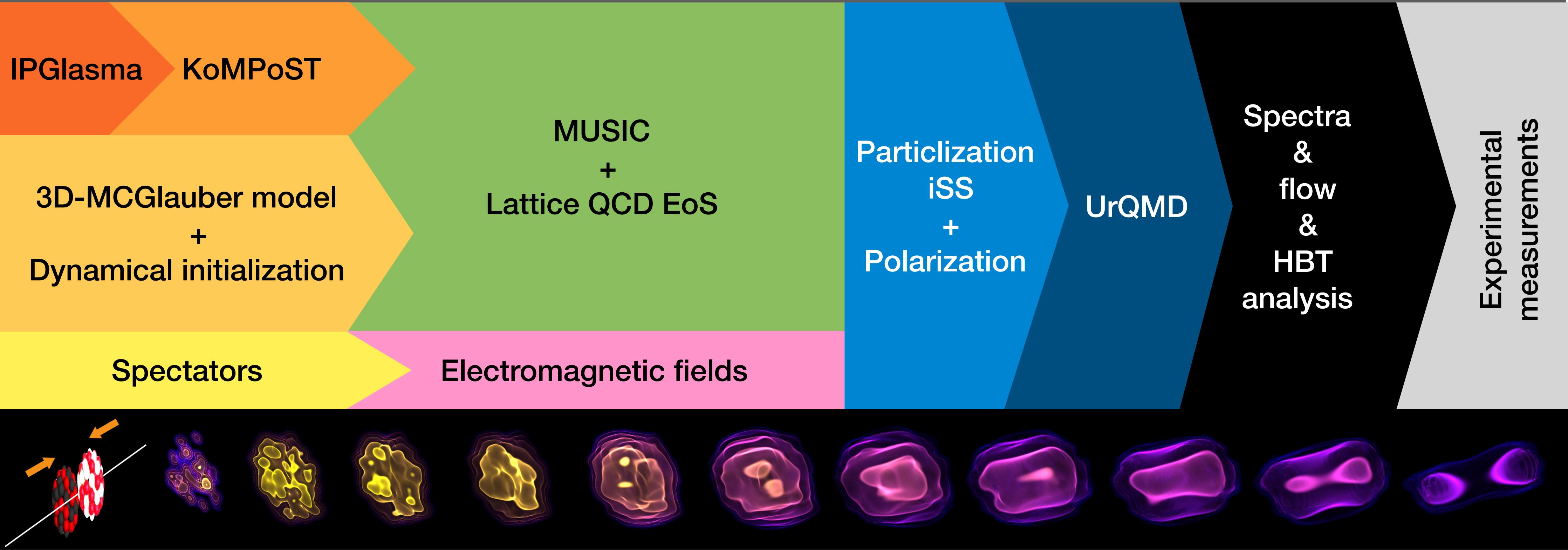


# AN OPEN SOURCE HYBRID FRAMEWORK – iEBE-MUSIC



<https://github.com/chunshen1987/iEBE-MUSIC>

## The iEBE-MUSIC Framework



State-of-the-art event-by-event simulations for relativistic heavy-ion collisions

ALSO SEE B. SCHENKE'S TALK (TUE 16:30)

# 3D MC-GLAUBER MODEL WITH STRING DECELERATION

- Transverse collision geometry is determined by MC-Glauber model
- 3 valence quarks are sampled from PDF with

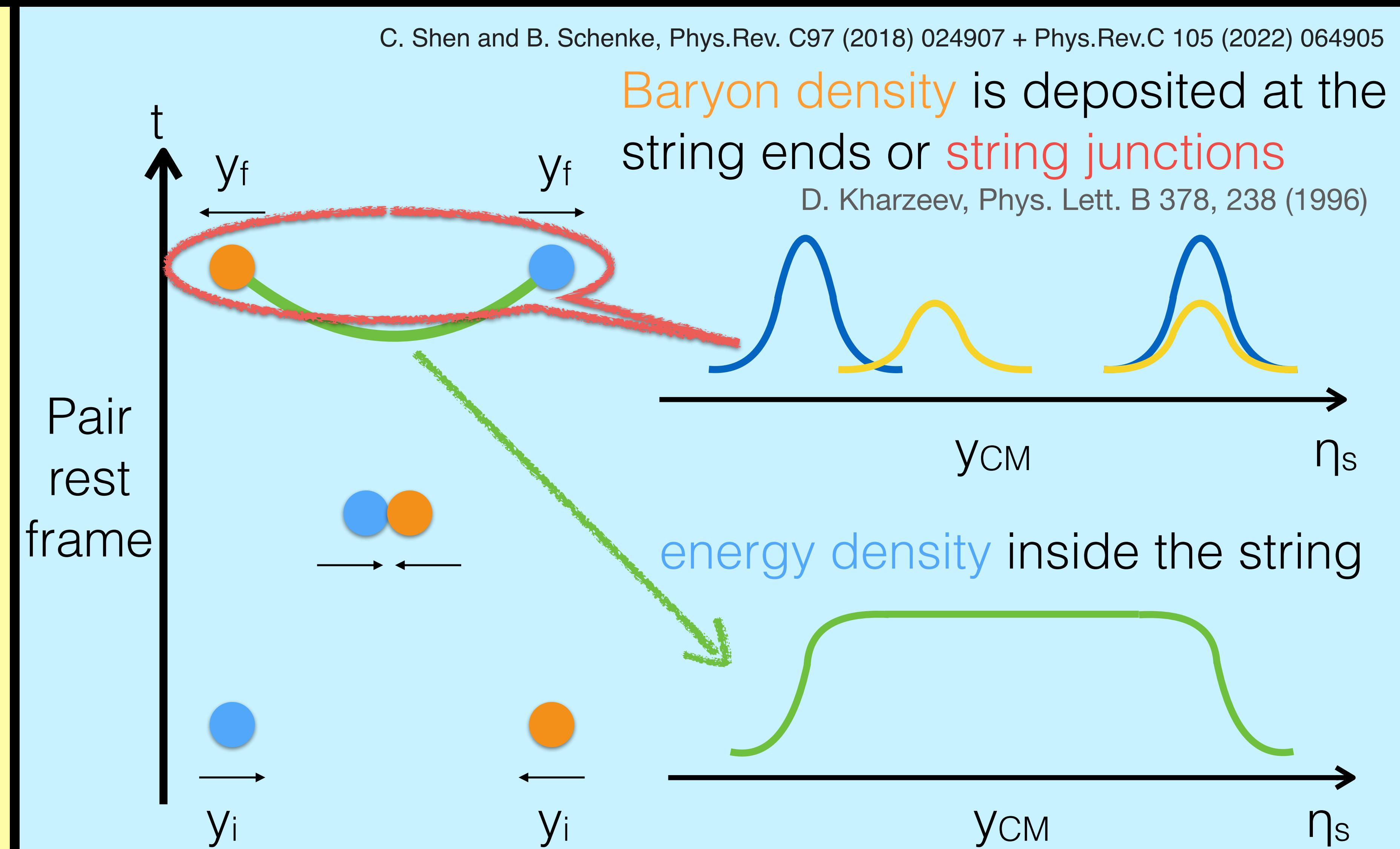
$$\sum_i x_i \leq 1$$

- Incoming quarks are decelerated with a string tension  $\sigma$ ,
- $dp^z/dt = -\sigma$

C. Shen and B. Schenke, Phys.Rev. C97 (2018) 024907 + Phys.Rev.C 105 (2022) 064905

Baryon density is deposited at the string ends or string junctions

D. Kharzeev, Phys. Lett. B 378, 238 (1996)



Imposed conservation for energy, momentum, and net baryon density

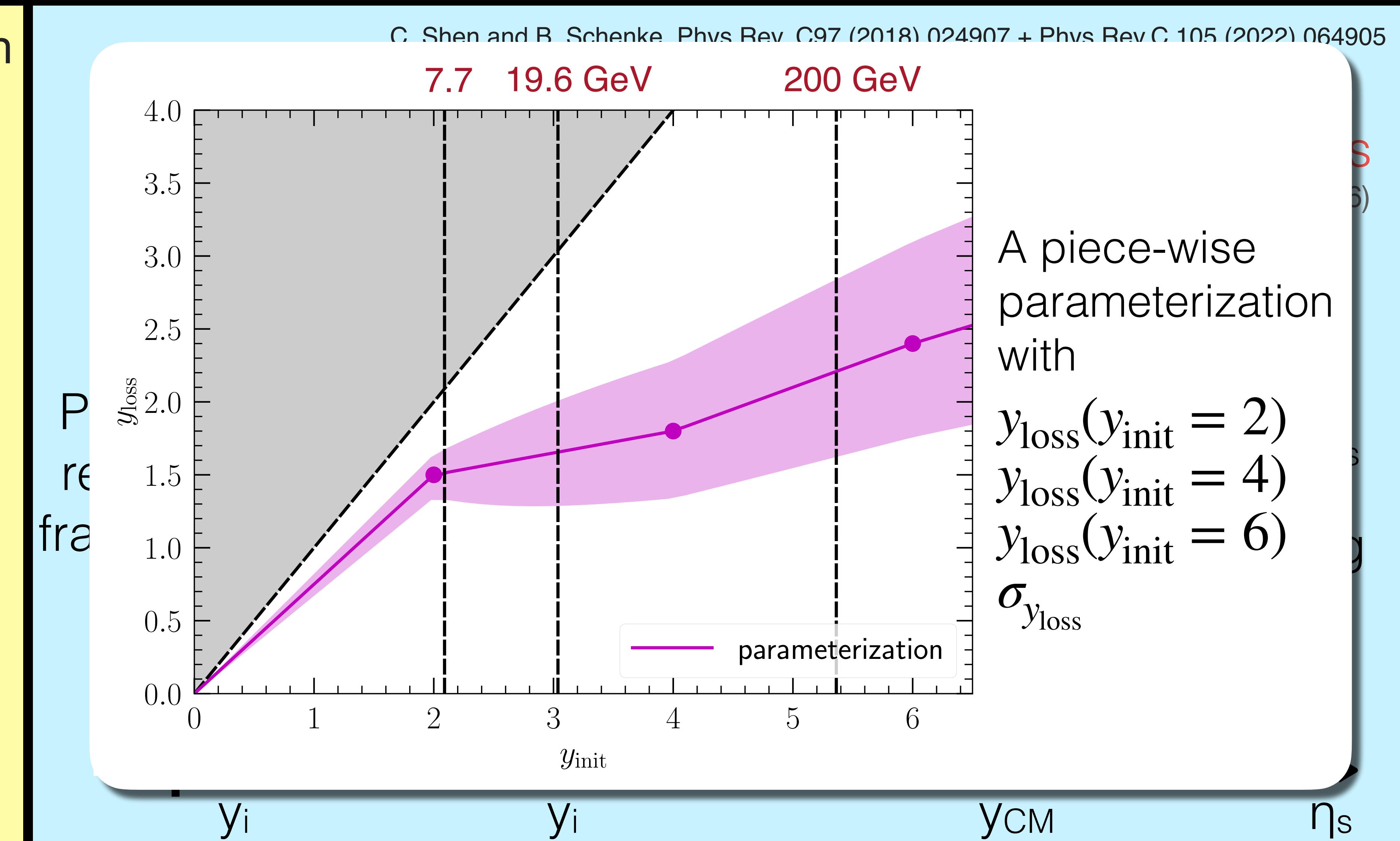
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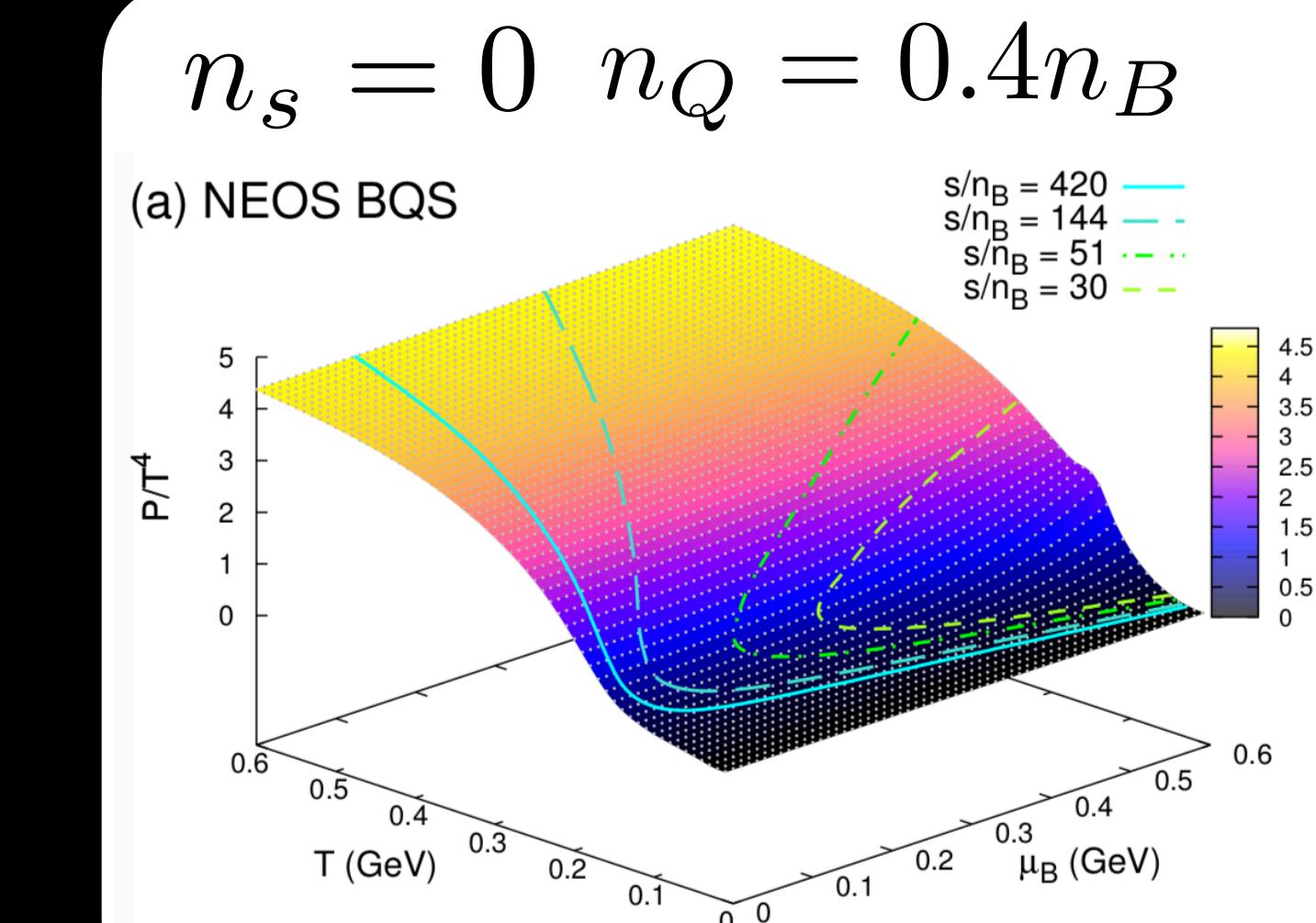
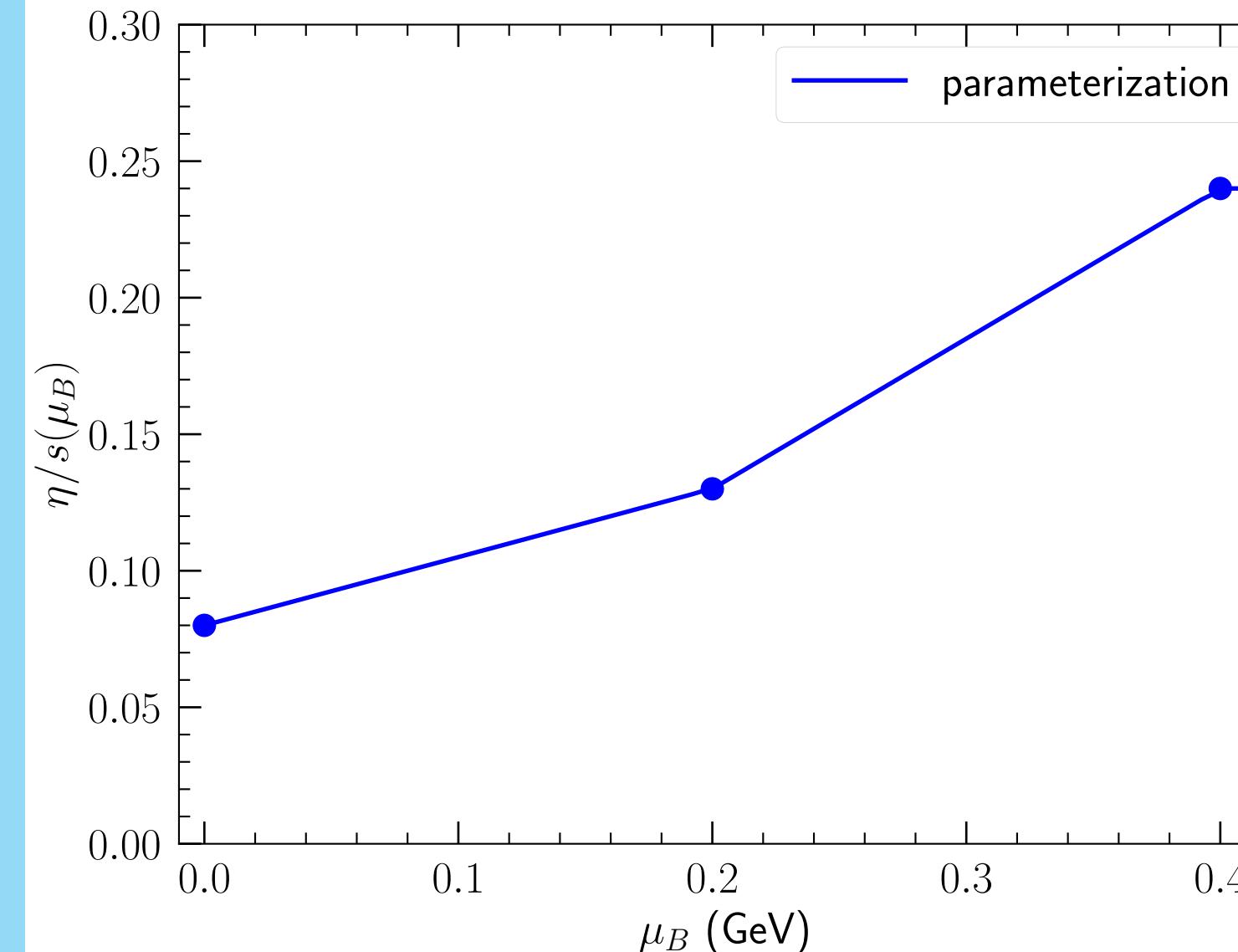


Imposed conservation for energy, momentum, and net baryon density

# 3D HYDRODYNAMICS WITH FINITE BARYON CURRENT

$$\partial_\mu T^{\mu\nu} = J^\nu_{\text{source}} + \partial_\mu J^\mu = \rho_{\text{source}}$$

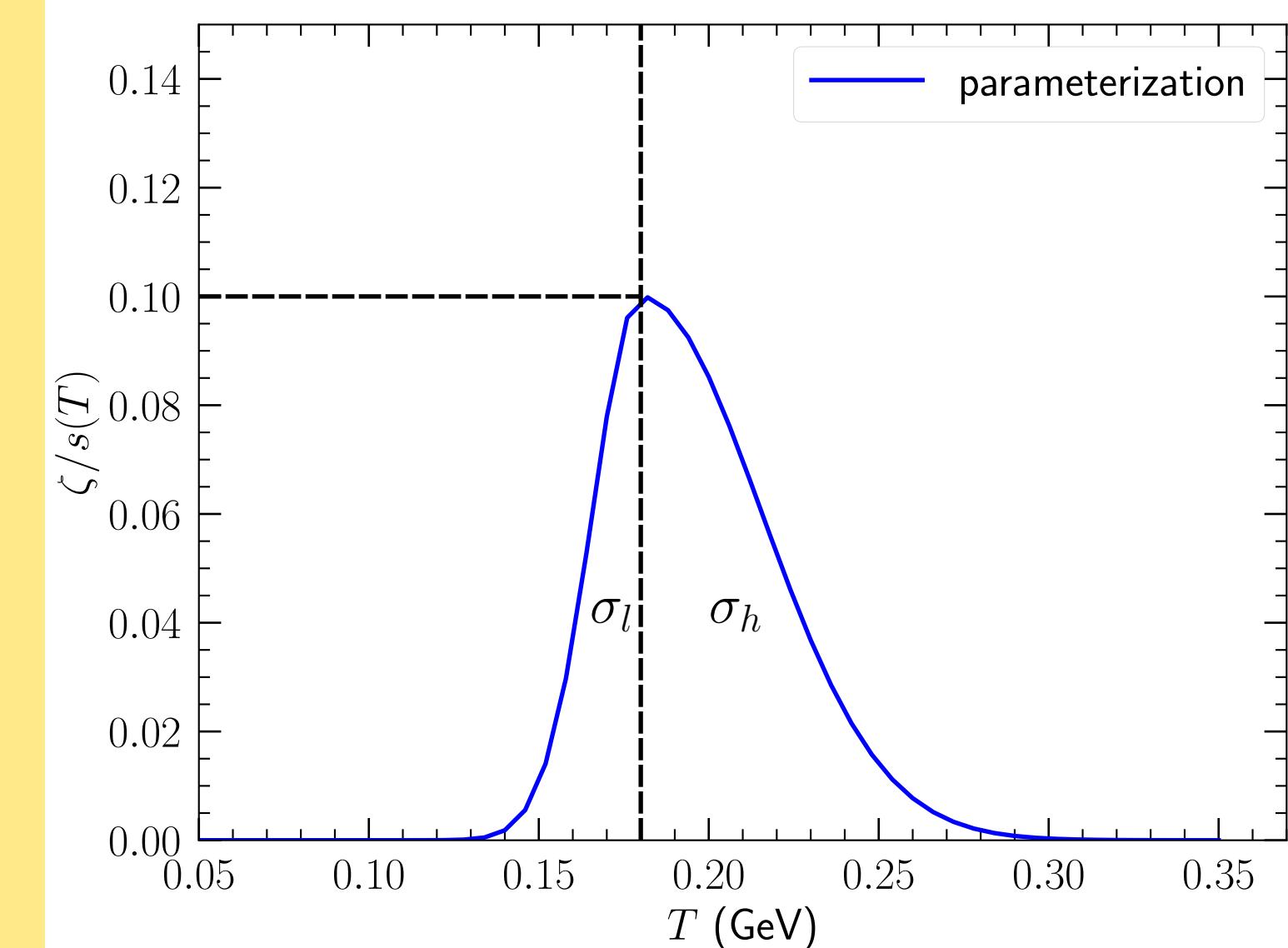
$\eta/s(\mu_B)$  has a piece-wise parameterization



A. Monnai, B. Schenke and C. Shen, Phys. Rev. C100, 024907 (2019)

SEE G. PIHAN'S TALK  
(TUE 12:00) FOR AN  
EXTENSION TO 4D EOS  
 $P(e, n_B, n_Q, n_S)$

$\zeta/s(T)$  is parameterized with a two-piece asymmetric Gaussian



# MODEL TRAINING & OBSERVABLE SELECTION

A 20-dimensional model parameter space with 1,000 training points

Au+Au	Hydro events per design	Avg. hadronic events per hydro
200 GeV	1,000	1,000
19.6 GeV	2,000	4,000
7.7 GeV	2,000	8,000

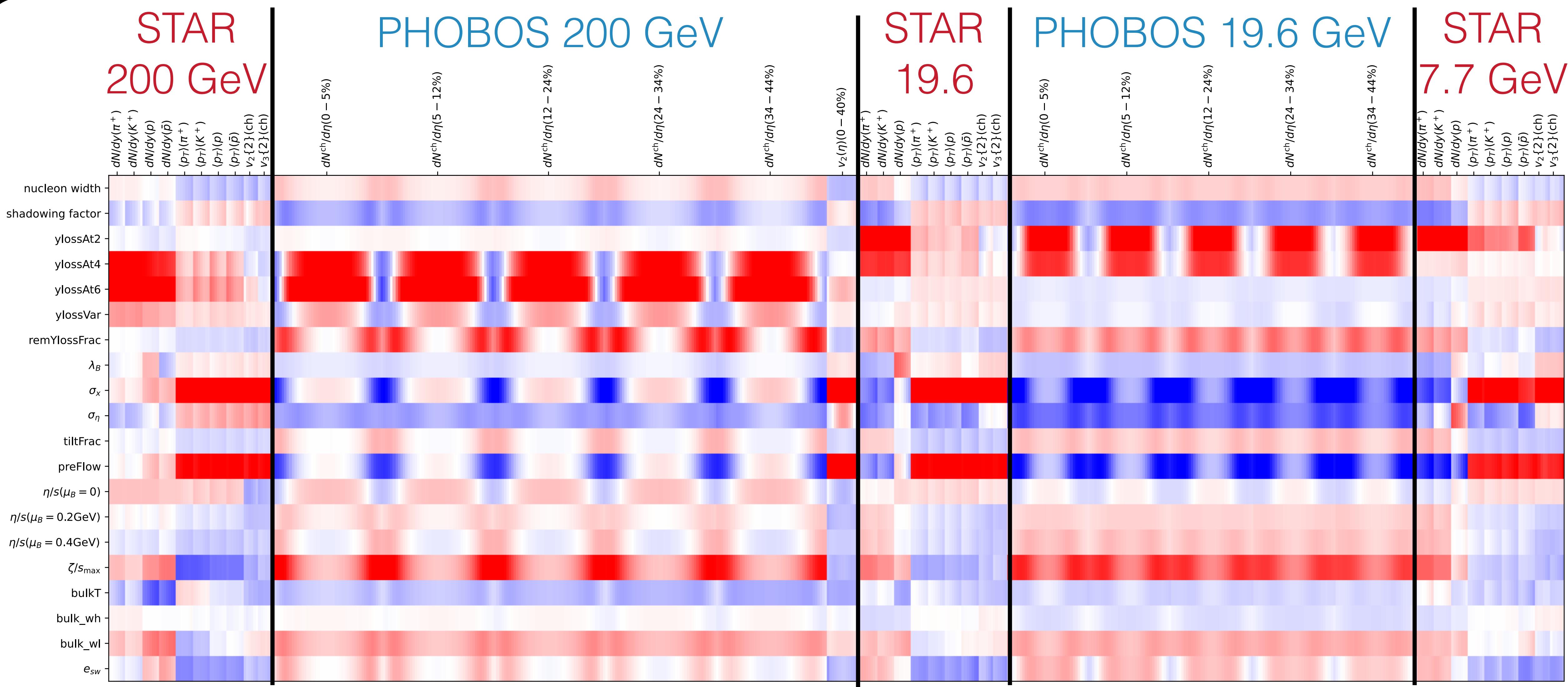
 Open Science Grid delivered 5 million CPU hours for the data generation

604 experimental data points

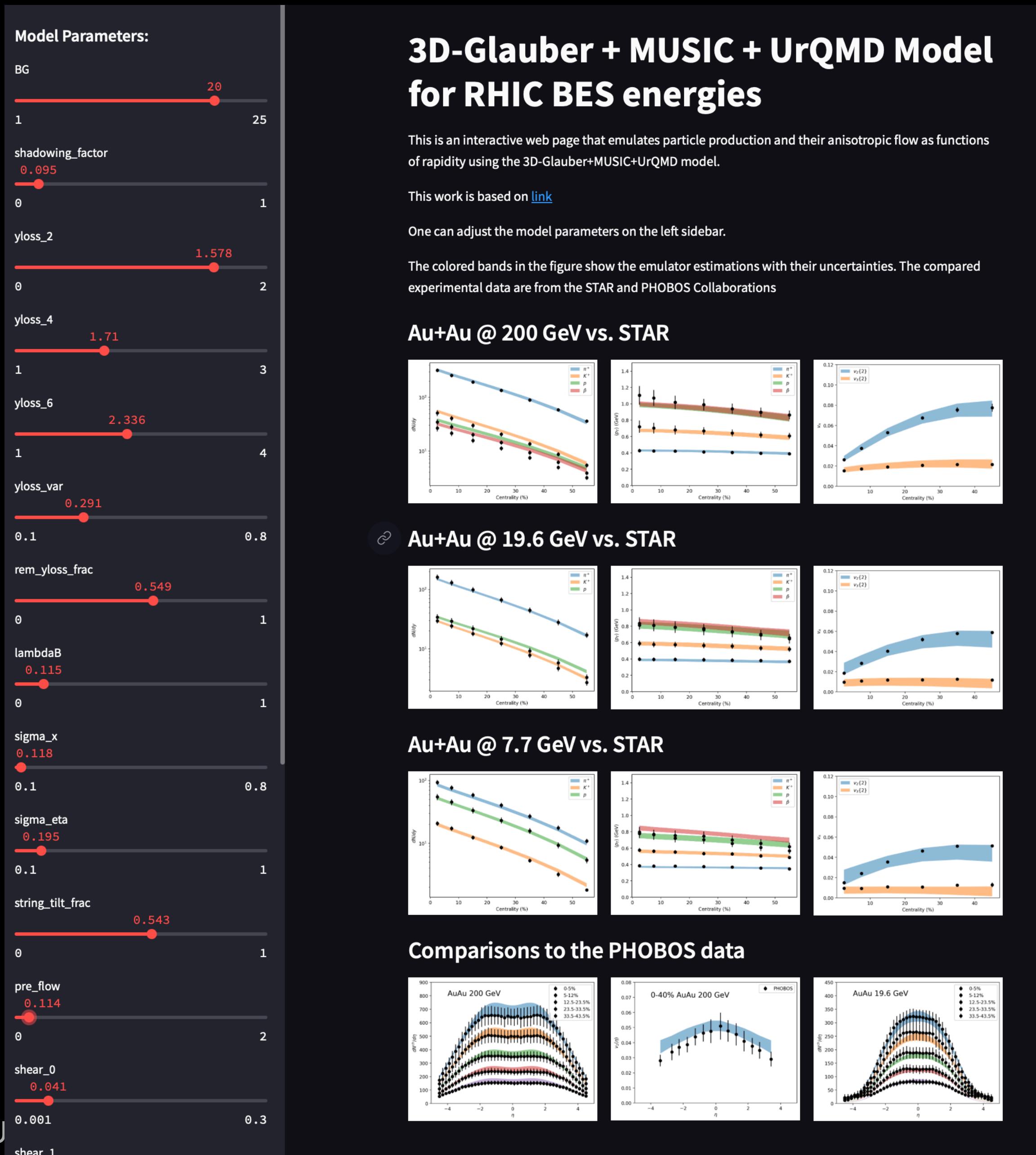
STAR Au+Au 200 GeV	midrapdity data vs. centrality	PHOBOS rapidity distribtion $dN^{\text{ch}}/d\eta$ $v_2(\eta)$
19.6 GeV	$dN/dy(\pi^+, K^+, p, \bar{p})$ $\langle p_T \rangle(\pi^+, K^+, p, \bar{p})$ $v_2^{\text{ch}}\{2\}, v_3^{\text{ch}}\{2\}$	
7.7 GeV	$dN/dy(\pi^+, K^+, p)$ $\langle p_T \rangle(\pi^+, K^+, p, \bar{p})$ $v_2^{\text{ch}}\{2\}, v_3^{\text{ch}}\{2\}$	$dN^{\text{ch}}/d\eta$

Phys. Rev. C79, 034909 (2009) Phys. Rev. C98, 034918 (2018)

# OBSERVABLE RESPONSES TO MODEL PARAMETERS



# PUBLIC INTERACTIVE MODEL EMULATOR



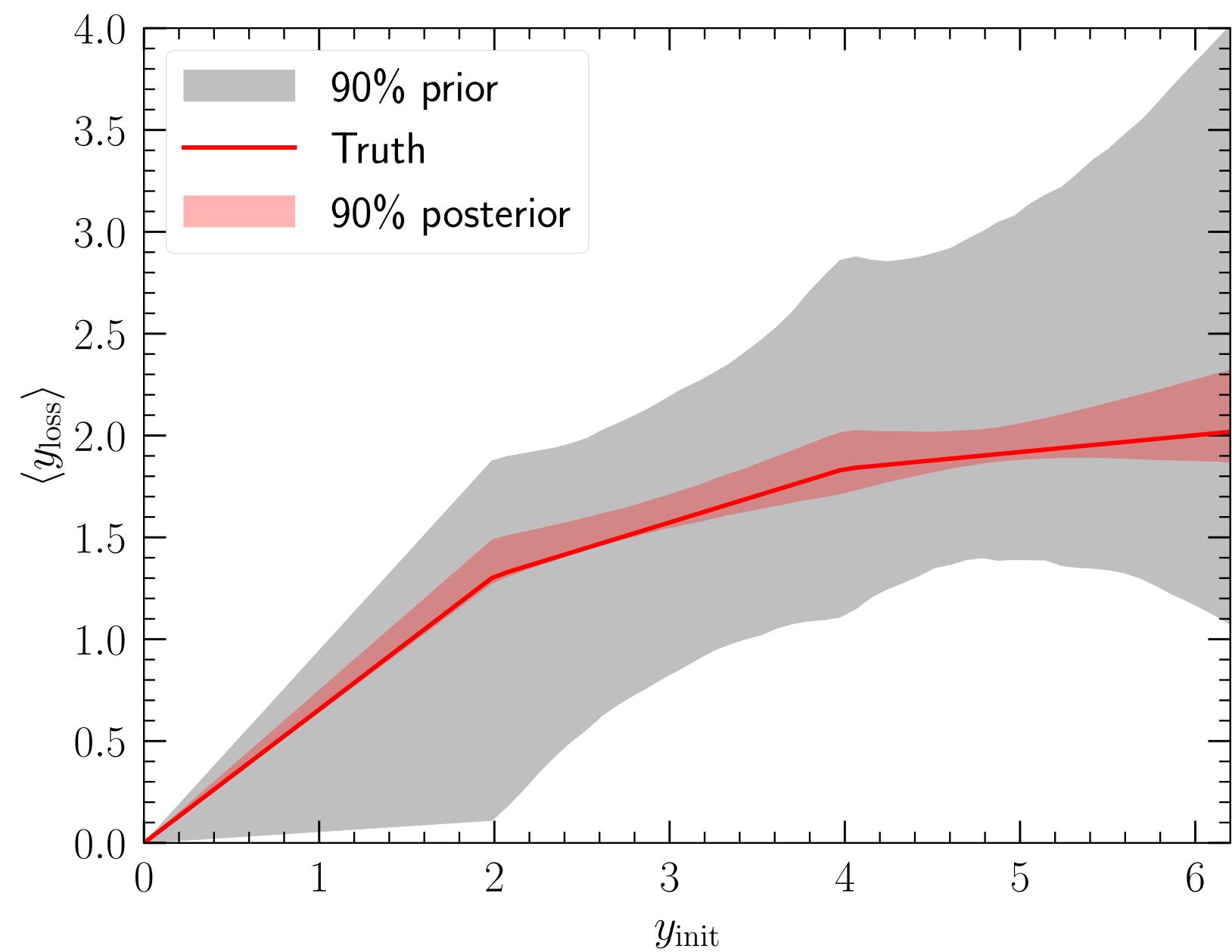
link



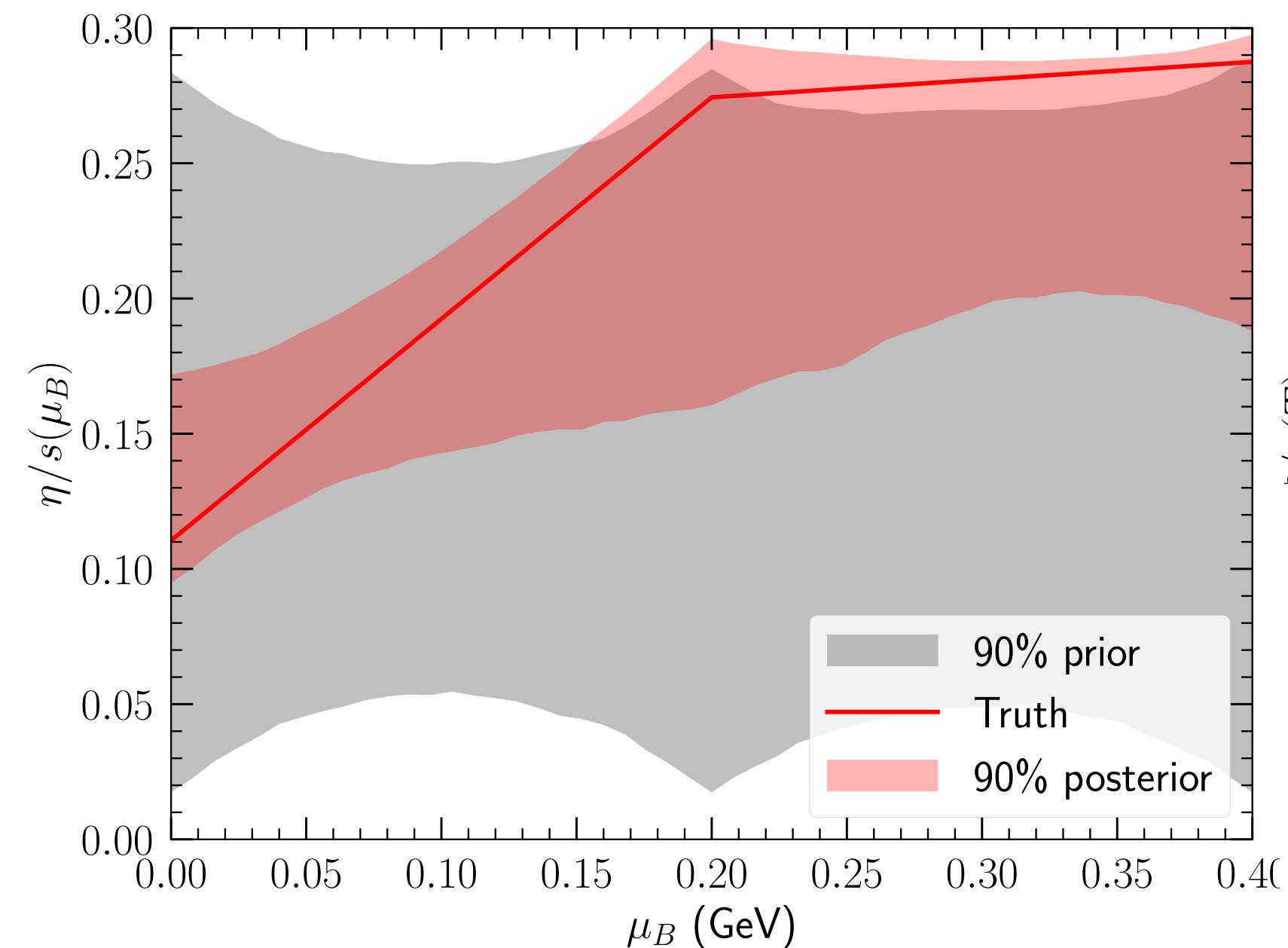
- An interactive webpage for emulating the full (3+1)D simulations for RHIC BES energies at real time
- Build your own intuition for heavy-ion phenomenology!

# BAYESIAN VALIDATION: CLOSURE TEST

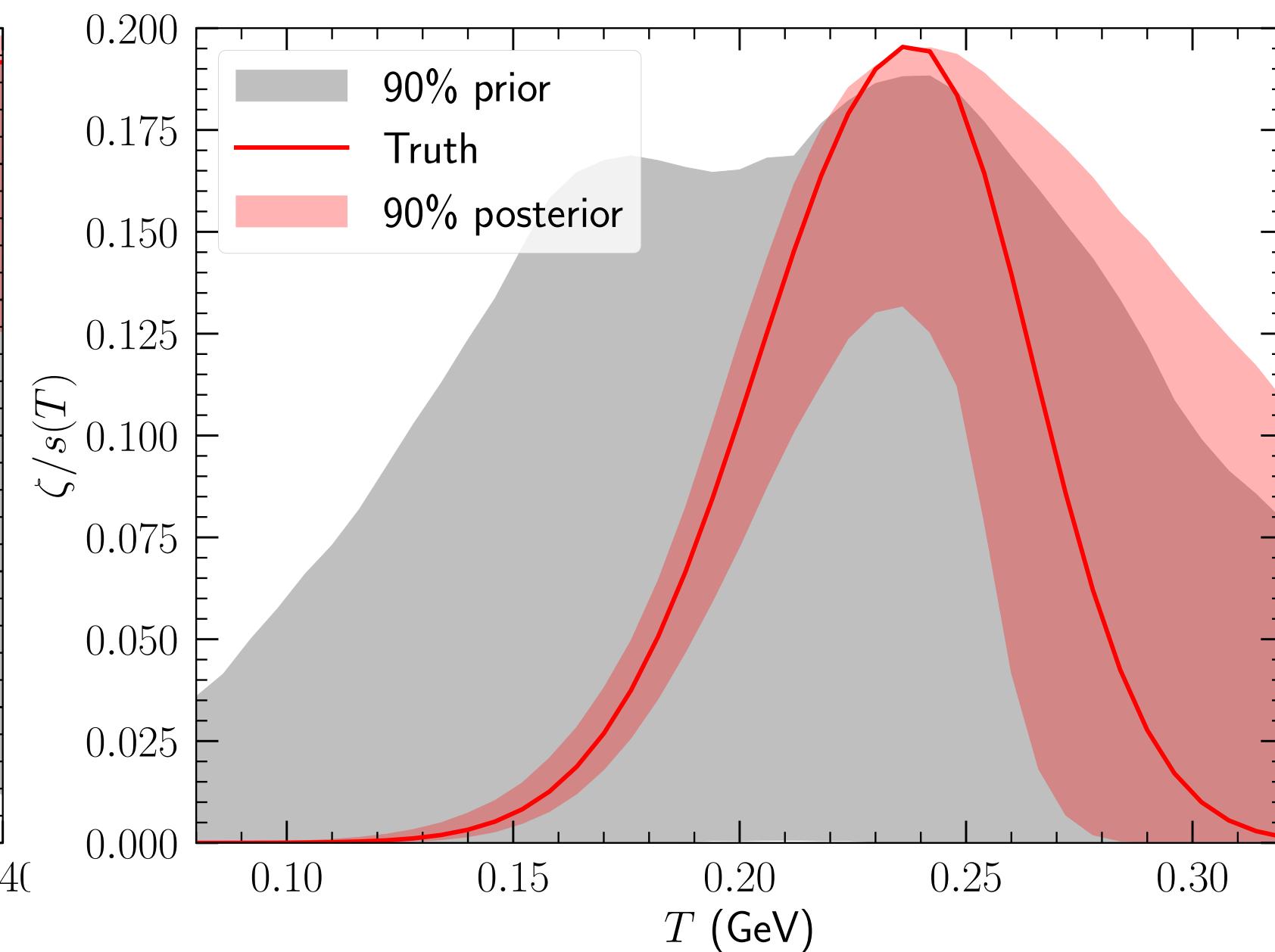
## Initial-state stopping



## $\eta/s(\mu_B)$



## $\zeta/s(T)$



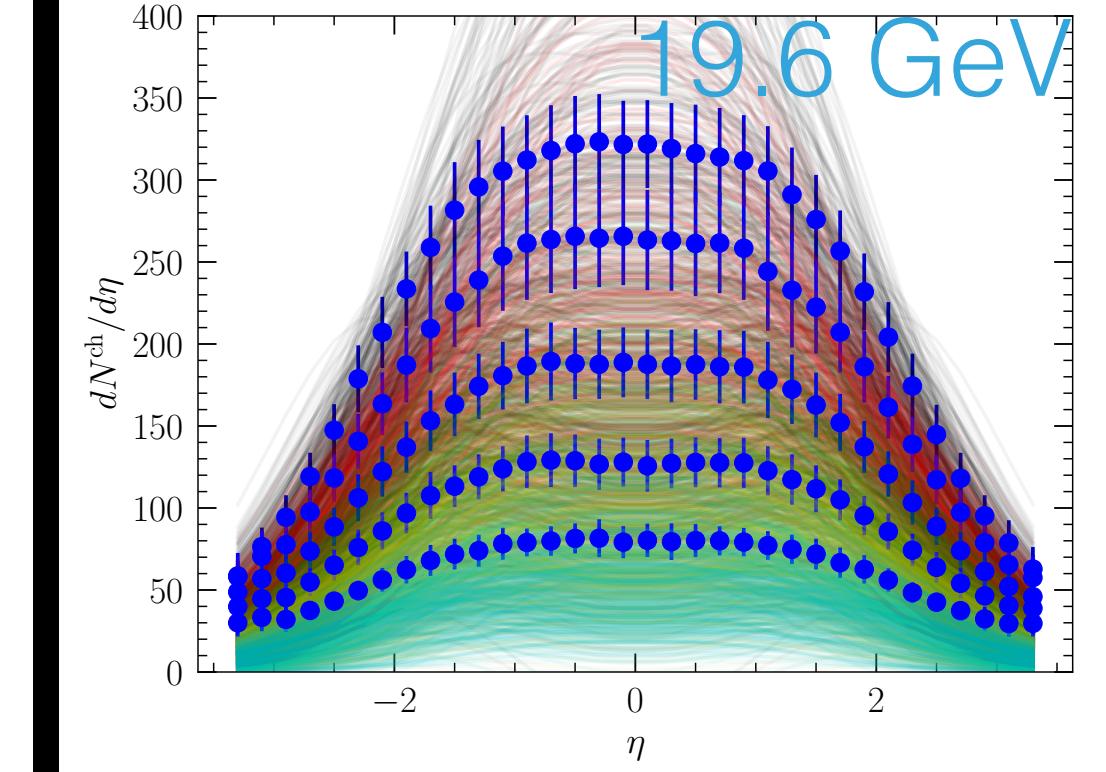
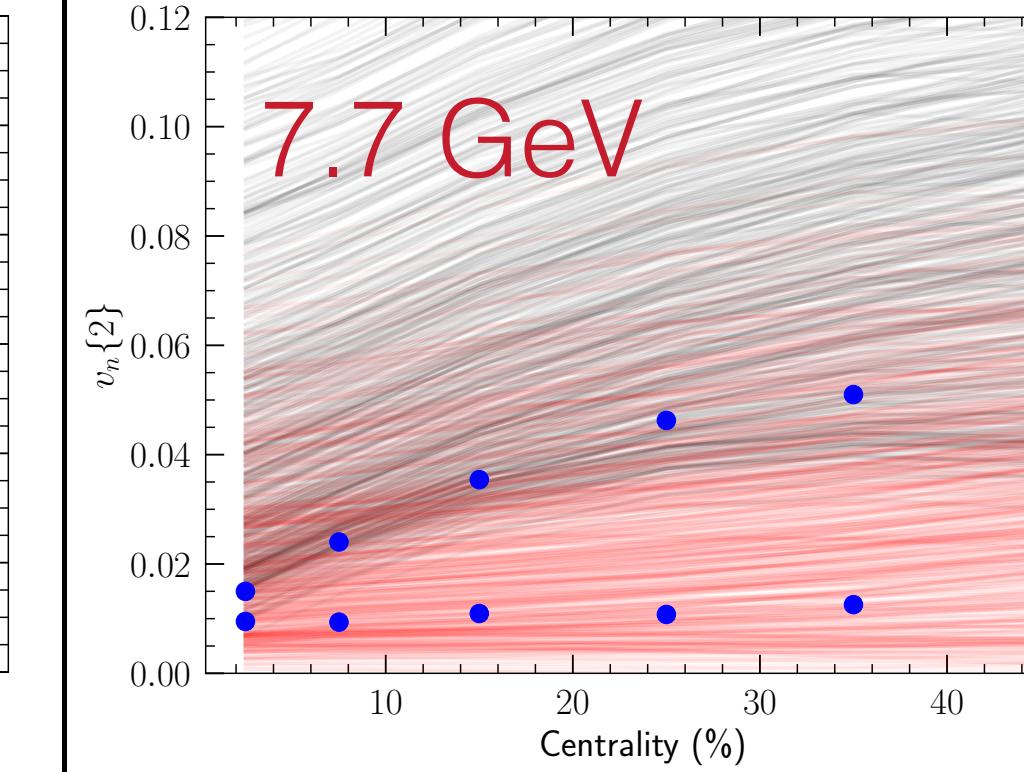
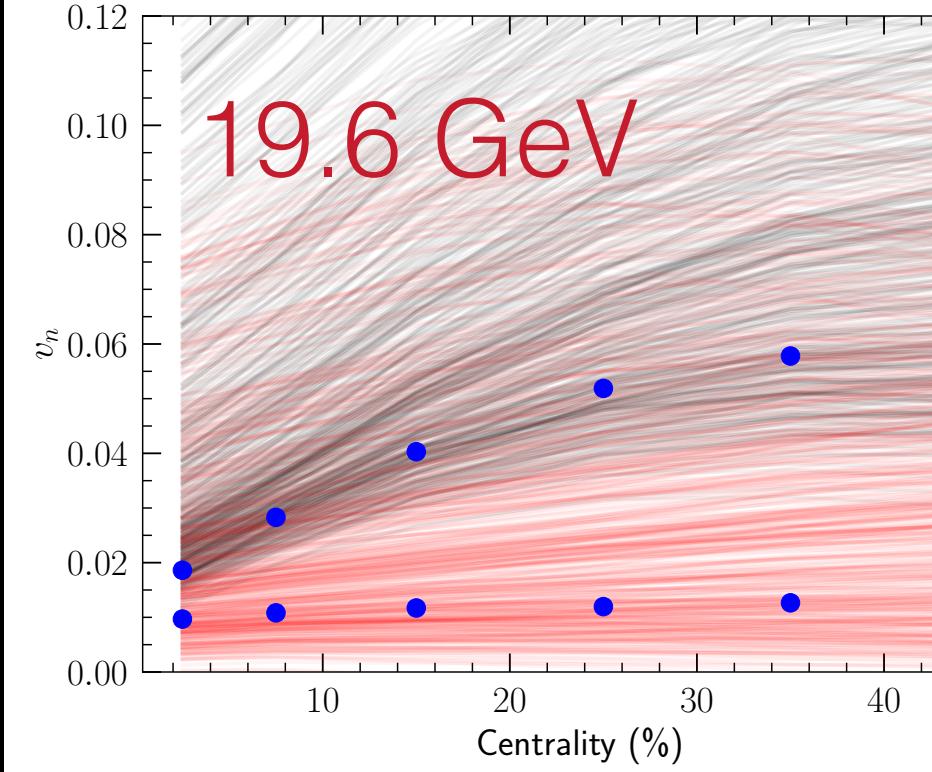
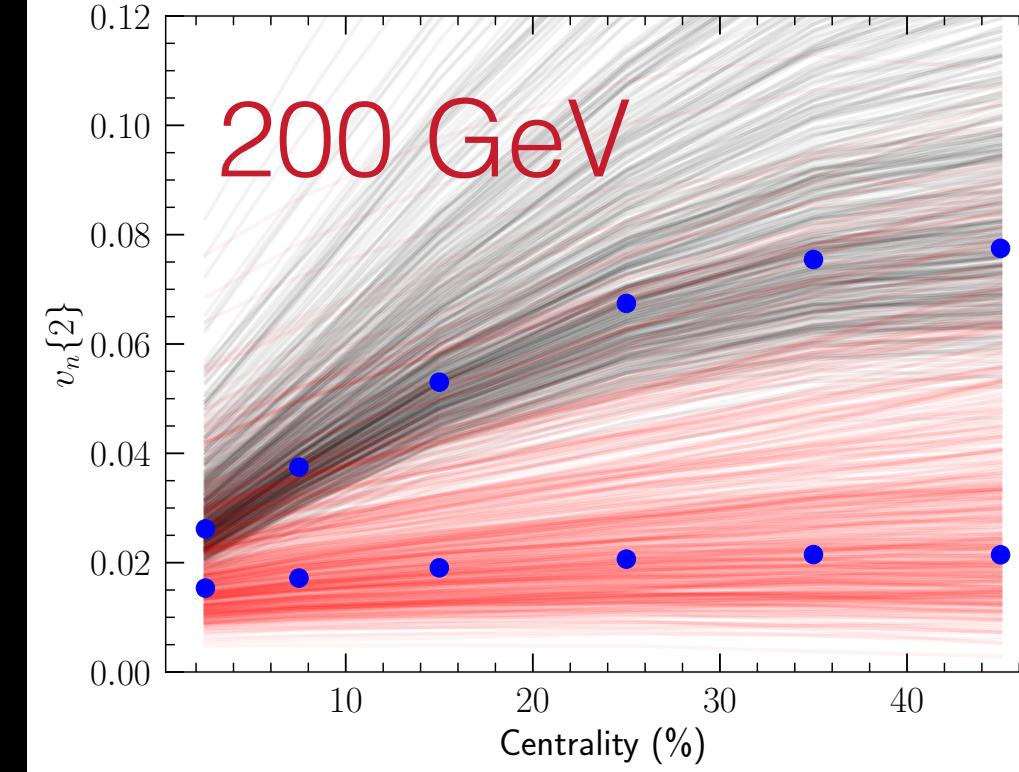
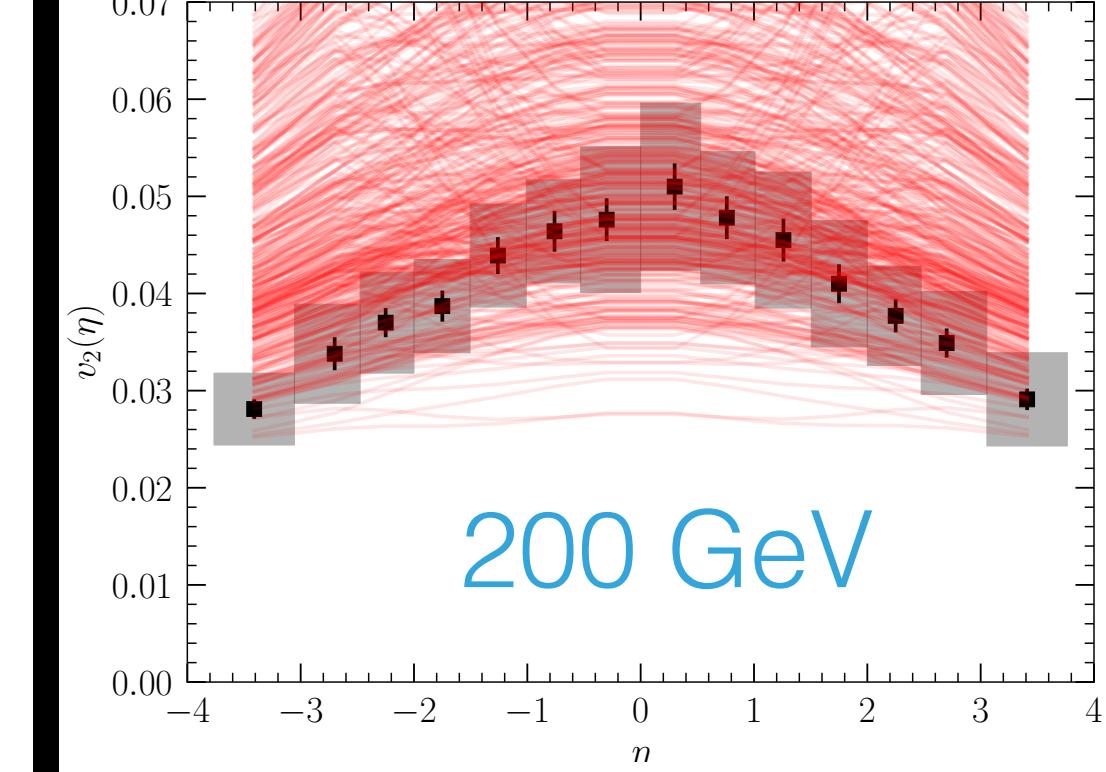
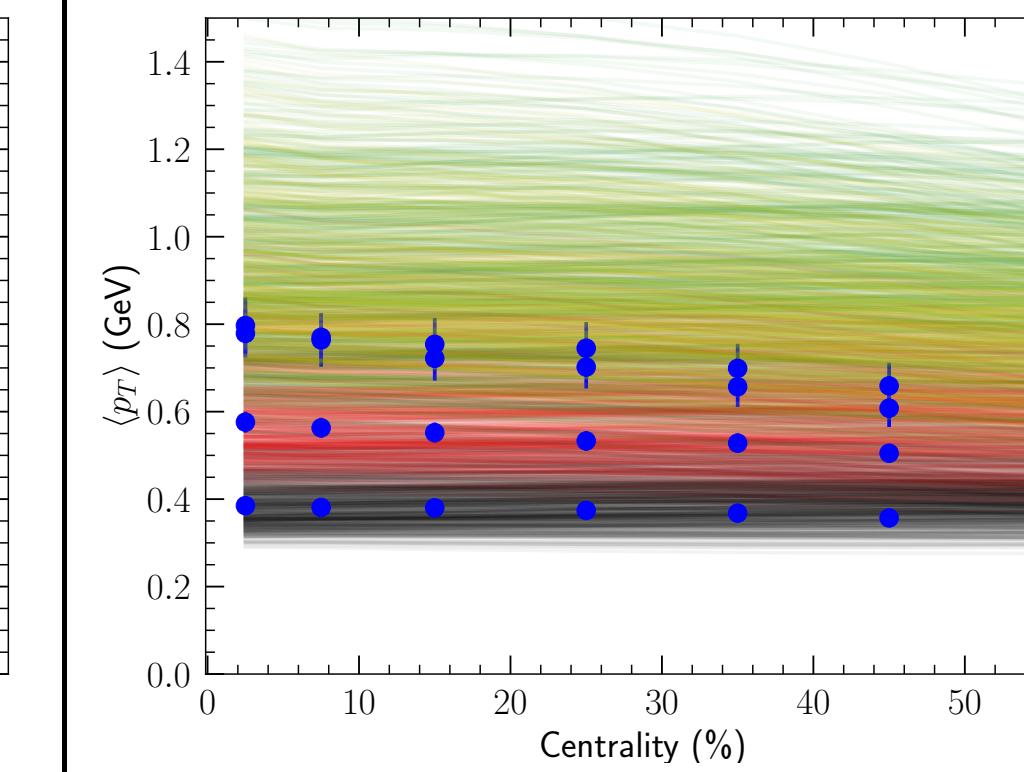
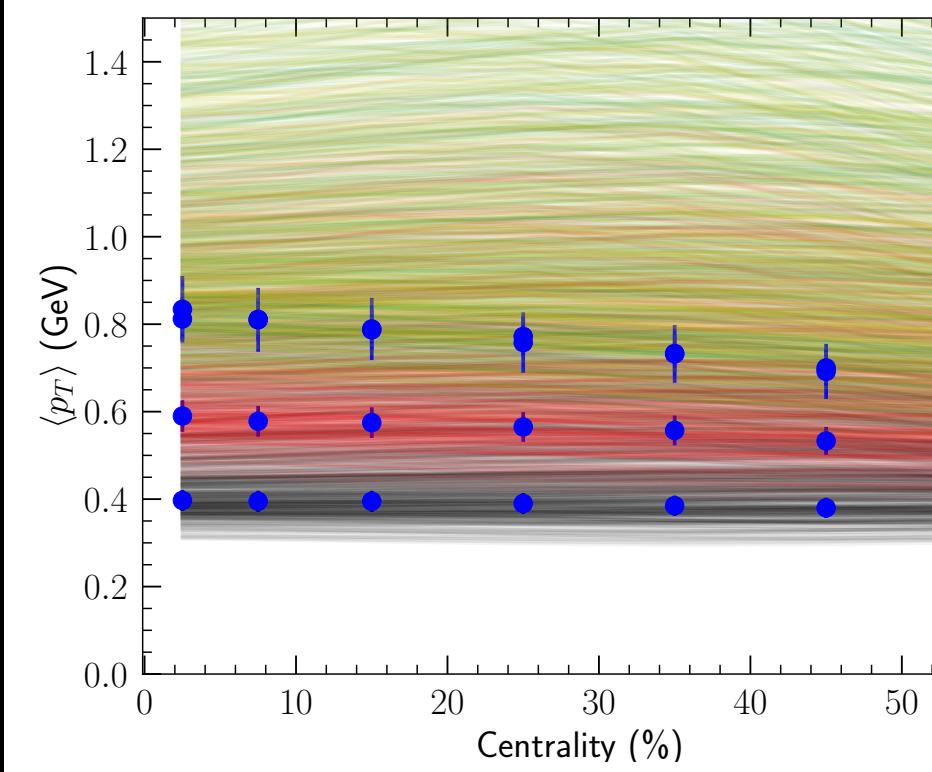
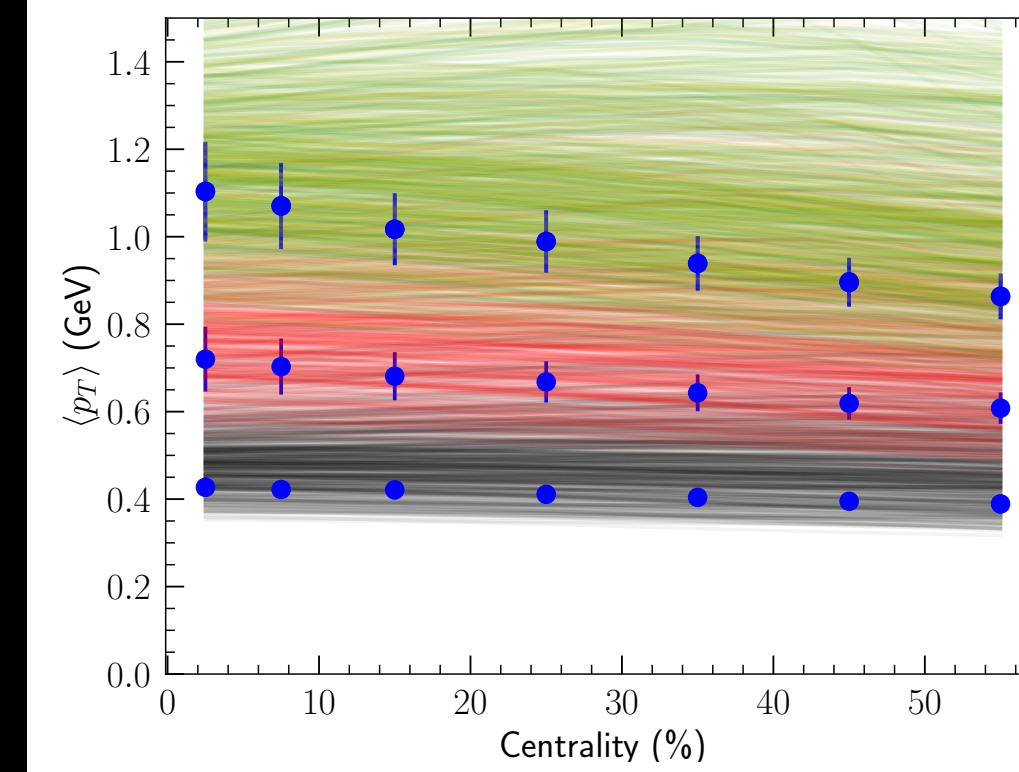
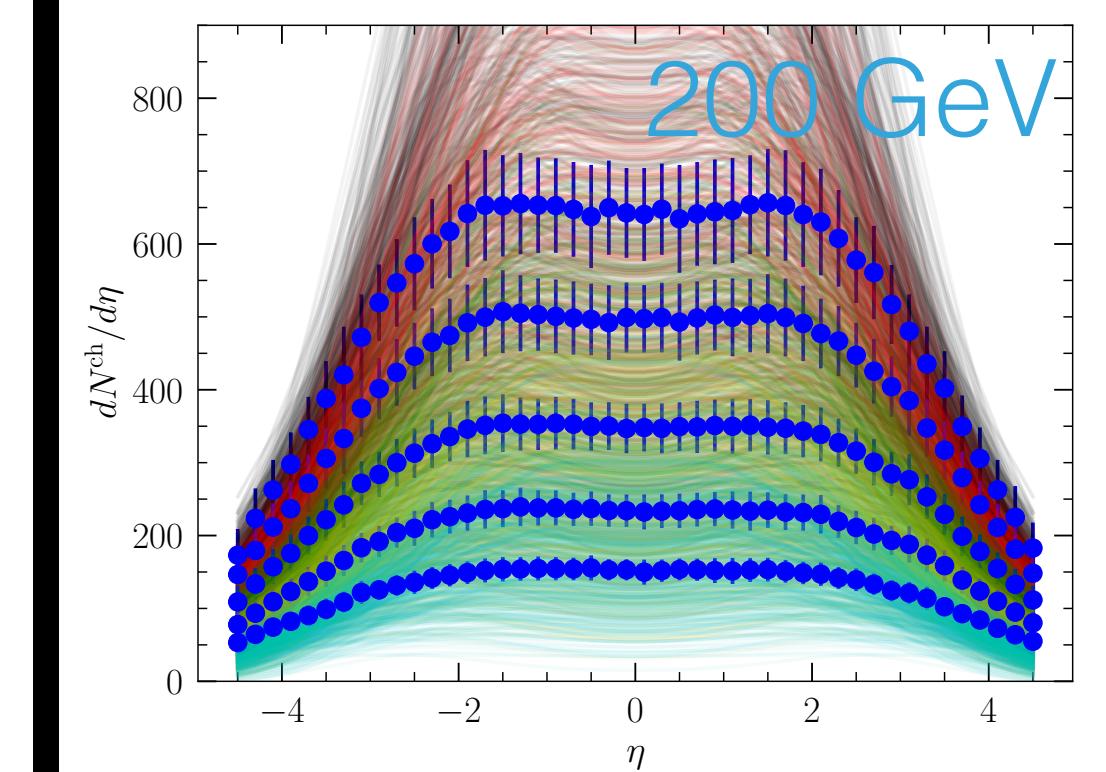
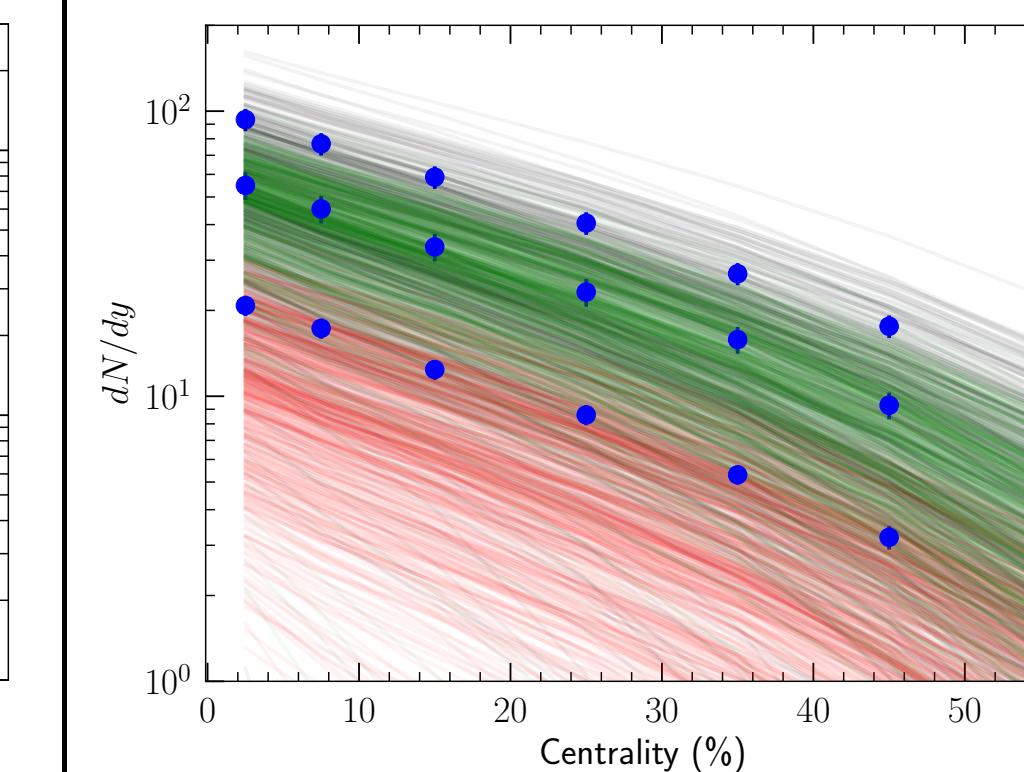
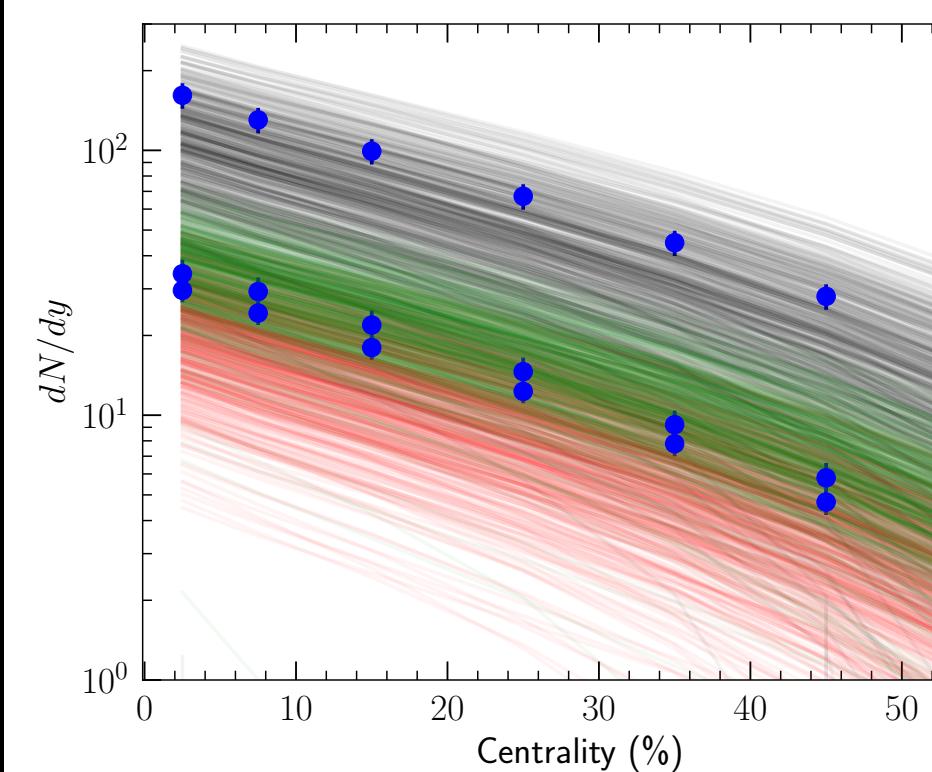
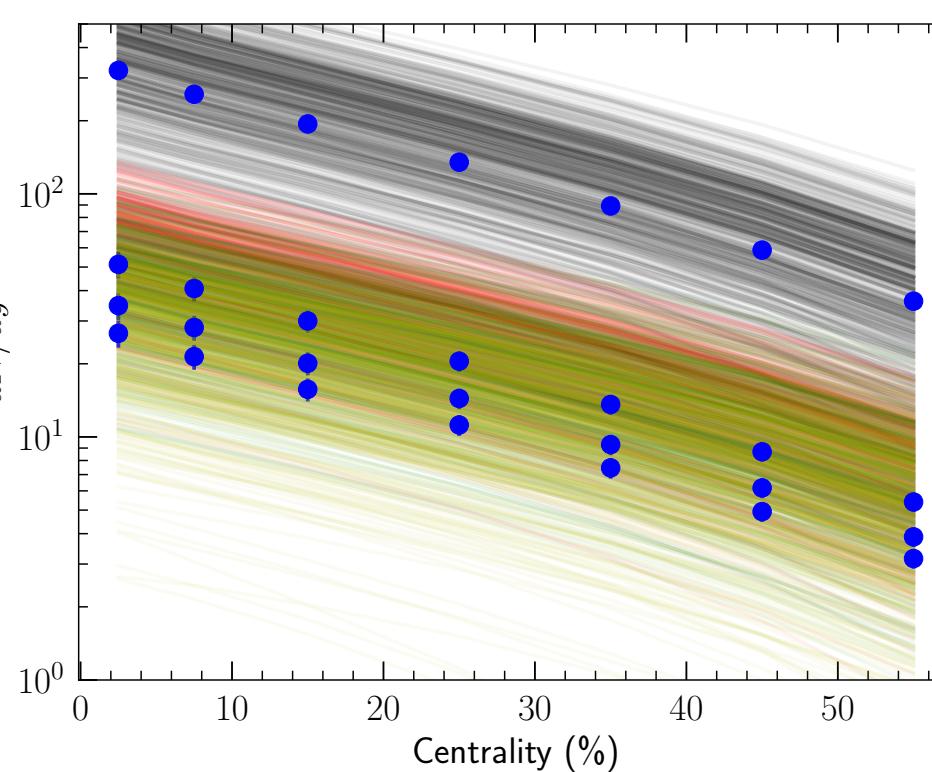
- Model emulation with Markov Chain Monte Carlo (MCMC) is verified with a closure test for initial-state stopping  $y_{\text{loss}}(y_{\text{init}})$ ,  $\eta/s(\mu_B)$ , and  $\zeta/s(T)$
- The selected observables can give strong constraints on the QGP properties at RHIC BES energies



PRIOR

STAR

# BAYESIAN INFERENCE AT RHIC BES ENERGIES

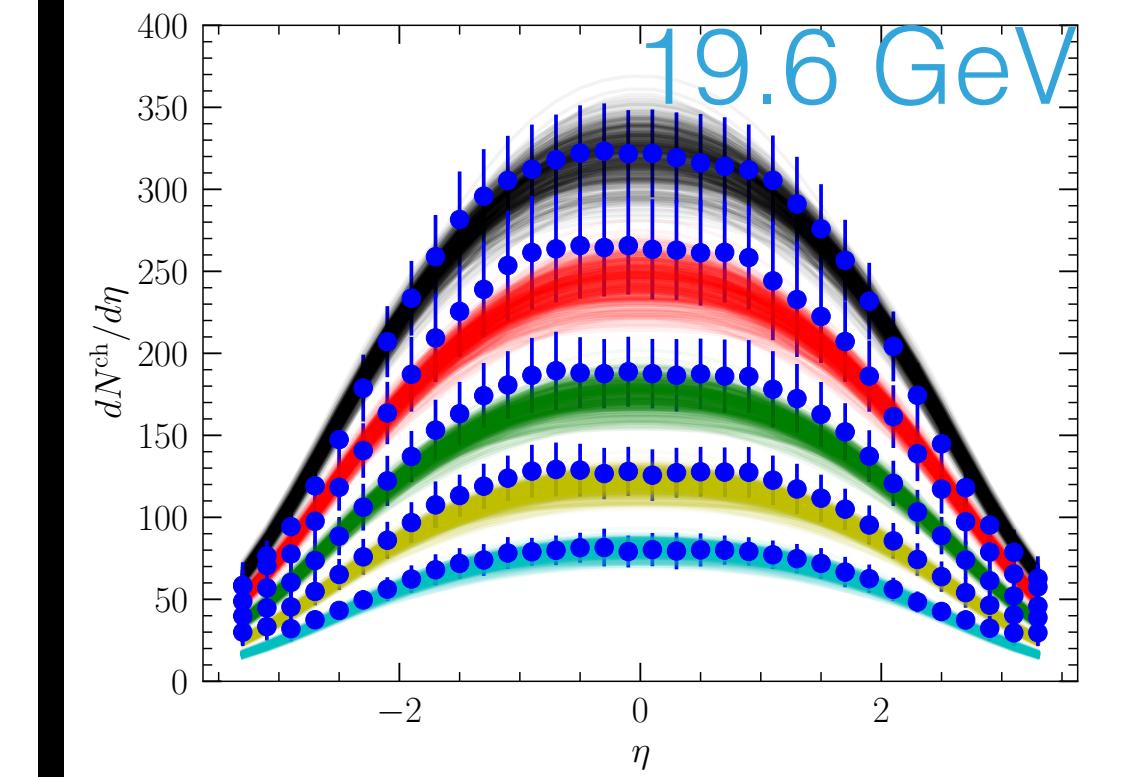
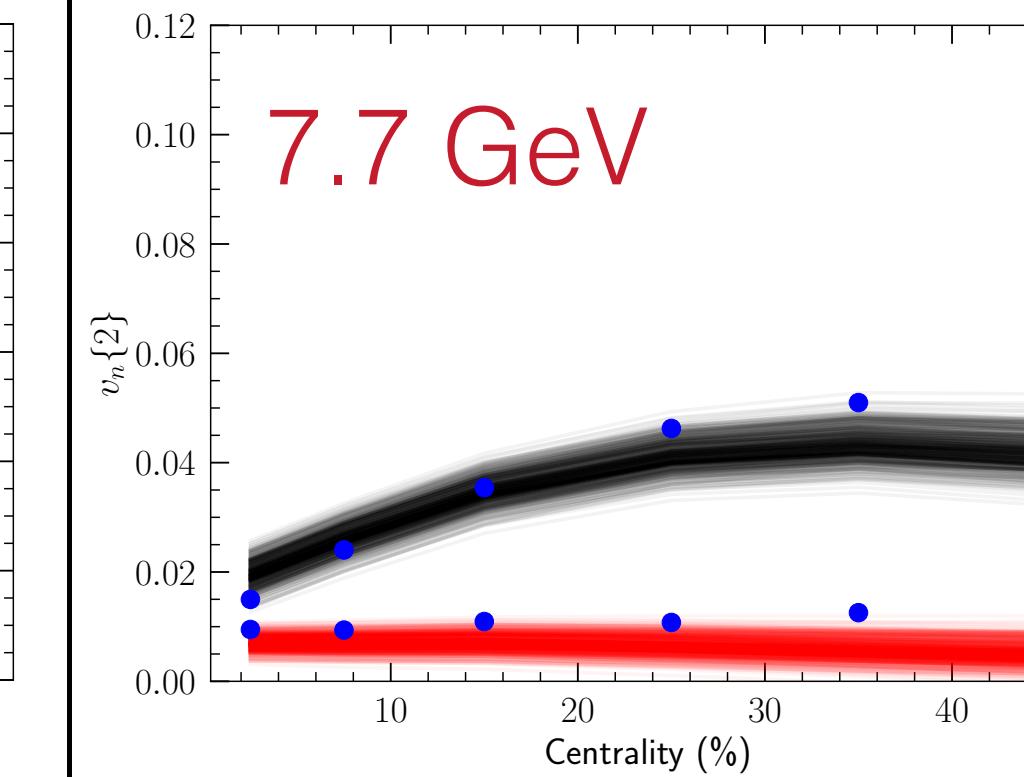
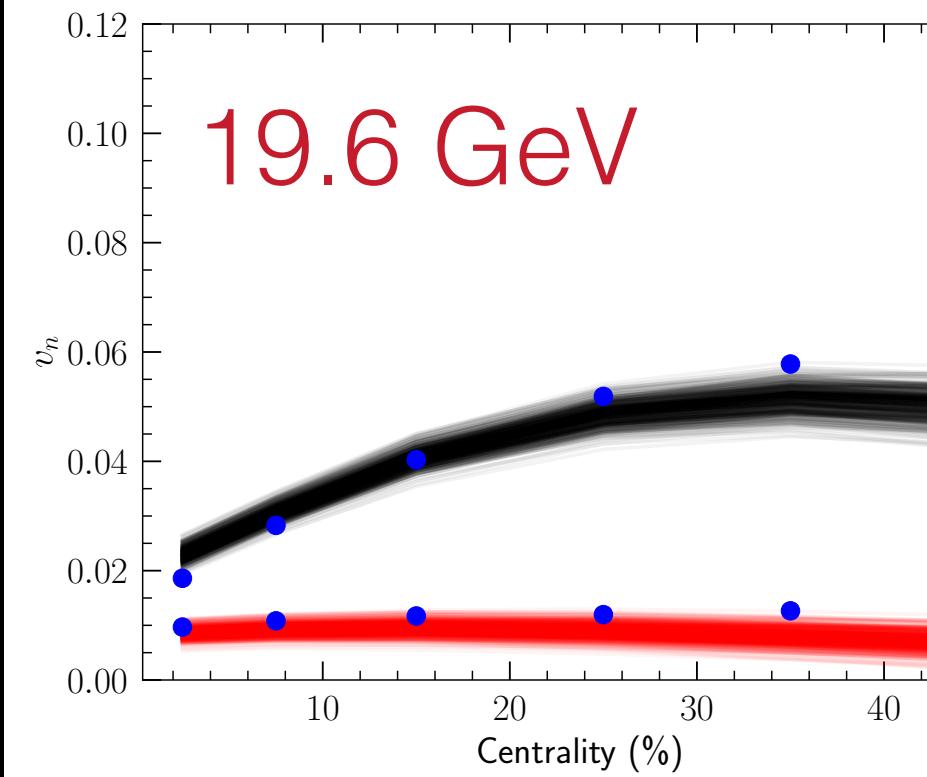
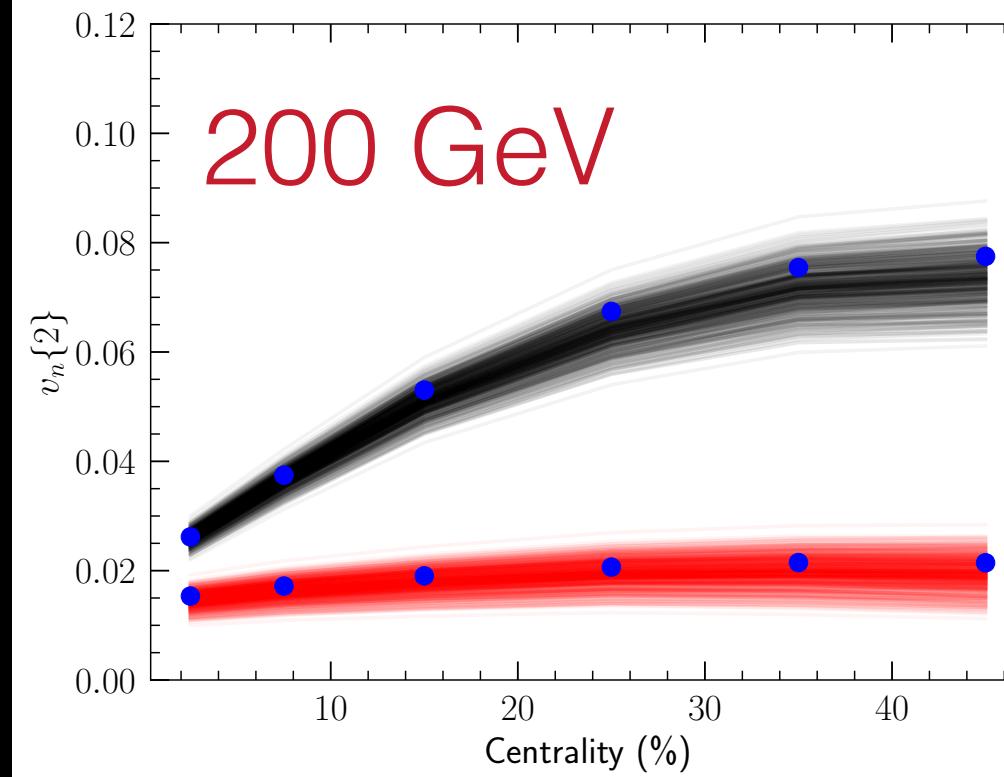
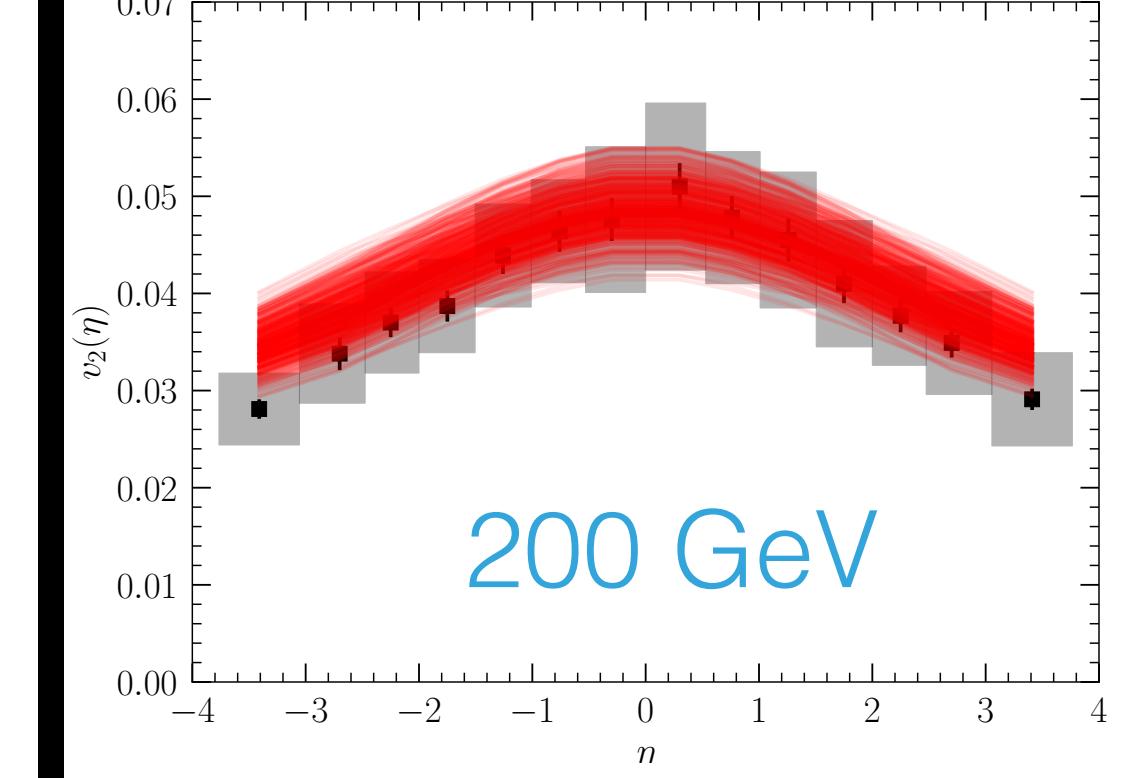
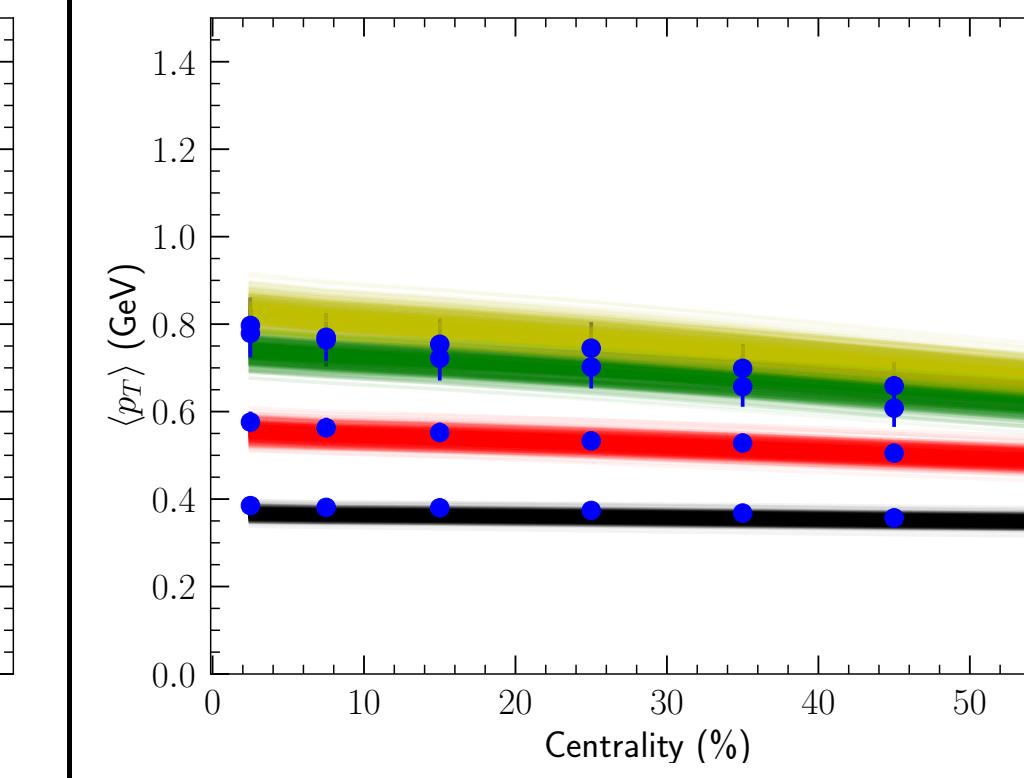
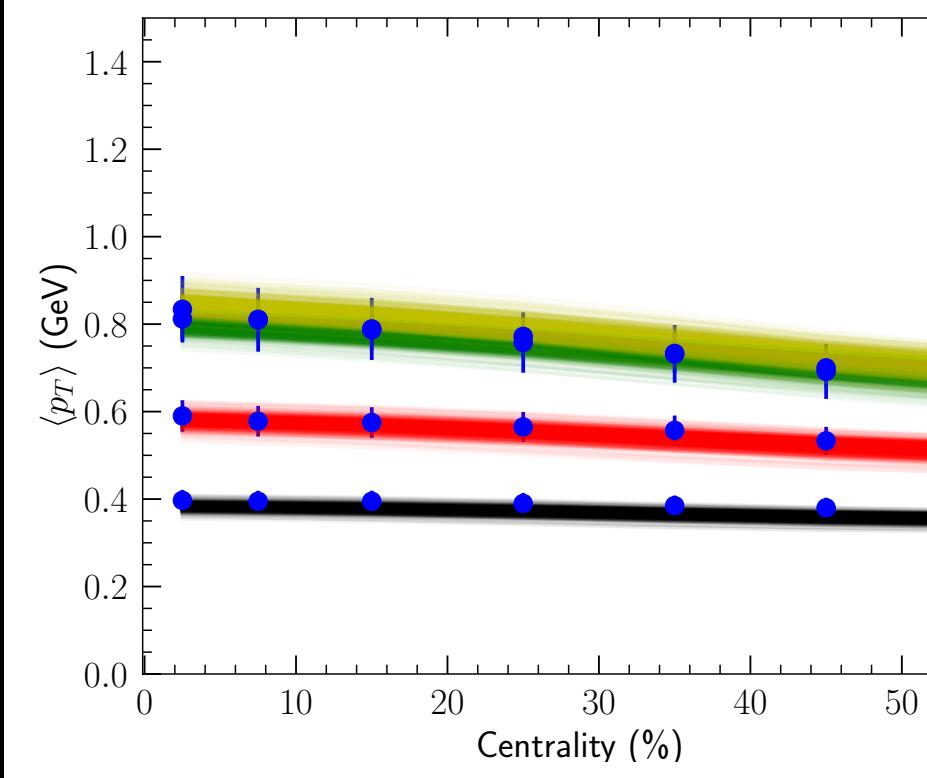
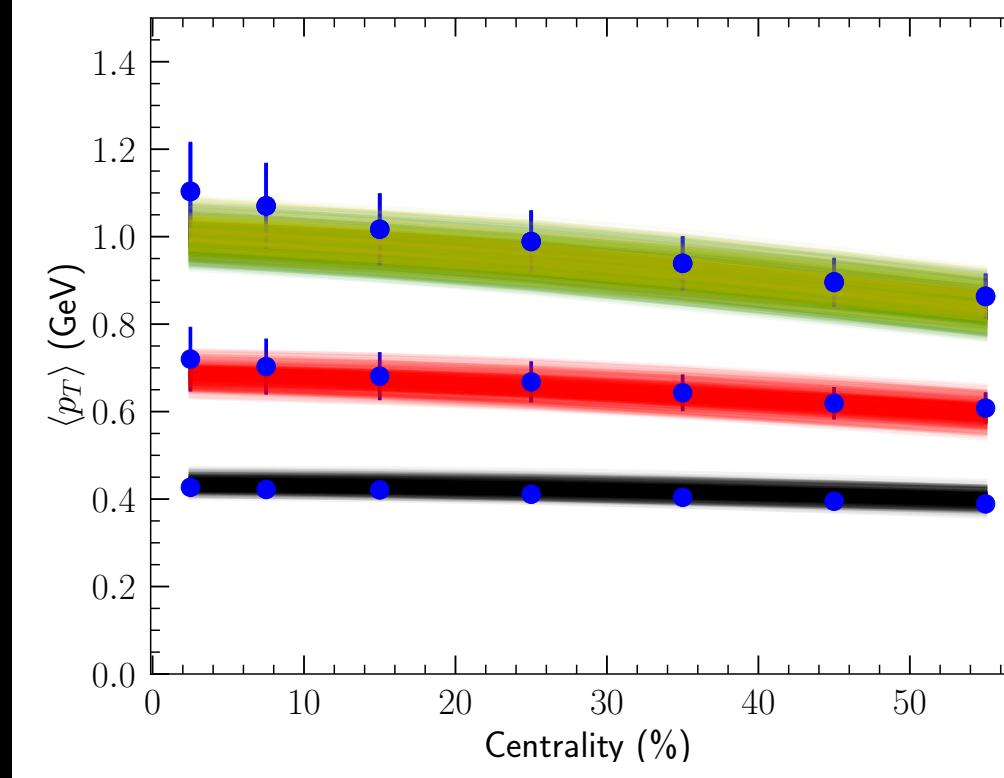
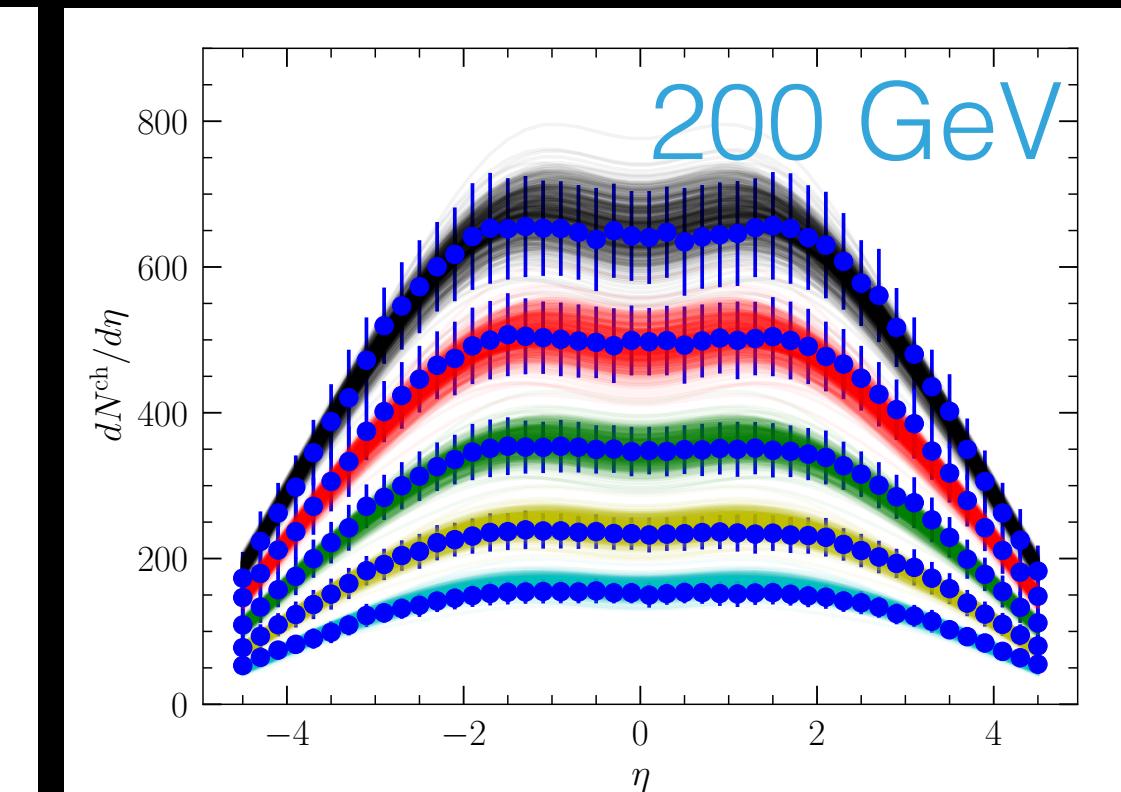
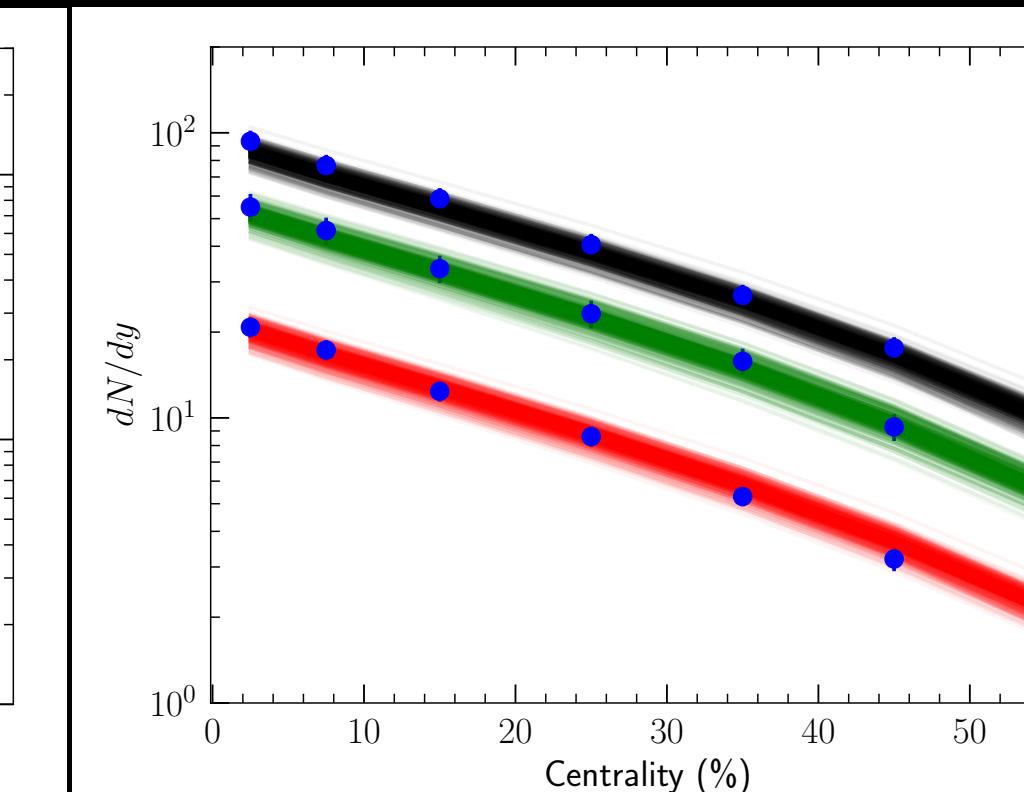
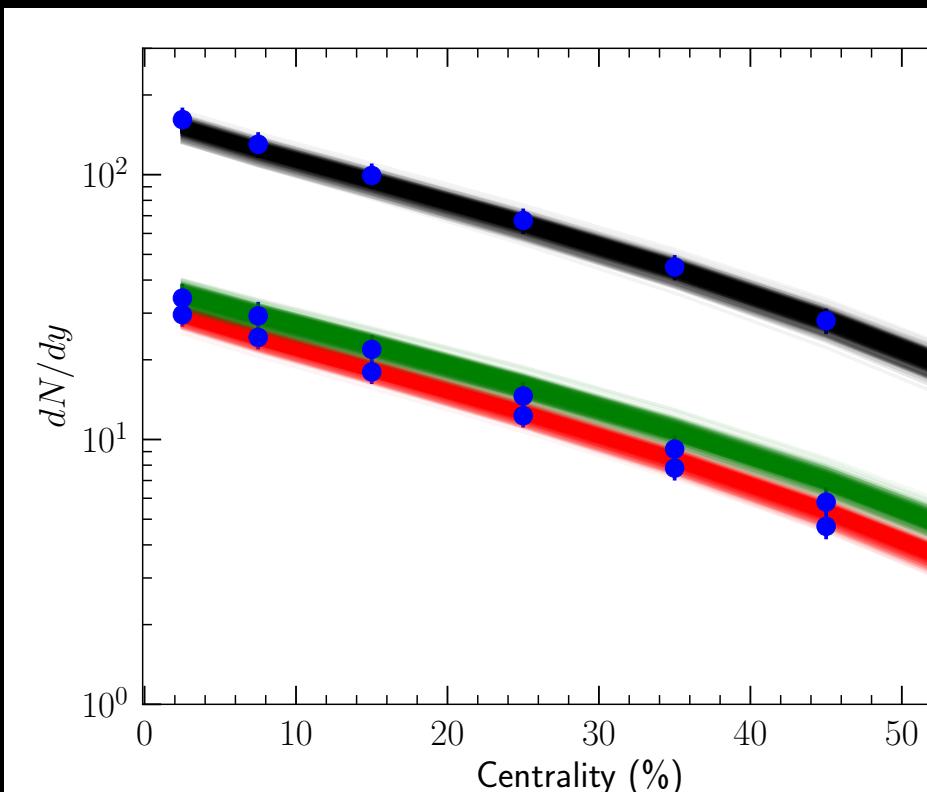
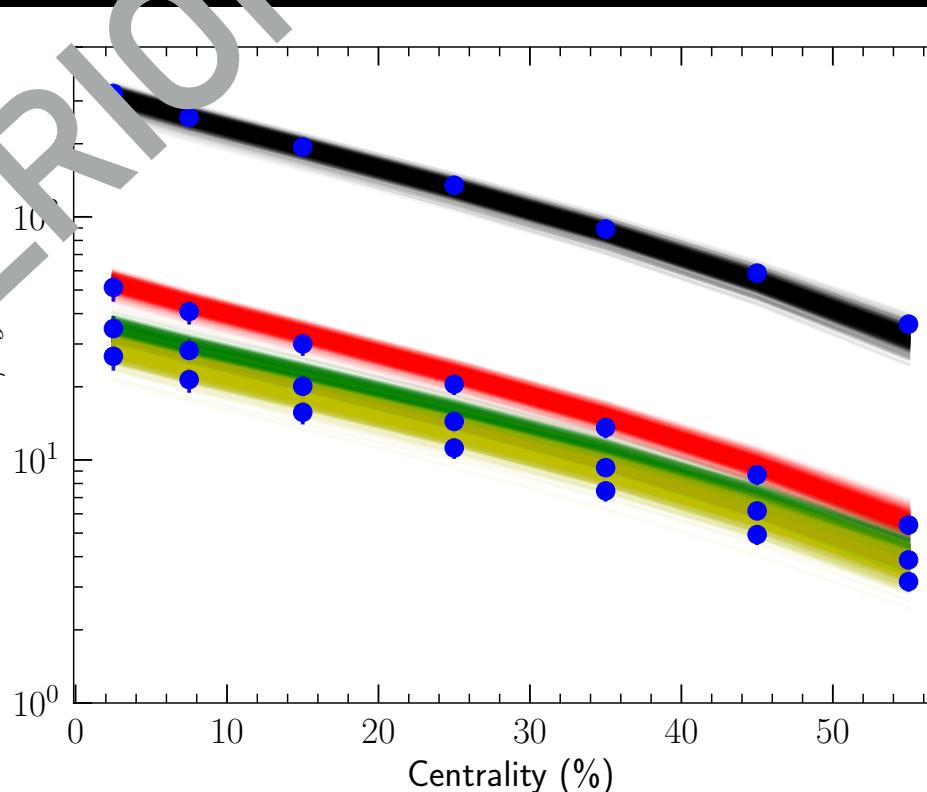


PHOBOS

# BAYESIAN INFERENCE AT RHIC BES ENERGIES

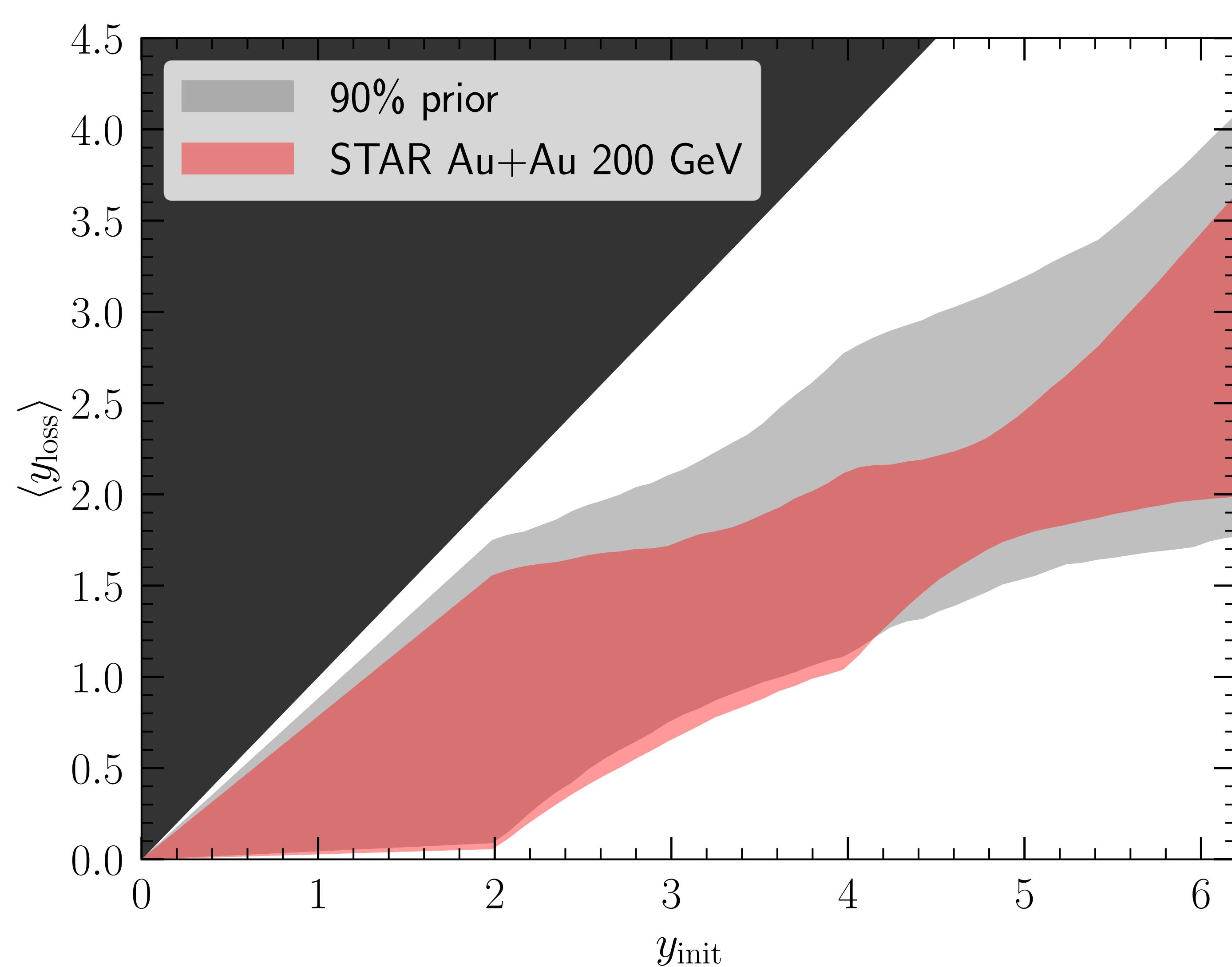
POSTERIOR

STAR



PHOBOS

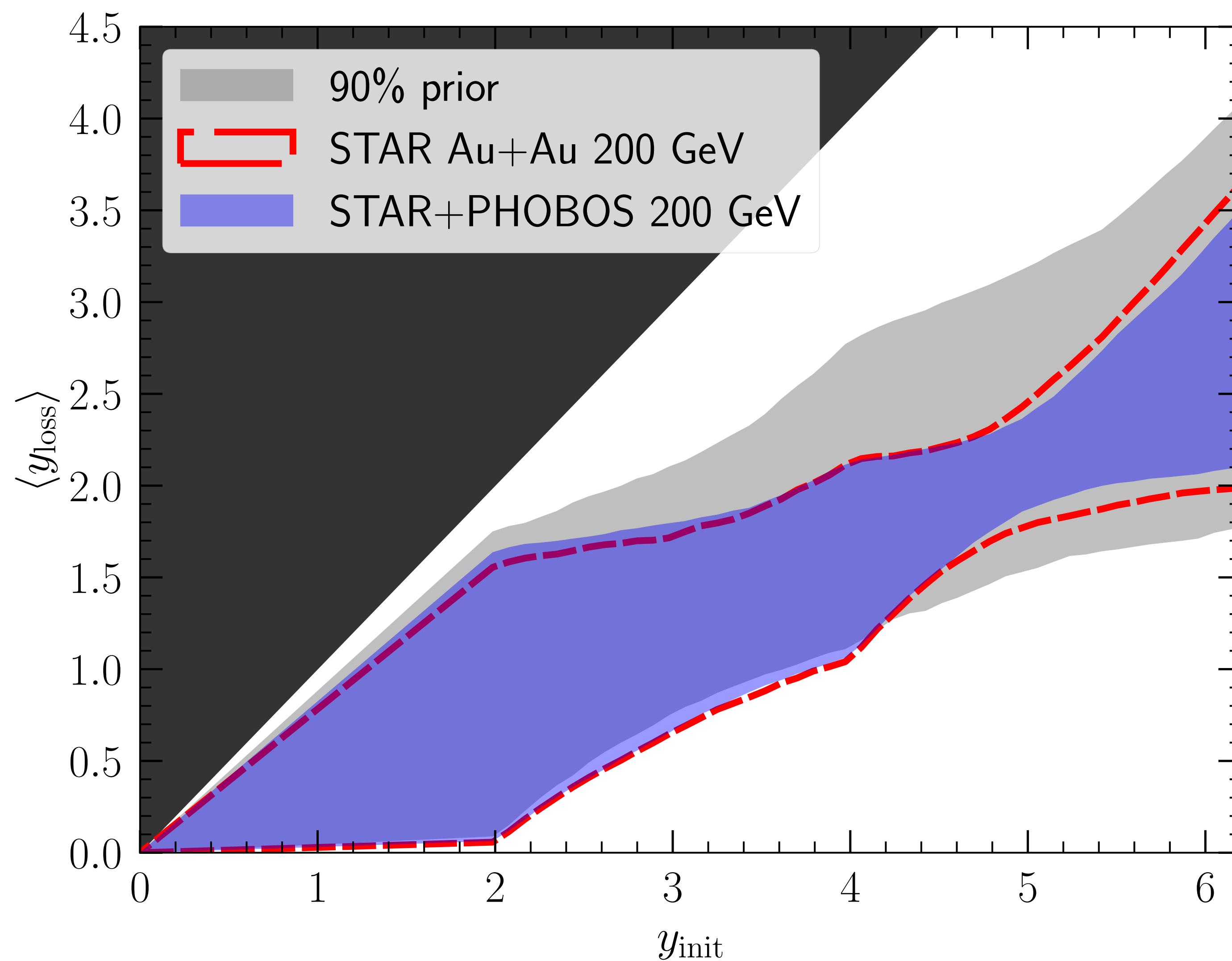
# INITIAL-STATE STOPPING



- Mid-rapidity particle productions at 200 GeV yields  $y_{\text{loss}} \sim 2$  for  $y_{\text{init}} \sim 5$

color bands indicate 90% credible interval in the posterior

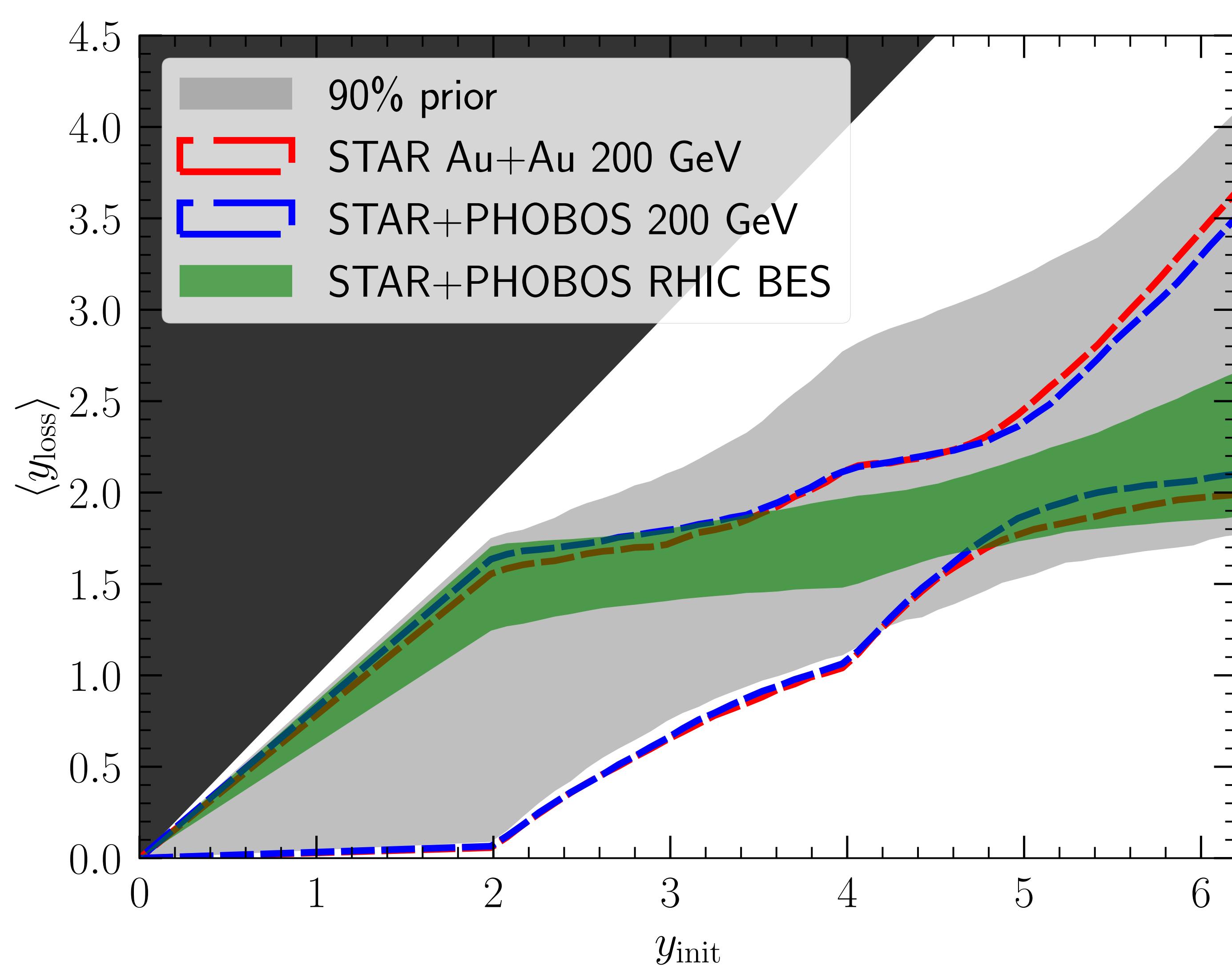
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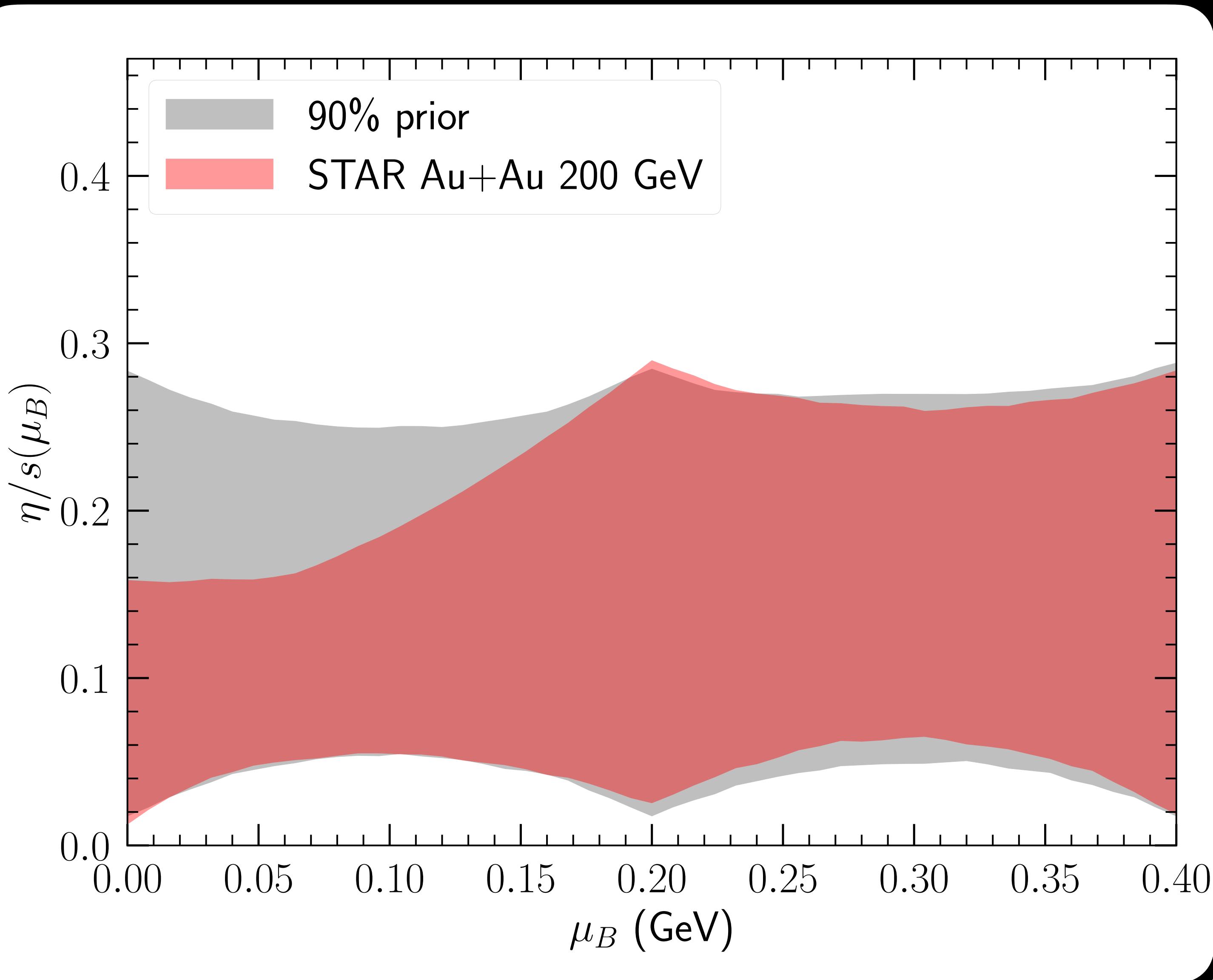
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- The rapidity distributions from PHOBOS give small improvements to the constraint
- Particle production from 7.7, 19.6, and 200 GeV sets **strong** constrain on  $y_{\text{loss}}(y_{\text{init}})$  for  $y_{\text{init}} \in [0,6]$

color bands indicate 90%  
credible interval in the posterior

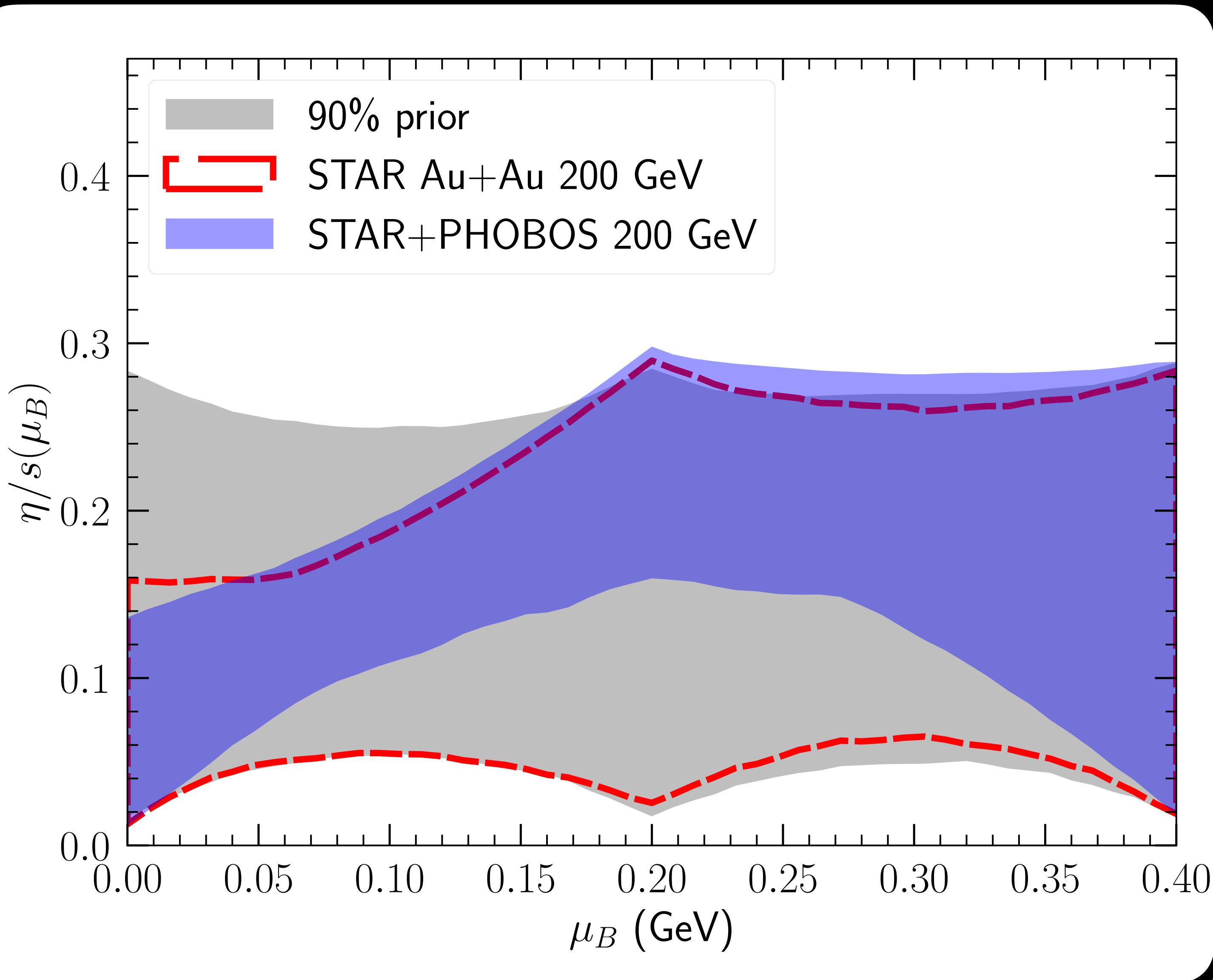
# SHEAR VISCOSITY $\eta/s(\mu_B)$



- Mid-rapidity data at 200 GeV can constrain  $\eta/s$  around  $\mu_B = 0$

color bands indicate 90%  
credible interval in the posterior

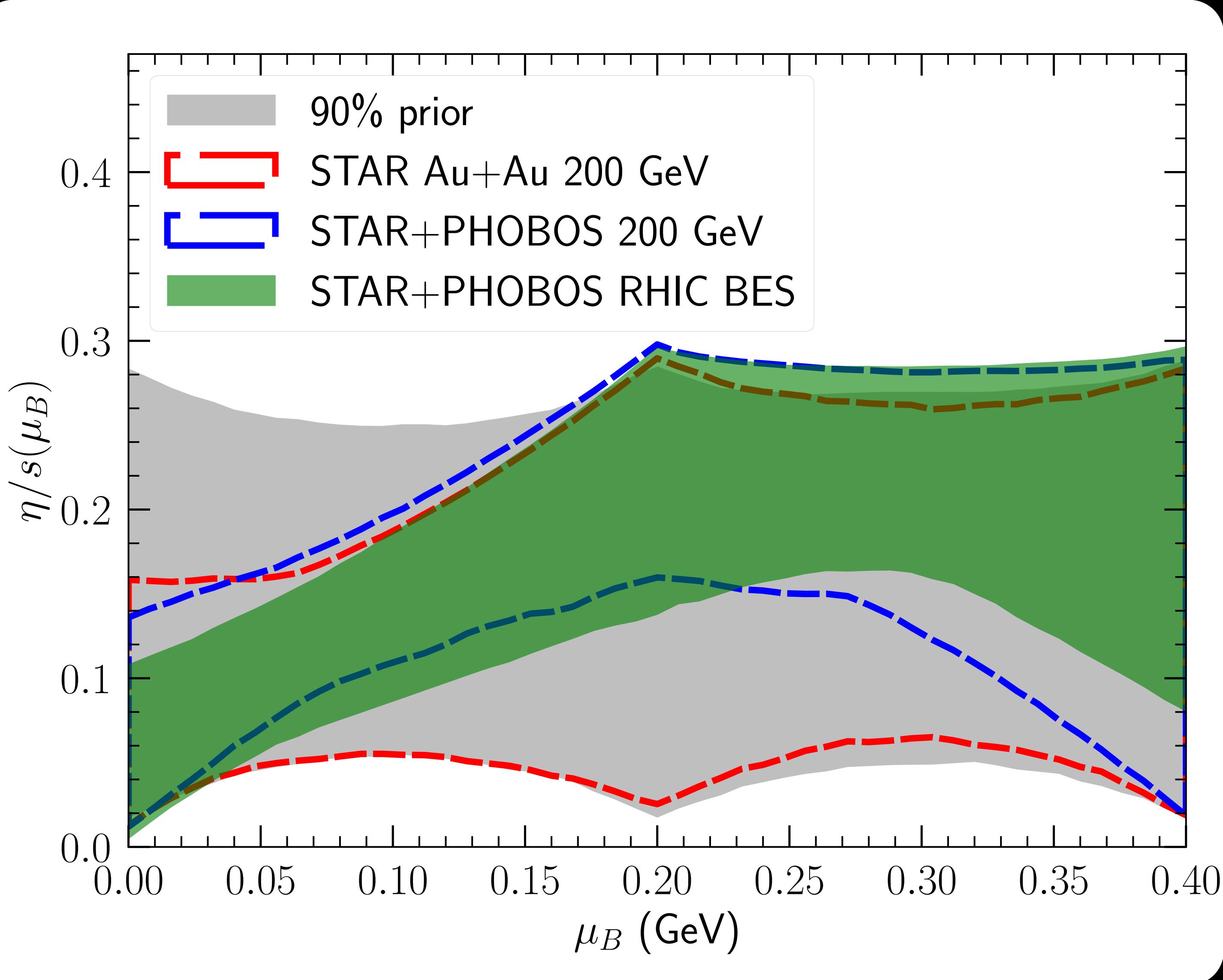
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- The  $dN^{\text{ch}}/d\eta$  and  $v_2(\eta)$  at 200 GeV significantly improve the  $\eta/s$  constraint at  $\mu_B \sim 0.2$  GeV

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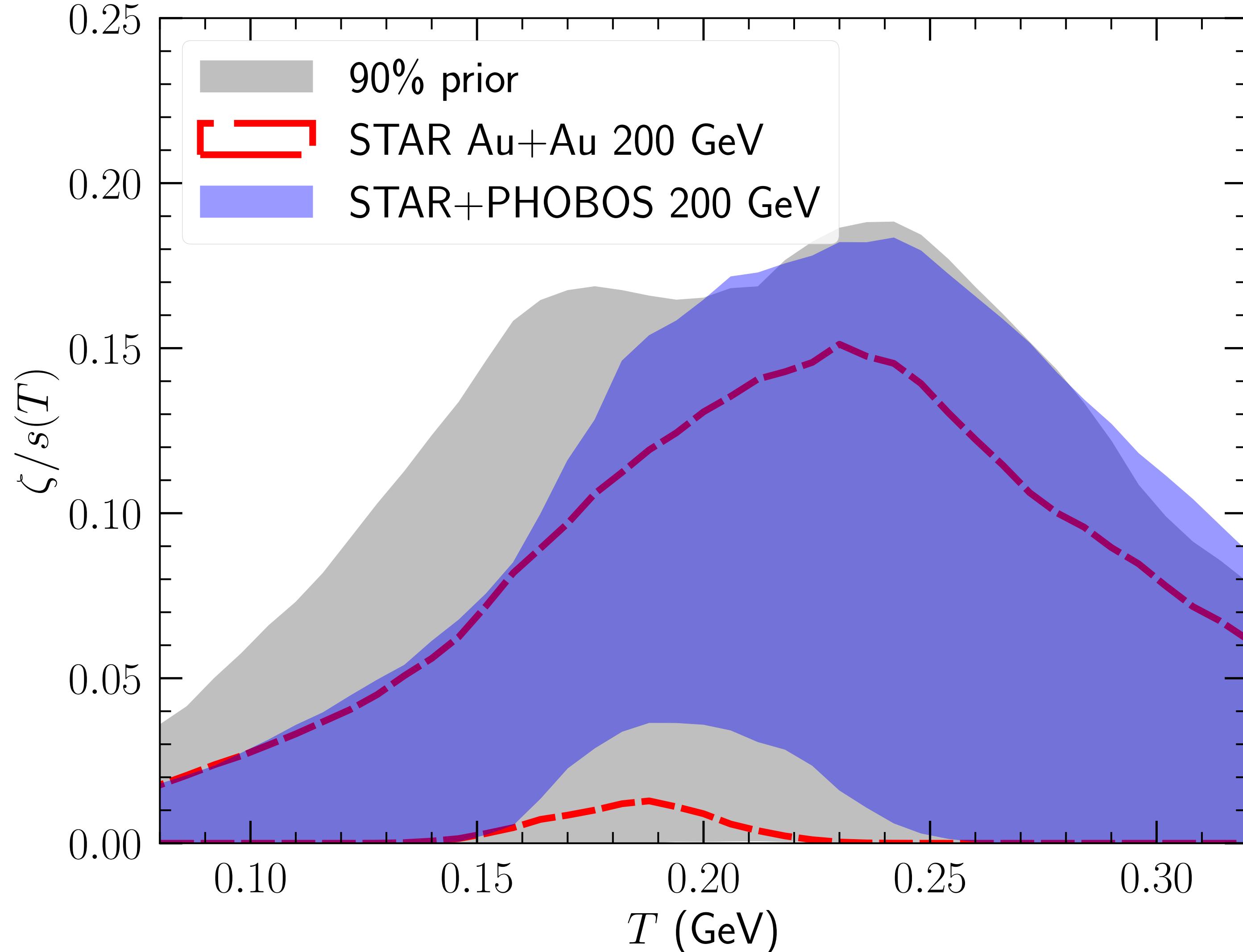
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- The full RHIC BES data (STAR+PHOBOS) shows that the QGP  $\eta/s$  is **larger** at finite  $\mu_B$  than that at  $\mu_B = 0$

color bands indicate 90%  
credible interval in the posterior

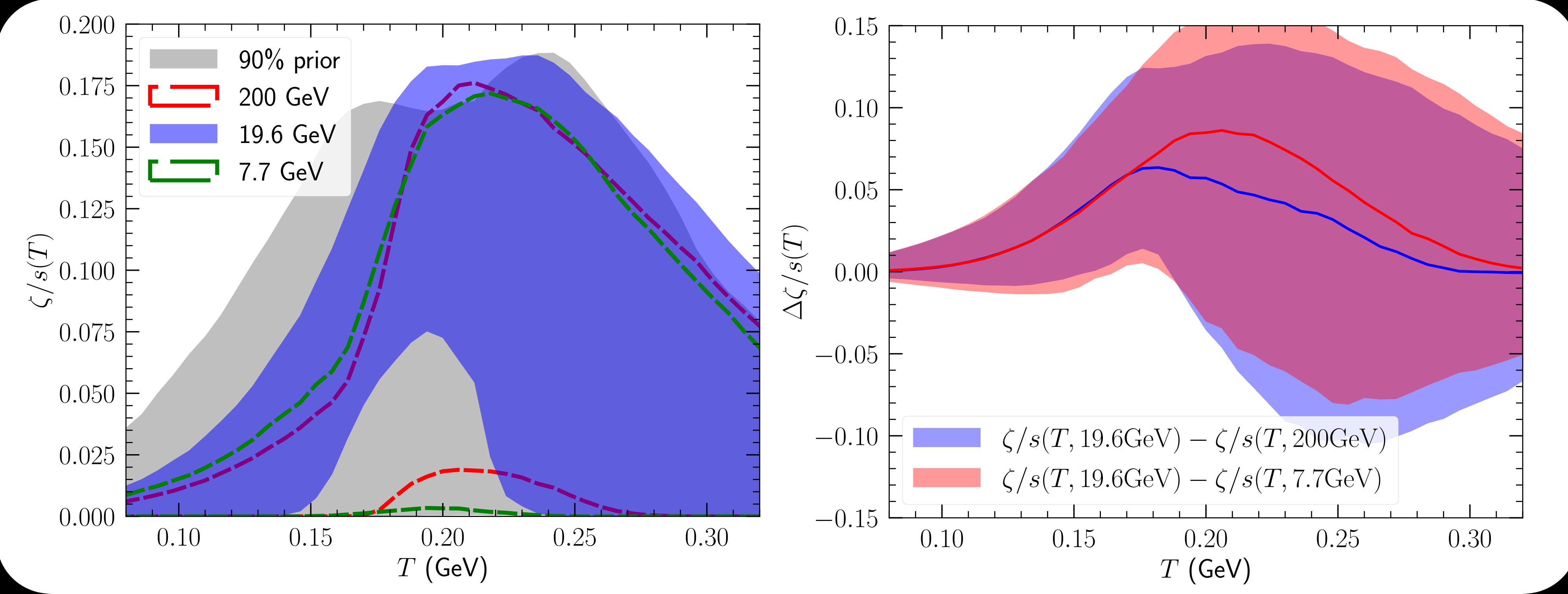
# BULK VISCOSITY $\zeta/s(T)$



- Mid-rapidity identified particle yields and their  $\langle p_T \rangle$  at 200 GeV set constraints on the temperature dependence of the QGP bulk viscosity
- The additional PHOBOS data shifts the posterior  $\zeta/s(T)$  to larger values

color bands indicate 90%  
credible interval in the posterior

# BULK VISCOSITY $\zeta/s(T, \sqrt{s})$

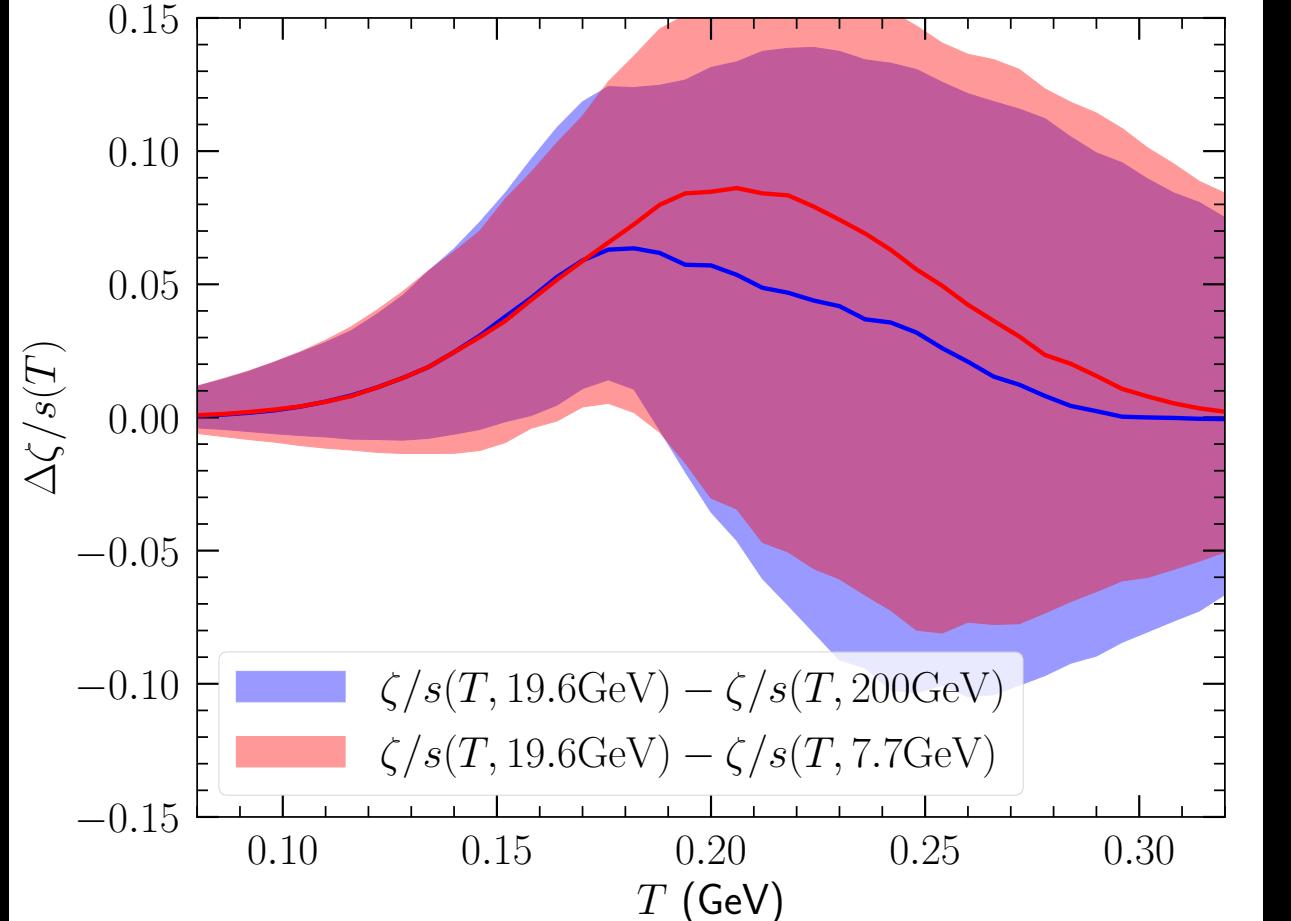
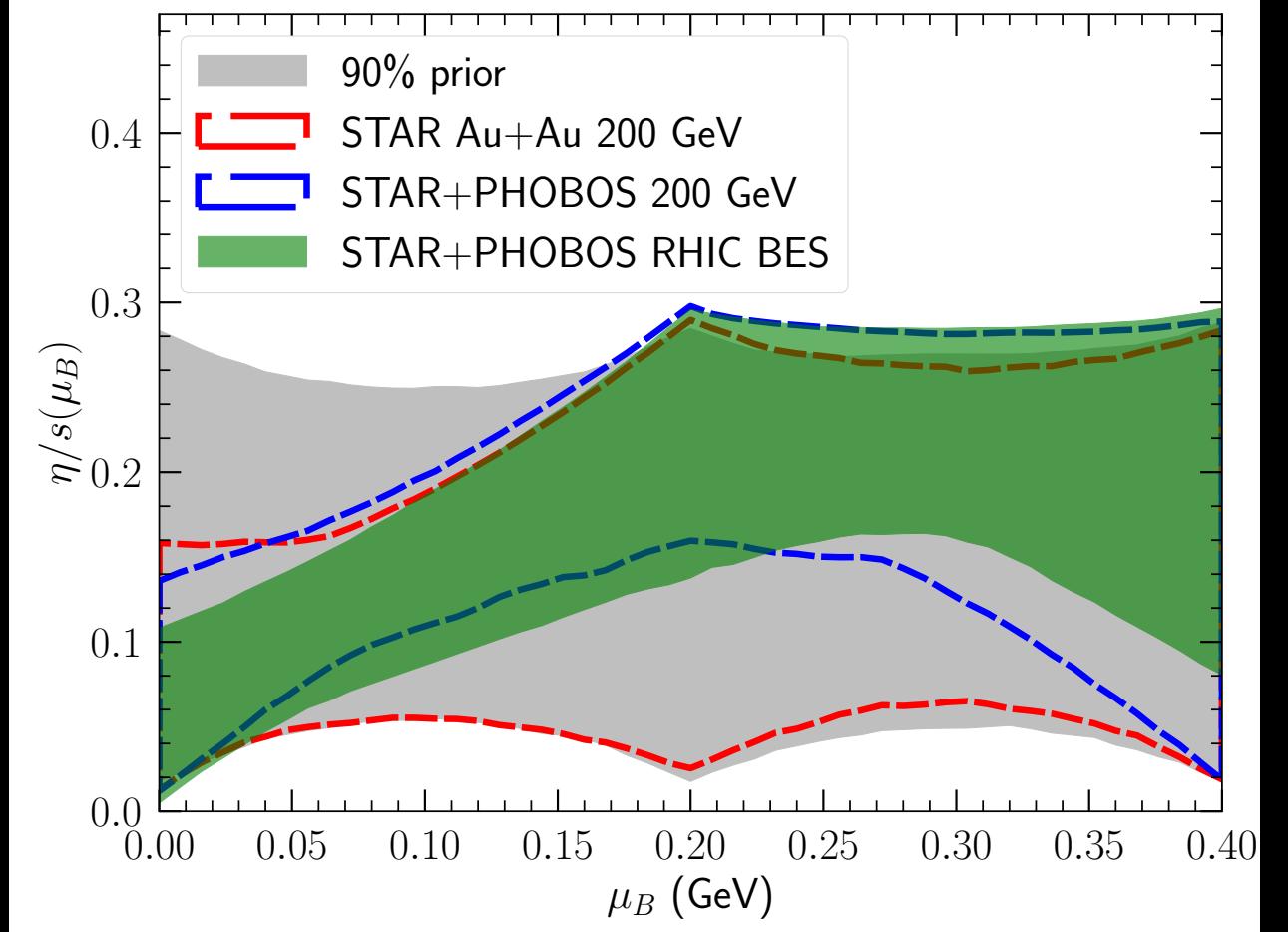
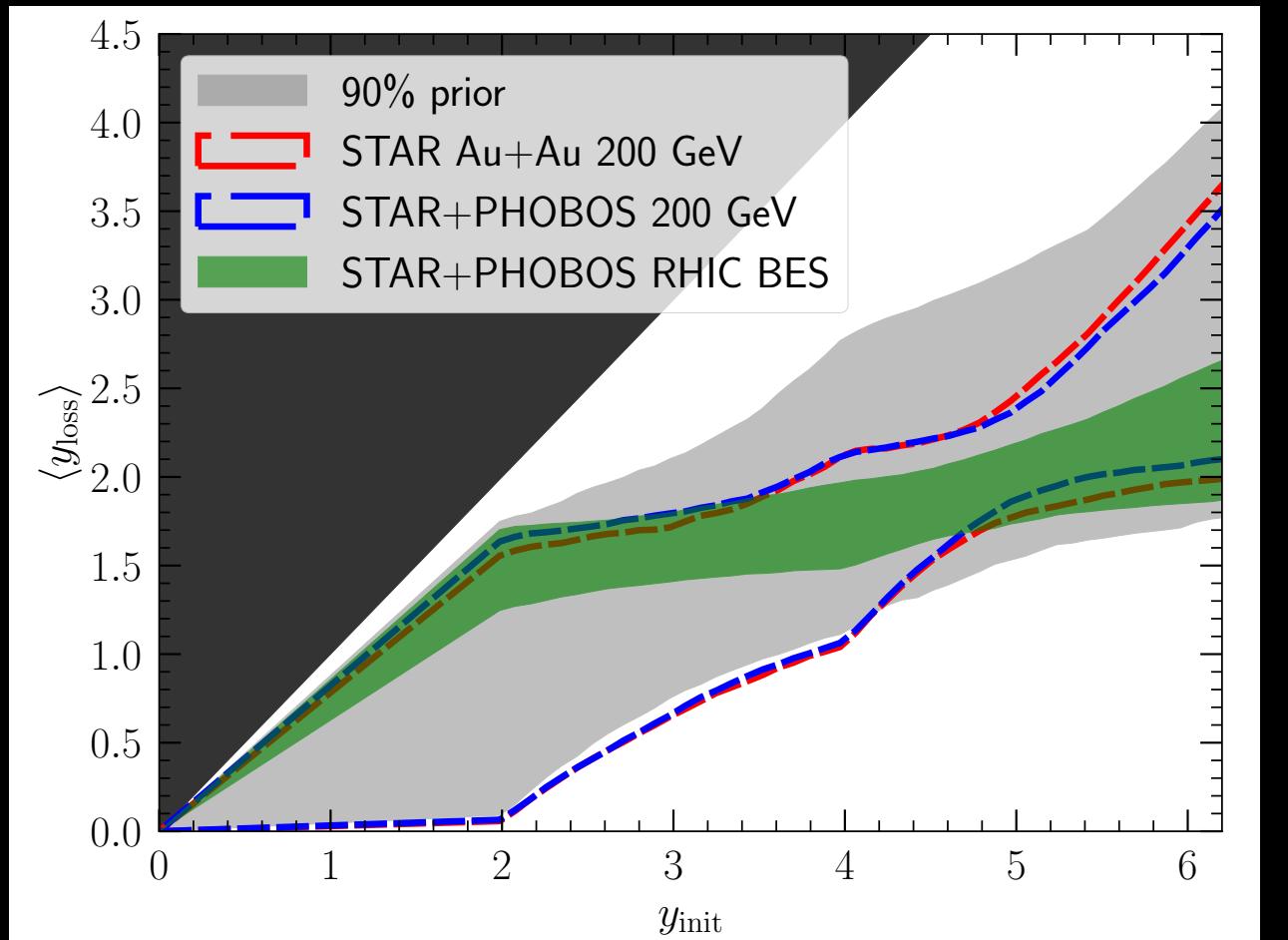


- Allowing  $\zeta/s(T)$  to be an independent function for the three collision energies, our calibration suggests a larger  $\zeta/s(T)$  at 19.6 GeV than those at 200 and 7.7 GeV for  $T \in [0.15, 0.2]$  GeV

*Hint for softening EoS/critical enhancement?*

# CONCLUSIONS

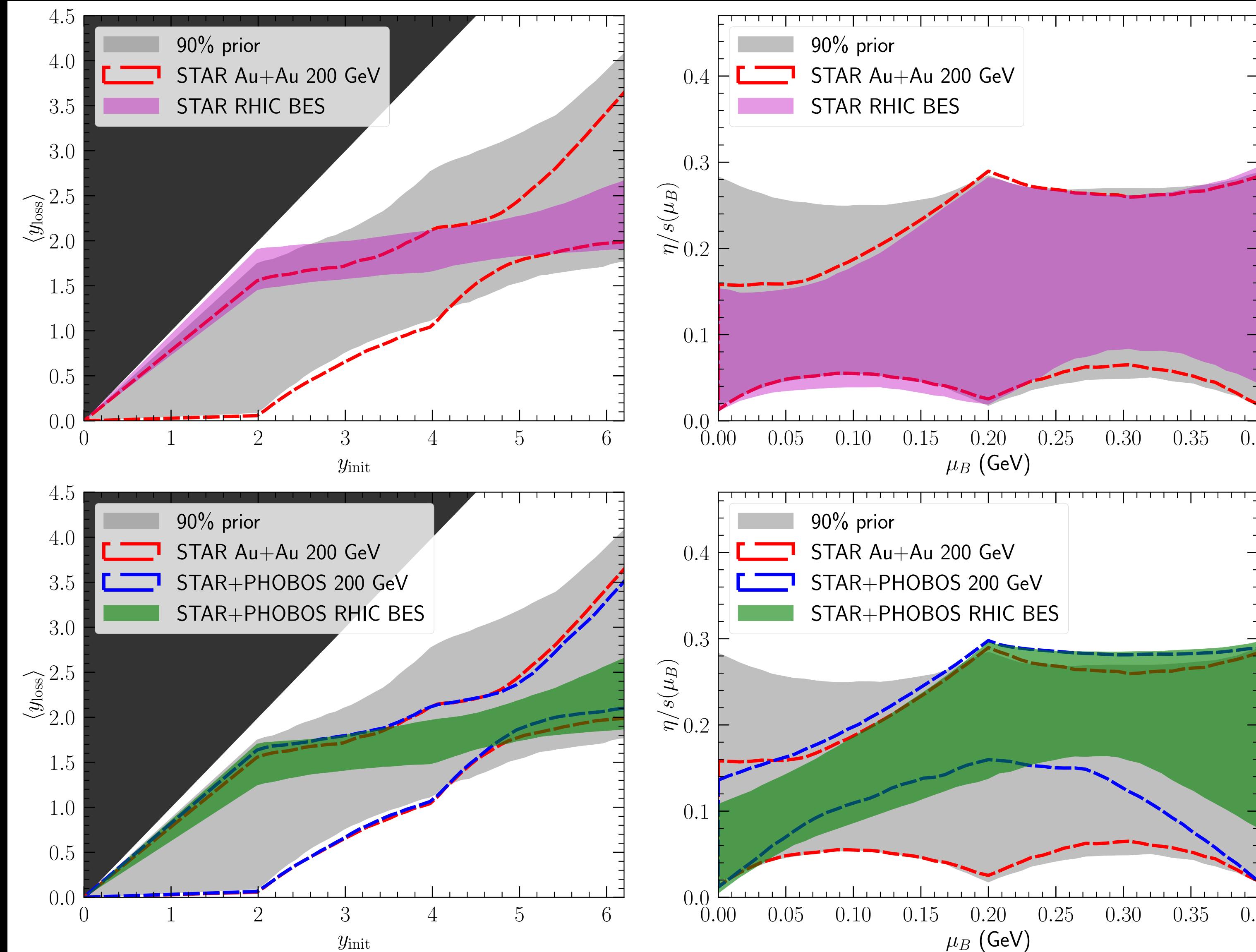
- We performed the **first** combined Bayesian Inference study at multiple RHIC BES energies with a state-of-the-art event-by-event (3+1)D hybrid framework
- With the RHIC BES phase I data, **robust constraints** are obtained for initial state stopping,  $\eta/s(\mu_B)$ , and  $\zeta/s(T, \sqrt{s})$
- The QGP effective  $\eta/s$  is larger at finite  $\mu_B$ , while  $\zeta/s(T)$  shows a hint for **non-monotonic** energy dependence around  $\sqrt{s} = 19.6$  GeV
- Our work marks an important step towards **quantitative characterization** of the QCD phase structure with the RHIC BES and future FAIR programs



Color bands indicate 90% credible interval in the posterior

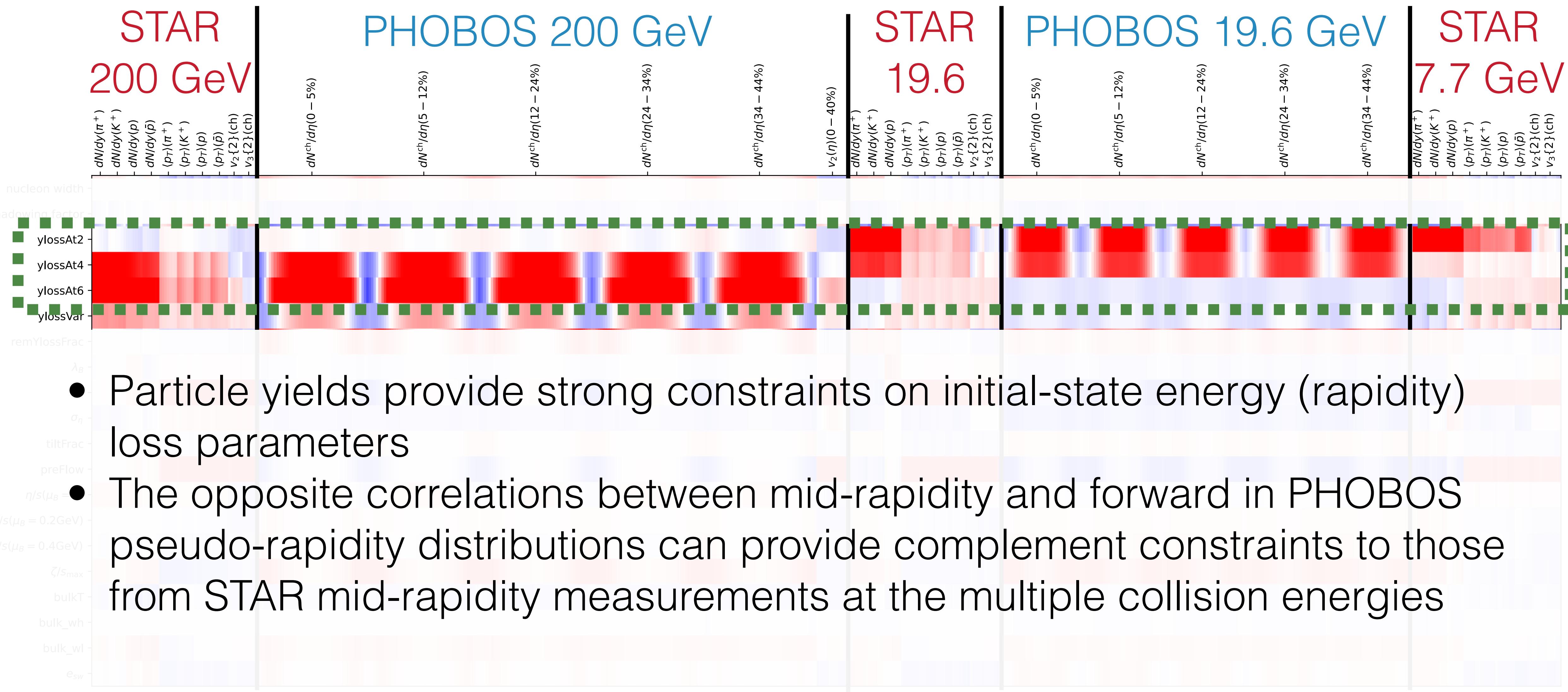


# INFERENCE WITH ONLY STAR MID-RAPIDITY DATA



- The mid-rapidity data from STAR BES energies can set a good constraint on  $y_{\text{loss}}(y_{\text{init}})$
- The rapidity dependent  $v_2(\eta)$  measurement imposes strong constraints on  $\eta/s(\mu_B)$

# OBSERVABLE RESPONSES TO MODEL PARAMETERS



- Particle yields provide strong constraints on initial-state energy (rapidity) loss parameters
- The opposite correlations between mid-rapidity and forward in PHOBOS pseudo-rapidity distributions can provide complement constraints to those from STAR mid-rapidity measurements at the multiple collision energies

Red: Positive correlation; Blue: Negative correlation

# OBSERVABLE RESPONSES TO MODEL PARAMETERS

STAR  
200 GeV

$dN/dy(\pi^+)$   
 $dN/dy(K^+)$   
 $dN/dy(p)$   
 $dN/dy(\bar{p})$   
 $\langle p_T \rangle(\pi^+)$   
 $\langle p_T \rangle(K^+)$   
 $\langle p_T \rangle(p)$   
 $\langle p_T \rangle(\bar{p})$   
 $v_2\{2\}\{ch\}$

PHOBOS 200 GeV

$dN^{ch}/d\eta(0 - 5\%)$

STAR  
19.6

$dN/dy(\pi^+)$   
 $dN/dy(K^+)$   
 $dN/dy(p)$   
 $\langle p_T \rangle(\pi^+)$   
 $\langle p_T \rangle(K^+)$   
 $\langle p_T \rangle(p)$   
 $\langle p_T \rangle(\bar{p})$   
 $v_2\{2\}\{ch\}$   
 $v_3\{2\}\{ch\}$

PHOBOS 19.6 GeV

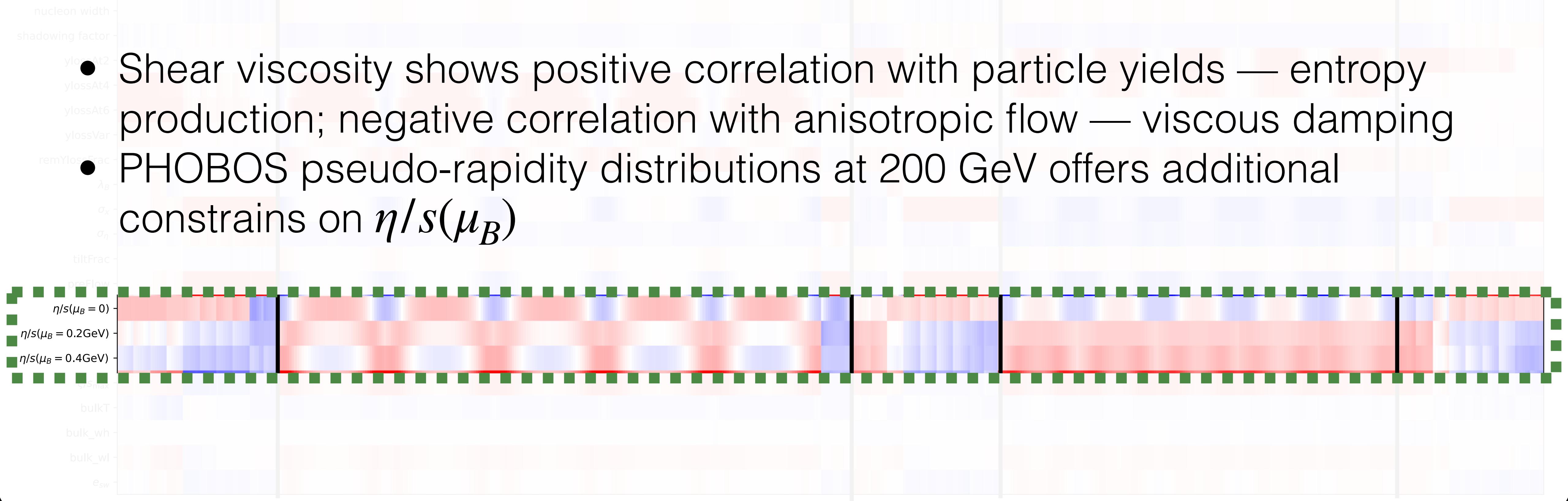
$dN^{ch}/d\eta(5 - 12\%)$

$dN^{ch}/d\eta(12 - 24\%)$

STAR  
7.7 GeV

$dN/dy(\pi^+)$   
 $dN/dy(K^+)$   
 $dN/dy(p)$   
 $\langle p_T \rangle(\pi^+)$   
 $\langle p_T \rangle(K^+)$   
 $\langle p_T \rangle(p)$   
 $\langle p_T \rangle(\bar{p})$   
 $v_2\{2\}\{ch\}$   
 $v_3\{2\}\{ch\}$

- Shear viscosity shows positive correlation with particle yields — entropy production; negative correlation with anisotropic flow — viscous damping
- PHOBOS pseudo-rapidity distributions at 200 GeV offers additional constraints on  $\eta/s(\mu_B)$



Red: Positive correlation; Blue: Negative correlation

# OBSERVABLE RESPONSES TO MODEL PARAMETERS

STAR  
200 GeV

$dN/dy(\pi^+)$   
 $dN/dy(K^+)$   
 $dN/dy(p)$   
 $dN/dy(\bar{p})$   
 $\langle p_T \rangle(\pi^+)$   
 $\langle p_T \rangle(K^+)$   
 $\langle p_T \rangle(p)$   
 $\langle p_T \rangle(\bar{p})$   
 $v_2\{2\}\{\text{ch}\}$

PHOBOS 200 GeV

$dN^{\text{ch}}/d\eta(0 - 5\%)$   
 $dN^{\text{ch}}/d\eta(5 - 12\%)$

STAR  
19.6

$v_2(\eta)(0 - 40\%)$   
 $dN/dy(\pi^+)$   
 $dN/dy(K^+)$   
 $\langle p_T \rangle(\pi^+)$   
 $\langle p_T \rangle(K^+)$   
 $\langle p_T \rangle(p)$   
 $\langle p_T \rangle(\bar{p})$   
 $v_2\{2\}\{\text{ch}\}$   
 $v_3\{2\}\{\text{ch}\}$

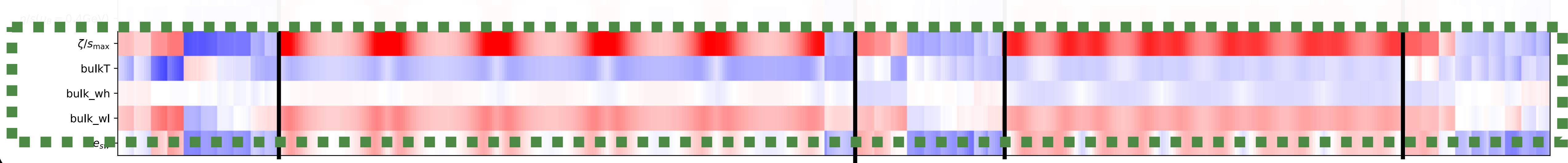
PHOBOS 19.6 GeV

$dN^{\text{ch}}/d\eta(0 - 5\%)$   
 $dN^{\text{ch}}/d\eta(5 - 12\%)$   
 $dN^{\text{ch}}/d\eta(12 - 24\%)$

STAR  
7.7 GeV

$dN/dy(\pi^+)$   
 $dN/dy(K^+)$   
 $dN/dy(p)$   
 $\langle p_T \rangle(\pi^+)$   
 $\langle p_T \rangle(K^+)$   
 $\langle p_T \rangle(p)$   
 $\langle p_T \rangle(\bar{p})$   
 $v_2\{2\}\{\text{ch}\}$   
 $v_3\{2\}\{\text{ch}\}$

- Bulk viscosity shows positive correlation with particle yields — entropy production; negative correlation with mean  $p_T$  — resistance to expansion
- Particle yields at forward and backward pseudo-rapidity shows stronger correlation with  $(\zeta/s)_{\max}$  than those with the mid-rapidity particle yields



Red: Positive correlation; Blue: Negative correlation

# QCD EQUATION OF STATE AT FINITE DENSITIES

M. Albright, J. Kapusta and C. Young, Phys. Rev. C90, 024915 (2014)

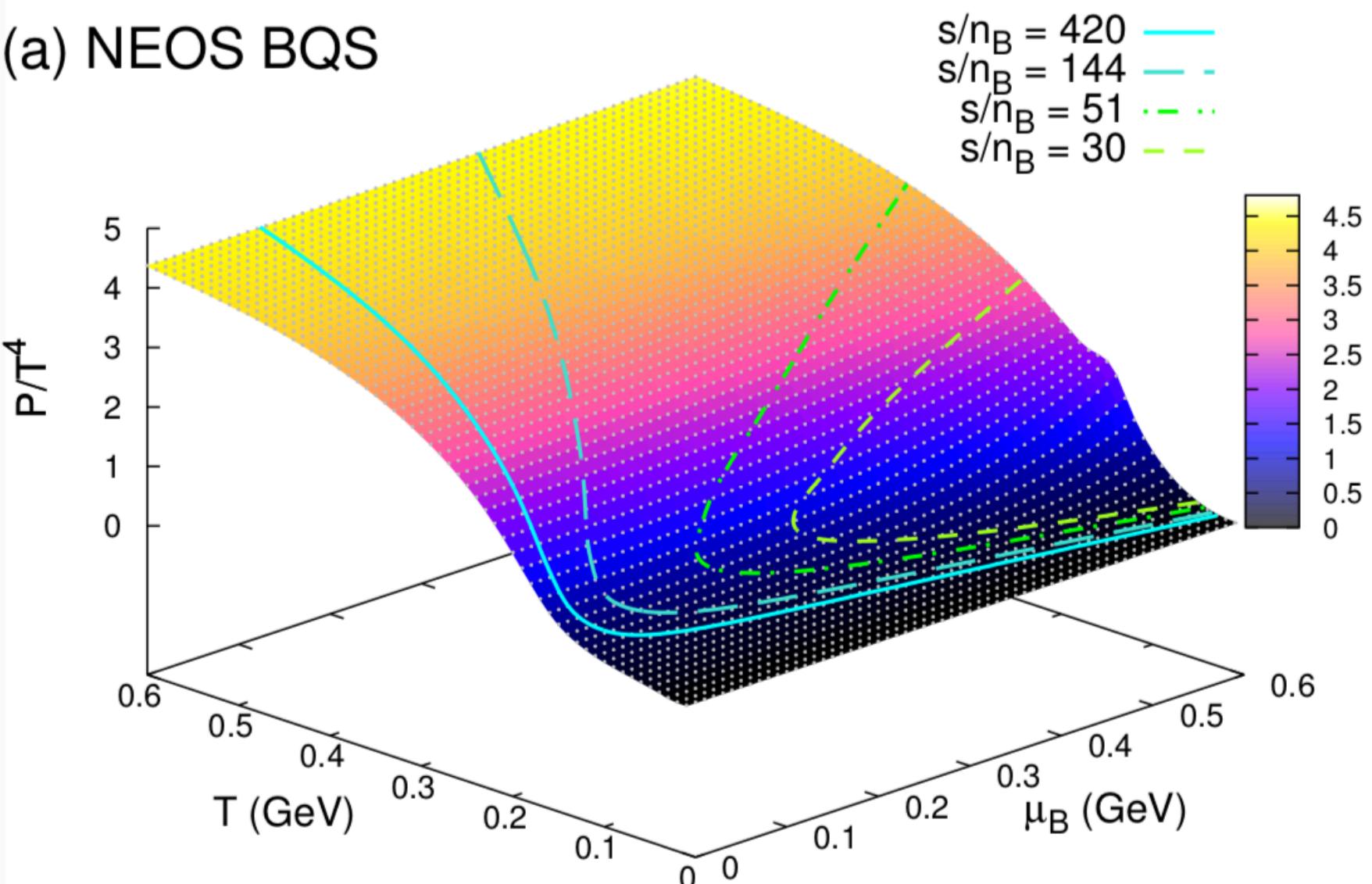
A. Monnai, B. Schenke and C. Shen, Phys. Rev. C100, 024907 (2019)

J. Noronha-Hostler, P. Parotto, C. Ratti and J. M. Stafford, Phys. Rev. C100, 064910 (2019)

J. M. Stafford *et. al*, arXiv:2103.08146 [hep-ph]

$$n_s = 0 \quad n_Q = 0.4n_B$$

(a) NEOS BQS



Lattice QCD: Taylor expansion up to the 4<sup>th</sup> order

$$\frac{P}{T^4} = \frac{P_0}{T^4} + \sum_{l,m,n} \frac{\chi_{l,m,n}^{B,Q,S}}{l!m!n!} \left(\frac{\mu_B}{T}\right)^l \left(\frac{\mu_Q}{T}\right)^m \left(\frac{\mu_S}{T}\right)^n$$

Match to Hadron Resonance Gas model at low T

$$\frac{P}{T^4} = \frac{1}{2}[1 - f(T, \mu_J)] \frac{P_{\text{had}}(T, \mu_J)}{T^4} + \frac{1}{2}[1 + f(T, \mu_J)] \frac{P_{\text{lat}}(T, \mu_J)}{T^4}$$

$$f(T, \mu_B) = \tanh[(T - T_c(\mu_B)) / \Delta T_c]$$

Enabled hydrodynamic simulations at finite  $\mu$