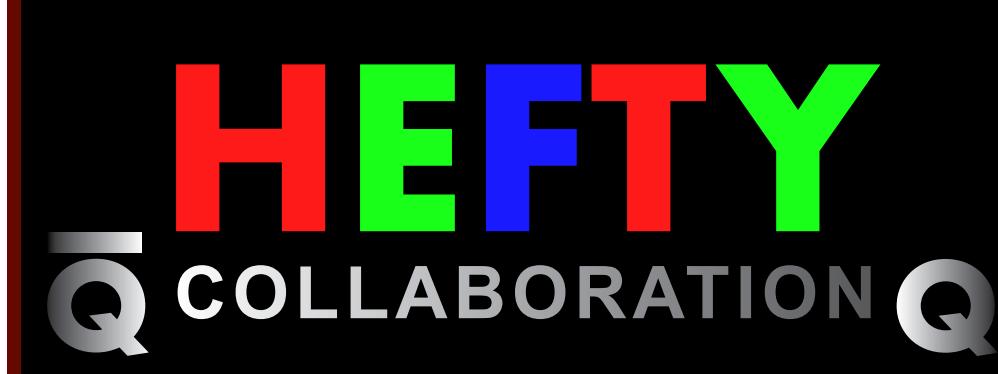
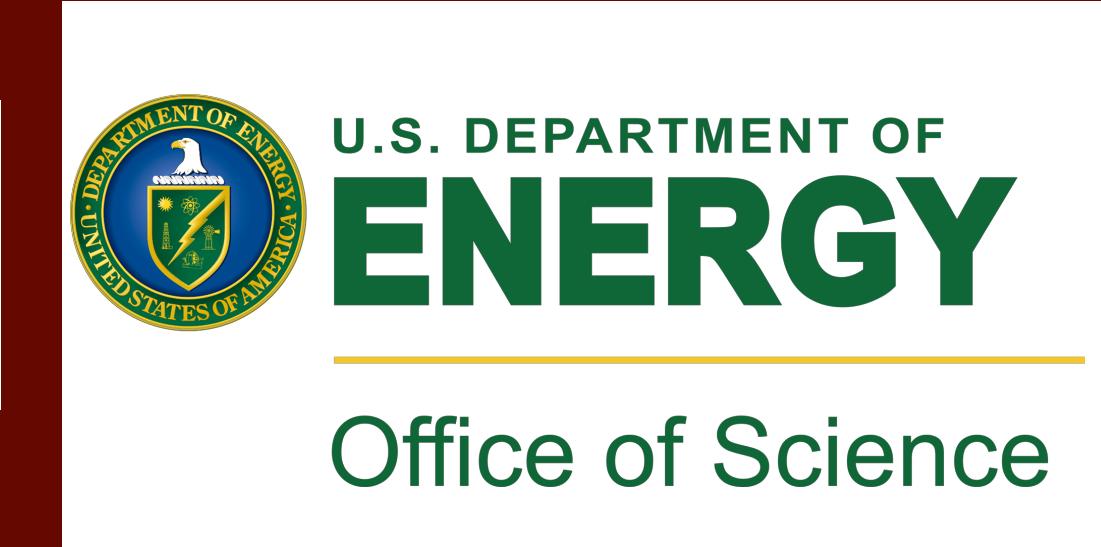




**CYCLOTRON INSTITUTE**  
TEXAS A & M UNIVERSITY



# **T-Matrix Analysis of Static Wilson Line Correlators from Lattice QCD at Finite Temperature**

**Zhanduo Tang<sup>1</sup>, Swagato Mukherjee<sup>2</sup>,  
Peter Petreczky<sup>2</sup>, Ralf Rapp<sup>1</sup>**

<sup>1</sup>Cyclotron Institute, Texas A&M University, College Station, TX, USA

<sup>2</sup>Physics Department, Brookhaven National Laboratory, Upton, New York, USA

# Outline

- 1. Introduction to T-matrix approach**
- 2. Wilson line correlators from T-matrix**
- 3. Fits to IQCD data for Wilson line correlators**
- 4. Heavy-quark transport coefficients with updated T-matrix**
- 5. Summary**

# 1.1 T-Matrix Approach to QGP

[Riek+Rapp '10, Liu+Rapp '18]

## ◆ Quantum many-body theory for heavy quarks, quarkonia and QGP structures

$$i = Q, \bar{Q} \quad j = Q, \bar{Q}$$

The diagram illustrates the decomposition of the T-matrix. On the left, a box labeled 'T' with indices  $i = Q, \bar{Q}$  and  $j = Q, \bar{Q}$  is shown. It is decomposed into two terms: a circle labeled 'V' with four external lines plus a circle labeled 'V' followed by a box labeled 'T'. A large grey arrow points to the right, leading to the Dyson-Schwinger equation.

$$T_{ij} = V_{ij} + \int V_{ij} G_i G_j T_{ij}$$

Dyson-Schwinger type self-consistent problem

- 1-body propagator:  $G_i = 1/(\omega - \omega_k - \Sigma_i)$
- Potential (**input** to T-matrix):  $V_{ij}$ 
  - Vacuum: Cornell potential,  $V = -\frac{4}{3}\alpha_s \frac{1}{r} + \sigma r$
  - QGP:  $V = -\frac{4}{3}\alpha_s \left( \frac{e^{-m_d r}}{r} + m_d \right) - \frac{\sigma}{m_s} \left( e^{-m_s r - (c_b m_s r)^2} - 1 \right)$ 
    - $m_{d/s}(T)$ : Debye screening masses for Coulomb/confining interaction
    - $c_b(T)$ : mimicks string breaking
- Heavy-quark mass:  $M_Q = M_Q^0 + V(r \rightarrow \infty)/2$ 

bare mass   Fock term (mass shift)

• Self-energy:

The diagram shows the self-energy  $\Sigma_i$  as a circle with a blue bracket above it labeled  $j=q,g$ , followed by a box labeled  $T_{ij}$ .

**Objective:**  
*Construct microscopic in-medium interaction based on constraints from lattice QCD*

## 1.2 Lorentz Structure of Potential

### ◆ Relativistic corrections to potential

- Breit correction:  $V = RV^{vec} + V^{sca}$

$$\text{with } R = \sqrt{\frac{\omega_i(p)\omega_j(p)}{M_i M_j}} \sqrt{1 + \frac{p^2}{\omega_i(p)\omega_j(p)}} \sqrt{\frac{\omega_i(p')\omega_j(p')}{M_i M_j}} \sqrt{1 + \frac{p'^2}{\omega_i(p')\omega_j(p')}}$$

- Lorentz structures

- ▶ Color-Coulomb:  $V^{vec} = V_{Coul}$

- ▶ Confining:

- Common assumption:  $V^{sca} = V_{conf}$

[Brambilla+Vairo '97]

[Szczeplaniak et al. '96, '97]

[Ebert et al. '98, '03]

- Improved assumption:  $V^{vec} = V_{Coul} + (1 - \chi)V_{conf}$ ,  $V^{sca} = \chi V_{conf}$

- **Vector component of confining potential** (for  $\chi < 1$ ) improves [Tang+Rapp '23]

- ▶ vacuum mass splittings in quarkonium spectroscopy

- ▶ agreement with heavy-quark transport coefficients from recent 2+1 flavor lattice QCD

[Altenkort et al '23]

## 2. Wilson Line Correlators in T-Matrix Framework

- ◆ **Wilson line correlator** (static limit of retarded  $Q\bar{Q}$  meson correlator)

$$W(r, \tau, T) = \int_{-\infty}^{\infty} d\omega e^{-\omega\tau} \rho_{Q\bar{Q}}(\omega, r, T)$$

- Static  $Q\bar{Q}$  spectral function:  $\rho_{Q\bar{Q}}(\omega, r, T) = \frac{-1}{\pi} \text{Im} \left[ \frac{1}{\omega - V(r, T) - \Phi(r, T)\Sigma_{Q\bar{Q}}(\omega, T)} \right]$
- Selfenergy:  $\Sigma_{Q\bar{Q}}$ , interaction between  $Q\bar{Q}$  and medium
- Interference function:  $\Phi(r, T)$  ( $\sim 0$  at small  $r$ ;  $\sim 1$  at large  $r$ )  
(parton scattering off  $Q$  and  $\bar{Q}$ )

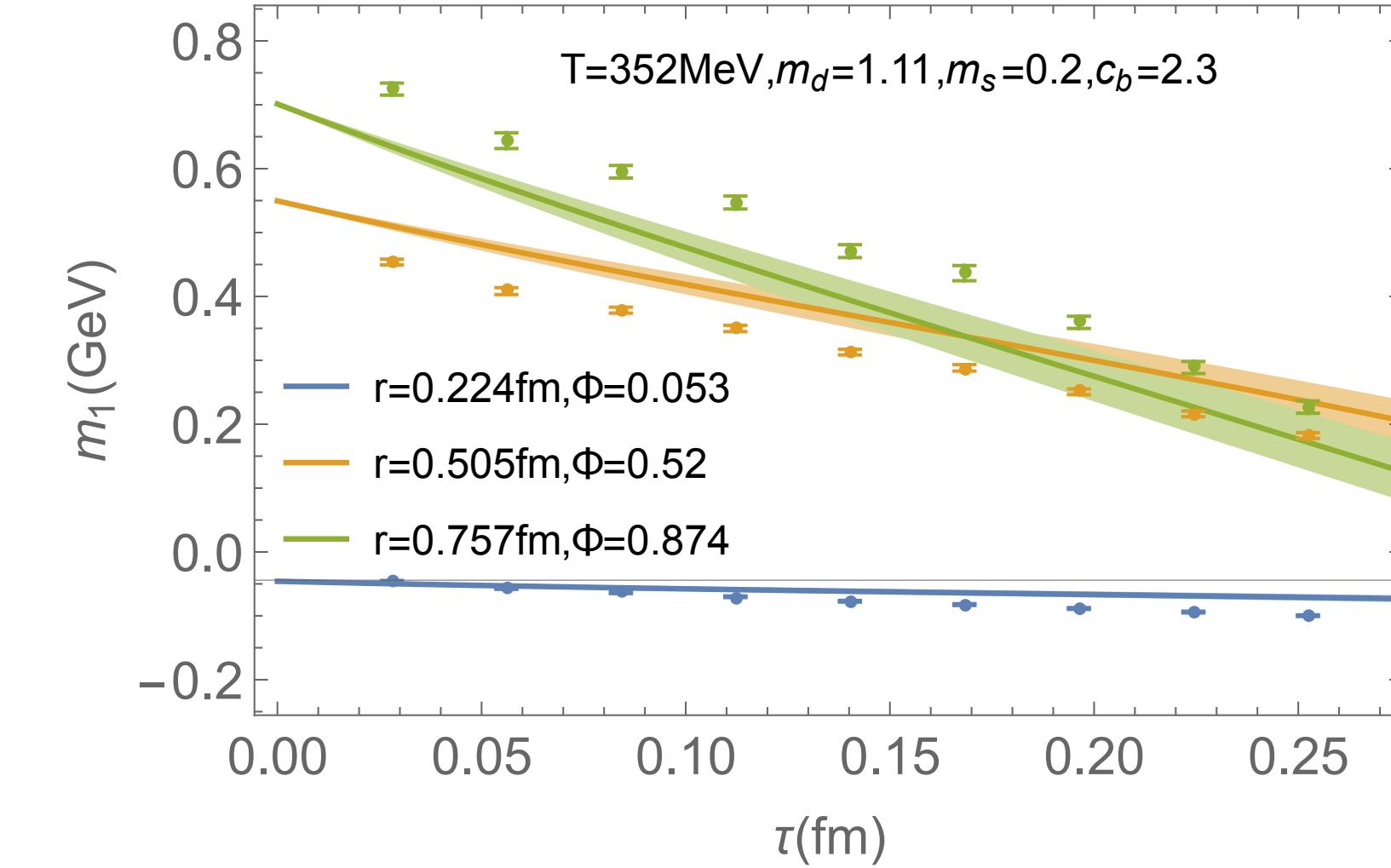
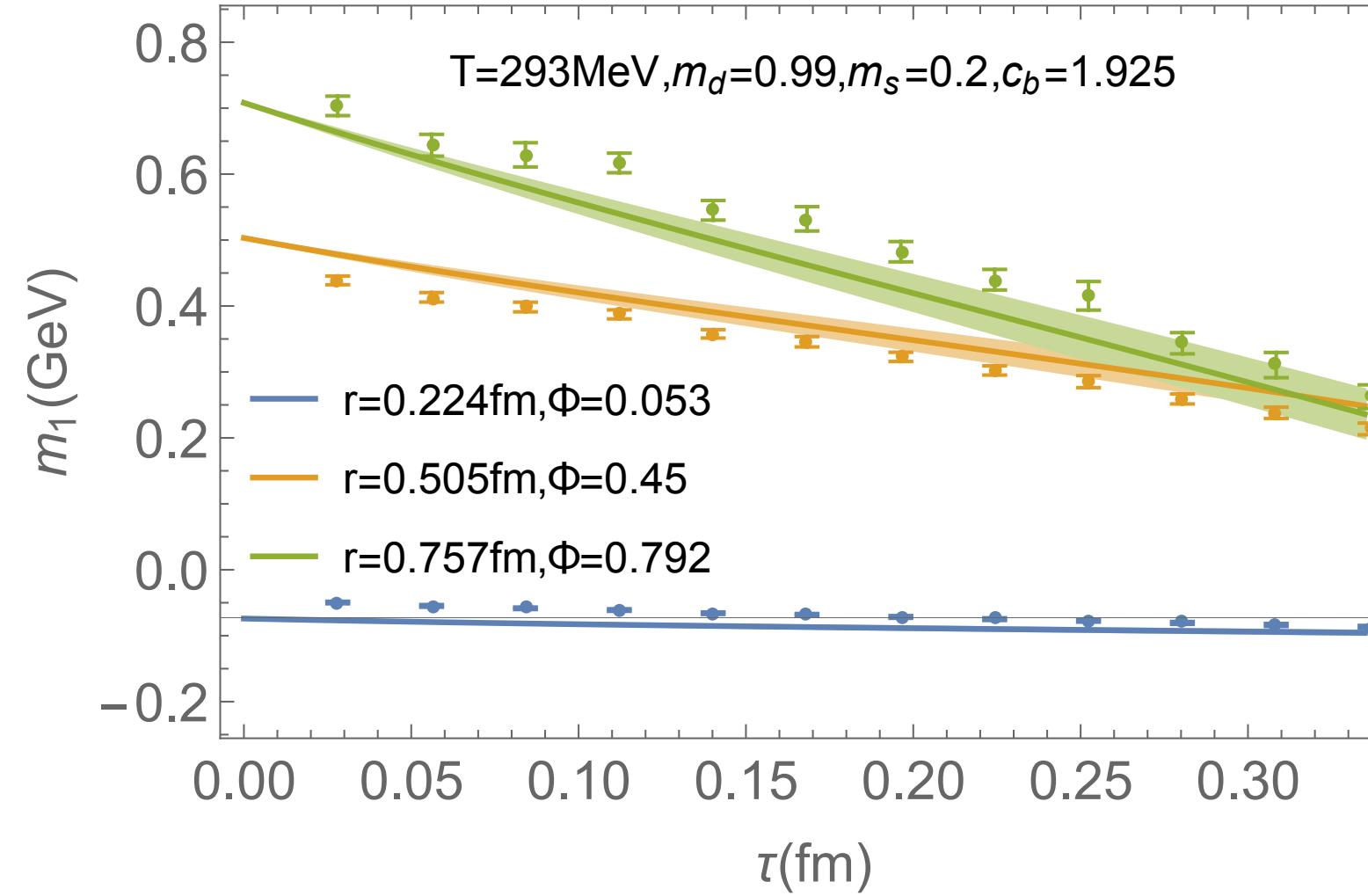
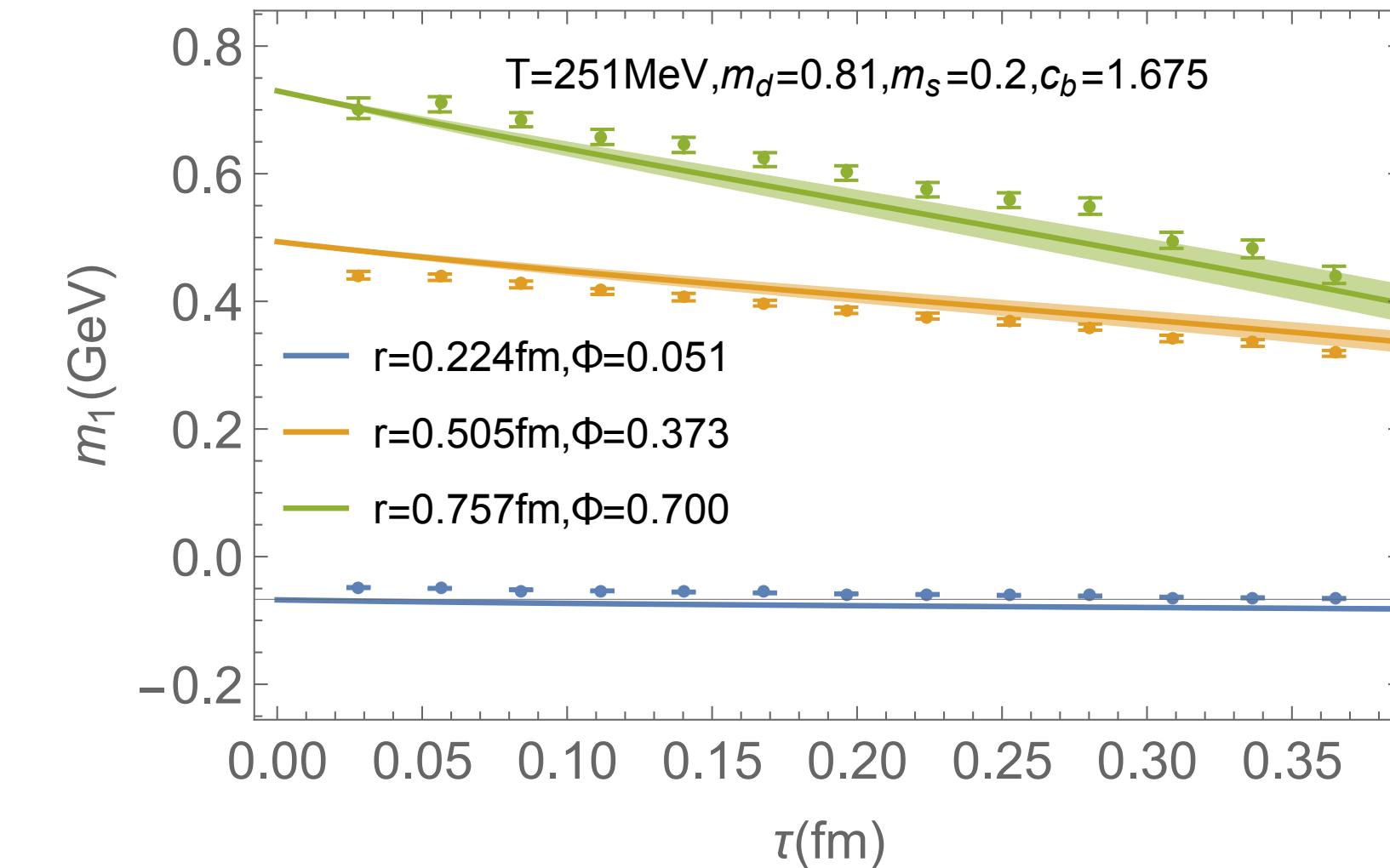
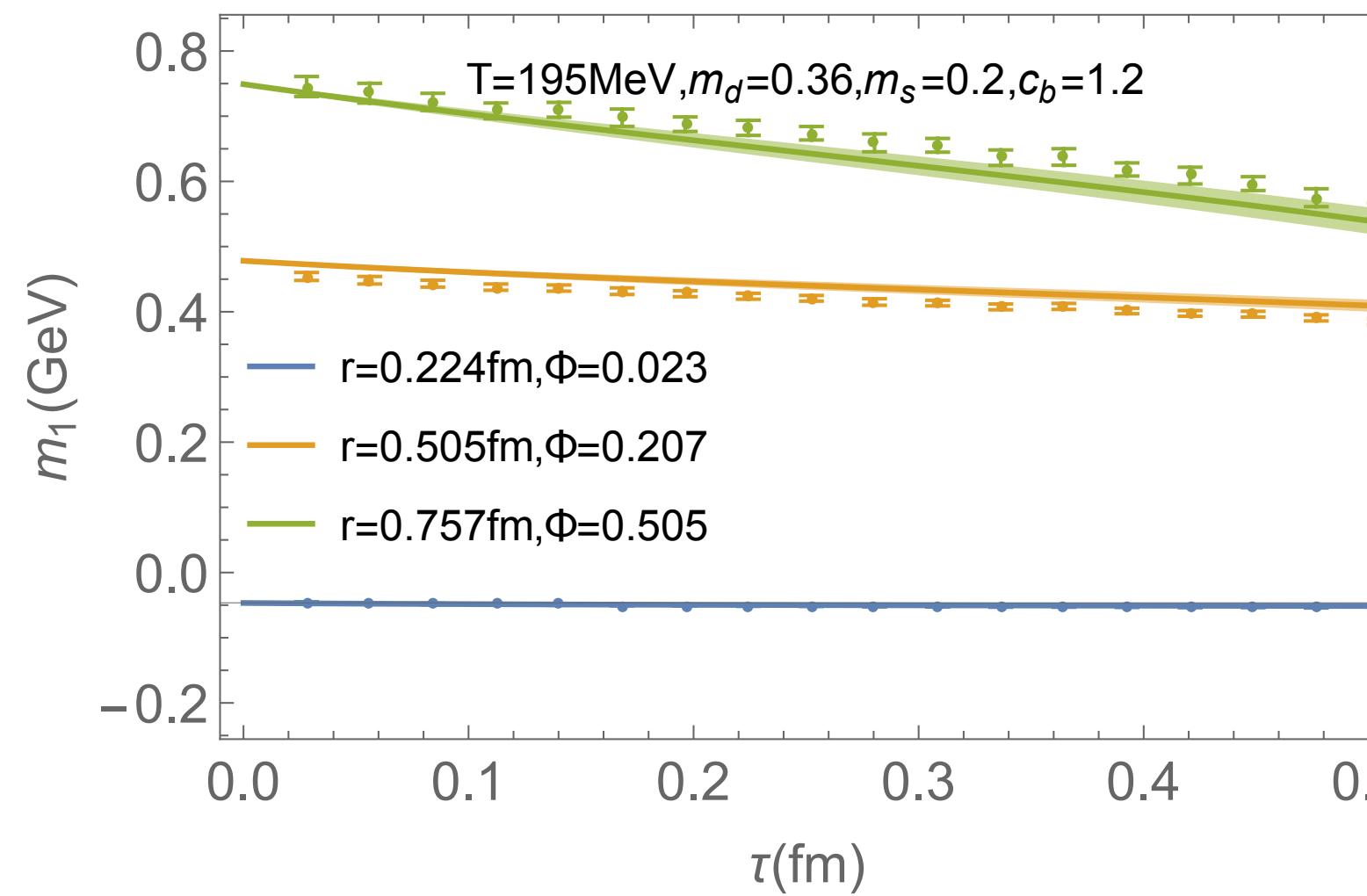
- ◆ **1st cumulant of Wilson line correlator**

$$m_1(r, \tau, T) = -\partial_\tau \ln W(r, \tau, T)$$

- $m_1(r, \tau = 0, T) = V(r, T) \rightarrow$  pole position of  $\rho_{Q\bar{Q}}$  (effective mass)
- Slope of  $m_1(r, \tau, T) \rightarrow$  width of  $\rho_{Q\bar{Q}}$  (interaction with medium)

# 3.1 Fits to LQCD Data for Wilson Line Correlators: $m_1$

[HotQCD Collaboration  
PRD 105, 054513 (2022)]

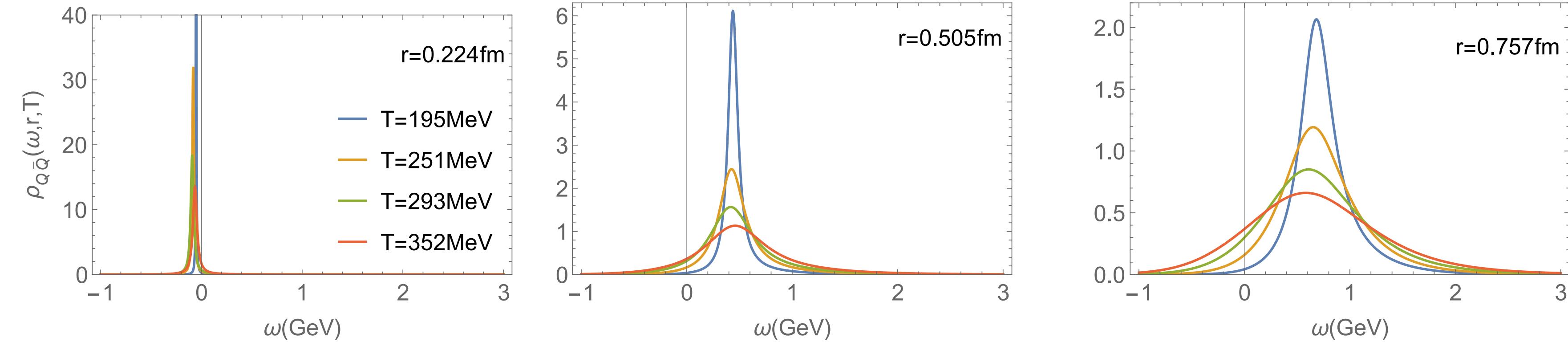


- Vary previously determined interference function  $\Phi(r, T)$  by  $\pm 10\%$

[Liu+Rapp '18]

## 3.2 Spectral Functions from Wilson Line Correlators

- ◆ Examples of Static  $Q\bar{Q}$  spectral functions ( $\omega$  shifted by  $2M_Q^0$ )

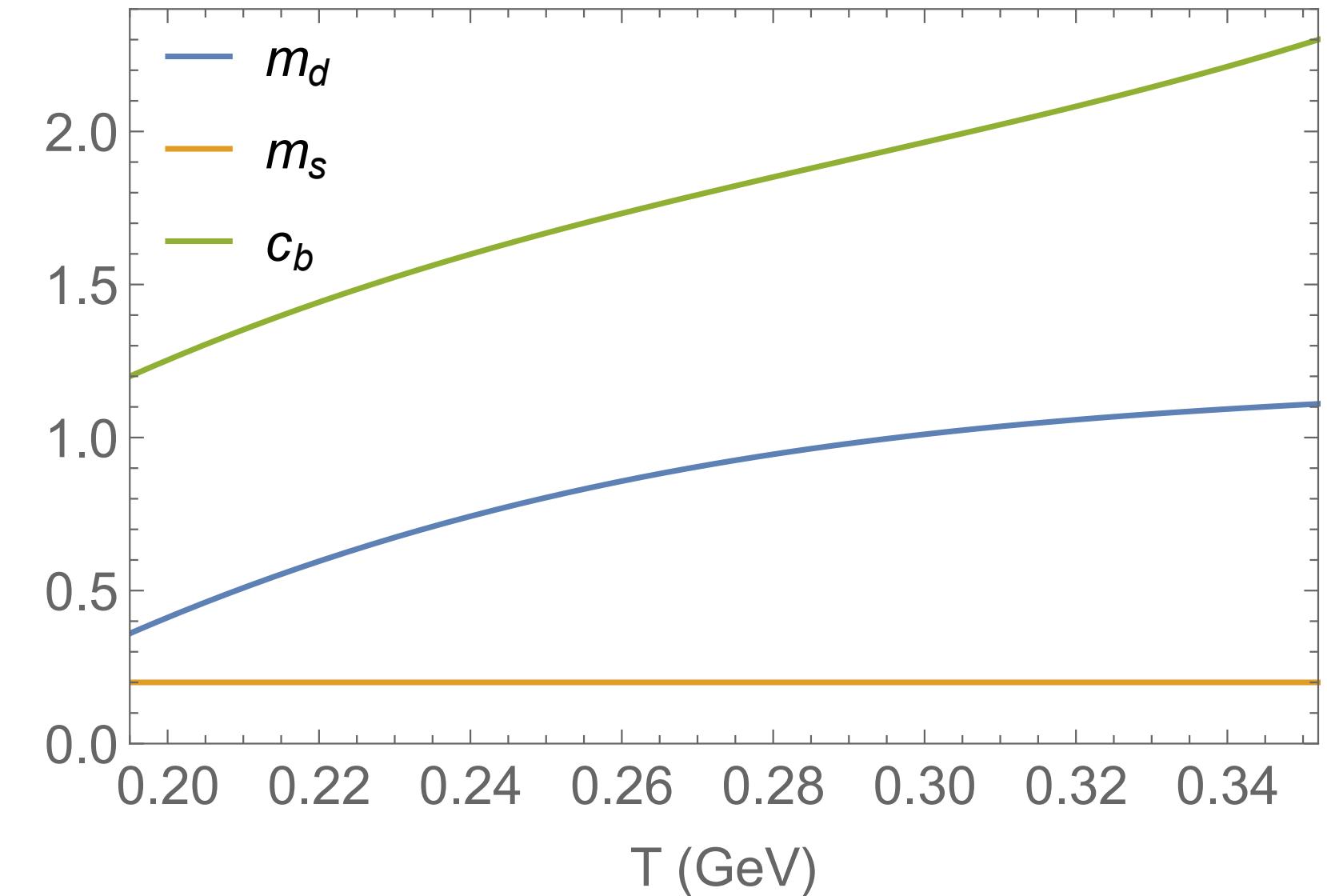


- As temperature increases:
  - ▶ mass almost unchanged: potential screening compensates for mass shift (supported by IQCD data)
  - ▶ width  $\uparrow$ : larger QGP density
- As distance increases:
  - ▶ mass  $\uparrow$ : less potential energy
  - ▶ width  $\uparrow$ :  $Q\bar{Q}$  interacting with more thermal partons

### 3.3 Screening Masses and Input Potential

#### ◆ Fit parameters

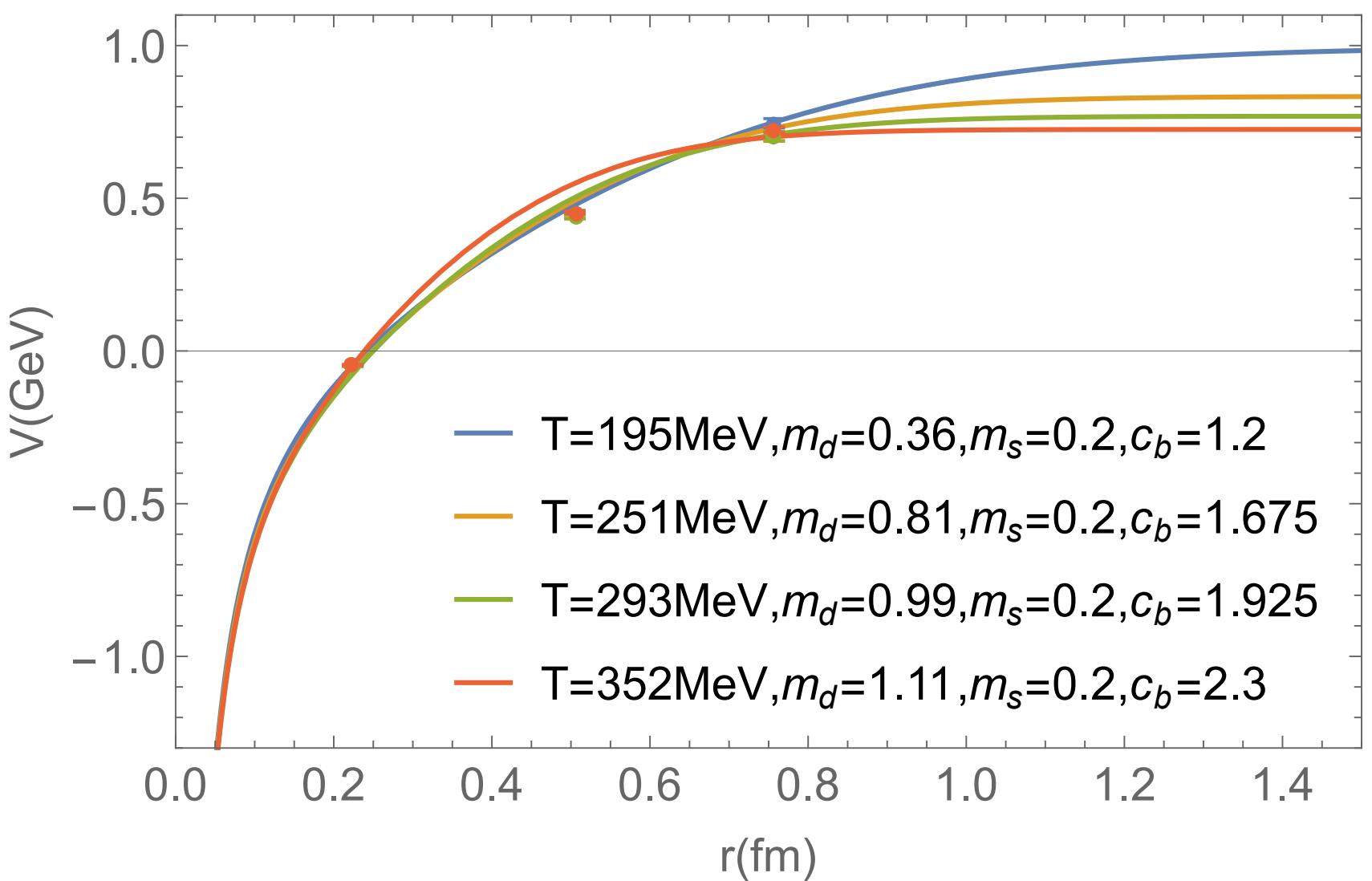
- Constant  $m_s$
- Increasing  $m_d \rightarrow m_1(\tau = 0)$  & slope decrease
- Increasing  $c_b \rightarrow m_1(\tau = 0)$  & slope decrease



#### ◆ Potentials

- Recall  $m_1(r, \tau = 0, T) = V(r, T)$
- LQCD suggest T-independent potential at small & intermediate  $r$

[HotQCD Collaboration, PRD 105, 054513 (2022)]



# 4.1 Heavy-Quark Transport in QGP

## ◆ Fokker-Planck equation

$$\frac{\partial}{\partial t} f(p, t) = \frac{\partial}{\partial p_i} \left\{ A(p)p_i f(p, t) + \frac{\partial}{\partial p_j} \left[ B_{ij}(p)f(p, t) \right] \right\}$$

thermal relaxation rate      momentum diffusion coefficient  
(friction coefficient)

- $A(p) \sim \sum_i \int d^4p' d^4q d^4q' \left| T_{Qi} \right|^2 \left( 1 - \frac{\mathbf{p} \cdot \mathbf{p}'}{\mathbf{p}^2} \right)$

- $T_{Qi}$ : heavy-light T-matrix

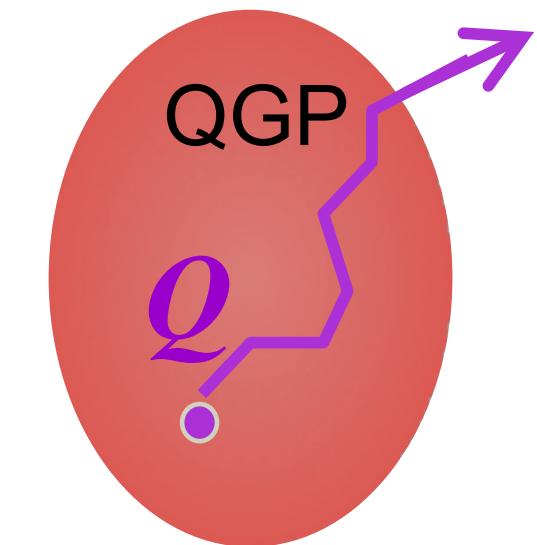


## ► Prediction for $D$ -meson channel

As temperature decreases...

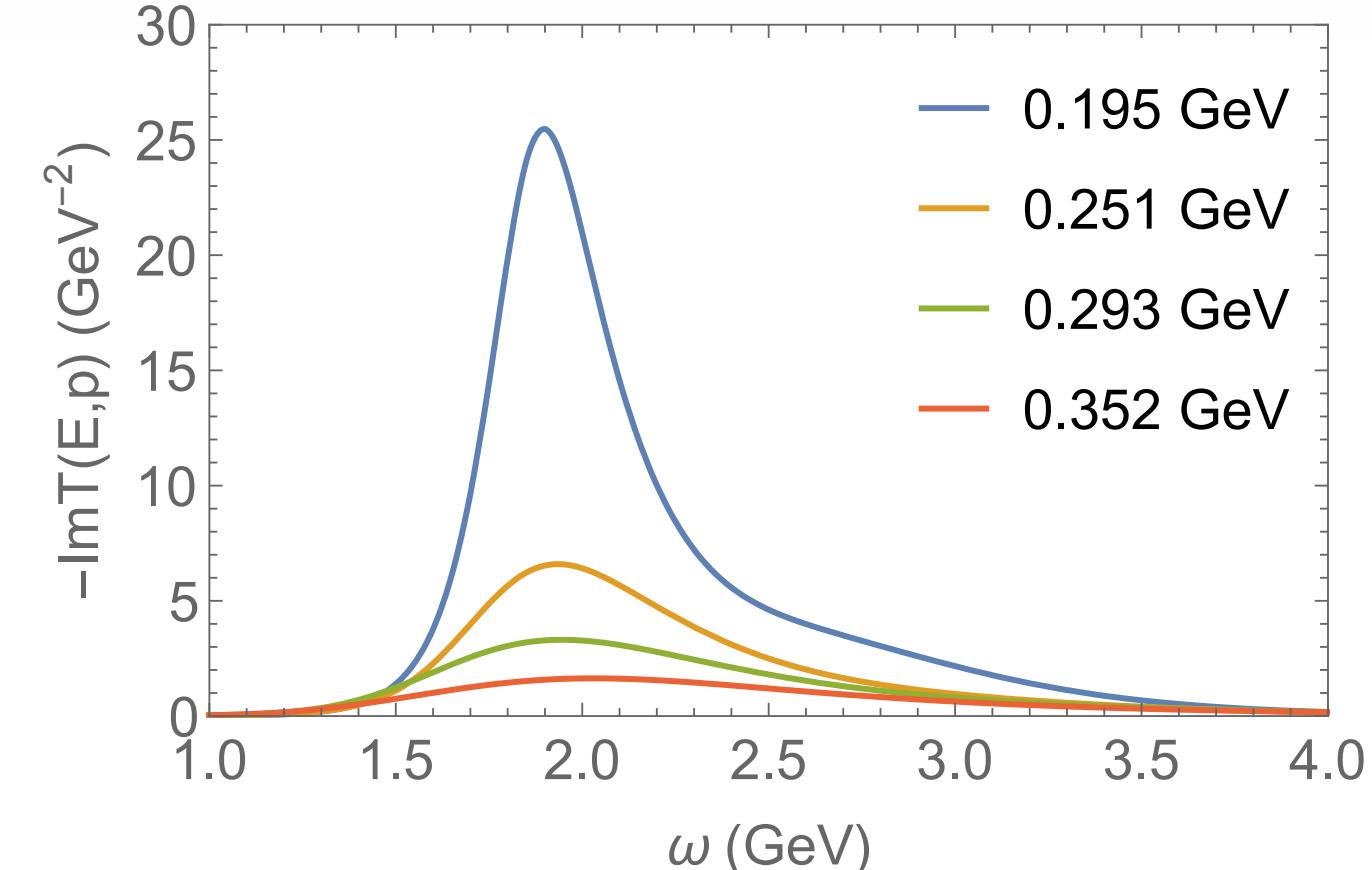
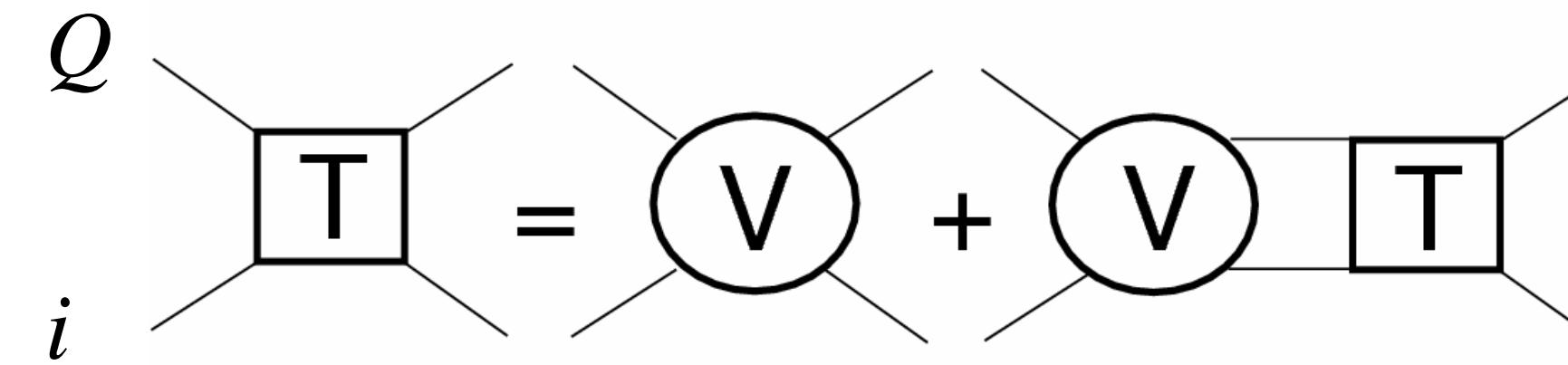
- Screening for potential decreases
- Confining force appears
- $D$ -meson emerges (with reasonable mass at  $\sim T_c$ )

Important for  $D_s$  at  $\sim T_c$

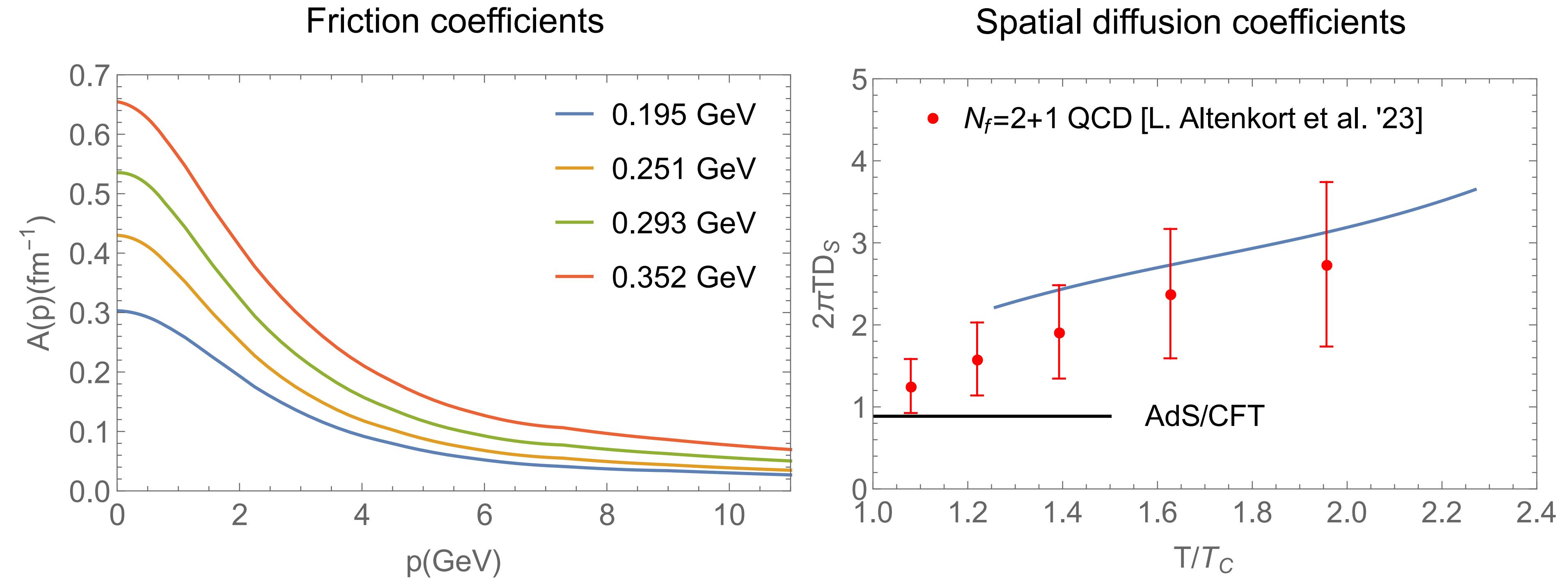


Brownian motion for heavy quarks through the QGP

- Spatial diffusion coefficient:  $2\pi TD_s = \frac{2\pi T^2}{M_Q A(p \rightarrow 0)}$



## 4.2 Transport Coefficients from Updated T-Matrix



- Harder momentum dependence of  $A(p)$  compared to previous study [Liu+Rapp '18] likely helpful for open heavy-flavor phenomenology [He+Rapp '20]
- Predicted  $2\pi TD_s$  consistent with recent IQCD data

## 5. Summary

- ◆ Applied T-matrix approach to fit static Wilson Line correlators to IQCD data, provides strong constraints on in-medium potential
- ◆ Selfconsistent T-matrix allows for fair agreement with IQCD data
- ◆ Calculated spatial diffusion coefficient agrees with IQCD data, consequences for phenomenology to be worked out

# **Thanks for Your Attention!**

Zhanduo Tang, Swagato Mukherjee, Peter Petreczky, Ralf Rapp

# Interference function

