Quantum to classical parton evolution in the QGP

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Mainly based on: 2305.10476, with J.-P. Blaizot, Y. Mehtar-Tani

Decoherence in the QGP

Goal: understand the decoherence mechanism for jets in the QGP

Why: in the vacuum decoherence for hardest subjects is related to IRC safety

$$\rho_n\left(\{p_i\}_{i=1}^n, \{p_i'\}_{j=1}^m\right) = \sum_{\{a_i,\lambda_i,f_i\}_{i=1}^n} \sum_{\{a_j',\lambda_j',f_j'\}_{j=1}^m} C_H^\dagger\left(p_1^{a_1\lambda_1 f_1}, \ldots, p_n^{a_n\lambda_n f_n}\right) \times I\left(p_1^{a_1\lambda_1 f_1}, \ldots, p_n^{a_n\lambda_n f_n}; p_1'^{\,a_1'\lambda_1' f_1'}, \ldots, p_m'^{\,a_m'\lambda_m' f_m'}\right) \times C_H\left(p_1'^{\,a_1'\lambda_1' f_1'}, \ldots, p_m'^{\,a_m'\lambda_m' f_m'}\right) + \ldots$$

Breuer, Petruccione; Nagy, Soper; Neill, Waalewijn

$$I\left(p_1^{a_1\lambda_1 f_1}, \ldots, p_n^{a_n\lambda_n f_n}; p_1'^{\,a_1'\lambda_1' f_1'}, \ldots, p_m'^{\,a_m'\lambda_m' f_m'}\right) = 0$$

unless $n = m, p_i = p_i'$ and $a_i = a_i'$ for all i,

How does this mechanism work in the presence of a QCD medium ?

Why: jets in the QGP can be described in terms of effective kinetic description

$$\left(\partial_t + \frac{\boldsymbol{p}}{|\boldsymbol{p}|} \cdot \nabla_{\boldsymbol{x}}\right) f(\boldsymbol{p}, \boldsymbol{x}, t) = -C[f]$$

How does this classical description emerge ?

Quark reduced density matrix in matter

The single parton wavefunction satisfies

Coupling to matter background

$$\left[i\partial_t + \frac{\partial_\perp^2}{2E} + gA(\boldsymbol{r},t)\right]\psi(\boldsymbol{r},t) = 0$$

Light front kinetic energy

The reduced density matrix can be defined as

$$\rho \equiv \mathrm{tr}_A(\rho[A]) = \left\langle |\psi_A(t)\rangle\langle\psi_A(t)| \right\rangle_A$$

We use the Gaussian approximation for the background field

$$g^2\left\langle A^a(\boldsymbol{q},t)A^{\dagger b}(\boldsymbol{q}',t')\right\rangle_A = \delta^{ab}\delta(t-t')(2\pi)^2\delta^{(2)}(\boldsymbol{q}-\boldsymbol{q}')\,\gamma(\boldsymbol{q})$$

Constructing the evolution equations

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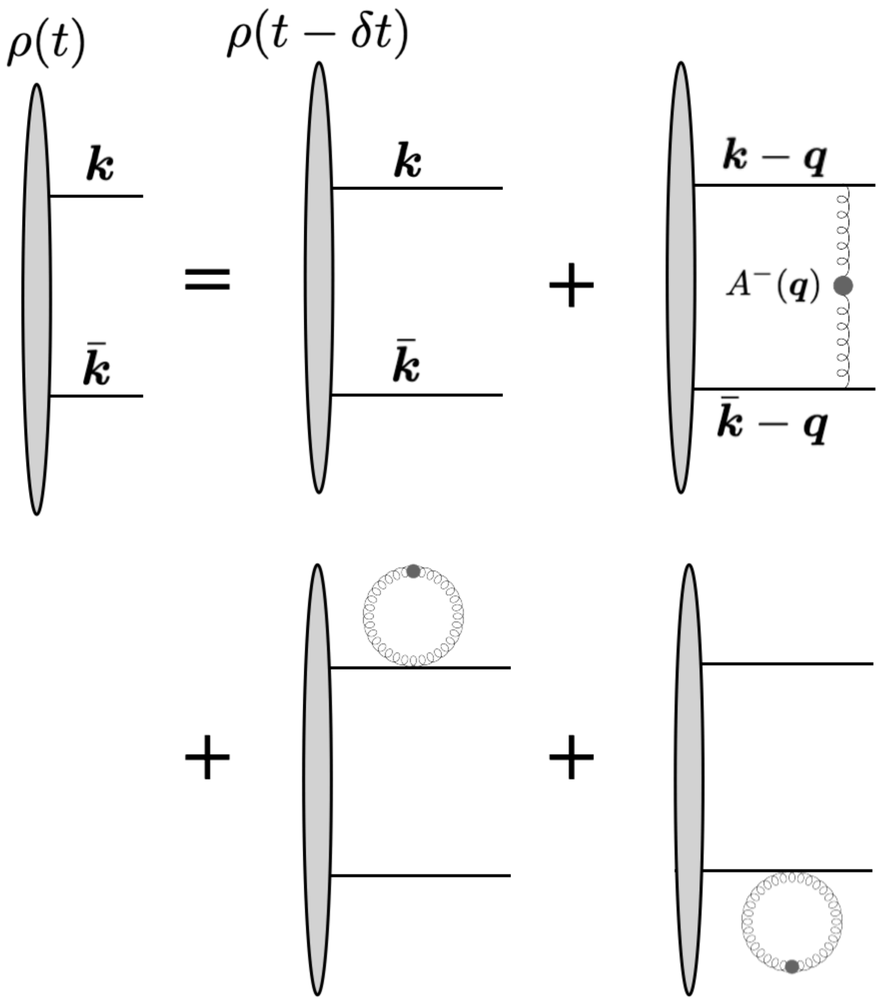

$$3 \otimes \bar{3} = 1 \oplus 8$$

$$\rho(t) \equiv \rho_{\rm s} + t^a \rho_{\rm o}^a = \frac{1}{N_c}{\rm Tr}_c(\rho) + 2\, t^a {\rm Tr}_c(t^a \rho)$$

For color singlet:

$$\langle \boldsymbol{k}|\rho_{\rm s}(t)|\bar{\boldsymbol{k}}\rangle = C_F \int_{\boldsymbol{q}} \int_0^t dt'\, e^{i\frac{(\boldsymbol{k}^2-\bar{\boldsymbol{k}}^2)}{2E}(t-t')} \times \gamma(\boldsymbol{q}) \left[\langle \boldsymbol{k}-\boldsymbol{q}|\rho_{\rm s}(t')|\bar{\boldsymbol{k}}-\boldsymbol{q}\rangle - \langle \boldsymbol{k}|\rho_{\rm s}(t')|\bar{\boldsymbol{k}}\rangle\right]$$

For color octet:

$$\langle \boldsymbol{k}|\rho_{\rm o}(t)|\bar{\boldsymbol{k}}\rangle = C_F \int_{\boldsymbol{q}} \int_0^t dt'\, e^{i\frac{(\boldsymbol{k}^2-\bar{\boldsymbol{k}}^2)}{2E}(t-t')} \times \gamma(\boldsymbol{q}) \left[\langle \boldsymbol{k}-\boldsymbol{q}|\rho_{\rm o}(t')|\bar{\boldsymbol{k}}-\boldsymbol{q}\rangle + \frac{1}{2N_c C_F}\langle \boldsymbol{k}|\rho_{\rm o}(t')|\bar{\boldsymbol{k}}\rangle\right]$$

$$\boldsymbol{b} \equiv \frac{\boldsymbol{r}+\bar{\boldsymbol{r}}}{2}, \quad \boldsymbol{x} \equiv \boldsymbol{r}-\bar{\boldsymbol{r}} \qquad \boldsymbol{K} = \frac{\boldsymbol{k}+\bar{\boldsymbol{k}}}{2}, \quad \boldsymbol{\ell} = \boldsymbol{k}-\bar{\boldsymbol{k}}$$

Constructing the evolution equations

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The matrix elements of the singlet and octet components satisfy Boltzmann transport

$$\gamma(\boldsymbol{q}) \approx g^4 n/\boldsymbol{q}^4$$

$$\partial_t \rho_{\mathrm{s,o}}(\boldsymbol{\ell}, \boldsymbol{x}, t) = -\left[\frac{\boldsymbol{\ell}\cdot\partial_{\boldsymbol{x}}}{E} + \Gamma_{\mathrm{s,o}}(\boldsymbol{x})\right]\rho_{\mathrm{s,o}}(\boldsymbol{\ell}, \boldsymbol{x}, t)$$

$$\Gamma_{\mathrm{s}}(\boldsymbol{x}) = C_F \int_{\boldsymbol{q}} \left(1 - e^{i\boldsymbol{q}\cdot\boldsymbol{x}}\right)\gamma(\boldsymbol{q})\,,$$

$$\Gamma_{\mathrm{o}}(\boldsymbol{x}) = \int_{\boldsymbol{q}} \left(C_F + \frac{1}{2N_c} e^{i\boldsymbol{q}\cdot\boldsymbol{x}}\right)\gamma(\boldsymbol{q})$$

This form allows to settle the evolution in color space

$$\rho_{\mathrm{s,o}}(\boldsymbol{b}, \boldsymbol{x}, t) = \rho^{(0)}_{\mathrm{s,o}}(\boldsymbol{b}, \boldsymbol{x})\, \mathrm{e}^{-t\,\Gamma_{\mathrm{s,o}}(\boldsymbol{x})} \qquad E \to \infty$$

$$\Gamma_{\mathrm{s}}(\boldsymbol{x}) \approx 4\pi\alpha_s^2 C_F n \log\left(\frac{Q^2}{m_D^2}\right)\frac{\boldsymbol{x}^2}{4} \equiv \frac{\hat{q}}{4}\boldsymbol{x}^2\,,$$

$$\Gamma_{\mathrm{o}}(\boldsymbol{x}) \approx \frac{4\pi\alpha_s^2 C_A n}{m_D^2}$$

$\boldsymbol{x} \to 0$ →

Singlet → Neutral to matter

Blaizot, Iancu, Braaten, Pisarski, …

Octet → Damping

Only true in the absence of coherent background fields

One can also show that singlet subspaces become equally probable

Zakharov, Blaizot, Escobedo, …

Singlet evolution 2305.10476

In the absence of interactions with the background, one obtains free streaming

$$\rho_W(\boldsymbol{b} - (\boldsymbol{K}/E)t, \boldsymbol{K}, 0)$$

which is dominated by the natural spreading of the wavepacket

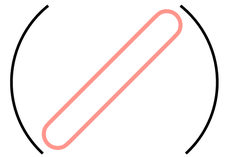
$$\rho(\boldsymbol{b}, t) = \frac{1}{\pi \langle \boldsymbol{b}^2 \rangle_t^{(0)}} \mathrm{e}^{-\frac{\boldsymbol{b}^2}{\langle \boldsymbol{b}^2 \rangle_t^{(0)}}}, \quad \langle \boldsymbol{b}^2 \rangle_t^{(0)} \equiv \frac{1}{\underline{\mu^2}} \left(1 + \frac{t^2}{t_0^2}\right)$$

Characteristic momentum scale of initial wave packet

This introduces the wave packet natural spreading scale

$$t_0 = \frac{E}{\mu^2}$$

Singlet evolution 2305.10476

When including interactions, more scales emerge. Consider first the **diagonal elements** $\rho =$ 

Momentum space:

$$\rho(\boldsymbol{\ell}, \boldsymbol{K}) = \frac{4\pi}{a} \exp\left\{ -\frac{1}{4a}\boldsymbol{K}^2 - \frac{1}{4E^2}\left(c - \frac{b^2}{4a}\right)\boldsymbol{\ell}^2 - i\frac{b}{4Ea}\boldsymbol{\ell}\cdot\boldsymbol{K} \right\}$$

For the sectors $\boldsymbol{\ell} = 0$ we recover the classical broadening distribution but with

$$a = \langle \boldsymbol{K}^2 \rangle_t, \quad \frac{c}{E^2} = \langle \boldsymbol{b}^2 \rangle_t, \quad \frac{b}{E} = -2\langle \boldsymbol{b}\cdot\boldsymbol{K} \rangle_t$$

$$D = \langle \boldsymbol{b}^2 \rangle \langle \boldsymbol{K}^2 \rangle - \langle \boldsymbol{K}\cdot\boldsymbol{b} \rangle$$

$$\langle \boldsymbol{k}^2 \rangle_t = \mu^2 + \hat{q}t = \mu^2\left(1 + \frac{t}{t_1}\right)$$

This introduces the scale where the wavepacket becomes sensitive to medium effects

$$t_1 = \frac{\mu^2}{\hat{q}}$$

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When including interactions, more scales emerge. Now let us look at the **off-diagonal elements** $\rho =$ 

Momentum space:

$$\rho(\boldsymbol{\ell}, \boldsymbol{K} = 0, t) = \frac{4\pi}{\mu^2(1 + (t/t_1))} \exp\left\{-\frac{\boldsymbol{\ell}^2}{4\mu^2} d(t)\right\} \qquad d(t) = 1 + \frac{1}{12}\left(\frac{t}{t_2}\right)^3 \frac{t + 4t_1}{t + t_1}$$

At late times, the initial condition is lost and off-diagonal terms vanish rapidly

$$\rho(\boldsymbol{\ell}, \boldsymbol{K} = 0, t) \approx \frac{4\pi}{\mu^2 + \hat{q}t} \exp\left\{-\frac{\boldsymbol{\ell}^2\, \hat{q}t^3}{48E^2}\right\}$$

The timescale after which diagonalization starts to take place reads

$$t_2^3 = \frac{E^2}{\hat{q}\mu^2} \qquad t_2^3 = t_1 t_0^2$$

Singlet evolution: numerical example

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$$t_2^3 = t_1 t_0^2$$

$$t_0 > t_2 > t_1$$

Medium-parton interactions dominate evolution

vs

$$t_0 < t_2 < t_1$$

Natural wave packet spreading determines evolution

$\hat{q} = 0.3\,\mathrm{GeV}^3$, $\mu = 0.3$ GeV, and $E = 200\,\mathrm{GeV}$

$t_1 \simeq 0.06$ fm $\quad t_2 \simeq 22.80$ fm $\quad t_0 \simeq 444.44$ fm

Singlet evolution: numerical example 2305.10476

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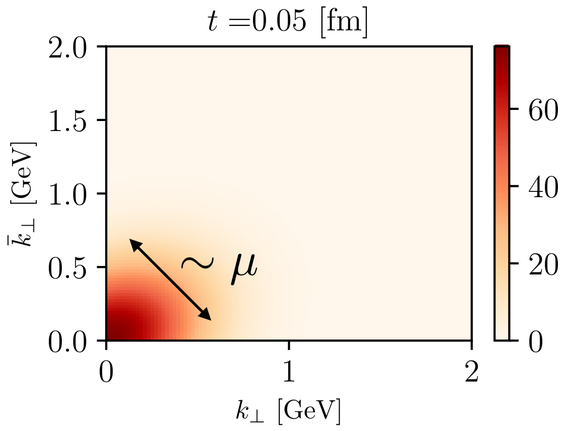

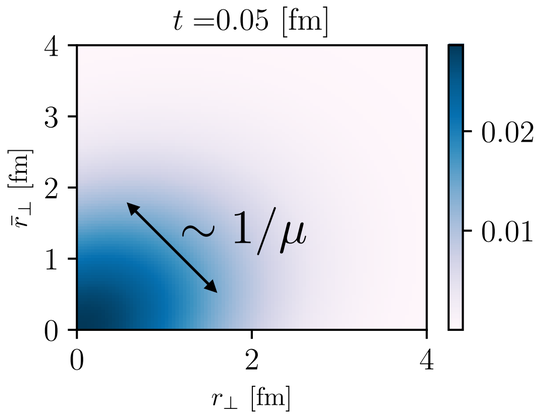

Singlet evolution: numerical example 2305.10476

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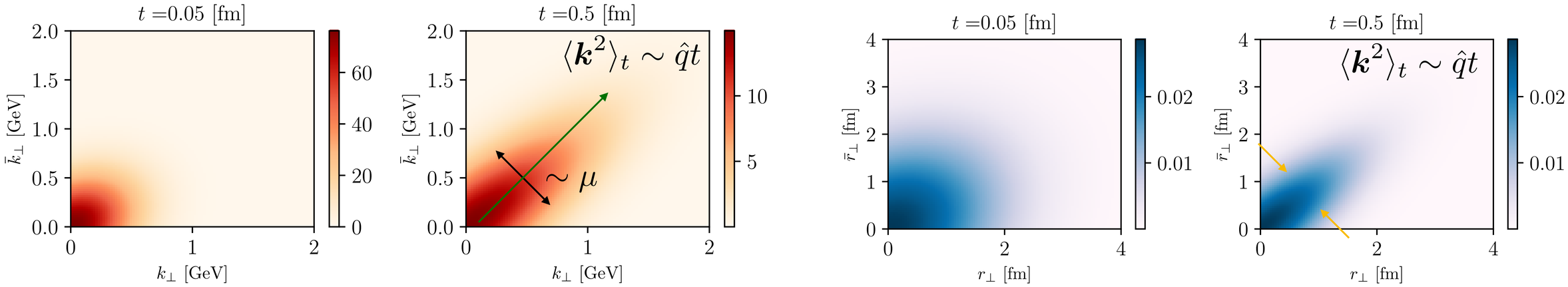

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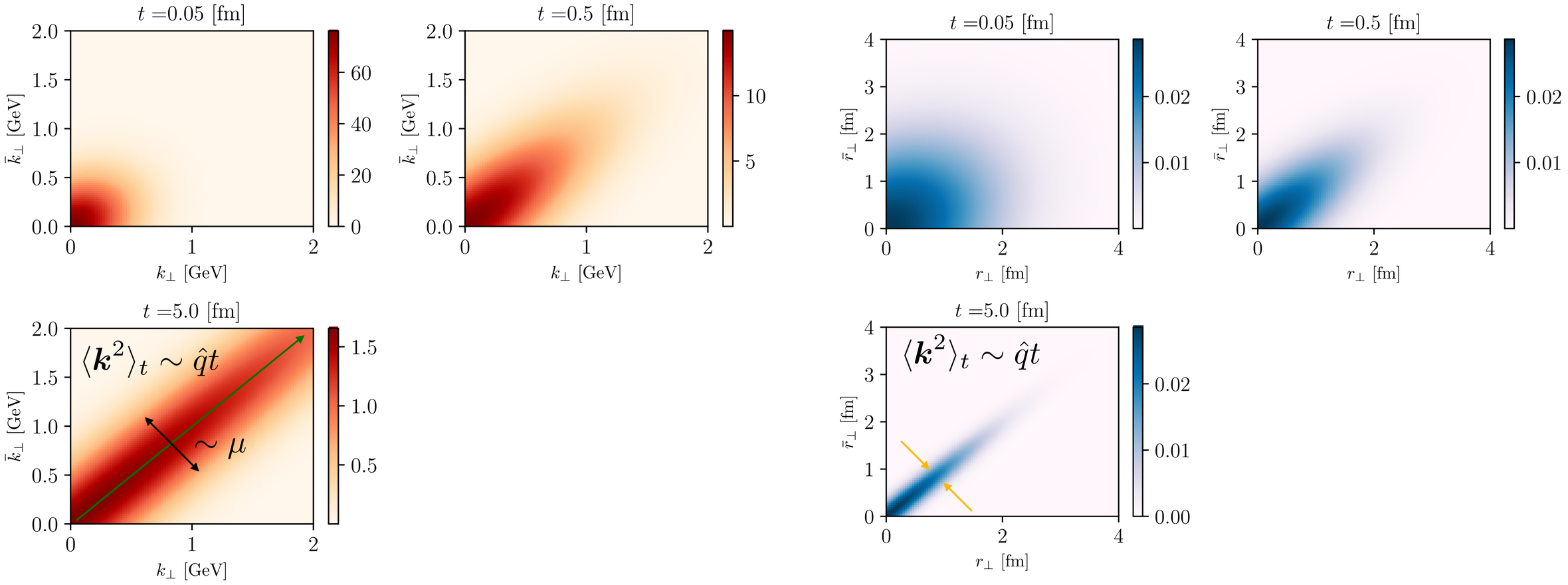

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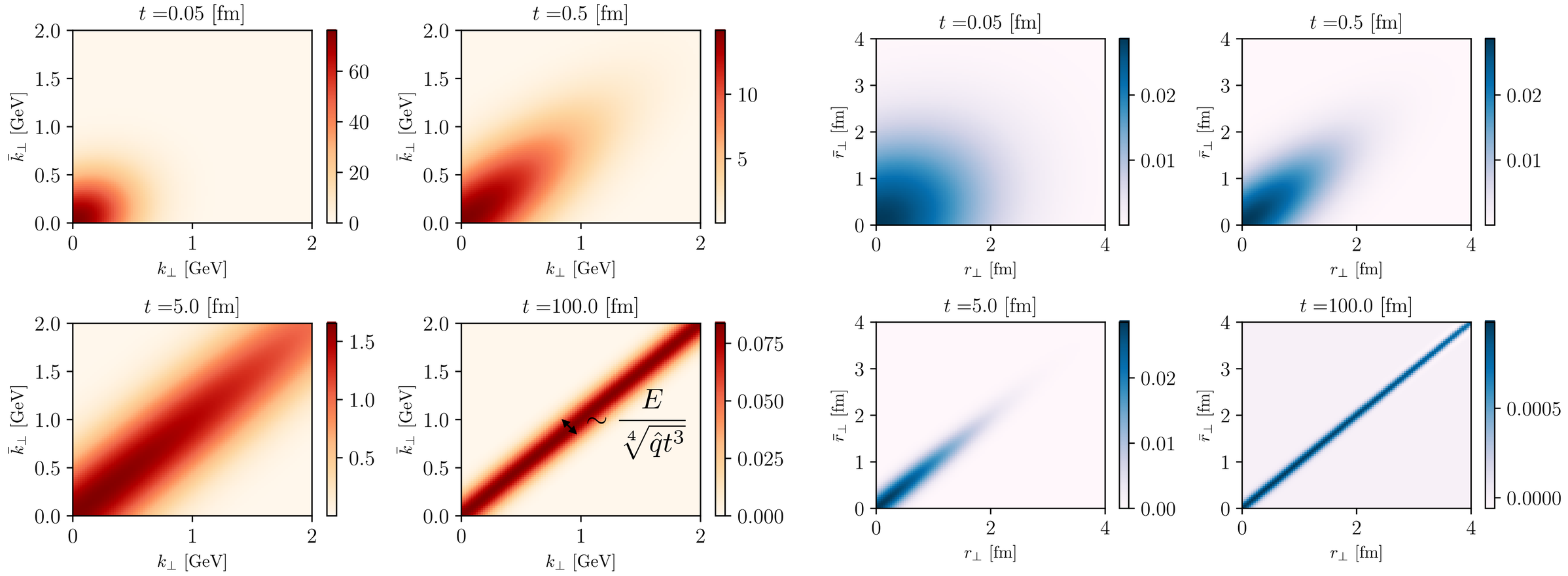
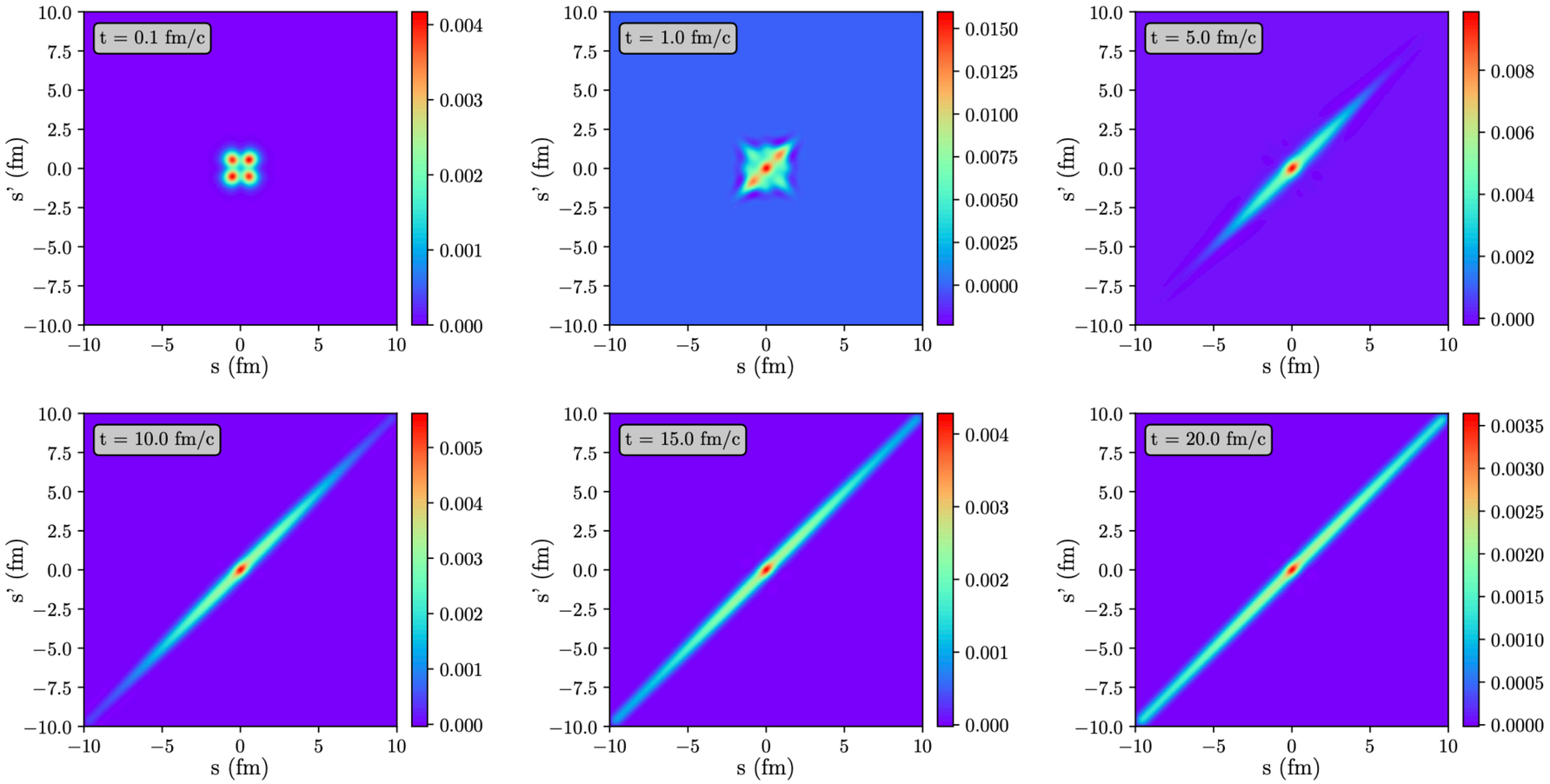


Singlet evolution: numerical example 2305.10476

Similar evolution to that seen for quarkonia

$\mathcal{D}_s$ **T = 300 MeV initial P-like octet**

Delorme, Gousset, Katz, Gossiaux



$$\frac{d}{dt}\begin{pmatrix}\mathcal{D}_s\\ \mathcal{D}_o\end{pmatrix} = \mathcal{L}\begin{pmatrix}\mathcal{D}_s(\mathbf{s},\mathbf{s}',t)\\ \mathcal{D}_o(\mathbf{s},\mathbf{s}',t)\end{pmatrix}$$

$$\mathcal{L} = \begin{pmatrix}\mathcal{L}_{ss} & \mathcal{L}_{so}\\ \mathcal{L}_{os} & \mathcal{L}_{oo}\end{pmatrix}$$

Entropy as a measure of quantum to classical transition

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Having access to the reduced density matrix we compute the associated **von-Neumann entropy**,

$$S_{\mathrm{vN}}[\rho] = -\mathrm{Tr}\rho \ln \rho \qquad S_{\mathrm{vN}} = \log\left(\frac{1-p}{4p}\right) + \frac{1}{\sqrt{p}} \ln \frac{1+p+2\sqrt{p}}{(1-p)}$$

the **purity**,

$$p \equiv \mathrm{Tr}\rho^2 \qquad \frac{1}{p} = \left(1+\frac{t}{t_1}\right)\left(1+\frac{t^3}{12t_2^3}\frac{t+4t_1}{t+t_1}\right)$$

and the **Wigner entropy**

$$S_{\mathrm{w}} \equiv -\int_{\boldsymbol{K},\boldsymbol{b}} \rho_{\mathrm{w}}(\boldsymbol{b},\boldsymbol{K}) \log \rho_{\mathrm{w}}(\boldsymbol{b},\boldsymbol{K}) \qquad S_{\mathrm{w}} = \ln\frac{1}{p} + 2 - \ln 4$$

Entropy as a measure of quantum to classical transition

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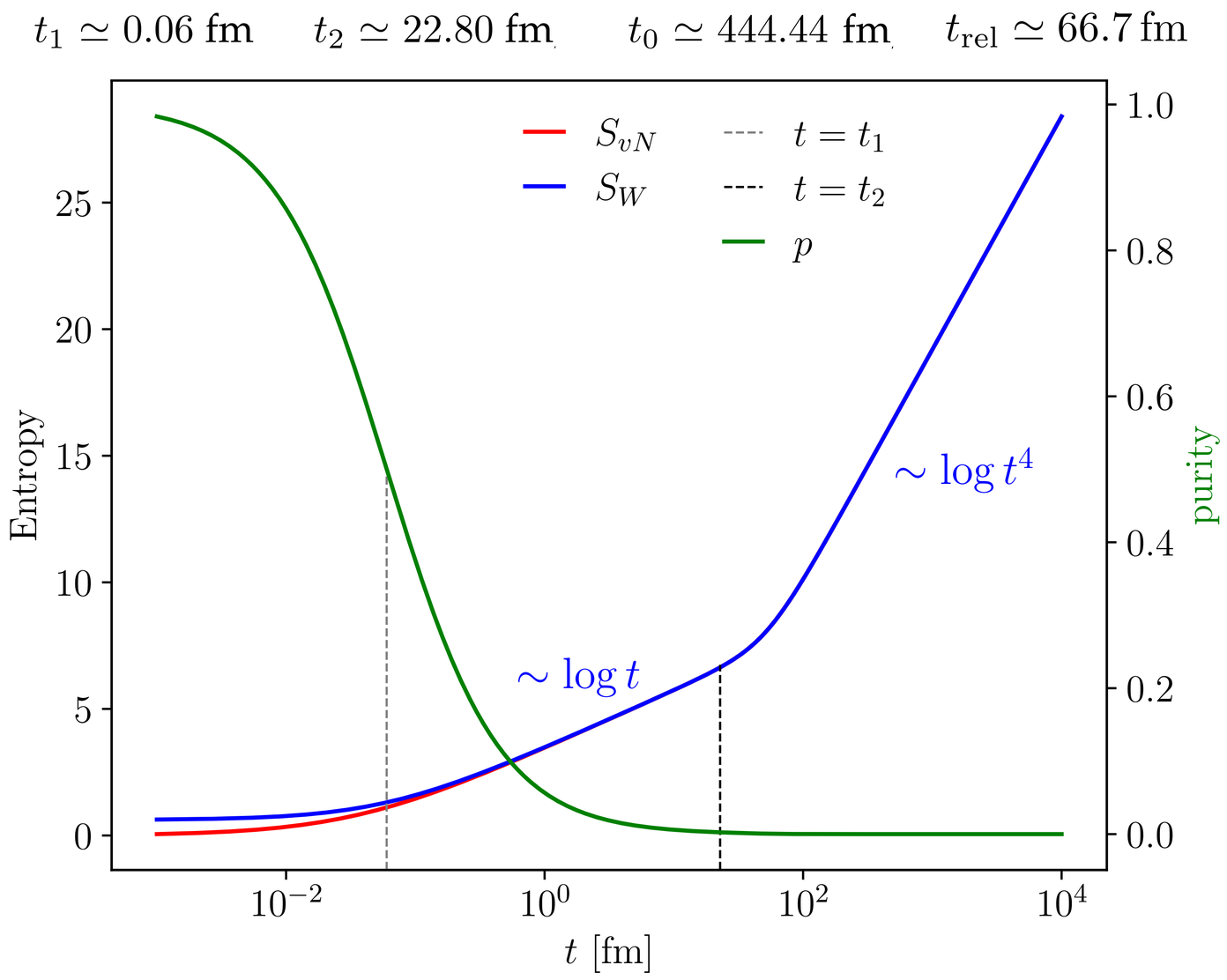


Asymptotically, one has that

$$S_{\rm vN} \simeq \ln\langle \boldsymbol{k}^2\rangle_t \langle \boldsymbol{b}^2\rangle_t \qquad \frac{S_{\rm w} - S_{\rm vN}}{S_{\rm w}} \approx \frac{\sqrt{p}}{\ln(1/p)}$$

thus, the **entropy content of the density matrix coincides with that of a classical distribution**

In reality, the entropy growth is bounded

$$\left[\frac{\partial}{\partial t} - \frac{\partial}{\partial \boldsymbol{K}}\left(\frac{\hat{q}}{4}\frac{\partial}{\partial \boldsymbol{K}} + \gamma_f \frac{\boldsymbol{K}}{E}\right)\right]\mathcal{P}(\boldsymbol{K}, t) = 0$$

This occurs roughly after a time $\gamma_f = \hat{q}/4T \sim T^2$

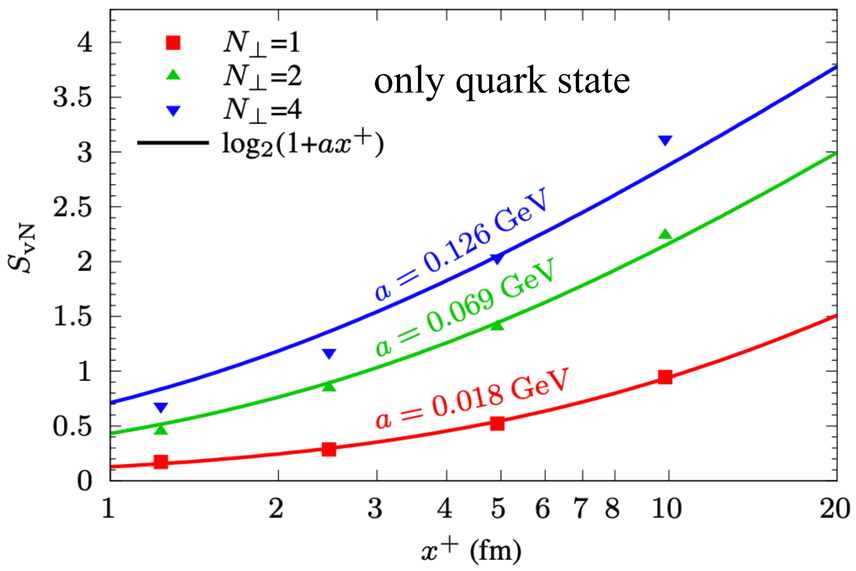
$$t_{\rm rel} \equiv ET/\hat{q}$$

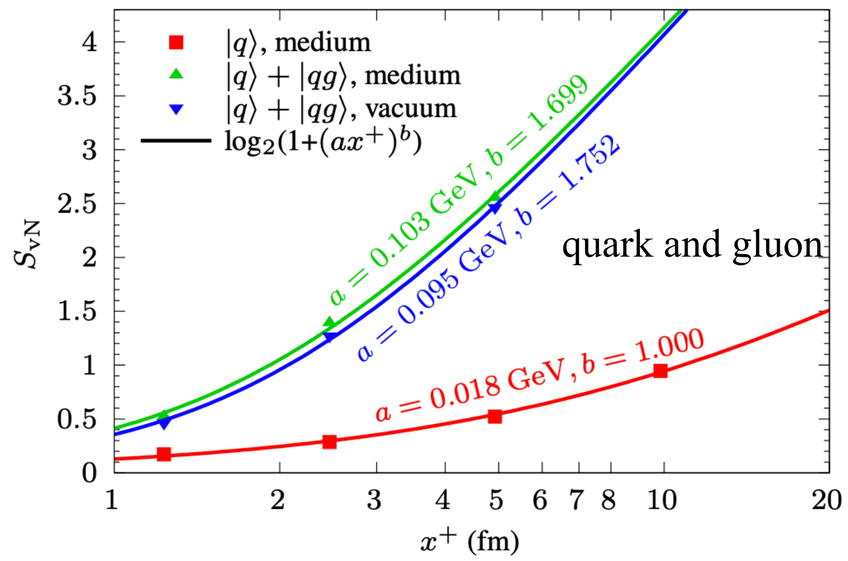
Jet entropy via quantum simulation

JB, X. Du, M. Li, W. Qian, C. Salgado, 2307.01792

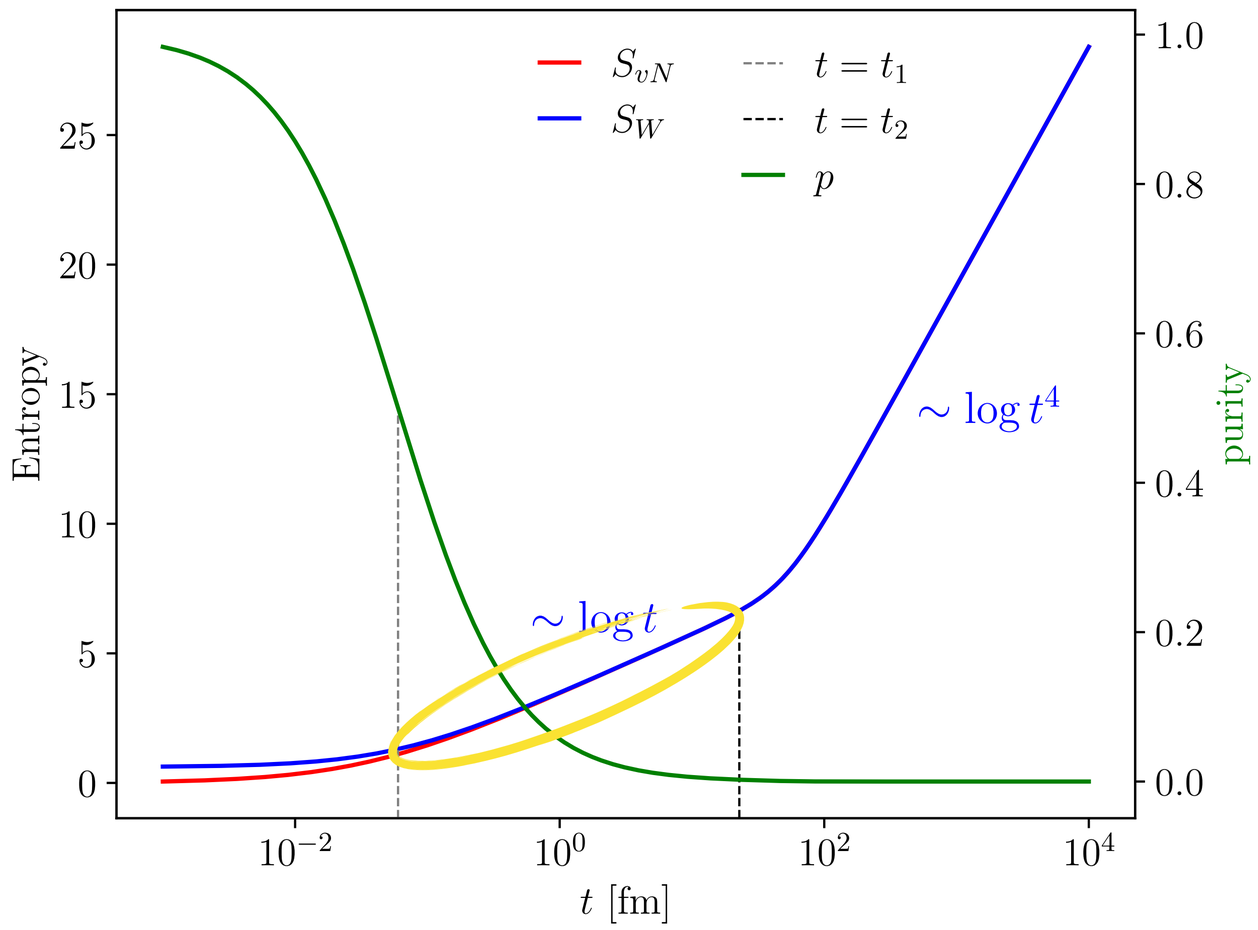
Brookhaven National Laboratory

Quantum simulation





Analytical calculation



Jet entropy in the QGP from pQCD

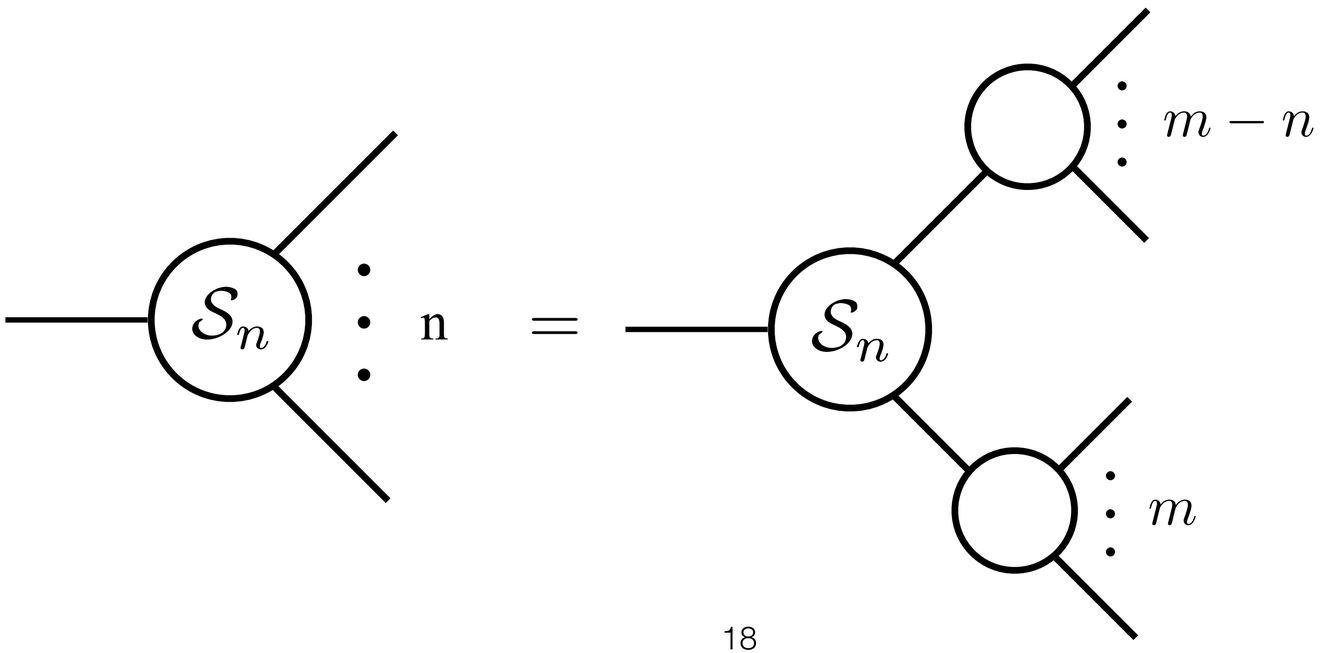
very preliminary, JB, J.-P. Blaizot, Y. Mehtar-Tani

Going beyond: assuming the decoherence mechanism in the QGP works at late times, we can write the entropy for the hardest subjets in a jet

$$\mathcal{S} = -\sum_n \int d\Pi_n \frac{dP}{d\Pi_n} \log \frac{dP}{d\Pi_n} = \sum_n \mathcal{S}_n$$

Nagy, Soper; Neill, Waalewijn

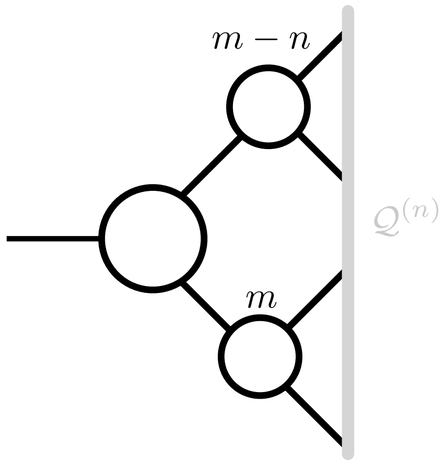
At leading logarithmic accuracy and using a physical gauge, we can then write



Jet entropy in the QGP from pQCD

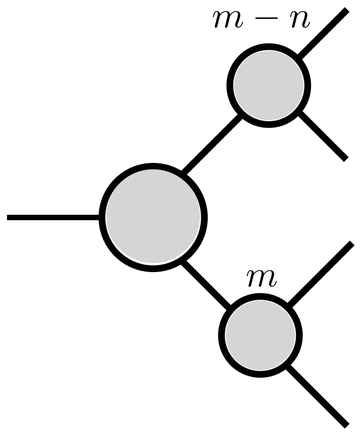
Going beyond: in matter jet entropy will be modified by different types of effects

Energy loss



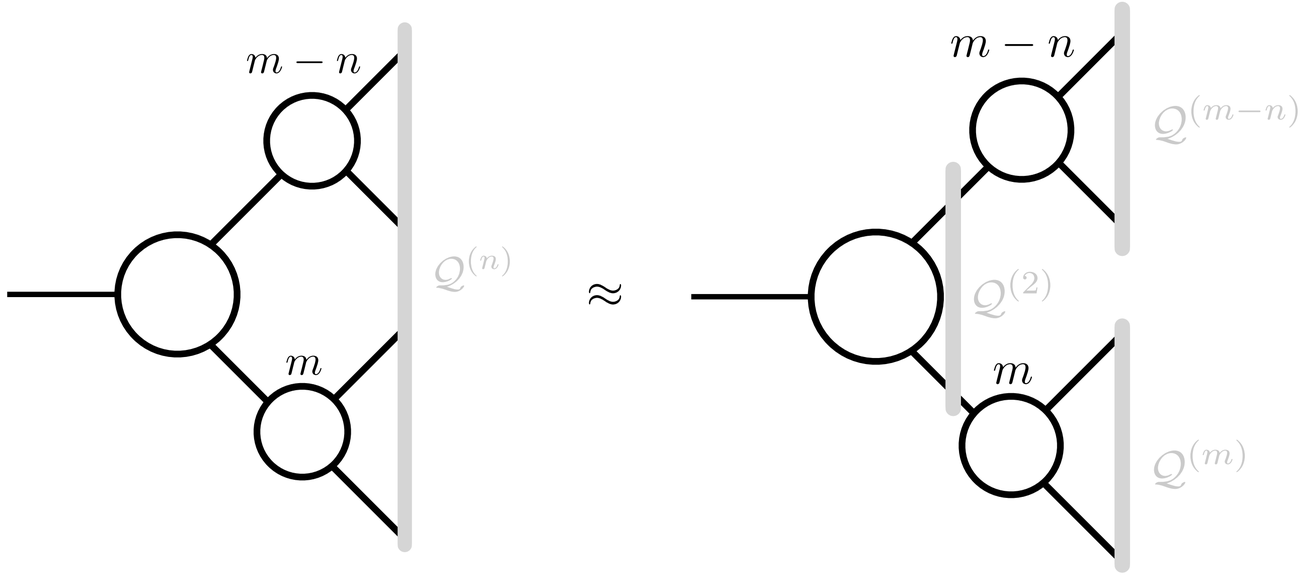
Phase space modifications

Medium induced radiation



Jet entropy in the QGP from pQCD

Energy loss: using simple quenching weight approximation and assuming



We find at leading order in the strong coupling

Entropy due to radiation going into matter

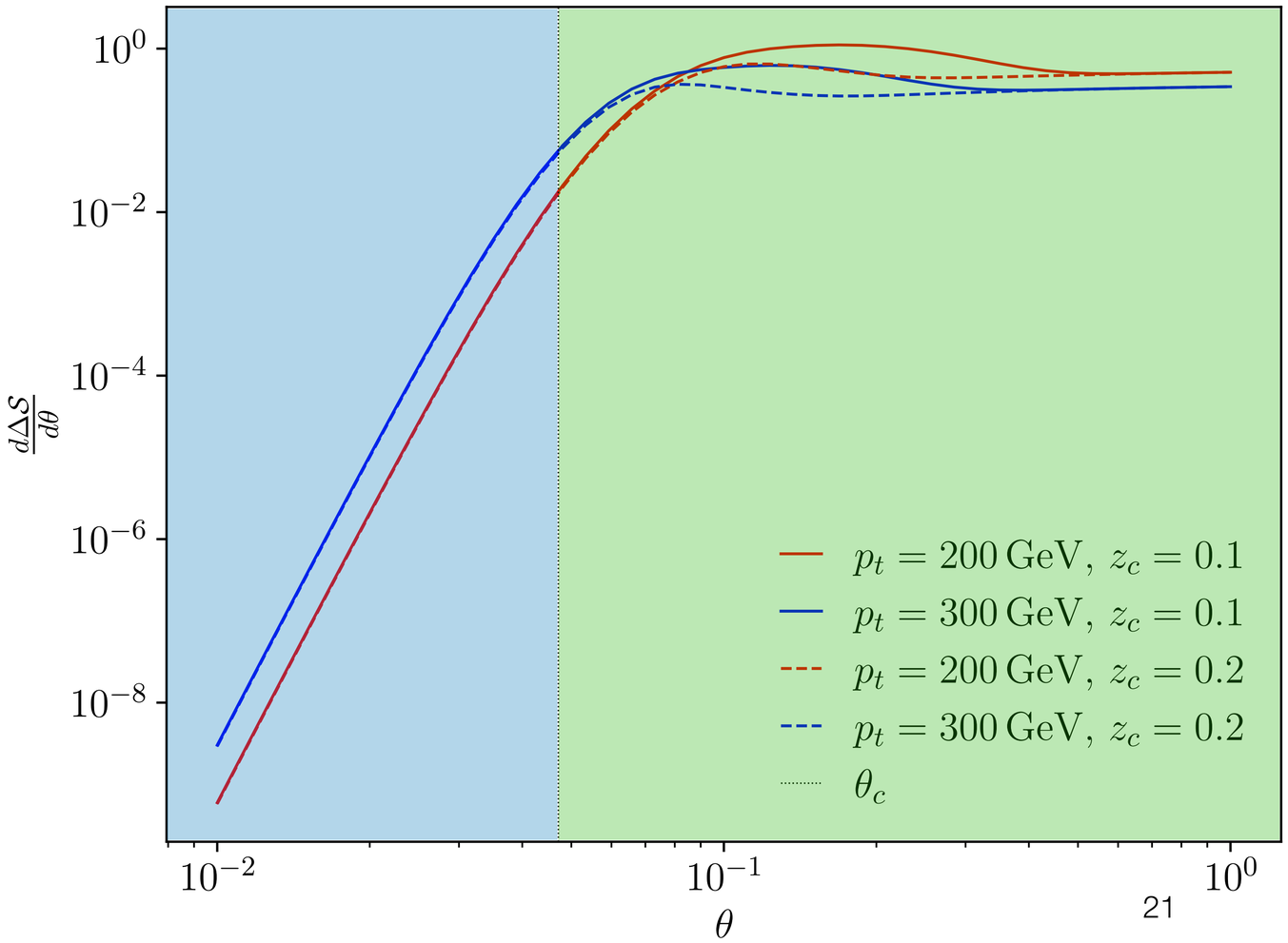
$$\mathcal{S}_Q(E,R) = \frac{2\alpha_s}{\pi}\left(Q^{(1)}\log\frac{1}{z_c}\log\frac{R}{R_c} + \mathcal{S}(Q^{(2)}) + \int_{z_c}^{1}\frac{dz}{z}\int_{R_c}^{R}\frac{d\theta}{\theta}Q^{(2)}(z,\theta)\log\frac{8\pi\alpha_s\Lambda^2}{z^2\theta^2E^2}\right) + \mathcal{O}(\alpha_s^2)$$

Entropy for single emission

Jet entropy in the QGP from pQCD

Medium induced radiation: at LO the modified splitting function can always be written as $\frac{d\sigma}{\sigma}(1+F_{\rm med})$

The **entropy variation with respect to the vacuum** then reads

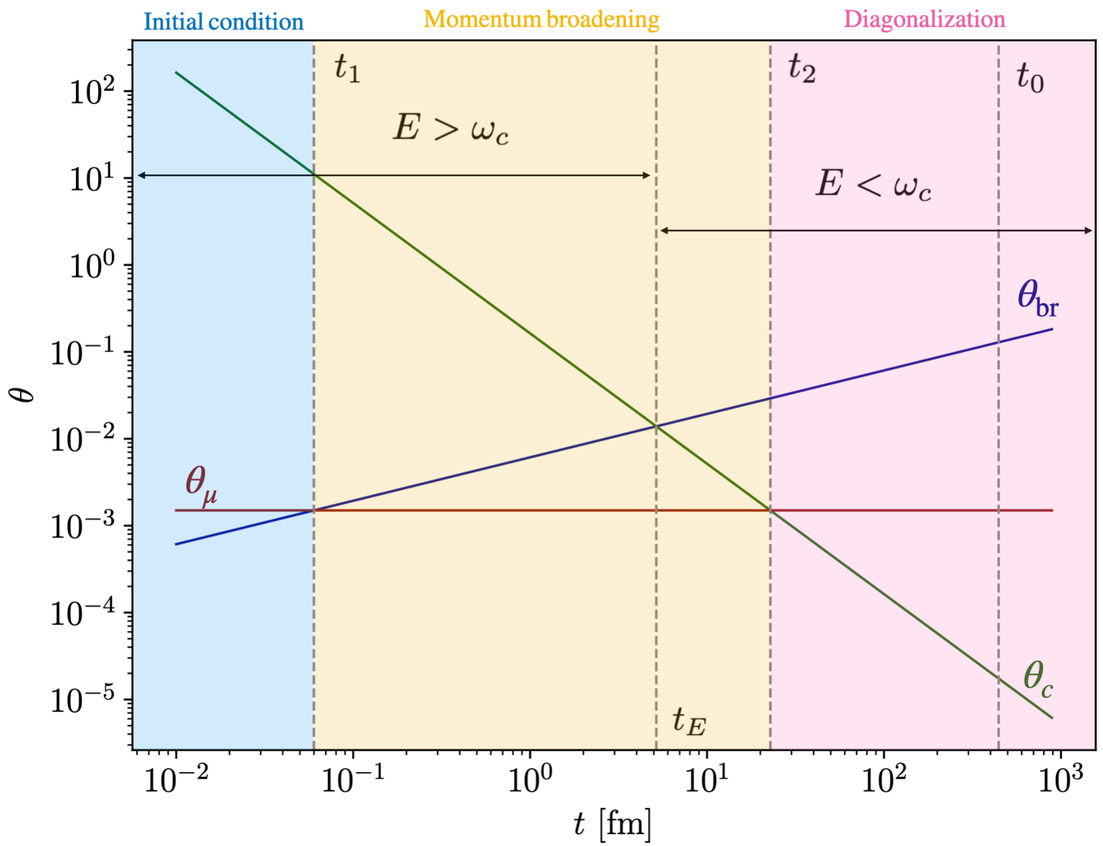


$$\frac{d[\Delta\mathcal{S}](p_t)}{d\theta} \approx \frac{2\alpha_s}{\pi\theta}\left\{\int_{z_c}^{1}\frac{dz}{z}\log\left[\frac{e^{F_{\rm med}(\theta,z)}}{1+F_{\rm med}(\theta,z)}\right]\right\}$$

which exhibits a transition between the coherent and decoherent regimes

Conclusion and Outlook

We studied how the decoherence takes place for a single parton in a dense QGP



New QIS techniques and measures might give new insight into jet quenching physics