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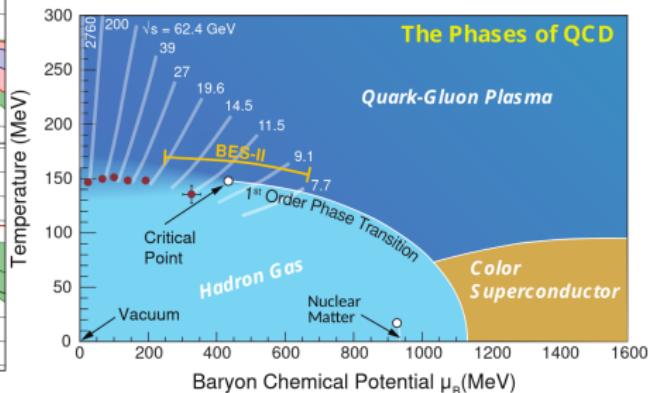
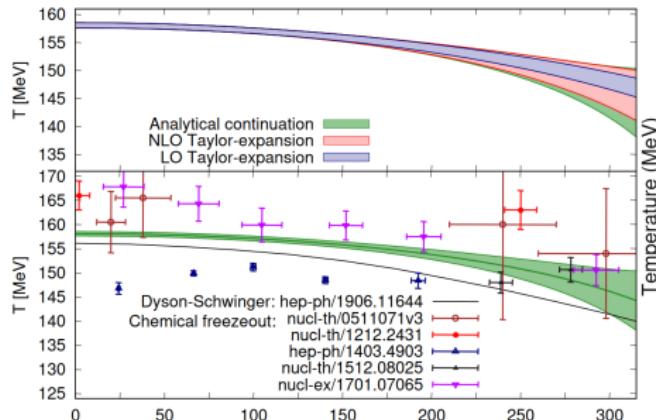
# FINITE VOLUME EFFECTS NEAR THE CHIRAL CROSSOVER

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and

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# Physical quark masses and finite chemical potential



## Assured knowledge

- Crossover at  $T = 158 \pm 0.6$  MeV  $\mu_B = 0$  [Aoki Nature (06)] [Borsanyi, R.K. PRL (20)]
- Nuclear liquid-gas transition low  $T$  and  $\mu_B \simeq 920$  MeV [Pochodzalla PRL 1995]
- Chem. freeze-out temp. close to  $T_{\text{ps}}$  [Andronic NPA 06] [Alba PLB 14] [Flor PLB 21]

# Chiral Observables

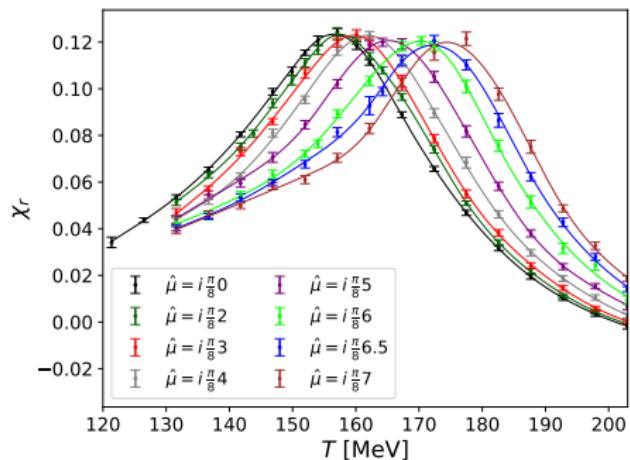
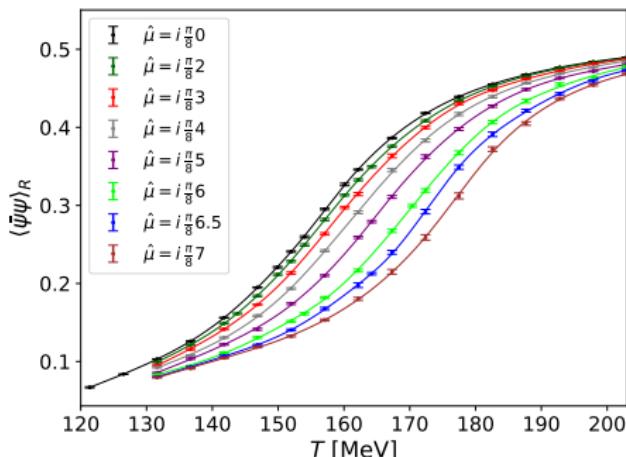
## Chiral condensate and susceptibility

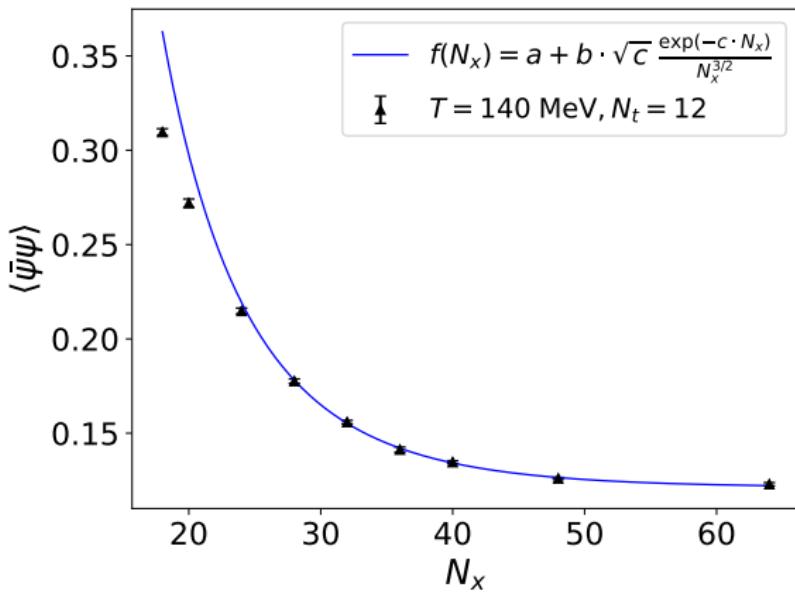
$$\langle \bar{\psi} \psi \rangle = \frac{T}{V} \frac{\partial \log Z}{\partial m}$$

$$\langle \bar{\psi} \psi \rangle_R = - [\langle \bar{\psi} \psi \rangle_T - \langle \bar{\psi} \psi \rangle_{T=0}] \frac{m}{f_\pi^4}$$

$$\chi = \frac{T}{V} \frac{\partial^2 \log Z}{\partial m^2}$$

$$\chi_R = [\chi_T - \chi_{T=0}] \frac{m^2}{f_\pi^4}$$





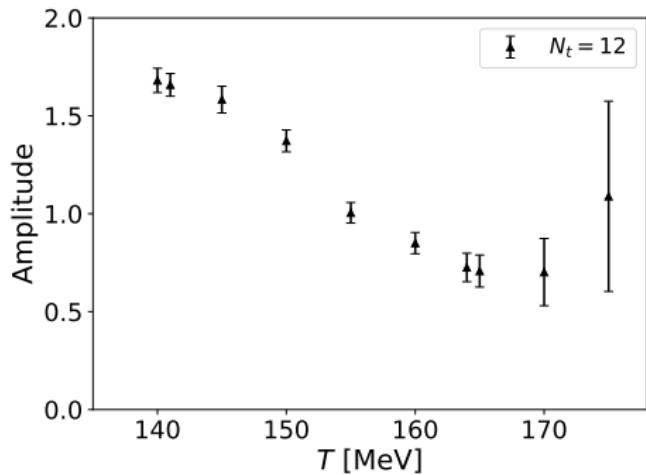
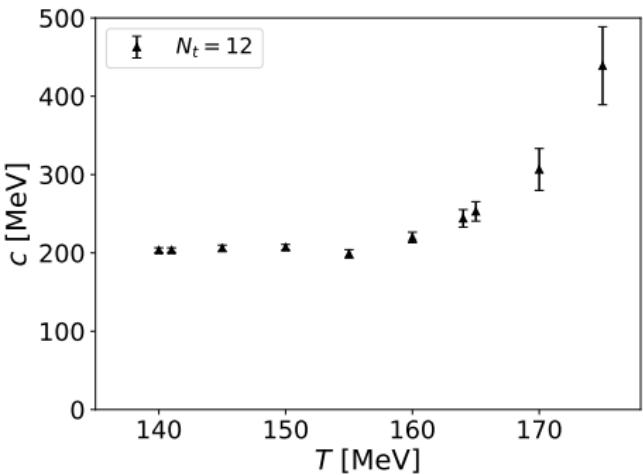
## Features

- Chiral PT prediction for  $T = 0$  [Adhikari PRD 2023]:  $4.5 \cdot \frac{\sqrt{m_\pi}}{F_\pi^2} \frac{e^{-m_\pi N_x}}{(2\pi N_x)^{3/2}}$
- Choose temperature below  $T_c$ :  $T = 140$  MeV
- Result:  $m_\pi = 131 \pm 10$  MeV in the range of  $N_x \in [28, 64]$

# Volume dependence of the condensate

## Features

- Solve  $\langle \bar{\psi} \psi \rangle$  for  $T \in [140, 180]$ : Exponential behaviour for full range
- $f(N_x) = a + b \cdot \exp(-c \cdot N_x)$ :  $c$  has mass dimension



# Precise determination of $T_c$

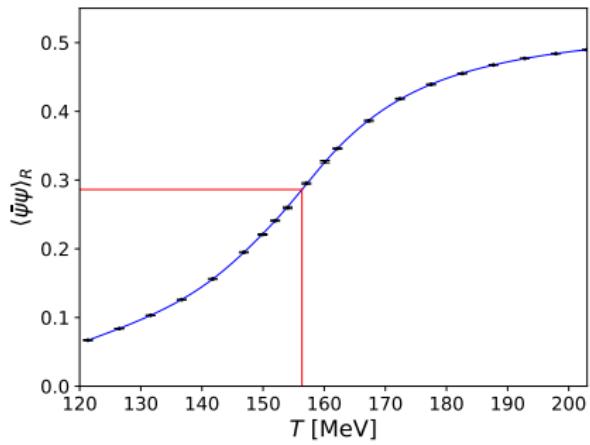
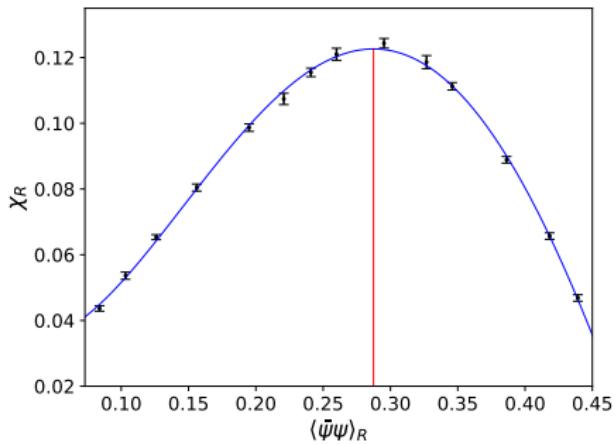
$$\langle \bar{\psi}\psi \rangle = \frac{T}{V} \frac{\partial \log Z}{\partial m}$$

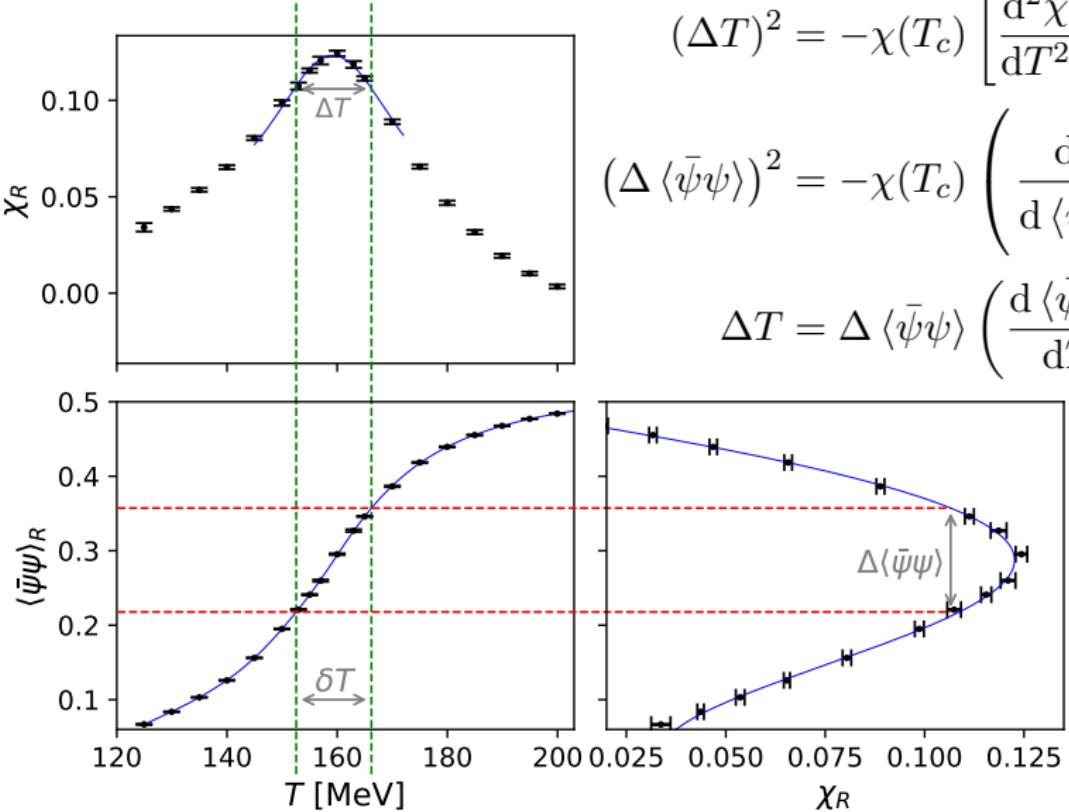
$$\langle \bar{\psi}\psi \rangle_R = - [\langle \bar{\psi}\psi \rangle_T - \langle \bar{\psi}\psi \rangle_{T=0}] \frac{m}{f_\pi^4}$$

$$\chi = \frac{T}{V} \frac{\partial^2 \log Z}{\partial m^2}$$

$$\chi_R = [\chi_T - \chi_{T=0}] \frac{m^2}{f_\pi^4}$$

How to get precisely the inflection point of  $\langle \bar{\psi}\psi \rangle$  or the maximum of  $\chi$ ?



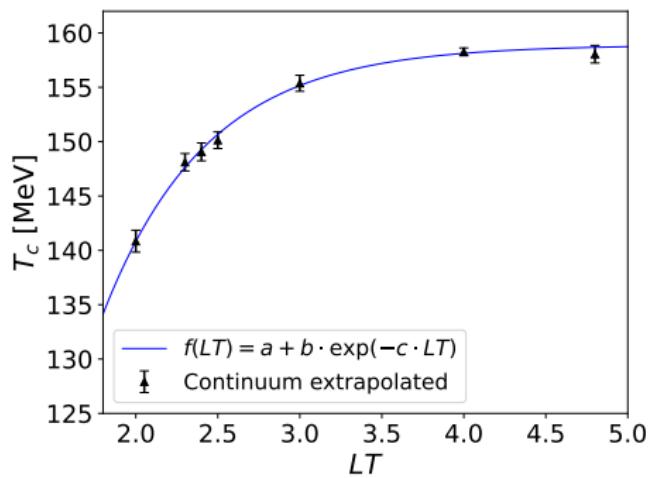
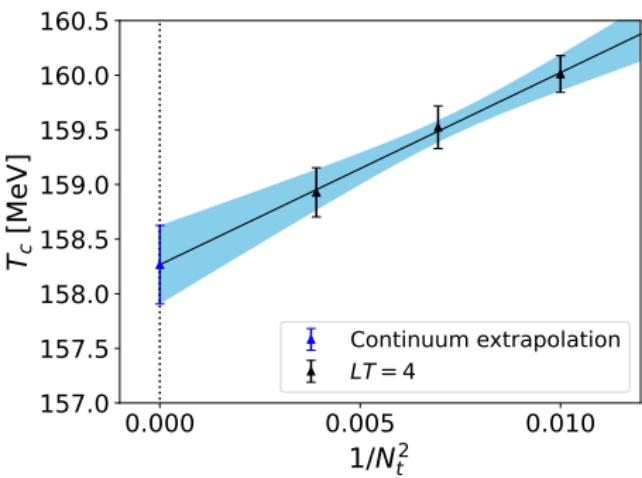


$$(\Delta T)^2 = -\chi(T_c) \left[ \frac{d^2 \chi}{dT^2}(T_c) \right]^{-1}$$

$$(\Delta \langle \bar{\psi}\psi \rangle)^2 = -\chi(T_c) \left( \frac{d^2 \chi}{d \langle \bar{\psi}\psi \rangle^2} \Big|_{\langle \bar{\psi}\psi \rangle_c} \right)^{-1}$$

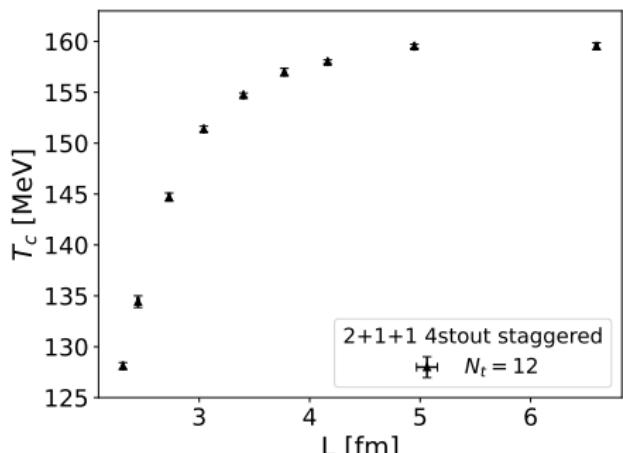
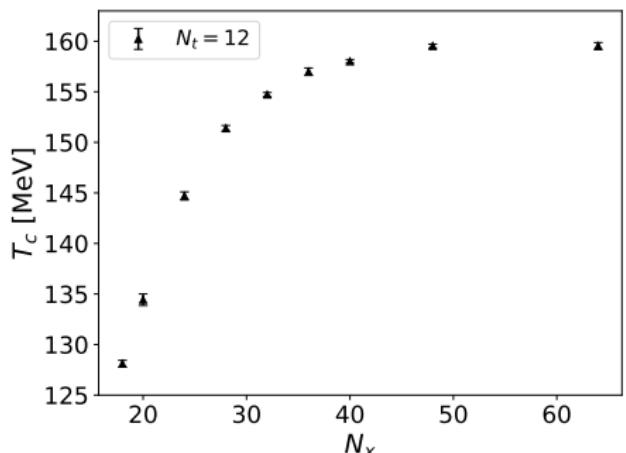
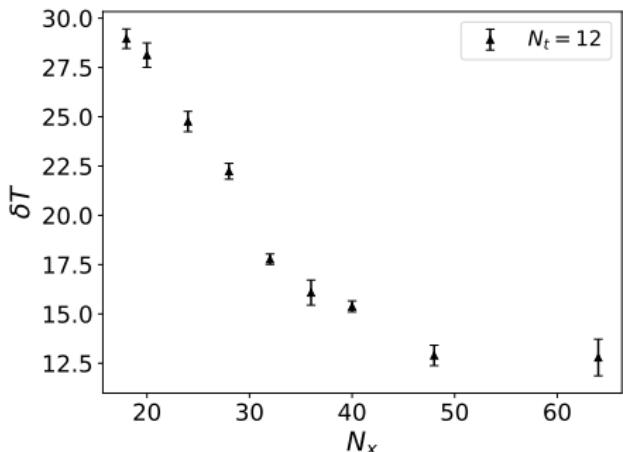
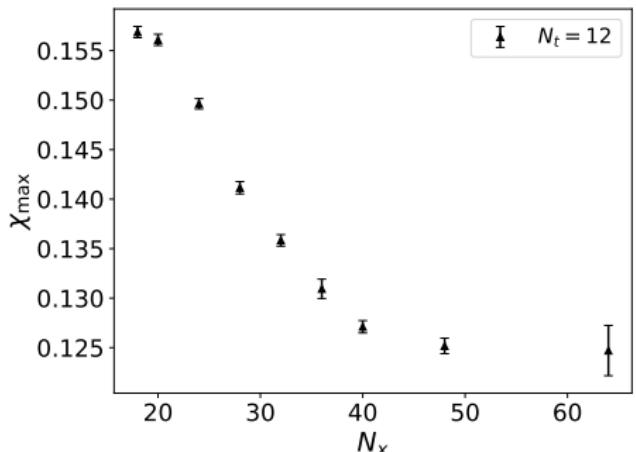
$$\Delta T = \Delta \langle \bar{\psi}\psi \rangle \left( \frac{d \langle \bar{\psi}\psi \rangle}{dT} \right)^{-1}$$

# Continuum and infinite volume extrapolated $T_c$



Infinite volume limit of  $T_c$  at  $\mu_B = 0$

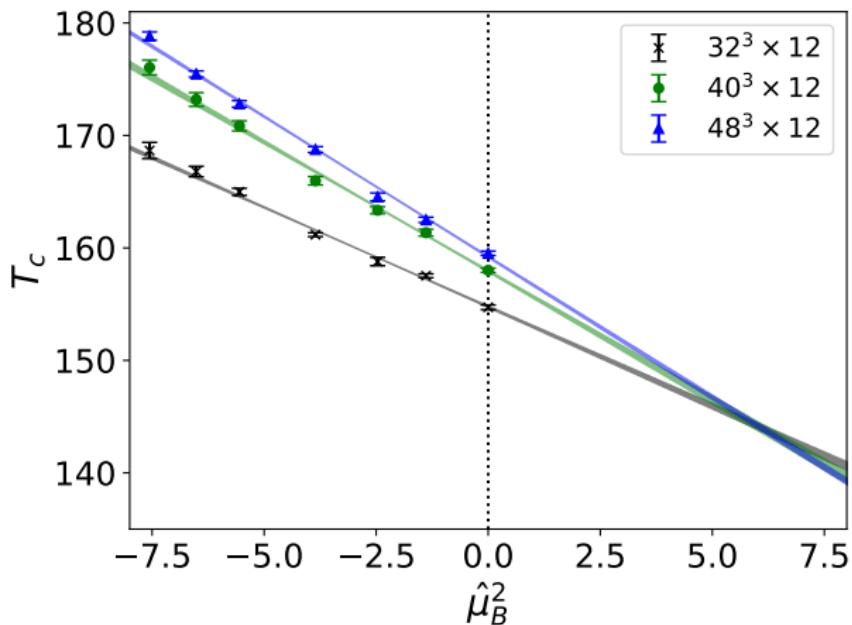
- Inf. vol. lim. on continuum extrapolated values  $\underline{T_c = 158.9 \pm 0.6 \text{ MeV}}$
- Compare to  $T_c(LT = 4) = 158.0 \pm 0.6 \text{ MeV}$  [Wuppertal-Budapest PRL 2020]



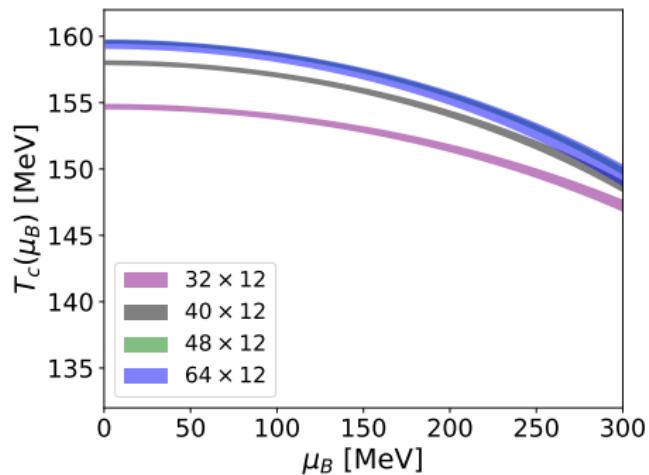
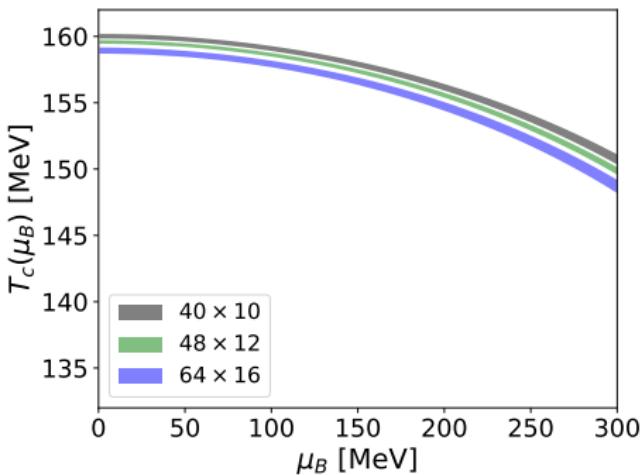
# Phase diagram in the strangeness neutral case

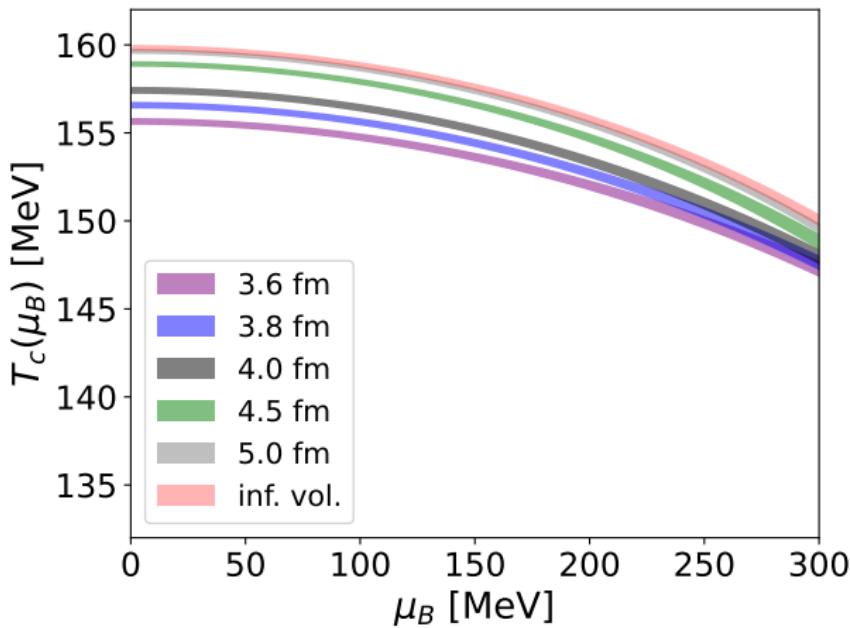
## Transition line in a finite box

- Determine transition line by  $\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa_2 \left( \frac{\mu_B}{T_c(\mu_B)} \right)^2$



# Cutoff vs. finite volume effects

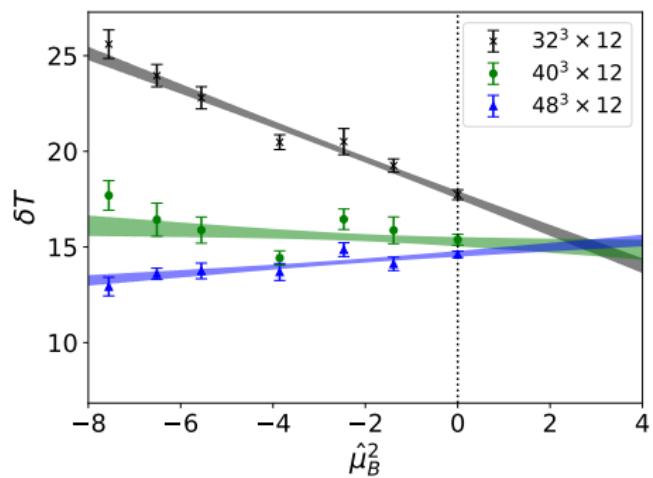
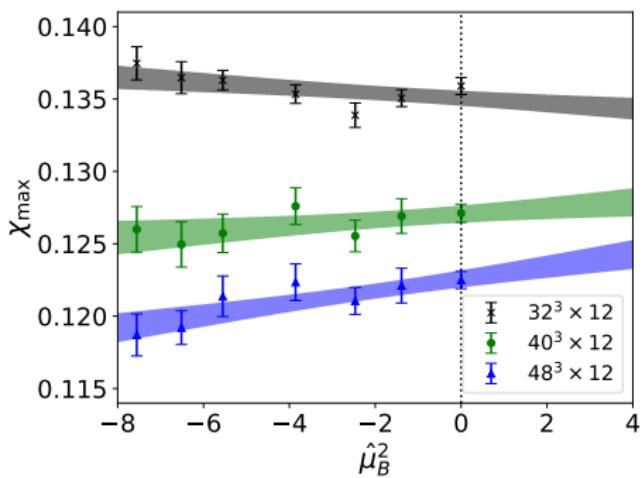




Transition line in a finite box with  $N_t = 12$

- Solve cubic equation for  $T_c(\mu_B)$  for every  $\mu_B$
- Box-size:  $L = \frac{N_x}{N_t T_c(\mu_B)}$ : Iterate  $T_c(N_x)$  for every  $\mu_B$  to match  $L$  [fm]
- Volume effects seem to decrease for larger  $\mu_B$

# Strength and width of the crossover at finite $\mu_B$

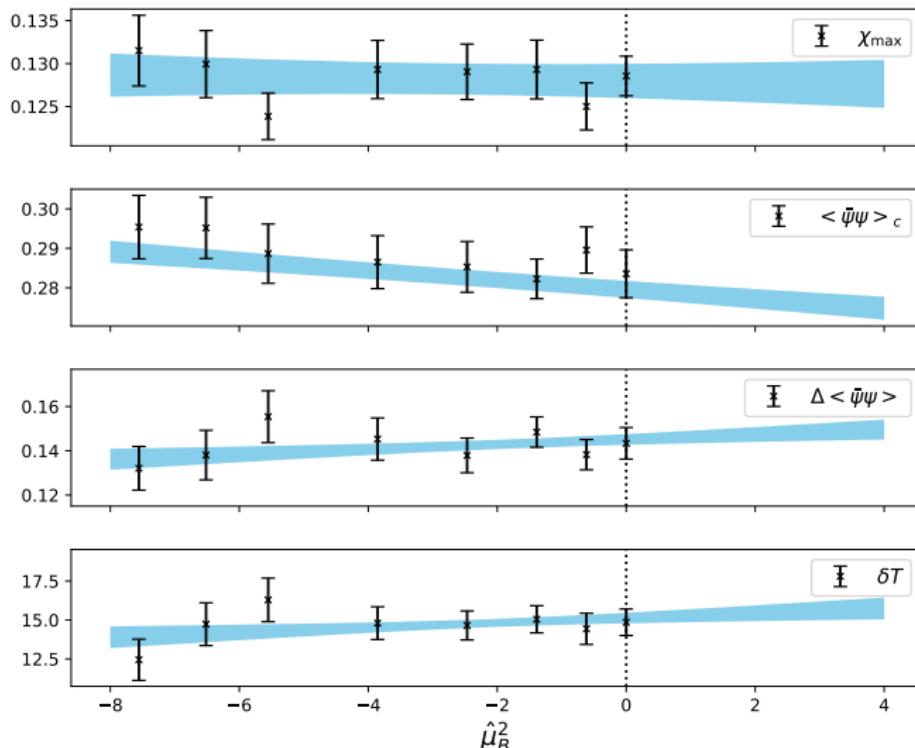


Does the crossover turn into a real transition?

- $\hat{\mu}_B^2 \leq 0$ :  $\chi_{\max}$  decreases in descending order of the vol.  $\Rightarrow$  weak crossover
- No sign of a stronger transition

Continuum extrapolated observables as functions of  $\hat{\mu}_B^2$  at  $LT = 4$ 

[Wuppertal-Budapest PRL 2020]



# Summary

## Phys. quark mass simulations at $\mu_B = 0$

- Observables show exponential behaviour at fixed  $N_t$  as functions of  $N_x$
- Strength and width of the transition increase for  $LT < 3$
- Condensate follows  $\chi$ -PT predictions for  $T < T_c$  :  $m_\pi = 131 \pm 10$  MeV
- Infinite volume limit on continuum extrapolated results:

$$\underline{T_c(\mu_B = 0) = 158.90 \pm 0.63 \text{ MeV}}$$

## Finite $\mu_B$ and continuum extrapolations

- Finite volume effects on  $T_c$  seem to get weaker for increasing  $\mu_B$
- $\hat{\mu}_B^2 < 0$  :  $\chi_{\max}$  decreases in descending order of the volume
- Mild  $\hat{\mu}_B^2$  dependence of all observables in the continuum limit

## Simulation setup

$\mu_B = 0$

- $N_t = 10$ :  $N_x = 20, 24, 28, 32, 40, 48$
- $N_t = 12$ :  $N_x = 18, 20, 24, 28, 32, 36, 40, 48, 64$
- $N_t = 16$ :  $N_x = 32, 40, 48, 64, 80$

imag.  $\mu_B$

- $N_t = 10$ :  $N_x = 40$
- $N_t = 12$ :  $N_x = 32, 40, 48, 64$
- $N_t = 16$ :  $N_x = 64$

## Details of the transition width

$$(\Delta T)^2 = -\chi(T_c) \left[ \frac{d^2 \chi}{dT^2}(T_c) \right]^{-1}$$

$$(\Delta T)^2 = -\chi(T_c) \left[ \frac{d\chi}{d\langle\bar{\psi}\psi\rangle} \Big|_{\langle\bar{\psi}\psi\rangle_c} \frac{d^2 \langle\bar{\psi}\psi\rangle}{dT^2} \Big|_{T_c} + \frac{d^2 \chi}{d\langle\bar{\psi}\psi\rangle^2} \Big|_{\langle\bar{\psi}\psi\rangle_c} \left( \frac{d\langle\bar{\psi}\psi\rangle}{dT} \Big|_{T_c} \right)^2 \right]^{-1}$$

First term in the bracket is zero, since  $\bar{\psi}\psi$  has inflection point at  $T_c$ .

$$\begin{aligned} \Delta T &= \sqrt{-\chi(T_c) \left( \frac{d^2 \chi}{d\langle\bar{\psi}\psi\rangle^2} \Big|_{\langle\bar{\psi}\psi\rangle_c} \right)^{-1} \left( \frac{d\langle\bar{\psi}\psi\rangle}{dT} \Big|_{T_c} \right)^{-1}} \\ &:= \Delta \langle\bar{\psi}\psi\rangle \left( \frac{d\langle\bar{\psi}\psi\rangle}{dT} \Big|_{T_c} \right)^{-1}. \end{aligned}$$