Isotropization of a longitudinally expanding plasma with the 2PI effective action

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Quark Matter 2023
September 6th 2023

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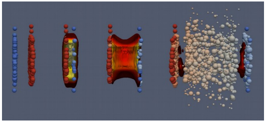
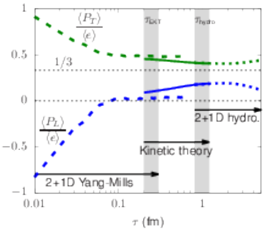
Introduction

- QGP produced in experiments is a fluid.
- How does hydrodynamization happen?
- State of the art: glasma (class. stat. field theory) + kinetic theory

  [See e.g. Berges, Heller, Mazeliauskas, Venugopalan (2020)]
- Transitioning between two models leaves questions open:
  - Vacuum fluctuations, instabilities, non-thermal fixed points...
- Both frameworks have shortcomings.
- Want a unified picture of initial stages of heavy-ion collisions.
  - Offered by two-particle irreducible (2PI) effective action.



[Bernhard et al. (2019)]



[Kurkela et al. (2018)]

Advantages of 2PI effective action $\Gamma[\phi, D]$

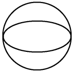
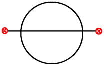
- **1. Close to underlying quantum field theory**
  - Evolves 1 and 2 pt. functions, $\phi = \langle\hat{\phi}\rangle$ and $D(x,y) = \langle\hat{\phi}(x)\hat{\phi}(y)\rangle$.

    [See e.g. Luttinger, Ward (1960); Baym, Kadanoff (1962); Berges (2004)]
  - Equations of motion come from

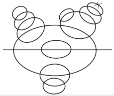
$$\frac{\delta\Gamma}{\delta\phi} = 0, \qquad \frac{\delta\Gamma}{\delta D} = 0.$$

$$\Gamma[\phi, D] = S[\phi] - \frac{i}{2}\mathrm{tr}\,\log D + \frac{i}{2}\mathrm{tr}\left(\left(D_0^{-1} - D^{-1}\right)D\right) + \Phi[\phi, D]$$

  - Have full quantum field theory if $\Phi[\phi, D]$ includes all 2PI bubble diagrams.
  - In practice do a truncation:

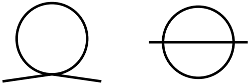
  - Still resums an infinite number of diagrams, respects e.g. energy conservation.



Advantages of 2PI effective action $\Gamma[\phi, D]$

- **2. Encompasses both kinetic th. and classical statistical th.**
  - Get equations of motion

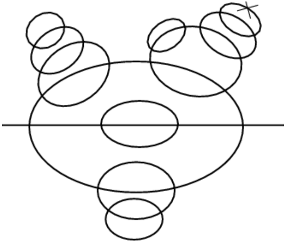
$$\left[\nabla_\mu \nabla^\mu + m^2 + \frac{\lambda}{2}\phi^2(x)\right] D(x,y) = \int d^4z\, \sqrt{-g_z}\, \Sigma(x,z) D(z,y)$$



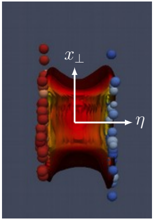
  - $\Sigma[D,\phi]$ and $\Pi[D,\phi]$.
  - D contains the spectral function $\rho(x,y) = \left\langle\, [\hat{\phi}(x), \hat{\phi}(y)]\, \right\rangle$ and the statistical function $F(x,y) = \frac{1}{2}\left\langle \{\hat{\phi}(x), \hat{\phi}(y)\}\right\rangle$.
  - For high occupancy $F \gg \rho$ get classical statistical theory. [Jeon (2004)]
    - Also includes vacuum fluctuations.
  - For slow variations and on-shell partons get kinetic theory.
    - Also includes off-shell particles and dynamical soft modes.
- **3. All scales evolve dynamically.**

Challenges in 2PI framework

- **1. Renormalization is challenging.**
  [ϕ^4: E.g. Blaizot, Iancu, Reinosa (2003)
  QED: Reinosa, Serreau (2009)]
  - Not known how to do for QCD.
    - Also right degrees of freedom not obvious, gauge invariance...
  - For now focus on ϕ^4 theory.
    - Both classical statistical and kinetic calculations exist.
    - Have isotropization, instabilities, interplay between hard and soft modes...
    - Big difference with QCD: Chemical equilibration slow.



- **2. Large memory requirements in computations.**
  - Need to include longitudinal expansion to describe HIC: 2+1 D $(\tau, \eta, x_\perp) \to (\tau, \nu, p_\perp)$. $\nabla_\mu \nabla^\mu \to \partial_\tau^2 + \frac{1}{\tau}\partial_\tau + p_\perp^2 + \frac{\nu^2}{\tau^2}$



- Goes beyond earlier calculations in 1+1 D setting.
  - Static box: [E.g. Aarts, Ahrensmeier, Baier, Berges, Serreau (2002)]
  - Expanding universe: [Tranberg (2008)]
  - Longitudinal expansion: [Hatta, Nishiyama (2011)]
- Assume axial symmetry, boost invariance etc.
- Need to store past propagator for memory integrals.

$$\partial_{\tau_x}^2 D(\tau_x, \tau_y; \nu, p_\perp) + ... = \int_{\tau_0}^{\tau_x} d\tau' \, \Sigma(\tau_x, \tau'; \nu, p_\perp) D(\tau', \tau_y; \nu, p_\perp).$$

- Requires RAM $N_{p_\perp} \times N_\nu \times N_\tau^2 \sim 256 \times 256 \times 1024^2 \to 250\,\mathrm{GB}$
- Calculate memory integrals on GPUs.

Details of implementation

$$\Big(\partial_\tau^2 + \frac{1}{\tau}\partial_\tau + p_\perp^2 + \frac{\nu^2}{\tau^2} + m^2 + \frac{1}{2}g^2\phi(\tau)^2 + \frac{1}{2}g^2\int_{p_\perp,\nu}(F - F_0)\Big)F(\tau_x,\tau_y;p_\perp,\nu) =$$
$$\int_{\tau_0}^{\tau_y} d\tau_z \tau_z\, \Sigma_F(\tau_x,\tau_z;p_\perp,\nu)\rho(\tau_z,\tau_y;p_\perp,\nu) + \int_{\tau_0}^{\tau_x} d\tau_z \tau_z\, \Sigma_\rho(\tau_x,\tau_z;p_\perp,\nu)F(\tau_z,\tau_y;p_\perp,\nu)$$

$$\Sigma_F(x,y) = g^4\left[\frac{1}{6}F(x,y)^3 - \frac{1}{8}\rho(x,y)^2F(x,y)\right] + g^4\phi(x)\phi(y)\left[\frac{1}{2}F(x,y)^2 - \frac{1}{8}\rho(x,y)^2\right]$$

$$\Sigma_\rho(x,y) = g^4\left[-\frac{1}{2}F(x,y)^2\rho(x,y) + \frac{1}{24}\rho(x,y)^3\right] - g^4\varphi(x)\varphi(y)\rho(x,y)F(x,y)$$

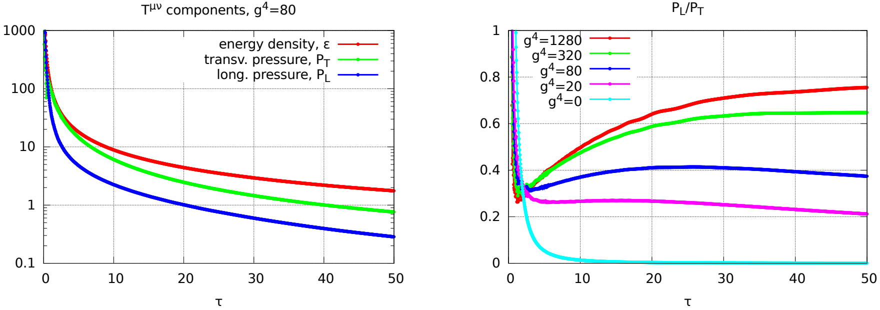
- In next few slides use $m = 2.5$, $\tau_0 = 0.004$, $\phi = 0$.
- Initialize with free ρ and with F with momentum distribution

$$F(\tau_0,\tau_0;p_\perp,\nu) = \left[\frac{1}{2} + f(p_\perp,\nu)\right]\frac{\pi e^{-\pi\nu}}{2}\left|H_{i\nu}^{(1)*}(m_\perp\tau_0)\right|^2$$

$$f(p_\perp,\nu) = 12.5\, e^{-p_\perp^2/4^2}e^{-\nu^2/4^2}$$

- For the time being focus on removing quadratic divergences.

Stress-energy tensor $T^{\mu\nu}$

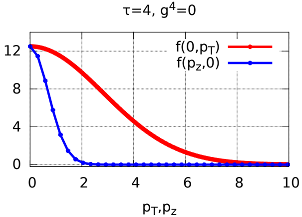


- For the first time see isotropization in full QFT!
  - $T^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta\Gamma}{\delta g_{\mu\nu}}$: includes memory integrals.
- Roughly $Q_s \sim 1$ so $\tau = 1/Q_s = 1 \to 0.1\,\mathrm{fm}$.
- Fairly low coupling, e.g. $g^4 = 1280 \to \alpha_s = 0.04$ at same η/s [Jeon (1994); Arnold, Moore, Yaffe (2000)]
- Still need further work on renormalization of $T^{\mu\nu}$: Some quadratic divergences still left.

Momentum distribution

- Extract $f(p_\perp, \nu)$ from F assuming quasiparticle picture. Get isotropization.

$$F(\tau_x, \tau_y; p_\perp, \nu) = \left[\frac{1}{2} + f(p_\perp, \nu)\right] \frac{\pi e^{-\pi\nu}}{4} \left(H^{(1)*}_{i\nu}(m_\perp \tau_x) H^{(1)}_{i\nu}(m_\perp \tau_y) + c.c.\right)$$



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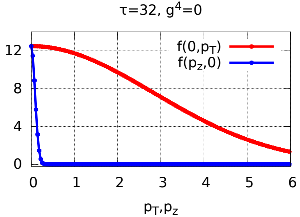

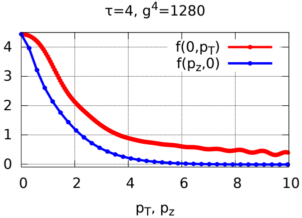

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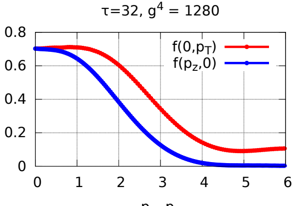
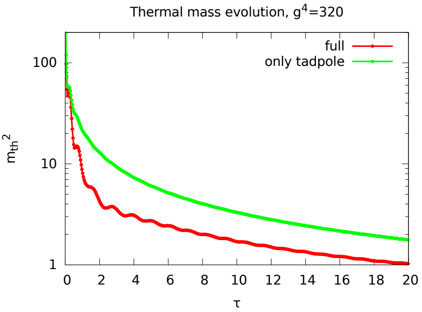
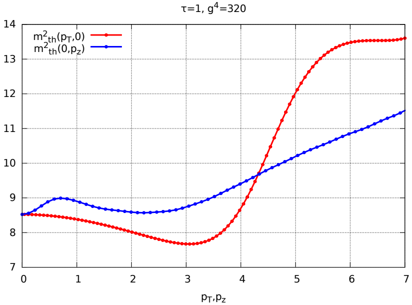


Thermal mass

- Soft scales are also dynamical: goes beyond kinetic theory.
- Can extract the thermal mass from time oscillations in F.

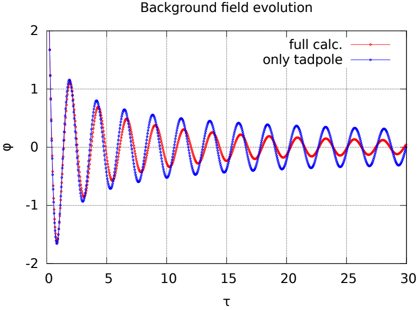


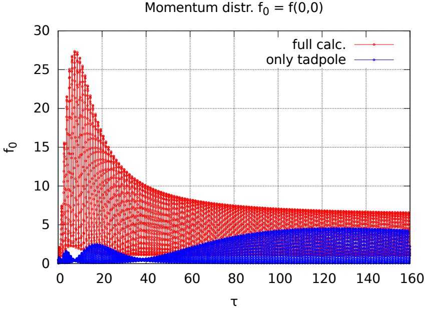


- Scattering reduces thermal mass.
- Mild dependence on $p_\perp$ and ν.

Classical fields decaying into particle

- HIC: classical fields decay into particles that form the medium.
- 2PI can do this rigorously.
  [See also e.g. Berges, Boguslavski, Schlichting (2012); Epelbaum, Gelis (2012)]
- Initialize with $\phi = 4.25$, F and ρ as in vacuum, $g^4 = 1280$.





- See rapid growth in particle content due to parametric resonance.

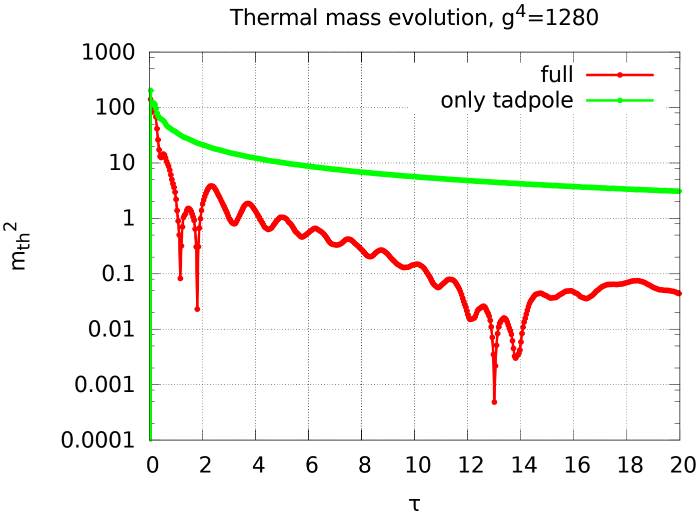
Conclusions

- 2PI effective action new framework in context of heavy-ion collisions.
  - Close to underlying QFT.
  - Encompasses all state-of-the-art descriptions.
  - All scales are dynamical, vacuum fluctuations fully included.
  - Don't know how to do for QCD: use ϕ^4.
  - Computationally challenging.
- Have seen isotropization on QFT level for first time.
  - Both in stress-energy tensor and momentum distributions.
- See how classical field decays into quasiparticles.
- Plenty of things to work on: full renormalization, large N resummation, virtuality of excitations, transport coefficients, QCD...

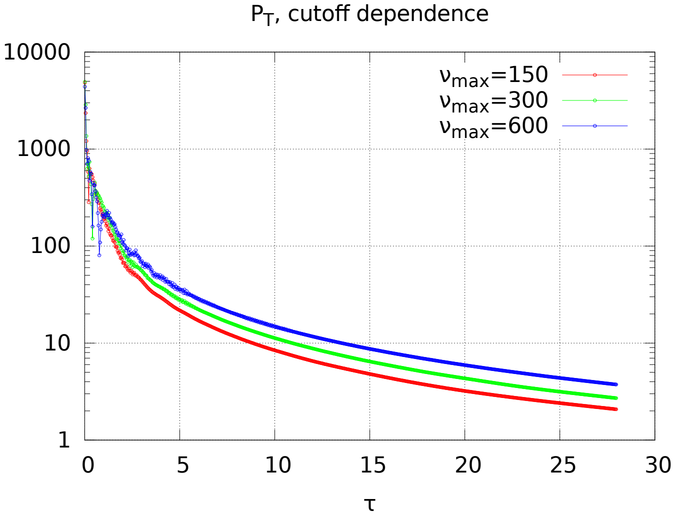
Stress energy tensor

$$\epsilon(\tau) = \frac{1}{2}\int \frac{d^2p_\perp d\nu}{(2\pi)^3}\left(m^2 + p_\perp^2 + \frac{\nu^2}{\tau^2} + \partial_\tau\partial_{\tau'}\right)F(\tau,\tau';p_\perp,\nu)\Big|_{\tau=\tau'}$$
$$+ \frac{\lambda}{8}F^2(x,x) + \frac{\lambda^2}{6}\int d\tau\tau\,[...]$$

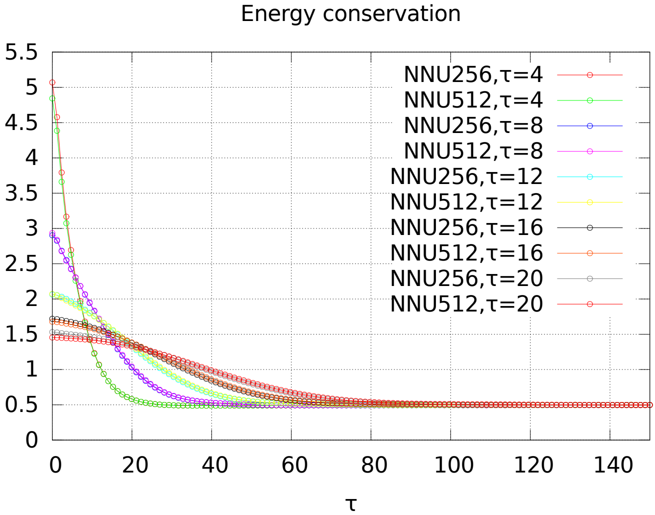
Thermal mass at higher coupling



Cutoff dependence for stress-energy tensor



Cutoff dependence for distribution



Cutoff dependence for tadpole