Event-by-event heavy-flavour dynamics: estimating the spatial diffusion coefficient D_s from charm to the infinite mass limit

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Outline

- Basic scales of Heavy Quarks

- Catania Quasi-Particle Model for charm quark dynamics:
  R_{AA} , $v_2 \rightarrow$ Spatial diffusion coefficient D_s (T) of charm.

- Initial state fluctuation $\rightarrow$ Event-Shape-Engineering
  Anisotropic flows $v_{n(=2,3,4)}$ and their correlations.

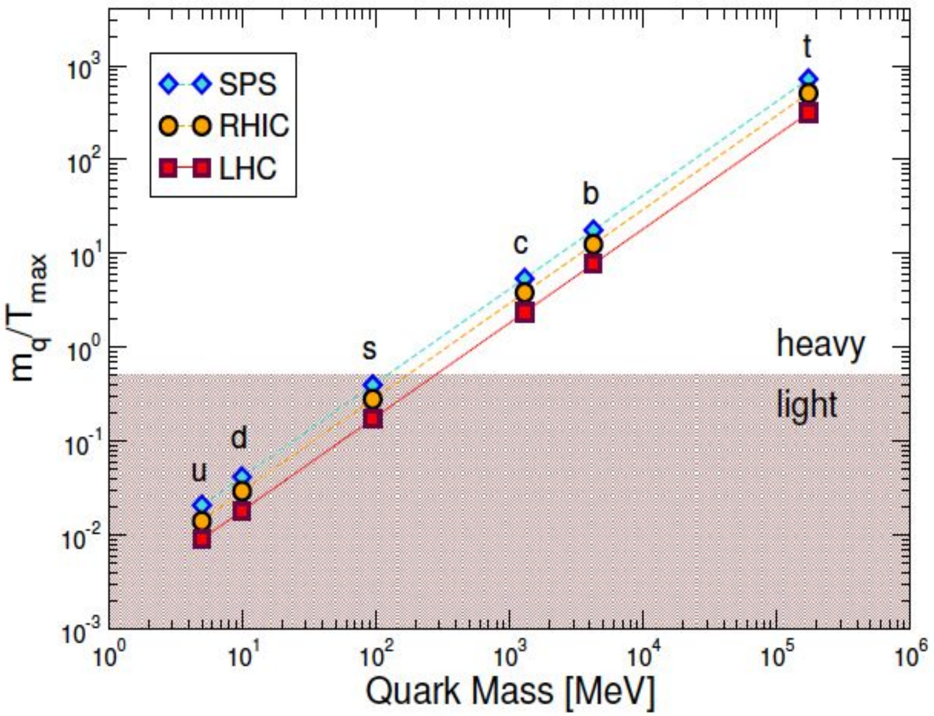
- Predictions for bottom quark
  R_{AA} , v_2 and v_3 of electrons from semileptonic B-meson decay.

- Spatial diffusion coefficient D_s (T): charm vs bottom and the infinite mass limit

- Conclusions and new perspectives

Basic scales of charm and bottom quarks

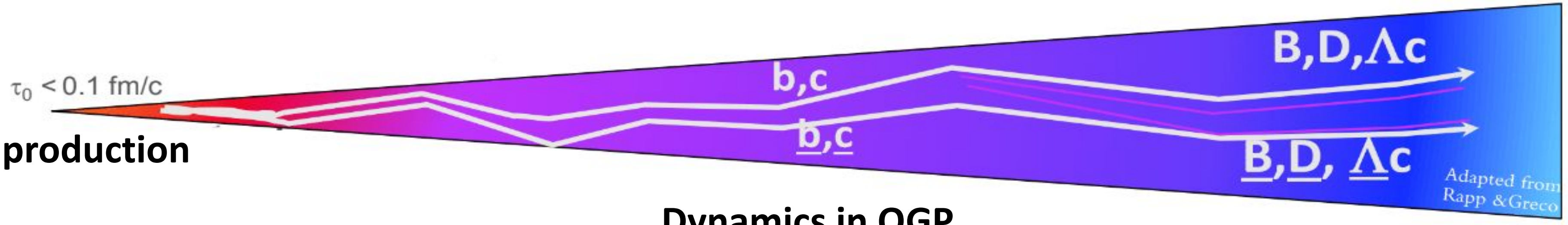
Charm $M_c \approx 1.3$ GeV and Bottom $M_b \approx 4.2$ GeV



- $m_{c,b} >> \Lambda_{QCD}$
  pQCD initial production
- $m_{c,b} >> T_{RHIC,LHC}$
  negligible thermal production
- $\tau_0 < 0{,}08$ fm/c $<< \tau_{QGP}$
- $\tau_{th} \approx \tau_{QGP} >> \tau_{g,q}$

They experience the full evolution of the QGP.

They carry more informations with respect to their light counterparts.



Initial production

Dynamics in QGP

Hadronization:
Final hadron Spectra and observables

Reviews:
1. X.Dong, V. Greco Prog. Part. Nucl. Phys. 104 (2019),
2. A.Andronic EPJ C76 (2016), 3) R.Rapp, F.Prino J.Phys. G43 (2016)

CATANIA QPM:

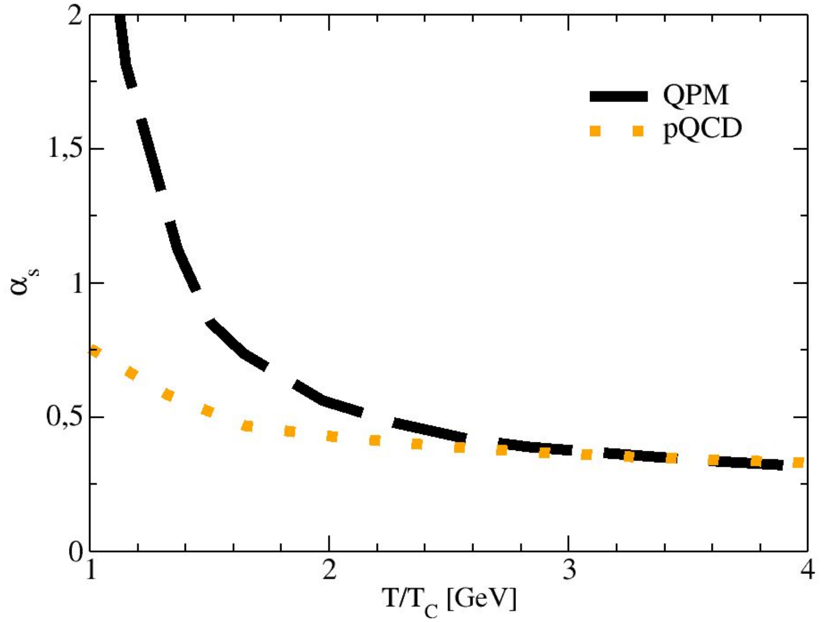
R_{AA}, v_n and v_n -v_m correlations

in charm sector

Quasi Particle Model (QPM) fitting lQCD

Non perturbative dynamics → M scattering matrices (q,g → Q) evaluated by Quasi-Particle Model fit to **lQCD thermodynamics**

N_f=2+1
Bulk:
u,d,s



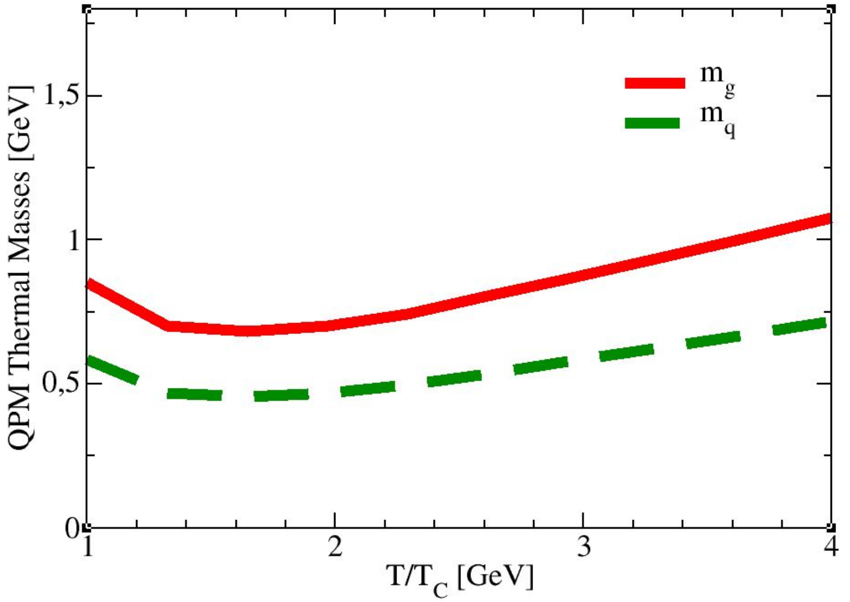
$$m_g^2(T) = \frac{2N_c}{N_c^2 - 1} g^2(T) T^2$$
$$m_q^2(T) = \frac{1}{N_c} g^2(T) T^2$$

→ **Thermal masses of gluons and light quarks**

g(T) from a fit to ε from lQCD data → good reproduction of P, ε-3P

$$g^2(T) = \frac{48\pi^2}{(11N_c - 2N_f)\ln\left[\lambda\left(\frac{T}{T_c} - \frac{T_s}{T_c}\right)\right]^2}$$

λ=2.6
T_s=0.57 T_c



Larger than pQCD especially as T → T_c

S. Plumari et al, *Phys.Rev.D* 84 (2011) 094004
H. Berrehrah,, PHYSICAL REVIEW C **93**, 044914 (2016)

Relativistic Boltzmann equation at finite η/s

Bulk evolution

$$p^{\mu}\partial_{\mu}f_{q}(x,p)+m(x)\partial^{x}_{\mu}m(x)\partial^{\mu}_{p}f_{q}(x,p)=C[f_{q},f_{g}]$$

$$p^{\mu}\partial_{\mu}f_{g}(x,p)+m(x)\partial^{x}_{\mu}m(x)\partial^{\mu}_{p}f_{g}(x,p)=C[f_{q},f_{g}]$$

Free-streaming

field interaction
$\varepsilon - 3p \neq 0$

Collision term
gauged to some η/s≠ 0

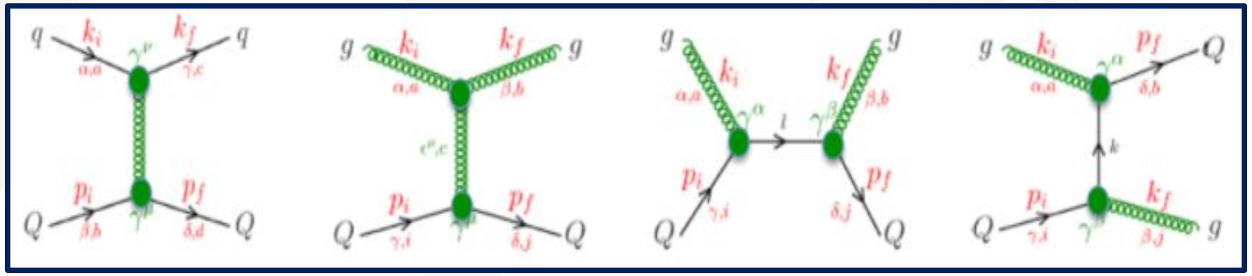
Equivalent to viscous hydro at η/s ≈ 0.1

HQ evolution

$$p^{\mu}\partial_{\mu}f_{Q}(x,p)=C[f_{q},f_{g},f_{Q}]$$

$$C[f_{q},f_{g},f_{Q}]=\frac{1}{2E_{1}}\int\frac{d^{3}p_{2}}{2E_{2}(2\pi)^{3}}\int\frac{d^{3}p_{1}'}{2E_{1}'(2\pi)^{3}}$$
$$\times[f_{Q}(p_{1}')f_{q,g}(p_{2}')-f_{Q}(p_{1})f_{q,g}(p_{2})]$$
$$\times|M_{(q,g)\to Q}(p_{1}p_{2}\to p_{1}'p_{2}')|$$
$$\times(2\pi)^{4}\delta^{4}(p_{1}+p_{2}-p_{1}'-p_{2}')$$

Feynman diagrams at first order pQCD for HQs-bulk interaction:



Scattering matrices $M_{g,q}$ by QPM fit to lQCD thermodynamics

Hybrid Hadronization Model for HQs

✔ **COALESCENCE**: Formula developed for the light sector **[Greco, Ko, Levai PRL 90 (2003)]**

$$\frac{dN_H}{d^2\boldsymbol{P}_T} = g_H \int \prod_{i=1}^{n} \frac{d^3p_i}{(2\pi)^3 E_i} p_i \cdot d\sigma_i \, f_{q_i}(x_i, p_i) \, f_W(x_1 \dots x_n; p_1 \dots p_n) \, \delta\left(\boldsymbol{P_T} - \sum_i^n p_{T,i}\right)$$

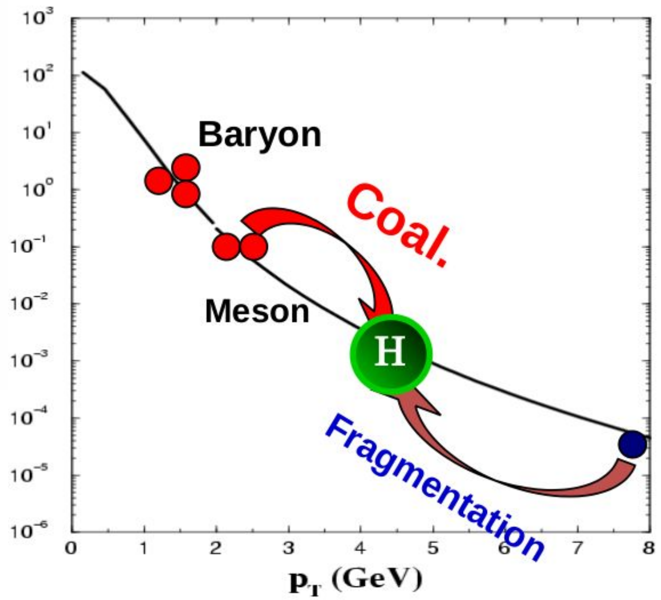
Statistical Factor
Color-spin-isospin

Parton Distribution Functions
(after Boltzmann evolution)

Hadron Wigner Function
(parameters fix according to quark model)
C.-W. Hwang, EPJ C23, 585 (2002)
C. Albertus et al., NPA 740, 333 (2004)

✔ **FRAGMENTATION**: HQs that do not undergo to Coalescence

$$\frac{dN_H}{d^2\boldsymbol{P}_T} = \sum_f \int dz \frac{dN_f}{d^2p_T} \frac{D_{f\to H}(z)}{z^2}$$



We use Peterson parametrization: $D_H(z) \propto \left[z\left(1 - \frac{1}{z} - \frac{\epsilon_c}{1-z}\right)^2\right]^{-1}$

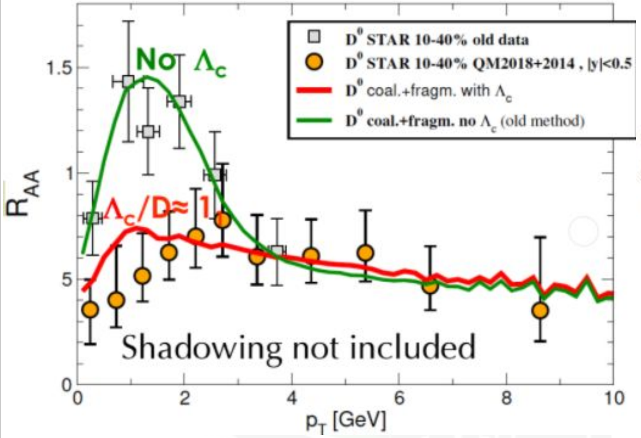
Peterson et al. PRD 27 (1983) 105

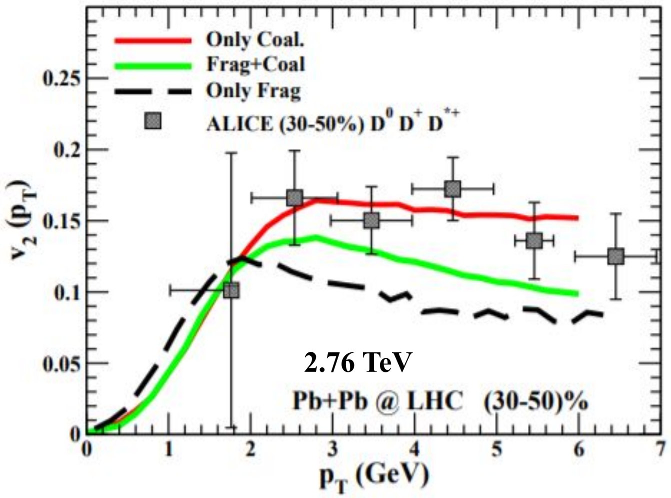
Parameter ϵ_c tuned to reproduce *D* and *B* meson spectra in pp collisions at high p_T.

Plumari, Minissale, Das , Coci , Greco, EPJ C 78 (2018) no.4

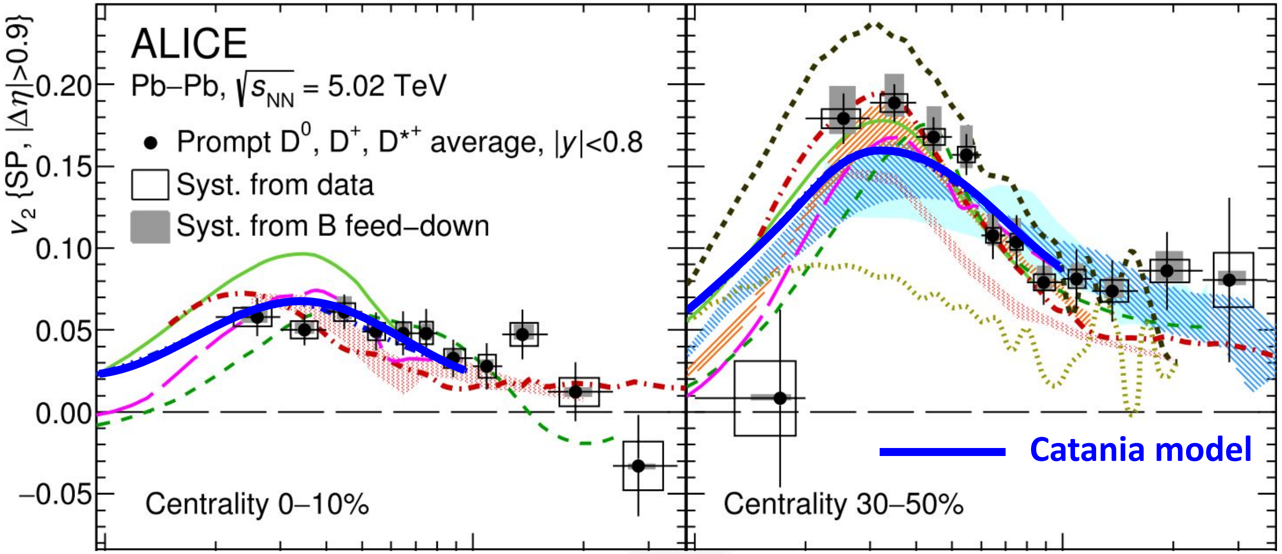
Catania Model (QPM+hadronization): some results for charm…

Scardina et al., PRC 97(2017)





Good description of R_{AA}, v_2 at RHIC & LHC energies

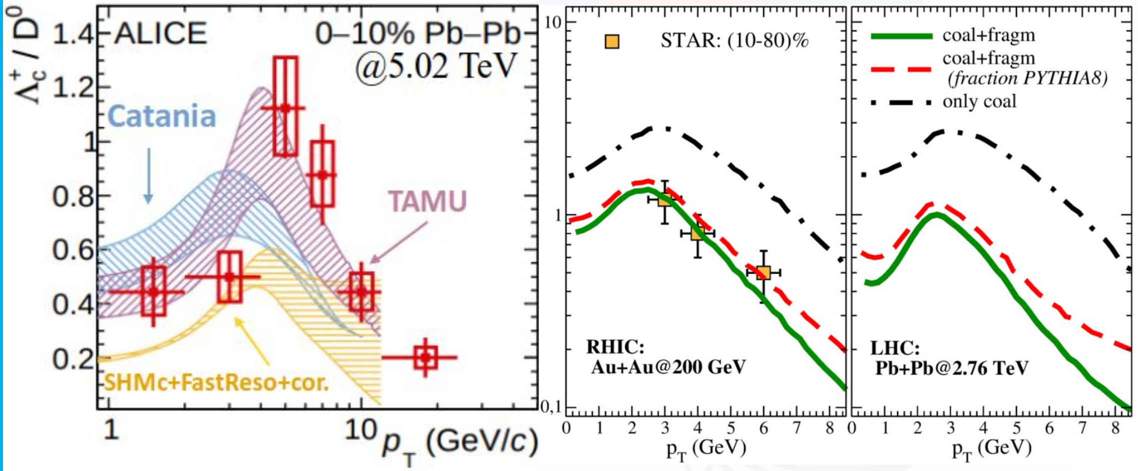


ALICE collaboration, *Phys.Lett.B* 813 (2021) 136054

Effect of the hadronization via coalescence+fragmentation

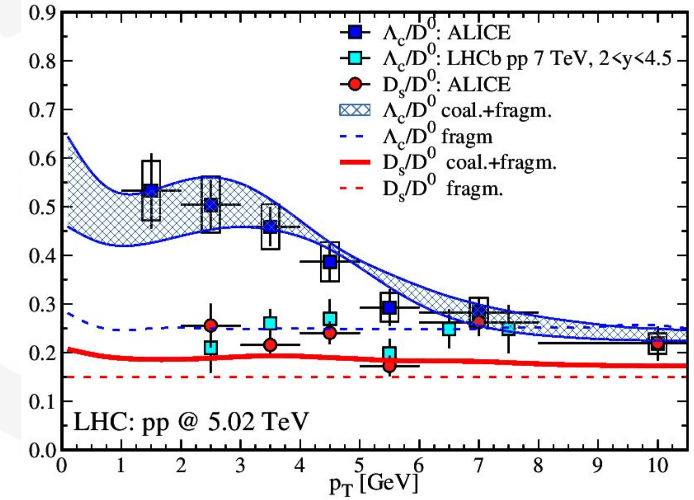
<u>in AA:</u>

S. Plumari, V. M. et al., Eur. Phys. J. C78 no. 4, (2018) 348

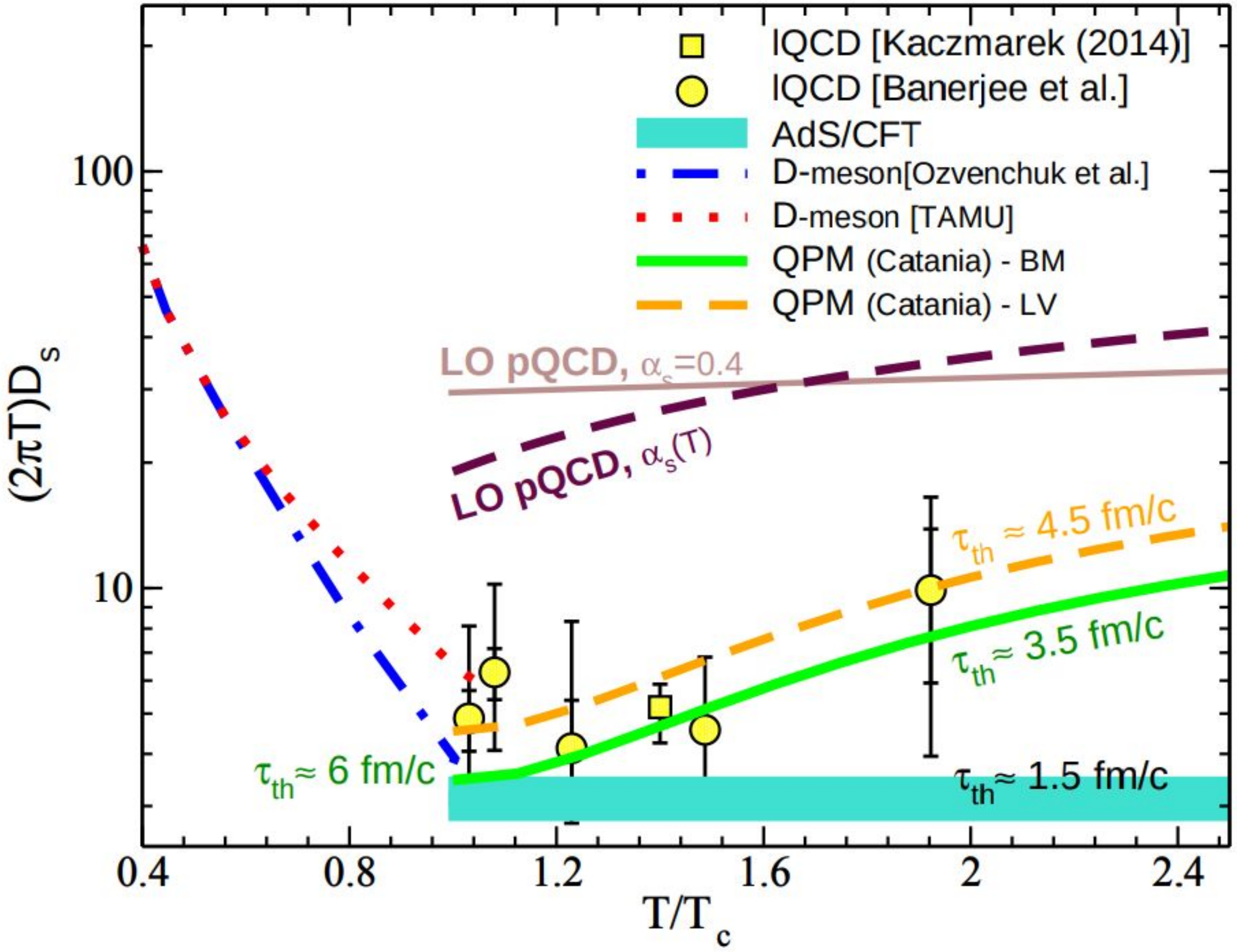


<u>in pp</u> (Charm parton distr. from FONLL w/o evolution):

V. M. et al. Physics Letters B 821 (2021) 136622



Spatial diffusion coefficient of charm quark



Not a model fit to lQCD data, but D_s estimate that comes from results of R_{AA} (p_T) and v_2 (p_T)

We have a probe with $\tau_{therm} \approx \tau_{QGP}$

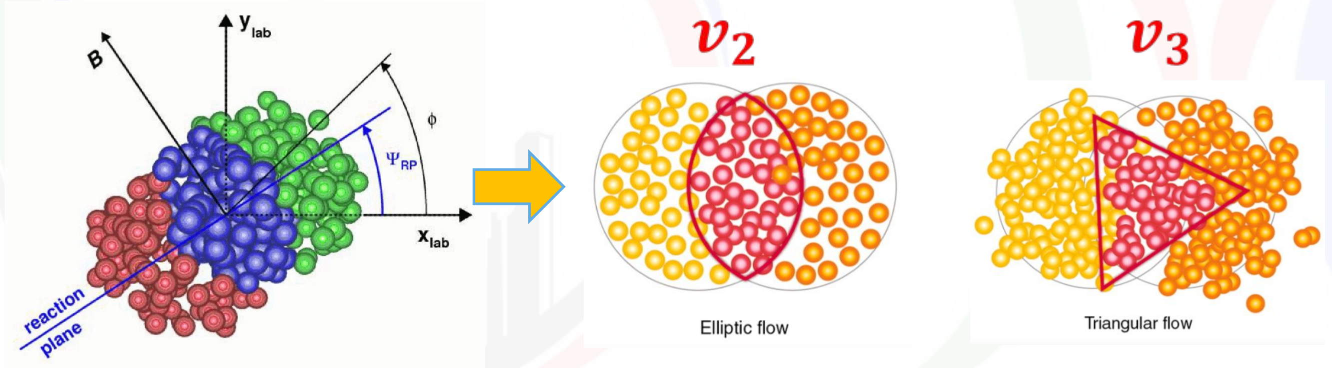
$$\tau_{th} = \frac{M}{2\pi T^2}(2\pi T D_s) \cong 1.8 \frac{2\pi T D_s}{(T/T_c)^2} \text{ fm/c}$$

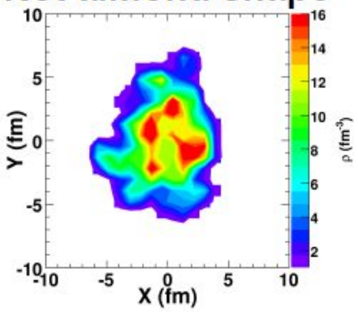
FUTURE:
-Access low p and precision data (detector upgrade)
-Better insight into hadronization
-***New observables***
-Bottom
Main focus of this talk

Reviews:
- F. Prino and R. Rapp, JPG(2019)
- X. Dong and V. Greco, Prog.Part.Nucl.Phys. (2019)
- Jiaxing Zhao et al., arXiv:2005.08277

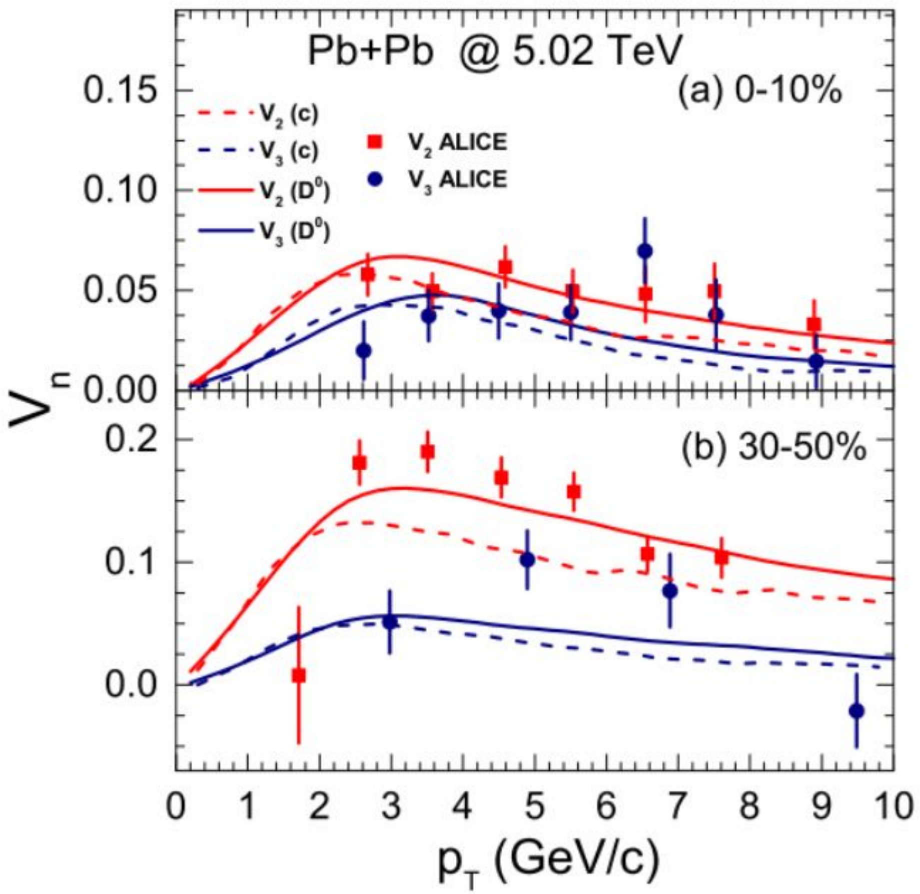
Extension to higher order anisotropic flows $v_n(p_T)$

$$E\frac{d^3N}{dp_T} = \frac{1}{2\pi}\frac{d^2N}{p_T dp_T dy}\left\{1 + \sum_{i=1}^{\infty} v_n \cos[n(\varphi - \Psi_n)]\right\}$$





Sambataro et al., Eur.Phys.J.C 82 (2022) 9, 833



Monte Carlo Glauber for initial condition of partons
S.Plumari et al, *Phys.Rev.C* 92 (2015) 5

Data taken from ALICE collaboration: *Phys.Lett.B* 813 (2021) 136054

In 30-50% centrality class → larger v_2 and comparable v_3

- ➢ v_2 mainly generated by the geometry of overlapping region. More elliptical shape in peripheral collision.
- ➢ v_3 mainly driven by the fluctuations that are independent of the centrality

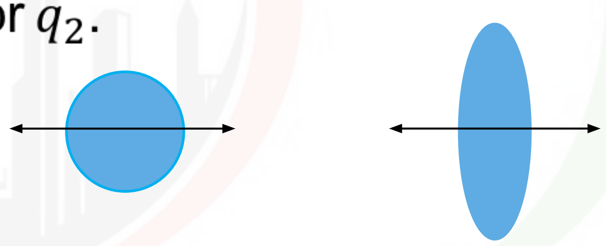
Extension to higher order anisotropic flows $v_n(p_T)$

ESE technique and v_n correlations

Selection of events with the same centrality but different initial geometry on the basis of the magnitude of the second-order harmonic reduced flow vector q_2.

$$q_2 = |\vec{Q}_2|/\sqrt{M}$$

$$\vec{Q}_2 = \sum_{j=1}^{M} e^{i2\varphi_j}$$



20 % small q_2 20 % large q_2

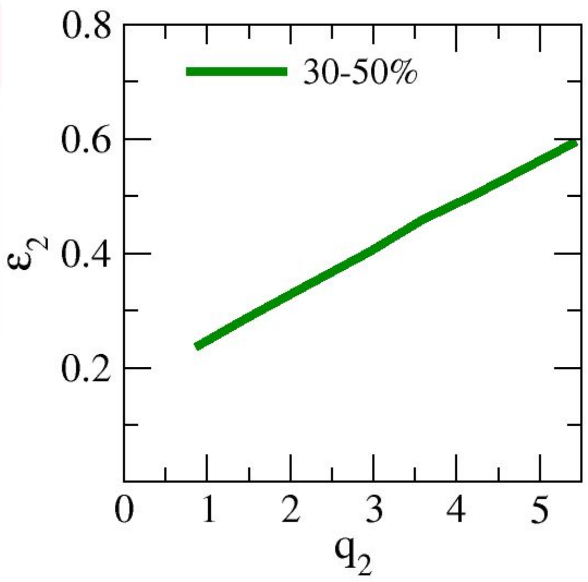
Large q_2 → large ε_2

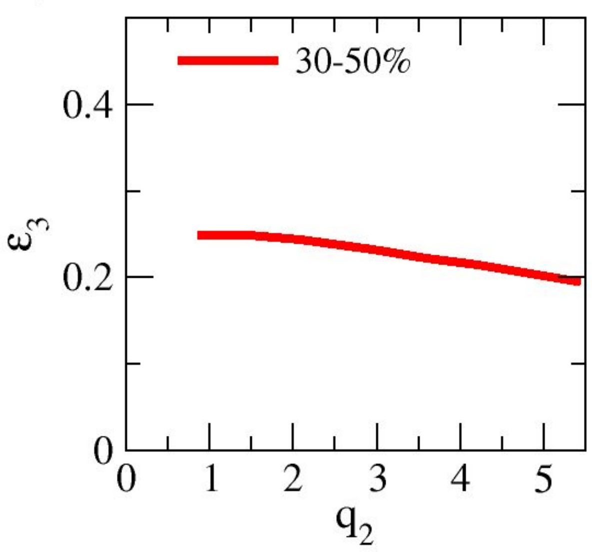
Study of events as a function of changing geometry at fixed centrality

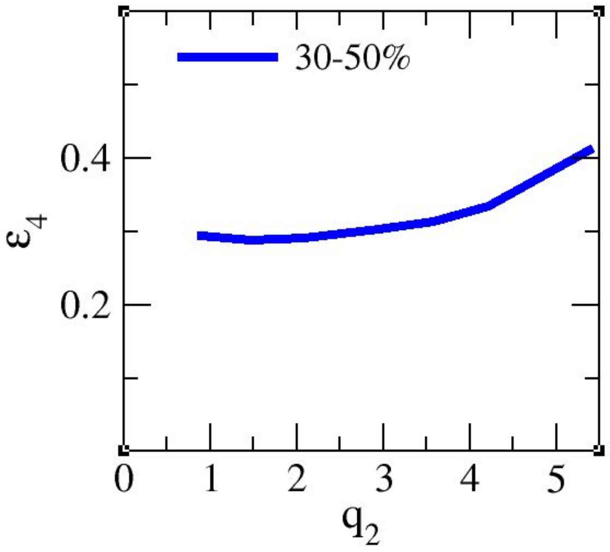
$$\epsilon_n = \frac{\langle r_\perp^n \cos[n(\varphi - \Phi_n)]\rangle}{\langle r_\perp^n \rangle} \qquad \Phi_n = \frac{1}{n}\arctan\frac{\langle r_\perp^n \sin(n\varphi)\rangle}{\langle r_\perp^n \cos(n\varphi)\rangle}$$

$$r_\perp = \sqrt{x^2 + y^2}, \qquad \varphi = \arctan(y/x)$$

Pb-Pb 5,02 TeV







Anti-correlation between ε_2 and ε_3

Non-linear correlation between ε_2 and ε_4

M.L. Sambataro, et al., *Eur.Phys.J.C* 82 (2022)

Extension to higher order anisotropic flows $v_n(p_T)$

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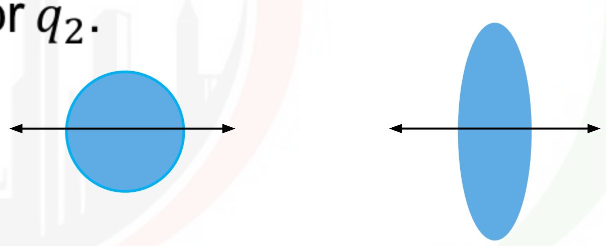


20 % small q_2 20 % large q_2

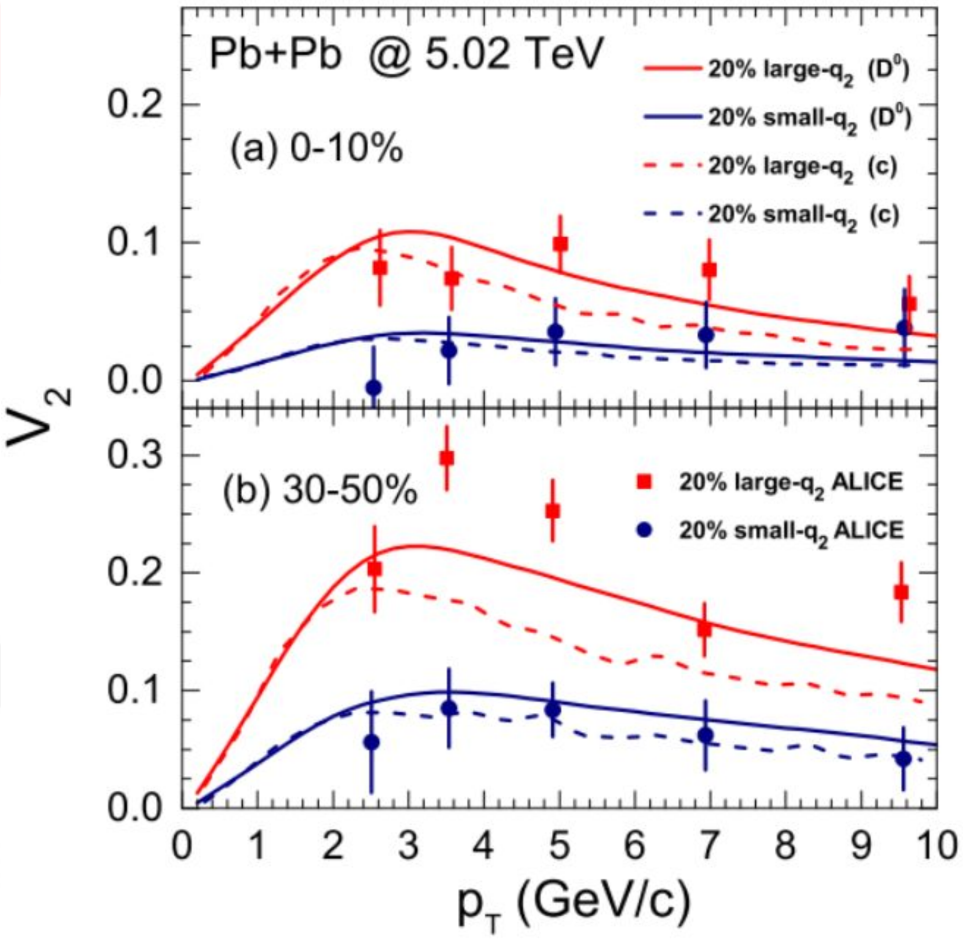
Large $q_2 \rightarrow$ large ε_2

Study of events as a function of changing geometry at fixed centrality

$$\epsilon_n = \frac{\langle r_\perp^n \cos[n(\varphi - \Phi_n)]\rangle}{\langle r_\perp^n \rangle} \qquad \Phi_n = \frac{1}{n}\arctan\frac{\langle r_\perp^n \sin(n\varphi)\rangle}{\langle r_\perp^n \cos(n\varphi)\rangle}$$

$$r_\perp = \sqrt{x^2 + y^2}, \qquad \varphi = \arctan(y/x)$$

$q_2$selected $v_2(p_T)$



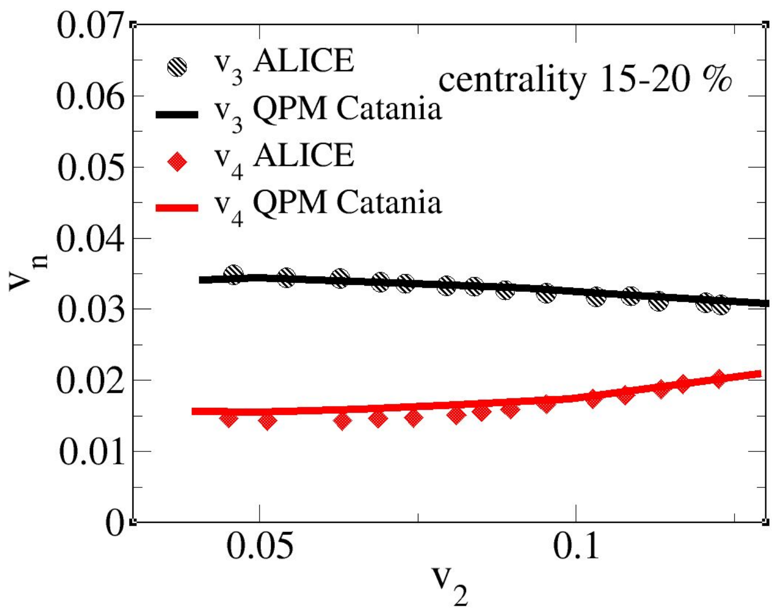
Data from ALICE collaboration: *Phys.Lett.B* 813 (2021) 136054

M.L. Sambataro, et al., *Eur.Phys.J.C* 82 (2022)

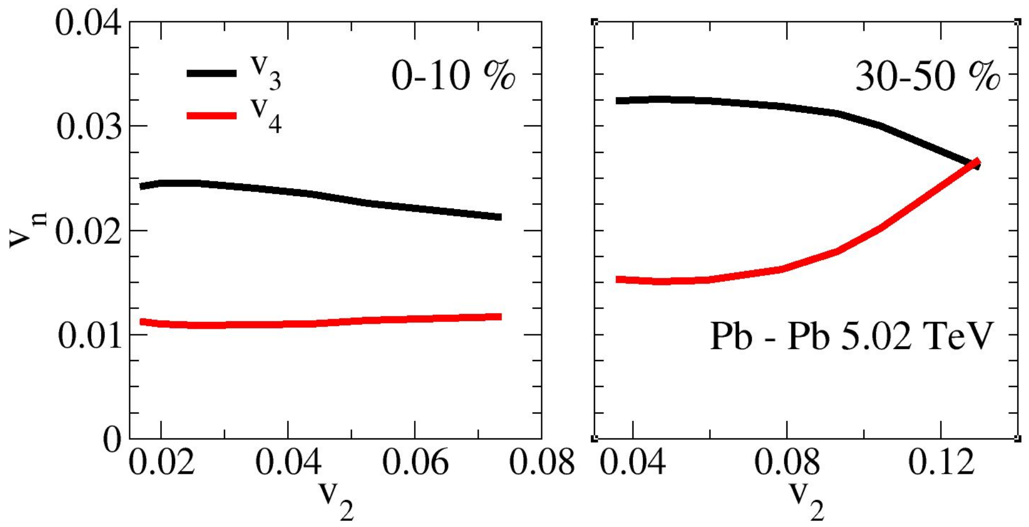
ESE: $v_n - v_m$ correlations

M.L. Sambataro, et al., *Eur.Phys.J.C* 82 (2022)

Charged particles



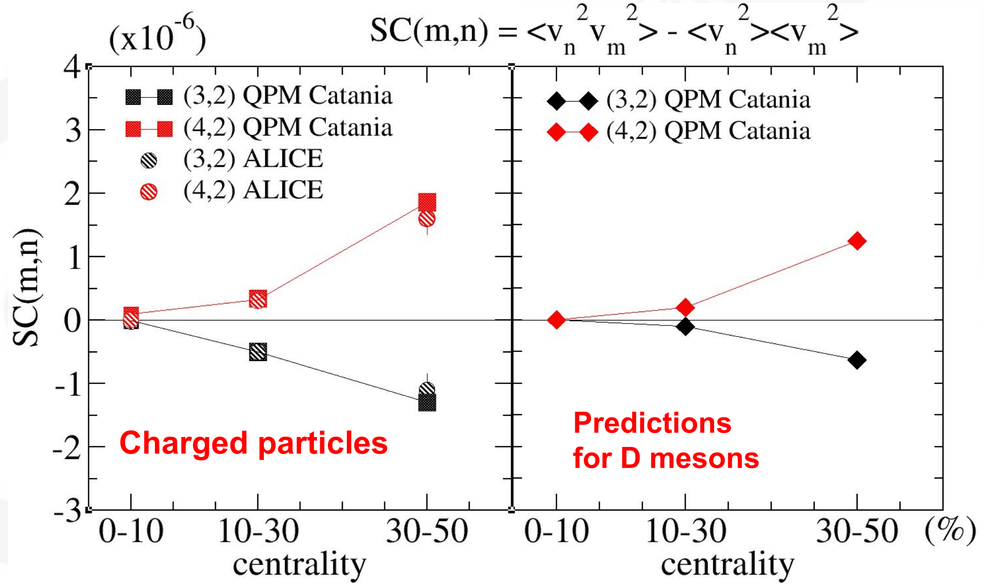
Predictions for D mesons



Correlations between the ε_n and ε_m present in the initial geometry → correlations between flow harmonics different orders, i.e. correlations v_n and v_m

- **Good description of v_{n-m} correlation for bulk**
- **Prediction for similar but weaker correlation for hard particles**
- **Correlation for D mesons provide insights on the interaction and its temperature dependence**

Plumari et al, Phys.Lett.B 805 (2020) 135460



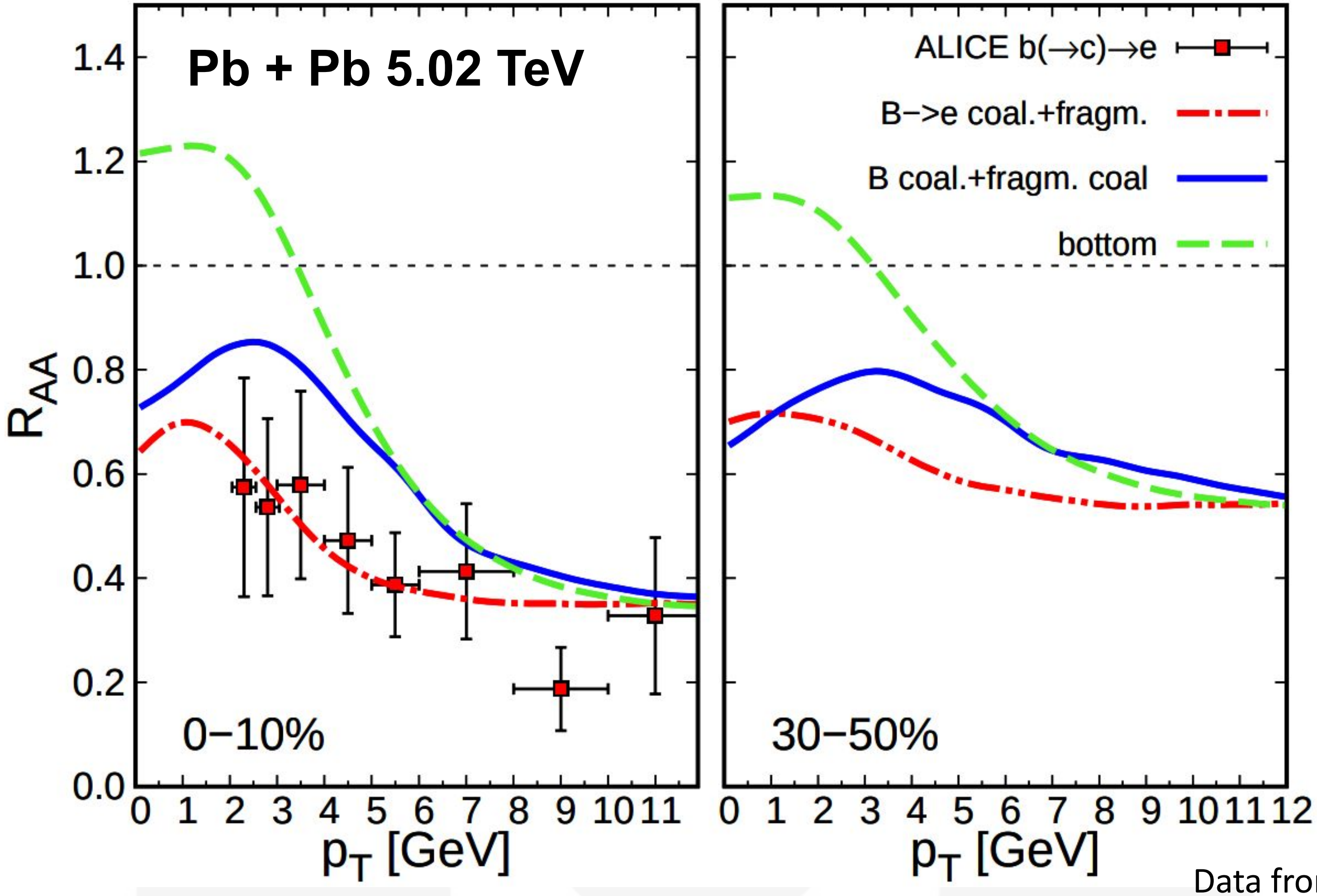
Data taken from: S. Mohapatra *Nucl.Phys.A* 956 (2016) 59-66

CHARM VS BOTTOM:
D_s IN THE INFINITE MASS LIMIT

Extension to bottom dynamics: R_{AA}

Hadronization with coalescence + fragmentation (Kartvelishvili FF) model

- ➢ Prediction for B meson R_{AA}
- ➢ R_{AA} of electrons from semileptonic B meson decay



No parameters changed with respect to charm dynamics → same interaction

- Shift of the peak to higher momenta → smaller with respect to the one for D mesons in the same model.

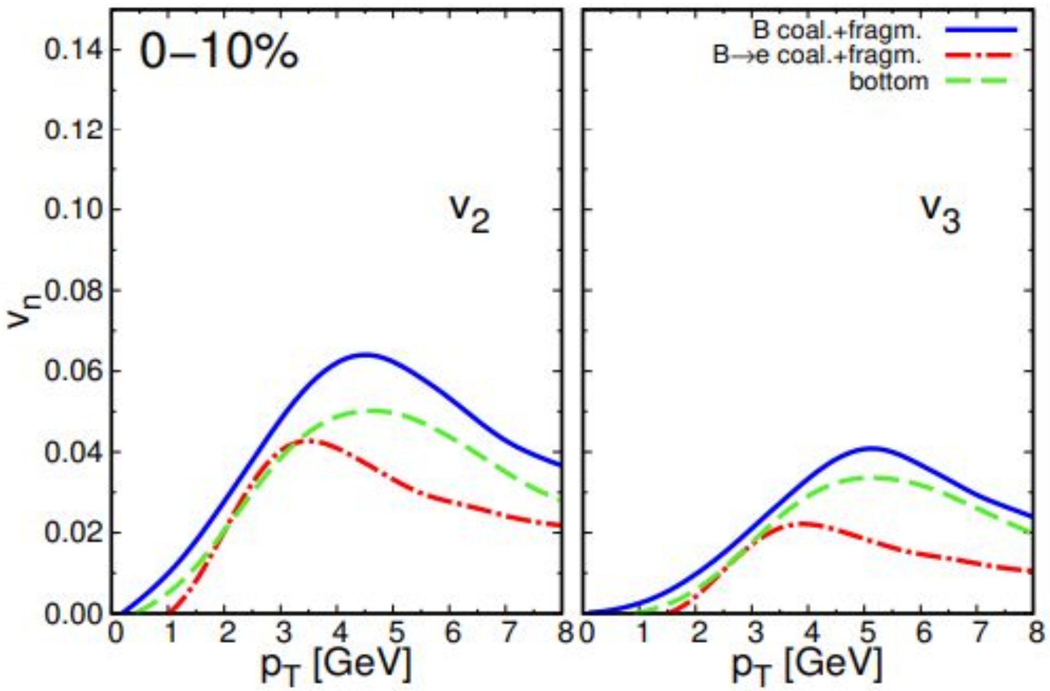
Data from: ALICE coll., arxiv:2211.13985

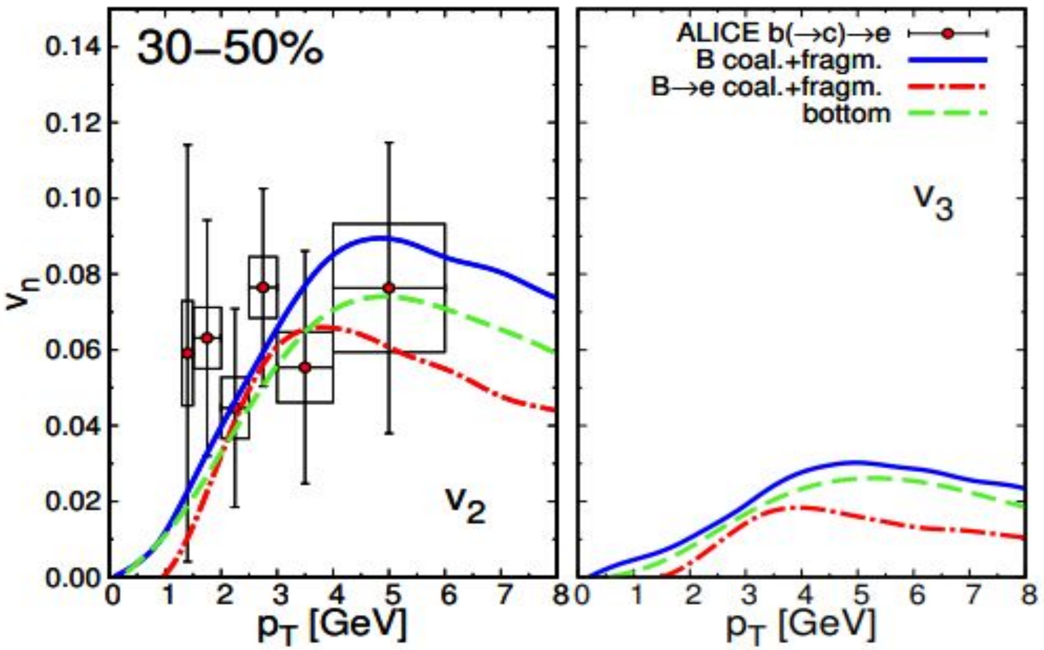
M.L. Sambataro, V. Minissale et al., e-Print: 2304.02953

Extension to bottom dynamics: $v_{(n=2,3)}$

Data from ALICE, PRL 126, 162001 (2021)

- ➢ Prediction for B meson
- ➢ electrons from semileptonic B meson decay within a coal + fragm model





The coupling of bottom quarks to the bulk medium is strong enough to collectively drag them in the expanding fireball.

Compared to charm quark:

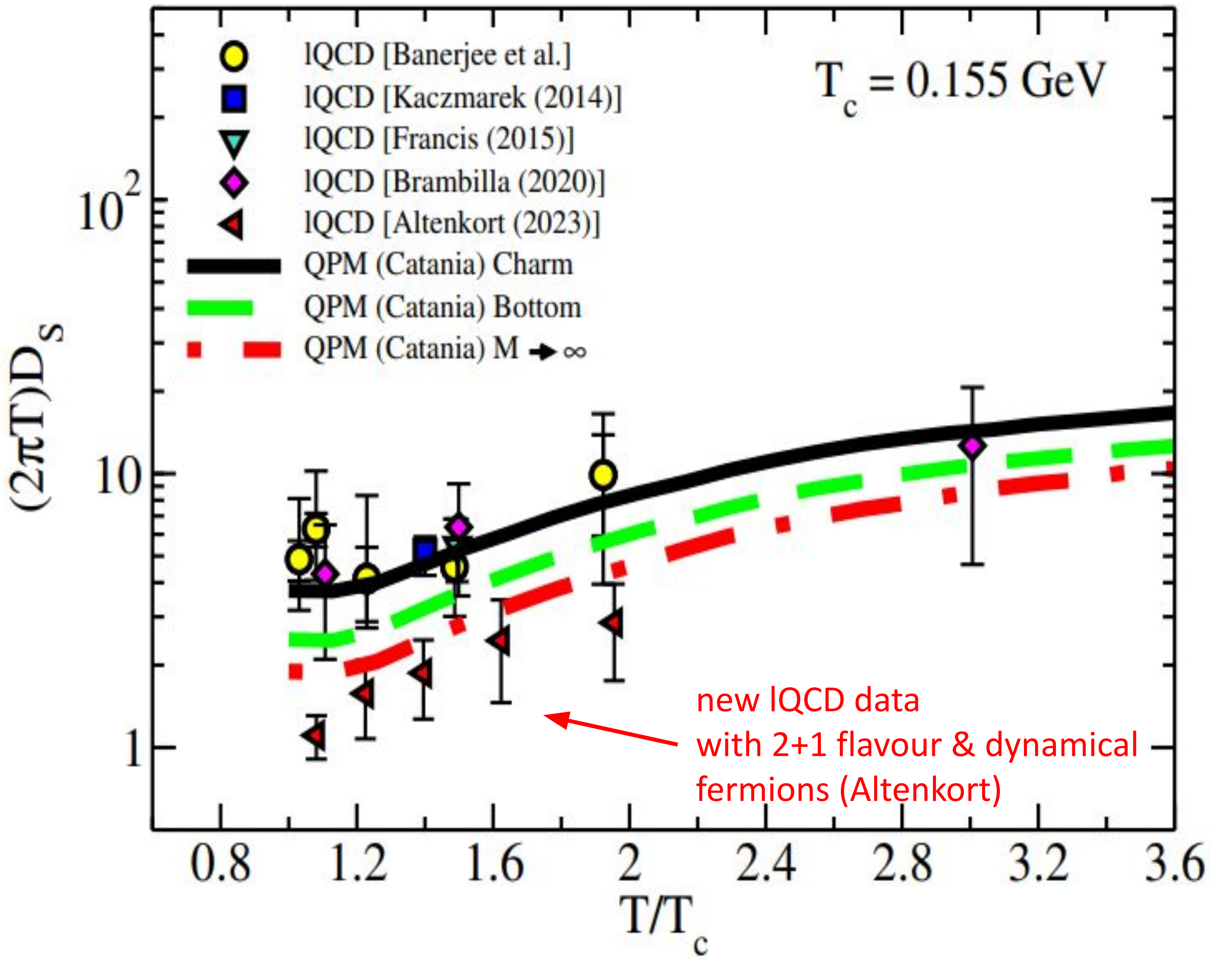
- Efficiency of conversion of ε_2 :
  - ➔ 15% smaller for v_2 in most central collisions.
  - ➔ 40% smaller for v_2 at 30−50% centrality.
- Efficiency of conversion of ε_3 :
  - ➔ 30% smaller for v_3 at both 0-10% and 30-50% centralities.

From central to peripheral:

- enhancement of v_2 (ε_2(0-10%)≃0.13 and ε_2(30-50%)≃0.42)
- similar v_3 (ε_3(0-10%)≃0.11 and ε_3(30-50%))≃0.21)

M.L. Sambataro, V. Minissale et al., e-Print: 2304.02953

$(2\pi T)D_s$: Charm quark vs Bottom quark



From D_s we obtain (in the 1-2T_c range):

- $\tau_{th}(c)$ ~ 5 fm/c
- $\tau_{th}(b)$ ~ 11 fm/c   breaking w.r.t. the relation:

  $\tau_{th}(b) = (M_b/M_c)\tau_{th}(c)$ ~ 3.3 $\tau_{th}(c)$ ~ 16.5 fm/c

- lQCD data are in $M_Q \to \infty$, so the D_s evaluated is mass independent + quenched medium
- QPM use finite mass and includes dynamical fermions

$$D_s = \frac{T}{M\,\gamma} = \frac{T}{M}\,\tau_{th}$$

From kinetic theory is expected that:

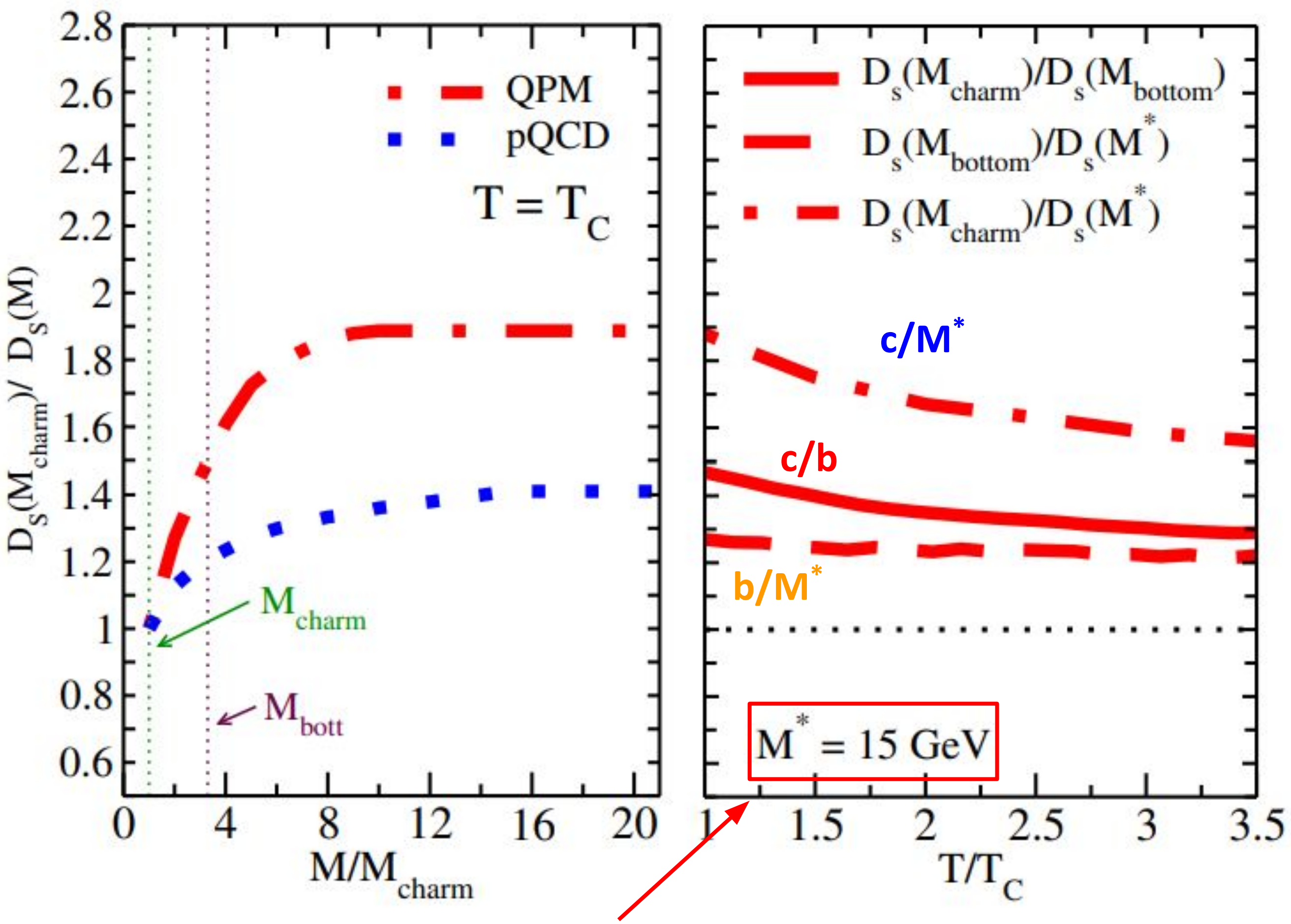
$$\tau_{th}(b)/\,\tau_{th}(c) \approx \gamma_c/\,\gamma_b \approx M_b/M_c$$

In QPM approach → $D_s(c)$ is 30-40% larger than $D_s(b)$ (no mass independence)

M→ ∞ limit is not reached for charm

M.L. Sambataro, V. Minissale et al., e-Print: 2304.02953

$(2\pi T)D_s$ ratios: Charm quark vs Bottom quark



fictitious super-heavy quark staying in the $M_Q \to \infty$ limit

- $D_s(M_{charm})/D_s(M)$ as a function of M/M_{charm} at T_C:

  Saturation scale of Ds for $M_Q \sim 8\, M_{charm} \gtrsim 10$ GeV

  $Ds(M_{charm})/Ds(M \to \infty) \simeq 1.9$ for QPM.

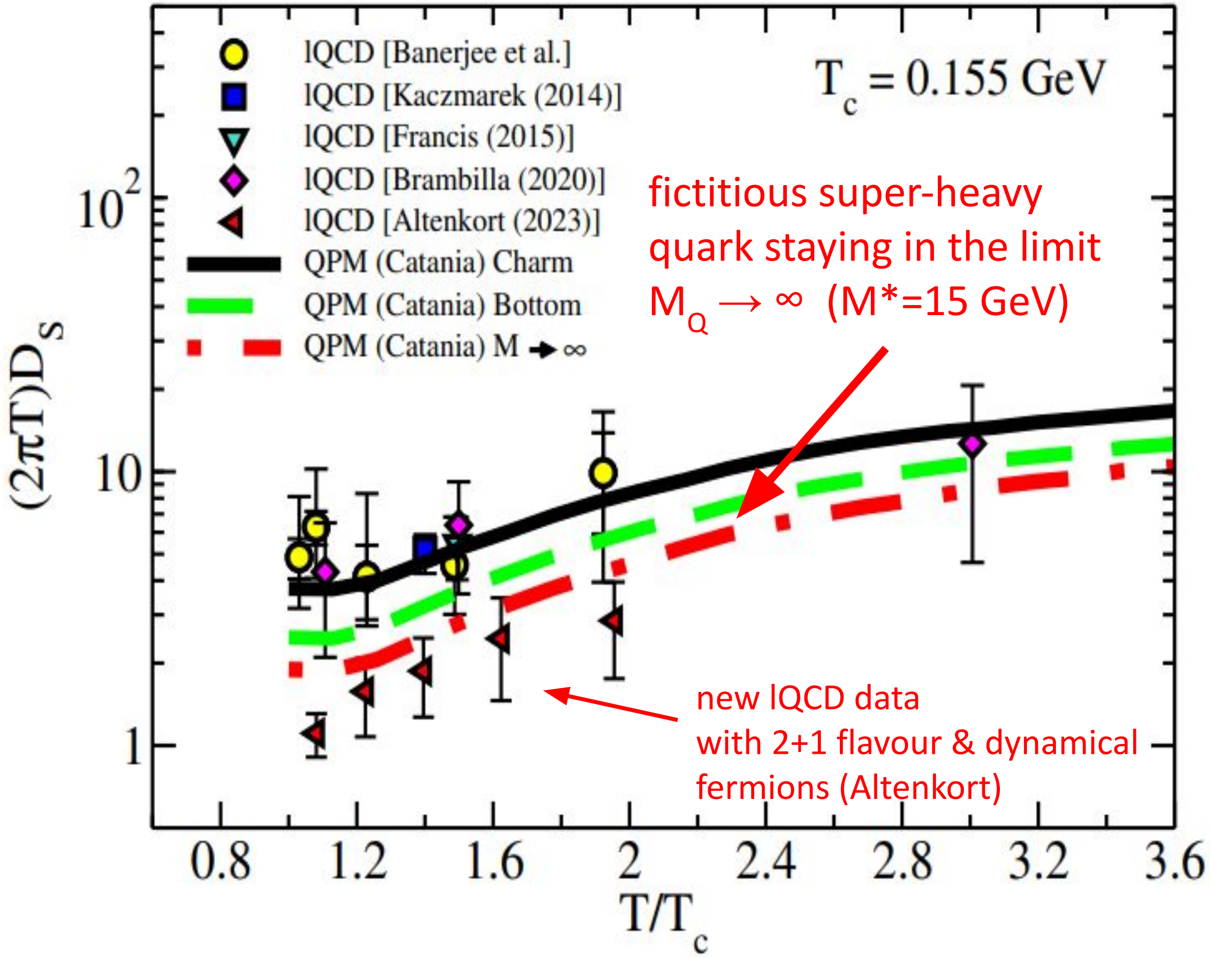
  In pQCD case $Ds(M_{charm})/Ds(M \to \infty) \simeq 1.4$

- Ratios at fixed mass as a function of T:

  - **b/M^*: about 25% in all T range**
  - **c/b: about 50% at T_C and not smaller than 30%**
  - **c/M^*: factor 1.5-2**

M.L. Sambataro, V. Minissale et al., e-Print: 2304.02953

$(2\pi T)D_s$: Charm quark vs Bottom quark



- lQCD data are in $M_Q \to \infty$ so D_s is mass independent

$$D_s = \frac{T}{M\,\gamma} = \frac{T}{M}\,\tau_{th}$$

- QPM use finite mass and includes dynamical fermions

From kinetic theory is expected that:

$$\tau_{th}(b)/\,\tau_{th}(c) \approx \gamma_c/\,\gamma_b \approx M_b/\,M_c$$

$D_s(T)$ from QPM in the infinite mass limit is the more pertinent to compare to lQCD simulations evaluated taking into account dynamical fermions

From D_s we obtain (in the 1-2T_c range):

- $\tau_{th}(c) \sim 5$ fm/c
- $\tau_{th}(b) \sim 11$ fm/c   breaking w.r.t. the relation:

$\tau_{th}(b) = (M_b/M_c)\tau_{th}(c) \sim 3.3\ \tau_{th}(c) \sim 16.5$ fm/c

M.L. Sambataro, V. Minissale et al., e-Print: 2304.02953

Conclusions

- $D_s(T)$ in QPM which reproduces D meson R_{AA} and v_2:
  - ➔ correct predictions for v_3, q_2 selected anisotropic flow and prediction for significant v_n - v_m correlation of D meson, similar correlation between soft and hard particles.

- Extension to bottom quark dynamics: good description of R_{AA} and v_2 of electrons from semileptonic B meson decay and prediction for v_3

- **Spatial diffusion coefficient $D_s(T)$: charm vs bottom and the infinite mass limit**
  - ➔ $D_s(c)$/ $D_s(b)$ ratio of about a factor of 1.5 at T ~ Tc and 1.3 at higher temperatures (T ~ 3 − 4 Tc)
  - ➔ For the **charm mass scale:** the infinite mass limit used in lQCD is not yet reached;
    For the **bottom mass scale**: discrepancy of only about a 20% w.r.t. the infinite mass limit

  - ➔ Taking into account mass scale dependence in QPM, we have satisfactory agreement with the most recent lQCD calculations that include dynamical fermions, differently from previous lQCD data in quenched approximation.

  - ➔ Thermalization time for bottom quark: τ_{th} ~ 10 − 12 fm/c which is about a factor of 2 larger than charm and so quite smaller than 3.3 as suggest by a simple M_Q/T scaling.

Thanks for the attention!

September 3 – 9, 2023
Houston, Texas