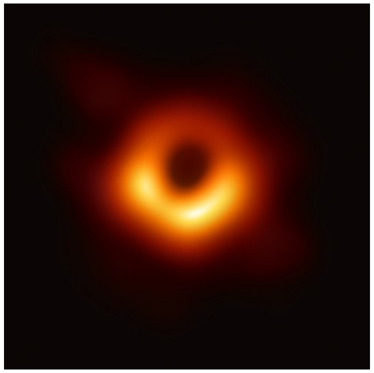
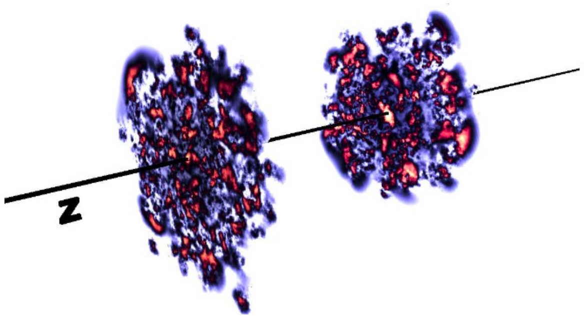
The Black Hole-CGC Double Copy:
Gravitational radiation from primordial Black Hole scattering



M_{BH}=(6.5 ± 0.2$_{stat}$ ± 0.7$_{sys}$) × 10^9 $M_\odot$ at center of Messier 87

Event Horizon Telescope image of photon ring

10^9 km ⟷ 10^{-19} km



Collisions of Color Glass Condensate gluon states in nuclei, arXiv:1206.6805

Raju Venugopalan

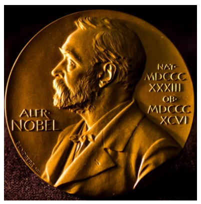
Brookhaven National Laboratory

Quark Matter 2023, Sept.4-9, 2023

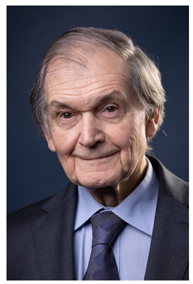
Work in preparation with Himanshu Raj: Saclay → Stony Brook

Black Holes "demystified"

A great achievement of physics in our lifespan: Black Holes from "exotic" solutions of General Relativity to observable phenomena



2020



Roger Penrose

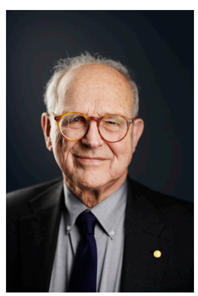
Reinhard Genzel

Andrea Ghez

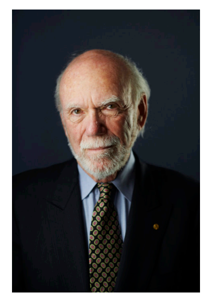
For the discovery that black hole formation is a robust prediction of the general theory of relativity

For the discovery of a supermassive compact object at the centre of our galaxy

2017



Reiner Weiss



Barry Barish

Kip Thorne

For decisive contributions to the LIGO detector and the observation of gravitational waves…from Black Hole mergers

Black Holes "demystified": The Black Hole N Portrait (BHNP)

Dvali, Gomez, arXiv:1203.6575
Dvali, Gomez, arXiv:1112.3359
Dvali,Guidice,Gomez,Kehagis, arXiv:1010.1415

Classical description: Macroscopic objects in GR with geometric, thermodynamic properties

$L_P^2 = \hbar\, G$

$M_P^2 = \frac{\hbar}{G}$

QFT understanding of BHNP: Black Holes are highly occupied "leaky" bound states of soft gravitons ($N = M_{BH}^2/M_P^2 >> 1$, $\sim 10^{66}$ for solar mass BHs)

Semi-classical limit: $N \to \infty$, $L_P \to 0$, Schwarzchild radius $R_S = 2\, G\, MBH = L_P\sqrt{N}$ = finite, $\hbar$ = finite

Event horizon, BH thermodynamics, no-hair theorem, understood simply in this limit

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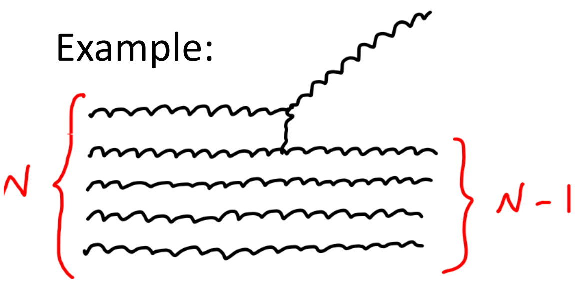
Example:



Black Hole evaporation Rate

$$\Gamma = 6\, n_h (1+N) \sim \cancel{\alpha_{gr}} R_S^2 \frac{\cancel{N^2}}{R_S^3}$$

$$= \frac{1}{R_S}$$

Hawking Temperature

$$T_H = \frac{\hbar}{R_S} = \frac{\hbar}{L_P\sqrt{N}}$$

Rate equivalently written as

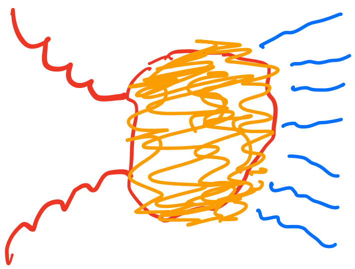
$$\frac{dM}{dt} = -\frac{T_H^2}{\hbar^2}$$

with Black Hole half-life ("Page time")

$$\boxed{t_{BH} = \frac{\hbar^2}{T_H^{\,3}\, G} = N^{3/2}\, L_P}$$

S-matrix picture of BHNP: $2 \rightarrow N$ graviton scattering

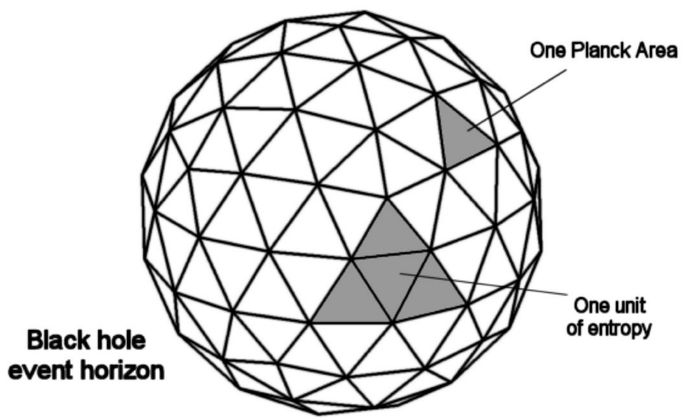
$2 \to N$



$P_{2\to N} \sim e^{S} \alpha_s^N N!$ $\Rightarrow$ If $N \sim \frac{1}{\alpha_s}$: $P_{2\to N} \sim e^{S} \cancel{\alpha_s^N} \cancel{\left(\frac{1}{\alpha_s}\right)^N} e^{-1/\alpha_s}$

Formation of a classical lump (aka BH)
exponentially suppressed, unless $S = \frac{1}{\alpha_S} = N$ $\boxed{\Rightarrow P_{2\to N} \sim O(1)}$

Quantum information perspective: BHNP saturates Bekenstein entropy bound $S \leq 2\pi\, ER/\hbar$



(for a nice discussion, see Bousso, arxlv:1810.01880)

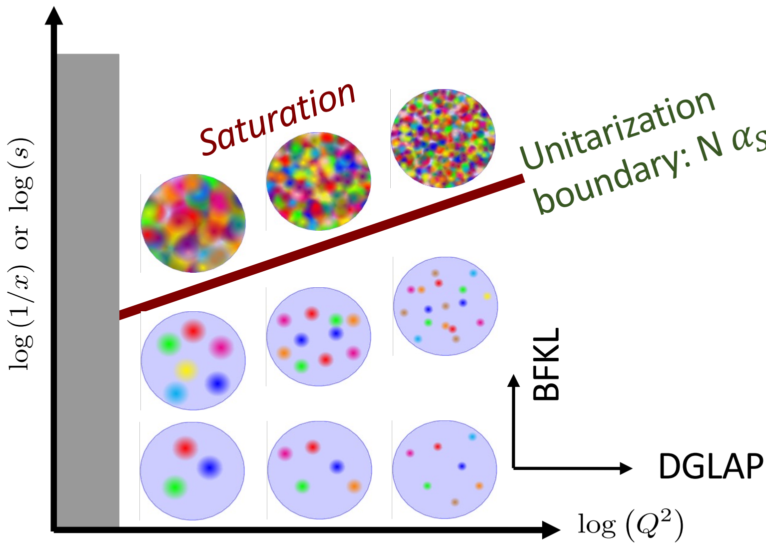
Define $E = N\, Q_S$ as energy in critically packed volume = R_S^3 of quanta ("qubits") saturating unitarity (maximal information) and $Q_S = 1/R_S$

Then, $S \leq 2\pi\, N\, Q_S\, R_S$ is saturated when $N = \frac{1}{\alpha_{gr}} \rightarrow S_{Bek} = \frac{1}{\alpha_{gr}}$

$$\boxed{S_{Bek} = \frac{1}{\alpha_{gr}} = \frac{R_S^2}{L_P^2} = \frac{Area}{4G} = S_{BH}}$$

Famous Bekenstein-Hawking area law

S-matrix picture of CGC: : $2 \rightarrow N$ gluon scattering



Classicalization and unitarization of $2 \rightarrow N$ cross-section occurs when $S_{CGC} = 1/\alpha_S$: saturated semi-classical "lump" is the CGC

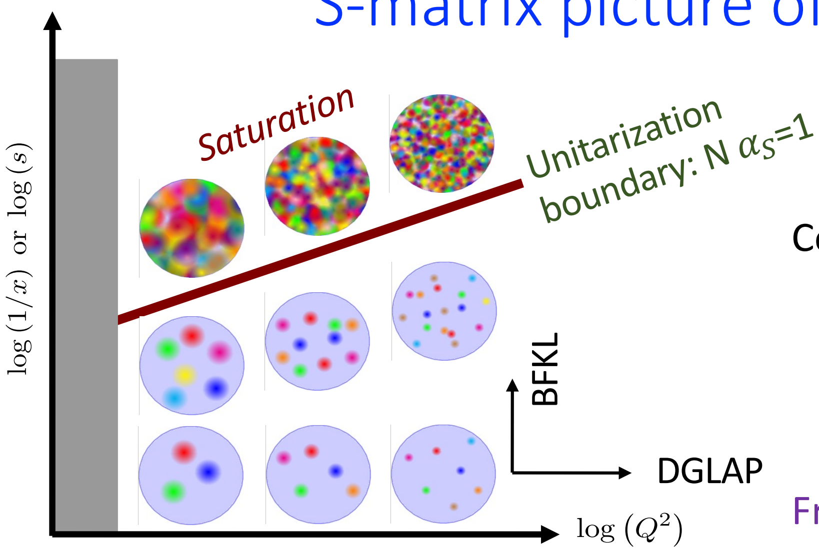
Condensate breaks Poincare invariance + global sub-group of $SU(3)_{color}$

$$\boxed{S_{CGC} = 1/\alpha_S = N = f_G^2 * \text{Area}}$$

The CGC satisfies the Bekenstein-Hawking area law in units of a Goldstone scale $f_G^2 = N\,Q_S^2$ For BH's this scale is the Planck mass

Dvali, RV, arXiv:2106.11989 (PRD, 2022)

S-matrix picture of CGC: : 2 → N gluon scattering



Classicalization and unitarization of 2 → N cross-section occurs when $S_{CGC} = 1/\alpha_S$: saturated semi-classical "lump" is the CGC

Condensate breaks Poincare invariance + global sub-group of $SU(3)_{color}$

$$\boxed{S_{CGC} = 1/\alpha_S = N = f_G^2 * \text{Area}}$$

Fresh insight: CGC is "classical" saddle point only up to 1/N corrections

- it is a leaky condensate and decays on a time scale $\tau \sim \frac{`1}{\alpha_S}\frac{1}{Q_S}$.
- Consistent with bottom-up QGP thermalization scenario

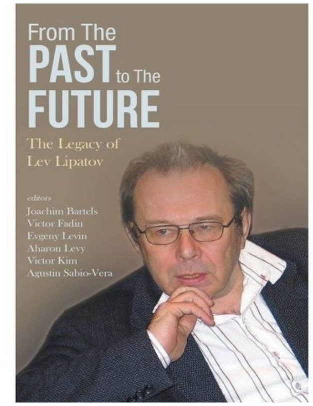
Baier,Mueller,Schiff,Son, hep-ph/0009237

Dvali-RV conjecture:

Both CGC / BH are semi-classical lumps (max occupancy/information) of gluons / gravitons (respectively) unitarizing cross-section at emergent scale Q_S

QCD Black disc/color screening saturation scale -> inverse of the Schwarzchild radius of Black Hole; both screen information
The physics at this scale is universal and independent of the microscopic details of the two theories

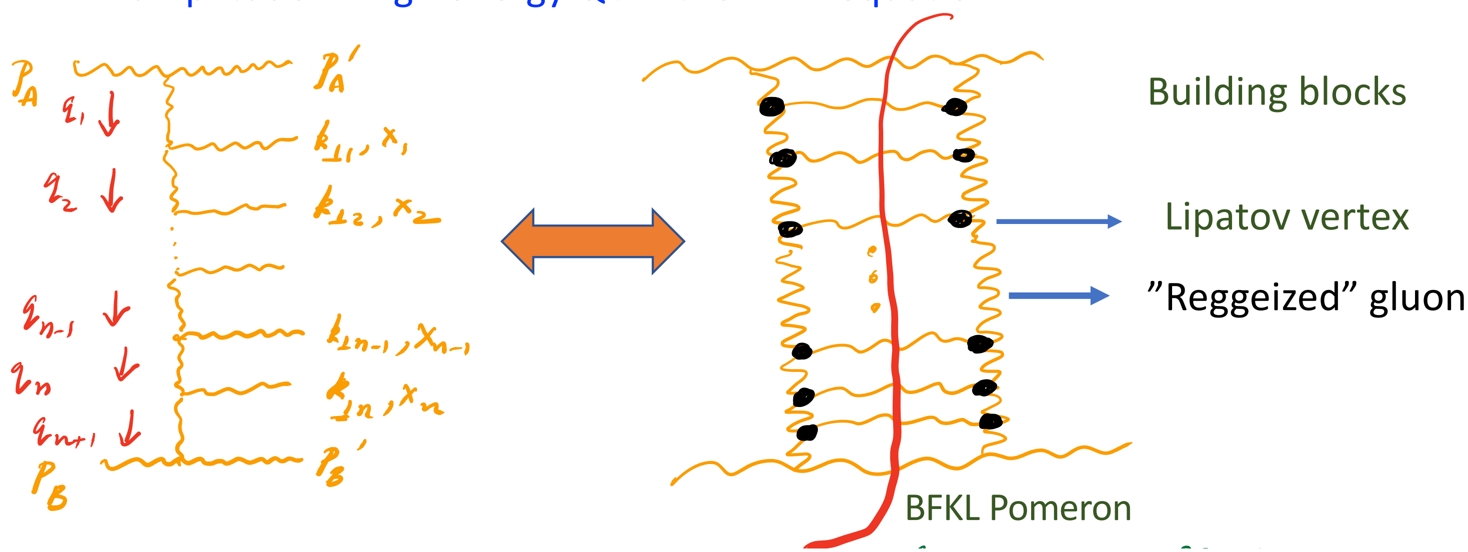
Lipatov’s EFT for wee partons in QCD and gravity



Lipatov, PLB 116B (1982) 411

Powerful double copy between QCD and gravity discovered by Lipatov 40 years ago

2 → N amplitude in high energy QCD: the BFKL equation



Identical construction for 2 → N amplitude in gravity

Gravitational Lipatov vertex: $\Gamma_{\mu\nu} \equiv \frac{1}{2} C_\mu C_\nu - \frac{1}{2} N_\mu N_\nu$

where C_μ is the QCD Lipatov vertex
and N_μ is the QED bremsstrahlung vertex

As in QCD, the graviton also “reggeizes” with Regge intercept ”2”, opposed to “1” in QCD

Double Copy: gluon → gravitational radiation in shockwave collisions

$$\begin{tikzcd}
\text{QCD at high occupancy} \arrow[d, swap, "\text{classical double copy (?)}"] & \text{perturbative QCD} \arrow[l] \arrow[d, "\text{BCJ double copy}"] \\
\text{Gravity at high occupancy} & \text{perturbative gravity} \arrow[l]
\end{tikzcd}$$

arrasco, Johannson,
arXiv: 1004.0476

Monteiro,O'Connell,White, arXIv:1410.0239
Goldberger, Ridgeway, arXiv:1611.03493

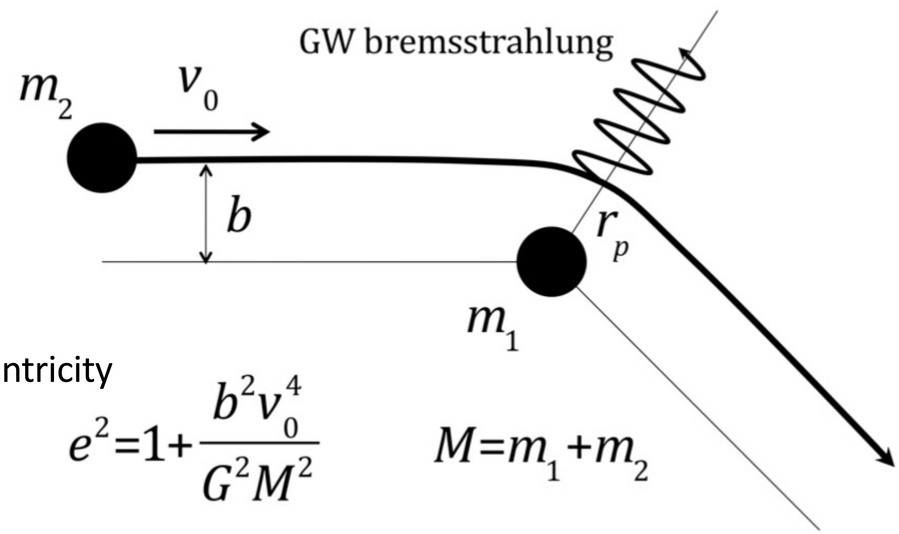
In QCD, in the CGC EFT, strong field semi-classical methods powerful alternative to amplitudes approach
- RG equations in rapidity allow for quantitative study of approach to gluon saturation

Can we do the same for gravity in the strong field regime of trans-Planckian scattering?
Can we compute gravitational wave radiation with varying frequency and impact parameter to extract quantum features of GR, and obtain insight into BH formation?

Gravitational radiation from primordial BH collisions

H. Raj, RV, in preparation

Close hyperbolic encounters (CHE) of BHs



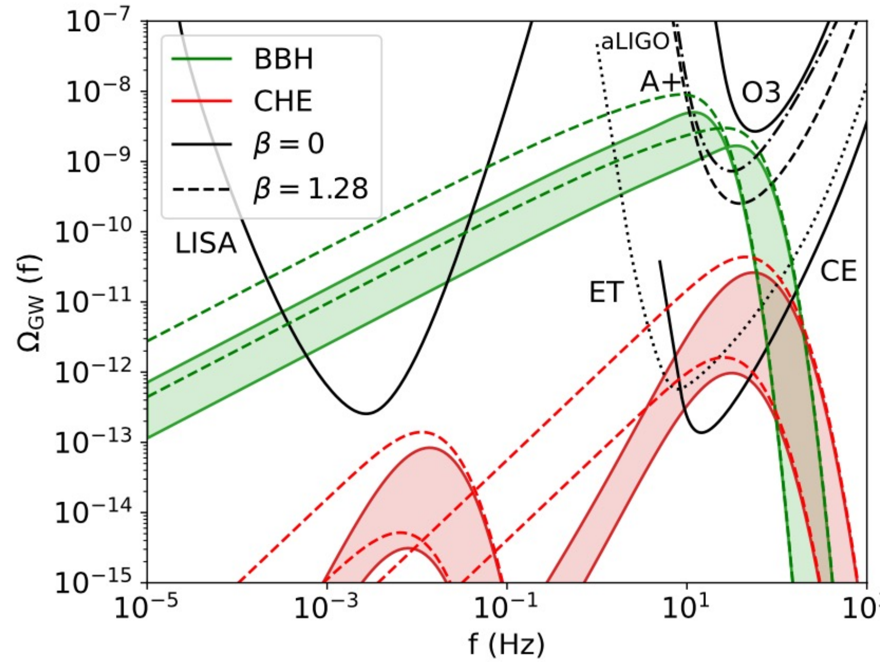
eccentricity

$$e^2 = 1 + \frac{b^2 v_0^4}{G^2 M^2} \qquad M = m_1 + m_2$$

Interesting question: what is the capture radius of GW radiation
In ultrarelativistic shockwave collisions in GR ?

Pretorius, Khurana, arXiv:gr-qc/0702084,
Sperhake,Berti,Cardoso,Pretorius, arXiv:1211.6114
Page, arXiv:2212.03890

Stochastic GW spectrum accessible at next gen. GWOs



Garcia-Bellido, Nesseris, arXiv:1706.02111;
Garcia-Bellido,Jaraba,Kuroyanagi, arXiv:2109.11376

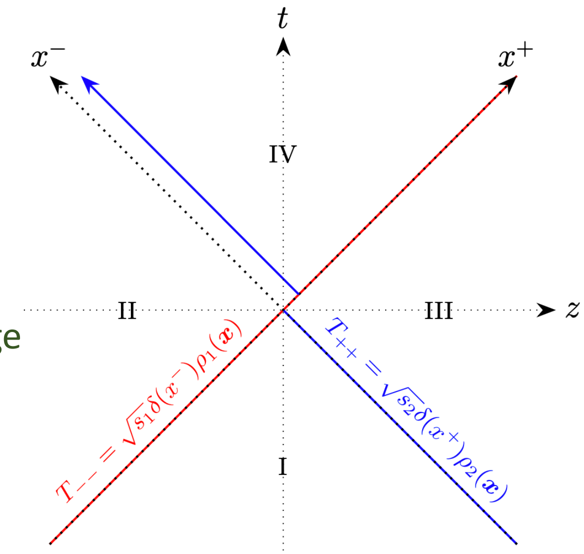
Dilute-Dense shockwave collisions in General Relativity

H. Raj, RV, in preparation

Solve Einstein's equations for linearized perturbations $h_{\mu\nu}$ around strong field shockwave metric

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

Light cone gauge: h_{+i}= 0



Dilute-dilute approx: expand eqns. to $O(\frac{\rho_A}{\partial^2{}_\perp}\frac{\rho_B}{\partial^2{}_\perp})$ – valid when R_S << b

$$\bar{g}_{--}\partial_+^2\tilde{h}_{ij} - \Box\tilde{h}_{ij} = \kappa^2\Big[(2\partial_i\partial_j - \Box_\perp\delta_{ij})\frac{1}{\partial_+^2}T_{++} + 2T_{ij} - \delta_{ij}T - \frac{2}{\partial_+}(\partial_i T_{+j} + \partial_j T_{+i} - \delta_{ij}\partial_k T_{+k})\Big]$$

$T_{\mu\nu}$ evolution in forward LC determined by geodesic equations and fully constrained by conservation laws – exactly analogous to current conservation for dilute-dense gluon shockwave collisions

Note: Studies of shockwave collisions in AdS/CFT, typically don't consider $T_{\mu\nu}$ evolution in the forward light cone in GR side of correspondence. Numerical GR studies typically limited to γ < 3

New result: first derivation of Lipatov vertex from Einstein's eqns.

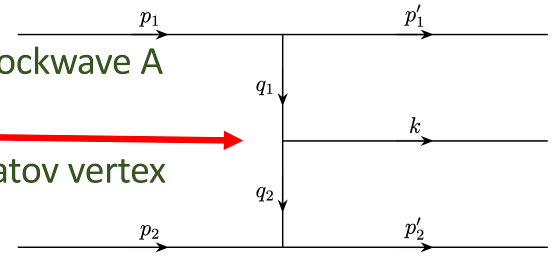
GW produced in CHE has the structure

$$-\boldsymbol{k}^2\tilde{h}_{ij} = \int \frac{d\boldsymbol{q}_2}{(2\pi)^2}\frac{\rho_A(\boldsymbol{q}_1)}{\boldsymbol{q}_1^2}\frac{\rho_B(\boldsymbol{q}_2)}{\boldsymbol{q}_2^2}\Gamma_{ij}$$

Shockwave A

GR Lipatov vertex

Shockwave B



$$\Gamma_{ij} = 2\left[\left(q_{2i} - k_i\frac{\boldsymbol{q}_{2\perp}^2}{\boldsymbol{k}_\perp^2}\right)\left(q_{2j} - k_j\frac{\boldsymbol{q}_{2\perp}^2}{\boldsymbol{k}_\perp^2}\right) - k_ik_j\frac{\boldsymbol{q}_{1\perp}^2\boldsymbol{q}_{2\perp}^2}{\boldsymbol{k}_\perp^4}\right]$$

with $\Gamma_{\mu\nu} \equiv \frac{1}{2}C_\mu C_\nu - \frac{1}{2}N_\mu N_\nu$

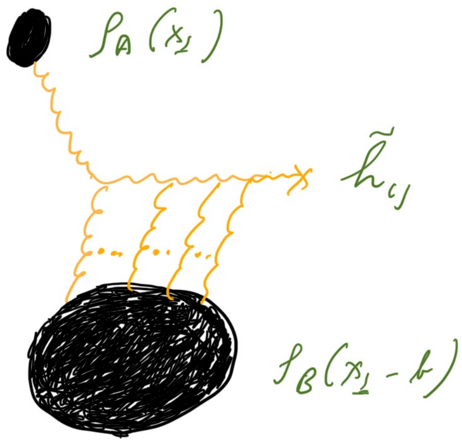
Recall C_μ is the QCD Lipatov vertex and N_μ is the QED Bremsstrahlung vertex

Thus a semi-classical computation in GR (completely analogous to prior QCD YM demonstration) can reproduce Lipatov's 2→ 3 Feynman diagram computation

In QCD, Lipatov vertex derived in
Blaizot, Gelis, RV, hep-ph/0402256
Gelis, Mehtar-Tani, hep-ph/0512079

Note: Amati, Ciafaloni, Veneziano and collaborators previously wrote down a 2-D EFT that captured this physics but did not derive it directly from GR shockwave solutions

New result: first derivation of Lipatov vertex from Einstein's eqns.



In dilute-dense case, usual QCD Wilson line (representing coherent mult. scattering) is replaced by its double copy counterpart…a la BCJ

$$\boxed{V(x^-, \boldsymbol{x}_\perp) \equiv \exp\left(\frac{1}{2}\int_{x_0^-}^{x^-} dz^- \underbrace{\bar{g}_{--}}_{A^{+,a}}(z^-, \boldsymbol{x}_\perp)\underbrace{\partial_+}_{T^a}\right)}$$

Yet, many subtle features of the correspondence remain to be understood

Prolegomenon to a future program…

A computation in progress is of GR shockwave (graviton-graviton-reggeon) propagators – a la CGC

This is necessary to understand the impact parameter and frequency dependence of
$2 \rightarrow N$ scattering (including such quantum effects) via an RG equation.

This framework may also help clarify (within the t'Hooft S-matrix ideology) fundamental issues related to Black Hole formation and decay.

Reminder: *Susskind's famous hologram paper is all about wee partons*

An outstanding goal would be quantitative studies to understand if next generation GWO's will be sensitive to this dynamics – the first of these will come online likely around the same time as the Electron-Ion Collider

Evolution and decay of Black Holes and CGC

We noted previously that the BH half life is t_{BH} = N R_S due to graviton loss from the leaky condensate (aka Hawking radiation)

This time scale is also inverse of $\omega = \frac{Q_S}{N}$, the energy gap of information carrying Bogoliubov Goldstone modes of the condensate
– microscopic understanding of Page time

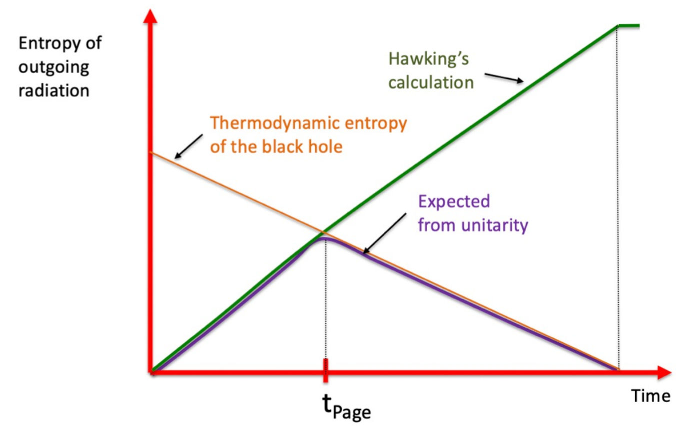
However the semi-classical BHNP description breaks down beyond Page time and genuine quantum effects take over

A similar process occurs in the Glasma (though now with 1-d expansion)
In the bottom-up thermalization picture, the quantum break time

is estimated to be $t_{quant} \sim \frac{\alpha_S^{-3/2}}{Q_S}$

In this case, the entropy continues to grow even after until the system thermalizes into a QGP

Page curve

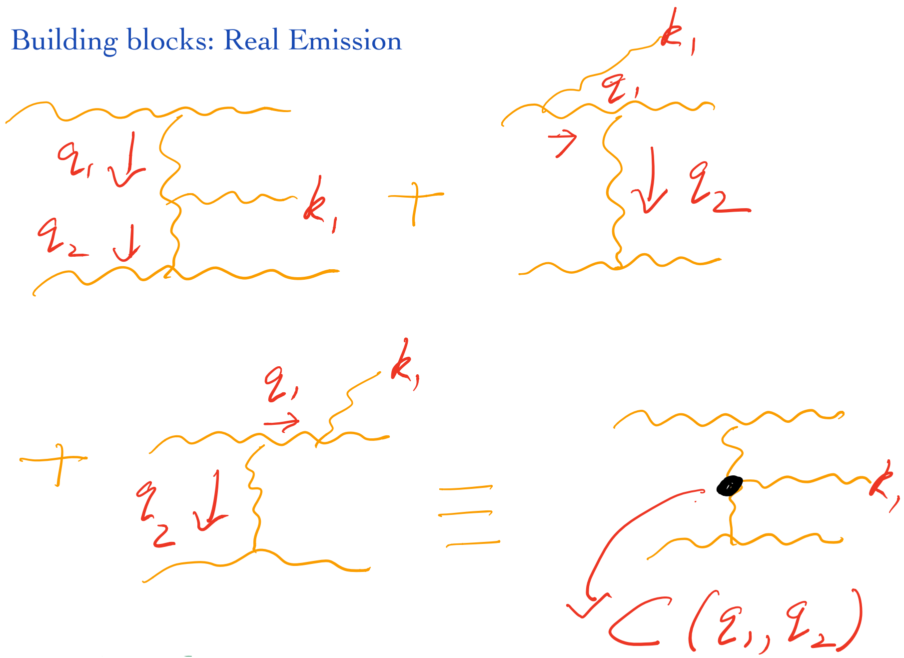


Almheiri et al., arXiv:2006.06872

$2 \rightarrow N + 2$ amplitude in the Regge limit of QCD: the BFKL equation

To build in real and virtual corrections to all orders in α_S, first focus on one rung of $2 \rightarrow$ N+2 ladder

Building blocks: Real Emission



Lipatov vertex

$$C(q_{i+1}, q_i) = -\underline{q}_{\perp i+1} - \underline{q}_{\perp i} + p_A \left(\frac{2 q_i^2}{\alpha_i s} + \beta_i \right) - p_B \left(\frac{2 q_{i+1}^2}{\beta_i s} + \alpha_i \right)$$

Non-local - gauge invariant!

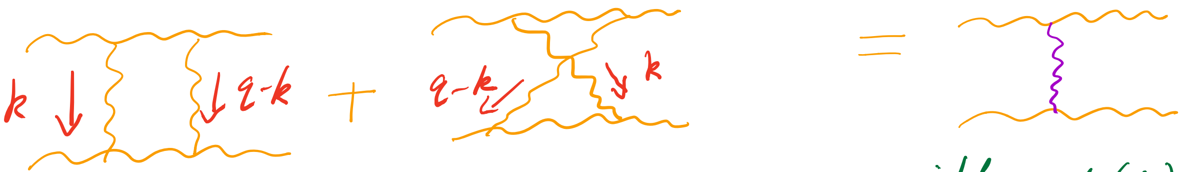
Ward identity: $k_i \cdot C = 0$

$$A^{2\rightarrow 3}(s,t) \sim 2s \frac{1}{t_1} C(q_2, q_1) \frac{1}{t_2}$$

BFKL pedagogical review:
DelDuca, hep-ph/9503226

$2 \rightarrow N+2$ amplitude in the Regge limit of QCD: the BFKL equation

Building Blocks: virtual corrections

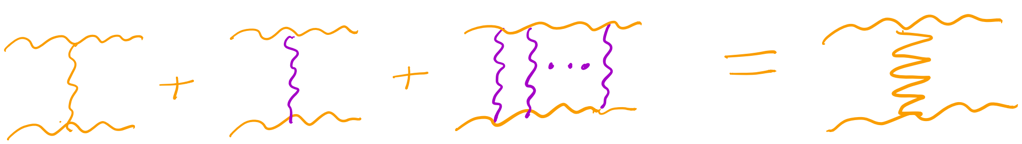


$\frac{1}{t} \to \frac{1}{t} \ln\frac{s}{-t}\,\alpha(t)$

with $\alpha(t) \propto \ln\frac{-t}{\mu^2}$

Infrared cutoff

Reggeization ansatz:



$\equiv \frac{1}{t_i} \to \frac{1}{t_i}\, e^{\alpha(t_i)(y_{i-1}-y_i)}$

$$\alpha(t) = \tilde{g}_S^2 \alpha^{(1)}(t) + \tilde{g}_S^4 \alpha^{(2)}(t) + \mathcal{O}(\tilde{g}_S^6)$$

Fadin, hep-ph/9807528

To this 2-loop order, this Regge trajectory can be obtained from the "cusp" anomalous dimension of the product of two Wilson lines: "Infrared factorization"

Korchemsky,Korchemskaya, hep-ph/9607229

Double log structure:
Sudakov form factor

→ infrared sensitive

→ $2 \to 2$ amp. vanishes for $\mu \to 0$

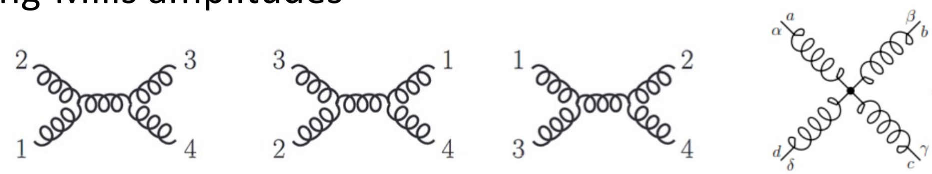
Gauge-Gravity correspondence

Double copy between QCD and Gravity amplitudes

Old idea (Kawai-Lewellyn-Tye) based on relations between closed and open string amplitudes – in "low energy" limit between Einstein & Yang-Mills amplitudes

$$M_4^{\rm tree}(1,2,3,4) = \left(\frac{\kappa}{2}\right)^2 s\, A_4^{\rm tree}(1,2,3,4)\, A_4^{\rm tree}(1,2,4,3)$$

$$\kappa = 32\,\pi^2\, G_N$$



Remarkable "BCJ" color-kinematics duality

Bern, Carrasco,Johansson,arXiv:0805.3993

Tree level $gg \to gg$ amplitudes (with on shell legs) can be written as

$$i\mathcal{A}_4^{\rm tree} = g^2\left(\frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u}\right)$$

with the s channel color factor $c_s = -2 f^{a_1 a_2 b} f^{b a_3 a_4}$

kinematic factor $n_s = -\frac{1}{2}\Big\{\Big[(\epsilon_1.\epsilon_2)p_1^\mu + 2(\epsilon_1.p_2)\epsilon_2^\mu - (1\leftrightarrow 2)\Big]\Big[(\epsilon_3.\epsilon_4)p_3^\mu + 2(\epsilon_3.p_4)\epsilon_4^\mu - (3\leftrightarrow 4)\Big] + s\Big[(\epsilon_1.\epsilon_3)(\epsilon_2.\epsilon_4) - (\epsilon_1.\epsilon_4)(\epsilon_2.\epsilon_3)\Big]\Big\}$

Tree level gravity amplitude obtained fron replacing color factors by kinematic factors

$$i\mathcal{A}_4^{\rm tree}|_{c_i\to n_i, g\to\kappa/2} = i\mathcal{M}_4^{\rm tree} = \left(\frac{\kappa}{2}\right)^2\left(\frac{n_s^2}{s} + \frac{n_t^2}{t} + \frac{n_u^2}{u}\right)$$

Significant on-going work on extension to loop amplitudes

Review: Bern et al., arXiv: 1909.01358

$2 \to N + 2$ amplitudes Trans-Planckian gravitation scattering: from wee partons to Black Holes

HIGH-ENERGY SCATTERING IN QCD AND IN QUANTUM GRAVITY AND TWO-DIMENSIONAL FIELD THEORIES

L.N. LIPATOV*

We construct effective actions describing high-energy processes in QCD and in quantum gravity with intermediate particles (gluons and gravitons) having the multi-Regge kinematics. The *S*-matrix for these effective scalar field models contains the results of the leading logarithmic approximation and is unitary. It can be expressed in terms of correlation functions for two field theories acting in longitudinal and transverse two-dimensional subspaces.

NPB 364 (1991) 614; 161 cites in INSPIRE

Effective action and all-order gravitational eikonal at planckian energies

AMATI,CIAFALONI,VENEZIANO **NPB403 (1993)707**

Building on previous work by us and by Lipatov, we present an effective action approach to the resummation of all semiclassical (i.e. $O(\hbar^{-1})$) contributions to the scattering phase arising in high-energy gravitational collisions. By using an infrared-safe expression for Lipatov's effective action, we derive an eikonal form of the scattering matrix and check that the superstring amplitude result is reproduced at first order in the expansion parameter R^2/b^2, where R, b are the gravitational radius and the impact parameter, respectively. If rescattering of produced gravitons is neglected, the longitudinal coordinate dependence can be explicitly factored out and exhibits the characteristics of a shock-wave metric while the transverse dynamics is described by a reduced two-dimensional effective action. Singular behaviours in the latter, signalling black hole formation, can be looked for.

The World as a Hologram

LEONARD SUSSKIND

Wee partons, by contrast, are not subject to Lorentz contraction. This implies that in the Feynman Bjorken model, the halo of wee partons eternally "floats" above the horizon at a distance of order $10^{-13} cm$ as it transversley spreads. The remaining valence partons carry the various currents which contract onto the horizon as in the Einstein Lorentz case.

By contrast, both the holographic theory and string theory require all partons to be wee. No Lorentz contraction takes place and the entire structure of the string floats on the stretched horizon. I have explained in previous articles how this behavior prevents the accumulation of arbitrarily large quantities of information near the horizon of a black hole. Thus we are led full circle back to Bekenstein's principle that black holes bound the entropy of a region of space to be proportional to its area.

***J.Math.Phys.* 36 (1995) 6377; 3242 cites**

In Acknowledgements:

Finally I benefitted from discussions with Kenneth Wilson and Robert Perry, about boosts and renormalization fixed points in light front quantum mechanics and Lev Lipatov about high energy scattering.