Heavy quark diffusion from 2+1 flavor lattice QCD

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Quark Matter 2023

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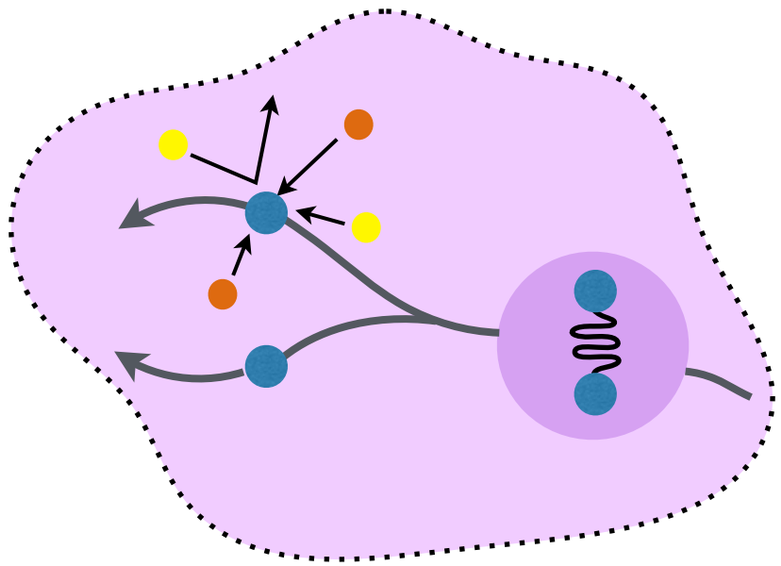
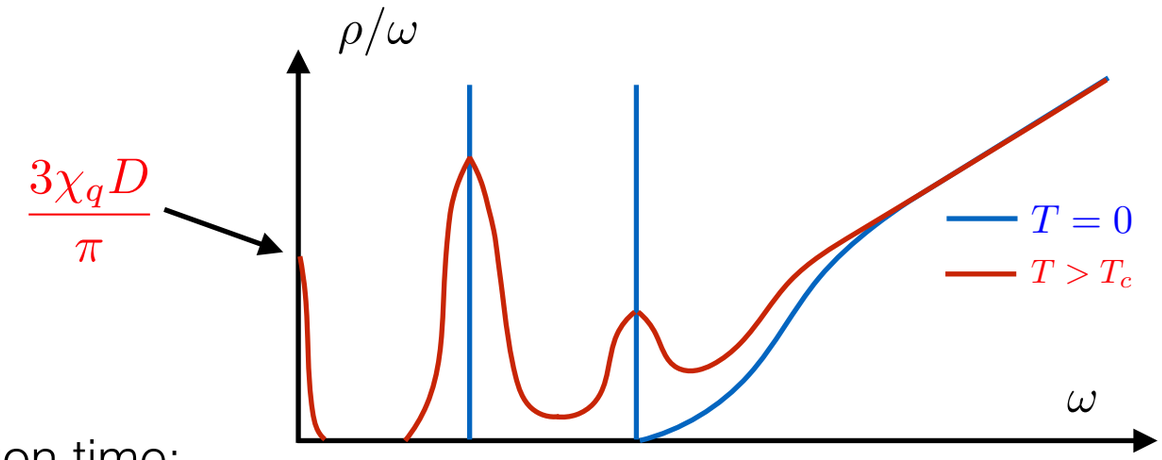
Heavy quark diffusion coefficient





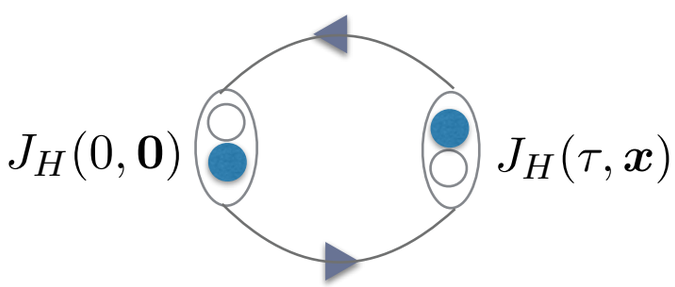
equilibration time:

$$\tau_{\rm kin} = \frac{1}{\eta_D} = \frac{DM}{T}$$

$$G_H(\tau,\vec{p}) = \sum_{x,y,z} exp(-i\,\vec{p}\cdot\vec{x})\langle J_H(0,\vec{0})J_H^\dagger(\tau,\vec{x}) = \int \frac{d\omega}{2\pi}\frac{\cosh(\omega(\tau T-1/2)/T)}{\sinh(\omega/2T)}\rho_H(\omega,\vec{p},T)$$

ill-posed

$K(\omega,\tau,T)$



- HQ diffusion is crucial input to the phenomenological studies
- HQ diffusion can be extracted from meson spectral function
- Reconstruction of meson spectral reconstruction is intractable

H.-T. Ding, **HTS**, et al., PRD 97, 094503

H.-T. Ding, **HTS**, et al., PRD 104 (2021) 11, 114508

Heavy quark diffusion in static limit

Heavy quark momentum diffusion coefficient in HQET

J. Casalderrey-Solana and D. Teaney, PRD 74, 085012
S. Caron-Huot et al., JHEP 0904 (2009) 053
A. Bouttefeux, M. Laine, JHEP 12 (2020) 150

$$\partial_t p_i = -\eta_D p_i + \xi_i(t)$$

$$\langle \xi_i(t)\xi_j(t')\rangle = \kappa\delta_{ij}\delta(t-t')$$

$$\partial_t \mathbf{p} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B}) = \mathbf{F}$$

- Mass dependent momentum diffusion coefficient

$$\kappa^{(M)} \equiv \frac{M^2\omega^2}{3T\chi_q}\sum_i \frac{2T\rho_V^{ii}(\omega)}{\omega}\Big|^{\eta\ll|\omega|\lesssim\omega_{\rm UV}}$$

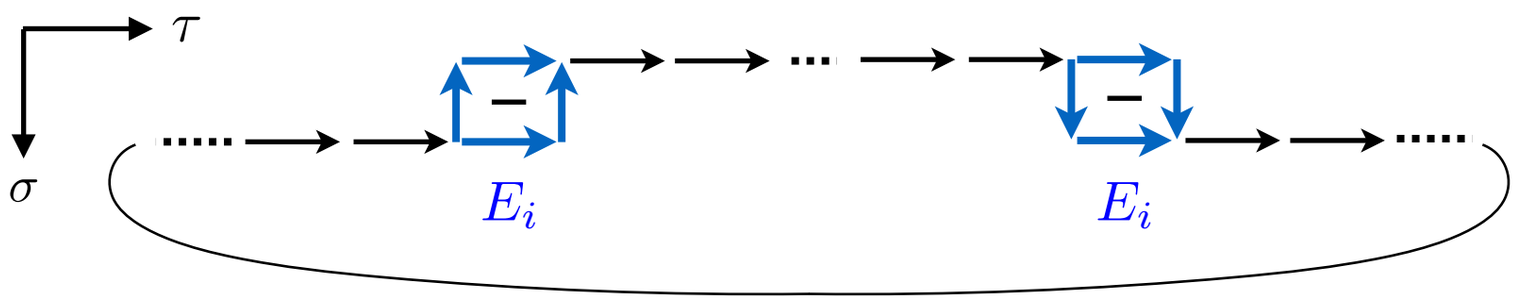
- Large quark mass limit in effective field theory

$$\kappa \equiv \frac{\beta}{3}\sum_{i=1}^{3}\lim_{\omega\to 0}\left[\lim_{M\to\infty}\frac{M^2}{\chi_q}\int_{-\infty}^{\infty}\mathrm{d}t\; e^{i\omega(t-t')}\int \mathrm{d}^3\vec{x}\,\left\langle\frac{1}{2}\{\mathcal{F}^i(t,\vec{x}),\mathcal{F}^i(0,\vec{0})\}\right\rangle\right]$$

Heavy quark momentum diffusion on the lattice

$$\frac{1}{2\pi T D} = \frac{\kappa}{4\pi T^3} = \frac{1}{2\pi T^2} \lim_{\omega \to 0} \frac{\rho(\omega)}{\omega}$$

$$\kappa = \kappa_E + \frac{2}{3}\langle \mathbf{v^2} \rangle \kappa_{\mathbf{B}}$$



Color-electric field correlation function

$$G(\tau, T) = \int \frac{d\omega}{\pi} K(\omega, \tau, T) \rho(\omega, T)$$

- Cheaper to measure on the lattice
- No peak structures in spectral functions
- Absence of transport peak

$$\langle \mathbf{v}^2 \rangle = \frac{3T}{M}$$

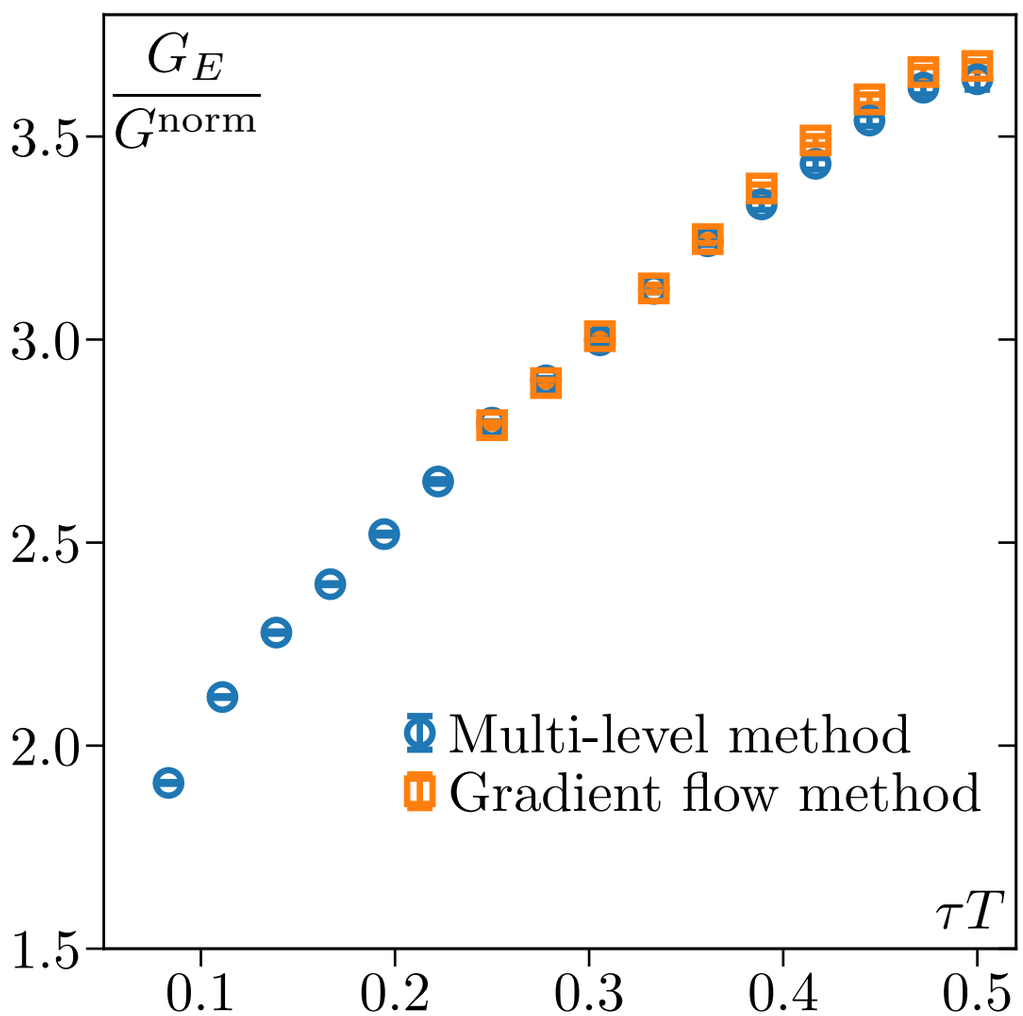
finite-mass correction:
~40% (charm) & ~15% (bottom) at 1.5Tc

Gradient flow

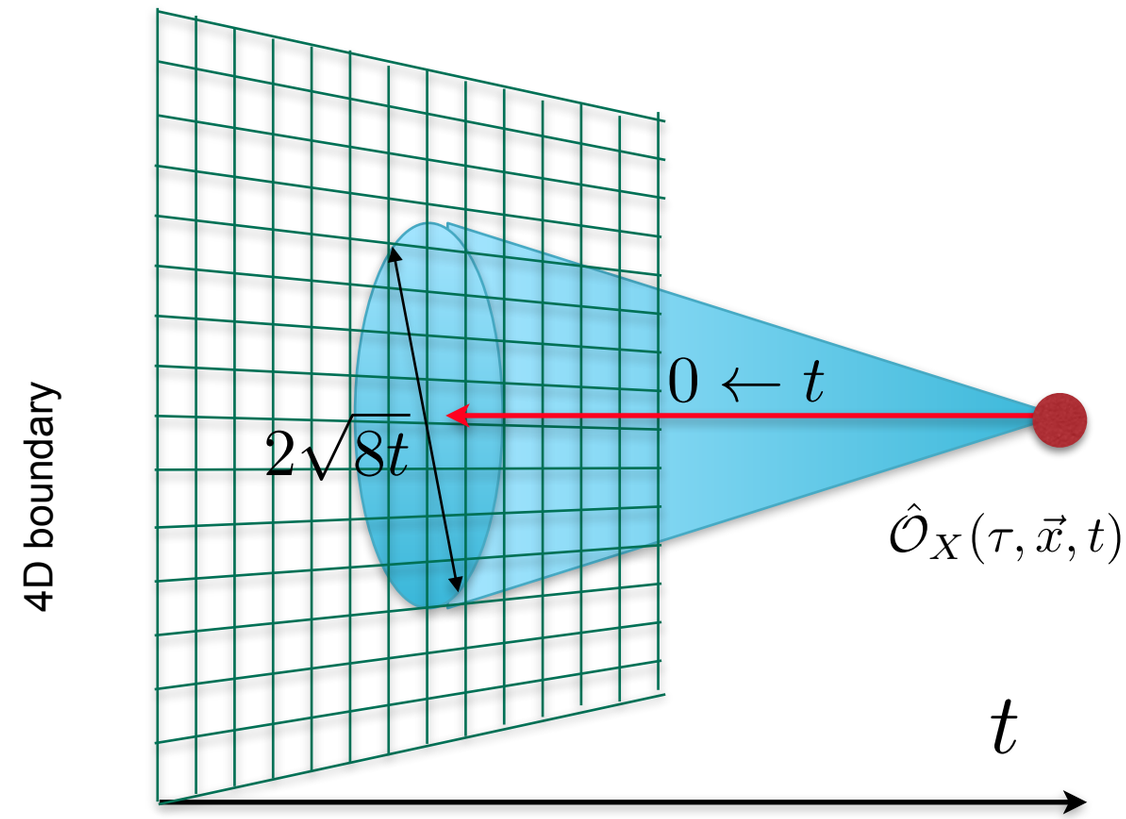
- Smear fields according to diffusion equation:

$$\frac{\mathrm{d}B_\mu(x,t)}{\mathrm{d}t} \sim -\frac{\delta S_G[B_\mu(x,t)]}{\delta B_\mu(x,t)} \sim D_\nu G_{\nu\mu}(x,t)$$

$$B_\nu(x,t)|_{t=0} = A_\nu(x)$$



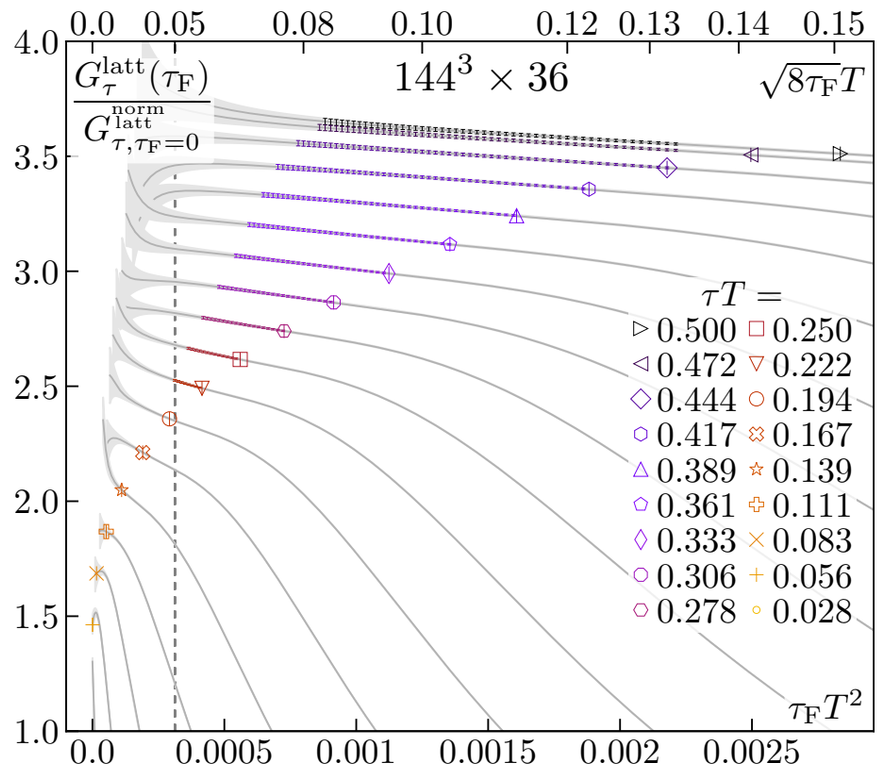
L. Altenkort, **HTS**, et al., PRD 103 (2021) 1, 014511

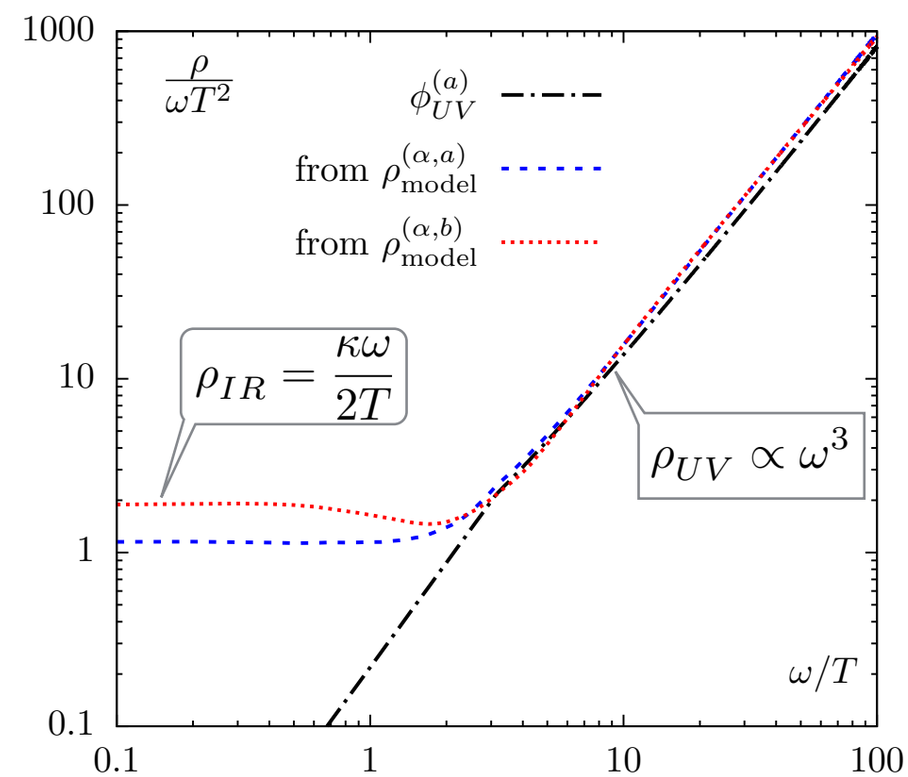


Luscher & Weisz, JHEP1102(2011)051
Narayanan & Neuberger, JHEP0603(2006)064

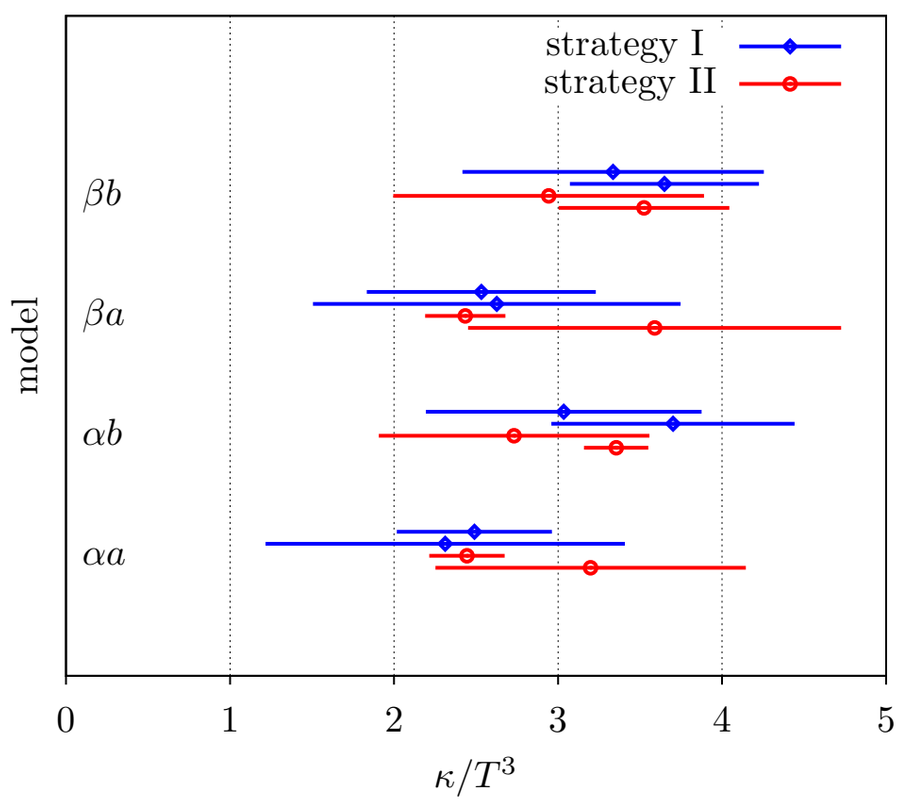
- Consistent quenched results from ML & GF
- Gradient flow paves the way to full QCD

A first taste in the quenched limit





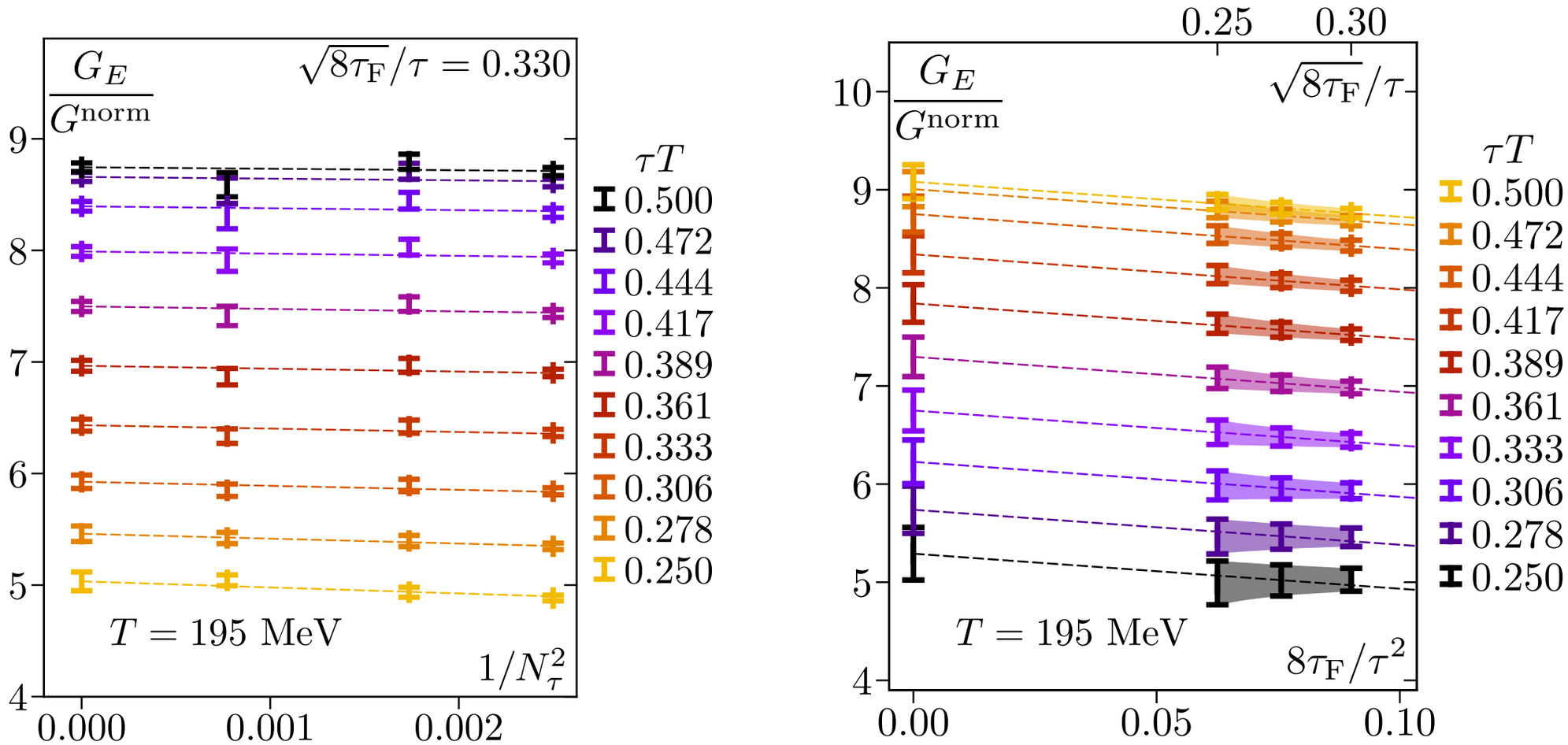
L. Altenkort, **HTS**, et al., PRD 103 (2021) 1, 014511



- First calculation of color-electric correlators using GF
- Good control for the SNR
- Robust extraction of kappa

Extension to QCD: double extrapolation

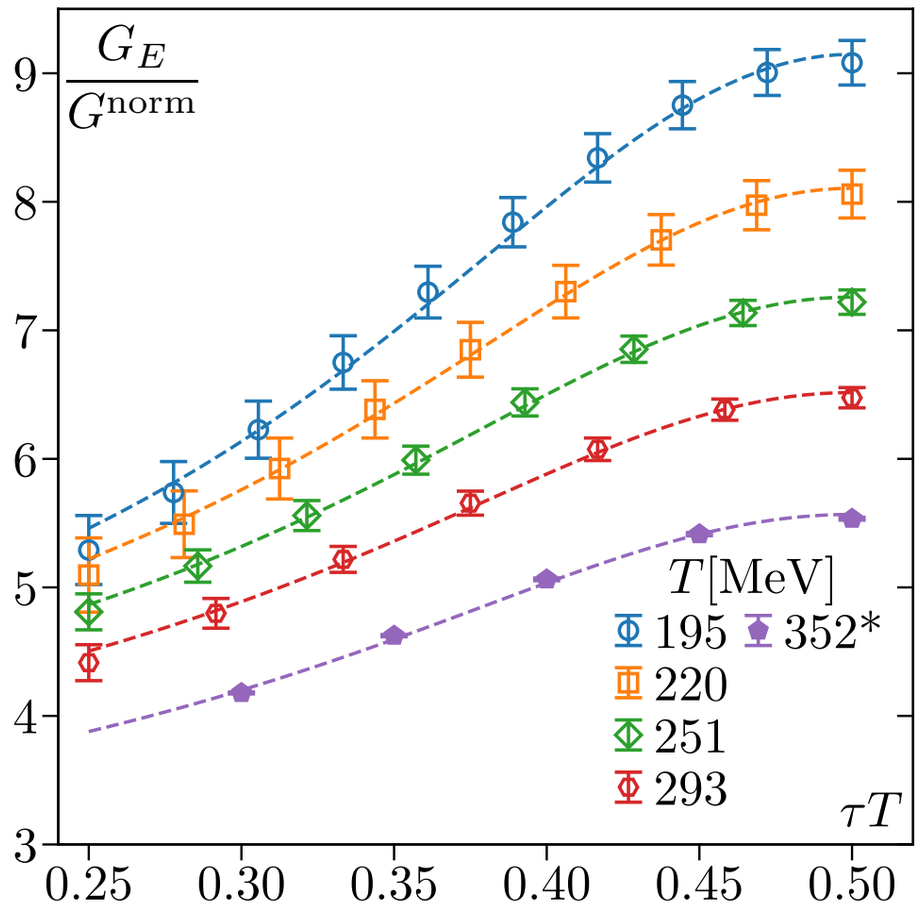
First full QCD calculation of kappa!

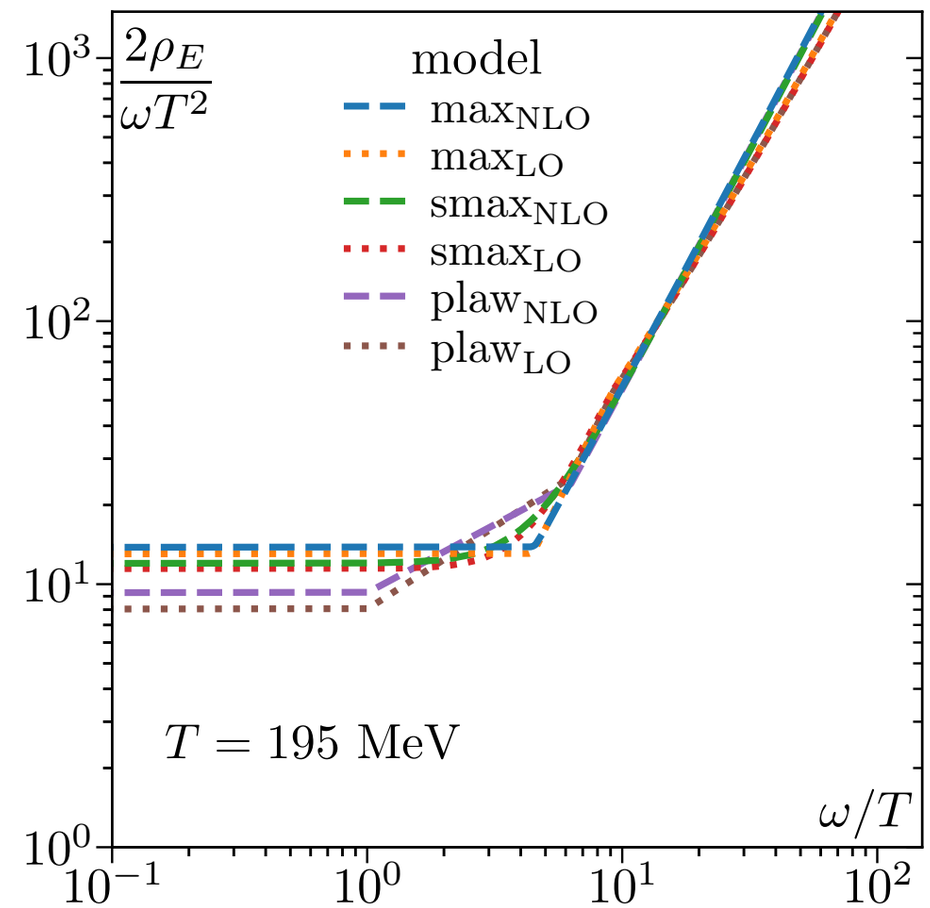


hotQCD, PRL 130 (2023) 23, 231902

- Wide temperature range with Mpion=320 MeV
- Extrapolation Ansatz describes lattice data well

Extraction of spectra from correlators





hotQCD, PRL 130 (2023) 23, 231902

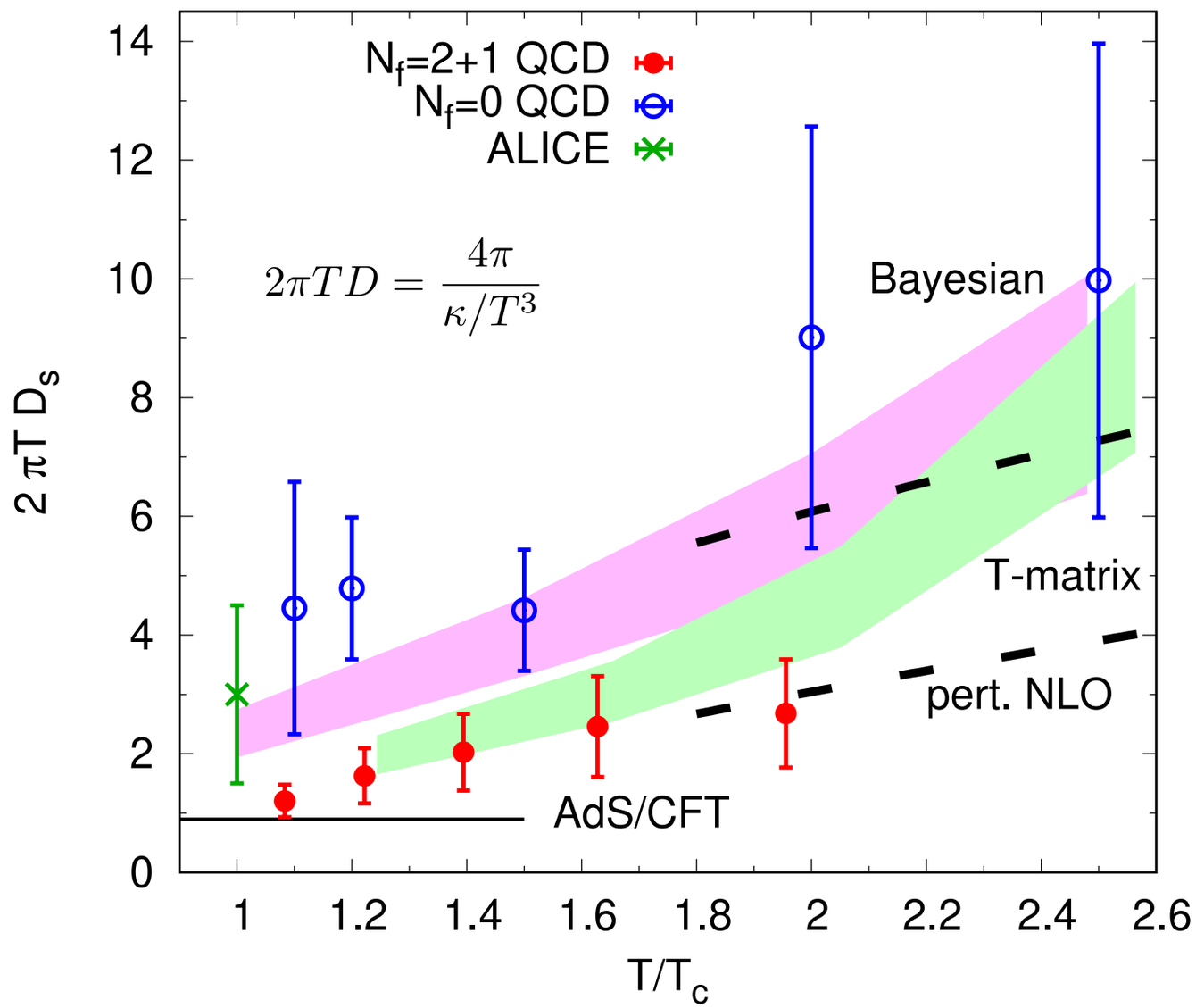
- Wide temperature range with Mpion=320 MeV
- Much larger magnitudes in QCD than in quenched
- Good description of lattice data using different models

$$\rho_{\rm max} \equiv \max(\phi_{\rm IR}, \phi_{\rm UV})$$

$$\rho_{\rm smax} \equiv \sqrt{\phi_{\rm IR}^2 + \phi_{\rm UV}^2}$$

$$\rho_{\rm plaw} \equiv \begin{cases} \phi_{\rm IR} & \omega \le \omega_{\rm IR}, \\ a\omega^b \quad \text{for} & \omega_{\rm IR} < \omega < \omega_{\rm UV}, \\ \phi_{\rm UV} & \omega \ge \omega_{\rm UV}, \end{cases}$$

Summary of heavy quark diffusion coefficient



Quenched results (blue):

N. Branbilla, et al., PRD 102 (2020) 7, 074503

L. Altenkort, **HTS**, et al., PRD 103 (2021) 1, 014511

D. Banerjee, et al., arXiv: 2206.15471

Full LQCD results (red):

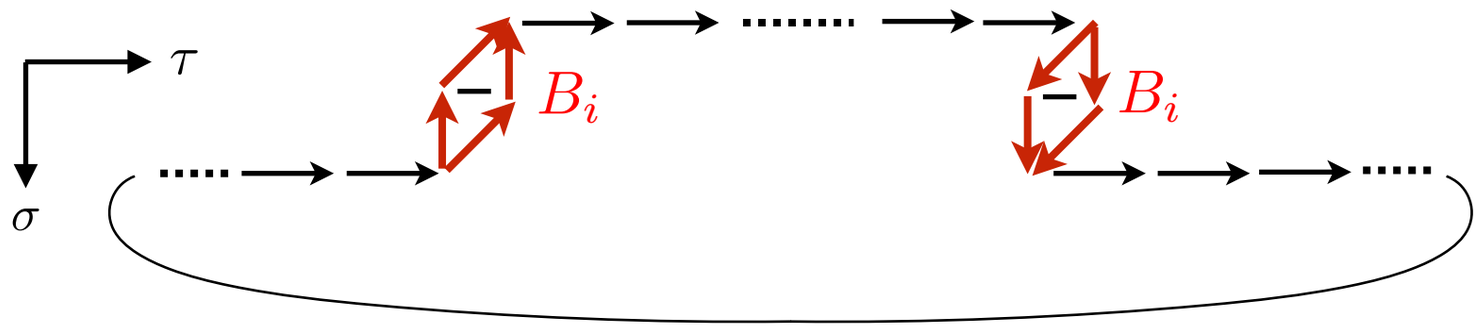
hotQCD, PRL 130 (2023) 23, 231902

Updated T-matrix results:

Zhanduo Tang, Wed. 5:30pm, D, [1072]

- Agree with AdS/CFT at ~Tc (rapid equilibrium)
- Agree with T-matrix estimate at moderate T
- Agree with NLO perturbative estimate at large T
- Smaller than quenched estimates

Finite mass correction to HQ momentum diffusion



Color-magnetic field correlation function

A. Bouttefeux, M. Laine, JHEP 12 (2020) 150

D. Guazzini, et al., JHEP 10 (2007) 081

Charm & bottom quark not heavy enough

Banerjee '22: D. Banerjee, et al., JHEP 08 (2022) 128

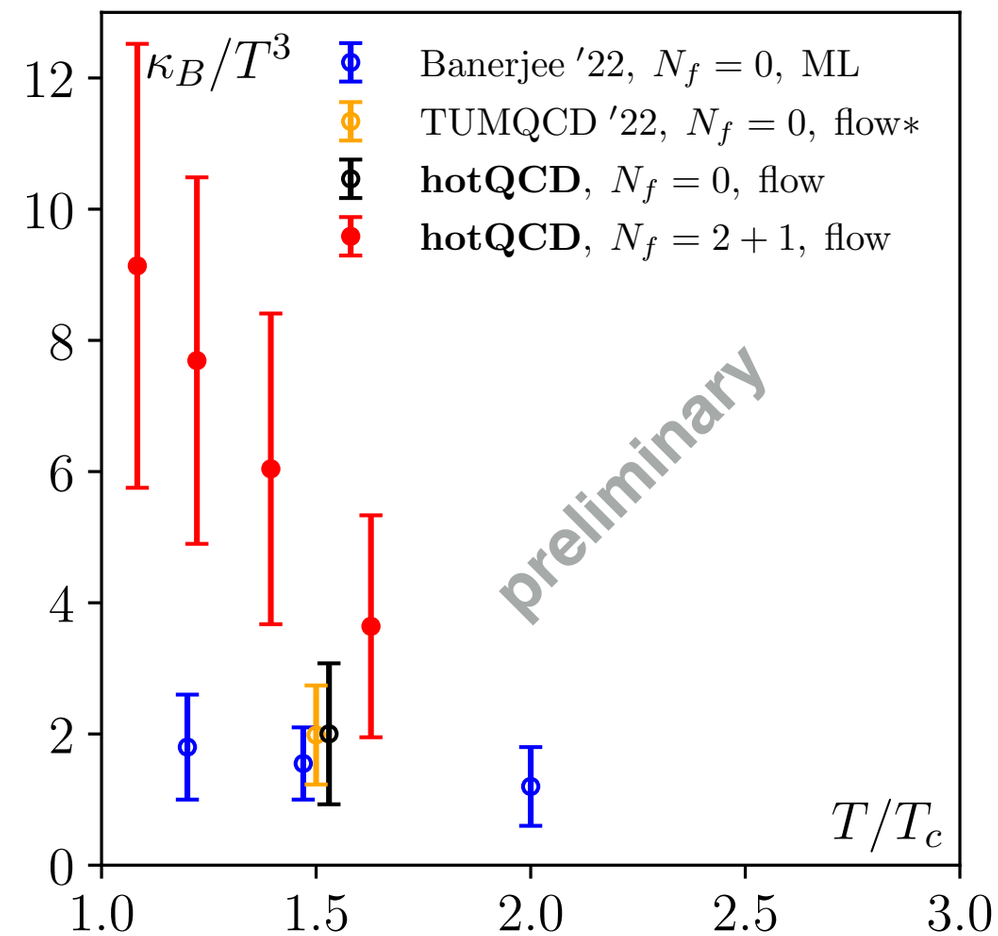
TUMQCD '22: N. Brambilla, et al., PRD107, 054508

Matching from GF scheme to MSbar scheme:

$$G_B^{\text{phys.}}(\tau_{\text{F}}) = Z_{\text{match}}(\bar{\mu}_{\text{ir}}, \bar{\mu}_{\text{uv}}, \mu_{\text{F}})\, G_B^{\text{flow}}(\tau_{\text{F}})$$

- Consistent quenched results for kappa_B among different calculations
- Similar dynamical-quark-effects between kappa_E & kappa_B

hotQCD, in preparation



Summary

HQ momentum diffusion coefficient from chromo-E/B field correlators

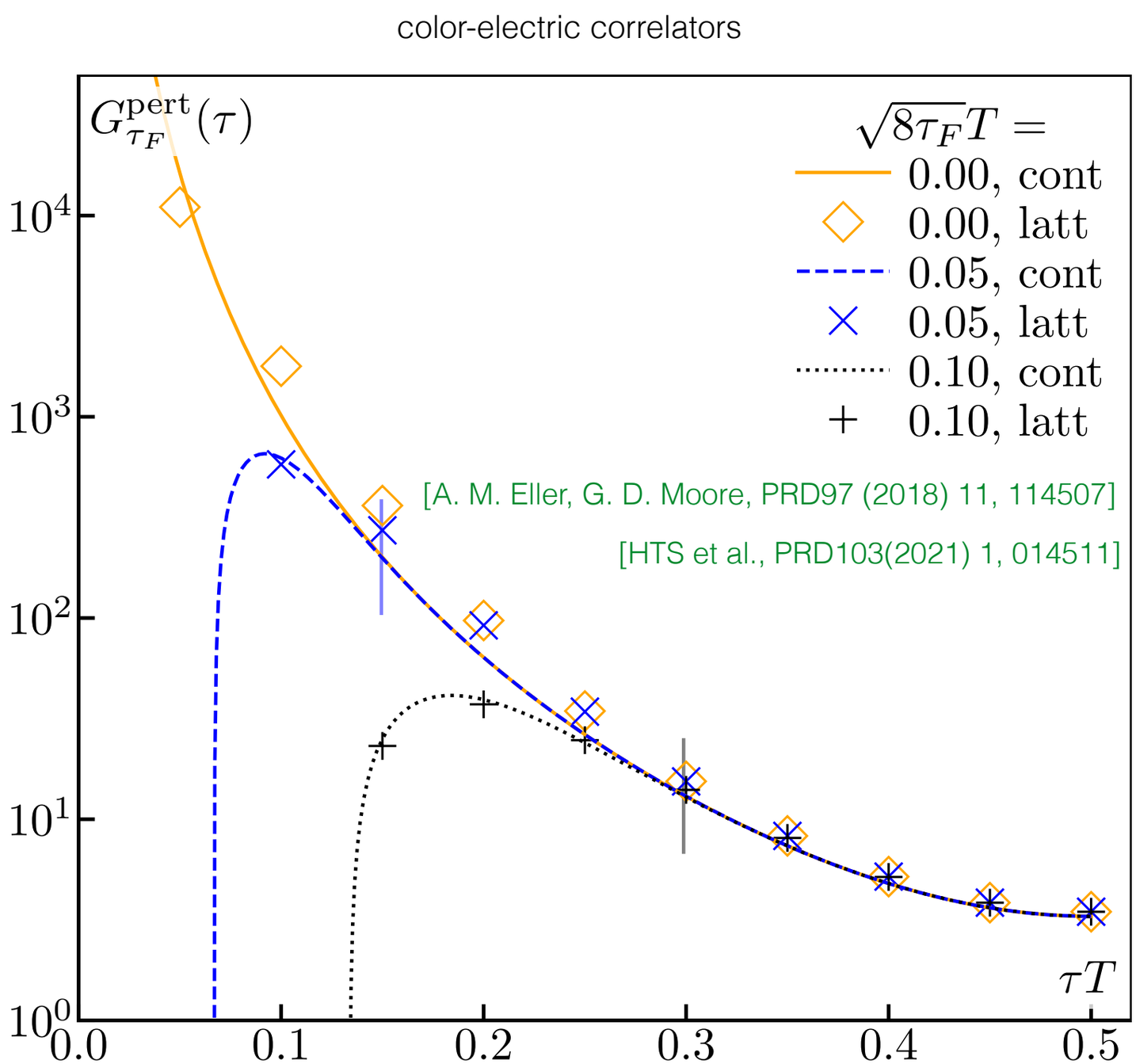
✓ Leading contribution is known in both quenched limit & full QCD

✓ Preliminary results for finite-mass correction are available in both quenched limit & full QCD

◉ Extension to physical pion mass

Lattice QCD is in a phase to provide accurate and realistic heavy flavor inputs for HIC phenomenology

Backup: flow time limits



- Flow destroys the signal at small distances
- Large distance parts are not affected
- More points are destroyed at larger flow times
- At most 1% deviation of flowed correlators from unflowed ones determines maximum flow time:

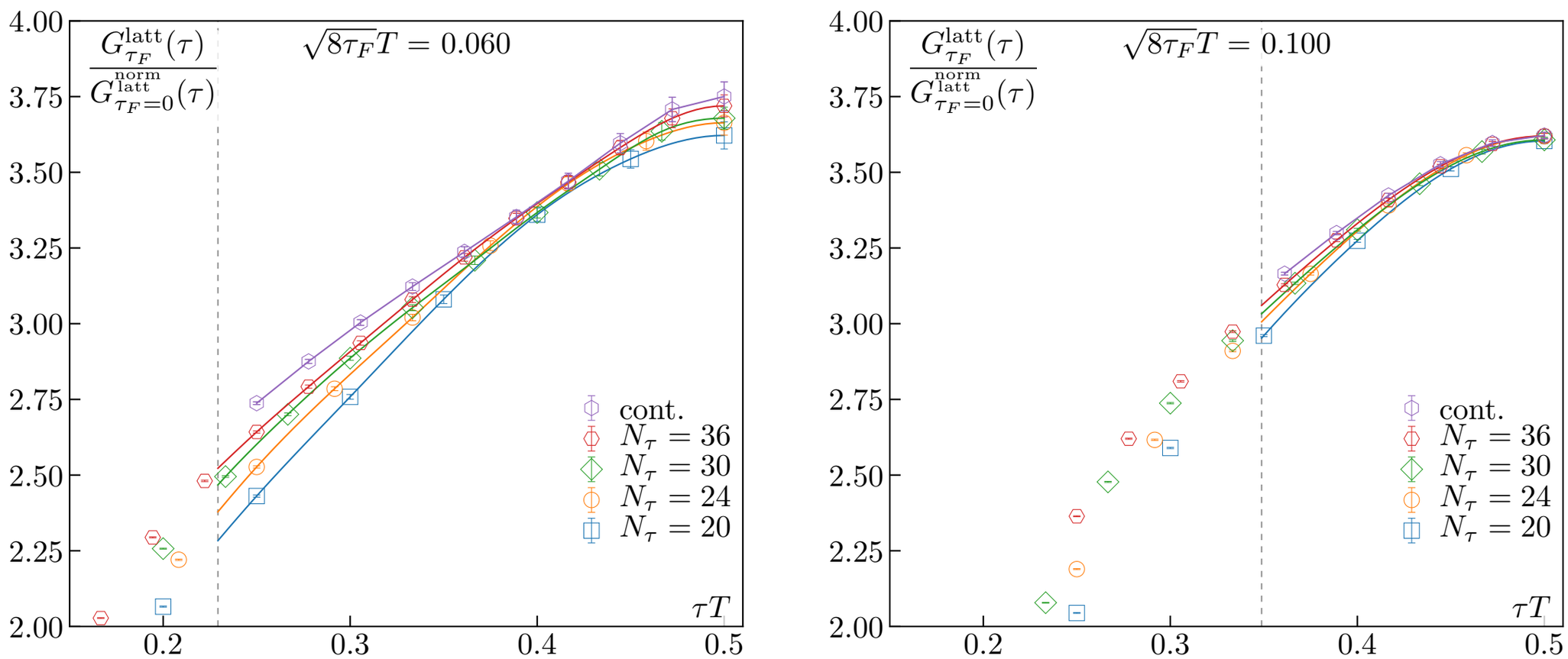
$$a \lesssim \sqrt{8t} \lesssim \frac{\tau - a}{3}$$

a : to suppress lattice effects

a : lattice version of the perturbative flow time limit

Backup: smearing effects of gradient flow

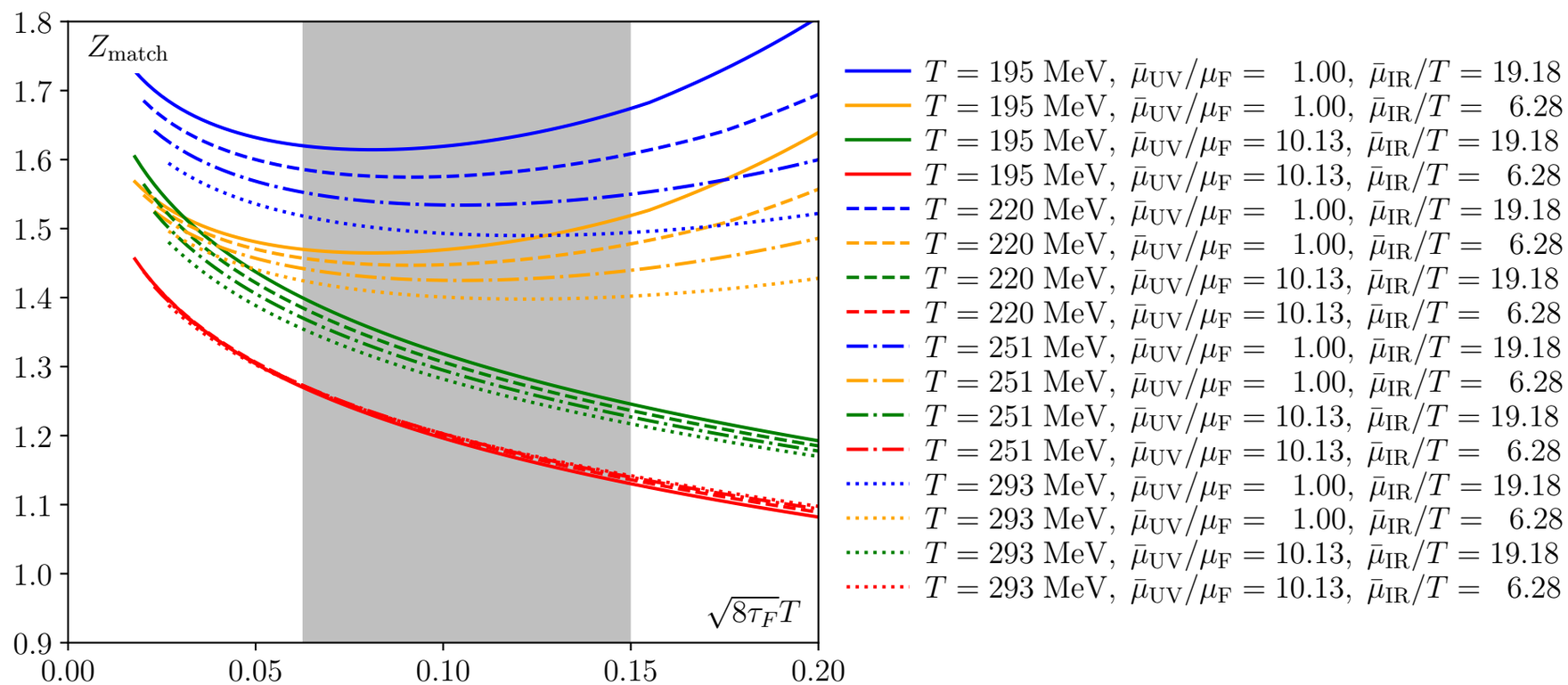
[HTS et al., PRD103(2021) 1, 014511]



- Gradient flow reduces the noise in correlators
- Gradient flow removes the lattice effects (disordering)
- Need proper flow time range

Backup: renormalization factor

$$\ln Z_{\rm match} = \int_{\bar\mu_{\rm ir}^2}^{\bar\mu_{\rm uv}^2} \gamma_0 g^2_{\overline{\rm MS}}(\bar\mu)\frac{d\bar\mu^2}{\bar\mu^2} + \gamma_0 g^2_{\overline{\rm MS}}(\bar\mu_{\rm ir})\left[\ln\frac{\bar\mu_{\rm ir}^2}{(4\pi T)^2} - 2 + 2\gamma_{\rm E}\right] - \gamma_0 g^2_{\overline{\rm MS}}(\bar\mu_{\rm uv})\left[\ln\frac{\bar\mu_{\rm uv}^2}{\mu_{\rm F}^2} - 2\ln(2) - \gamma_{\rm E} - \frac{8}{3}\right]$$

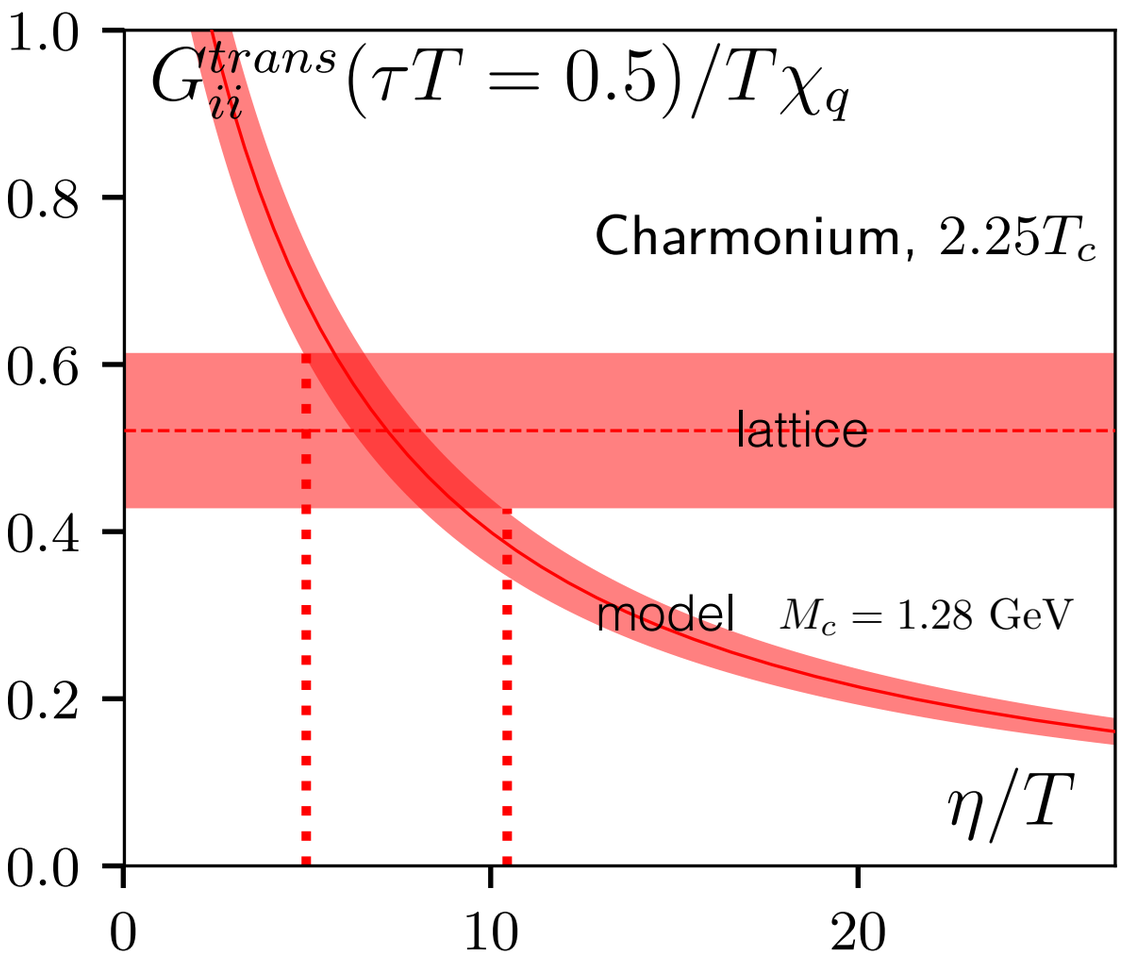


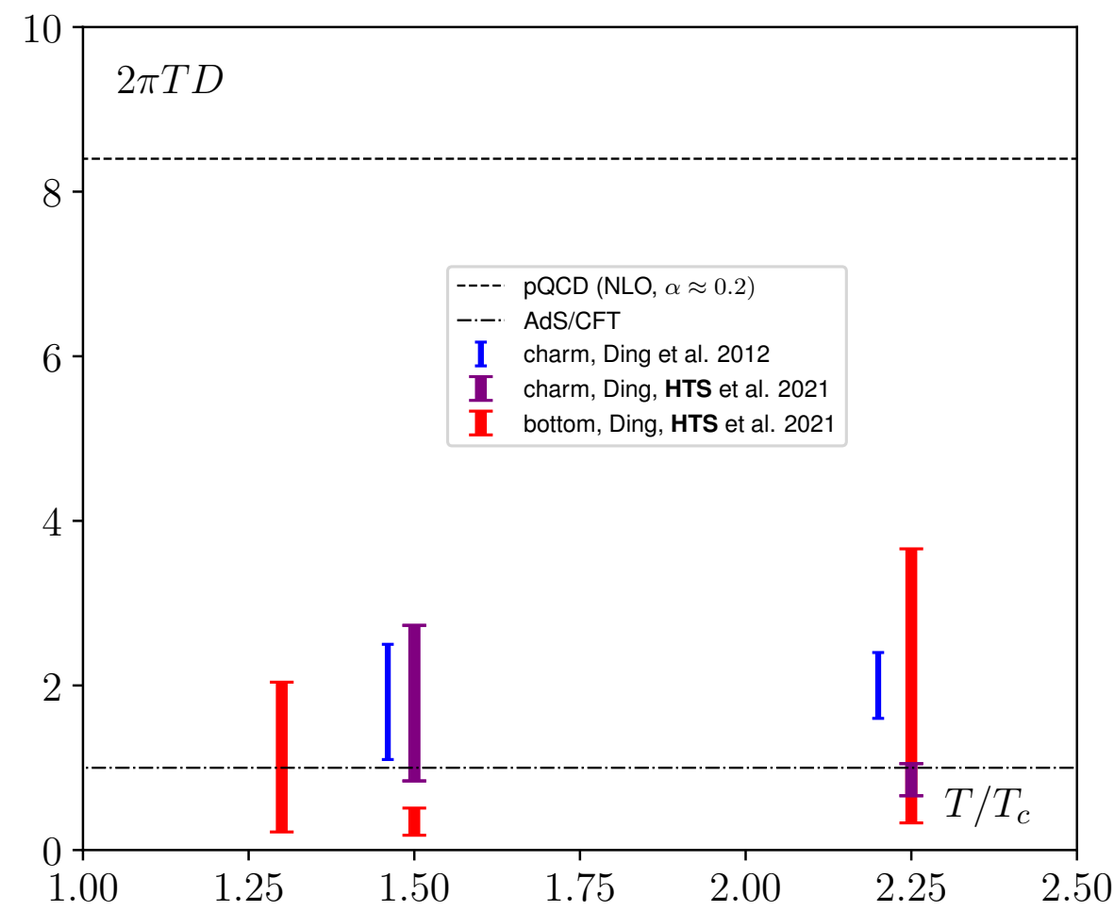
Charm & bottom quark diffusion (quenched)

Model the transport peak contribution of quarkonium correlators

$$G_{\rm trans}(\tau) = \int \frac{d\omega}{\pi} 3\chi_q D \frac{\omega\eta}{\omega^2+\eta^2} K(\omega,\tau,T)$$

H.-T. Ding, **HTS**, et al., PRD 104 (2021) 11, 114508





- Consistent with AdS/CFT

Backup: full QCD setup

$$N_f = 2+1,\ \text{HISQ},\ m_\pi = 320\ \text{MeV}$$

T [MeV]	β	am_s	am_l	N_σ	N_τ	# conf.
195	7.570	0.01973	0.003946	64	20	5899
	7.777	0.01601	0.003202	64	24	3435
	8.249	0.01011	0.002022	96	36	2256
220	7.704	0.01723	0.003446	64	20	7923
	7.913	0.01400	0.002800	64	24	2715
	8.249	0.01011	0.002022	96	32	912
251	7.857	0.01479	0.002958	64	20	6786
	8.068	0.01204	0.002408	64	24	5325
	8.249	0.01011	0.002022	96	28	1680
293	8.036	0.01241	0.002482	64	20	6534
	8.147	0.01115	0.002230	64	22	9101
	8.249	0.01011	0.002022	96	24	688
352	8.249	0.01011	0.002022	96	20	2488

- Wide temperature range
- Different lattice spacings
- Large lattices towards thermodynamic limit