INTERFEROMETRY IN A MOAT REGIME

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

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[FR, Pisarski, Rischke, PRD 107 (2023)]

CRC-TR 211
Strong-interaction matter under extreme conditions

HFHF

QUARK MATTER 2023
HOUSTON - 06/09/2023

QM2022 IN KRAKOW

MOAT REGIMES & THEIR SIGNATURES IN HEAVY-ION COLLISIONS

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[Pisarski, FR, PRL 127 (2021)]

- QUARK MATTER 2022 -

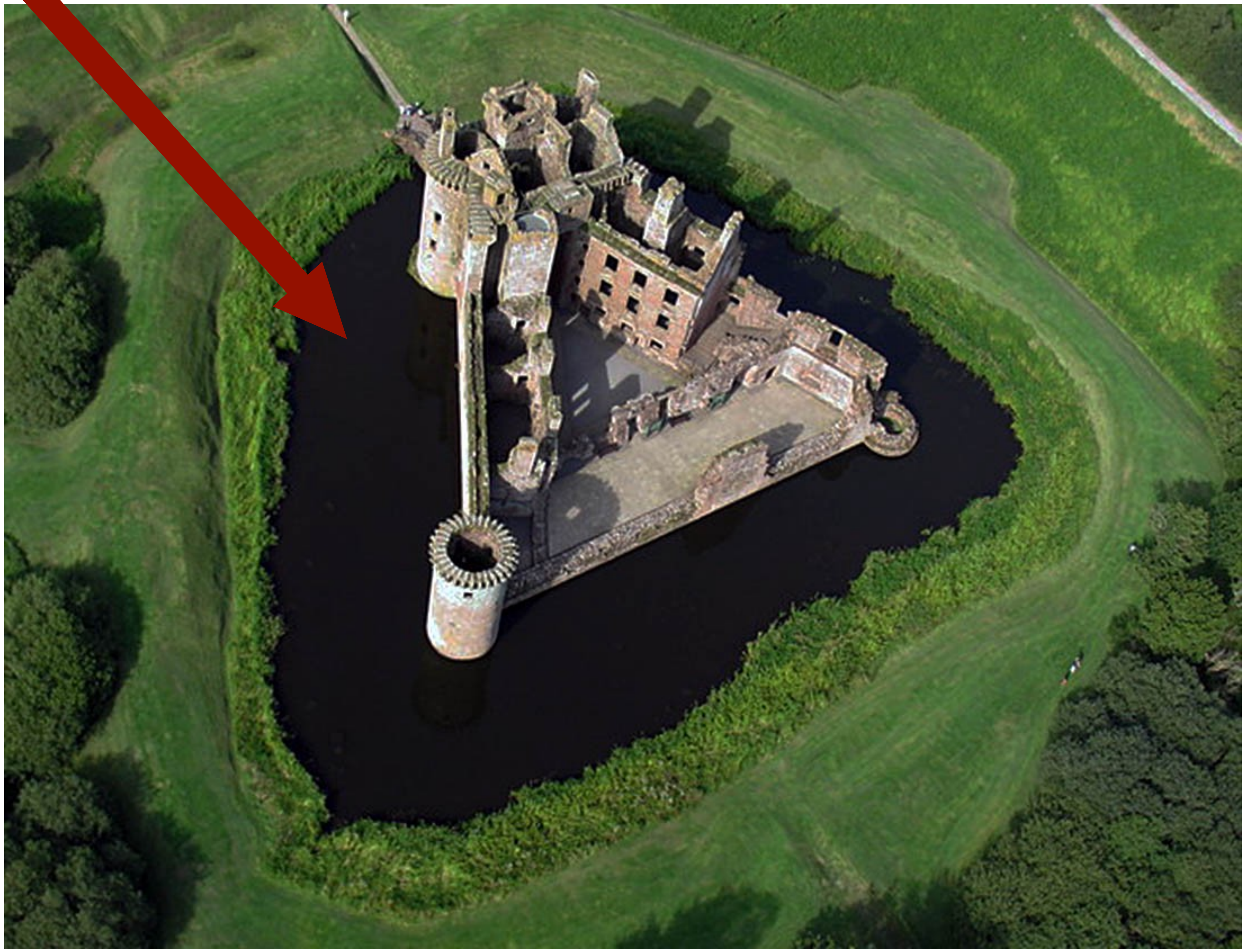
KRAKOW - 06/04/2022

we studied correlations arising from thermodynamic fluctuations

many of you asked:
what about HBT?

this talks answer your question!

A MOAT

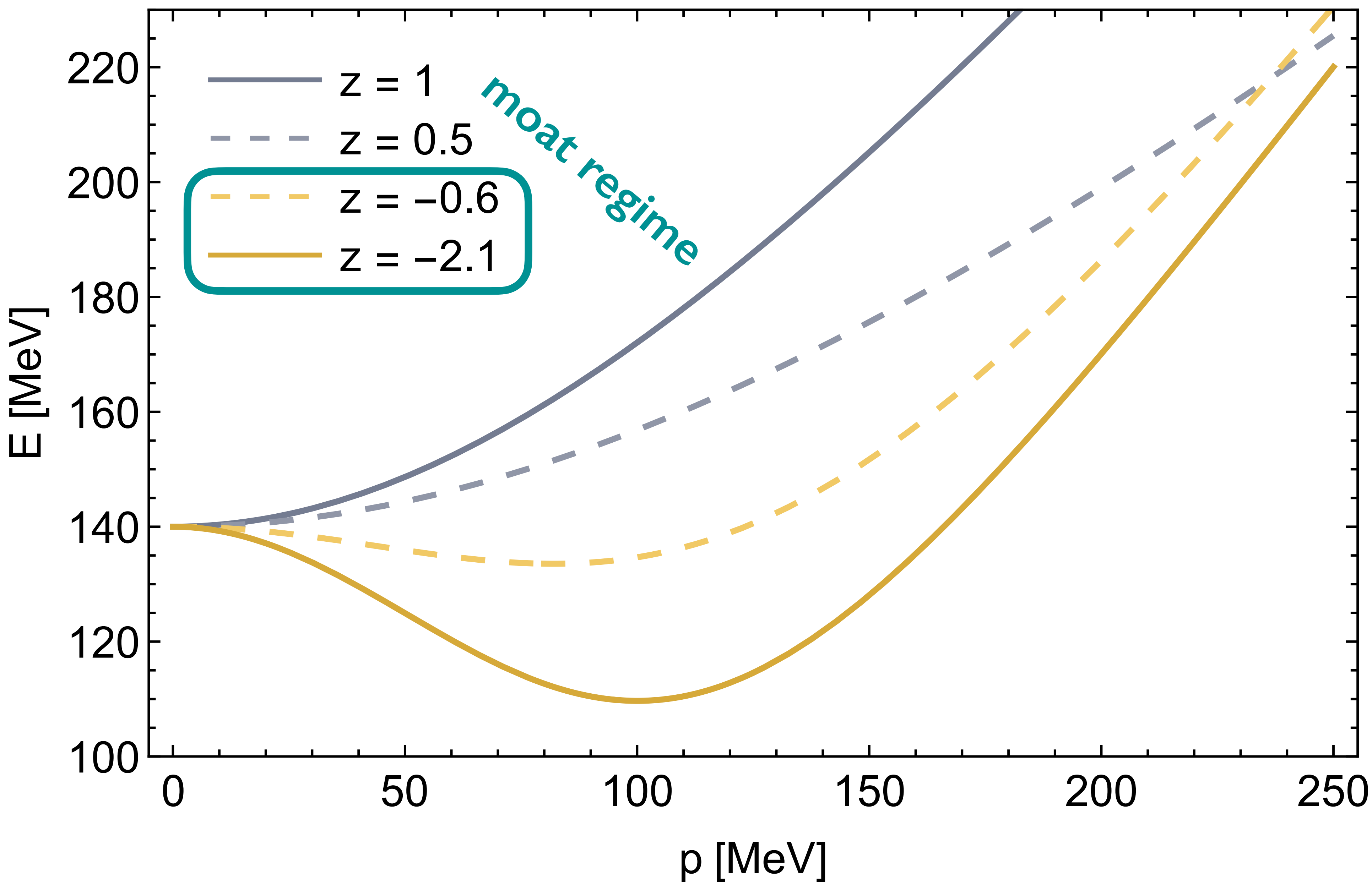


[Caerlaverock Castle, Scotland (source: Wikipedia)]

A MOAT

energy of particle ϕ:

$$E(\mathbf{p}^2) = \sqrt{Z(\mathbf{p^2})\,\mathbf{p^2} + m^2} = \sqrt{z\,\mathbf{p}^2 + w\mathbf{p}^4 + \mathcal{O}(\mathbf{p^6}) + m^2}$$



→ particles are favored to have nonzero momentum

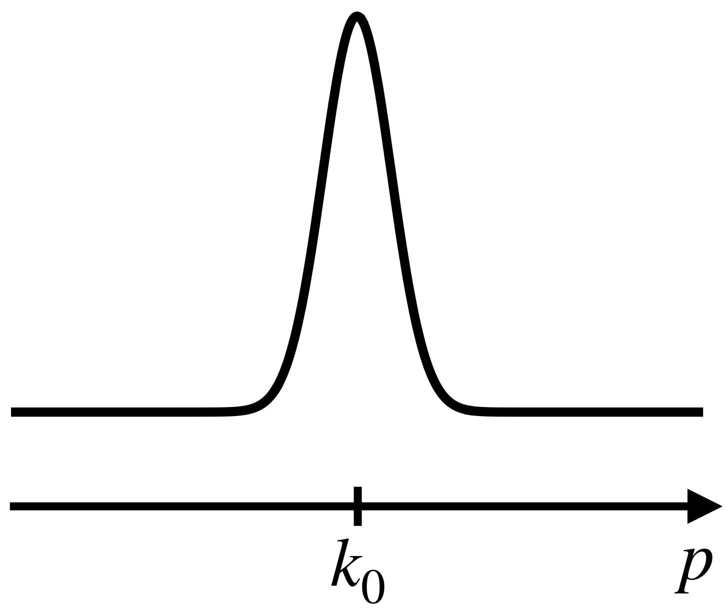
"gain energy by going faster"

WHAT DOES THE MOAT MEAN?

heuristic picture:

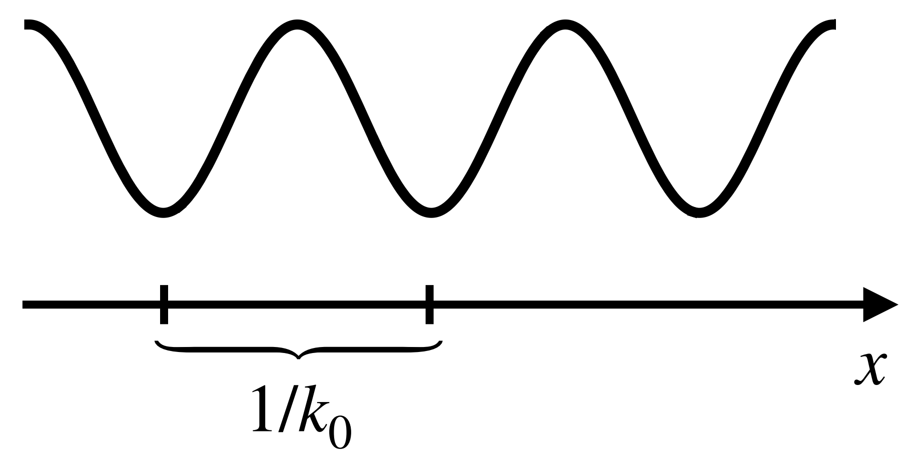
particle distribution for a deep moat:

$$n_B\big(E(\mathbf{p}^2)\big) \sim \delta(p - k_0)$$



spatial oscillation in position space

$$\mathrm{FT}\Big[n_B\big(E(\mathbf{p}^2)\big)\Big] \sim \sin(2\pi k_0 x)$$



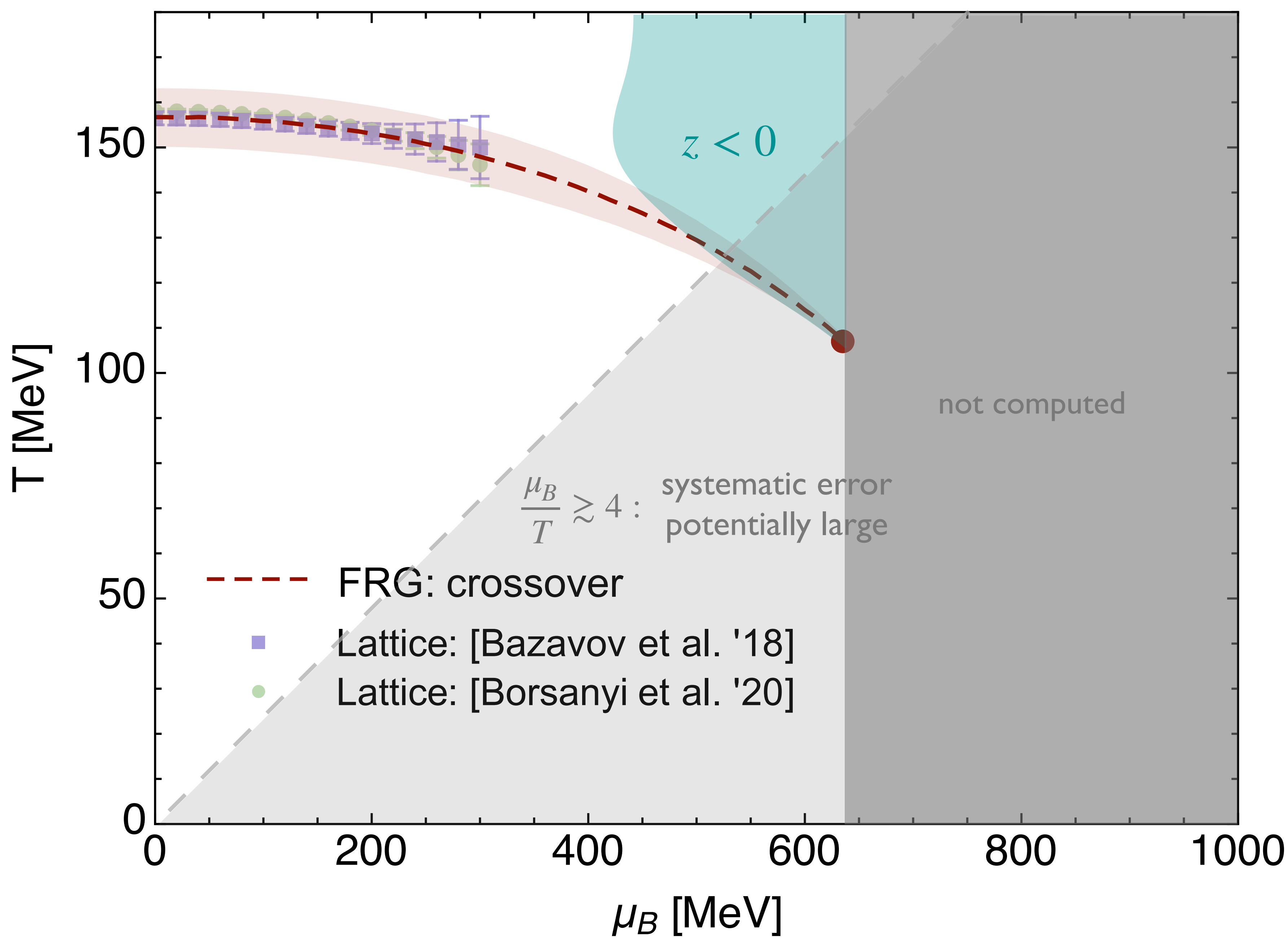
moat energy dispersion (minimal energy at k_0) $\longleftrightarrow$ spatial modulations (with wavenumber k_0)

- typical for inhomogeneous/crystalline phases or a quantum pion liquid (QπL)

WHERE CAN MOAT REGIMES APPEAR?

- some examples in low-energy models at large μ
- first indications also in QCD:

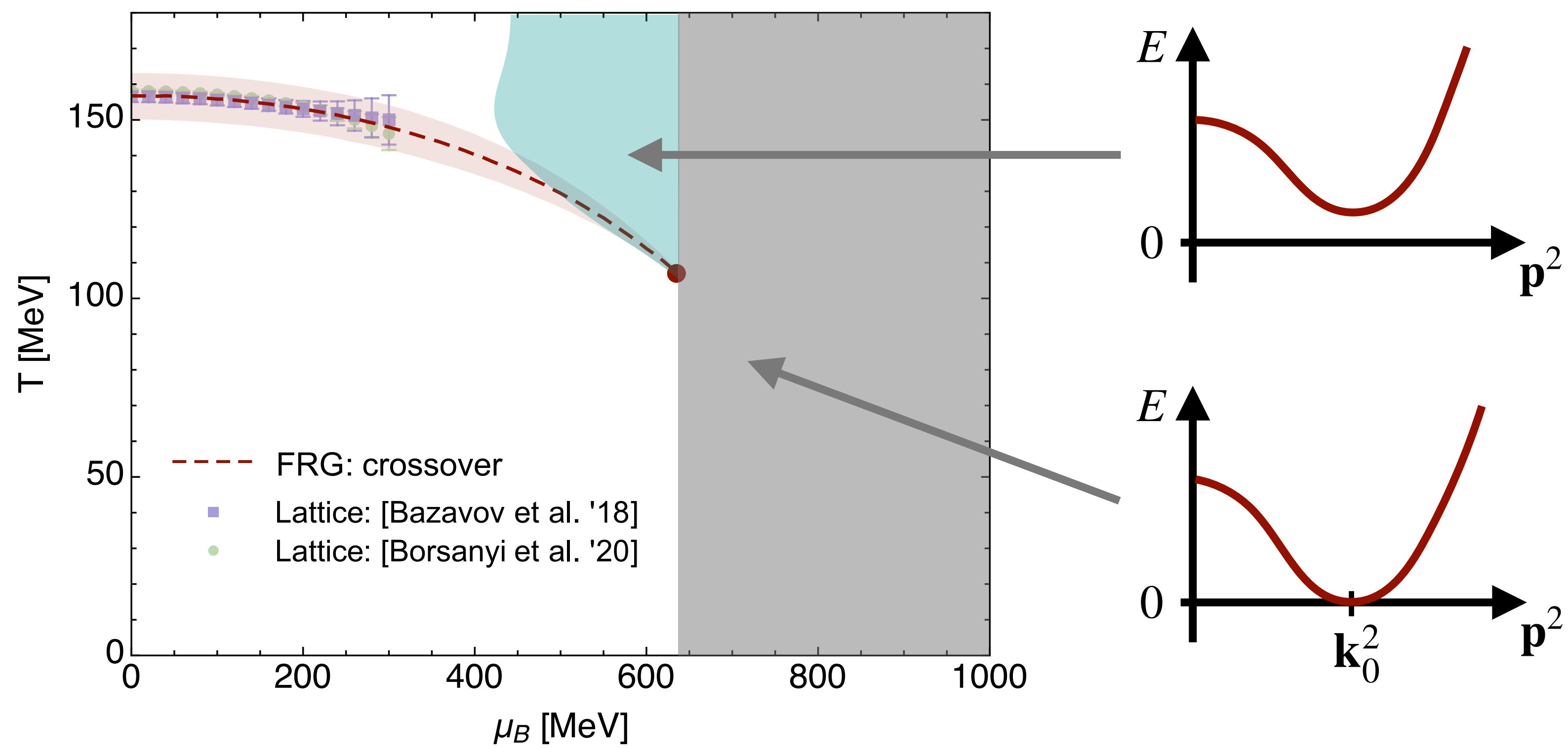
[Fu, Pawlowski, FR, PRD 101 (2020)]



→ indication for extended region with $z < 0$ in QCD: **moat regime**

IMPLICATIONS OF THE MOAT

The energy gap might close at lower T and larger μ_B :



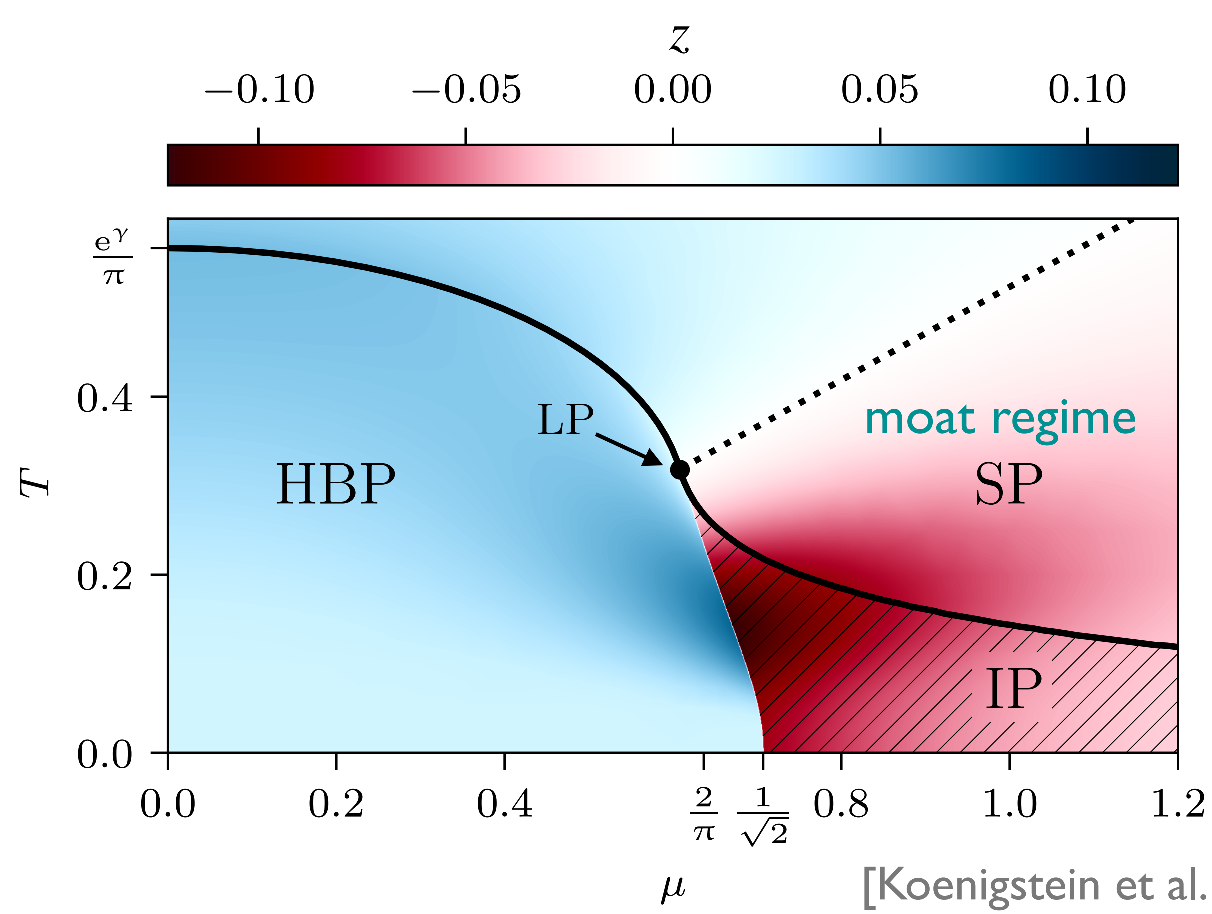
$E > 0$ for all $\mathbf{p}^2$

$E = 0$ at $\mathbf{p}^2 > 0$:

Zero energy cost to condense particles with nonzero momentum k_0

→ instability towards formation of an **inhomogeneous condensate**

- Example: Gross-Neveu Model in 1+1 dim. at large N_f



[Koenigstein et al. (2021)]

IMPLICATIONS OF THE MOAT

BUT: formation of inhomogeneous phases depends on dynamics of soft (massless) modes.

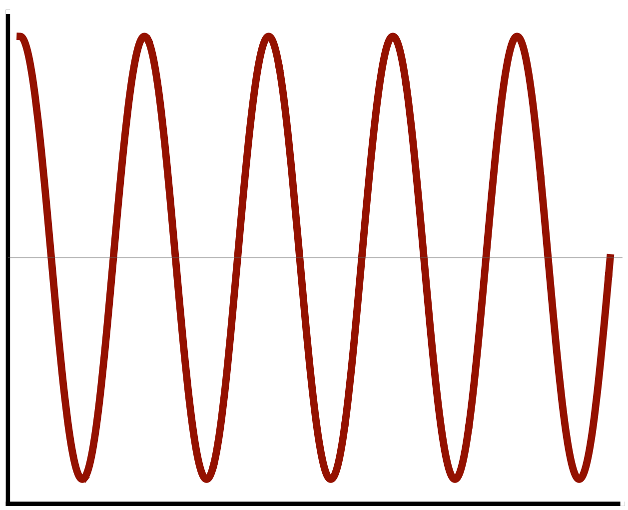
→ fluctuation-induced instabilities of inhomogeneous phases

→ other types of phases possible (possibly without long-range order!)

inhom. phase

no instability
(typical in mean-field)

$$\langle \phi(x)\phi(0)\rangle \sim \sin(k_0\, x)$$

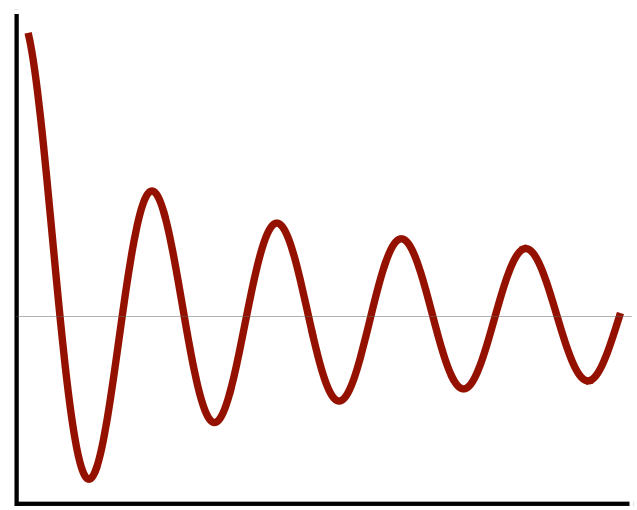


[Fukushima, Hatsuda, RPP 74 (2010)]
[Buballa, Carignano, PPNP 81 (2014)]

liquid crystal

Landau-Peierls instability
(Goldstones from spatial SB)

$$\langle \phi(x)\phi(0)\rangle \sim \sin(k_0 x)\, x^{-\alpha}$$

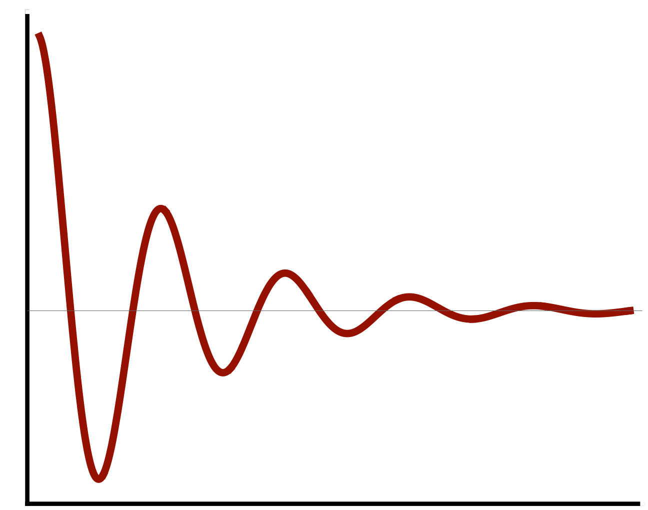


[Landau, Lifshitz, Stat. Phys. I, §137]
[Lee et al., PRD 92 (2015)]
[Hidaka et al., PRD 92 (2015)]

quantum pion liquid

PTV instability
(Goldstones from flavor SB)

$$\langle \phi(x)\phi(0)\rangle \sim \sin(k_0 x)\, e^{-mx}$$

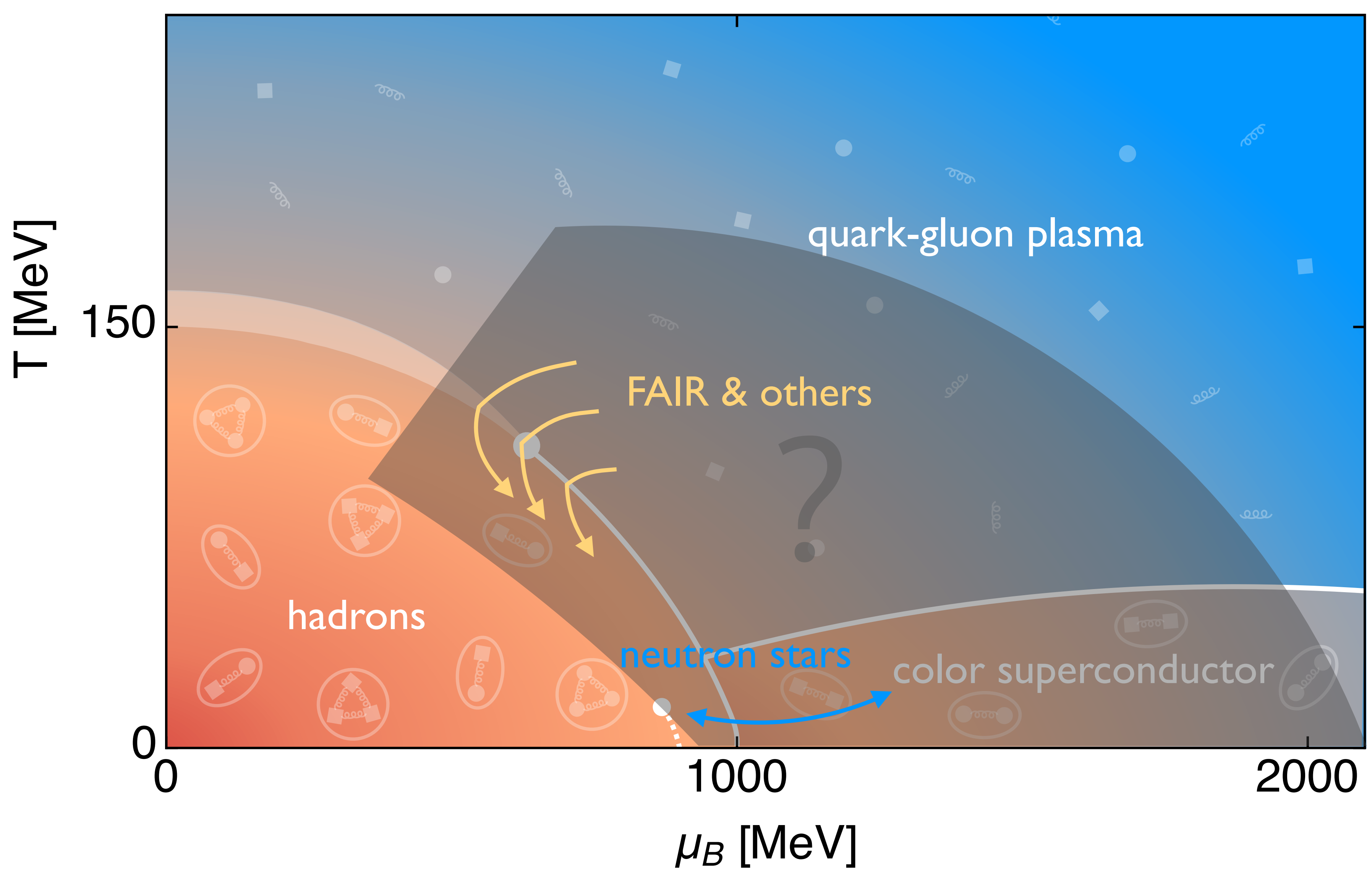


[Pisarski, Tsvelik, Valgushev, PRD 102 (2020)]
[Pisarski, PRD 103 (2021)]
[Schindler, Schindler, Ogilvie (2021)]

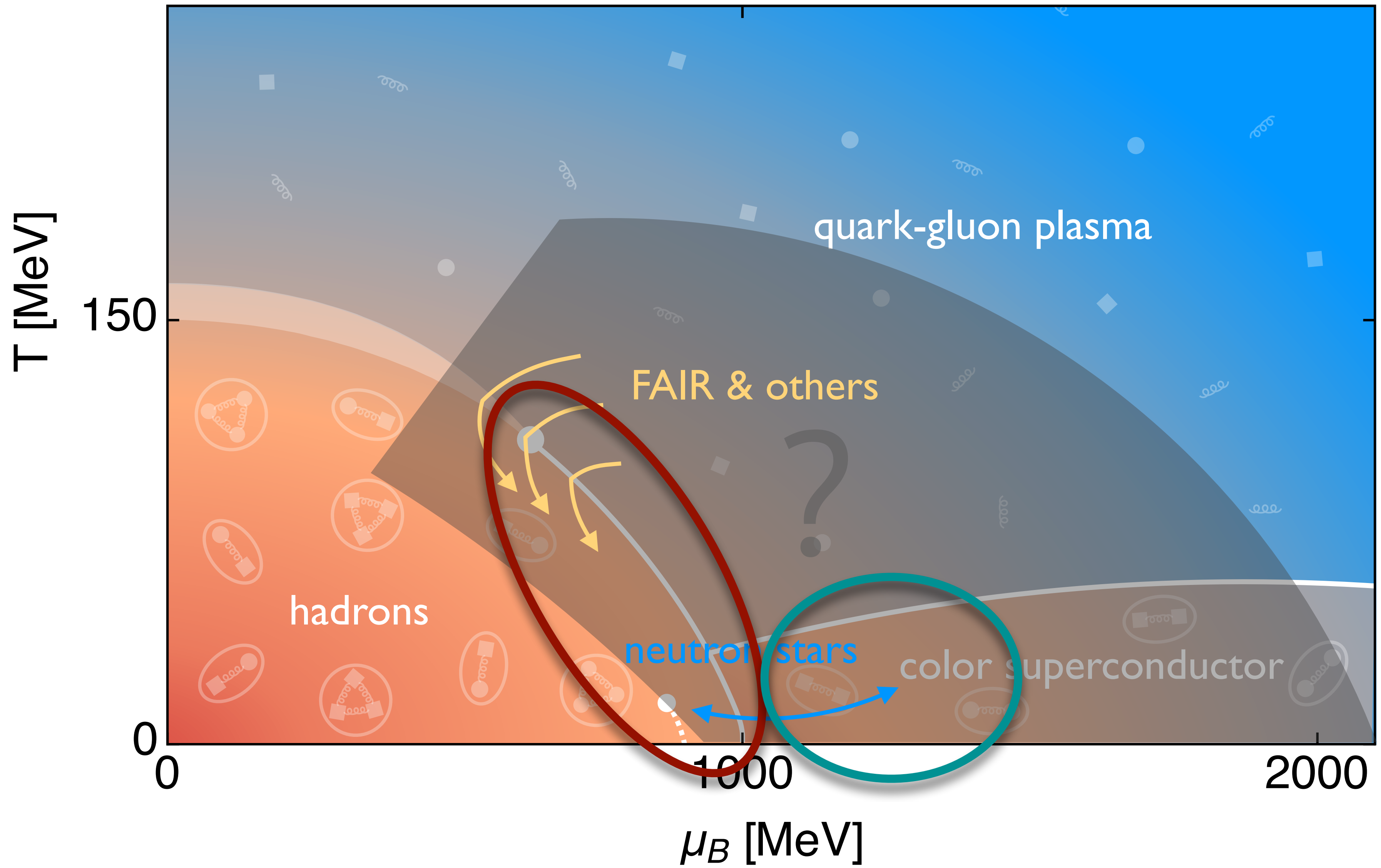
either way...

the moat is a common feature of regimes with spatial modulations

THE MOAT REGIME



THE MOAT REGIME



These phases are expected in the "unknown" region of the phase diagram

this is/will be covered by fixed target experiments

→ **search for moats in heavy-ion collisions!**

SEARCH FOR MOAT REGIMES

intuitive idea:

Characteristic feature of a moat regime: minimal energy at nonzero momentum

$\Rightarrow$ **enhanced particle production at nonzero momentum**

⟶ look for signatures in the **momentum dependence** of particle correlations

(first proposed in [Pisarski, FR, PRL 127 (2021)])

To do:

- develop new formalism to study particle correlations in moat regime
- consider two-particle correlations: **interference**

SPECTRA & INTERFERENCE

experiments count particles $\longrightarrow$ particle number correlations

- compute **particle spectra**, e.g., $$n_1(\mathbf{p}_\perp) = \omega_{\mathbf{p}_\perp} \left\langle \hat{N}_1 \right\rangle = \omega_{\mathbf{p}_\perp} \left\langle a^\dagger_{\mathbf{p}_\perp} a_{\mathbf{p}_\perp} \right\rangle$$ $$n_2(\mathbf{p}_\perp, \mathbf{q}_\perp) = \omega_{\mathbf{p}_\perp}\omega_{\mathbf{q}_\perp} \left\langle \hat{N}_1 \hat{N}_2 \right\rangle = \omega_{\mathbf{p}_\perp}\omega_{\mathbf{q}_\perp} \left\langle a^\dagger_{\mathbf{p}_\perp} a_{\mathbf{p}_\perp} a^\dagger_{\mathbf{q}_\perp} a_{\mathbf{q}_\perp} \right\rangle$$
- most elementary correlation: interference (follows from identical particles; no other fluctuations necessary)
- interference from two-particle scattering: encoded in n_2
- Gaussian approximation captures relevant effects:

$$\boxed{\begin{aligned} n_2(\mathbf{p}_\perp, \mathbf{q}_\perp) &\sim \langle a^\dagger_{\mathbf{p}_\perp} a_{\mathbf{p}_\perp}\rangle\langle a^\dagger_{\mathbf{q}_\perp} a_{\mathbf{q}_\perp}\rangle + \left|\langle a^\dagger_{\mathbf{p}_\perp} a_{\mathbf{q}_\perp}\rangle\right|^2 + \left|\langle a_{\mathbf{p}_\perp} a_{\mathbf{q}_\perp}\rangle\right|^2 \\ &= n_1(\mathbf{p}_\perp)\, n_1(\mathbf{q}_\perp) + \left|n_1(\mathbf{p}_\perp, \mathbf{q}_\perp)\right|^2 + \left|\bar{n}_1(\mathbf{p}_\perp, \mathbf{q}_\perp)\right|^2 \end{aligned}}$$

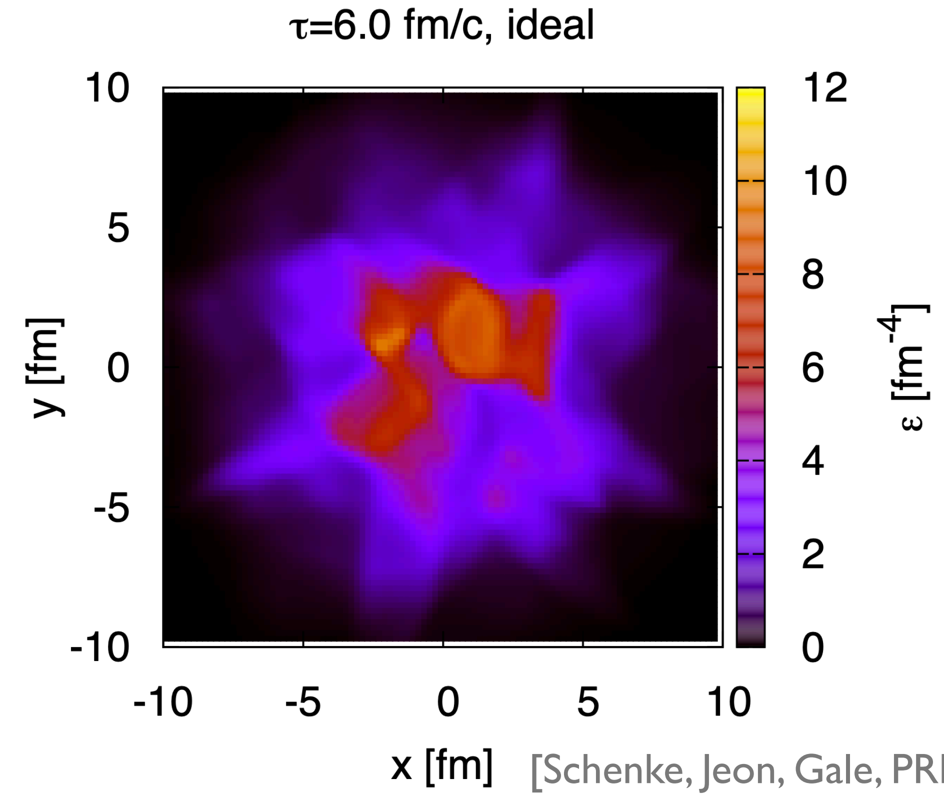
particle-particle interference
(Hanbury-Brown Twiss correlation)

particle-antiparticle interference
(negligible here)

$\longrightarrow$ study interference in a moat regime

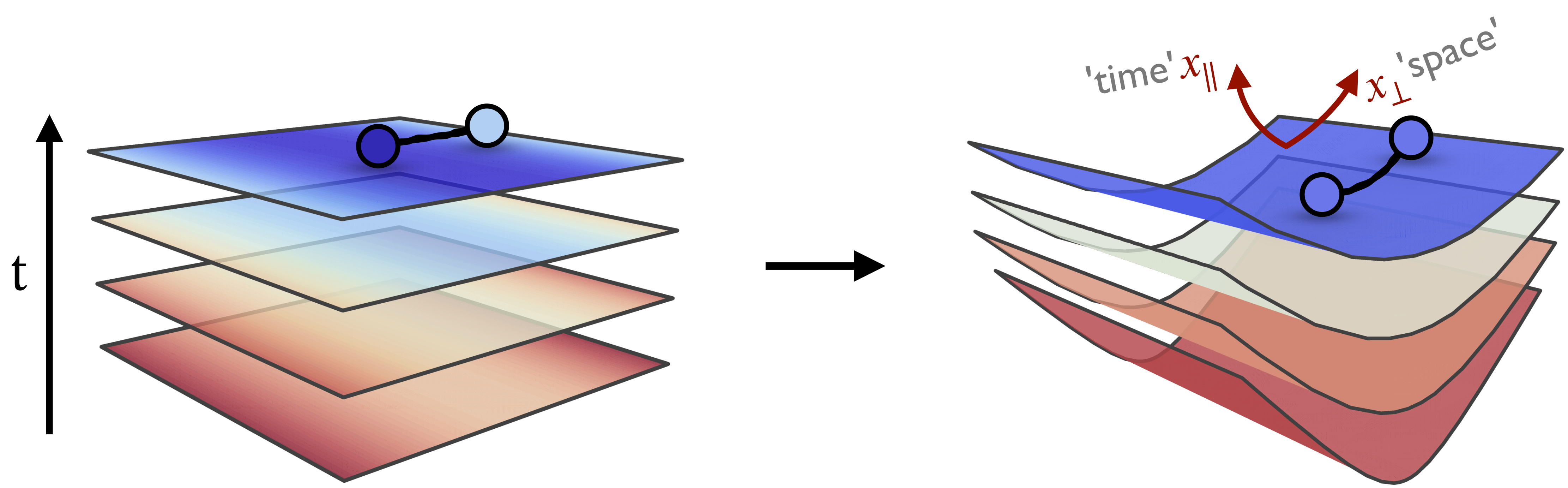
HYPERSURFACES IN HEAVY-ION COLLISIONS

- fixed thermodynamic conditions on 3d hypersurfaces $\Sigma \neq \mathbb{R}^3$
- freeze-out typically on fixed T (or ϵ) hypersurface



[Schenke, Jeon, Gale, PRL 106 (2010)]

instead of correlations on $\mathbb{R}^3$ consider appropriate **foliation** of spacetime



INTERFERENCE ON A HYPERSURFACE

[FR, Pisarski, Rischke, PRD 107 (2023)]

- use ladder operators of in-medium state on curved hypersurface Σ

$$a_{\mathbf{p}_\perp} = i \int d\Sigma^\mu \, e^{i\bar{p}\cdot x} \frac{1}{\sqrt{2\omega_{\mathbf{p}_\perp}}} \left(\partial_\mu - i\bar{p}_\mu\right) \phi(x)$$

$d\Sigma^\mu = \sqrt{|\det G|}\, d^3w\, \hat{v}^\mu$

on-shell momentum $\bar{p}_\parallel = \omega_{\mathbf{p}_\perp}$

- energy of an on-shell particle:

$$\omega_{\mathbf{p}_\perp} = \sqrt{Z(\mathbf{p}_\perp^2)\,\mathbf{p}_\perp^2 + m^2}$$

→ express n-particle spectra in terms of real-time correlations of $2n$ fields

Interference in local thermal equilibrium (fluctuation-dissipation relation + sufficiently isotropic system)

average and relative pair momentum

single-particle distribution, e.g., Bose-Einstein

$$\boxed{n_1(\mathbf{P}, \mathbf{\Delta P}) = \frac{1}{2} \int d\Sigma_X \, e^{-i\overline{\Delta P}\cdot X} \int \frac{dP_\parallel}{2\pi} \left[\left(P_\parallel + \bar{P}_\parallel\right)^2 - \frac{1}{4} \overline{\Delta P}_\parallel^2 \right] f\left(X; P_\parallel, \mathbf{P}_\perp\right) \rho\left(X; P_\parallel, \mathbf{P}_\perp\right)}$$

average position

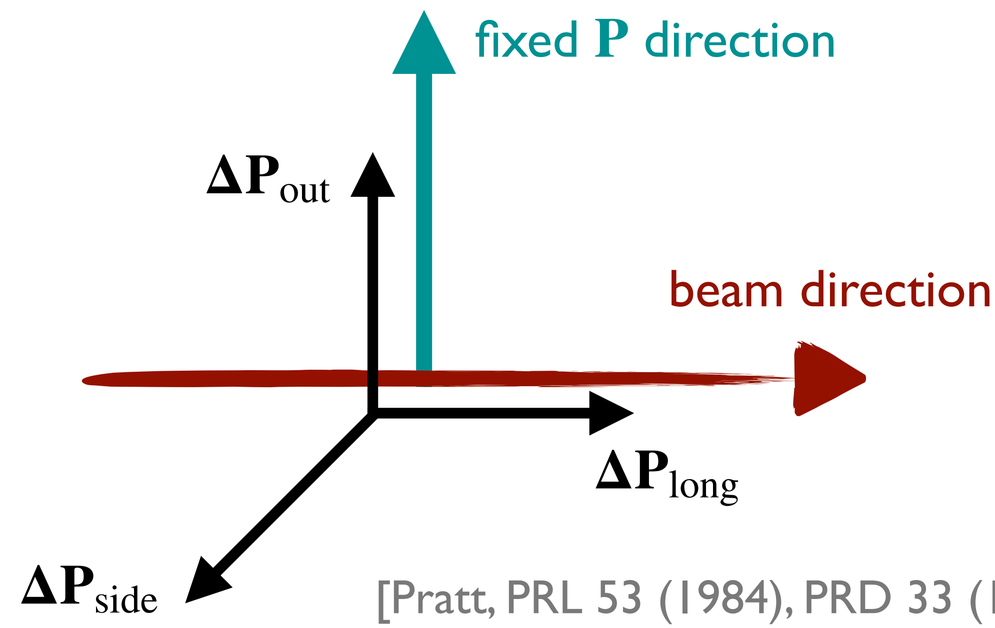
→ in-medium effects enter through P-dependence of the spectral function $\rho(x,y) = \langle [\phi(x), \phi(y)] \rangle$

- not most general expression: involves statistical function and gradients in X
- single particle spectrum for $p = q$ (cf. also [D. Anchishkin, J. Phys. G (2022)])

INTERFEROMETRY

Compute two-particle spectrum in illustrative model

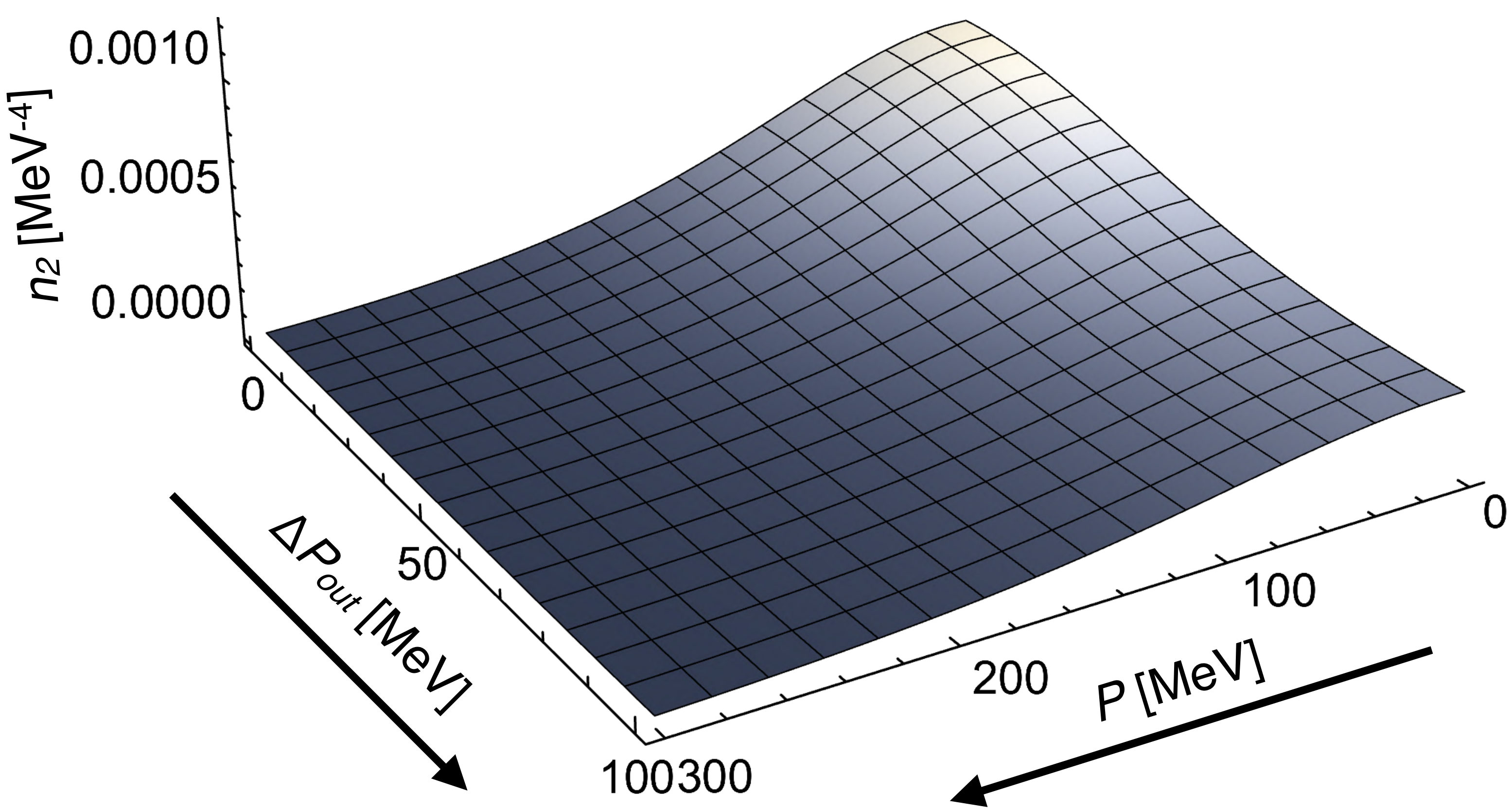
- moat quasi-particle with $k_0 = 100\,\mathrm{MeV}$
- hypersurface at fixed proper time



[Pratt, PRL 53 (1984), PRD 33 (1986)]

Remember: in-medium effects in P-dependence!

normal phase: $\omega_{\mathbf{P}_\perp} = \sqrt{\mathbf{P}^2 + m^2}$



→ correlation peaks at $|\mathbf{P}| = 0$

(side- and long-correlations qualitatively the same)

INTERFEROMETRY

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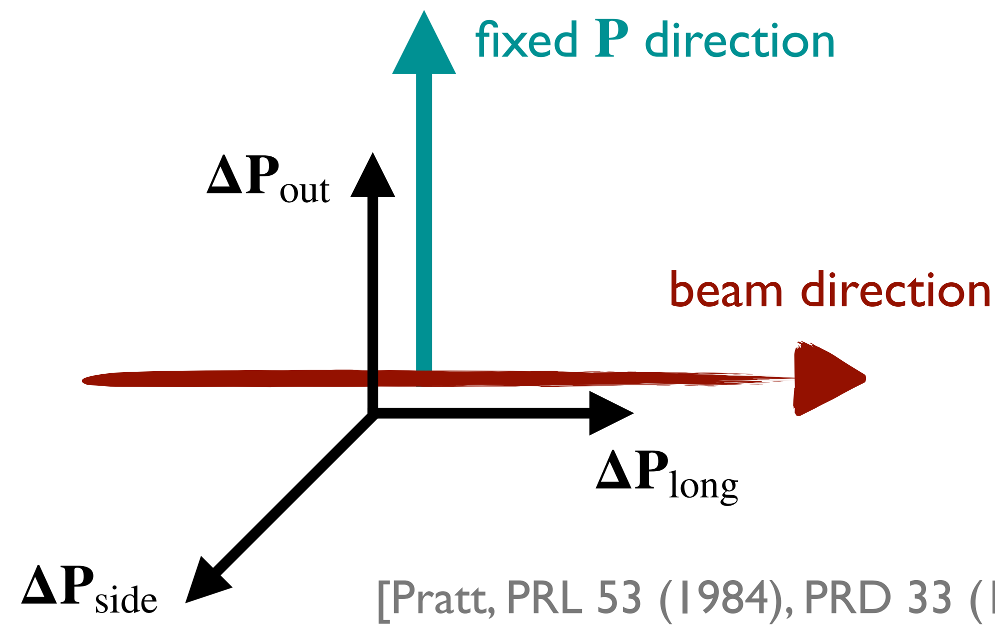
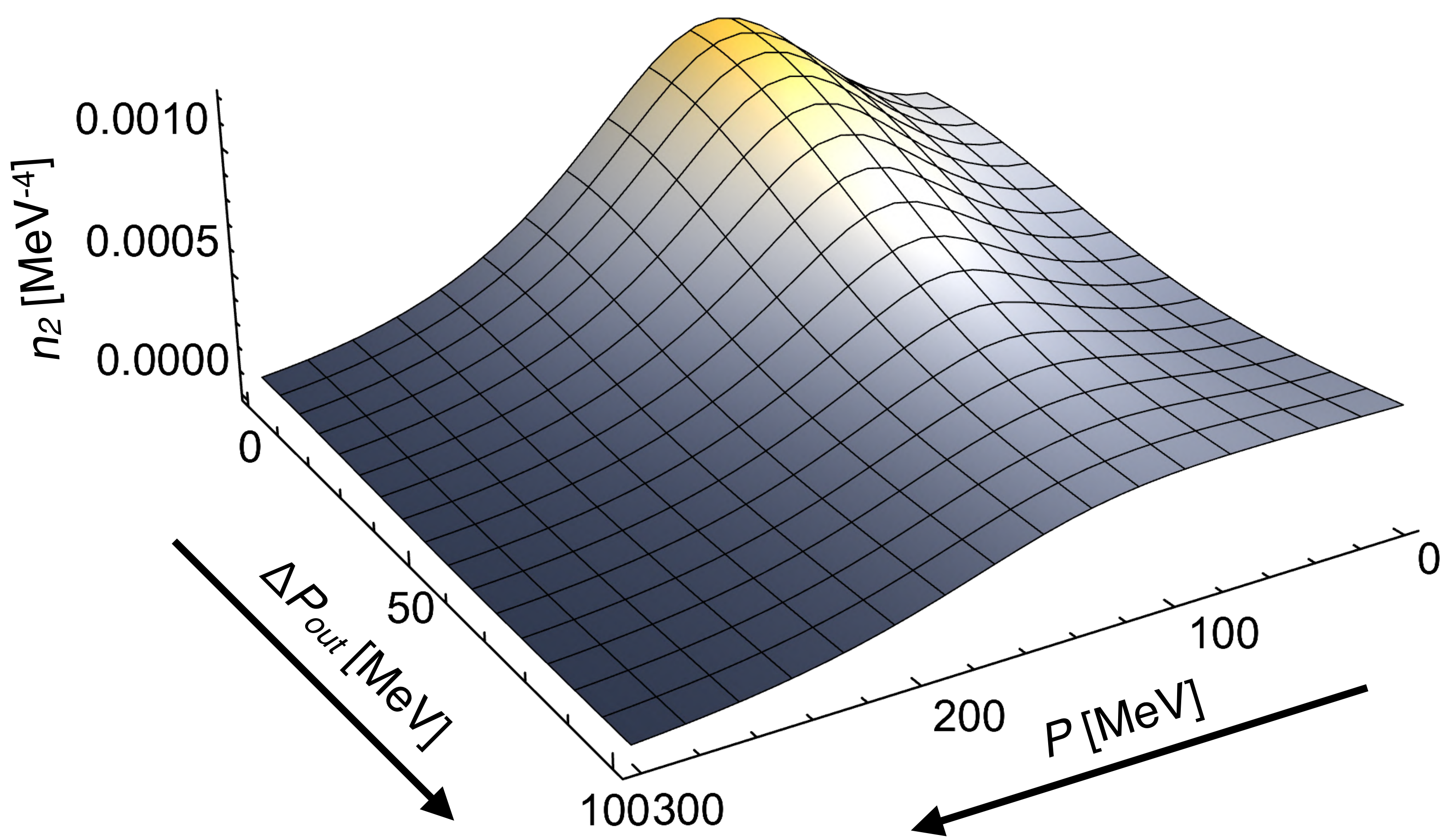


[Pratt, PRL 53 (1984), PRD 33 (1986)]

Remember: in-medium effects in P-dependence!

moat regime: $\omega_{\mathbf{P}_\perp} \sim \sqrt{z\,\mathbf{P}^2 + w\,\mathbf{P}^4 + m^2}\,, \quad z < 0$



→ correlation peaks at $|\mathbf{P}| = k_0 > 0$

(related to the wave number of underlying spatial modulation)

signature of a moat regime

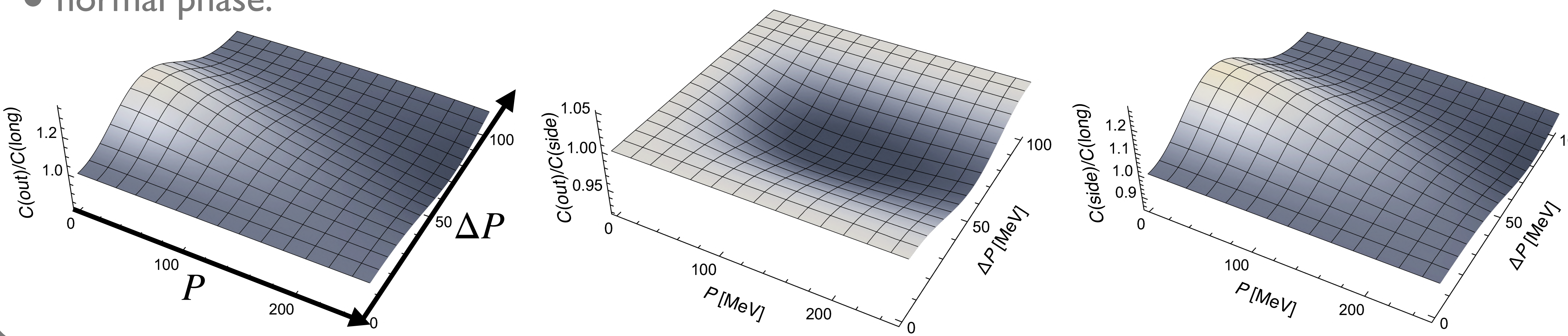
(side- and long-correlations qualitatively the same)

NORMALIZED TWO-PARTICLE CORRELATION

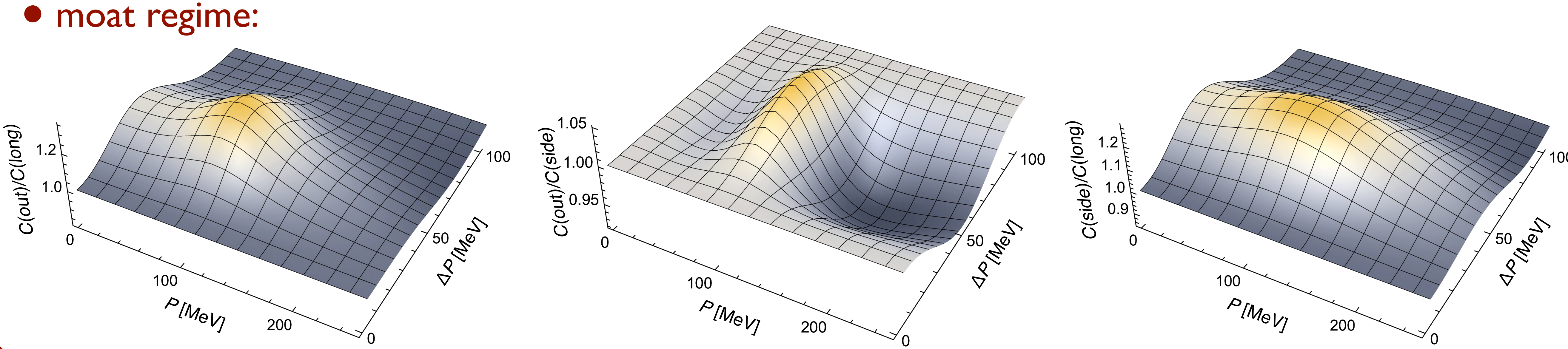
Usually measured in experiments: $$C(\mathbf{P},\mathbf{\Delta P}) = \frac{n_2(\mathbf{P},\mathbf{\Delta P})}{n_1\left(\mathbf{P}+\frac{1}{2}\mathbf{\Delta P}\right) n_1\left(\mathbf{P}-\frac{1}{2}\mathbf{\Delta P}\right)}$$

We propose to look at **ratios**: $C_{\rm out}/C_{\rm long}$, $C_{\rm out}/C_{\rm side}$ and $C_{\rm side}/C_{\rm long}$

- normal phase:



- moat regime:



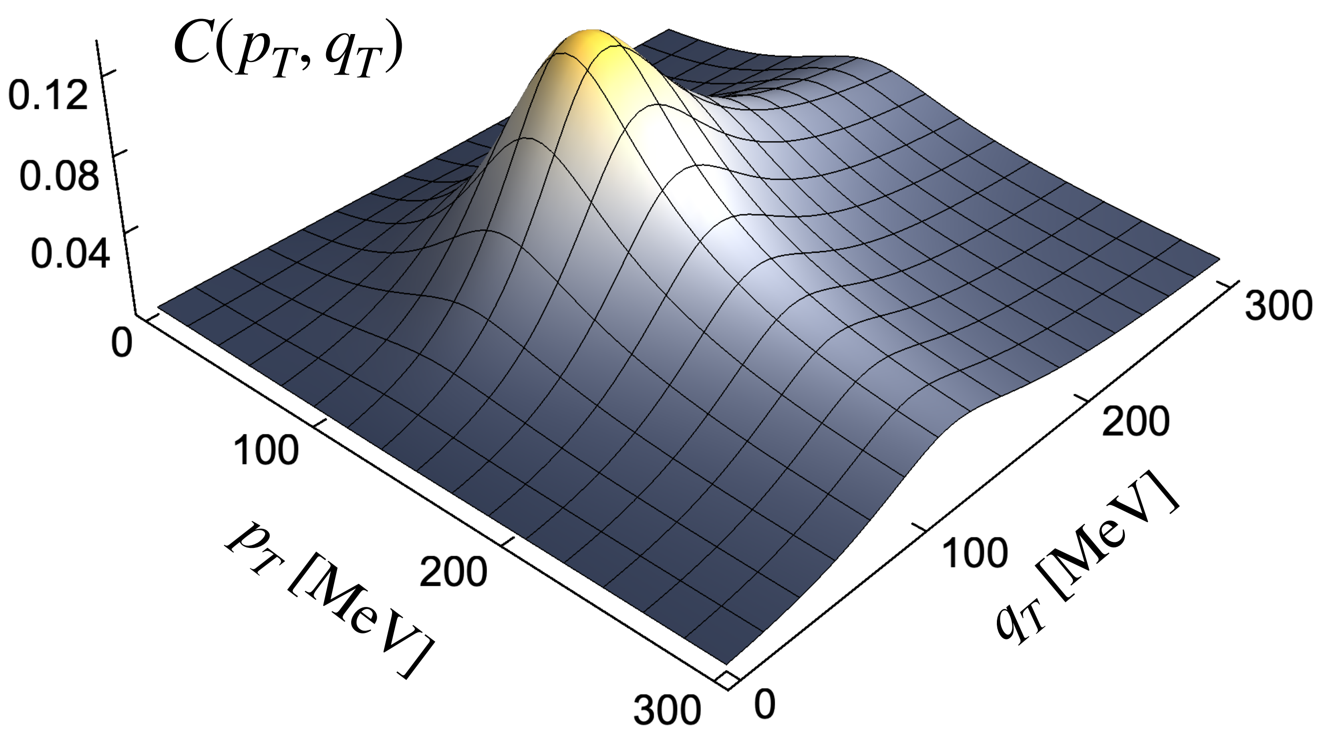
MORE SIGNALS FROM THE MOAT REGIME

Here: 2-particle correlation from identical particle interference in a moat regime

But qualitative result appears to be generic

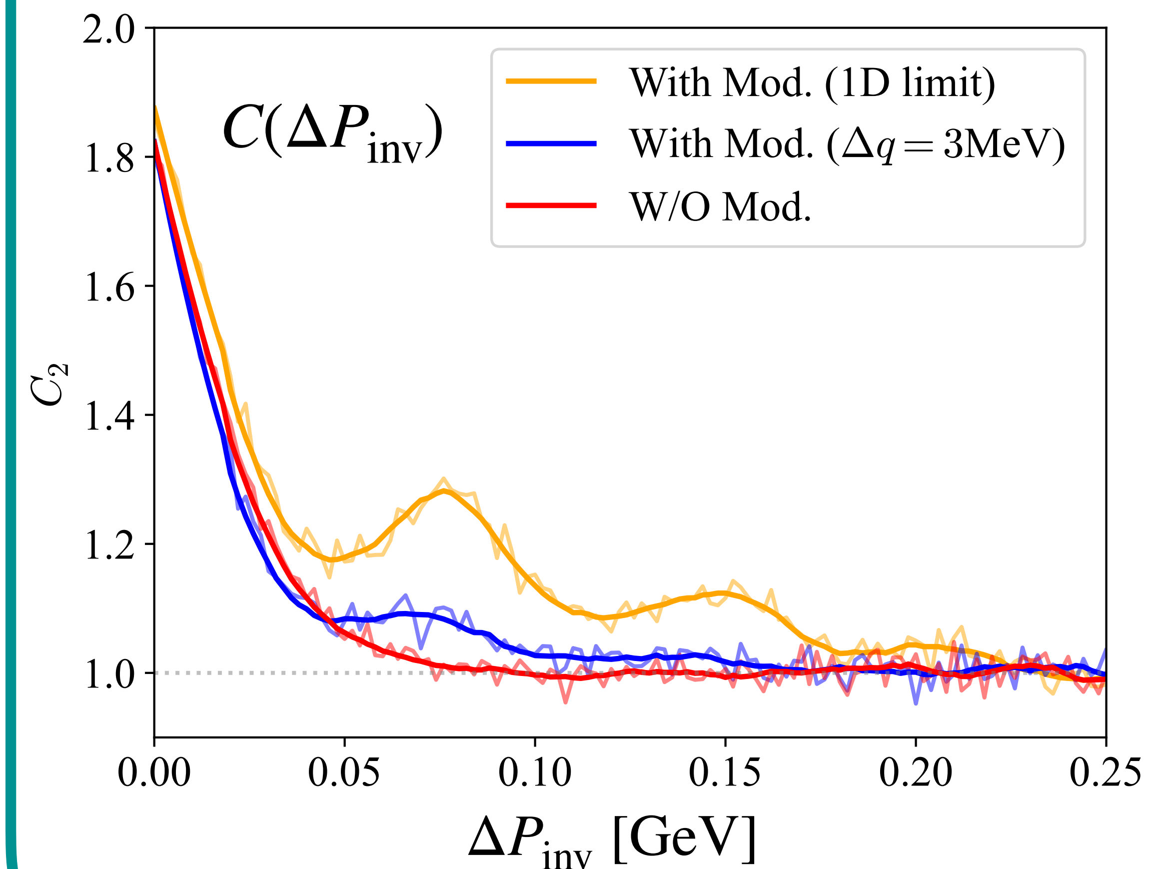
thermodynamic fluctuations in a moat regime

[Pisarski, FR, PRL 127 (2021)]



interference from primordial inhomogeneity (with AMPT transport)

[Fukushima et al., arXiv:2306.17619 (2023)]



→ **peak position related to wavenumber of modulation**

SUMMARY

Moats arise in regimes with spatial modulations

- expected to occur at $\mu_B \gtrsim 400$ MeV
- precursors for inhomogeneous-, liquid-crystal-like or quantum pion liquid phases

Signatures of a moat regime in particle interferometry

- characteristic peaks at nonzero momentum
- propose to measure ratios of normalized correlations to detect a moat regime
- in fixed-target HIC range!

Opportunity to discover novel phases with heavy-ion collisions through measurement of particle correlations

- So far: basic description of qualitative effects at intermediate stage of collision
- To do: quantitative description of moat regimes & propagation of signal to the detector

BACKUP

A HYPERSURFACE

- hypersurface Σ defined through parametric equations:

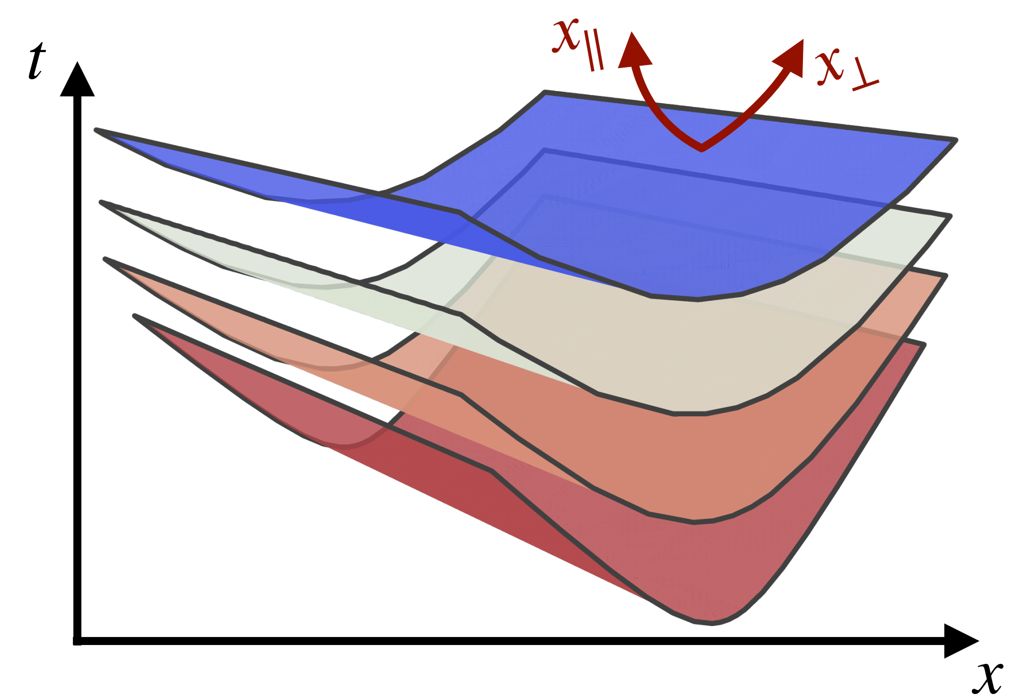
$$x^\mu = x^\mu(w^i)$$

coordinates of ambient spacetime

intrinsic coordinates of Σ $(i = 1, 2, 3)$
e.g., angles φ, ϑ on a 3-sphere

- define tangent and normal vectors of Σ:

$$e_i^\mu = \frac{\partial x^\mu}{\partial w^i}\,, \qquad \hat{v}^\mu \sim \bar{\epsilon}^{\mu\alpha\beta\gamma}\, e_{1\alpha} e_{2\beta} e_{3\gamma}$$



- decompose spacetime metric as

$$g^{\mu\nu} = \hat{v}^\mu \hat{v}^\nu - G^{ij} e_i^\mu e_j^\nu$$

induced metric on Σ: $G_{ij} = -\, g_{\mu\nu}\, e_i^\mu e_j^\nu$

- define '**time**' and '**space**' : $x_\parallel = \hat{v}^\mu x_\mu$ and $\mathbf{x}_\perp = \mathbf{e}^\mu x_\mu$

→ **foliation** of spacetime: $\{x_\parallel\} \times \Sigma$ instead of $\{t\} \times \mathbb{R}^3$

INTERFERENCE IN FULL GLORY

- introduce average and relative coordinates

$$X = \frac{1}{2}(x+y)\,, \qquad \Delta X = x - y$$
$$P = \frac{1}{2}(p+q)\,, \qquad \Delta P = p - q$$

- spectral and statistical function as **Wigner transformed** two-point functions

$$\rho(X,P) = \int d\Delta X_{\parallel} \int d\Sigma_{\Delta X}\, e^{iP\cdot\Delta X} \left\langle \left[\phi\left(X + \frac{1}{2}\Delta X\right), \phi\left(X - \frac{1}{2}\Delta X\right) \right] \right\rangle$$
$$F(X,P) = \frac{1}{2}\int d\Delta X_{\parallel} \int d\Sigma_{\Delta X}\, e^{iP\cdot\Delta X} \left\langle \left\{ \phi\left(X + \frac{1}{2}\Delta X\right), \phi\left(X - \frac{1}{2}\Delta X\right) \right\} \right\rangle$$

The particle-particle interference term then is general:

$$n_1(\mathbf{p}_\perp, \mathbf{q}_\perp) = \frac{1}{2}\int d\Sigma_X\, e^{-i\overline{\Delta P}\cdot X} \int \frac{dP_{\parallel}}{2\pi} \left[\frac{1}{4}\partial^2_{X_\parallel} + \frac{i}{2}\overline{\Delta P}_{\parallel}\partial_{X_\parallel} + \left(P_\parallel + \bar{P}_\parallel\right)^2 - \frac{1}{4}\overline{\Delta P}^2_{\parallel} \right] \left[F(X,P) - \frac{1}{2}\rho(X,P) \right]$$

AN ILLUSTRATIVE MODEL I

highlight qualitative effects

Particle in a moat regime:

- bosonic quasi-particle:

$$\rho(P) = 2\,\mathrm{Im}D_R(P) = \frac{\pi}{\omega_{\mathbf{P}_\perp}}\left[\delta(P_\parallel - \omega_{\mathbf{P}_\perp}) - \delta(P_\parallel + \omega_{\mathbf{P}_\perp})\right] \quad \text{with} \quad \omega_{\mathbf{P}_\perp} = \sqrt{Z(\mathbf{P}_\perp^2)\,\mathbf{P}_\perp^2 + m^2}$$

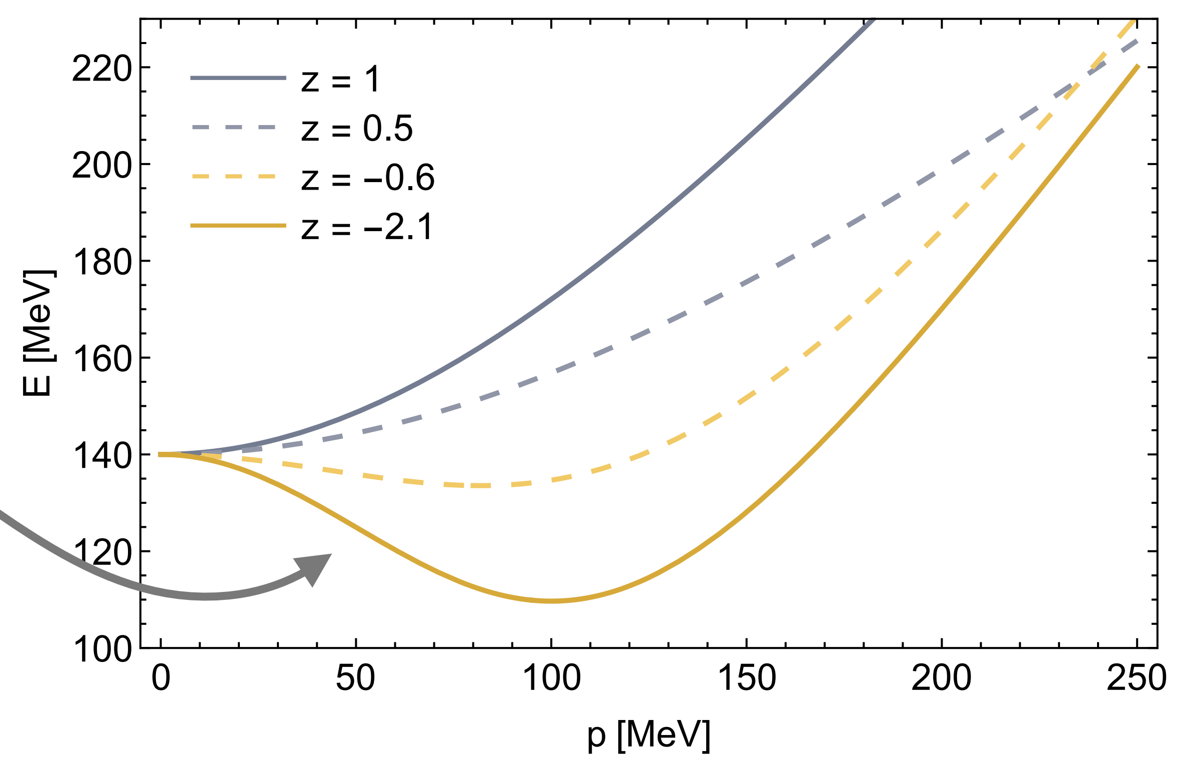
$\longrightarrow$ puts the average pair momentum on-shell

- single-particle distribution: $f\left(X; P_\parallel, \mathbf{P}_\perp\right) = n_B\left(P_\parallel\right) = \dfrac{1}{e^{P_\parallel/T} - 1}$

Wave function renormalization:

- moat spectrum, but well-defined large momentum limit (free relativistic dispersion at large $\mathbf{p}^2$)

$$Z(\mathbf{P}^2) = 1 - \frac{\lambda^2}{\mathbf{P}^2 + M^2}$$
$$\approx \underbrace{1 - \frac{\lambda^2}{M^2}}_{\mathbf{p}^2\text{-coefficient } z \text{ in dispersion}} + \frac{\lambda^2}{M^4}\mathbf{P}^2 + \mathcal{O}(\mathbf{P}^4)$$



AN ILLUSTRATIVE MODEL 2

highlight qualitative effects

Parameters:

- interferometry measurements typically use pions: $m = m_\pi = 140\,\mathrm{MeV}$
- pions show indications for a moat dispersion in QCD for $\mu_B \gtrsim 450\,\mathrm{MeV}$

[Fu, Pawlowski, FR, PRD 101 (2020)]

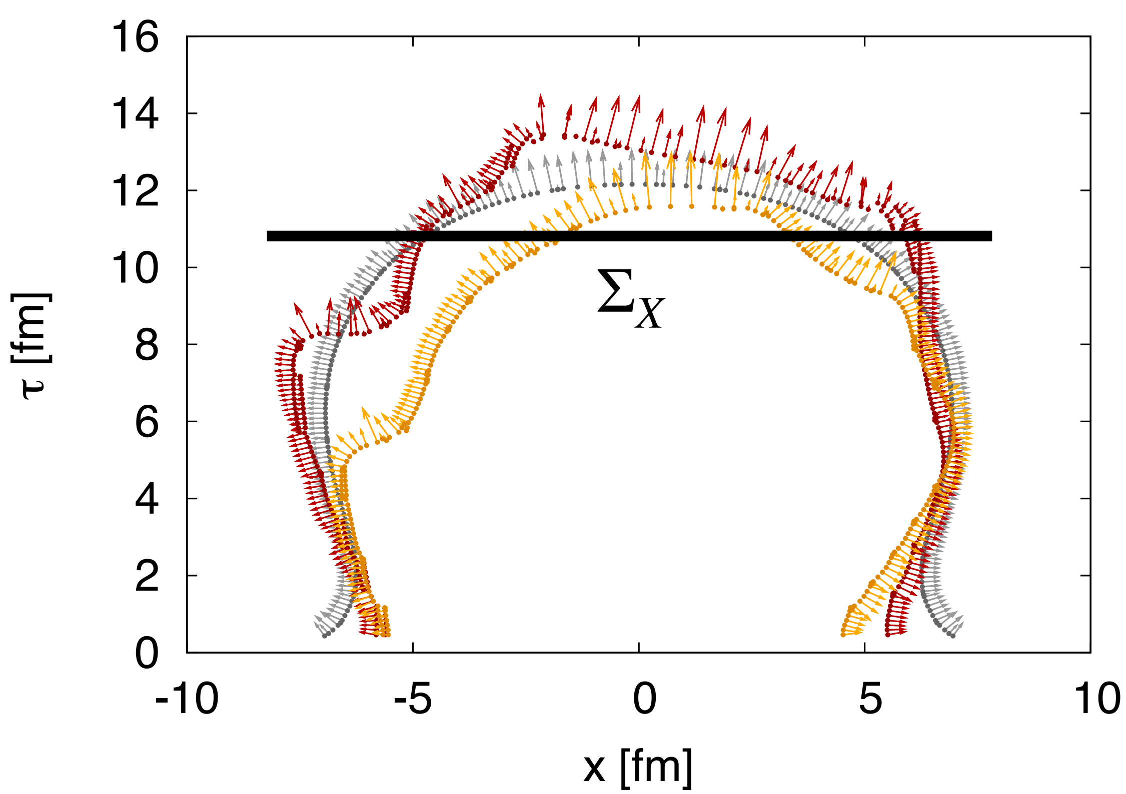
- choose wavenumber (min. of the energy) $\mathcal{O}(m_\pi)$: $|\mathbf{P}_{\mathrm{min}}| = 100\,\mathrm{MeV}$

Hypersurface:

beam direction

- fixed T hypersurfaces in high-energy HICs approx. at fixed proper time $\tau = \sqrt{X_0^2 - X_3^2}$

$\longrightarrow$ very successful in describing transverse momentum spectra



fixes temporal and spatial coordinates on Σ_X

$$X_\parallel = \tau\,, \qquad \mathbf{X}_\perp = \begin{pmatrix} 0 \\ -r \\ 0 \end{pmatrix}$$

and the induced metric

$r = \sqrt{X_1^2 + X_2^2}$

$$G^{ij} = \begin{pmatrix} \tau^{-2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & r^{-2} \end{pmatrix}$$

HANBURY-BROWN TWISS RADII

Original idea: use intensity interferometry to measure size of astronomical objects

original experiment in Narrabri, Australia

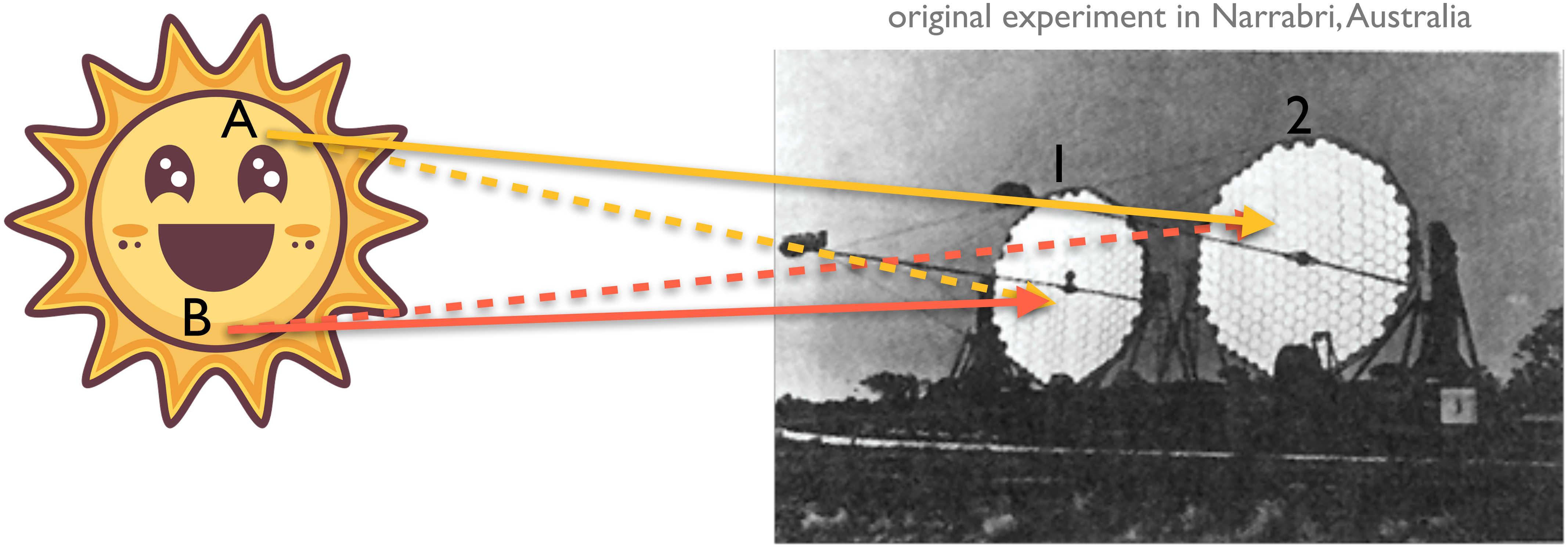


Figure 2. Picture of the two telescopes used in the HBT experiments. The figure was extracted from Ref.[1]. [Goldhaber (1991)]

- interference term (approximately) the Fourier trafo of the **emission function** $S(x, \mathbf{P})$

$$n_1(\mathbf{P}, \Delta\mathbf{P}) \approx \int d^4x \, e^{-i\overline{\Delta P}\cdot x} \, S(x, \mathbf{P})$$

- emission function: distribution of spacetime position x and momentum $\mathbf{P}$ of particles

→ range of correlation in $\Delta\mathbf{P}$ related to inverse size of the source

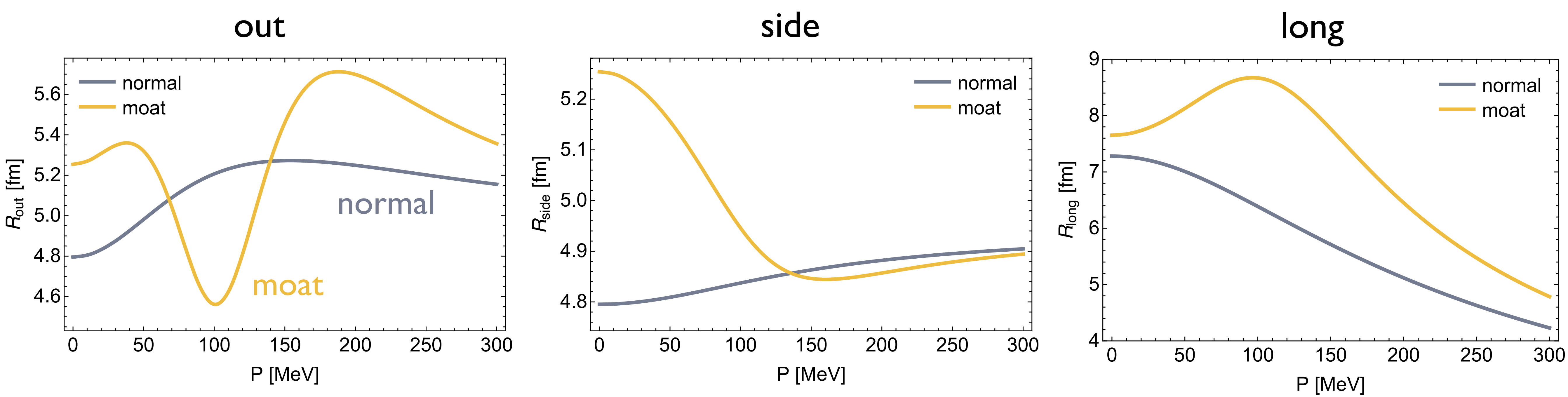
HBT RADII IN A MOAT REGIME

- define HBT radius R through range of correlation in $\mathbf{\Delta P}$

correlation is max. at $\mathbf{\Delta P} = \mathbf{0}$

$$R = \frac{1}{|\mathbf{\Delta P}^*|}\ , \ \text{with} \ \ C(\mathbf{P}, \mathbf{\Delta P}^*) = \frac{1}{2}\, C(\mathbf{P}, \mathbf{0})$$

- yields $R(|\mathbf{P}|)$:



→ **HBT radii modified in moat regime**

THERMODYNAMIC FLUCTUATIONS

n-particle correlation:

$$\left\langle \prod_{i=1}^{n} n_1(\mathbf{p}_i) \right\rangle \sim \left[\prod_{i=1}^{n} \int d\Sigma_i^\mu \int \frac{dp_i^0}{2\pi} (p_i)_\mu \Theta(\breve{p}_i^0) \right] \left\langle \prod_{i=1}^{n} f(\breve{p}_i)\, \rho(x, \breve{p}_i) \right\rangle$$

thermodynamic average

[Pisarski, FR, PRL 127 (2021)]

- fluctuations, e.g., of thermodynamic quantities lead to fluctuations of F_ϕ
- consider small fluctuations in T, μ_B, u
- normalized two-particle correlation (without interference):