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# Fluid dynamics of charm quarks in the quark-gluon plasma

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Based on  
Capellino, Dubla, Floerchinger, Grossi, Kirchner, Masciocchi, e-Print: [2307.15580](https://arxiv.org/abs/2307.15580) [hep-ph] (2023)  
Capellino, Beraudo, Dubla, Floerchinger, Pawlowski, Masciocchi, Selyuzhenkov PRD 106 (2022) 3, 034021



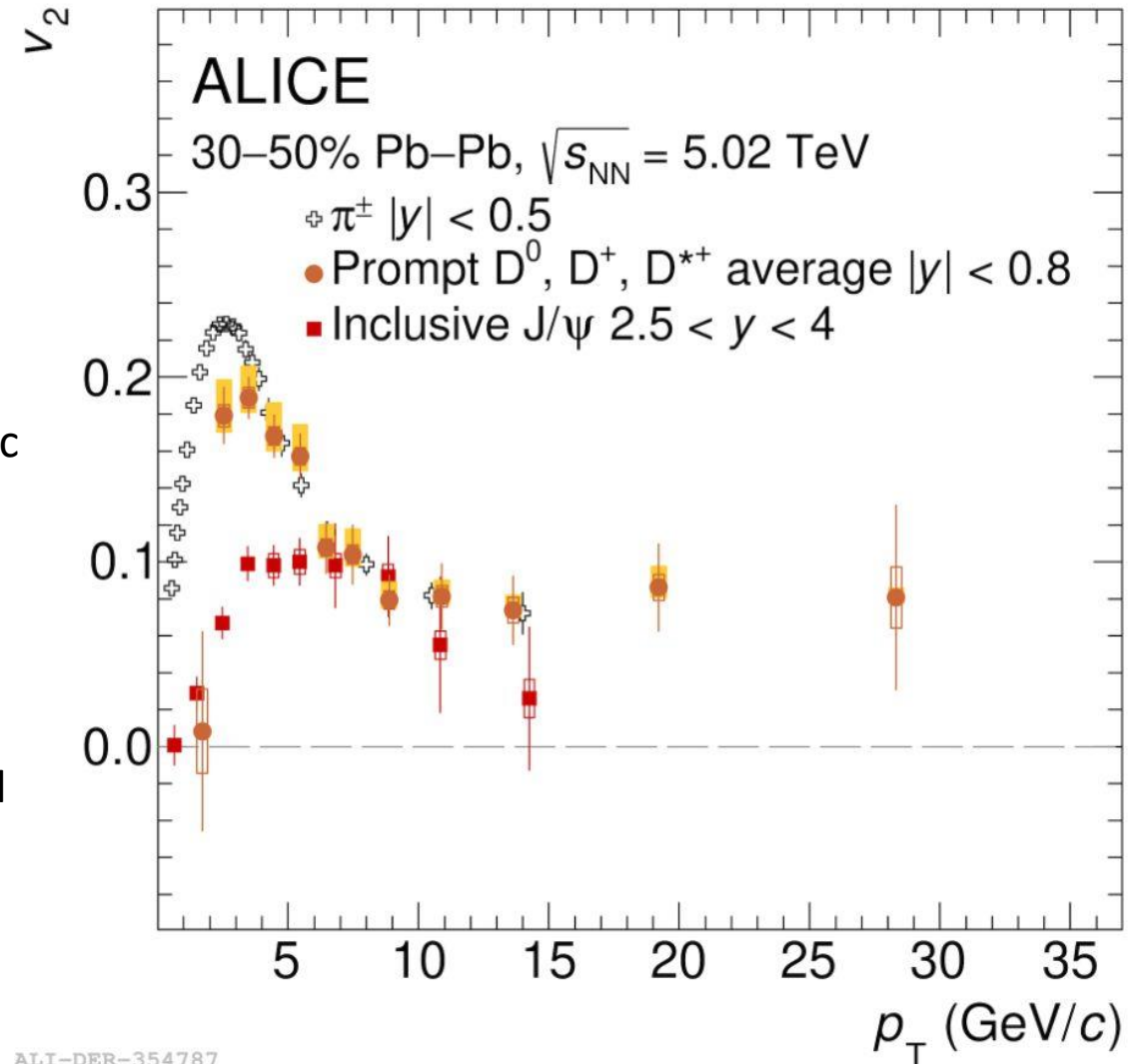
# Heavy quarks as probes of the QGP

ALICE PLB 813 (2021) 136054

- Produced via hard scatterings at the beginning of the collision before the QGP is formed: **they go through all the stages of the expanding fireball**
- In the low  $p_T$  region they provide a window to study **equilibration processes**

Significant measurements of  $J/\psi$  and D mesons of positive elliptic flow

- How strongly are **charm quarks** interacting with the partons in the QGP?
- Do they interact long enough with the medium to be considered part of the medium itself?



ALI-DER-354787

# Thermalization

If particles have enough time to interact with each other they will eventually relax to (at least local) thermal equilibrium.

- **Chemical equilibrium:**

the particle multiplicity is given by a thermal distribution at a **unique (local) chemical potential** ( $\mu = 0$ )

HQs: initial hard production far from chemical equilibrium with the QGP! **Fugacity factor**  $e^{q\alpha}$  needed.

- **Kinetic equilibrium:**

the momentum distribution of the particles approaches a Maxwell-Boltzmann distribution described by a **unique (local) temperature**.

HQs: possibly get quite close to local kinetic equilibrium in the QGP within the lifetime of the fireball.

$$f(E, x) \sim \underbrace{e^{-E/T(x)} e^{q\alpha}}_{\text{in kinetic equilibrium}} + \underbrace{\delta f(E, x)}_{\text{out of kinetic equilibrium}}$$

# A new approach: fluid dynamics for heavy quarks

Supported by IQCD calculations [Altenkort *et al.* PRL 130 (2023) 23, 231902], we assume heavy quarks had enough time to interact with the light thermal partons of the medium and to approach local kinetic equilibrium

→ we treat them with a **fluid-dynamic approach!**

We write down a current associated to the **conservation of QQbar pairs** in the medium: “accidental” symmetry

→ **Number of QQbar pairs fixed by initial hard production!**

$$N^\mu = n u^\mu + \nu^\mu$$

HQ density

HQ diffusion current

$$\nabla_\mu N^\mu = 0$$

Equation of motion (Israel-Stewart type):

$$\tau_n \partial_t \nu^i + \nu^i = \kappa_n \nabla^i \alpha$$

Relaxation time

HQ diffusion coefficient

gives rise to the fugacity factor

# Fluid-dynamic transport coefficients

We computed the **relaxation time** and **diffusion coefficient** associated to charm quarks by integrating the first moment of the Fokker-Planck equation

$$p^\mu \partial_\mu f(\mathbf{p}, \mathbf{x}, t) = \frac{\partial}{\partial p^i} \left[ A(p) p^i f(\mathbf{p}, \mathbf{x}, t) - g^{ij} \frac{\partial}{\partial p^j} D(p) f(\mathbf{p}, \mathbf{x}, t) \right]$$

Drag coefficient

Momentum-diffusion  
coefficient

$$\tau_n = \frac{D_s I_{31}}{T P_0}$$

$$\kappa_n = \frac{T^2}{D} n = D_s n$$

Where the **spatial diffusion coefficient** is defined as

$$D_s = \lim_{k \rightarrow 0} \frac{T}{M A(k)}$$

# Fluid-dynamic transport coefficients

$$\tau_n = \frac{D_s I_{31}}{T P_0}$$

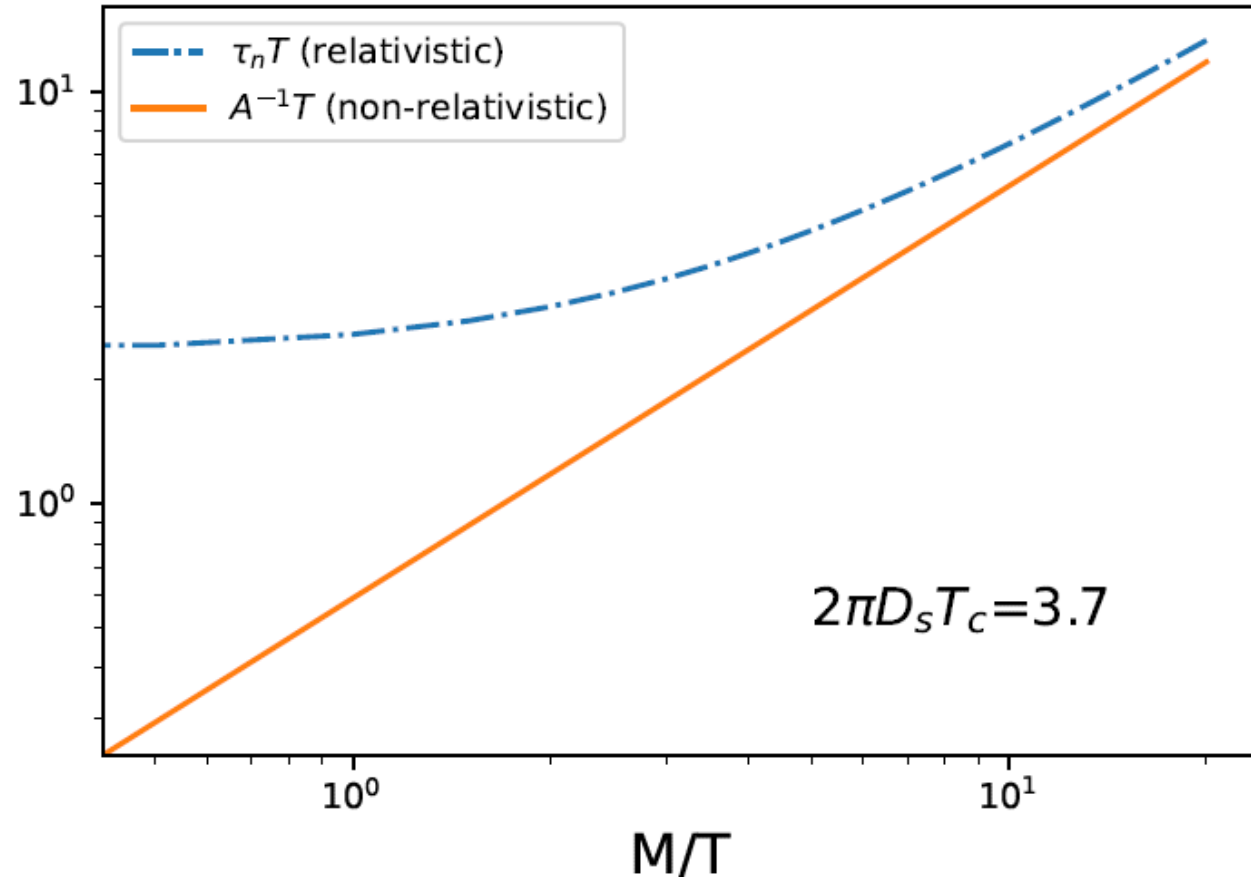
$$\kappa_n = \frac{T^2}{D} n = D_s n$$

$$I_{31} = \frac{1}{3} \int dP p^0 p^2 f_0(p)$$

$$p^0 \sim M \quad I_{31} \sim M P_0$$

$$\tau_n \sim \frac{D_s M P_0}{T P_0} = D_s \frac{M}{T}$$

$$\tau_n = (2\pi D_s T) \frac{1}{2\pi} \frac{M}{T} = A^{-1} T$$



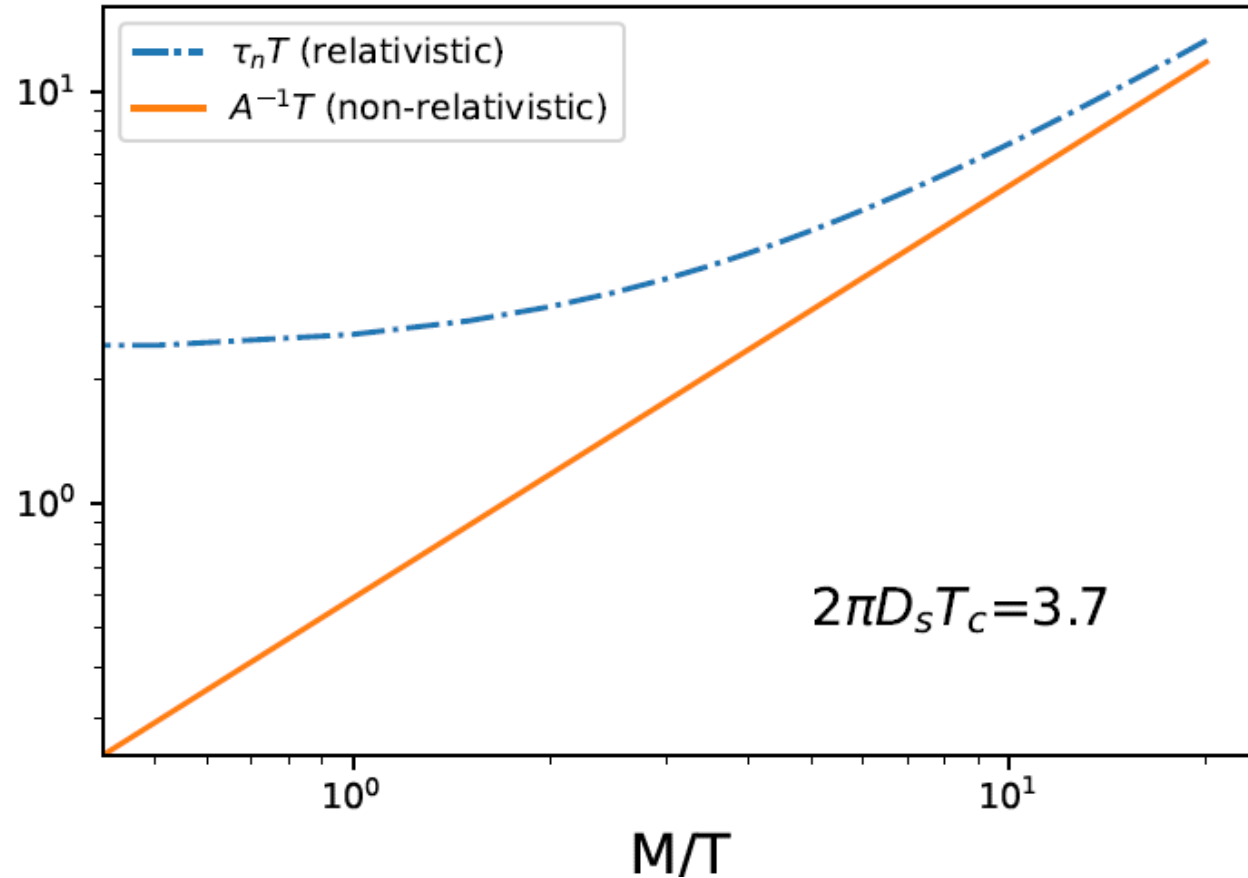
Important check: the hydrodynamic **relaxation time** is consistent with the relaxation time found within the Fokker-Planck approach in the non-relativistic limit.

# Fluid-dynamic transport coefficients

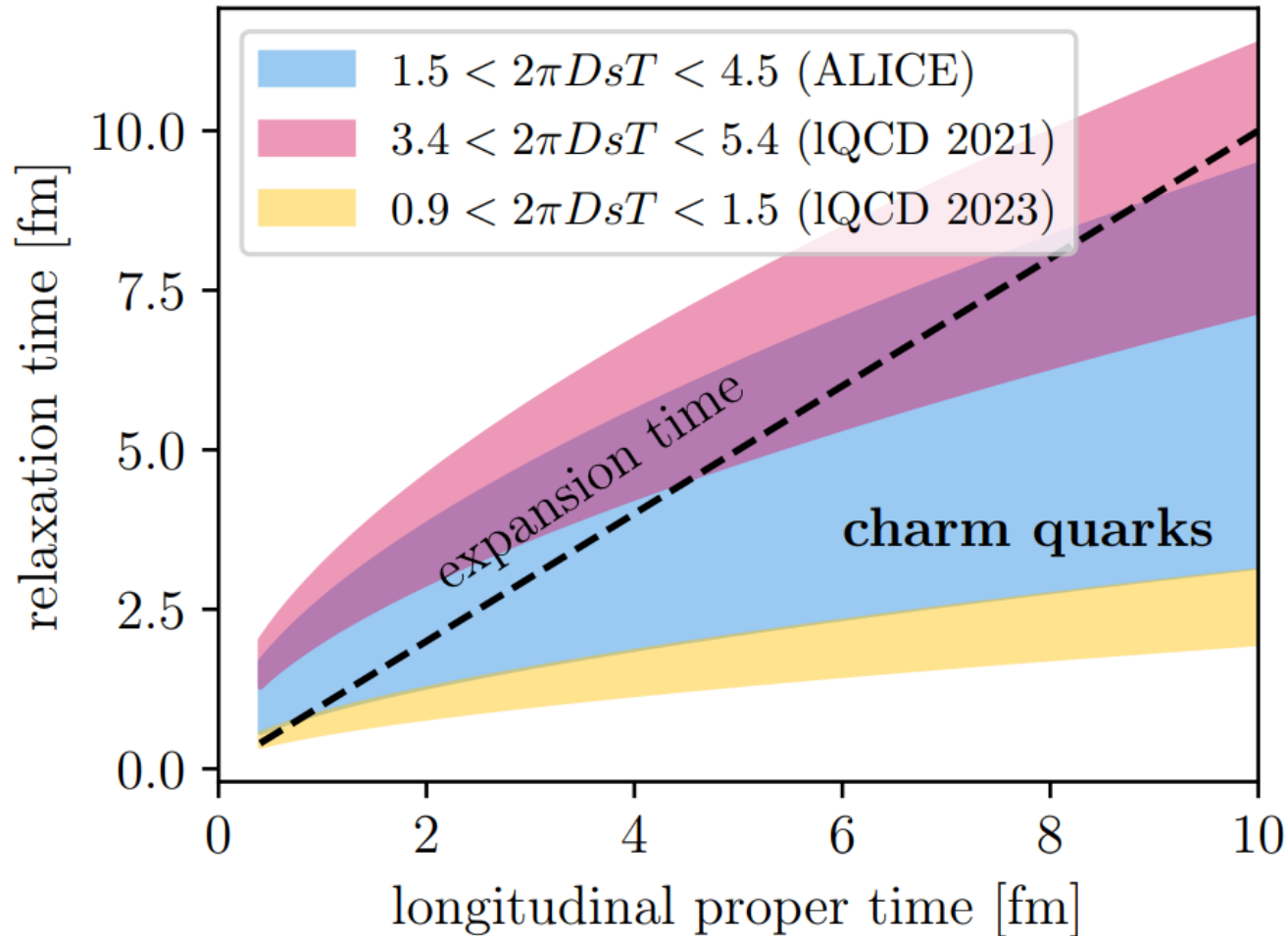
$$\tau_n = \frac{D_s I_{31}}{T P_0}$$

$$\kappa_n = \frac{T^2}{D} n = D_s n$$

The relation between **spatial diffusion coefficient** and **momentum-diffusion coefficient** - usually found in a non-relativistic setup - arises naturally also in a relativistic framework.



# Charm quark relaxation time



Capellino *et al.* PRD 106 (2022) 034021 (updated with new IQCD results)

$$\tau_n = \frac{D_s I_{31}}{T P_0}$$

$$\kappa_n = \frac{T^2}{D} n = D_s n$$

Bjorken flow:  $v_x = v_y = 0$   $v_z = z/t$

IQCD 2021: Altenkort *et al.* PRD 103 (2021) 014511

IQCD 2023: Altenkort *et al.* PRL 130 (2023) 23, 231902

ALICE fits to data: ALICE JHEP 01 (2022) 174

Relaxation time is much shorter than typical expansion time of the QGP in Bjorken flow  
→ **Fluid-dynamic description of charm looks meaningful!**



# Fluid dynamics with a conserved charge

We implement the conservation of the energy-momentum tensor and charm current

$$\nabla_\mu T^{\mu\nu} = 0 \qquad \nabla_\mu N^\mu = 0$$

together with a IQCD-inspired Equation of State and Israel-Stewart equations of motion for the dissipative currents.

Equation of State: **Hadron Resonance Gas of charm states**  $n(T, \alpha) = \frac{T}{2\pi^2} \sum_{i \in \text{HRGc}} q_i M_i^2 e^{q_i \alpha} K_2(M_i/T)$

We assume that the **charm fugacity** and **diffusion current** are a **perturbation** with respect to  $T, u^\mu, \pi^{\mu\nu}, \Pi$

→ they don't contribute significantly to the thermodynamics of the bulk evolution of the system!

E.g. in the non-dissipative case

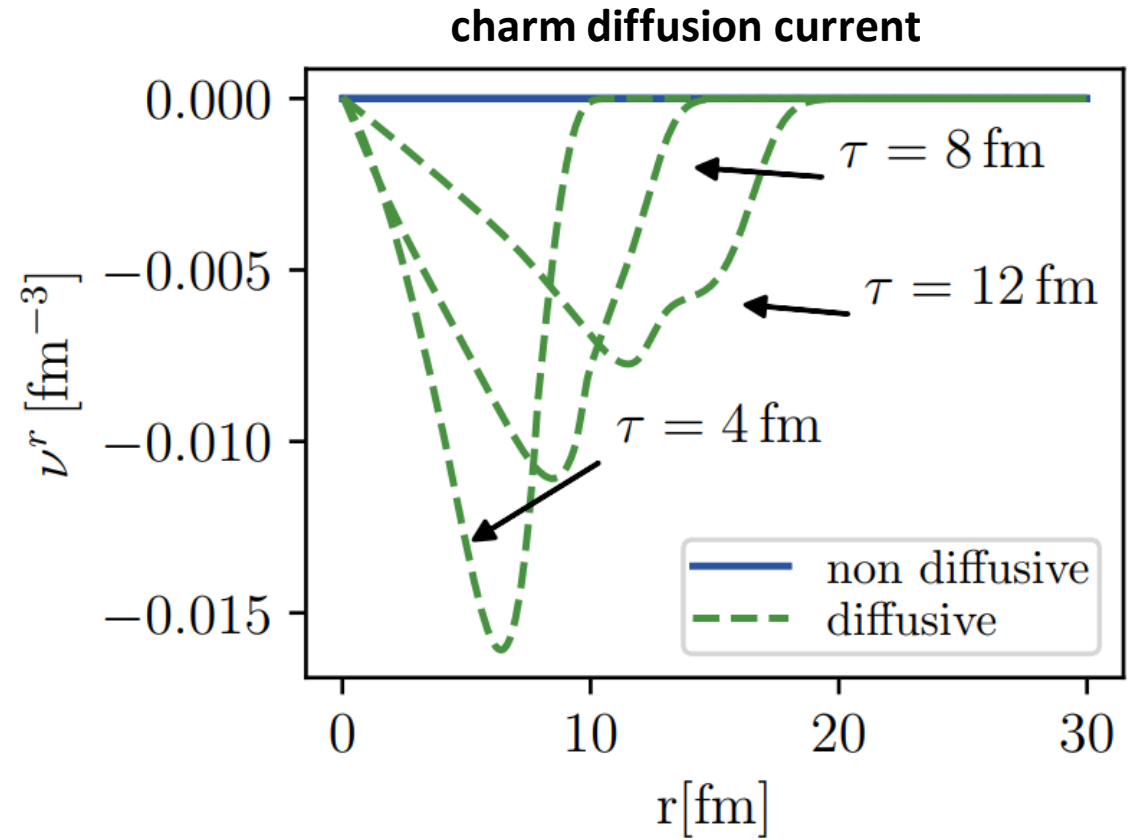
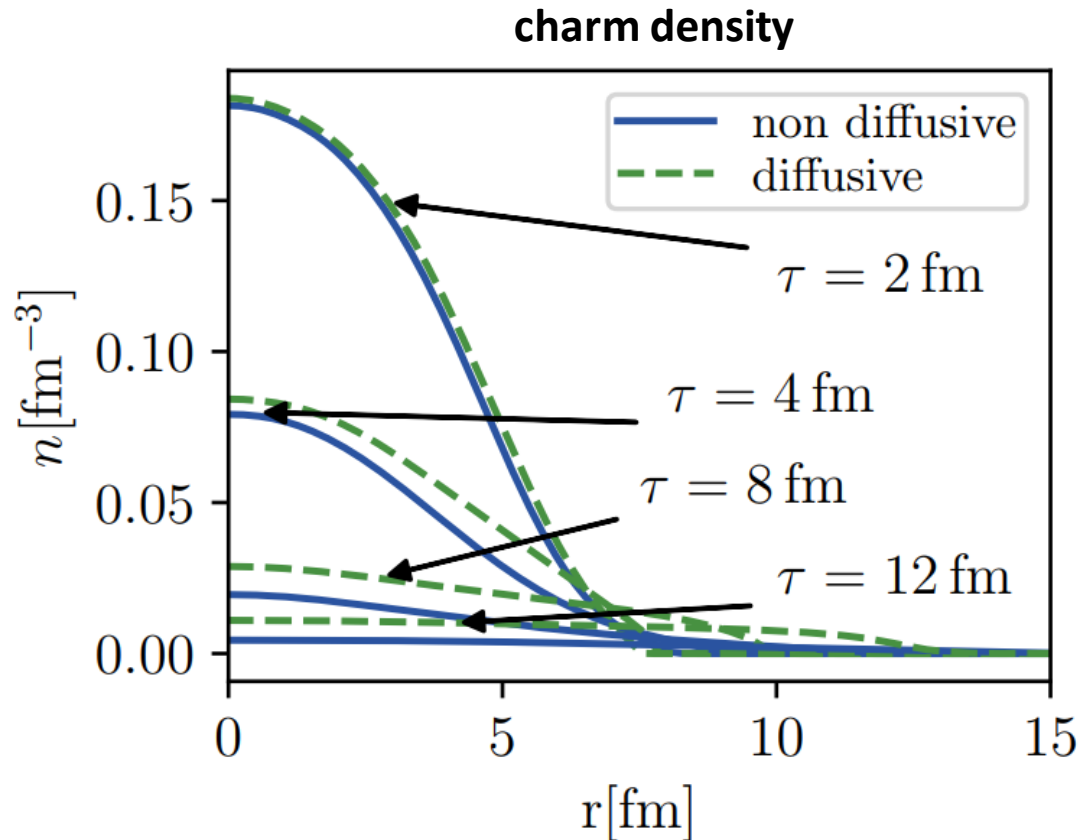
$$\begin{aligned} P &= P(T) & n &= n(T, \alpha) \\ \epsilon &= \epsilon(T) \end{aligned}$$

→ The evolution of  $T$  and  $u^\mu$  is not influenced by  $\alpha$

# Evolution of charm fields

The evolution of the charm density and diffusion current depend on the **spatial diffusion coefficient**  $D_s$ .

The ratio  $|\nu^r|/n$  is not always  $\ll 1$   $\rightarrow$  The larger  $D_s$ , the larger the out-of-equilibrium corrections coming from  $\nu^r$  are throughout the QGP evolution.



# Charm-hadron integrated yields $dN/dy$

- ☐ Resonance decays are taken into account

Mazeliauskas *et al.* Eur. Phys. J. C (2019) 79: 284

- ☐ Mesons are compatible with the experimental data within uncertainties

- ☐ Deviation of  $2.4\sigma$  for  $\Lambda_c^+$ :

- missing higher resonance states?

He, Rapp, PLB 795, 117 (2019)

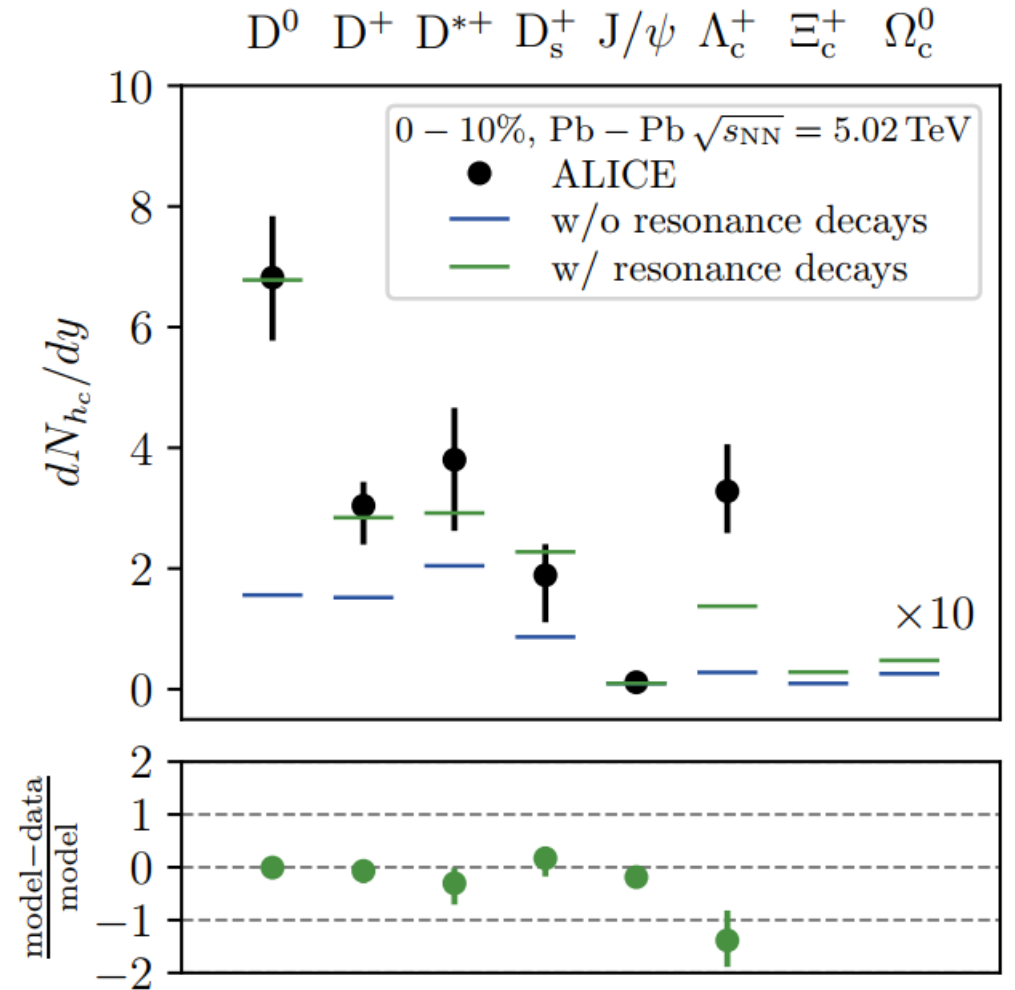
Andronic *et al.* JHEP 07, 035 (2021)

- coalescence mechanisms?

Plumari *et al.* Eur. Phys. J. C 78, 348 (2018)

Beraudo *et al.*, Eur. Phys. J. C 82, 607 (2022)

- ☐ Prediction for not-yet-measured open-charm states  $\Xi_c^+$ ,  $\Omega_c^0$



Capellino *et al.* [2307.15580](#) [hep-ph]

ALICE JHEP 01 (2022) 174, ALICE (2023) arXiv:2303.13361 [nucl-ex], ALICE PLB 839 137796 (2023), ALICE PLB 827 136986 (2022)

# Momentum distributions

We compute spectra with a **Cooper-Frye prescription** at  $T_{fo} = 156.5$  MeV

**Notice: dissipative corrections on the freeze-out surface are missing!**

→ Expanding in terms of

$$f^{(hc)} = f_{eq}^{(hc)} + \delta f_{bulk}^{(hc)} + \delta f_{shear}^{(hc)} + \delta f_{diff}^{(hc)}$$

requires **multi-fluid setup** (not yet implemented)

❑ The fluid-dynamic description of charm captures the physics of **D mesons up to 4-5 GeV** and **J/ψ up to 3 GeV**!

❑ Expected deviation for  $\Lambda_c^+$  as seen in integrated yields

❑ deviation for **J/ψ**: primordial J/ψ

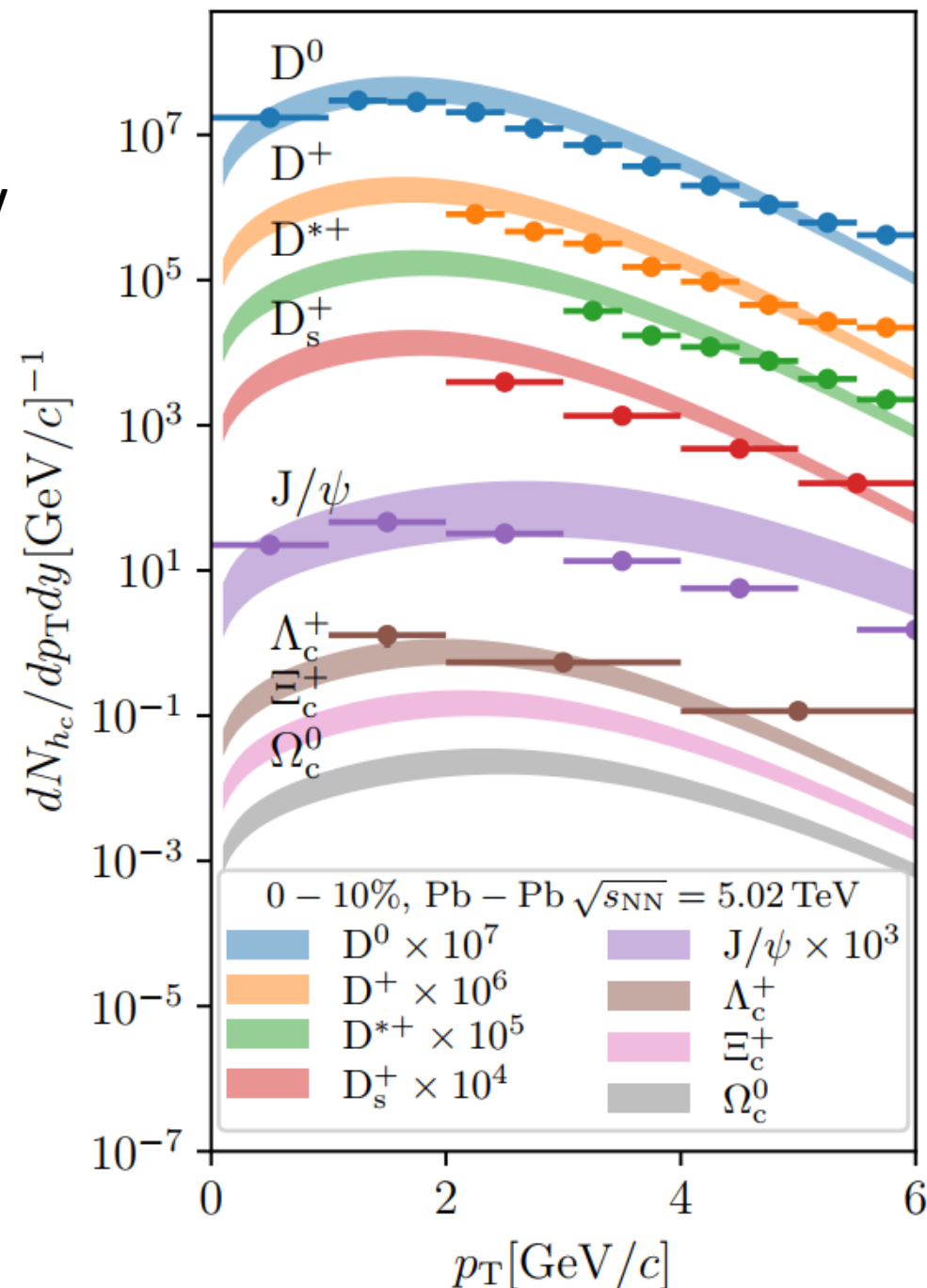
Capellino *et al.* (2307.15580 [hep-ph])

ALICE JHEP 01 (2022) 174

ALICE (2023), arXiv:2303.13361 [nucl-ex]

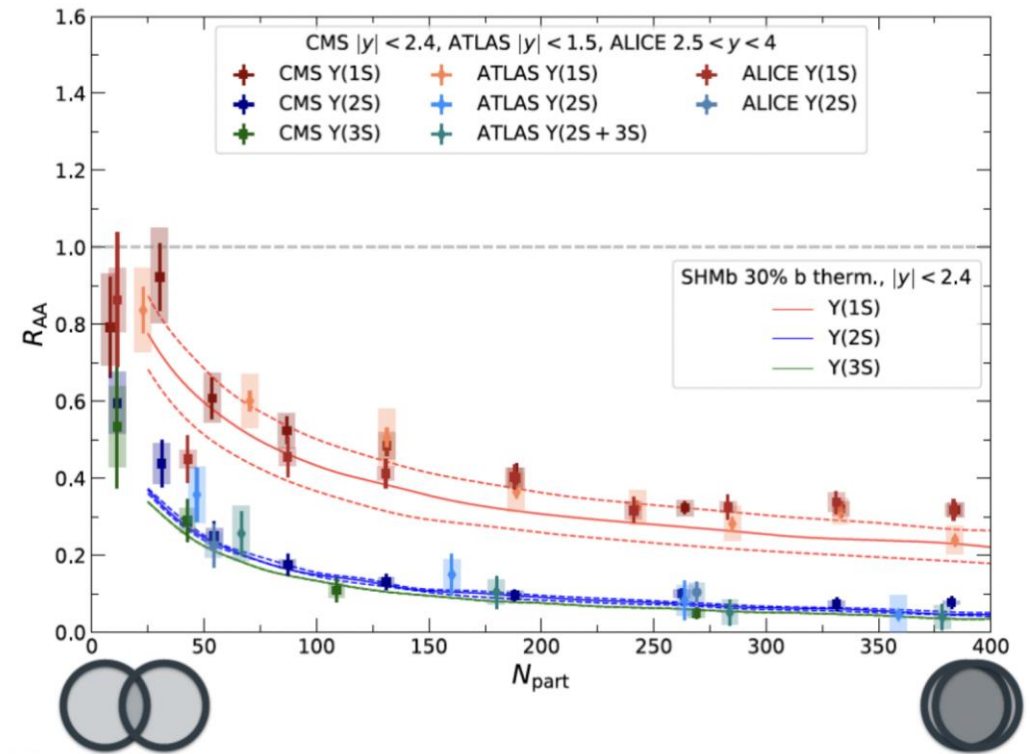
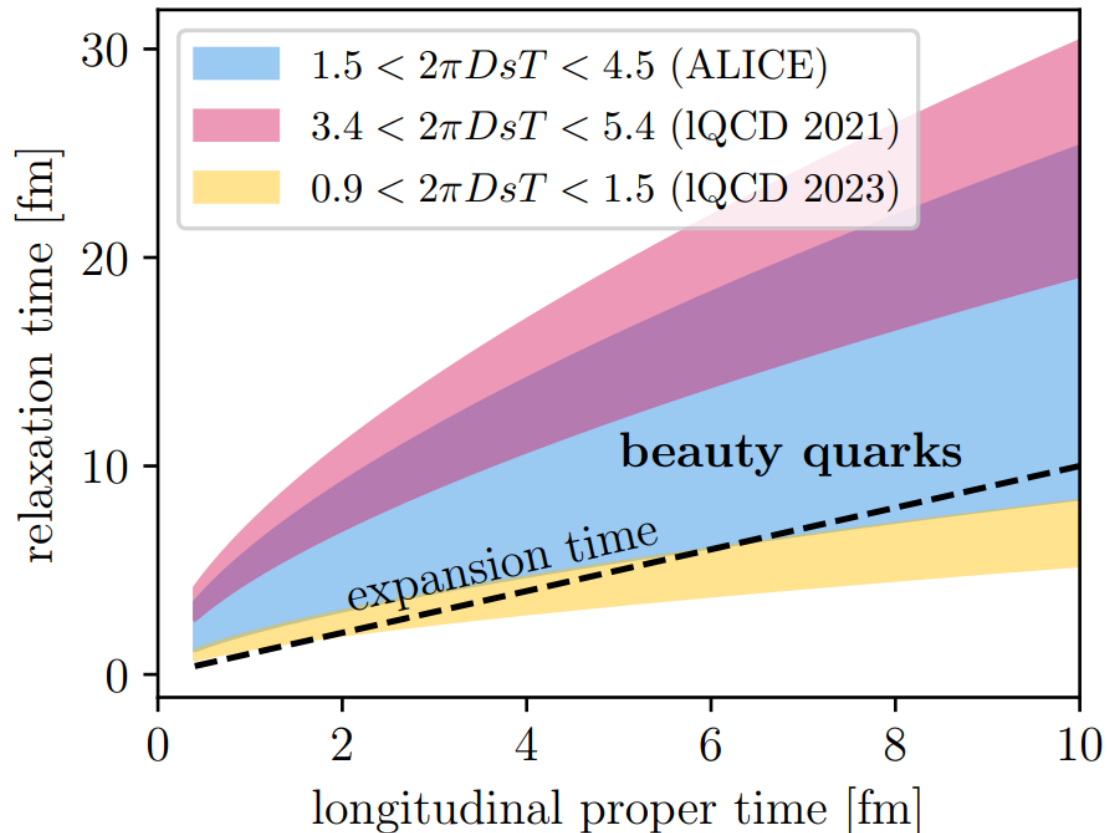
ALICE PLB 839, 137796 (2023)

ALICE PLB 827, 136986 (2022)



# What about bottom quarks?

- Need of precise measurements for spectra and flow coefficients to study thermalization of bottom quarks in the QGP → Run3 at the LHC: new data for pp and Pb-Pb collisions
- $\Upsilon$  described by SHM if 30% of bottom quarks assumed to thermalize → **partial thermalization?**
- Presence of currently unknown open bottom states will lead to a reduction of the bottomonia yields



**Fluid dynamics of beauty quarks  
WORK IN PROGRESS**

# Summary and outlook

## Summary

- ✓ We presented a new approach based on **fluid dynamics to describe charm quark dynamics in the QGP**
- ✓ Theoretical studies and experimental data point towards **full thermalization of charm quarks** within the lifetime of the QGP at LHC energies.
- ✓ Integrated yields as well as the momentum distributions of charm hadrons are **consistent with experimental data up to  $p_T$  of 4-5 GeV** → **tension in the charm baryon sector**

## Outlook

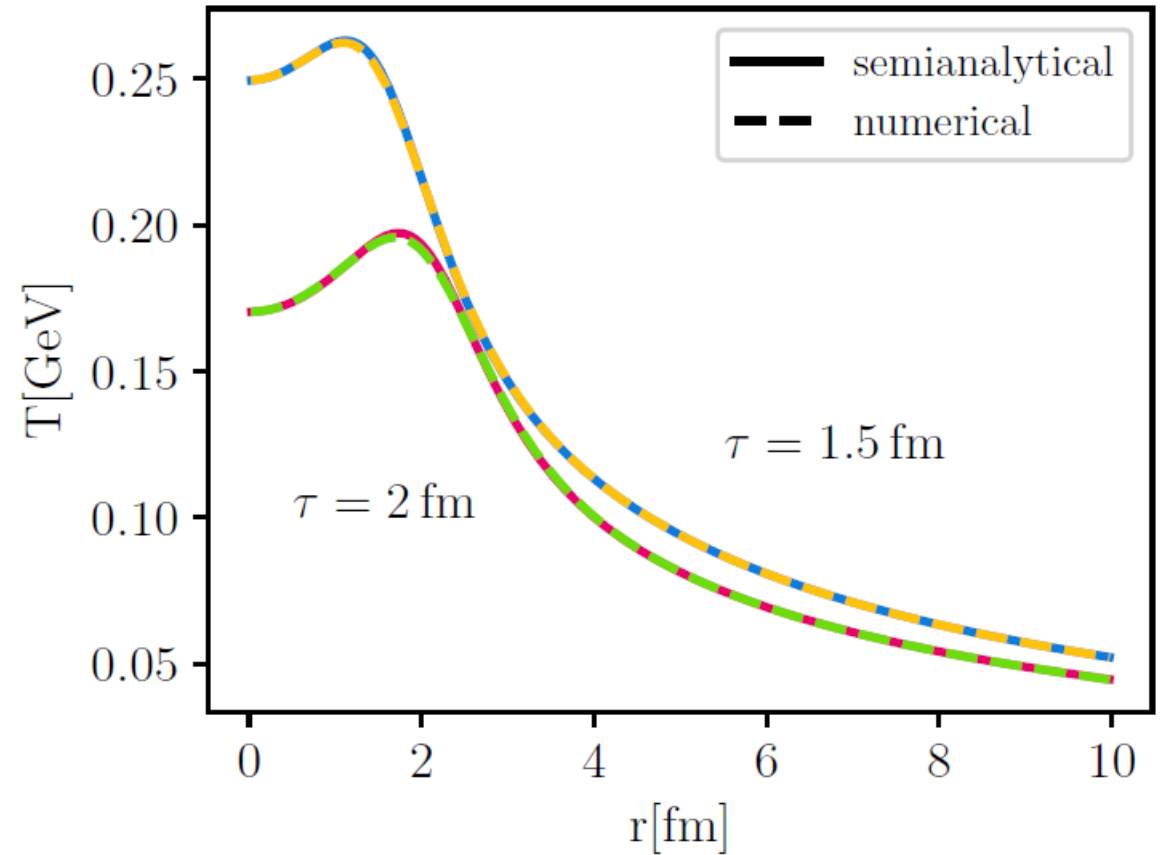
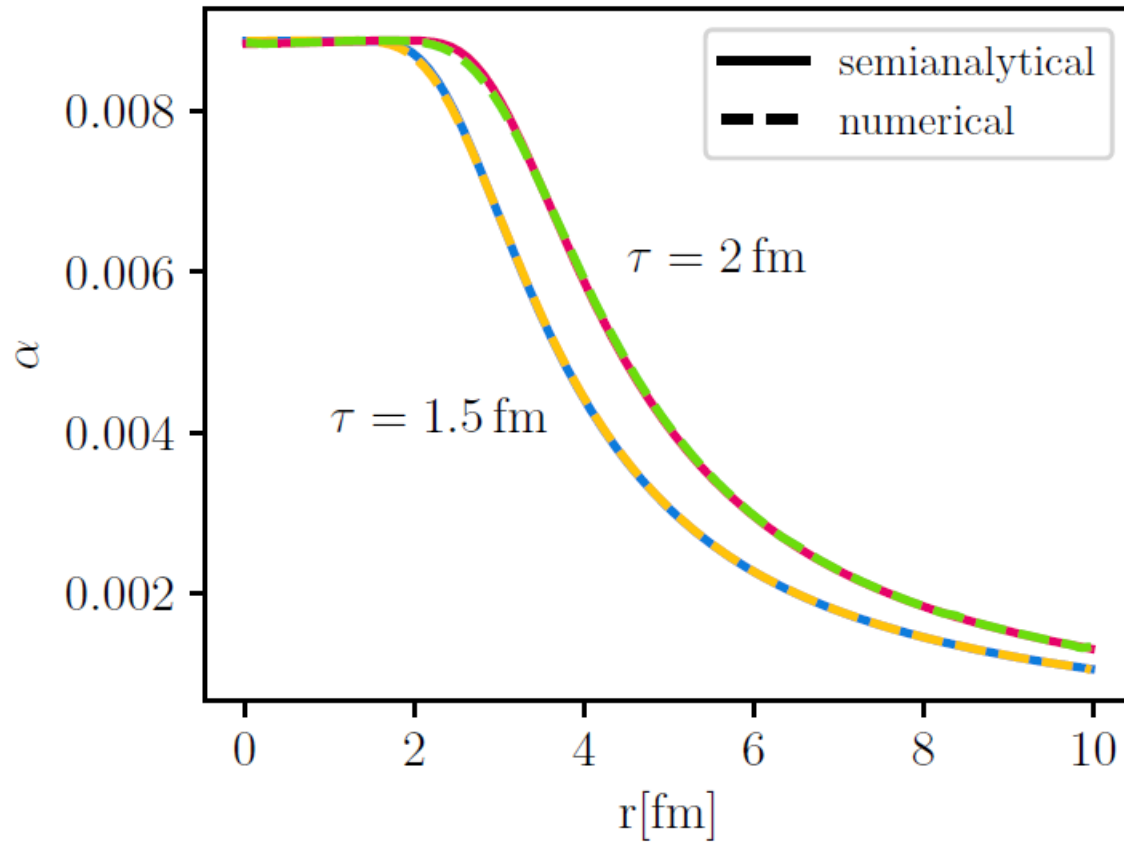
- ❑ Full treatment with **viscous corrections** at the freeze-out is necessary
- ❑ Extension to **flow coefficients + Bayesian analysis** to understand the full «thermalization of charm» picture
- ❑ Extension to **beauty quarks**
- ❑ Investigation of thermalization at lower energies → **top RHIC energies?**

**Thank you for your attention!**

Back up

# Code validation

We validated our numerical framework against Gubser flow.



The solid lines correspond to the semianalytic Gubser solution, while the dashed lines are the numerical result with  $N = 200$  discretization points. We have here chosen the maximal radius to be 10 fm.



# Equation of motion for the HQ diffusion current

We derive hydrodynamic equations of motion from kinetic theory (Boltzmann) in a **Fokker-Planck approximation**

$$p^\mu \partial_\mu f(\mathbf{p}, \mathbf{x}, t) = \frac{\partial}{\partial p^i} \left[ A(p) p^i f(\mathbf{p}, \mathbf{x}, t) - g^{ij} \frac{\partial}{\partial p^j} D(p) f(\mathbf{p}, \mathbf{x}, t) \right]$$

By integrating the first moment of the equation

$$\int dP p^\nu p^\mu \partial_\mu f(\mathbf{p}, \mathbf{x}, t) = \int dP p^\nu \frac{\partial}{\partial p^i} \left[ A(p) p^i f(\mathbf{p}, \mathbf{x}, t) - g^{ij} \frac{\partial}{\partial p^j} D(p) f(\mathbf{p}, \mathbf{x}, t) \right]$$

We obtain a relaxation-type equation for the diffusion current

$$\tau_n \partial_t \nu^i + \nu^i = \kappa_n \nabla^i \left( \frac{\mu}{T} \right)$$

Relaxation time

HQ diffusion coefficient

$$\tau_n = \frac{D_s I_{31}}{T P_o}$$
$$\kappa_n = \frac{T^2}{D} n = D_s n$$