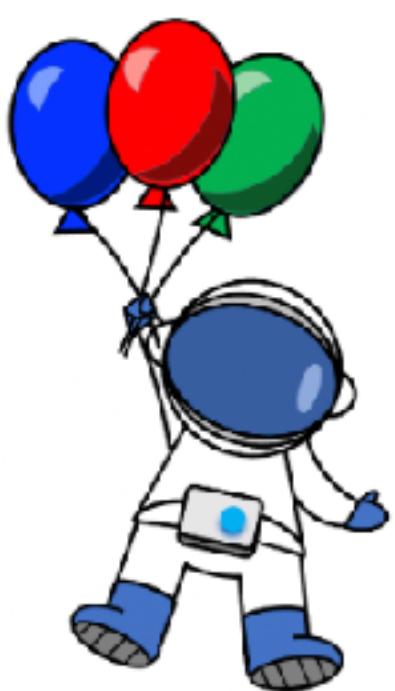




QM 2023

SEPTEMBER 3-9, 2023 | HOUSTON, TEXAS

# Harmonics of Parton Saturation in inclusive and diffractive Lepton-jet correlation at EIC



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*Collaborator:* Xuan-Bo Tong, Bo-Wen Xiao

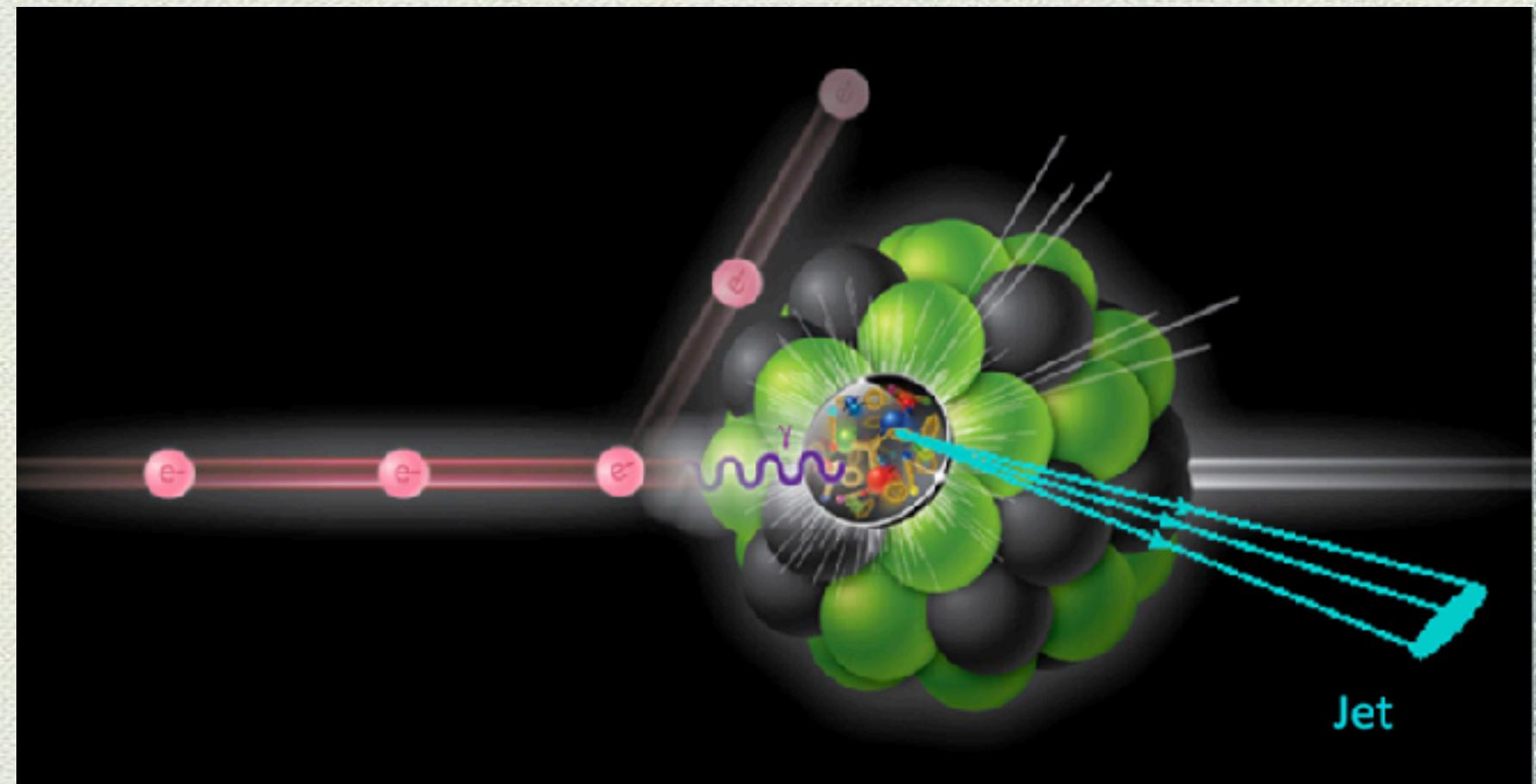
PRL.130.151902(2023)

Second Paper in preparation

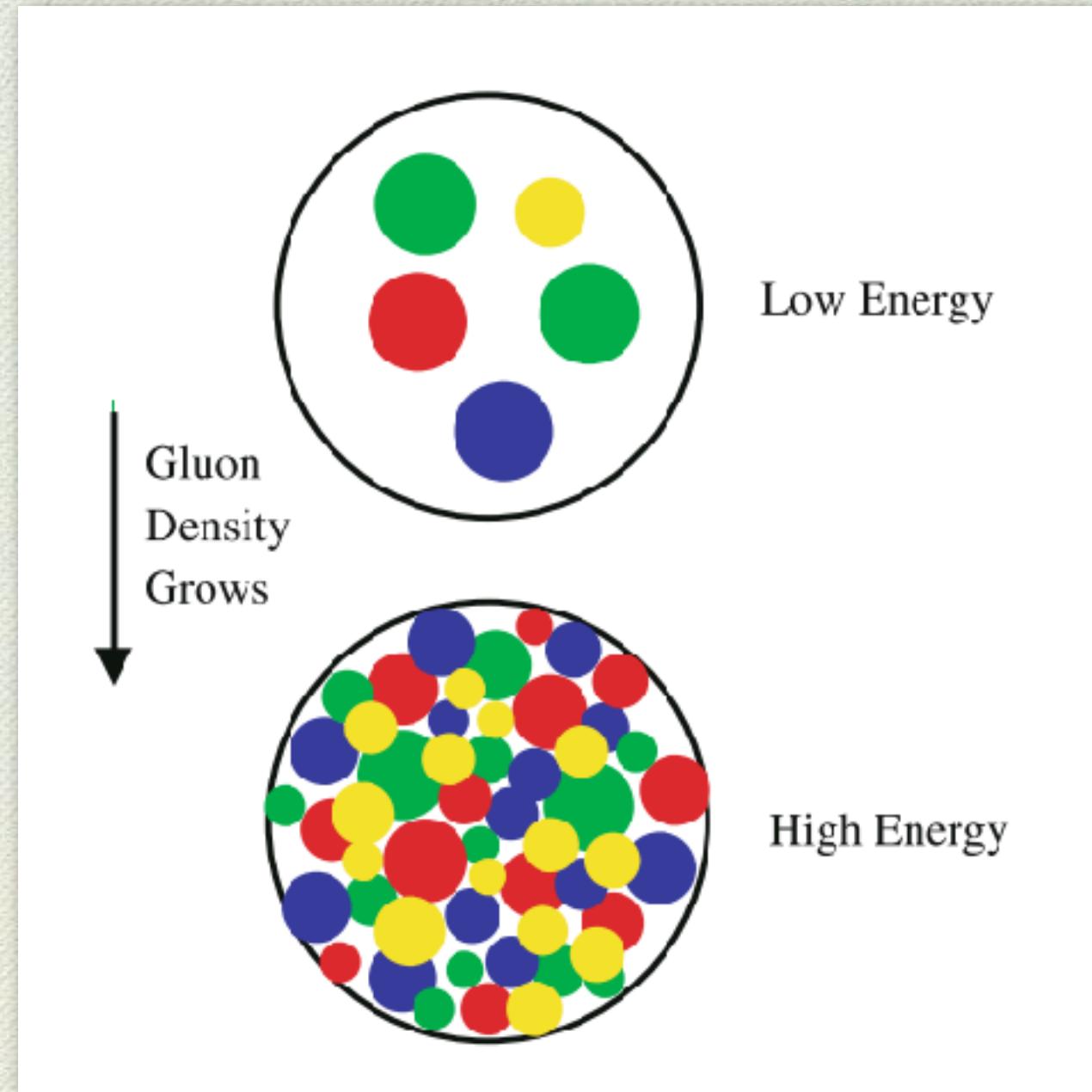


# Outline

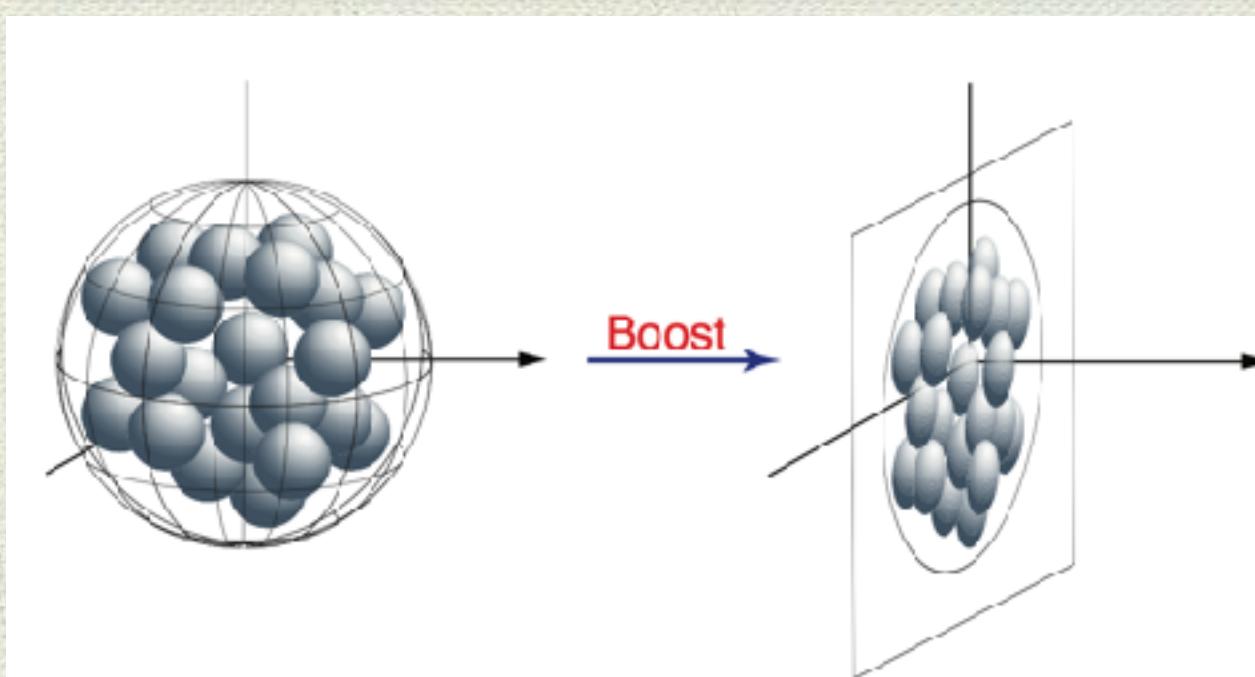
- ◆ Parton Saturation
- ◆ Lepton-Jet Correlation — New Channel for Saturation
- ◆ Harmonics of inclusive Lepton-Jet Correlation
- ◆ Harmonics of diffractive Lepton-Jet Correlation
- ◆ Summary



# Parton Saturation



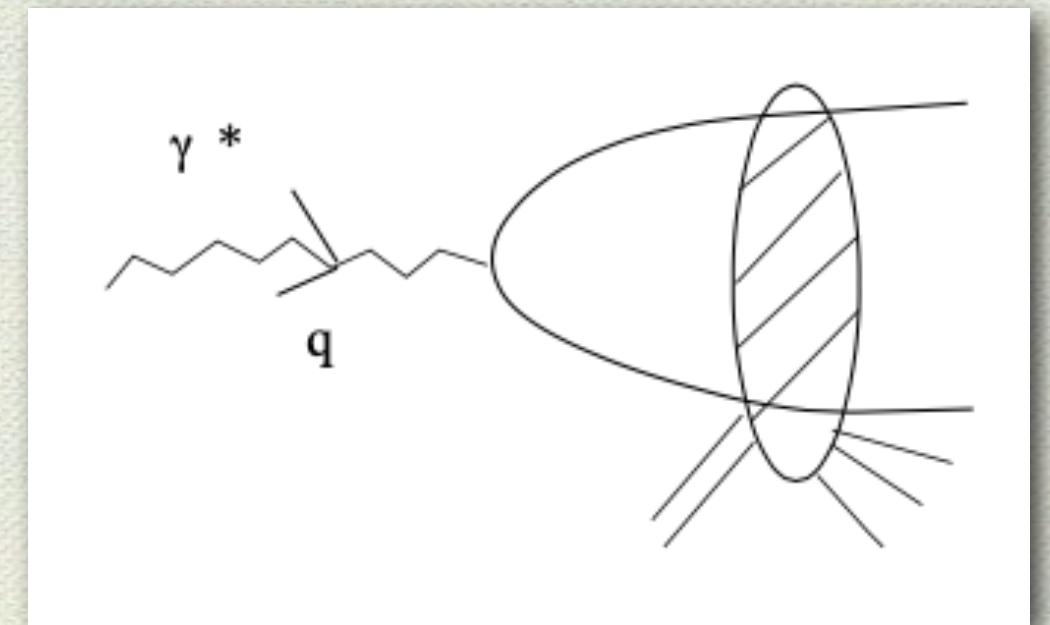
- ◆ In high energy proton / nucleus, large-x parton radiates small-x gluon ( $\uparrow$ ), high density gluon fusion( $\downarrow$ ), gluon density saturates
- ◆ When gluon density saturates, typical transverse size of gluons  $\frac{1}{Q_s}$ , saturation scale  $Q_s$
- ◆  $Q_s^2 \sim A^{1/3}$ , same transverse area Au has more gluons than p, smaller transverse size, larger  $Q_s$



# Color Glass Condensate effective theory

- ◆ Dipole Picture of DIS

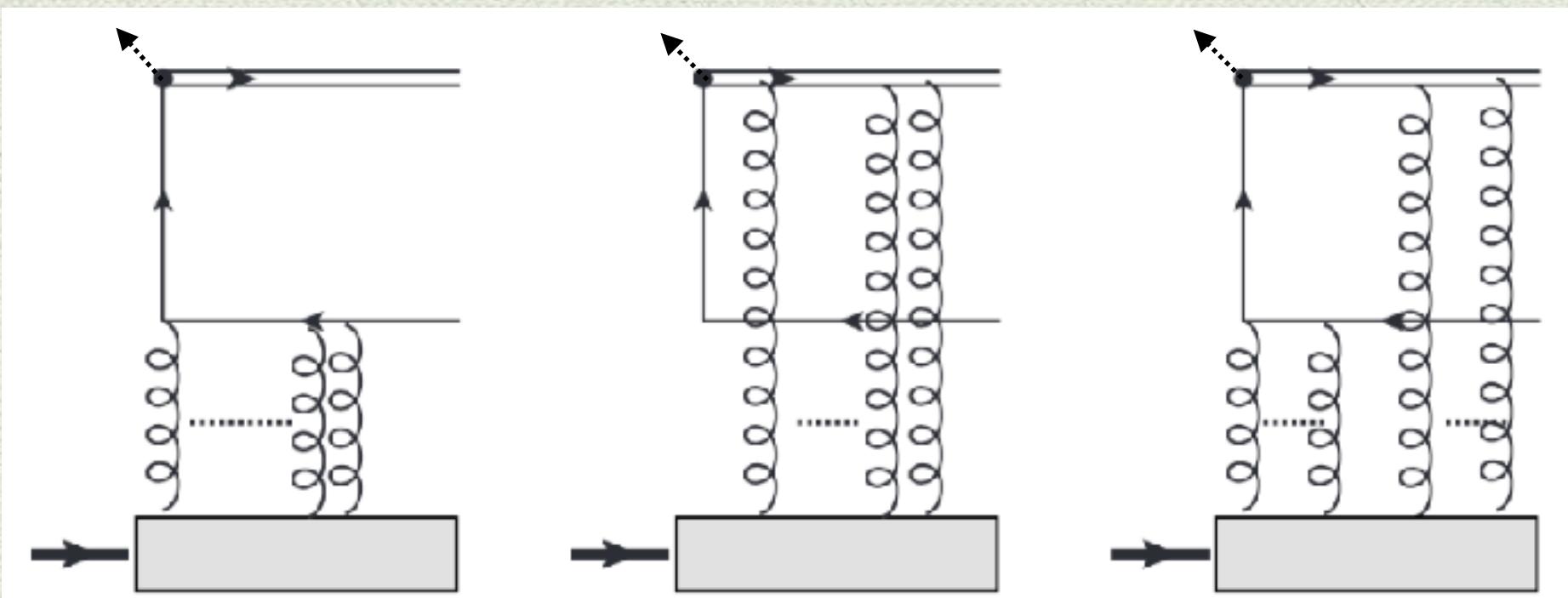
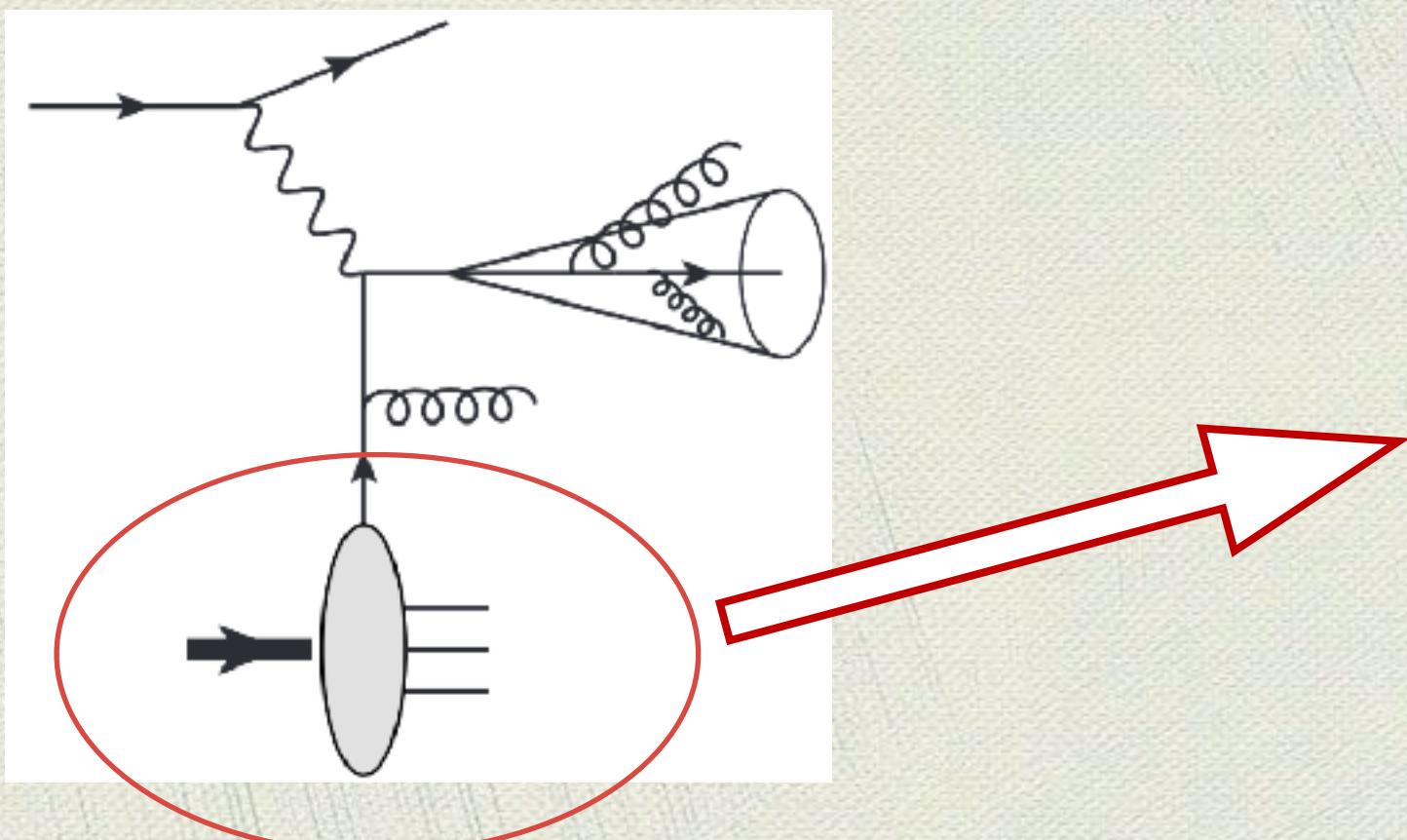
Proton/Nucleus Rest Frame  $\gamma^* \rightarrow q + \bar{q}$



$q\bar{q}$  (dipole) scatter with Proton/Nucleus, scattering matrix  $S_x$

- ◆ Small-x quark TMD distribution

Marquet-Xiao-Yuan, PLB 682, 207-211 (2009)  
Xiao-Yuan-Zhou, NPB 921, 104-126 (2017)



$$xf_q(x, b_\perp) \propto S_x$$

# Two-particle correlation at EIC

## ◆ Various two-particle correlation

- di-jet Dominguez-Xiao-Yuan PRL106, 022301 (2011) Mäntysaari-Mueller-Salazar-Schenke PRL124,112301(2020)
- di-hadron Zheng-Aschenauer-Lee-Xiao, PRD89,074037(2014) Bergabo-Jalilian-Marian 2301.03117
- photon-jet Kolb  -Roy-Salazar-Schenke-Venugopalan JHEP01(2021)052

## ◆ Back-to-back configuration

$$\vec{q}_\perp \ll \vec{P}_\perp \text{ correlation limit } \left\{ \begin{array}{l} \vec{q}_\perp = \vec{k}_{1\perp} + \vec{k}_{2\perp} \quad \text{Imbalance momentum} \\ \vec{P}_\perp = (\vec{k}_{1\perp} - \vec{k}_{2\perp})/2 \quad \text{Relative momentum} \end{array} \right.$$

$q_\perp \sim Q_s$  probe saturation through small  $q_\perp$

# Lepton-Jet Correlation

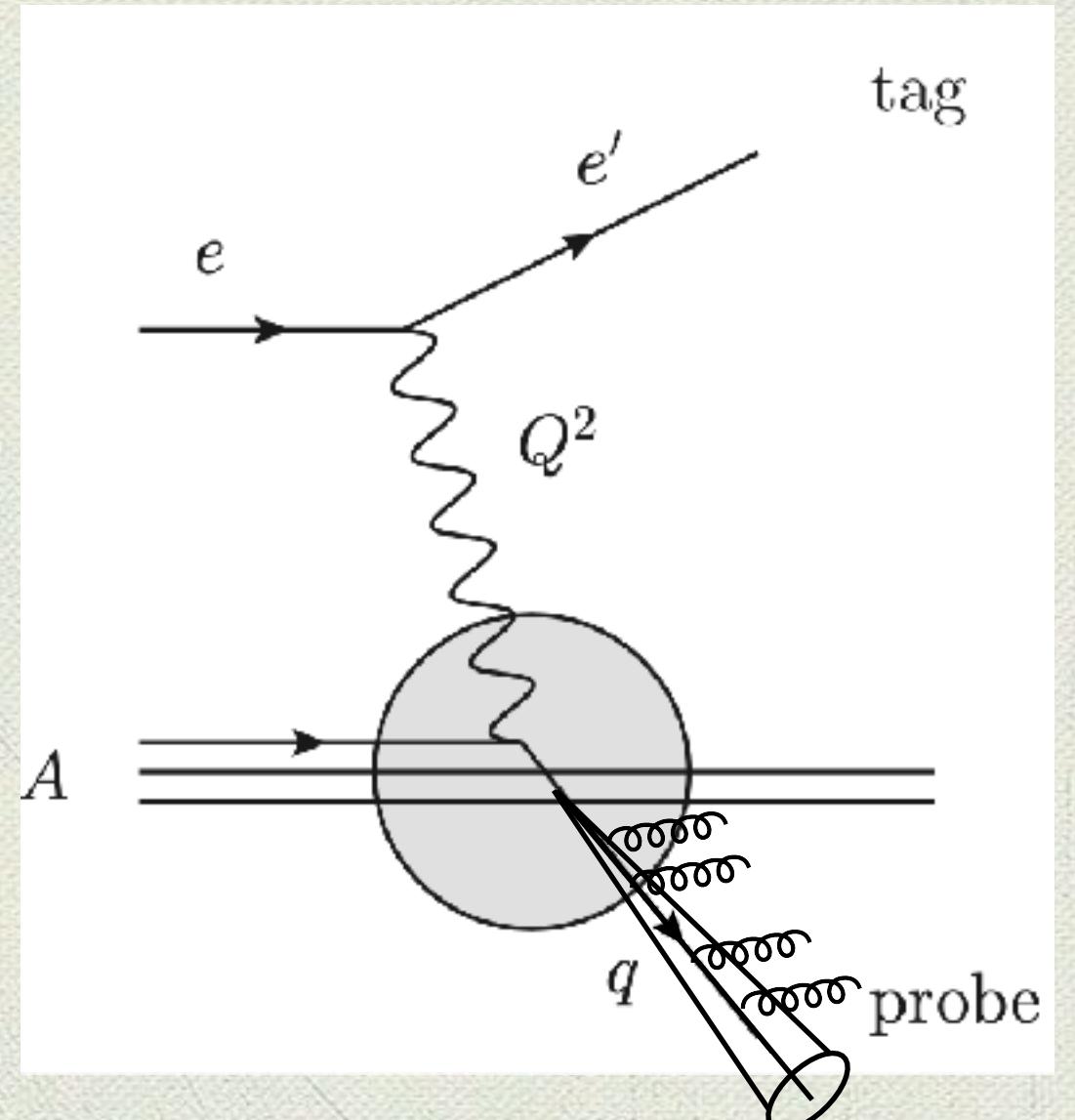
Liu-Ringer-Vogelsan-Yuan PRL22,192003 (2019); PRD102,094022 (2020)

- ◆ TMD factorization at one-loop order

$$\frac{d^5\sigma}{dy_\ell d^2k_{\ell\perp} d^2q_\perp} \propto xf_q(x, k_\perp) \otimes H_{\text{TMD}} \otimes S_J(\lambda_\perp)$$

Hard factor      Soft factor

- Resum the soft gluon radiation, average over azimuthal angle  $\phi$
- Easy to measure in experiment, electron easy to tag
- At small- $x$ , small- $x$  quark TMD PDF encodes gluon saturation
- Other application : Sivers asymmetry, Collins asymmetry, ...  
Kang-Liu-Mantry-Shao PRL 125, 242003(2020)  
Arratia-Kang-Prokudin-Ringer PRD102,074015 (2020)

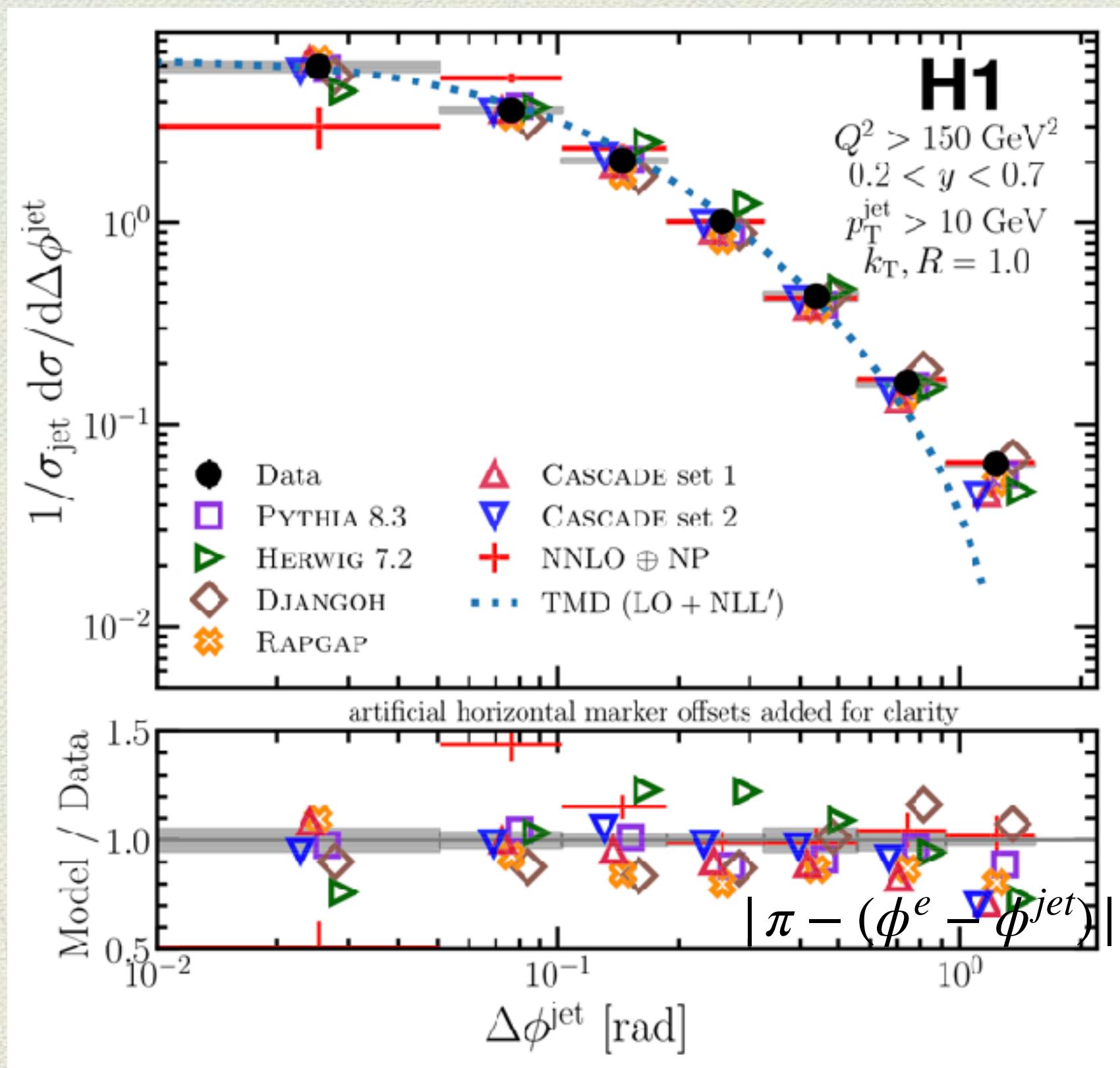


[Figure from Arratia et al PRC101,065204(2020)]

# Experimental study of Lepton Jet Correlation

## Measurement of LJC at HERA(H1)

H1 Collaboration, PRL 128, 132002(2022)

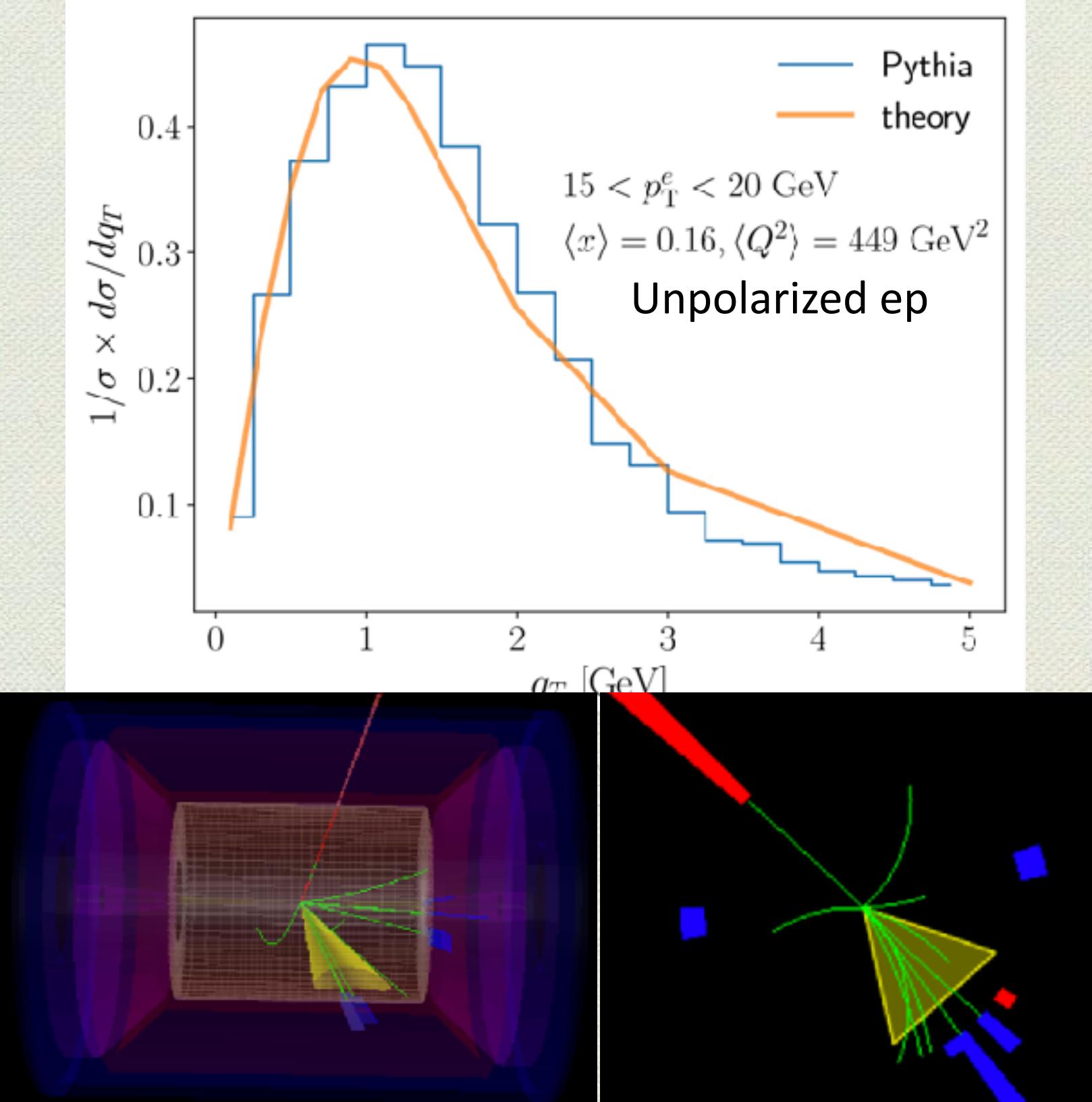


• H1 datas agree with TMD calculation

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## Simulation studies at the EIC

Arratia-Song-Ringer-Jacak PRC101,065204(2020)  
Arratia-Kang-Prokudin-Ringer PRD102,074015



• Indicate the probability to measure LJC at EIC  
lepton-jet correlation

# $\phi$ dependent Lepton Jet Correlation

Azimuthal angle  $\phi = \angle(\vec{q}_\perp, \vec{P}_\perp)$

One soft gluon radiation  $\frac{d^5\sigma}{dy_\ell d^2P_\perp d^2q_\perp} \propto (S_{\text{iso}} + S_{\text{aniso}}) \quad \left\{ \begin{array}{ll} S_{\text{iso}} & \text{angle-independent part} \\ S_{\text{aniso}} & \text{angle-dependent part} \end{array} \right.$

Soft gluon radiation to all orders  $\frac{d^5\sigma}{dy_\ell d^2k_{\ell\perp} d^2q_\perp} \propto e^{S_{\text{iso}}} (1 + S_{\text{aniso}}) \quad \begin{aligned} S_{\text{iso}} &= -\text{Sud}_P \\ &\text{Sudakov factor} \end{aligned}$

- Only keep  $e^{S_{\text{iso}}}$  — TMD factorization formula
- $S_{\text{aniso}}$  contains  $\cos n\phi$ , all the asymmetry

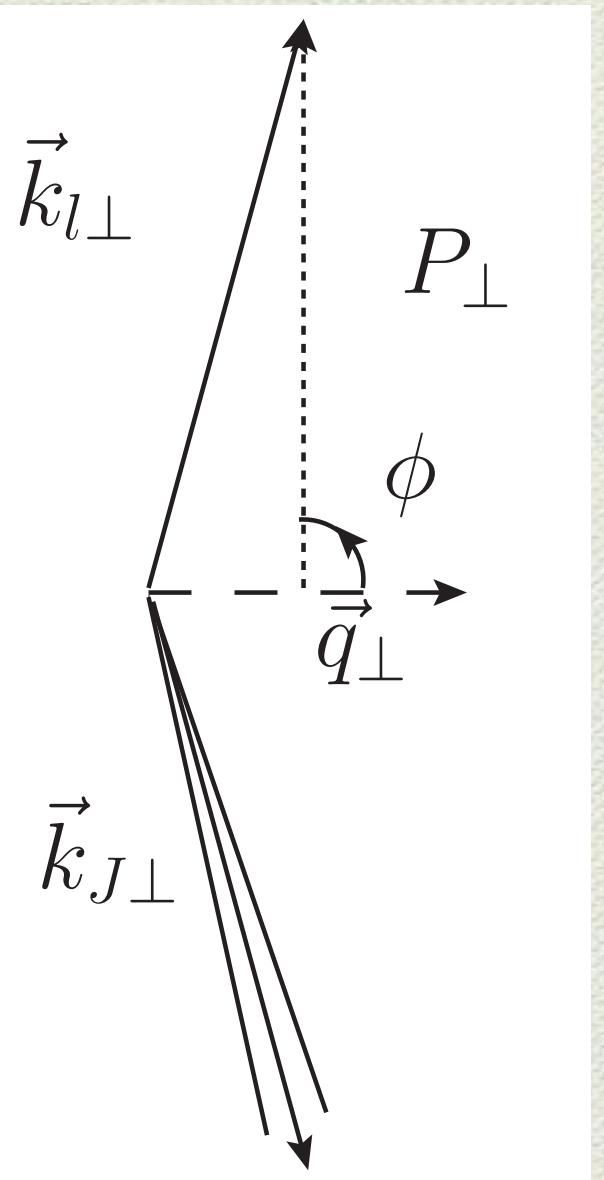
# Harmonics of Lepton Jet Correlation

- Harmonics— Measure of azimuthal angle asymmetry

$$\frac{d\sigma}{dP_\perp dq_\perp d\phi} = \sigma_0 + 2\sigma_1 \cos(\phi) + 2\sigma_2 \cos(2\phi) + \dots$$

Harmonics: the normalized Fourier coefficient  $\langle \cos n\phi \rangle = \frac{\sigma_n}{\sigma_0}$

- $\langle \cos n\phi \rangle^2$  as “how much” the  $\cos n\phi$  contribute to LJC
- soft gluon radiation of final jet  $\rightarrow \phi$  asymmetry



transverse plane

$$\vec{q}_\perp = \vec{k}_{l\perp} + \vec{k}_{J\perp}$$

$$\overrightarrow{P}_\perp = \frac{1}{2}(\vec{k}_{J\perp} - \vec{k}_{l\perp})$$

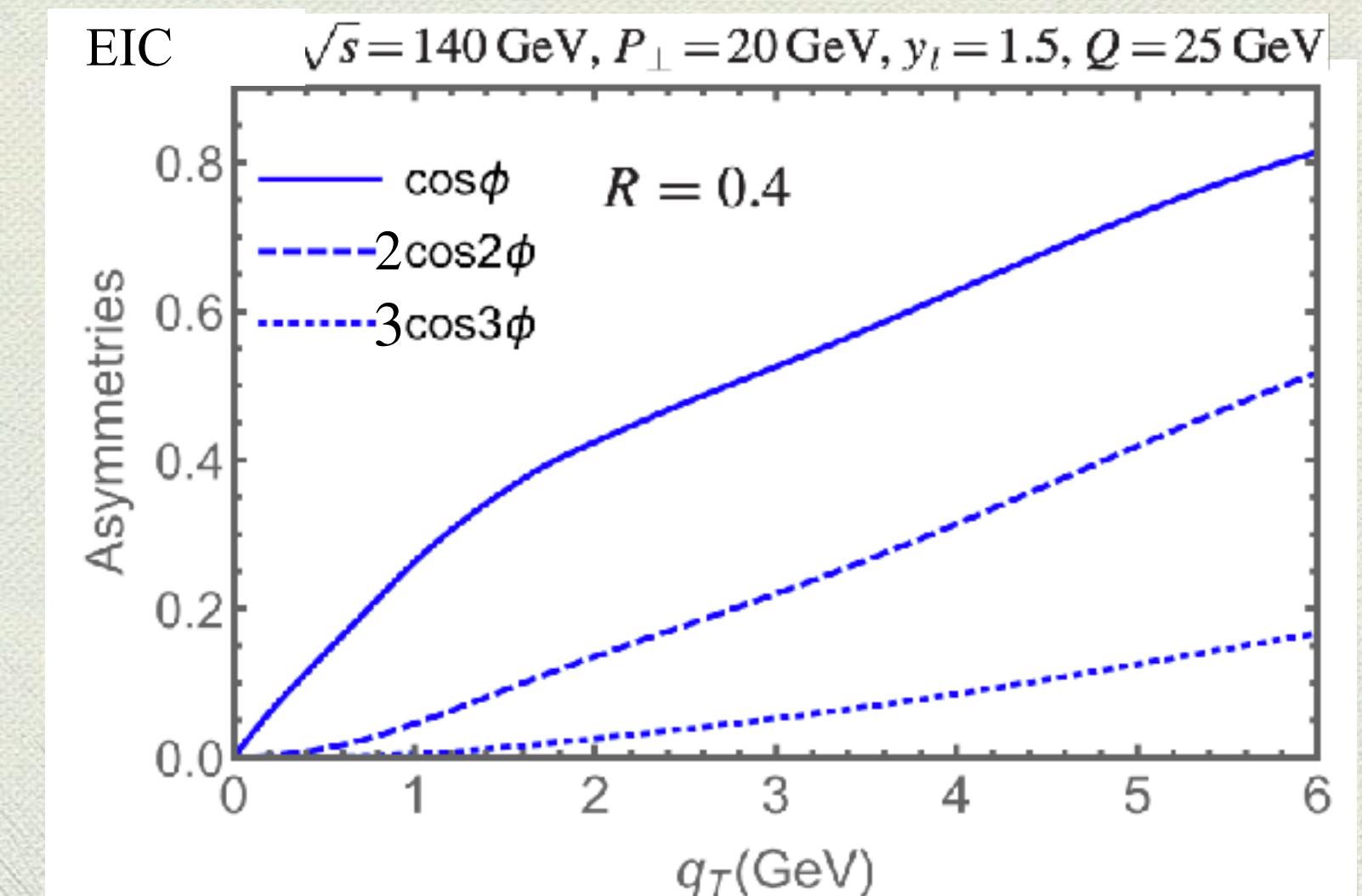
# Harmonics:non-saturation framework

Hatta-Xiao-Yuan-Zhou, PRD104,054037 (2021)

- TMD factorization + Collins-Soper-Sterman(CSS) resummation

$$\frac{d^5\sigma}{dy_\ell d^2k_{\ell\perp} d^2q_\perp} \propto xf(x, \mu_b) \otimes e^{-S_{\text{ud}}} \otimes [J_0(q_\perp b_\perp) + \sum_{n=1}^{\infty} 2 \cos(n\phi) \alpha_s(\mu_b) \frac{C_F c_n(R)}{n\pi} J_n(q_\perp b_\perp)]$$

- The harmonics  $\langle \cos n\phi \rangle \propto q_\perp^n$  at small-x
- x in collinear PDF  $xf(x, \mu_b)$  can be large-x or small-x
- $xf(x, \mu_b)$  include valence and sea quarks



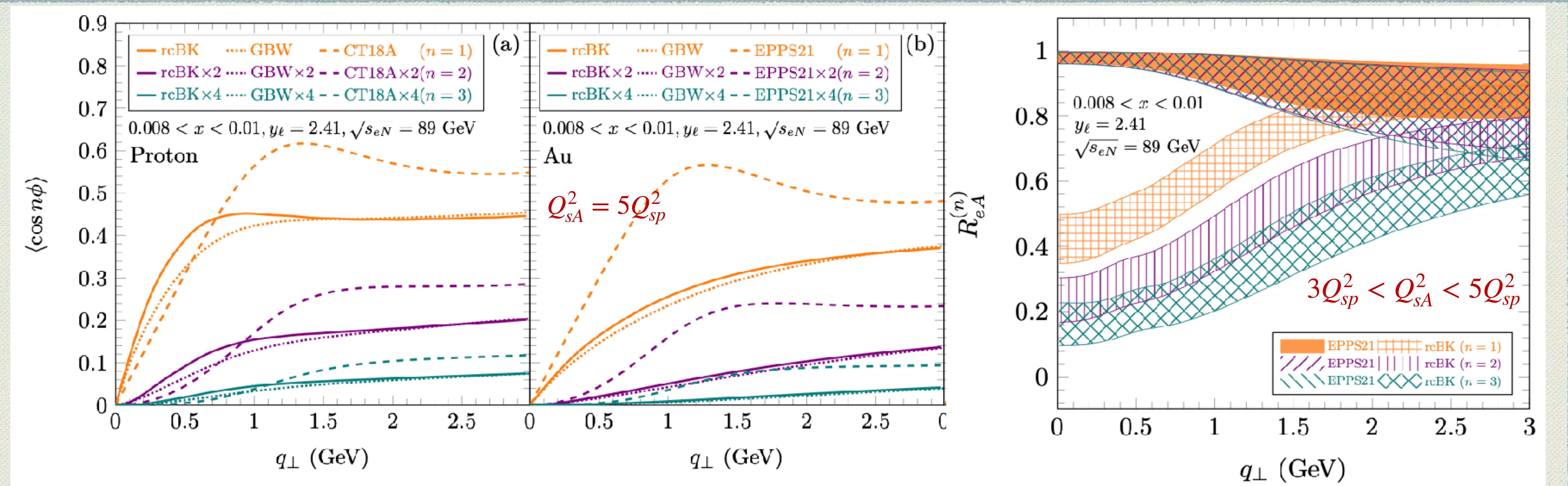
# Harmonics: Saturation framework

## Small- $x$ factorization and resummation

$$\frac{d^5\sigma}{dy_\ell d^2k_{\ell\perp} d^2q_\perp} \propto xf(x, b_\perp) \otimes e^{-S_{\text{ud}}} \otimes [J_0(q_\perp b_\perp) + \sum_{n=1}^{\infty} 2 \cos(n\phi) \alpha_s(\mu_b) \frac{C_F c_n(R)}{n\pi} J_n(q_\perp b_\perp)]$$

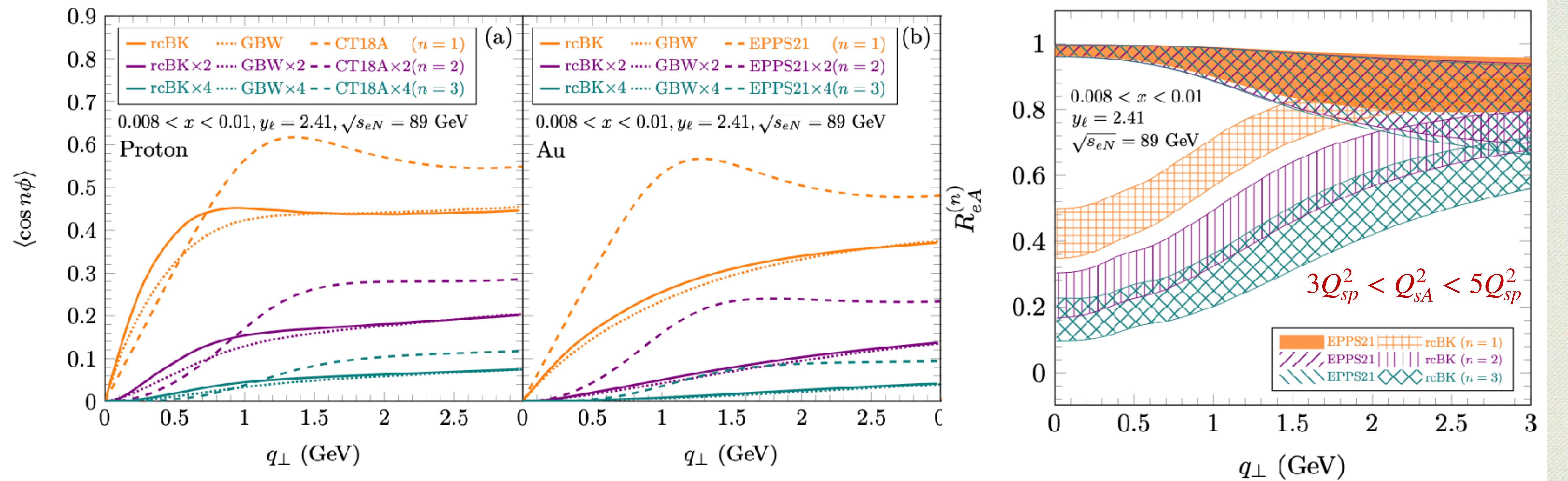
- Unintegrated quark distribution  $xf(x, b_\perp)$  related to dipole scattering matrix  $S_x$
- CGC framework, initial quark  $k_\perp$  determined by  $Q_s$ , has no angular preference
- Soft gluon radiation produce asymmetry, saturation suppress asymmetry

# Numerical Results of Harmonics $\langle \cos n\phi \rangle$



- Harmonics sizable decrease from ep to eA → nuclear modification factor  $R_{eA}^{(n)} = \frac{\langle \cos n\phi \rangle_{eA}}{\langle \cos n\phi \rangle_{ep}}$
- Suppression is significant at small  $q_\perp$  region ( $q_\perp \lesssim Q_s$ )

# Numerical Results of Harmonics $\langle \cos n\phi \rangle$



- Higher Harmonics more sensitive,  $R_{eA}^3 < R_{eA}^2 < R_{eA}^1$

Striking difference between saturation and non-saturation model

# Analytical analysis $\langle \cos n\phi \rangle$ at small $q_\perp$

$$\langle \cos n\phi \rangle = \frac{\int b_\perp db_\perp J_n(q_\perp b_\perp) xf_q(x, b_\perp) e^{-\text{Sud}(b_\perp)} \alpha_s(\mu_b) \frac{C_F c_n(R)}{n\pi}}{\int b_\perp db_\perp J_0(q_\perp b_\perp) xf_q(x, b_\perp) e^{-\text{Sud}(b_\perp)}} \approx \mathcal{C}_n q_\perp^n$$

$\approx \frac{(q_\perp b_\perp)^n}{2^n \Gamma(n+1)}$

- In correlation limit  $\vec{q}_\perp \ll \vec{P}_\perp \lesssim Q$ , use saddle point approximation

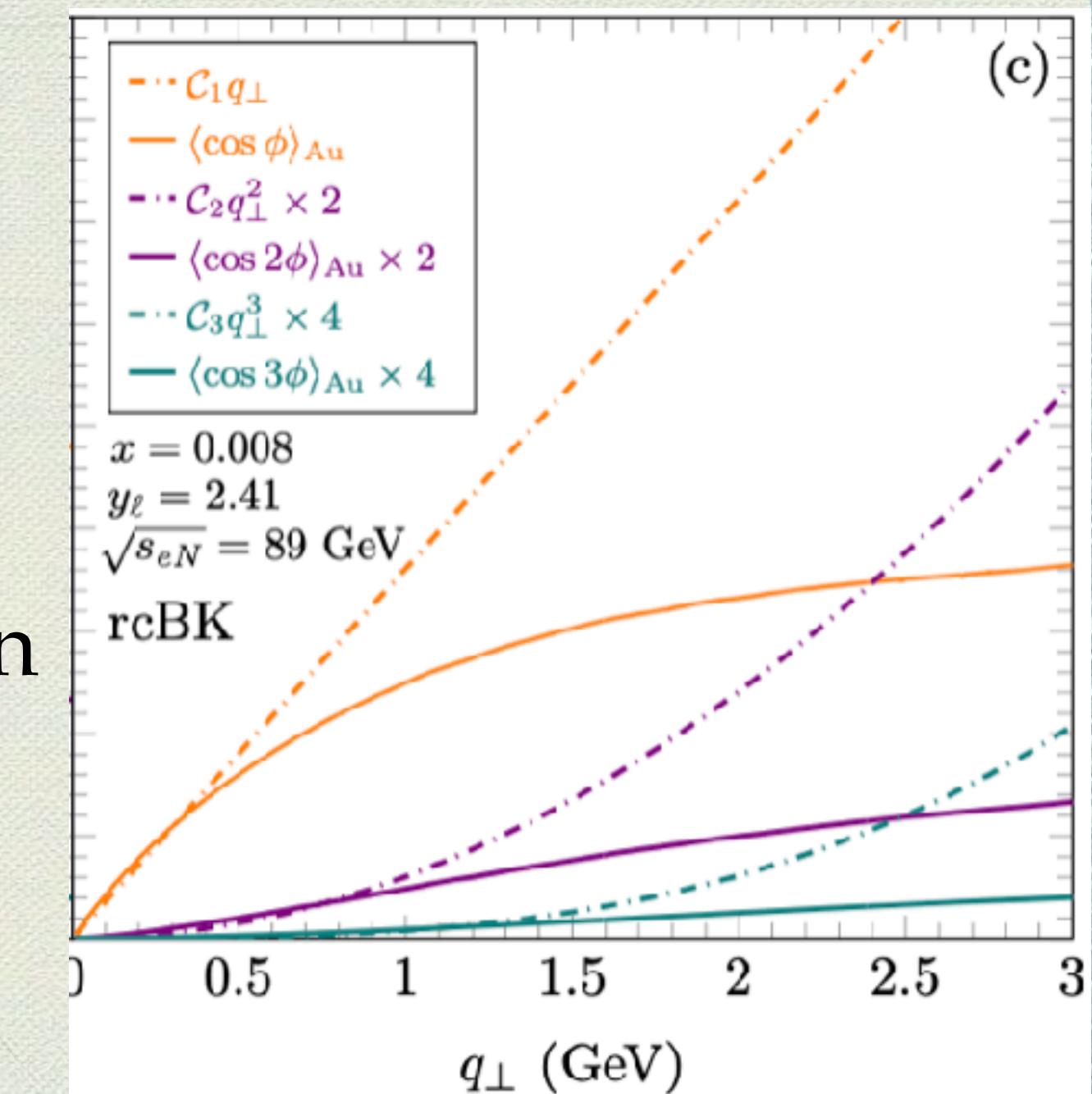
saddle point of numerator  $b_{\perp n}^{\text{sp}}$  of denominator  $b_{\perp 0}^{\text{sp}}$

Plot the  $\mathcal{C}_n q_\perp^n$  and  $\langle \cos n\phi \rangle$ , agree well in small  $q_\perp$  region

- Use small  $b_\perp$  approximation of quark TMD

$$\langle \cos n\phi \rangle \propto \frac{f_q(x, b_{\perp n}^{\text{sp}})}{f_q(x, b_{\perp 0}^{\text{sp}})} \approx \frac{\ln(Q_s b_{\perp n}^{\text{sp}})}{\ln(Q_s b_{\perp 0}^{\text{sp}})}$$

(1) with  $Q_s (\uparrow)$ ,  $\langle \cos n\phi \rangle \downarrow$  (2)  $R_{eA}^3 < R_{eA}^2 < R_{eA}^1$

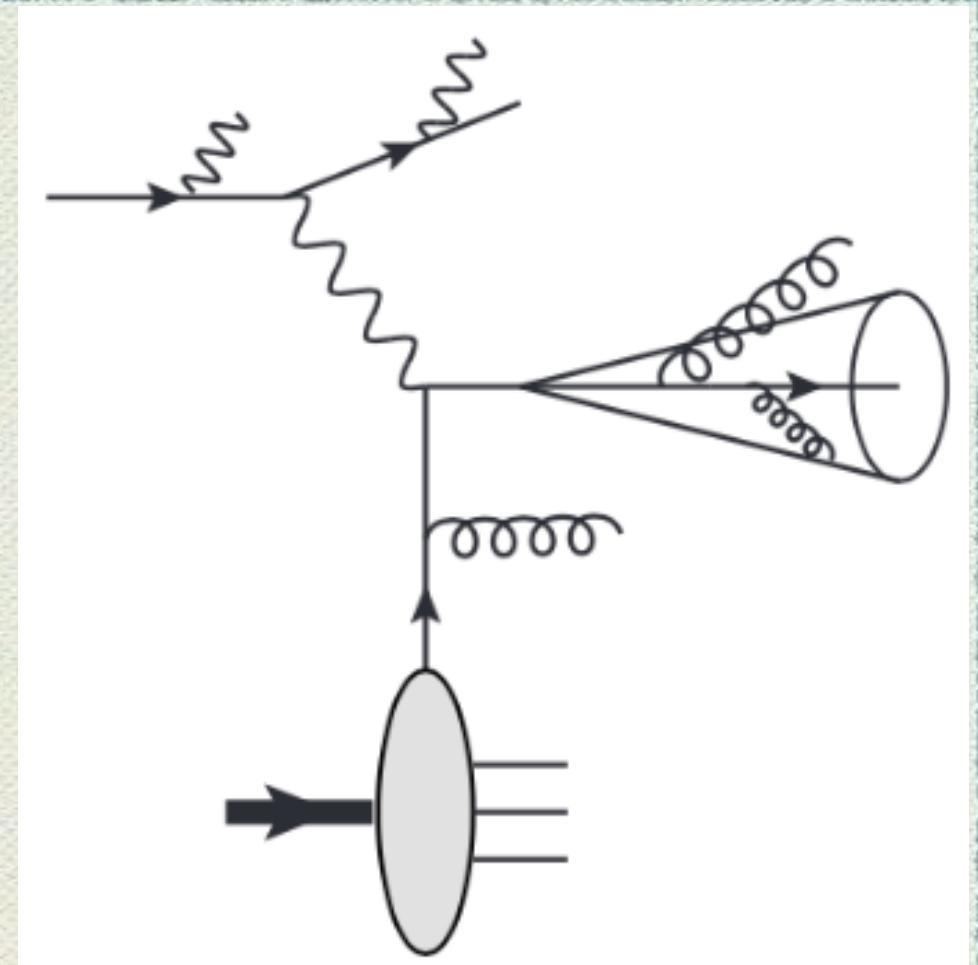


# QED corrections

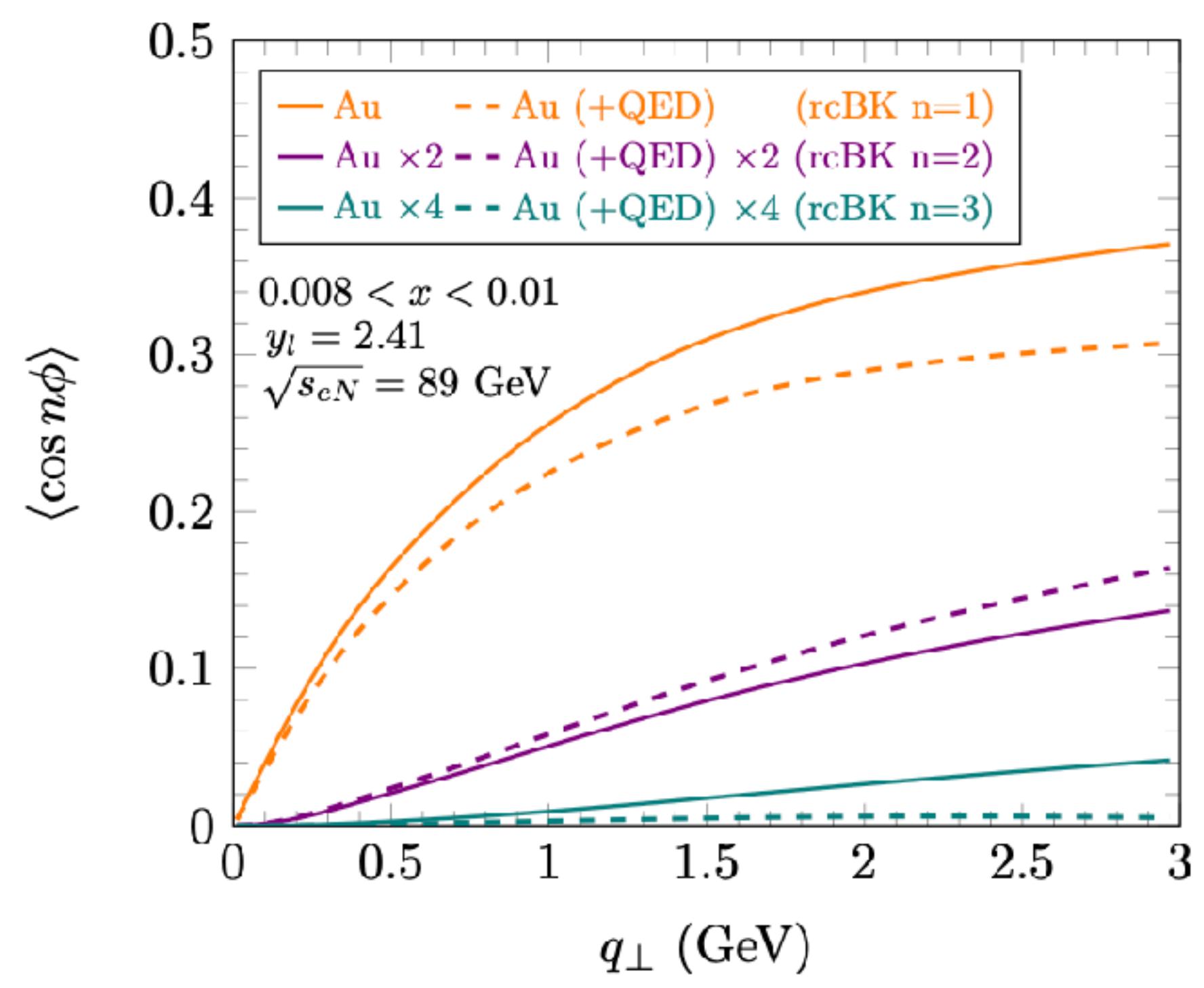
$$\langle \cos n\phi \rangle = \frac{\int b_\perp db_\perp J_n(q_\perp b_\perp) xf_q(x, b_\perp) e^{-(\text{Sud}^g + \text{Sud}^\gamma)} \frac{(\alpha_s C_F c_n(R) + \alpha_e c_n^\gamma)}{n\pi}}{\int b_\perp db_\perp J_0(q_\perp b_\perp) xf_q(x, b_\perp) e^{-\text{Sud}(b_\perp)}}$$

- ◆ Two corrections:
  - 1)  $\alpha_e \ll \alpha_s \rightarrow \text{Sud}^\gamma \ll \text{Sud}^g$
  - 2) Even  $\alpha_e \ll \alpha_s$ ,  $c_n^\gamma$  is large,  $\alpha_e c_n^\gamma$  sizable correction
- ◆ QED radiation favor lepton direction, the away side of jet direction reduce odd harmonics, increase even harmonics at small R

$$c_n^\gamma = (-1)^n \left[ \ln \frac{P_\perp^2}{m_e^2} + \frac{2}{\pi} \int_0^\pi d\phi (\pi - \phi) \frac{\cos \phi}{\sin \phi} (\cos n\phi - 1) \right]$$

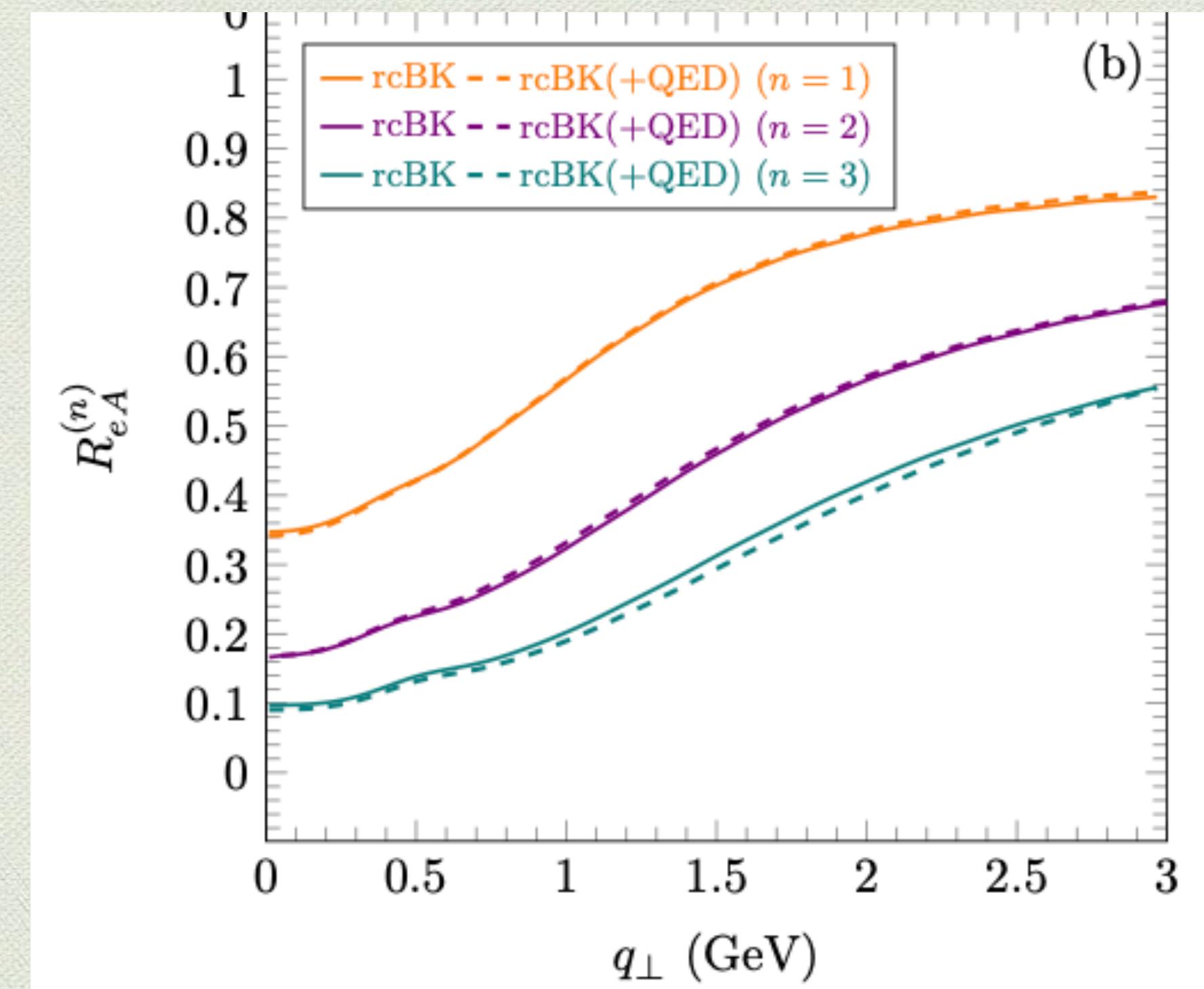


# QED corrections: numerical results



Sizable correction to  $\langle \cos n\phi \rangle$

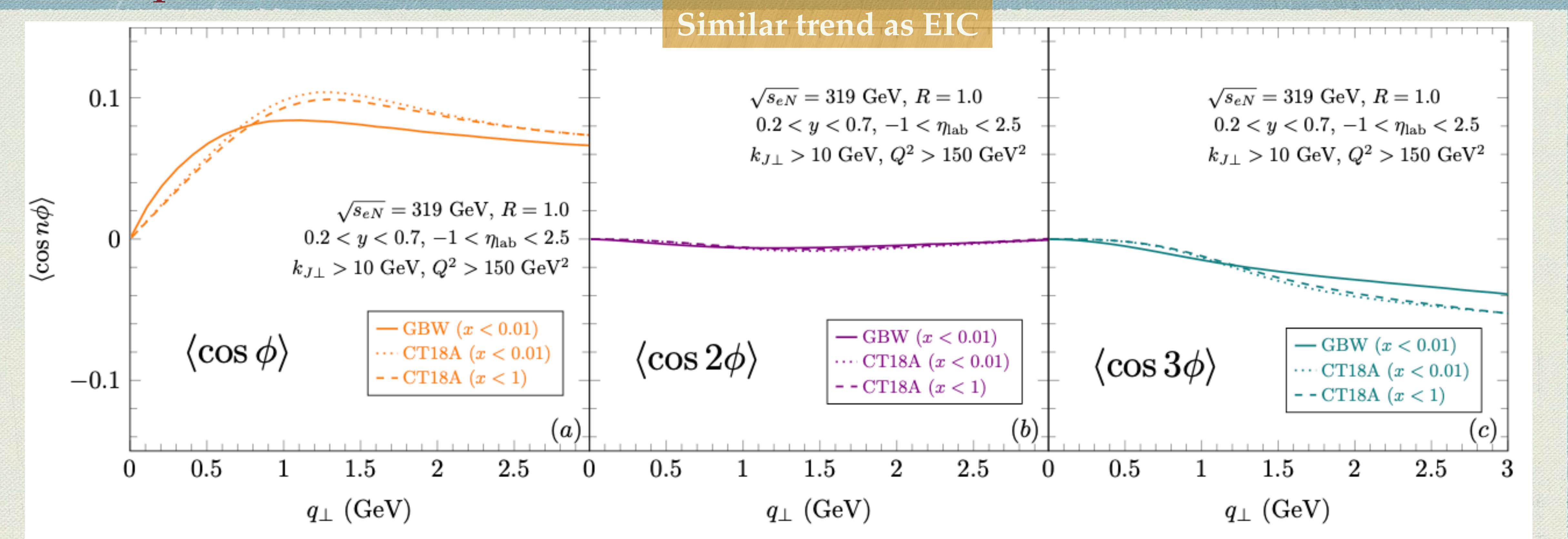
$$\langle \cos n\phi \rangle \propto (\alpha_s C_F c_n(R) + \alpha_e c_n^\gamma)$$



Negligible correction to  $R_{eA}^{(n)}$

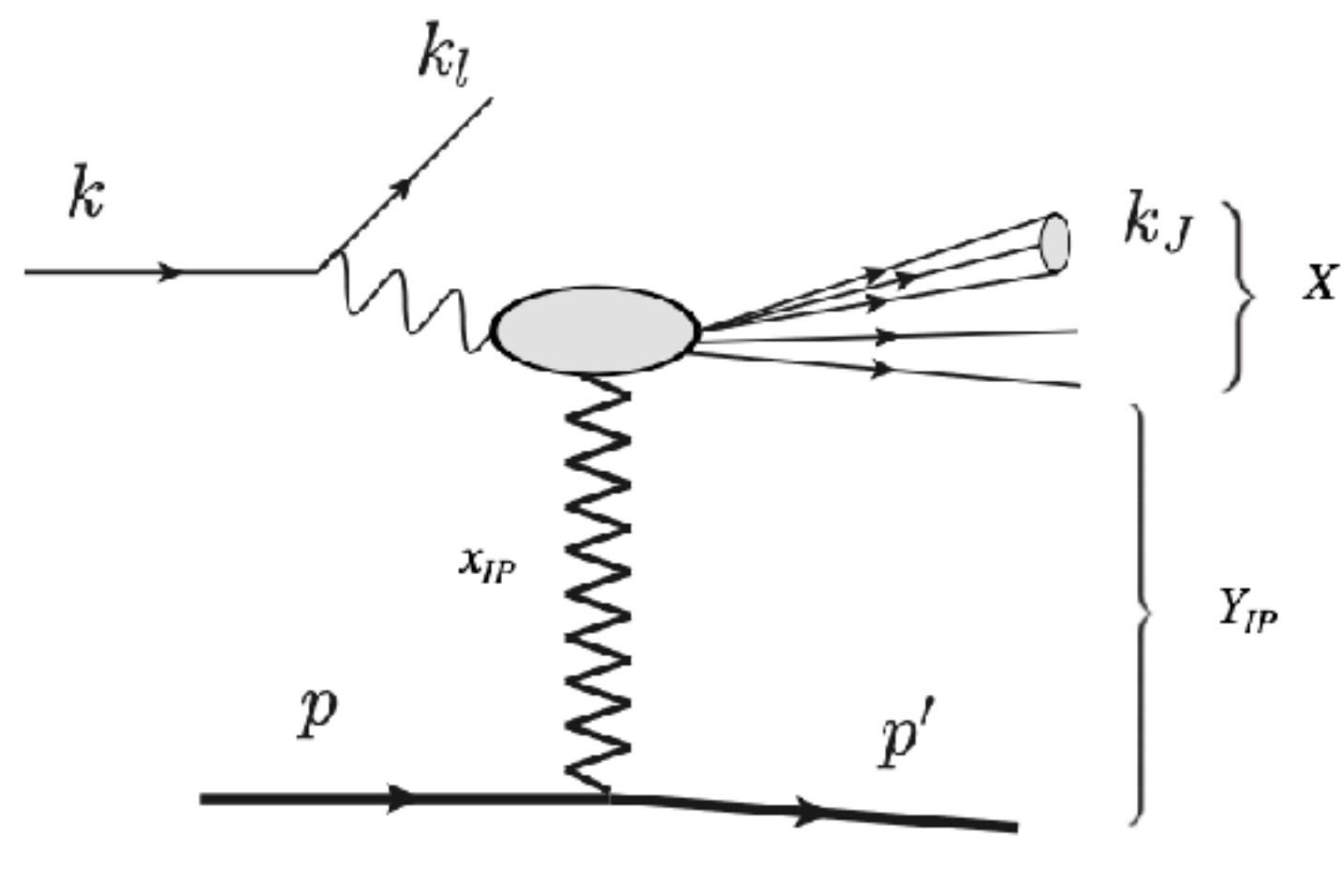
$$\frac{\langle \cos n\phi \rangle_{eA}}{\langle \cos n\phi \rangle_{ep}} \propto \frac{(\cancel{\alpha_s C_F c_n(R)} + \cancel{\alpha_e c_n^\gamma})}{(\cancel{\alpha_s C_F c_n(R)} + \cancel{\alpha_e c_n^\gamma})}$$

# HERA calculation: inclusive harmonics (e+p)



- ◆ GBW model (Saturation model) and non-saturation model with CT18A (proton) PDF
- ◆ Small- $x$  ( $x < 0.01$ ) and entire ( $x < 1$ ) region

# Diffractive Lepton-Jet Process



Besides lepton-jet, large rapidity gap between hard interaction product  $X$  and remnant proton/nucleus  $p'$

$$\langle \cos n\phi \rangle = \frac{\int b_\perp db_\perp J_n(q_\perp b_\perp) x \frac{df_q^D(\beta, b_\perp; x_{IP})}{dY_{IP}dt} e^{-\text{Sud}(b_\perp)} \alpha_s(\mu_b) \frac{C_F c_n(R)}{n\pi}}{\int b_\perp db_\perp J_0(q_\perp b_\perp) x \frac{df_q^D(\beta, b_\perp; x_{IP})}{dY_{IP}dt} e^{-\text{Sud}(b_\perp)}}$$

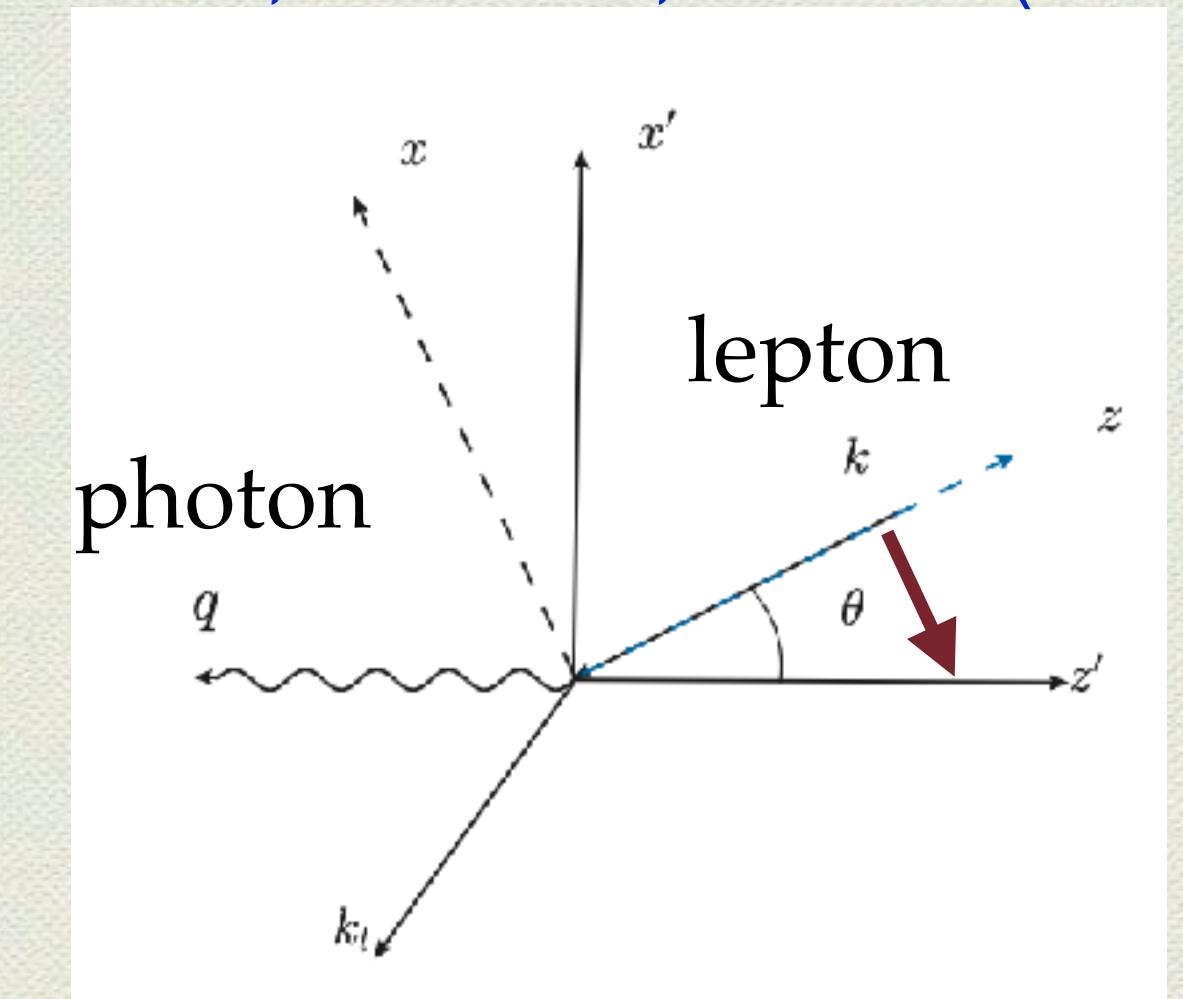
Replace quark TMD PDF by quark TMD Diffractive PDF

# Rapidity Gap for diffractive lepton-jet

Same as semi-inclusive diffractive DIS :  $Y_{IP} \sim \ln(1/x_{IP})$

Hatta-Xiao-Yuan, PRD 106, 094015 (2022)

- ◆ Lorentz transformation between these two frames involve boost and rotation
  - Diffractive lepton-jet process — lepton-nucleon rest frame
  - Semi-inclusive diffractive DIS — photon-nucleon rest frame
- ◆ Rapidity gap invariant under Lorentz boost
- ◆ Lorentz rotation matrix nearly identity matrix



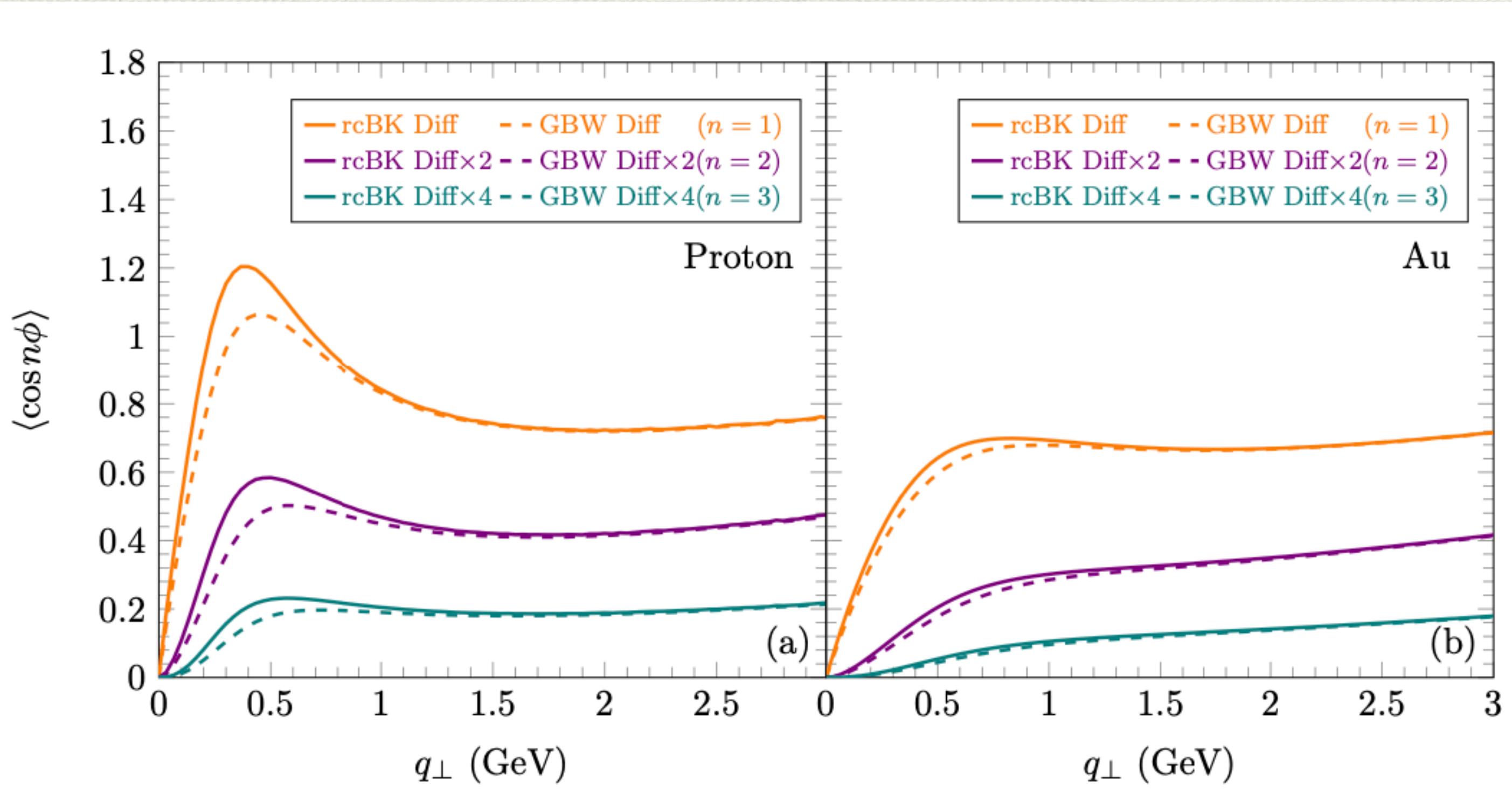
$$\sqrt{s_{eN}} = 89\text{GeV}, y_\ell = y_J = 2.41, x = 0.008$$

$$P_\perp = 4\text{GeV}, Q = 5.6\text{GeV}, \beta = 0.94$$

$$\rightarrow \theta = 0.00187$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & 0 & -\sin \theta \\ 0 & 0 & 1 & 0 \\ 0 & \sin \theta & 0 & \cos \theta \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

# Diffractive harmonics



- ◆ The suppression of harmonics from Proton to Au
- ◆  $\langle \cos n\phi \rangle_{\text{diff}}$  two times the value of  $\langle \cos n\phi \rangle$

$$\langle \cos n\phi \rangle_{\text{diff}} \approx 2 \langle \cos n\phi \rangle$$

Analytical expression  $\langle \cos n\phi \rangle \propto \frac{f_q(x, b_{\perp n}^{\text{sp}})}{f_q(x, b_{\perp 0}^{\text{sp}})}$

1)  $b_{\perp 0}^{\text{sp}} < b_{\perp n}^{\text{sp}}$  With EIC kinematics, we have

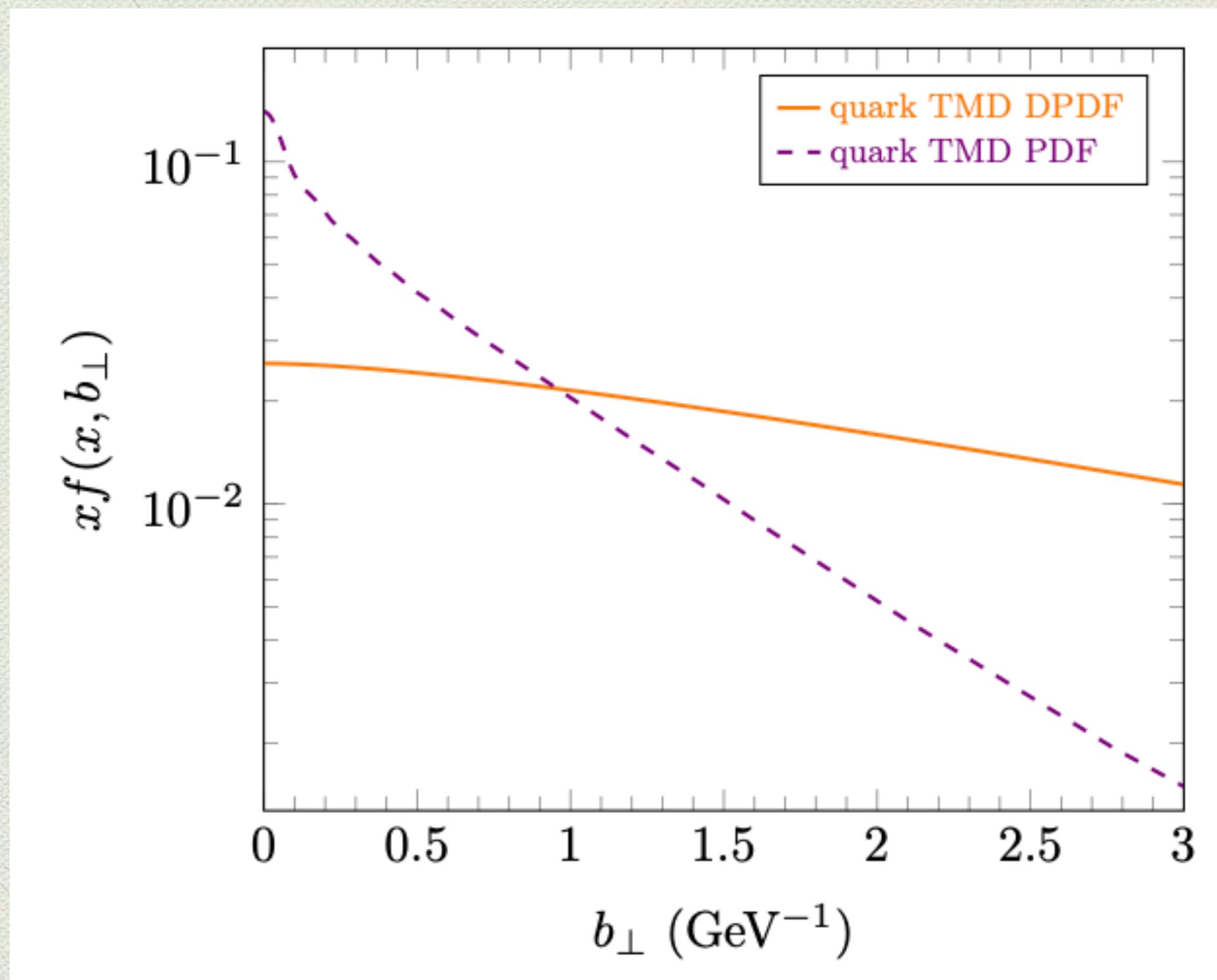
$$b_{\perp 0}^{\text{sp}}, b_{\perp n}^{\text{sp}} \in [0, 3] \text{ GeV}^{-1}$$

2) Flat diffractive PDF  $\rightarrow$

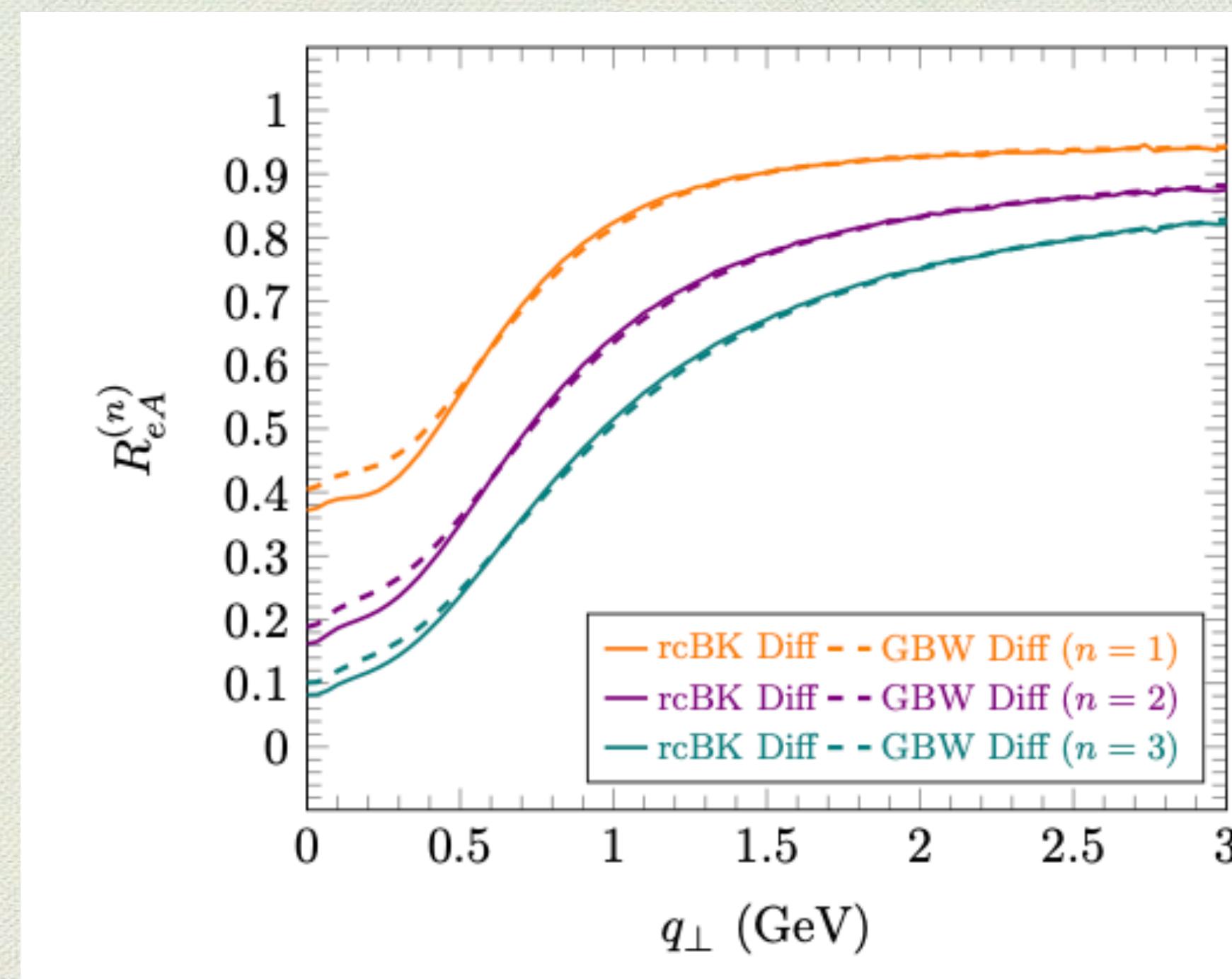
$$\frac{df_q^D(\beta, b_{\perp n}^{\text{sp}}; x_{IP})}{dY_{IP}dt} / \frac{df_q^D(\beta, b_{\perp 0}^{\text{sp}}; x_{IP})}{dY_{IP}dt} \approx 1$$

3) Steep PDF  $\rightarrow$

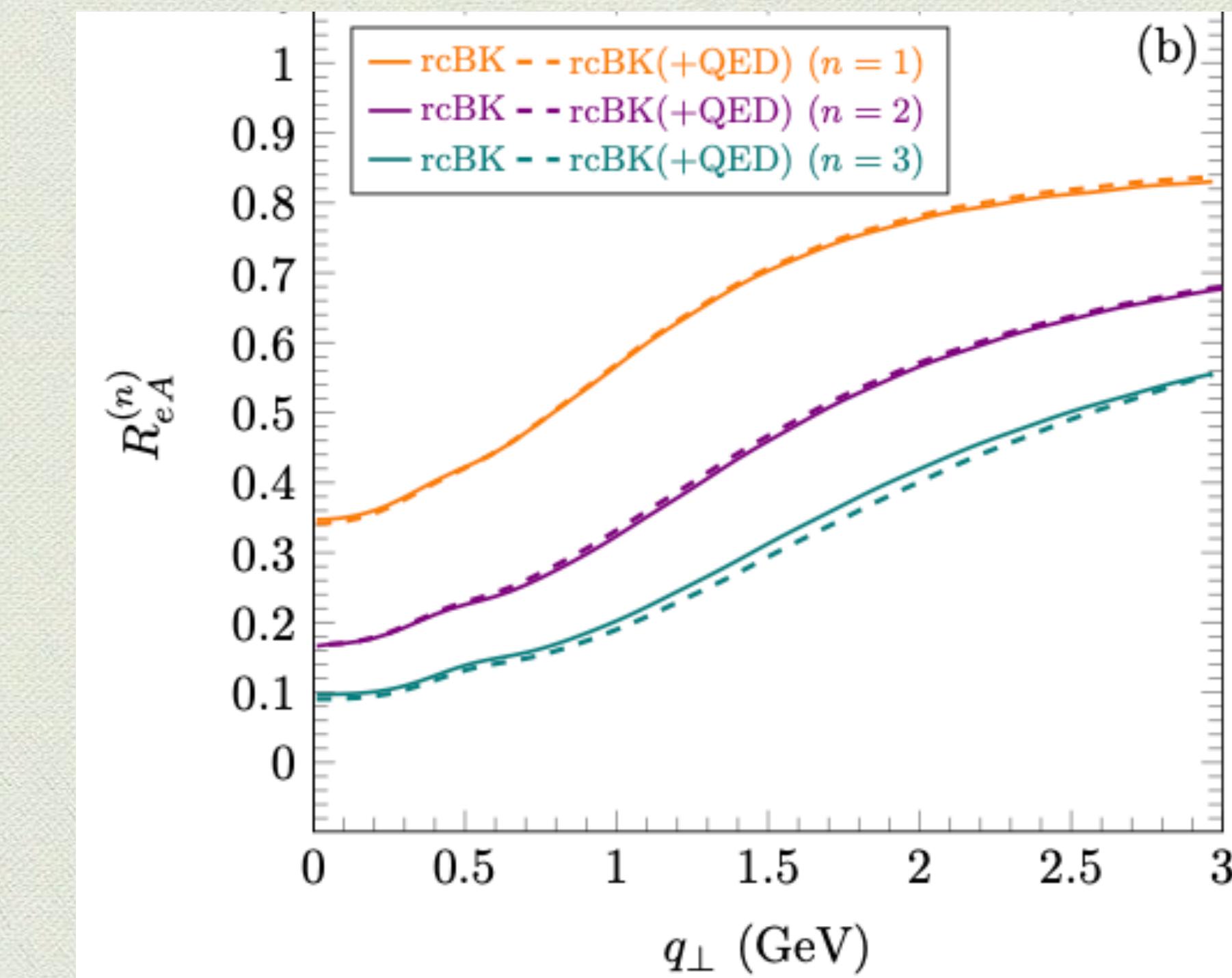
$$\frac{f(x, b_{\perp n}^{\text{sp}})}{f(x, b_{\perp 0}^{\text{sp}})} \ll 1$$



# Diffractive nuclear modification factor



Diffractive  $R_{eA}^n$



Inclusive  $R_{eA}^n$

- ◆ Diffractive  $R_{eA}^n$  almost the same as inclusive  $R_{eA}^n$

# t-dependence of $\langle \cos n\phi \rangle_{\text{diff}} \rightarrow$ nucleus density profile

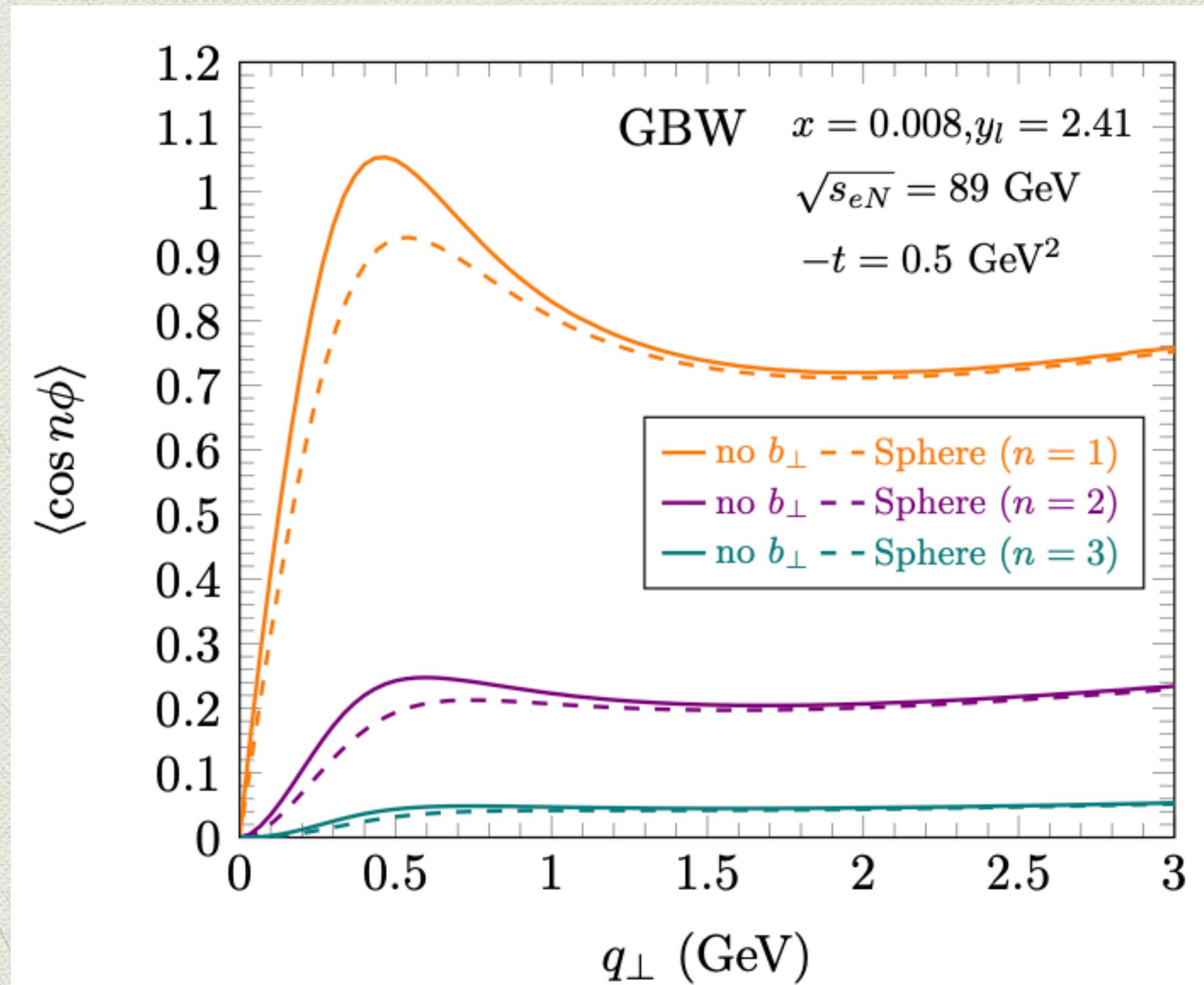
$$\frac{df_q^D(\beta, k_\perp; x_{IP})}{dY_{IP}dt} \text{ involves } \int \frac{d^2 b_\perp d^2 r_\perp}{(2\pi)^4} e^{i \vec{k}_{1\perp} \cdot \vec{r}_\perp + i \vec{\Delta}_\perp \cdot \vec{b}_\perp} \mathcal{S}_x(r_\perp, b_\perp) \quad (-t = \Delta_\perp^2)$$

1. No  $b_\perp$  dependence

$$S_x(r_\perp) = e^{-\frac{r_\perp^2 Q_s^2}{4}} \quad (\text{GBW})$$

2. Sphere Shape

$$S_x(r_\perp, b_\perp) = e^{-\frac{r_\perp^2}{4} C_s \sqrt{1 - b_\perp^2/r_p^2}}$$



# Summary

- ◆ Parton saturation one of three pressing question for future EIC
- ◆ Lepton Jet Correlation can be a new channel for gluon saturation
- ◆ Fourier analysis of LJC help to construct observable  $\langle \cos n\phi \rangle$ ,  $R_{eA}^{(n)} = \frac{\langle \cos n\phi \rangle_{eA}}{\langle \cos n\phi \rangle_{ep}}$   
sensitive to saturation
- ◆ Larger diffractive Lepton-Jet harmonics  $\langle \cos n\phi \rangle_{\text{diff}}$  — better probes for saturation, new probes for nucleus density profile