

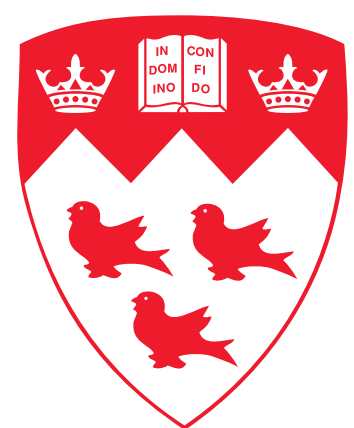


30th International Conference on Ultra-relativistic Nucleus-Nucleus Collisions (QM 2023)

Exploring the Freeze-out Hypersurface with a Rapidity- dependent Thermal Model

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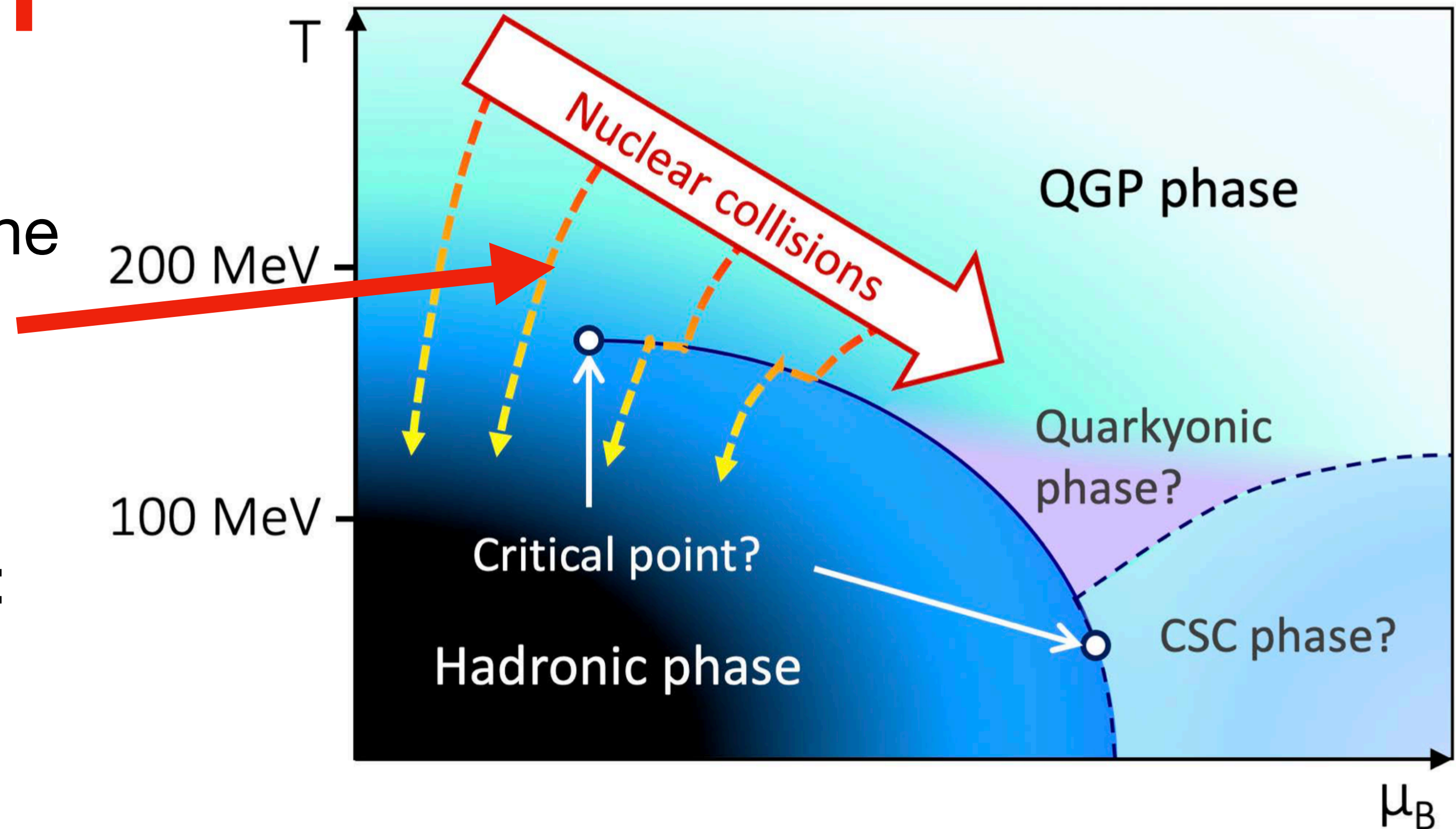
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Based on

Lipei Du, HG, Sangyong Jeon & Charles Gale, 2302.13852
HG, Lipei Du, Sangyong Jeon & Charles Gale, 23xx.xxxx

QCD phase diagram

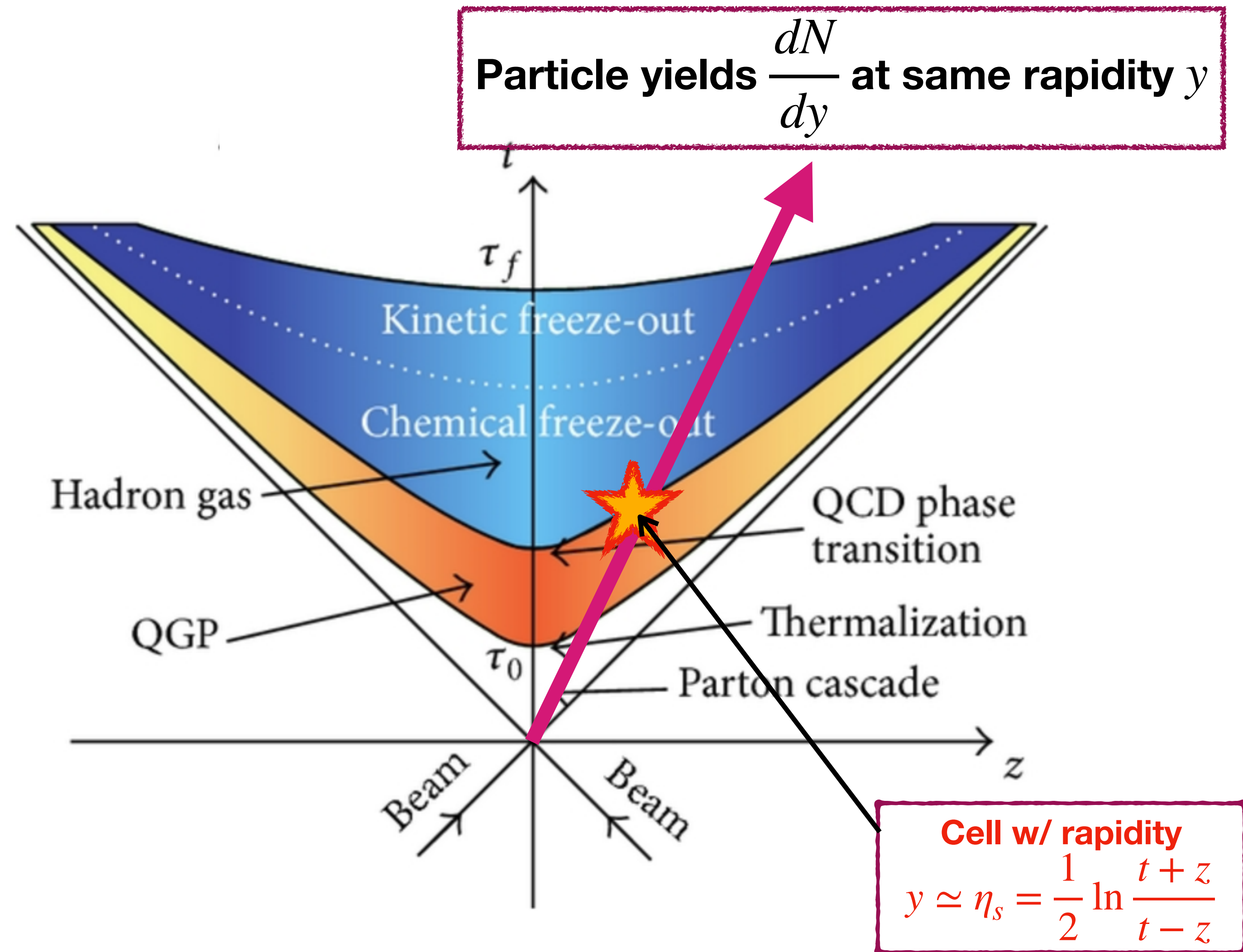
- Beam energy scan: Mapping out the phase diagram by varying the collision energies.
- **Thermal Models:** particle multiplicities \rightarrow thermodynamics: including Hadron Resonance Gas model (HRG).
- Tools available on the market, e.g. [V. Vovchenko & H. Stoecker, *Comput. Phys. Commun.* (2019)]



A. Monnai, et al, IJMPA (2021)

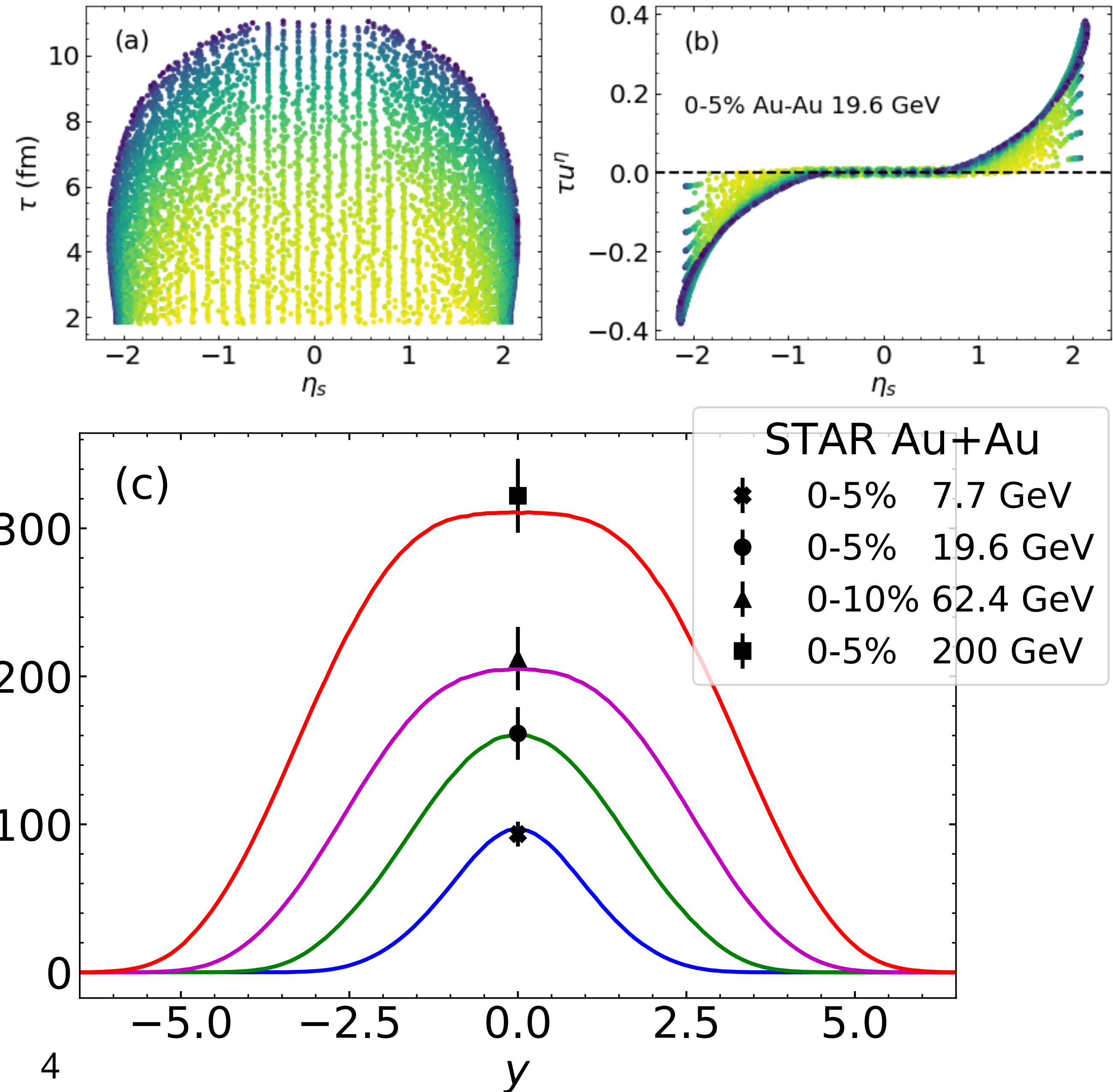
Rapidity scan along the freeze-out surface

- Freeze-out surface is not homogenous.
- **Rapidity scan:** inferring freeze-out thermodynamics for cells at different rapidity η_s from particle yields dN/dy .
- Commonly used practice: using HRG for *each rapidity bin independently*. See, e.g. [V. Begun, et al, PRC (2018)]



About large rapidity?

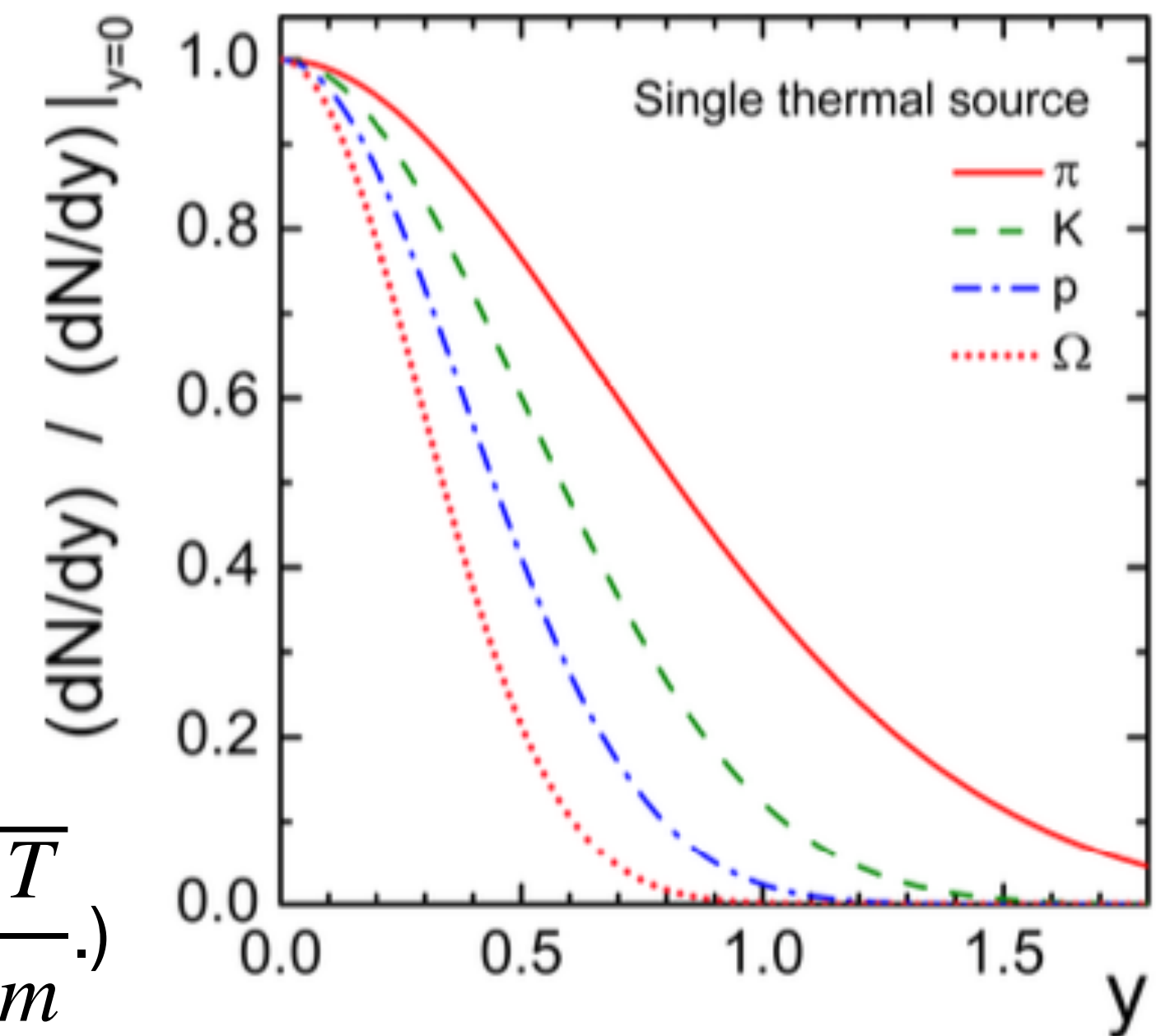
- Multistage hydro => freeze-out cells live within a limit range of η_s .
- E.g., right fig. $|\eta_s| < \eta_{max} \approx 2$ for $\sqrt{s} = 19.6$ GeV.
- Particle yields reach $y \approx 4$
- How to improve the “commonly used practice” (independent y bins) for large rapidity (tail) region?



Two effects

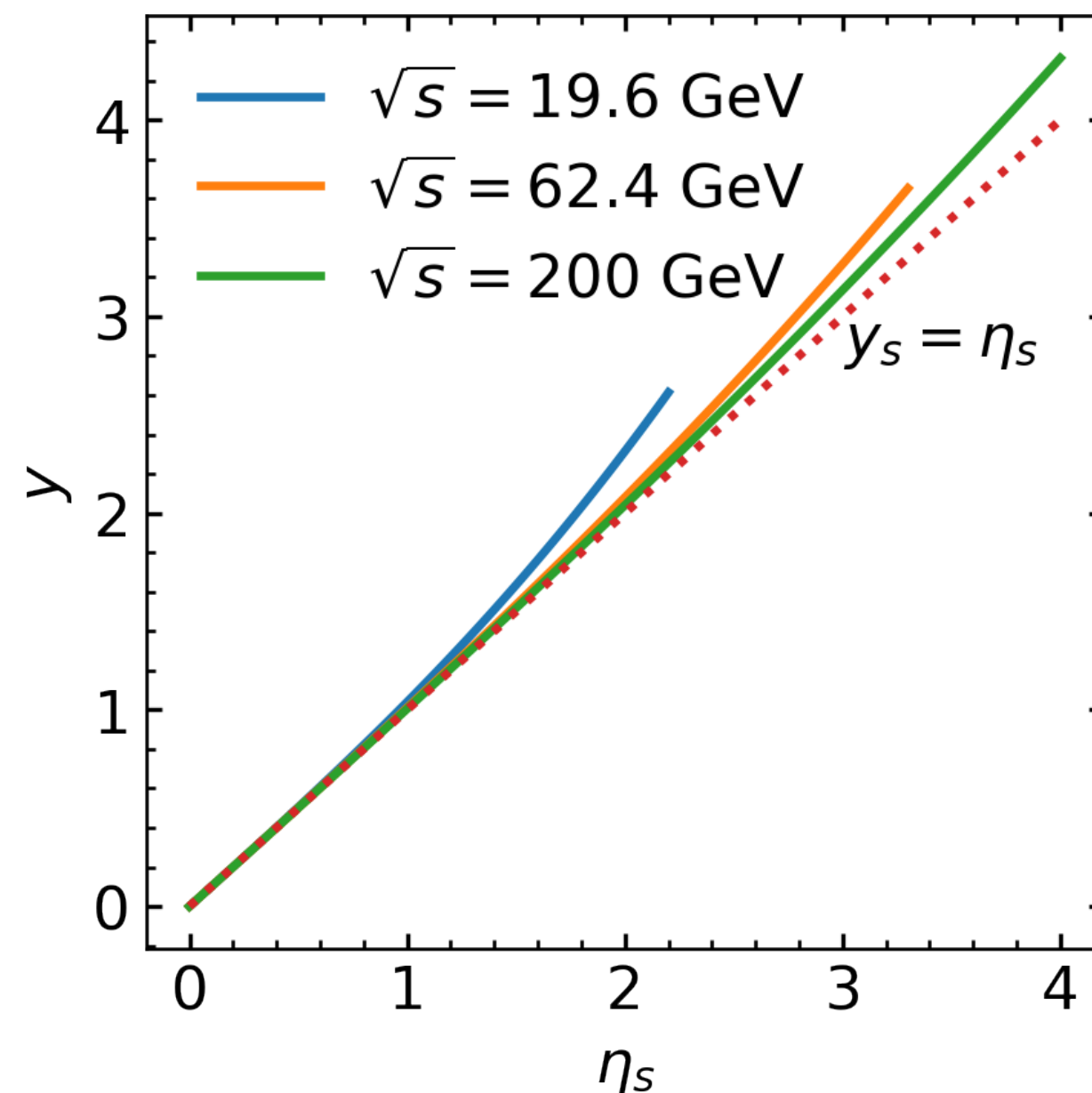
- **Thermal smearing:** a thermal source with rapidity y_s contributes to particle yields at other rapidities.

Fig: yields from a resting source ($y=0$).



[V. Begun, et al, PRC (2018)]

Significant smearing effect for lighter particles (e.g. π . smearing width $\Delta y \sim \sqrt{\frac{T}{m}}$.)



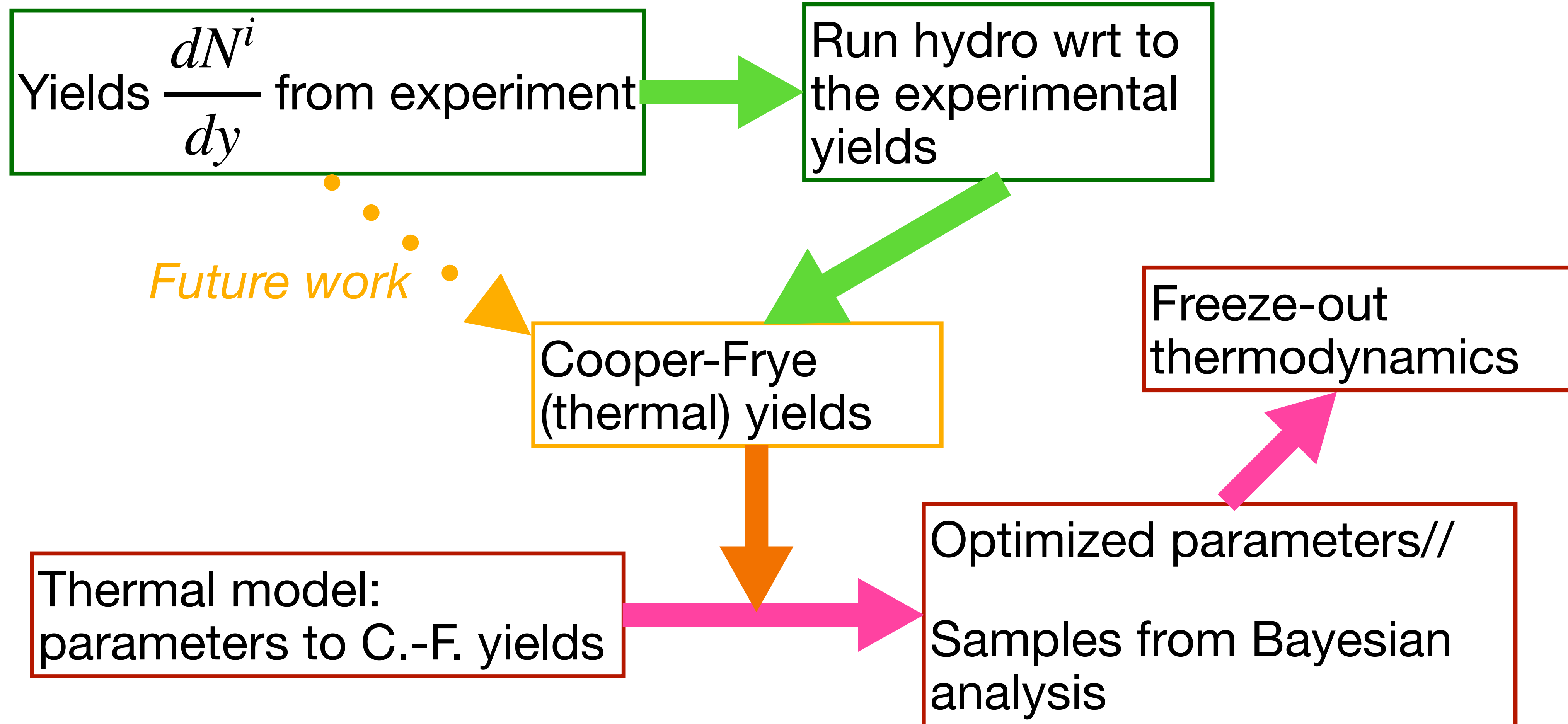
- **Longitudinal boost:** Deviation from Bjorken
Flow as $\sqrt{s} \downarrow : \eta_s < y_{s^*}$ subscript s = source, here FO cells
- Particles produced as a freeze-out cell with small η_s can be boosted to a large rapidity y .

A thermal model with smearing effect + longitudinal flow

- Parametrizing $T(\eta_s) = T_0 + T_2\eta_s^2 + \dots$, $V(\eta_s)$, $\mu(\eta_s)$;
 - Convert cell's space-time rapidity η_s to rapidity y_s by kinematics $\tau u^\eta = \boxed{\alpha} \eta_s^3$.
 - For a cell with $y_s = 0$, thermal yields worked out as $\frac{dN^i}{dy} \equiv K^i(y; T, \mu, V)$.
* $i = \pi^+, K^+, p - \bar{p}$
 - Integrating over all cells (with different η_s) by

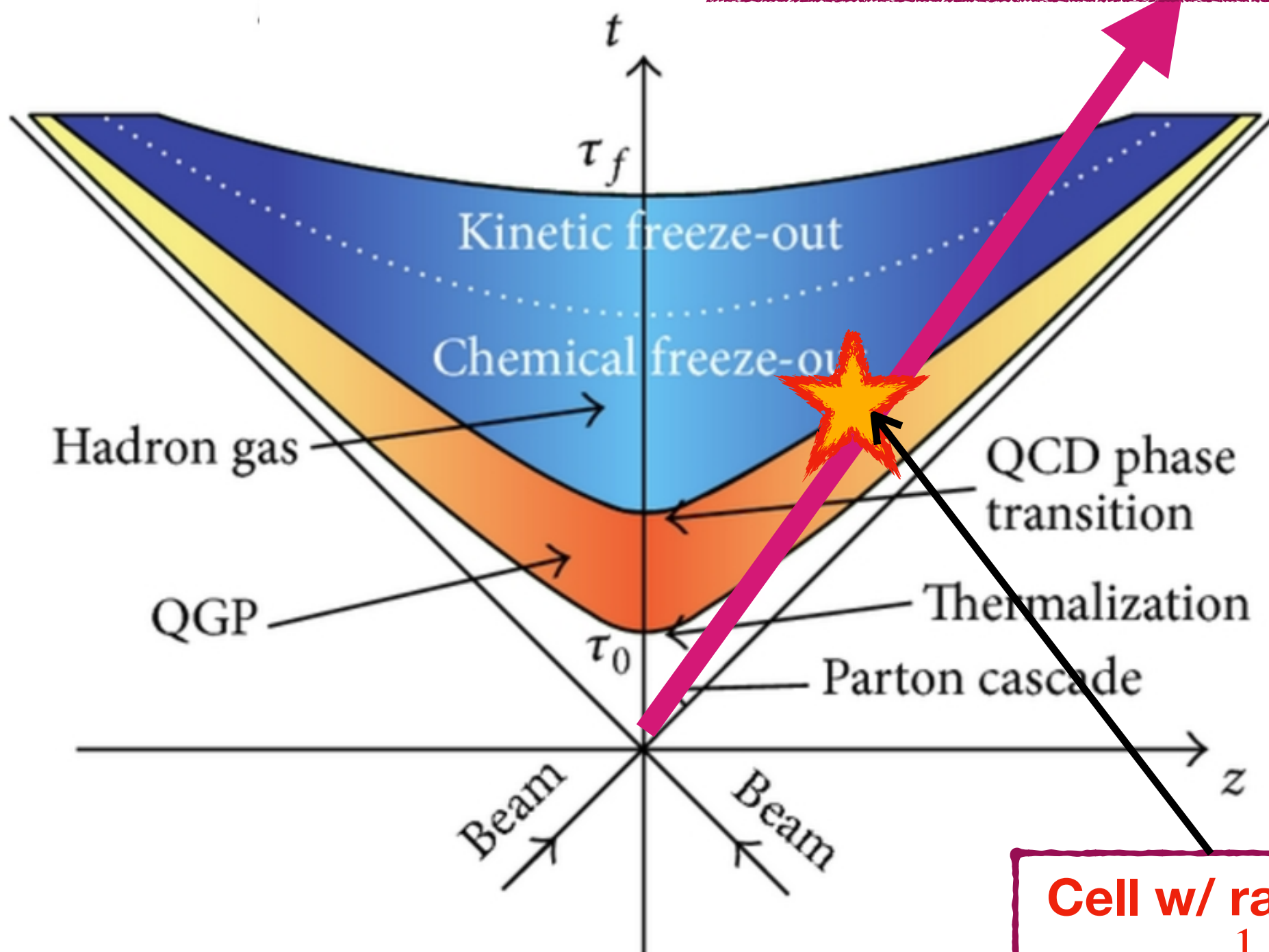
$$\frac{dN^i}{dy} = \int_{|\eta_s| < \boxed{\eta_{max}}} d\eta_s K^i(y - y_s(\eta_s); T(\eta_s), \mu(\eta_s), V(\eta_s))$$
- Longitudinal dynamics

Workflow: implementing the model



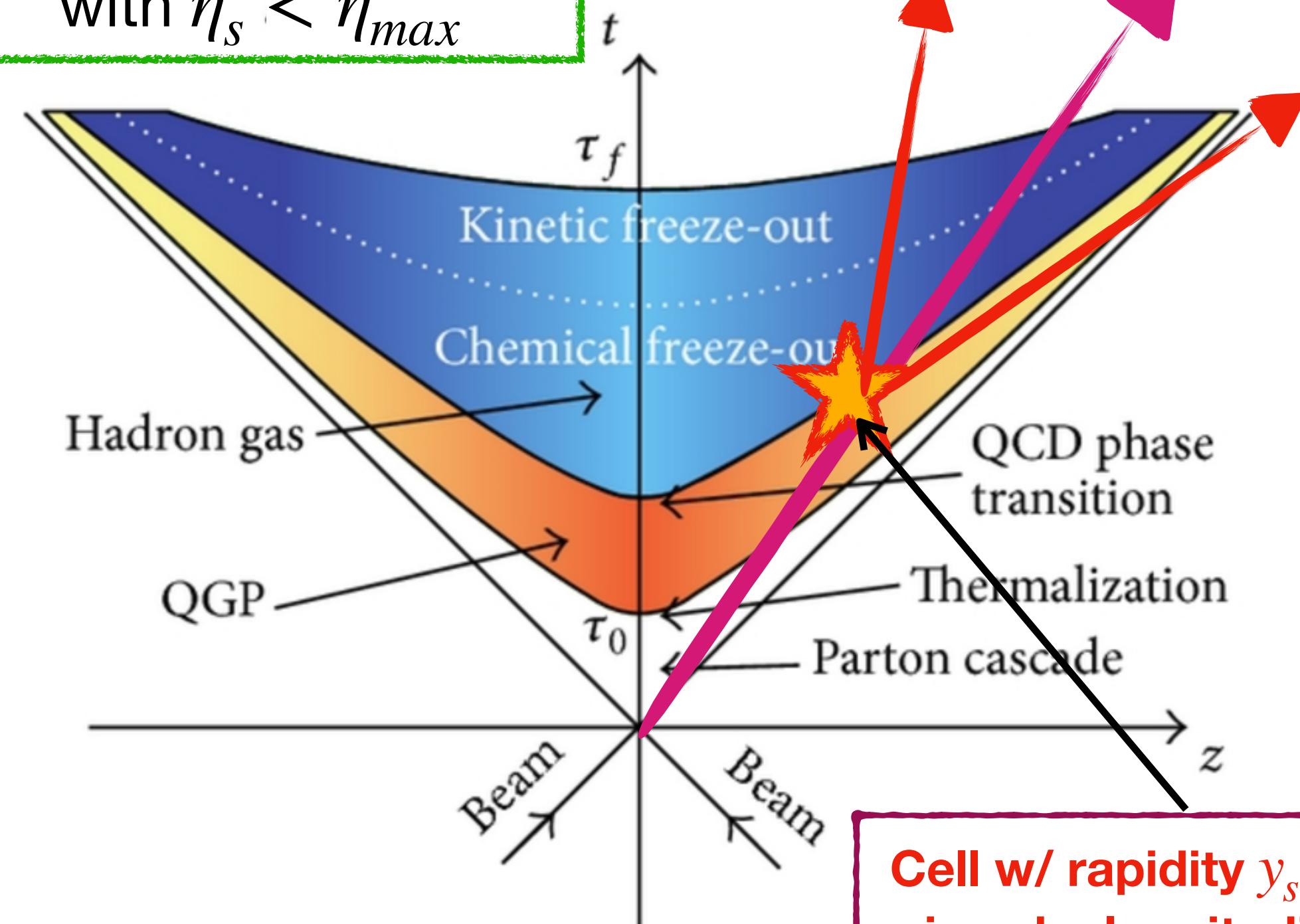
“Discrete” v.s. “Continuous” thermal model

Particle yields $\frac{dN}{dy}$ at
same rapidity $y = y_s$



Cell w/ rapidity
 $y_s \simeq \eta_s = \frac{1}{2} \ln \frac{t+z}{t-z}$

Final $\frac{dN}{dy}$ obtained by
summing over all cells
with $\eta_s < \eta_{max}$

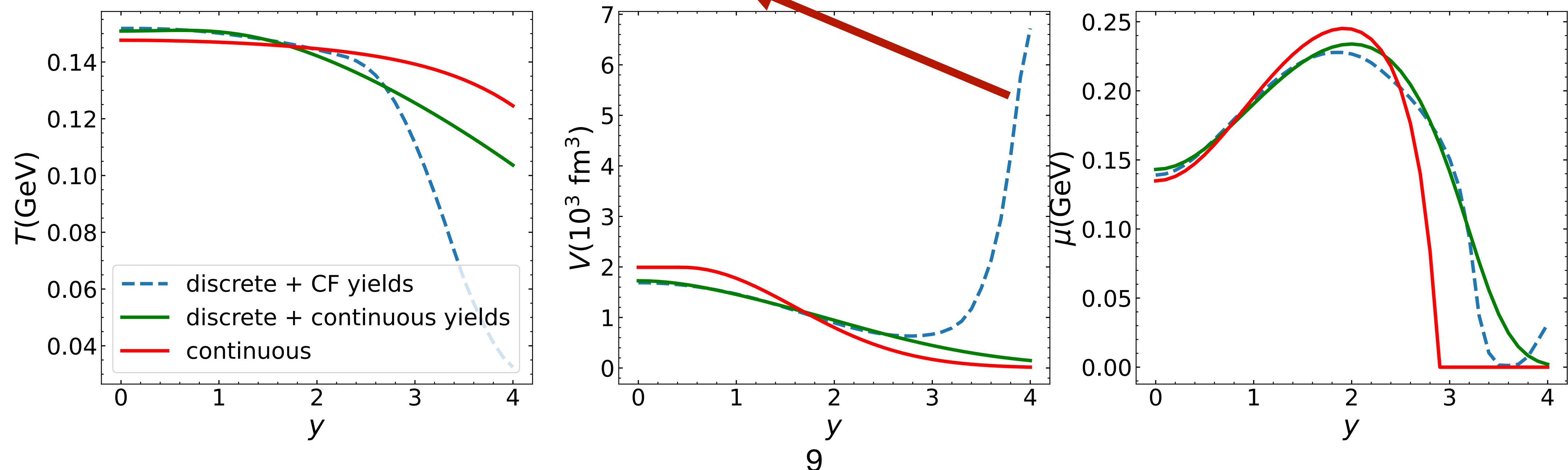


Cell emits particles
with all rapidity y ,
centred around y_s

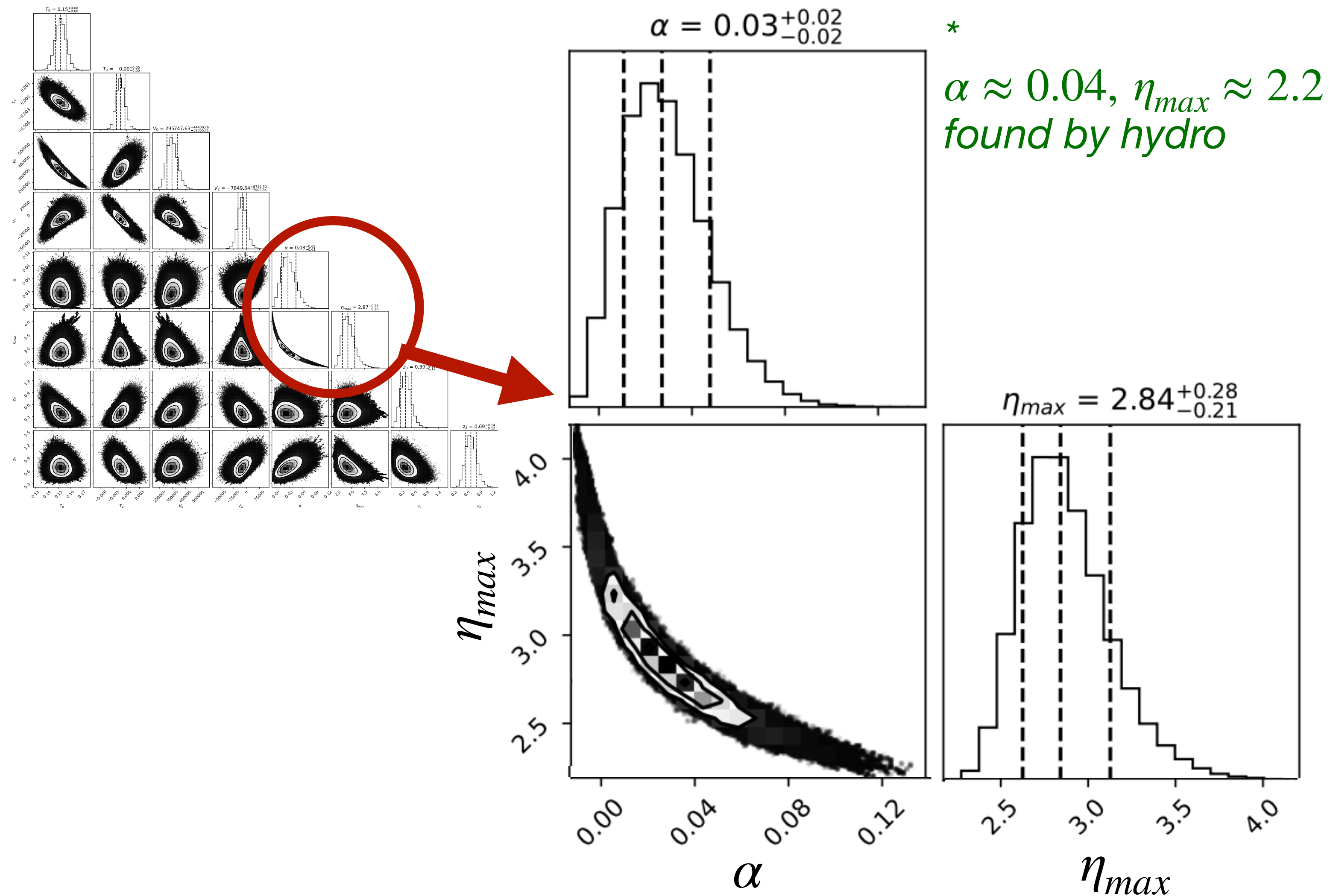
Cell w/ rapidity $y_s(\eta_s) > \eta_s$
given by longitudinal flow

Comparison: “discrete” and “continuous” models

- Fit **C.-F. yields** from a multistage hydro @ 19.6 GeV; longitudinal flow turned off in thermal model. Red/Green lines: two models applied to the same yields.
- Similar (T, μ_B) given around mid-rapidity $|y_s| < 2$ from both models => can safely use the independent-rapidity-bin method for mid-rapidity.
- Large uncertainty and unphysical result given by discrete model at large rapidity.



A Bayesian study: Longitudinal dynamics

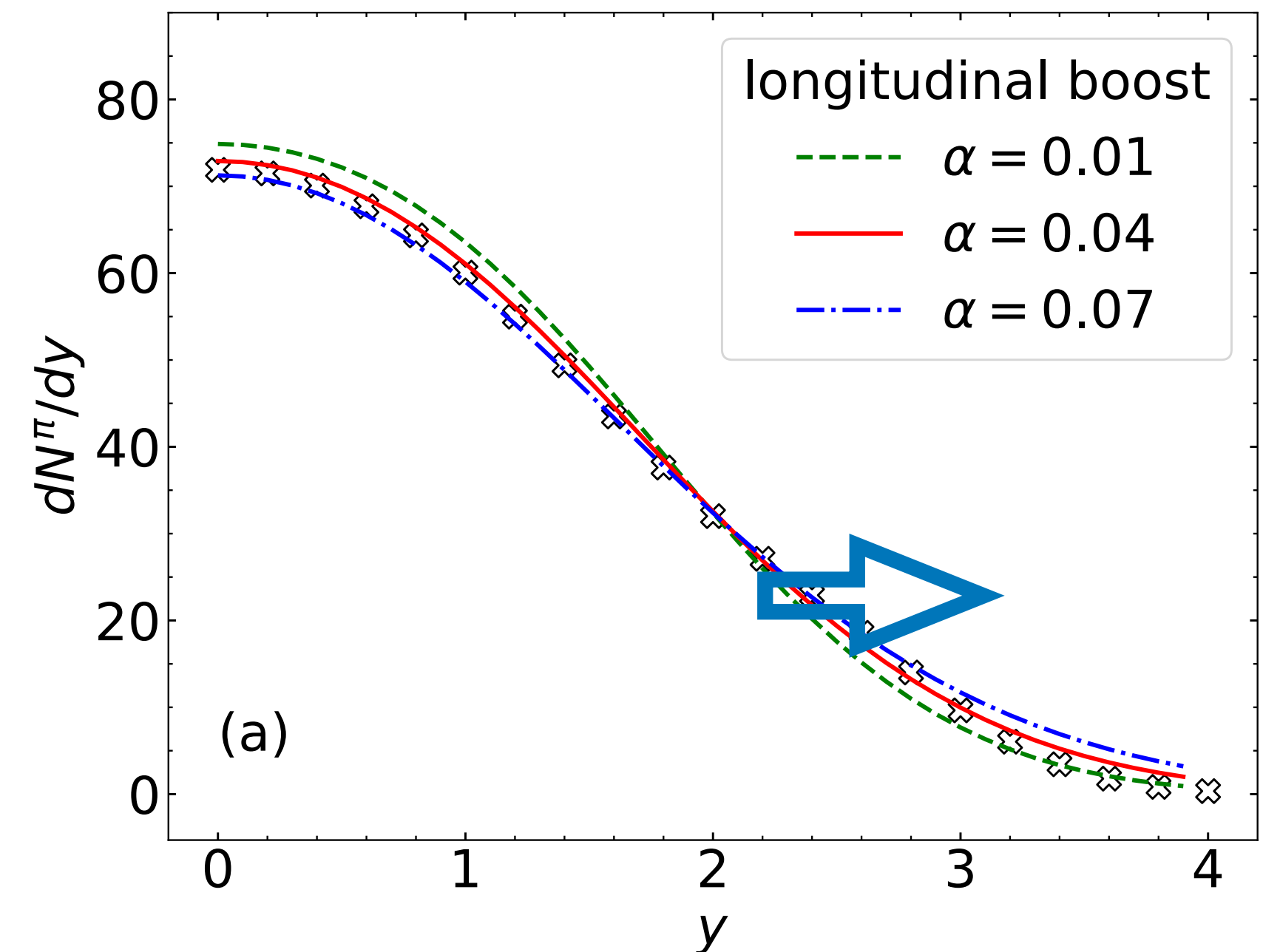
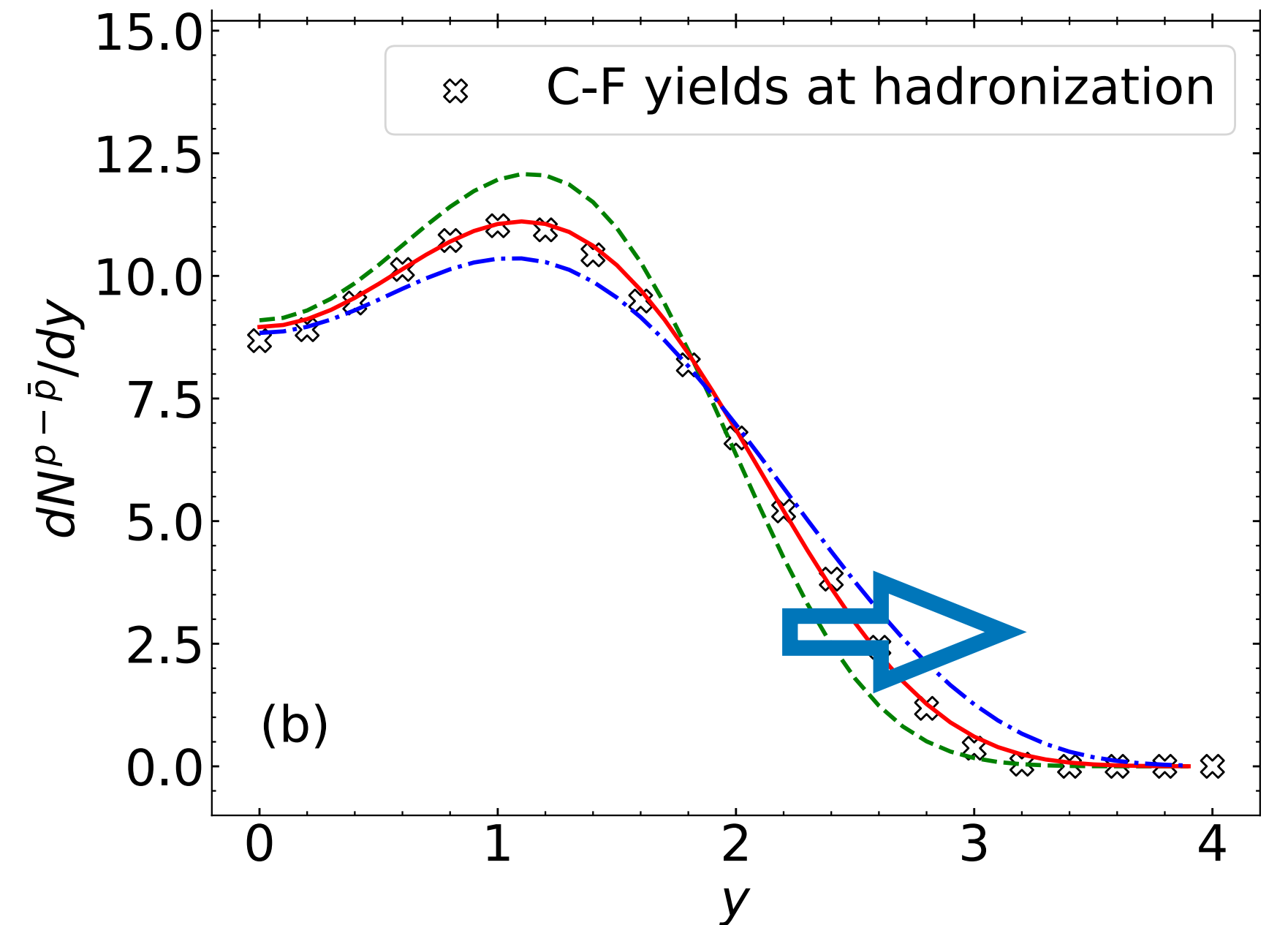


- A strong correlation between system size η_{max} and flow strength α .
- Flow parameters can still be constrained.
- A positive α is favoured => see longitudinal dynamic from a thermal model!

Effects on yields from the flow

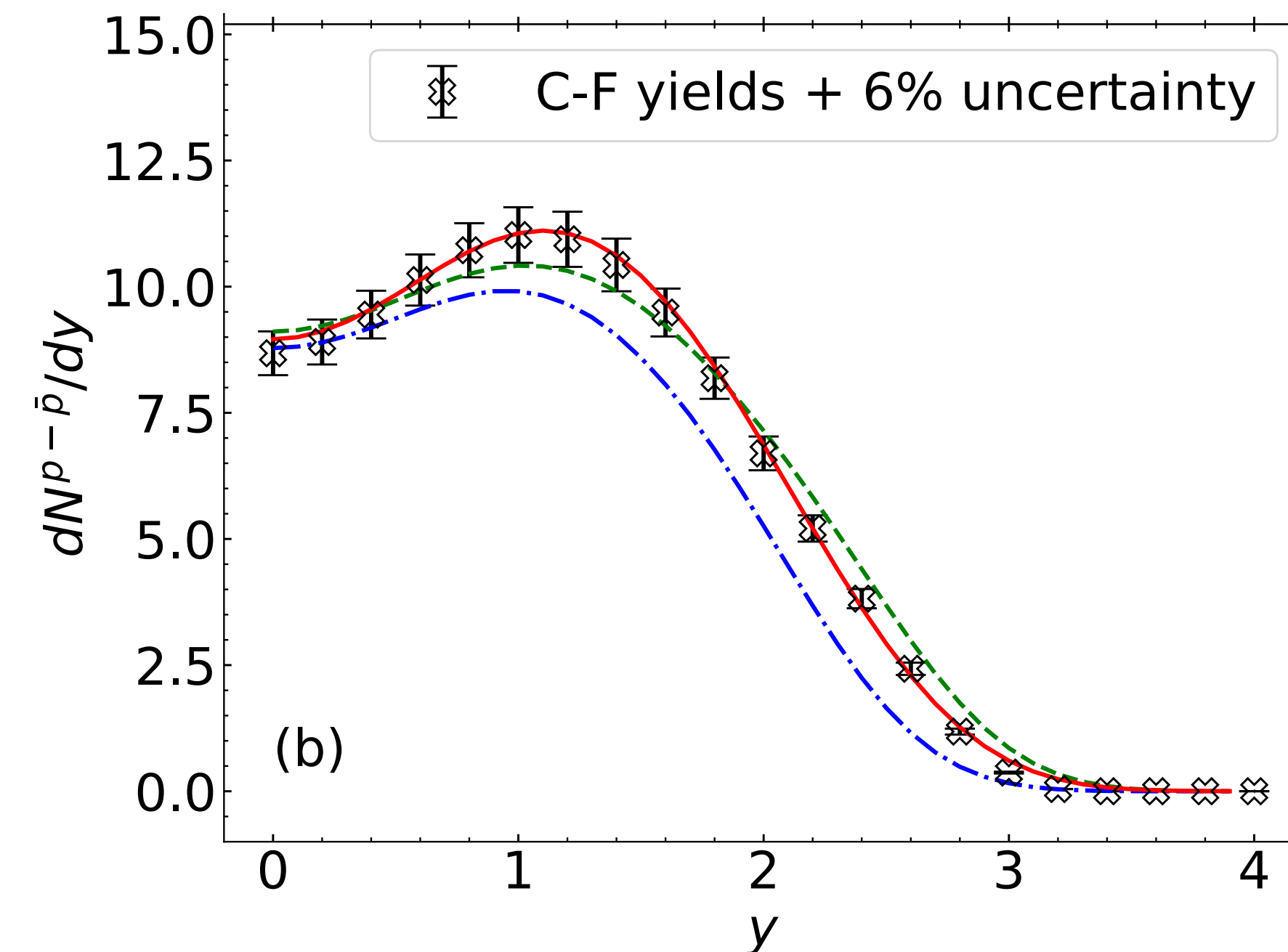
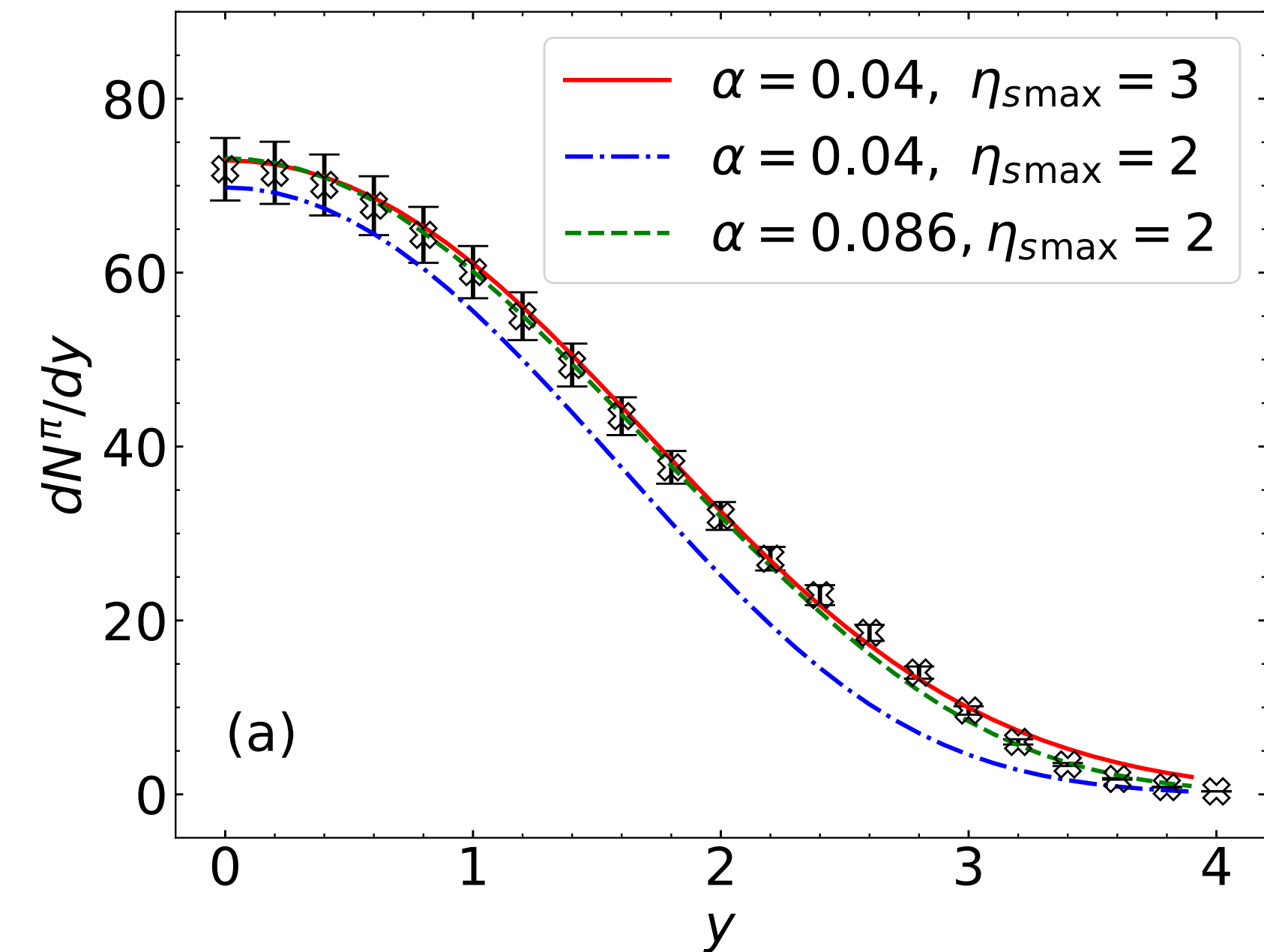
- From hydro profile @19.6 GeV: longitudinal flow $\alpha = 0.04$.
- Fix α but $\eta_{max} = \infty \Rightarrow$ Obtaining $T(\eta_s), \mu(\eta_s), V(\eta_s)$ profile by fitting the Cooper-Frye yields.
- Keeping the $T(\eta_s), \mu(\eta_s), V(\eta_s)$ profile obtained and varying $\alpha \Rightarrow$ Exploring the role of longitudinal flow

Larger flow \Rightarrow More particles boost to large y from mid rapidity

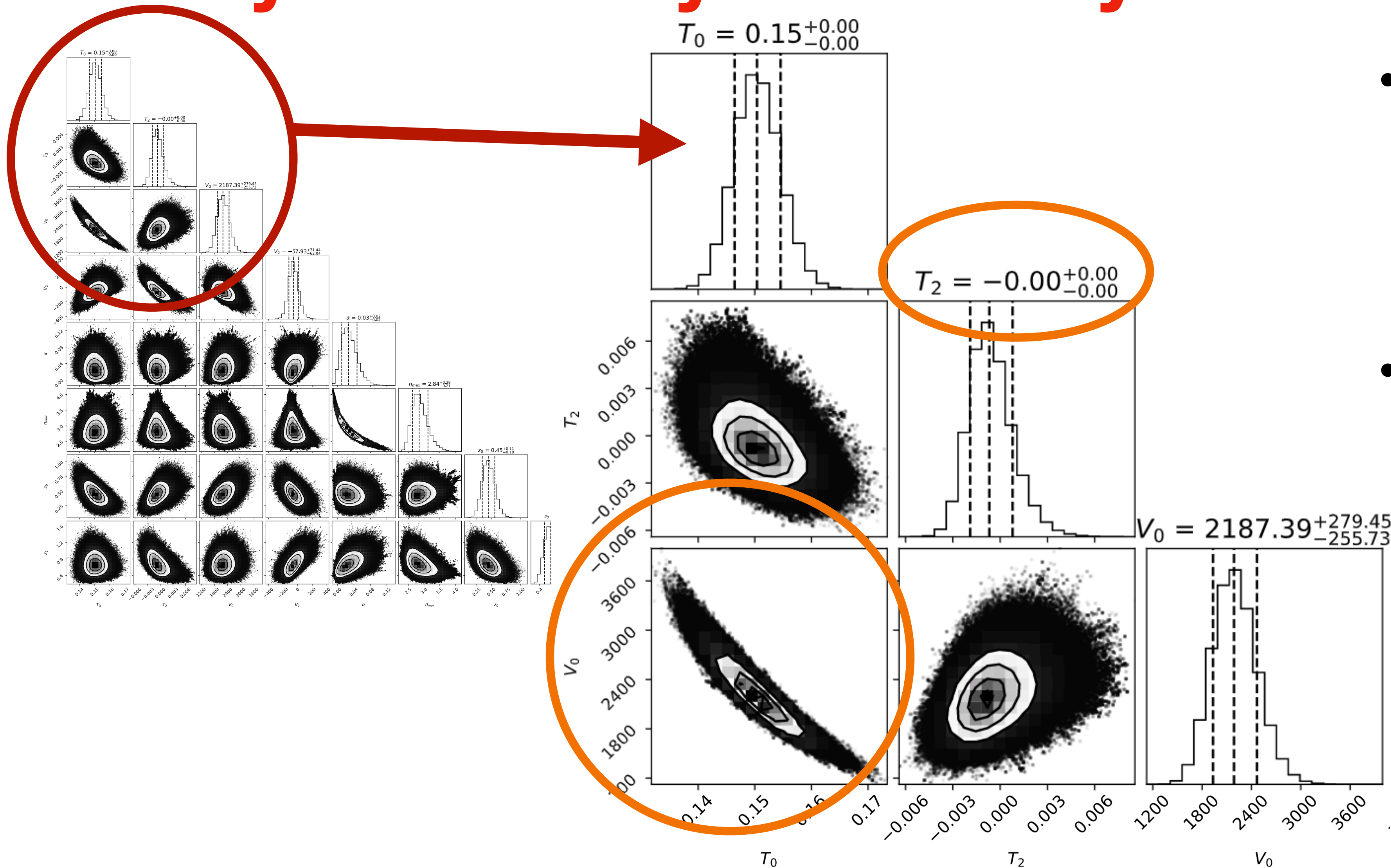


Coupling between system size and flow

- Now we turn on a finite system size $\eta_{max} = 2 \Rightarrow$ yields overall smaller \Rightarrow smearing effect manifested.
- Smaller **system size** can be partially compensated by a **stronger flow** \Rightarrow coupling between α and η_{max} .



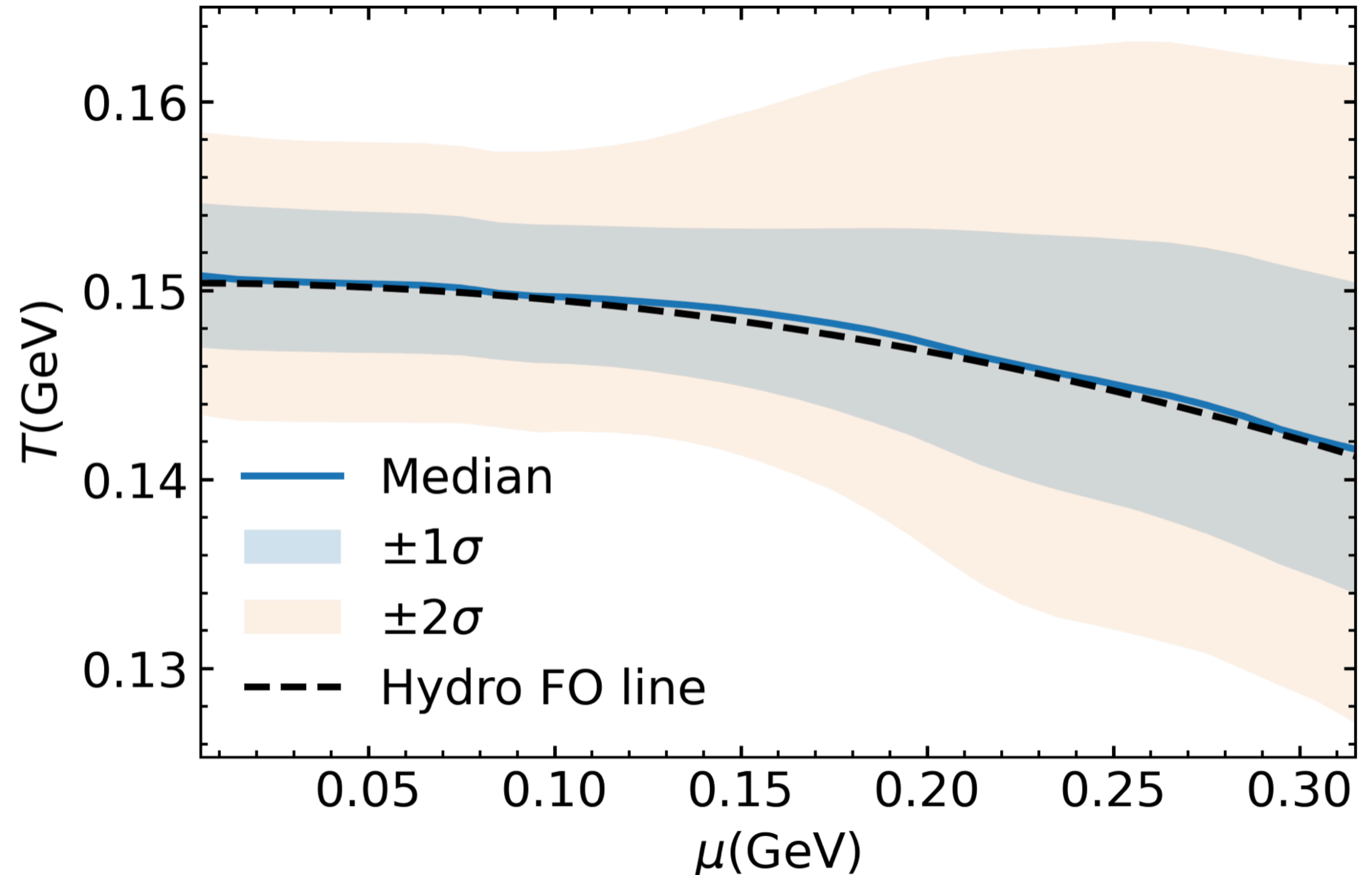
A Bayesian study: Thermodynamics



- Almost **isothermal** freeze-out surface:
 $T(\eta_s) = T_0 + T_2 \eta_s^2$
w/ a very small T_2 .
- Large correlation between mid-rapidity temp T_0 and transverse system size V_0 : total **entropy** $\sim VT^3$ should be conserved.

Consistency check w/ hydro freeze-out

- Freeze-out condition used in our hydro:
constant energy density
 $e_{fo} = 0.26 \text{ GeV/fm}^3$.
- Yields are generated by hydro => Hydro FO line should be respected by the thermal-model samples.
- After considering two effects, a good match is indeed achieved.



Summary and Outlook

- Thermal model is a popular and intuitive way to extract freeze-out thermodynamics. Inspired by hydro, we incorporated both smearing effect and longitudinal flow into thermal model. Applied to C.-F. yields from a multistage hydro.
- Large rapidity: yields get contributions from mid-rapidity, by both effects => Can't use independent-rapidity-bin approach.
- Mid-rapidity: smearing effect doesn't give a significant correction in extracting freeze-out thermodynamics.
- Correlation between longitudinal system size and flow strength.
- A Bayesian analysis favours the existence of a longitudinal flow.
- **To do:** applying the model directly to experimental data (BRAHMS, BES...) => need to deal with the feed-down effect. Confirming our findings w/ hydro yields.

Backup

Uncertainty of discrete model for small yields

T uniquely given by the ratio $\frac{n^\pi}{n^K} = \frac{m_\pi^2}{m_K^2} \frac{\sum_{n=1}^{\infty} (-1)^{n-1} K_2(nm_\pi/T)}{\sum_{n=1}^{\infty} (-1)^{n-1} K_2(nm_K/T)}$.

$$\delta T = \frac{dT}{dr_{\pi/K}} \delta \left(\frac{n^\pi}{n^K} \right) = \frac{dT}{dr_{\pi/K}} \frac{n^K \delta n^\pi - n^\pi \delta n^K}{(n^K)^2}.$$

$$\frac{dT}{dr_{\pi/K}} \sim O(0.01 \text{ GeV})$$

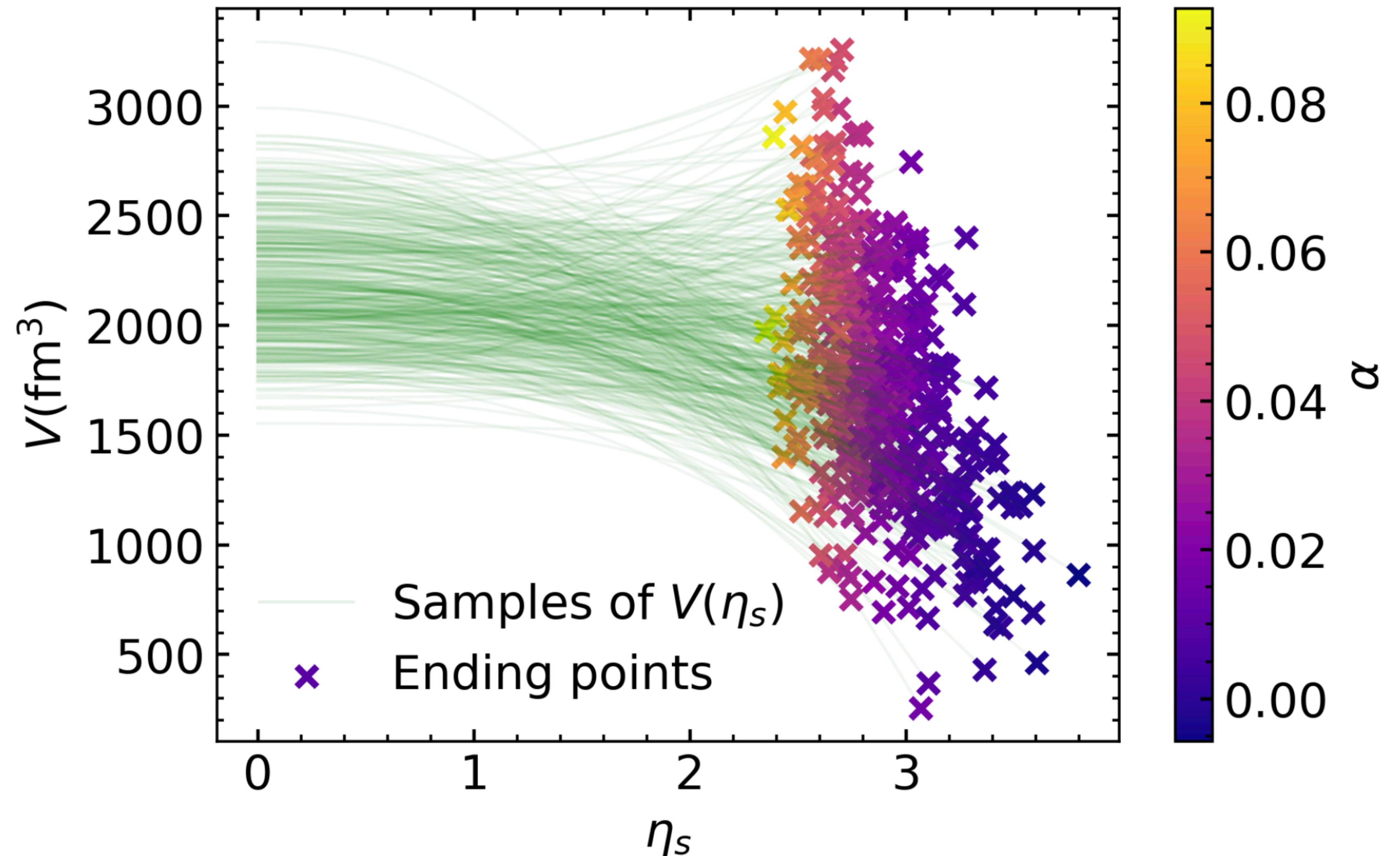
Tail region $n^K \rightarrow 0 \Rightarrow$ Significant δT

$VT^3 \sim \text{const.} \Rightarrow \delta V/V \sim 3\delta T/T \Rightarrow \delta V$ expected to be even larger.

Discrete model gives thermodynamics that is too sensitive to any kinds of uncertainty in yields \Rightarrow the unphysical result is actually “not to be believed”

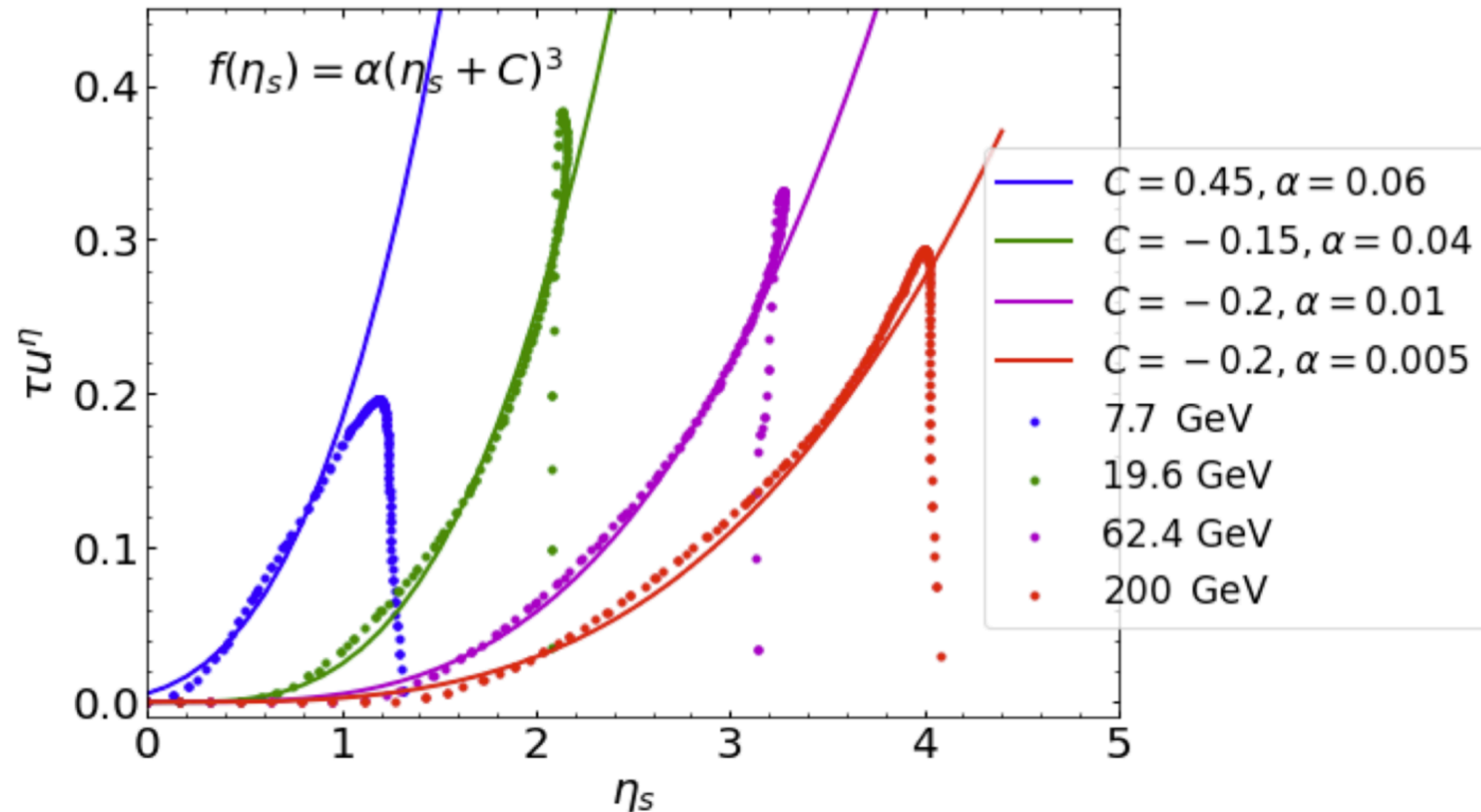
System size and the flow

- System size limited within $\eta_s < \eta_{max}$, marked with “x”.
- Small η_{max} compensated by large α .
- Decreasing $V(\eta_s)$ suggested by most samples.



Longitudinal flow: parametrization

- Evident in hydro: parametrized as $\tau u^\eta = \alpha \eta_s^3, \eta_s < \eta_{max}$.



Longitudinal flow: $y - \eta_s$ conversion

$$ds^2 = dt^2 - dz^2 = d\tau^2 - \tau^2 d\eta_s^2, \quad \Rightarrow \quad u^\tau \equiv \frac{d\tau}{ds} = \sqrt{1 + (\tau u^\eta)^2}.$$

$$\begin{pmatrix} dt \\ dz \end{pmatrix} = \begin{pmatrix} \cosh \eta_s & \tau \sinh \eta_s \\ \sinh \eta_s & \tau \cosh \eta_s \end{pmatrix} \begin{pmatrix} d\tau \\ d\eta_s \end{pmatrix} \Rightarrow v^z = \frac{\tanh \eta_s d\tau + \tau d\eta_s}{d\tau + \tau \tanh \eta_s d\eta_s} = \frac{\tanh \eta_s \sqrt{1 + (\tau u^\eta)^2} + \tau u^\eta}{\sqrt{1 + (\tau u^\eta)^2} + \tau u^\eta \tanh \eta_s}.$$

$$y = \frac{1}{2} \ln \frac{E + p^z}{E - p^z} = \frac{1}{2} \ln \frac{1 + v^z}{1 - v^z}. \Rightarrow y(\eta_s) = \frac{1}{2} \ln \frac{(\sqrt{1 + (\tau u^\eta)^2} + \tau u^\eta)(1 + \tanh \eta_s)}{(\sqrt{1 + (\tau u^\eta)^2} - \tau u^\eta)(1 - \tanh \eta_s)}.$$


 Rapidity of the source

Backup slides: thermal models

$$\frac{d^3 N}{d^3 \vec{p}} = \frac{V}{(2\pi)^3} f(\vec{p}; T, \mu)$$

$f(\vec{p}; T, \mu)$: Fermi-Dirac/Bose-Einstein distribution; expanded as series of Boltzmann/Maxwell dist.

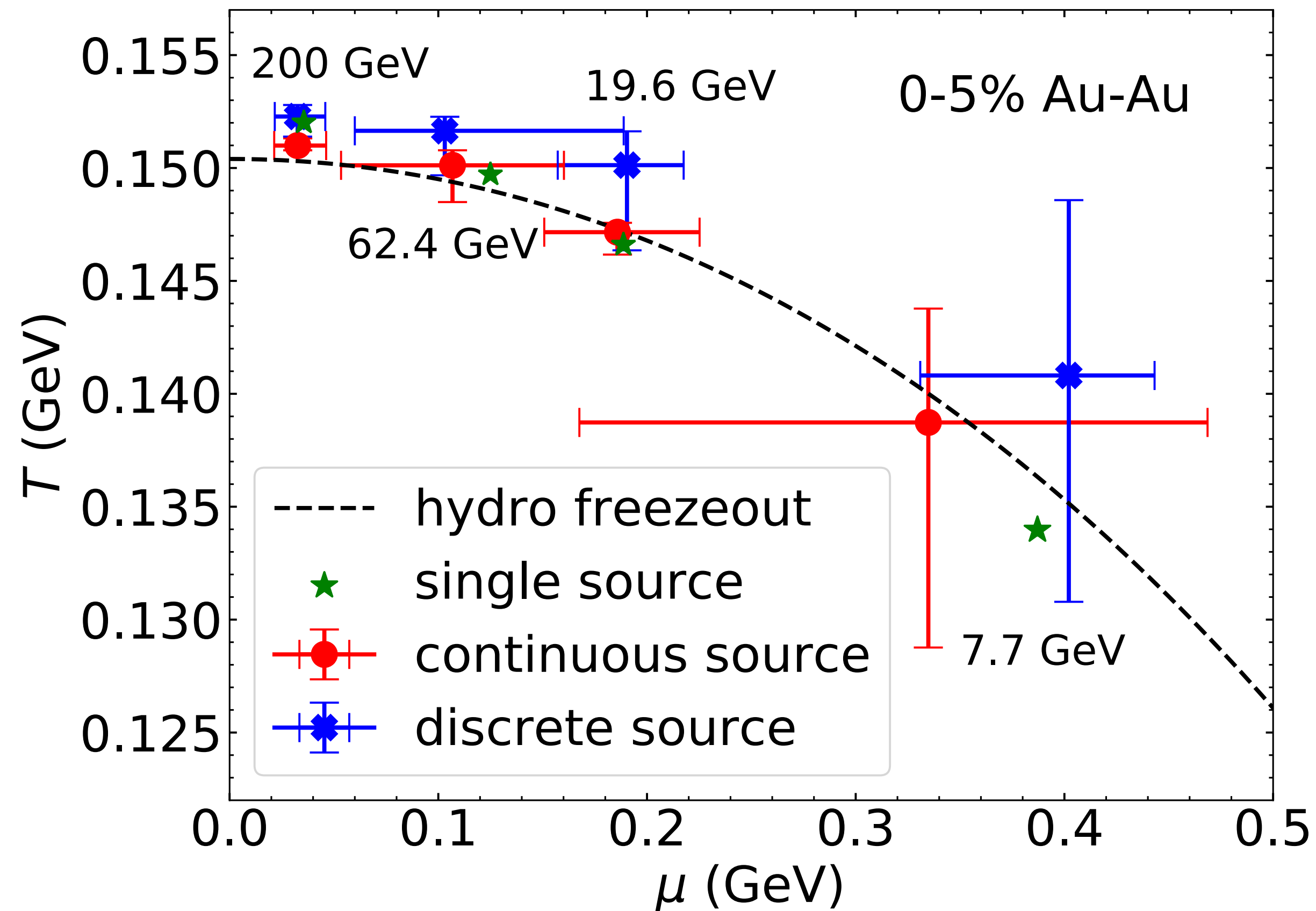
Discrete model: $N = \int d^3 \vec{p} \frac{d^3 N}{d^3 \vec{p}} \Rightarrow (T, \mu, V) \rightarrow (N^\pi, N^K, N^{p-\bar{p}})$

Continuous model (with smearing): $(p_x, p_y, p_z) = (p_T \cos \phi, p_T \sin \phi, m_T \cosh y)$

Integrating over $(\phi, m_T) \Rightarrow$

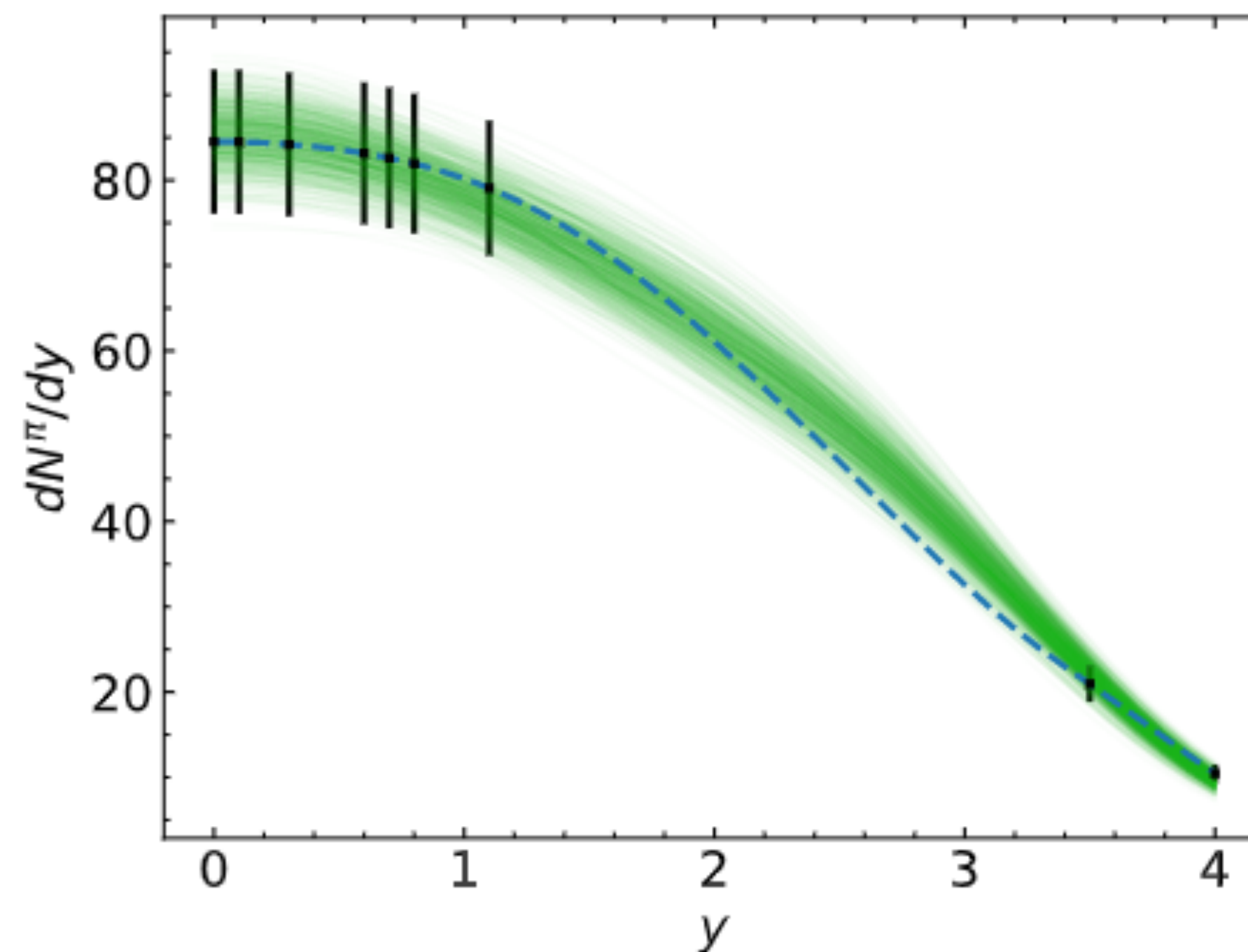
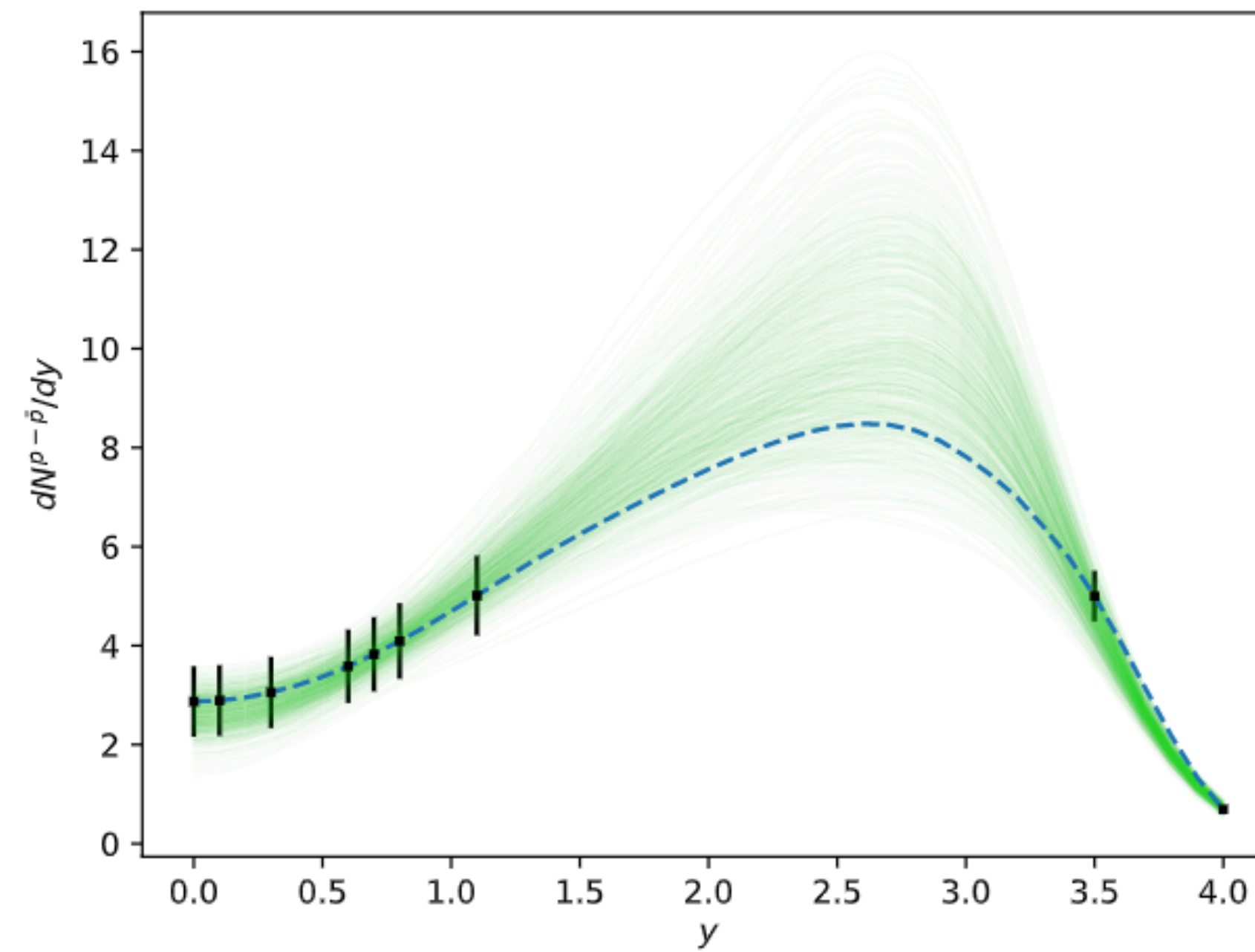
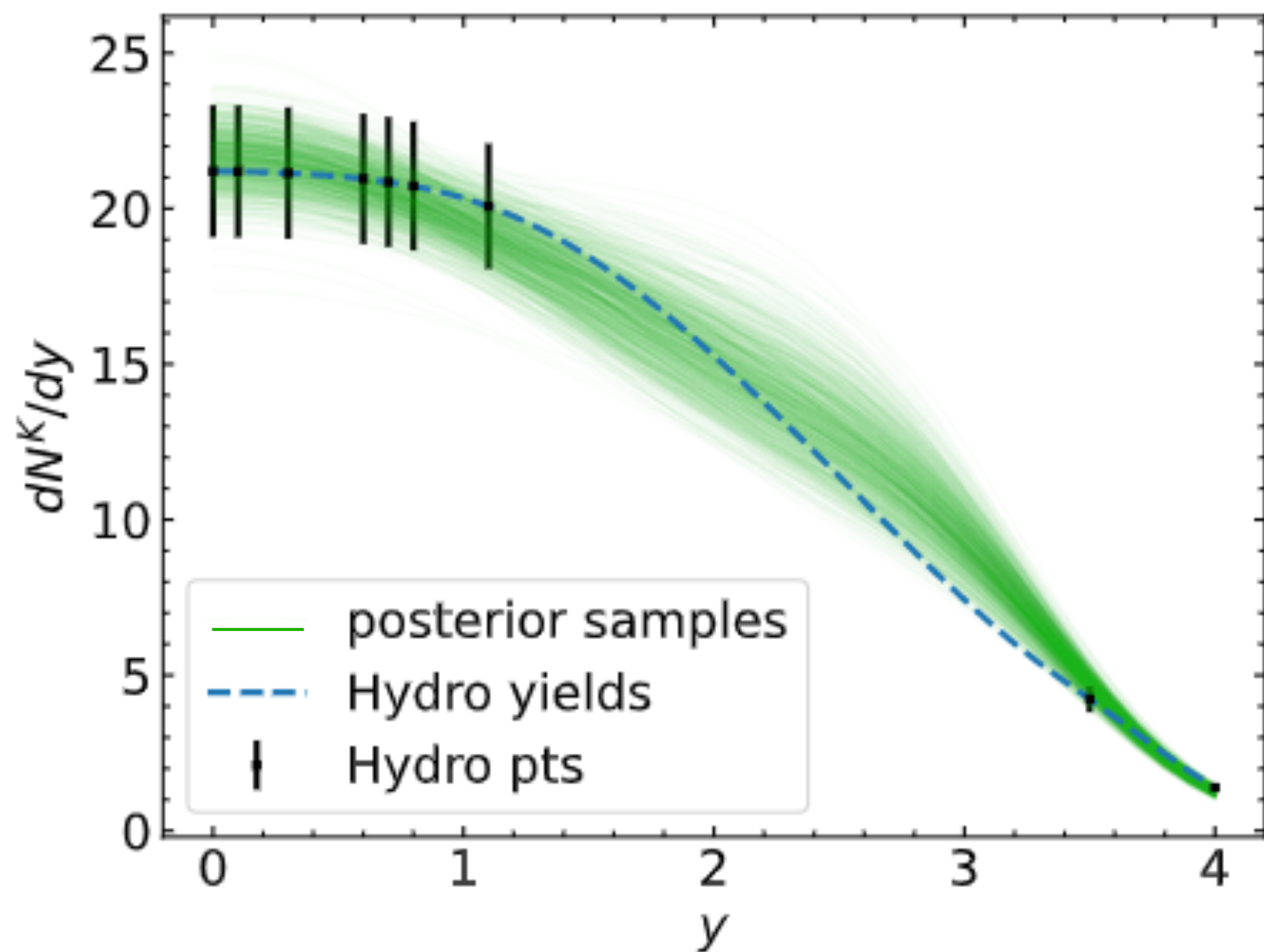
$$\left. \frac{dN_i}{dy} \right|_{y_s=0} = \frac{g_i V T^3}{(2\pi)^2} \sum_{n=1}^{\infty} \left(\frac{1}{n} \right)^3 \left(\frac{2}{\cosh^2 y} + \frac{n m_i}{T} \frac{2}{\cosh y} + \frac{n^2 m_i^2}{T^2} \right) \exp \left(-\frac{n m_i \cosh y}{T} \right), \quad (10)$$

Distribution of freeze-out cells on (T, μ_B) diagram

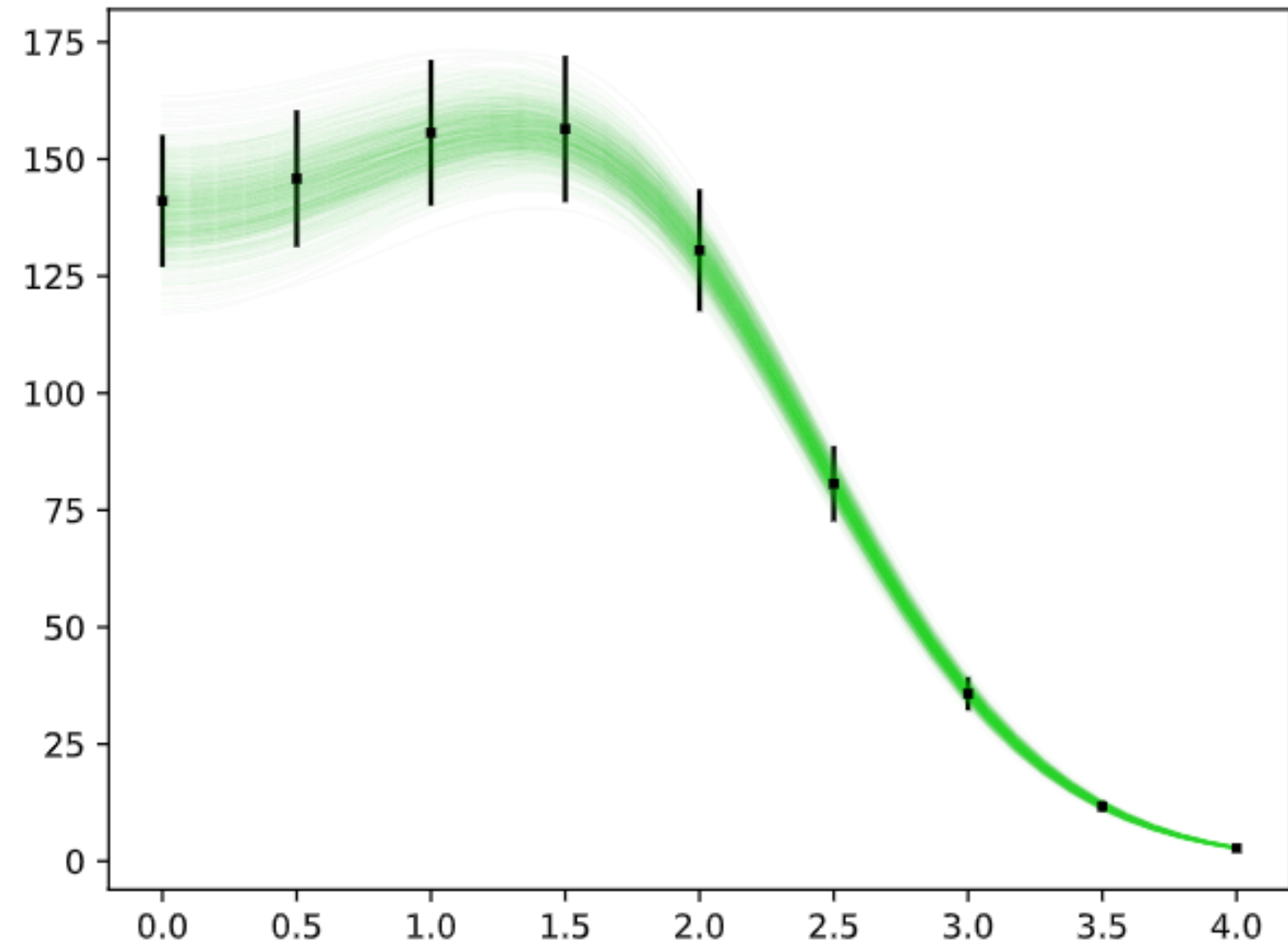
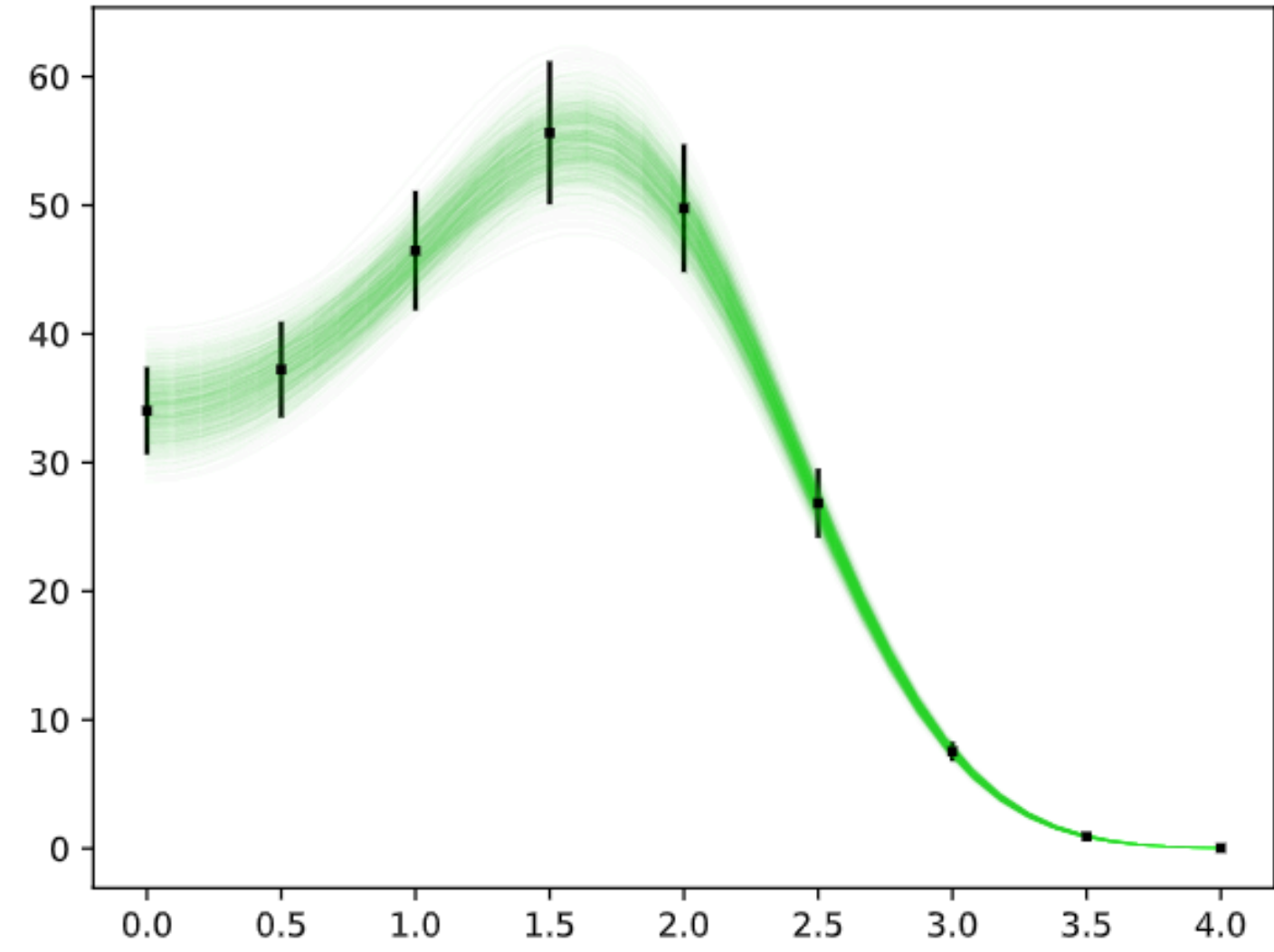
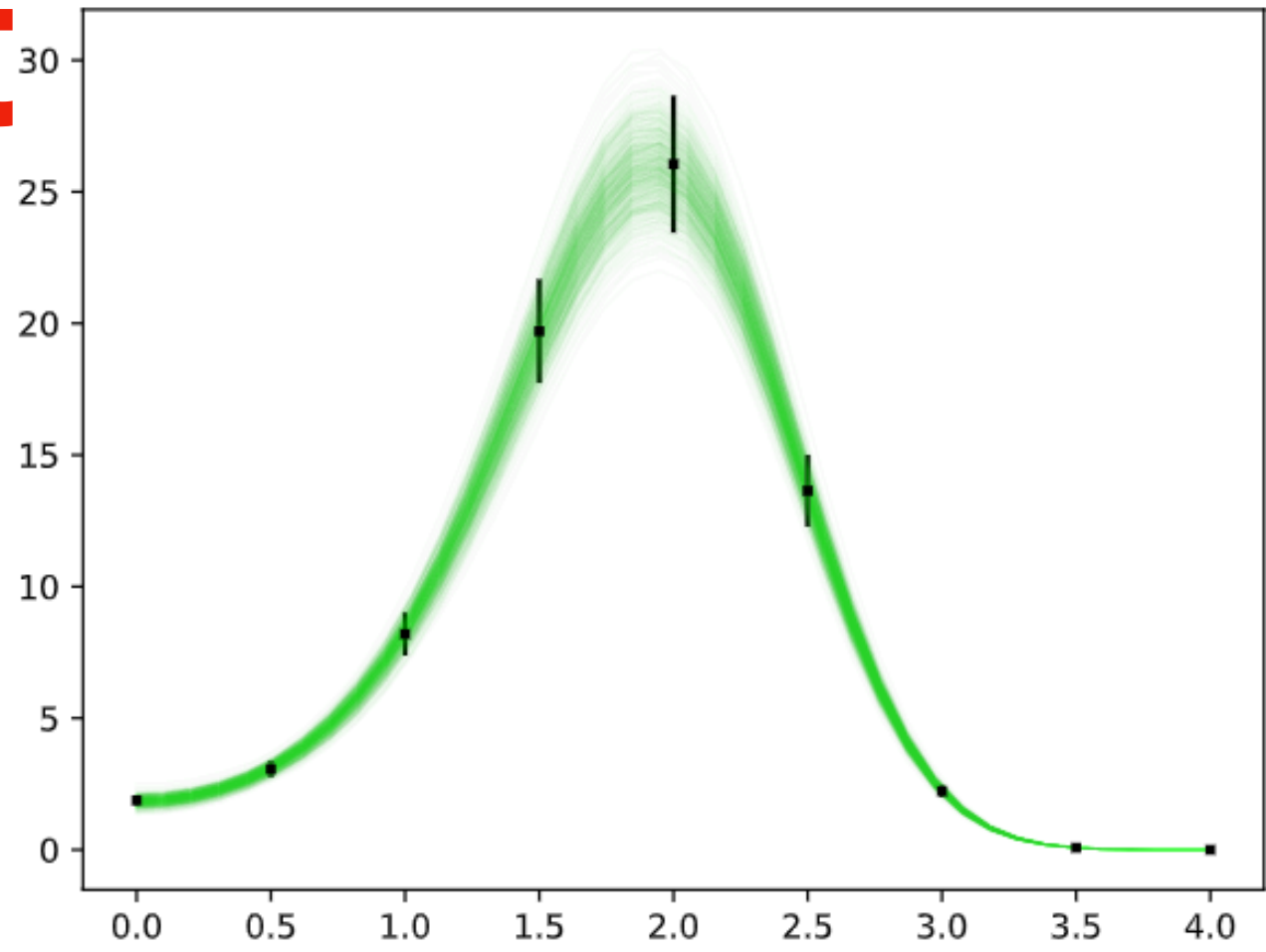
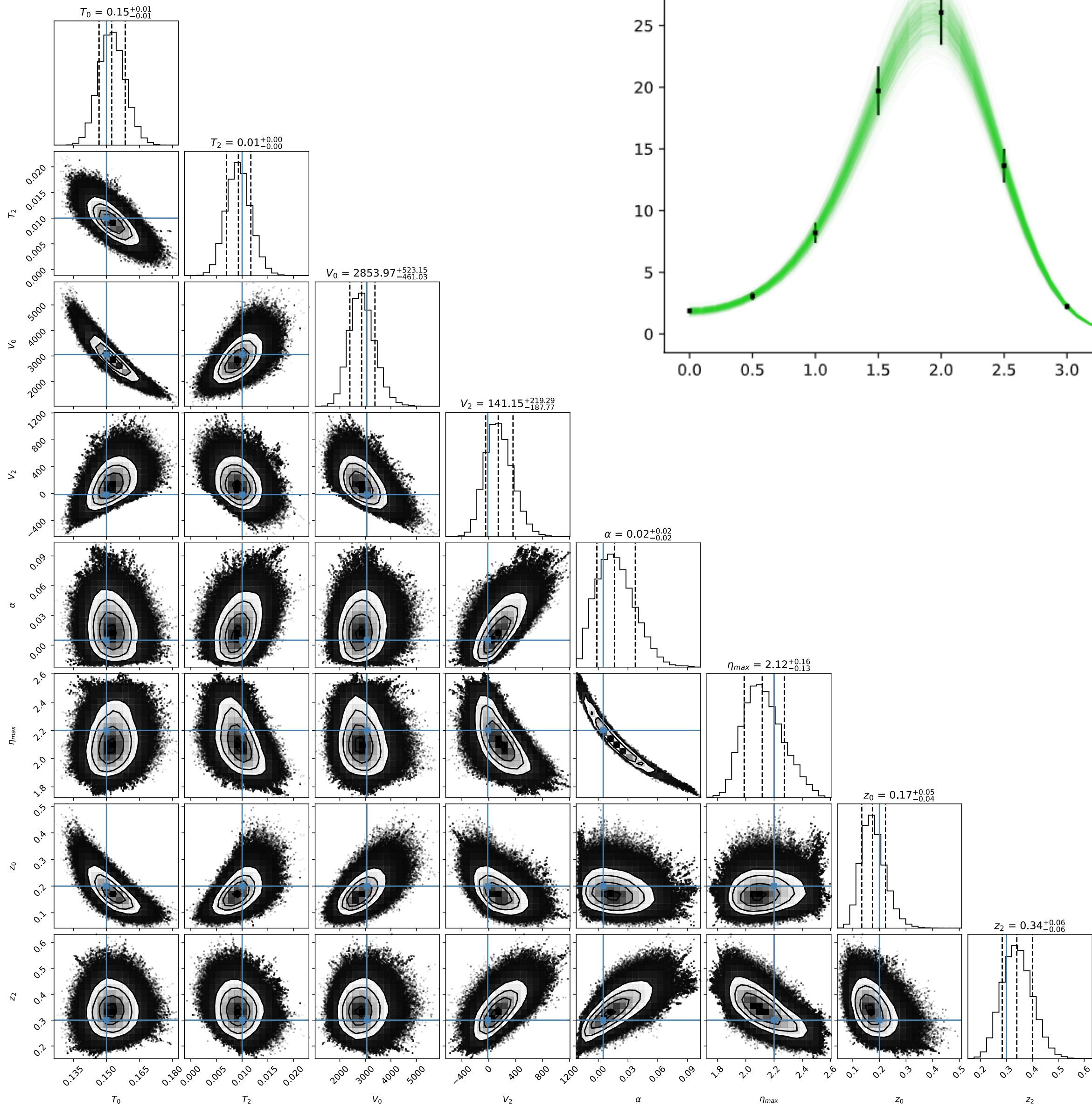


- Errorbars: median and 25% and 75% percentiles of freeze-out cells' (T, μ_B) distribution.
- Continuous model gets result closer to the hydro freeze-out line. ✓
- Qualitatively similar traits by both models: as $\sqrt{s} \uparrow$
higher T , more homogenous,...

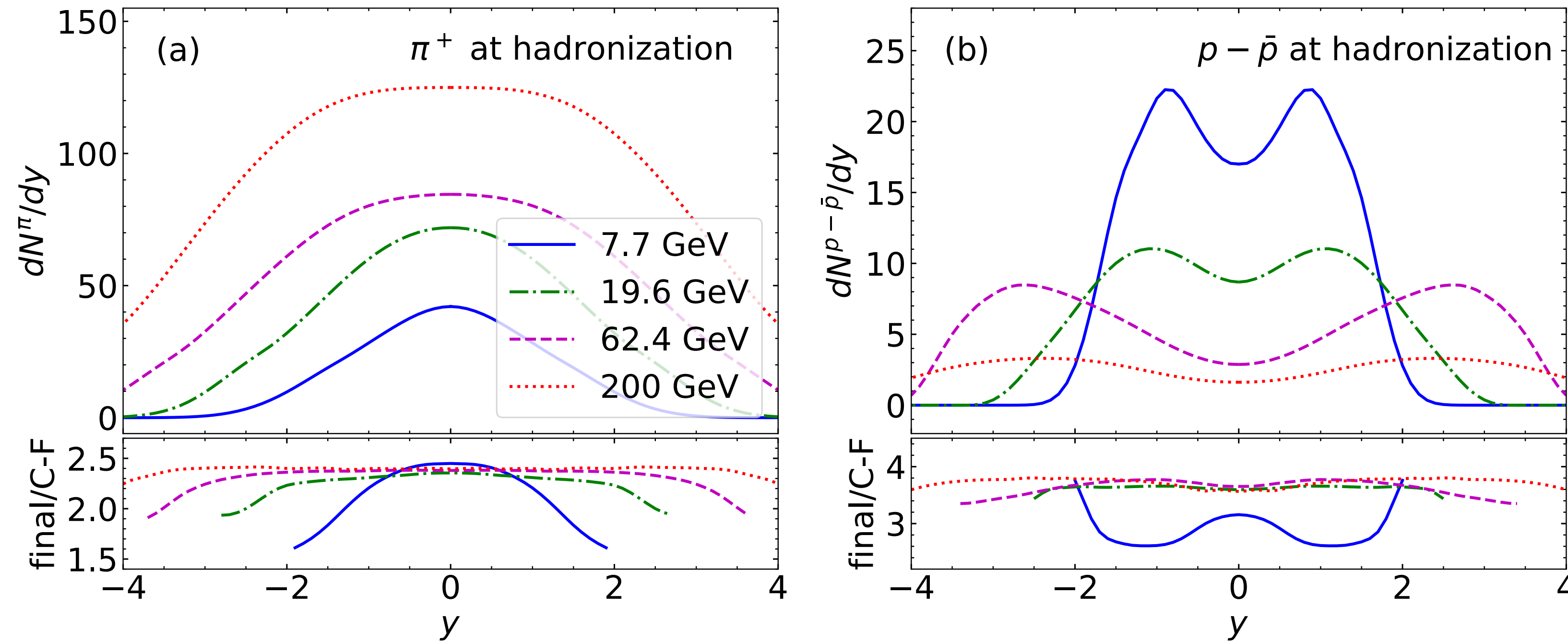
Posterior validation



Closure test

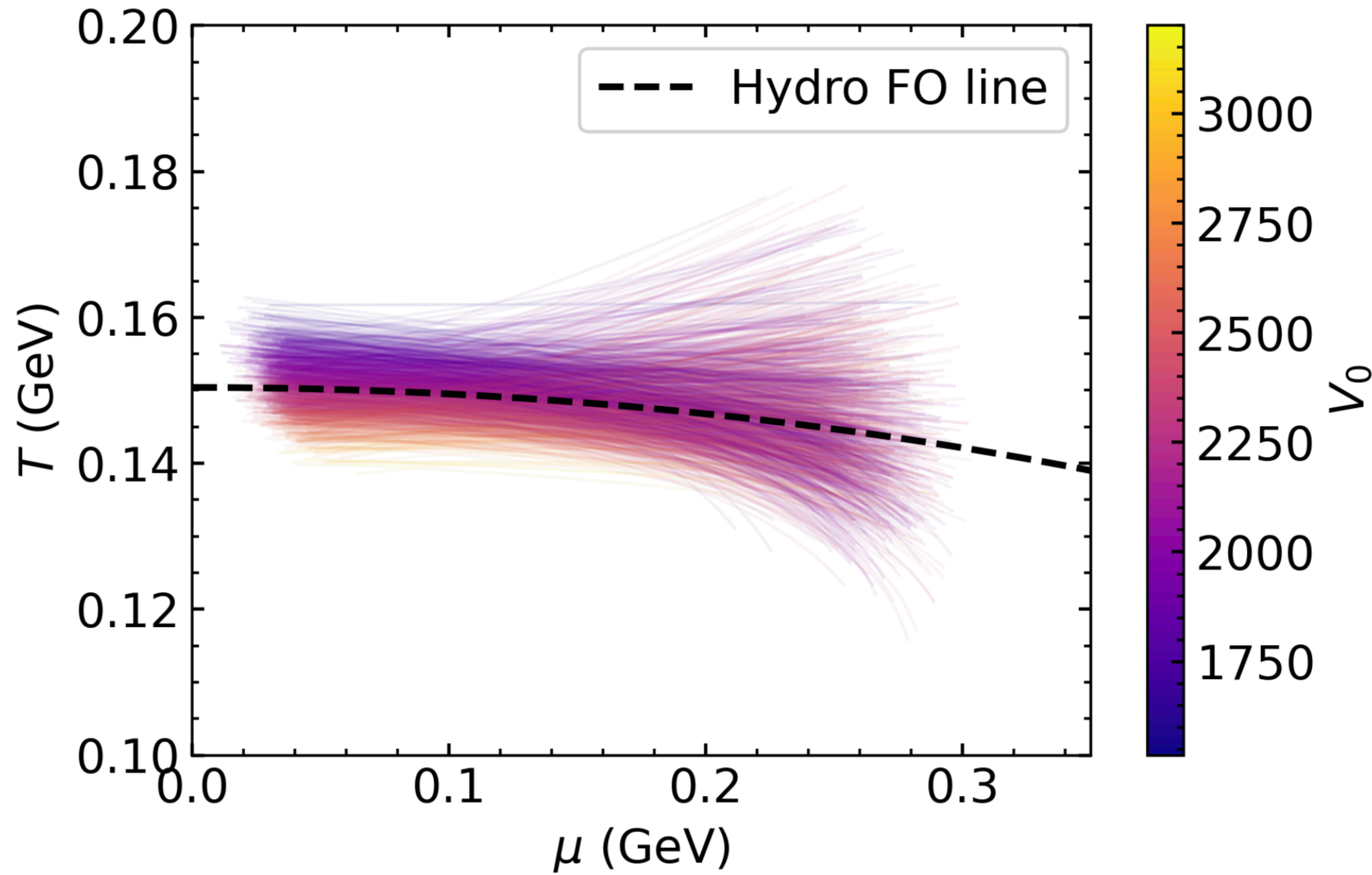


Yields at chemical freeze-out



- Particle yields differ from “purely thermal yields” because of resonance decays.
 - Thermal model considering both smearing and decay is hard! \Rightarrow Make use of final yields
- multistage hydro, find $\frac{\text{Cooper - Frye yields}}{\text{Cooper - Frye yields}}$

“Freeze-out” phase diagram



- Samples centred around the Hydro FO line (by which the yields for our MCMC is generated).
- Lower temperature compensated by larger volume.
- Flat FO line for small $\mu_B \Rightarrow$ Isothermal