

# High order fluctuations of conserved charges in the continuum limit

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# Fluctuations of conserved charges

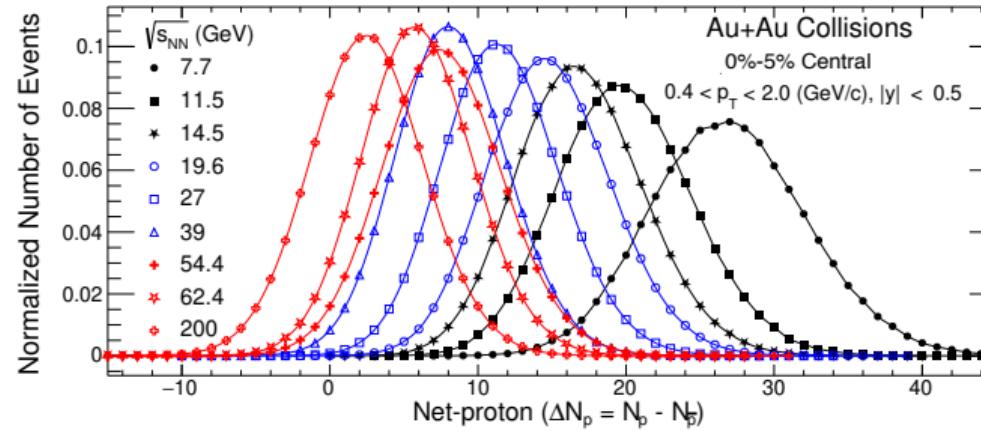
Conserved in QCD

- the baryon number  $B$ ,
- strangeness  $S$ ,
- the electric charge  $Q$  (*addressed in my QM22 talk*),
- charm  $C$  (*ignored in this talk*)

(*net number of quarks ( $u, d, s, c$ ) are conserved*).

These net charges can fluctuate in sub-systems.

*Proxies (e.g. protons) have been measured in the Beam Energy Scan program:*



# Fluctuations of conserved charges

Grand canonical ensemble introduces *chemical potentials* as a thermodynamic force to favour positive expectation value of a net charge (e.g. finite baryon density).

$$Z(V, T, \mu_B, \mu_Q, \mu_S) = \sum_{B, Q, S} e^{B\mu_B/T} e^{Q\mu_Q/T} e^{S\mu_S/T} \mathcal{Z}(V, T, B, Q, S)$$

Derivatives of the logarithm (*grand potential*) give the cumulants:

$$\langle B \rangle = \frac{1}{VT^2} \frac{\partial \log Z(V, T, \mu_B, \mu_Q, \mu_S)}{\partial \mu_B} = \chi_1^B$$

$$\langle B^2 \rangle - \langle B \rangle^2 = \frac{1}{VT} \frac{\partial^2 \log Z(V, T, \mu_B, \mu_Q, \mu_S)}{\partial \mu_B^2} = \chi_2^B$$

$$\langle BQ \rangle - \langle B \rangle \langle Q \rangle = \frac{1}{VT} \frac{\partial^2 \log Z(V, T, \mu_B, \mu_Q, \mu_S)}{\partial \mu_B \partial \mu_Q} = \chi_{11}^{BQ}$$

# Fluctuations of conserved charges

Where do they play a role?

- Contact to experiment (*not straightforward* [Bzdak,Koch,Skokov 1203.4529; Braun-Munzinger et al 2007.02463])
- Experimental signature of critical end point [Stephanov 0809.3450]
- Fluctuations are the Taylor coefficients of the equation of state [D. Clarke We 14:40]

$$\frac{p(\mu_B)}{T^4} = \frac{p(0)}{T^4} + \frac{1}{2!} \frac{\mu_B^2}{T^2} \chi_2^B(T) + \frac{1}{4!} \frac{\mu_B^4}{T^4} \chi_4^B(T) + \frac{1}{6!} \frac{\mu_B^6}{T^6} \chi_6^B(T) + \dots$$

- Higher fluctuations are the Taylor coefficients of lower fluctuations

$$\chi_2^B(\mu_B) = \chi_2^B(\mu_B = 0) + \frac{1}{2!} \frac{\mu_B^2}{T^2} \chi_4^B(\mu_B = 0) + \frac{1}{4!} \frac{\mu_B^4}{T^4} \chi_6^B(\mu_B = 0) + \dots$$

- Taylor coefficients can be used to reveal analytic structure of the thermodynamic potential
  - Repulsive interactions beyond ideal HRG [Huovinen&Petreczky 1708.00879] [Vovchenko et al 1708.02852]
  - Searching the critical end point [J. Goshwami We 15:20]
- Hints for chiral O(4) universality [P. Petreczky We 17:10]

# Baryon and strangeness fluctuations

## Leading order

*Continuum since 2011 [Wuppertal-Budapest 1112.4416]*

$$\chi_2^B(T) \quad \chi_{11}^{BS}(T) \quad \chi_2^S(T)$$

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## Next-to-leading order

*Continuum starting at 2015 [Wuppertal-Budapest 1507.04627] [HotQCD 1708.04897, ..., 2212.09043]*

$$\chi_4^B(T) \quad \chi_{31}^{BS}(T) \quad \chi_{22}^{BS}(T) \quad \chi_{13}^{BS}(T) \quad \chi_4^S(T)$$

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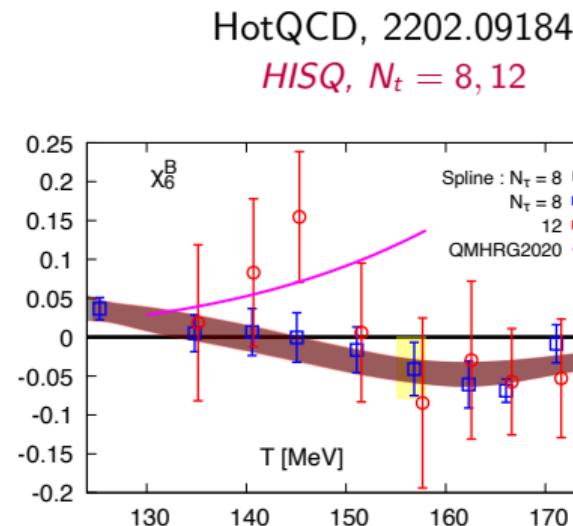
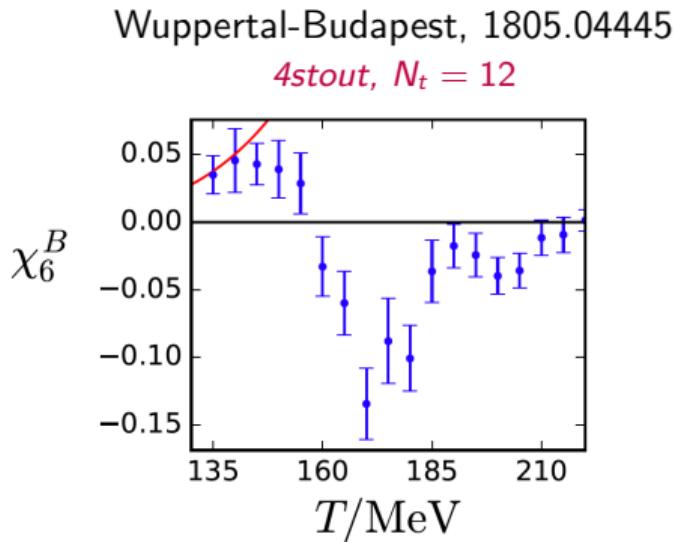
## Next-to-next-to-leading order

*Continuum results in this talk*

$$\chi_6^B(T) \quad \chi_{51}^{BS}(T) \quad \chi_{42}^{BS}(T) \quad \chi_{33}^{BS}(T) \quad \chi_{24}^{BS}(T) \quad \chi_{15}^{BS}(T) \quad \chi_6^S(T)$$

# Baryon fluctuations in the literature

Two results without continuum extrapolation

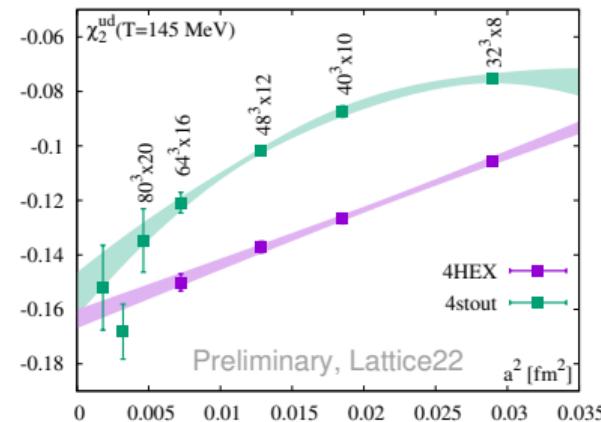
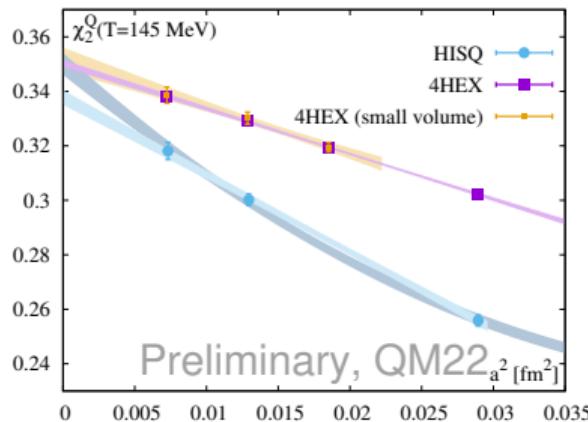


Is  $\chi_6^B$  positive or negative at  $T \approx 150$  MeV?  
Does QCD agree with HRG at  $T \approx 140$  MeV?

# Continuum extrapolation with the 4HEX action

We use the new 4HEX staggered action with strongly reduced taste breaking.

Continuum extrapolation  $T = 145$  MeV



4stout results: [\[Wuppertal-Budapest \[1507.04627\]\]](#)

HISQ results: [\[HotQCD \[2107.10011\]\]](#)

4HEX results: [\[Wuppertal-Budapest QM2022\]](#)

Results are at a lattice size of  $LT = 4$ . The label "small volume" is  $LT = 3$  in the plot, we will use  $LT = 2$  for high order fluctuations.

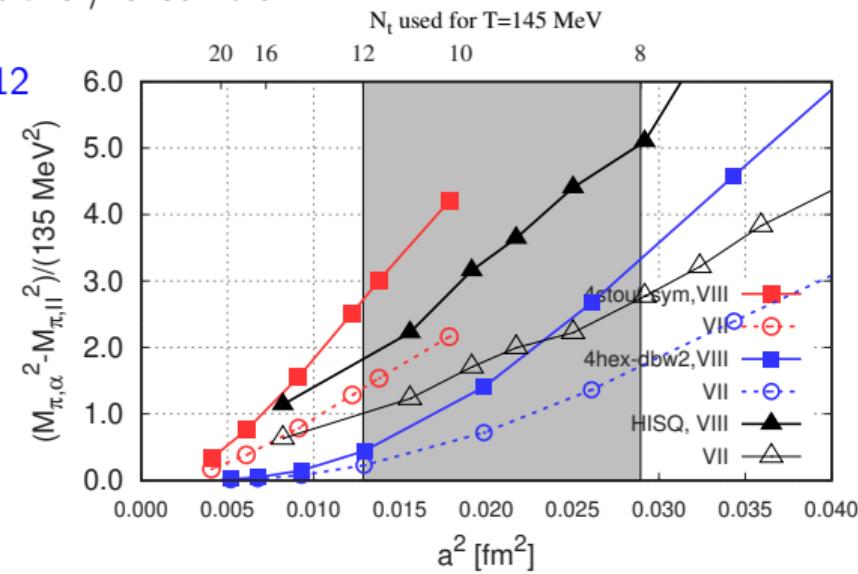
# Thermodynamics with the 4HEX action

- 4 steps of HEX smearing + DBW2 gauge action
- Physical point defined by  $m_\pi/f_\pi = 1.0337$ ,  $m_s/m_{\text{light}} = 27.63$
- $m_{\text{light}}$  tuned in the  $a$  range:  $0.22 \dots 0.072$  fm
- Thermodynamics runs: cca 80000 configurations / ensemble  
10 Temperatures  $130 \dots 175$  MeV  
3 lattice spacings  $16^3 \times 8, 20^3 \times 10, 24^3 \times 12$

**Taste breaking**  
*(smaller is better)*

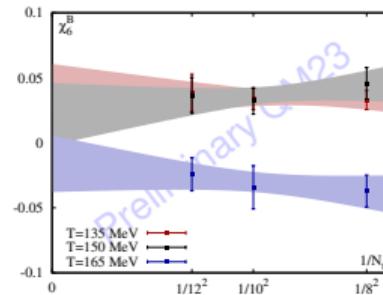
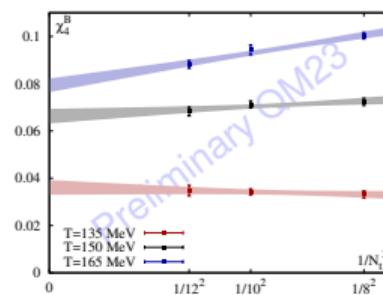
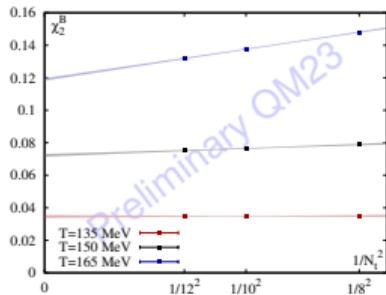
*show relative quadratic discretization error on  $M_\pi^2$*

(HISQ: thanks to Peter Petreckzy)

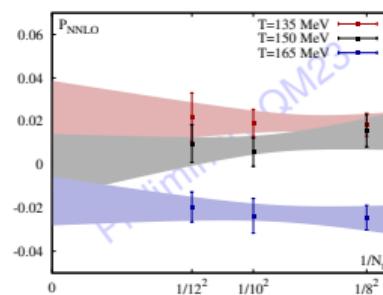
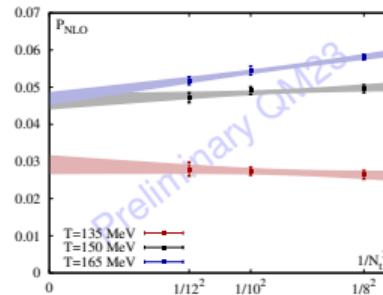
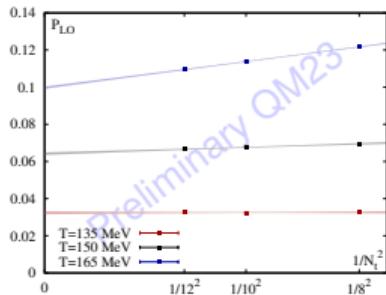


# 4HEX continuum extrapolations

## Baryon Taylor coefficients

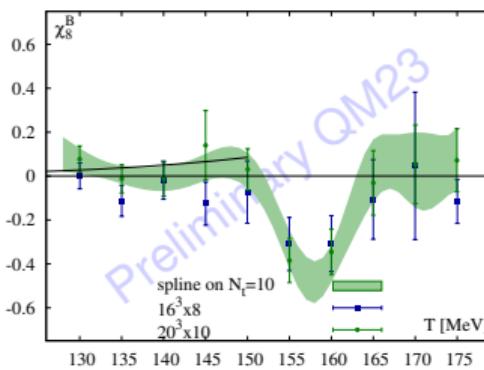
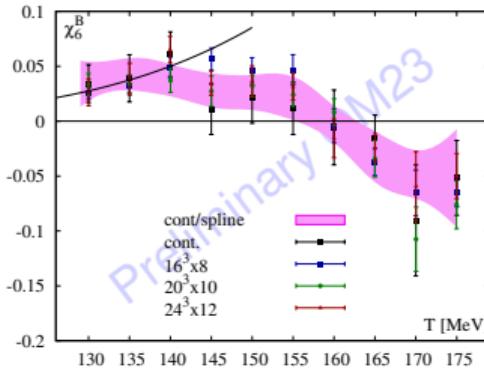
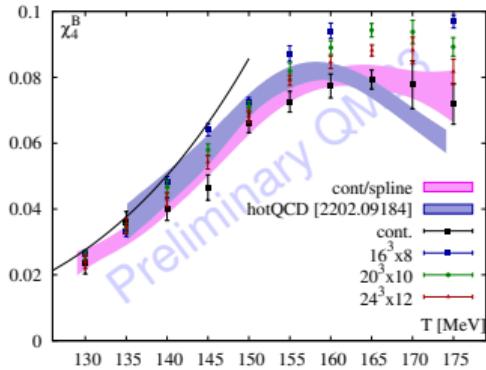


## Strangeness neutral Taylor coefficients

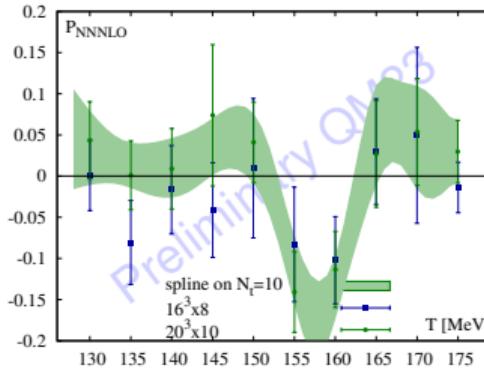
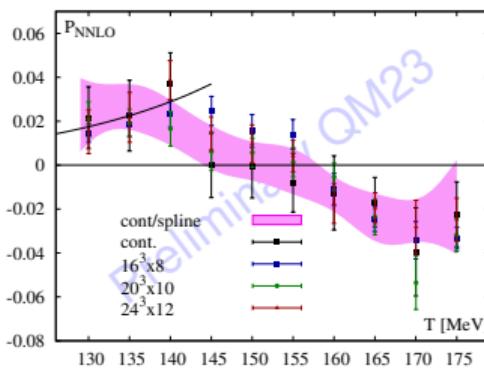
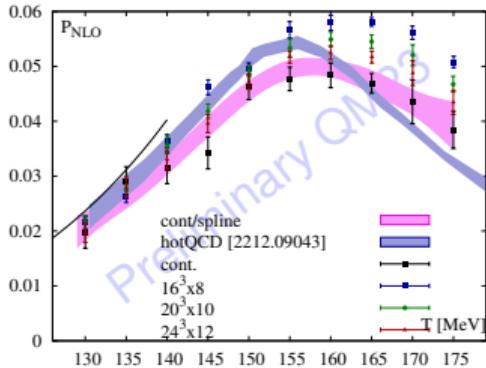


# High order coefficients

## Baryon Taylor coefficients



## Strangeness neutral Taylor coefficients



# What is strangeness neutrality?

We extrapolate :  $B(\mu_B, \mu_S, T)$      $S(\mu_B, \mu_S, T)$

Strangeness neutrality means that we solve for  $\mu_S$  at any  $\mu_B$  and  $T$

$$S(\mu_B, \mu_S, T) \equiv 0$$

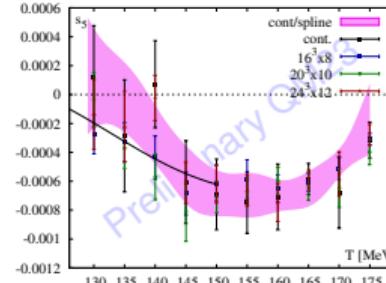
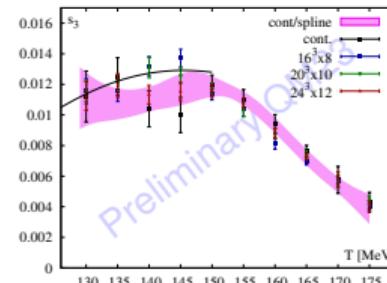
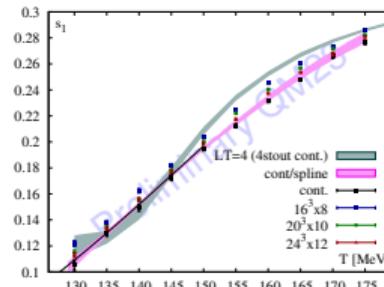
That requires a tuning of  $\mu_S^* = \mu_S^*(\mu_B, T)$ . [ For simplicity  $\mu_Q \equiv 0$ . ]

$$\mu_S^*(\mu_B, T) = s_1(T)\mu_B + s_3(T)\mu_B^3 + s_5(T)\mu_B^5 + \dots$$

One obtains  $s_1(T)$ ,  $s_3(T)$  and  $s_5(T)$  from the standard Taylor coefficients

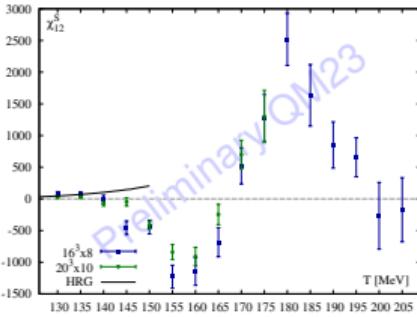
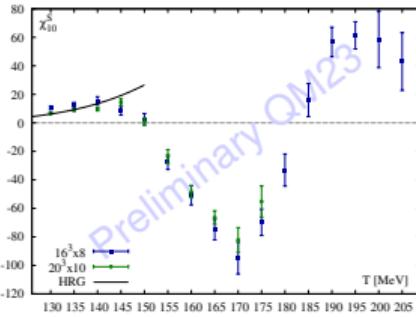
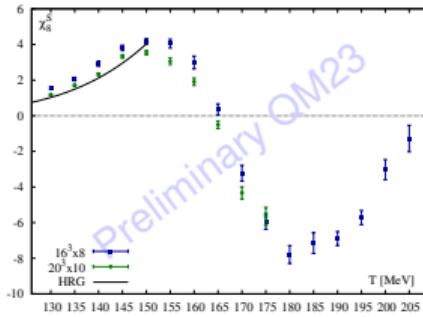
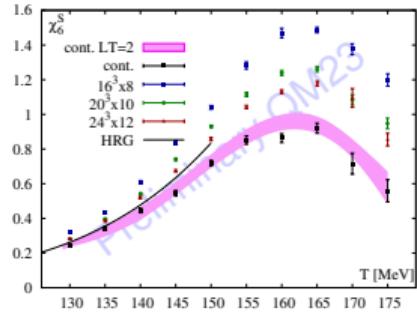
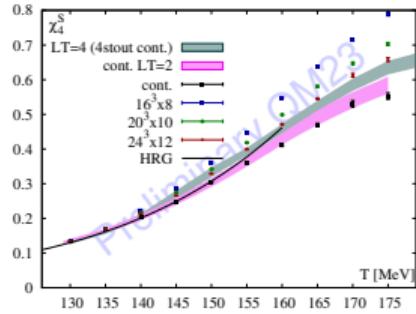
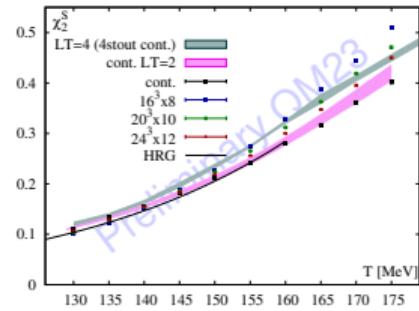
[HotQCD 1208.1220; 1701.04325]

4HEX results:



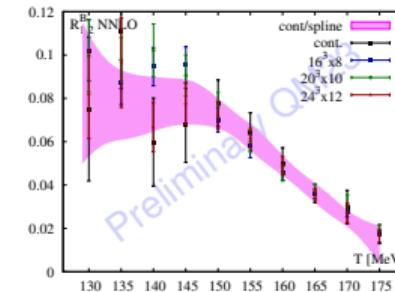
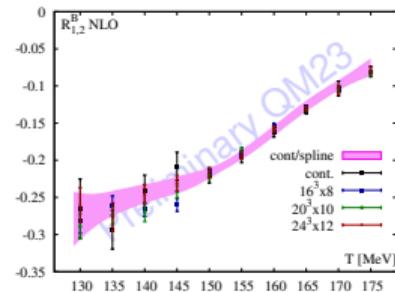
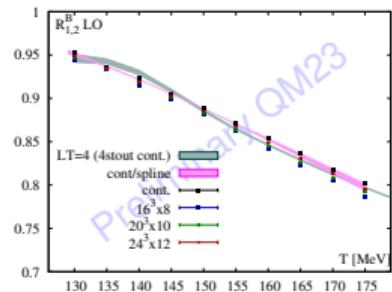
# Strangeness fluctuations

$$\chi_n^S = \frac{\partial^n(p/T^4)}{\partial \mu_S/T^n}$$

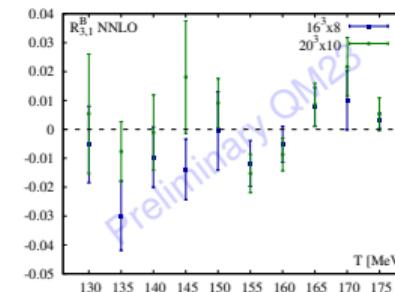
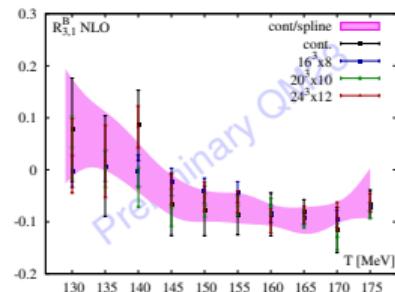
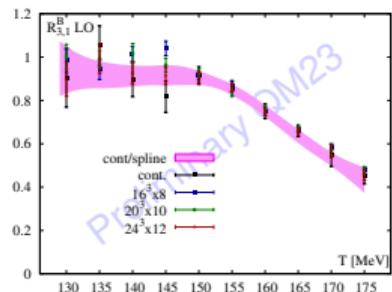


# Baryon cumulant ratios

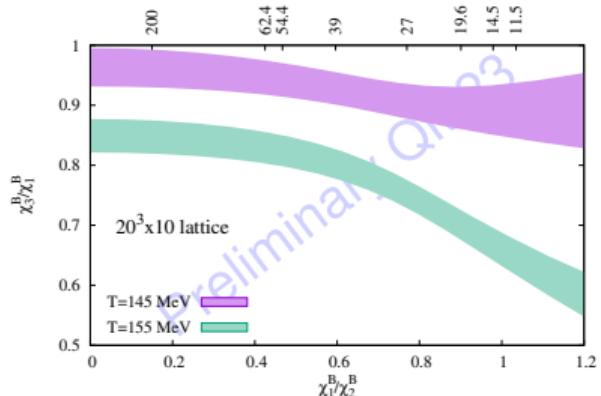
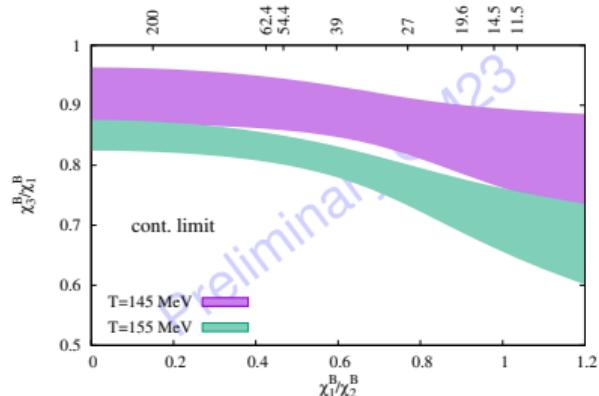
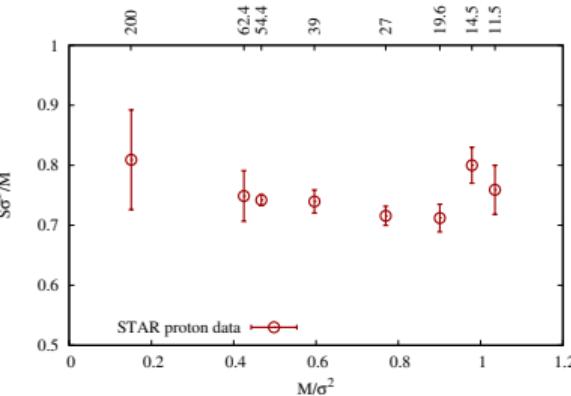
$$\frac{M}{\sigma^2}(\mu_B, T) = \frac{\mu_B}{T} f_{LO}(T) + \frac{\mu_B^3}{T^3} f_{NLO}(T) + \frac{\mu_B^5}{T^5} f_{NNLO}(T) + \dots$$



$$\frac{S\sigma^3}{M}(\mu_B, T) = g_{LO}(T) + \frac{\mu_B^2}{T^2} g_{NLO}(T) + \frac{\mu_B^4}{T^4} g_{NNLO}(T) + \dots$$



# Plotting cumulant ratios with STAR data



- Experimental data shows proton fluctuations at various collision energies [STAR [2001.02852]]
- Lattice data refers to
  - baryon fluctuations
  - in a grand canonical ensemble
  - in equilibrium

*STAR data motivates further research on very high order cumulants.*

High order fluctuations from lattice is an expensive endeavour

- HotQCD has been pursuing a *decade* long campaign on  $32^3 \times 8$  lattices (*cca*  $5 \times 10^6$  configurations)
- Earlier (2018) we exploited analiticity in the (imaginary) chemical potential to substantially cut costs on  $48^3 \times 12$  lattices *two years of INCITE access*
- Today I presented continuum extrapolated fluctuations in a smaller volume ( $LT = 2$ ) thanks to the 4HEX discretization using a fraction of resources  $\frac{1}{2}$  *years in Jülich*
  - Cut-off effects are significant (*staggered fermions*)  
New action (*4HEX*) permits continuum extrapolation from  $N_t = 8, 10$  and  $12$ .
  - Large volume kills the signal (*as expected in the presence of a sign problem*); small volume could already show hints for criticality