

Universal cumulants from fluctuating width of rapidity distributions

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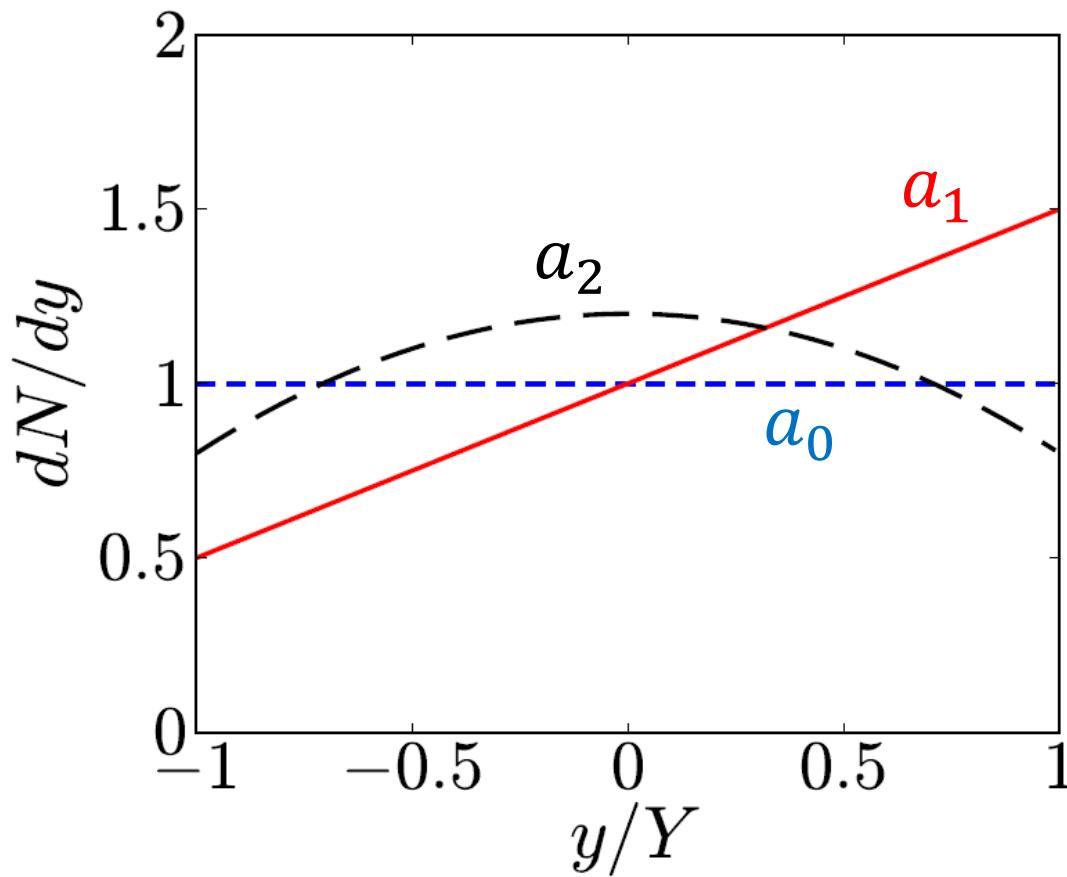
Outline

- long-range rapidity density fluctuations
- correlations
- width fluctuations
- results
- summary

Fireball shape in rapidity fluctuates

AB, D. Teaney,
PRC 87 (2013) 024906

ATLAS Coll.,
PRC 95 (2017) 064914



here $dN/dy \equiv \rho_{\text{event}}(y)$

In other words

$$\rho_{\text{event}}(y) = \langle \rho(y) \rangle \left[1 + a_0 + a_1 \frac{y}{Y} + \dots \right]$$

↑
single particle distribution
in an event (neglecting
statistical fluctuations)

↑
average single
particle distribution

a_0 is rapidity independent fluctuation of fireball as a whole
multiplicity distribution

a_1 is an event-by-event rapidity asymmetry
e.g. asymmetry in the number of left- and right-going constituents (nucleons, quarks,
diquarks, etc.)

Y - measurement is from $-Y$ to Y

$$\rho_{\text{event}}(y) = \langle \rho(y) \rangle \left[1 + \sum_{i=0}^{\infty} a_i T_i(y/Y) \right]$$

↑
orthogonal polynomials

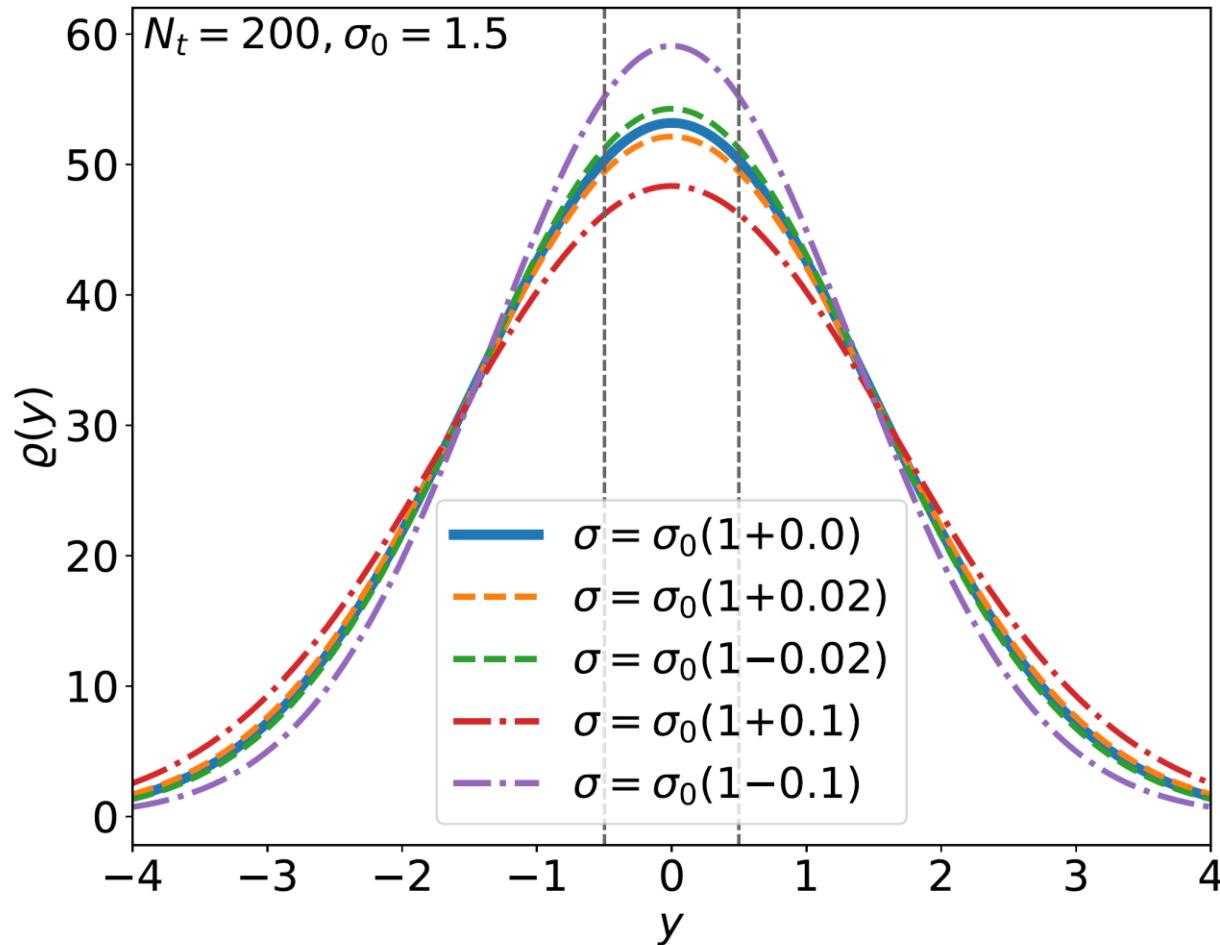
$$\frac{C_2(y_1, y_2)}{\langle \rho(y_1) \rangle \langle \rho(y_2) \rangle} = \sum_{i,k} \langle a_i a_k \rangle T_i(y_1/Y) T_k(y_2/Y)$$

$$\frac{C_2(y_1, y_2)}{\langle \rho(y_1) \rangle \langle \rho(y_2) \rangle} \sim \langle a_0^2 \rangle + \langle a_1^2 \rangle \frac{y_1 y_2}{Y^2} + \dots$$

New approach, fluctuating width, $p(\sigma)$

$$\varrho(y) = \frac{N_t}{\sqrt{2\pi}\sigma} \exp\left(-\frac{y^2}{2\sigma^2}\right)$$

$$\varrho(y) \approx \frac{N_t}{\sqrt{2\pi}\sigma} \left(1 - \frac{y^2}{2\sigma^2}\right)$$



Two-particle correlation and second factorial cumulant

$$\sigma_0 \equiv \langle \sigma \rangle = \int_0^{+\infty} d\sigma \sigma p(\sigma)$$

$$\varrho_{\text{meas}}(y) = \int_0^{+\infty} d\sigma \varrho(y, \sigma) p(\sigma)$$

$$\varrho_{\text{meas},2}(y_1, y_2) = \int_0^{+\infty} d\sigma \varrho(y_1, \sigma) \varrho(y_2, \sigma) p(\sigma)$$

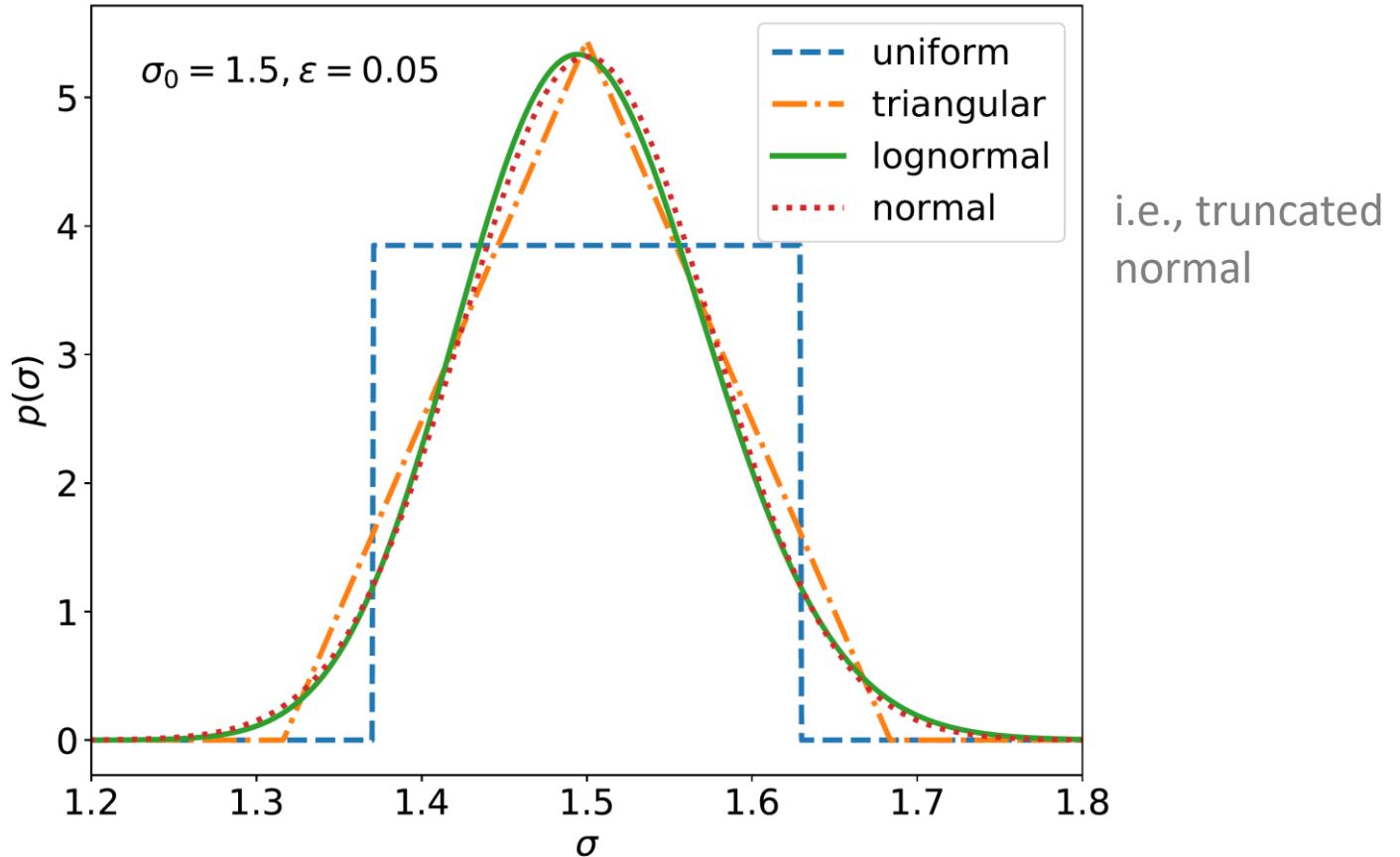
$$C_2(y_1, y_2) = \varrho_{\text{meas},2}(y_1, y_2) - \varrho_{\text{meas}}(y_1) \varrho_{\text{meas}}(y_2)$$

$$\hat{C}_2 = \int_{-Y}^Y dy_1 \int_{-Y}^Y dy_2 C_2(y_1, y_2)$$

And analogously for higher-order correlation functions

Examples of studied width distributions

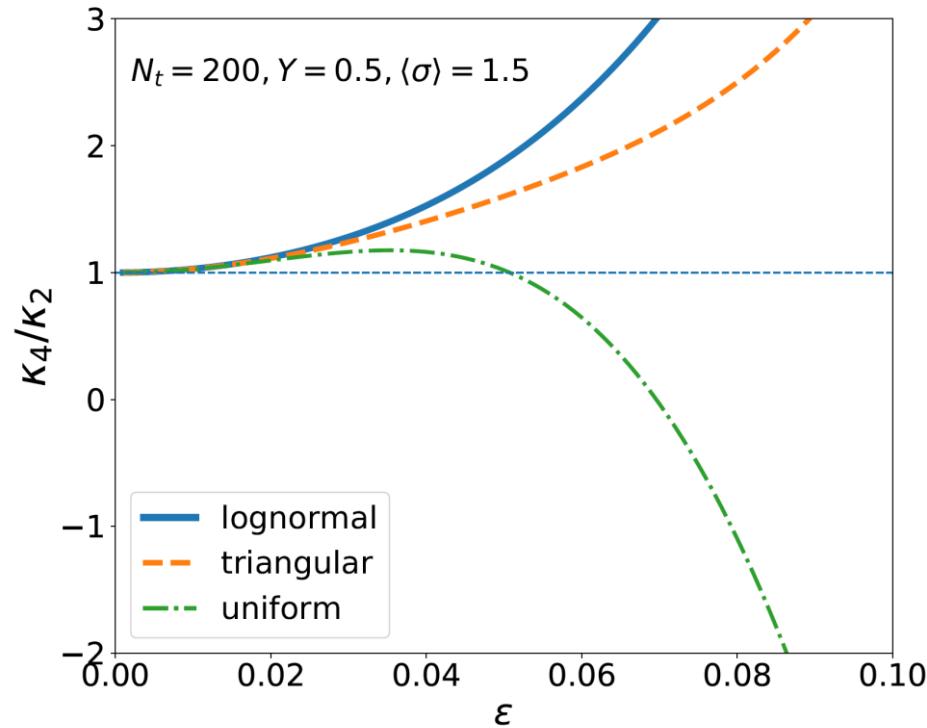
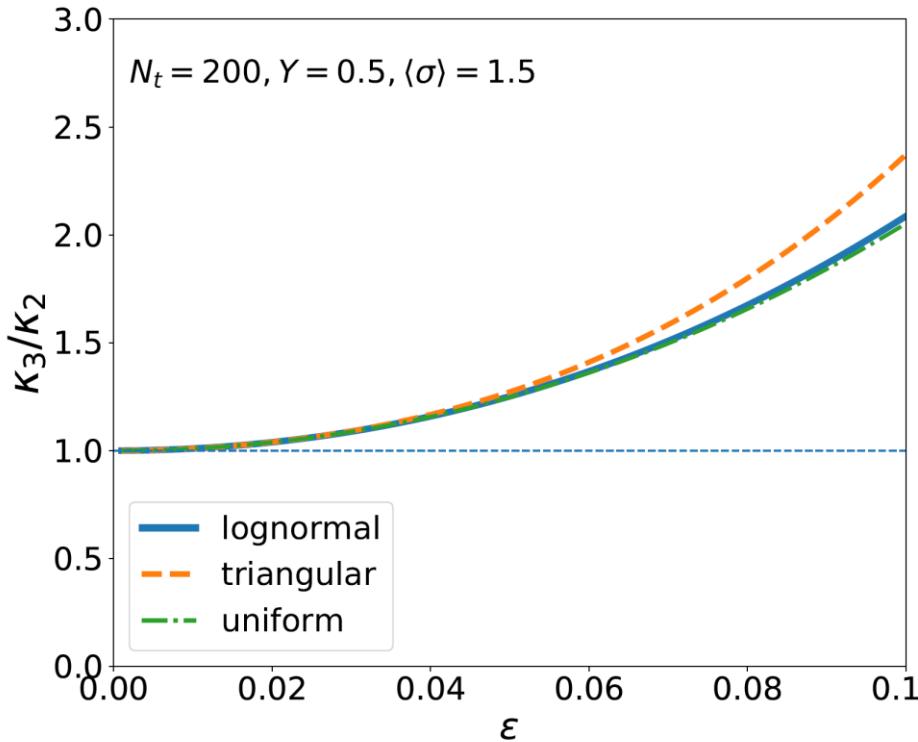
$$\langle \sigma \rangle = \sigma_0 \quad \sqrt{\langle (\sigma - \langle \sigma \rangle)^2 \rangle} = \underline{\varepsilon} \underline{\langle \sigma \rangle}$$



$$\text{lognormal } p(\sigma) = \frac{1}{\sqrt{2\pi} b \sigma} \exp\left(-\frac{(\ln \sigma - a)^2}{2b^2}\right)$$

Results (examples)

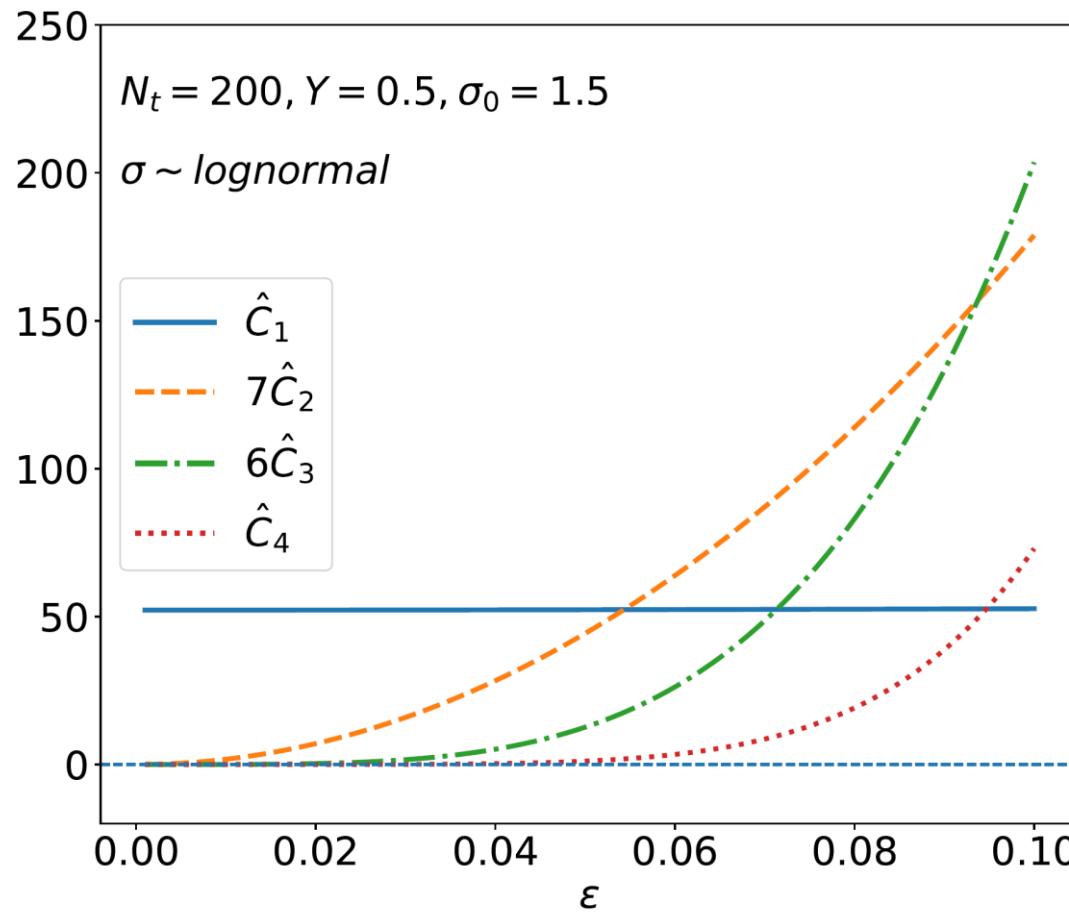
Results for truncated normal $p(\sigma)$ look similar to lognormal



Universality for small fluctuations (see paper for general arguments)

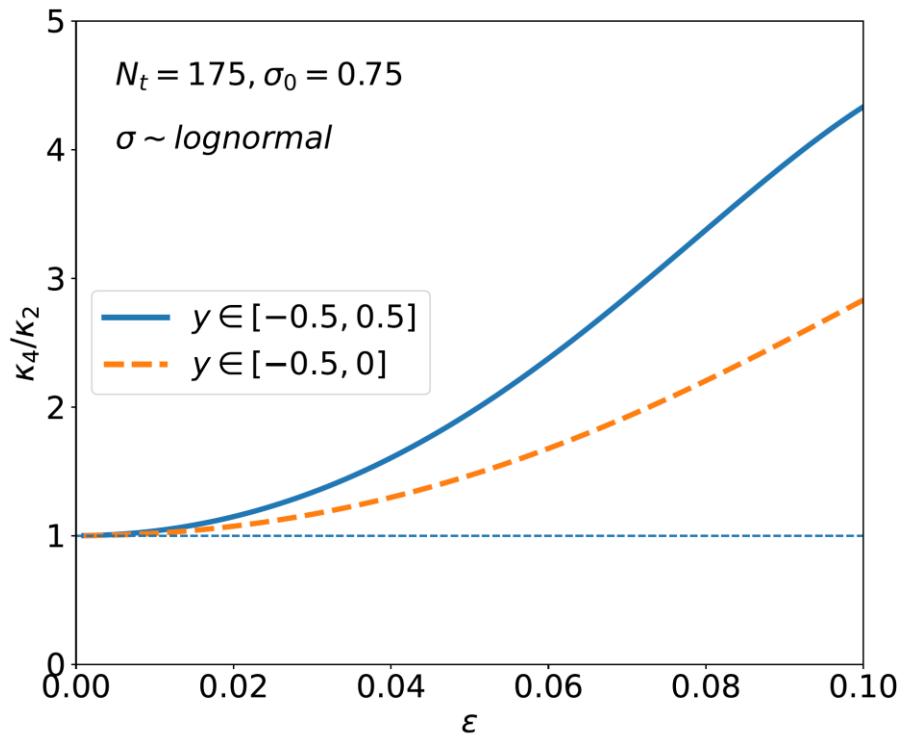
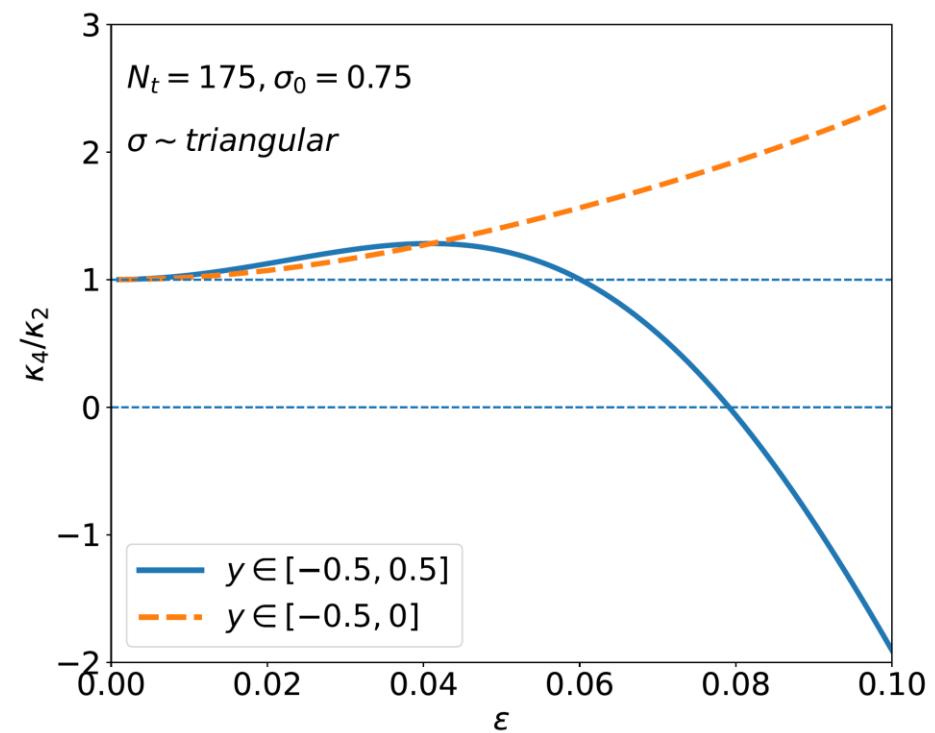
Cumulants are driven by two-particle correlation

$$\hat{C}_1 = \langle N \rangle$$



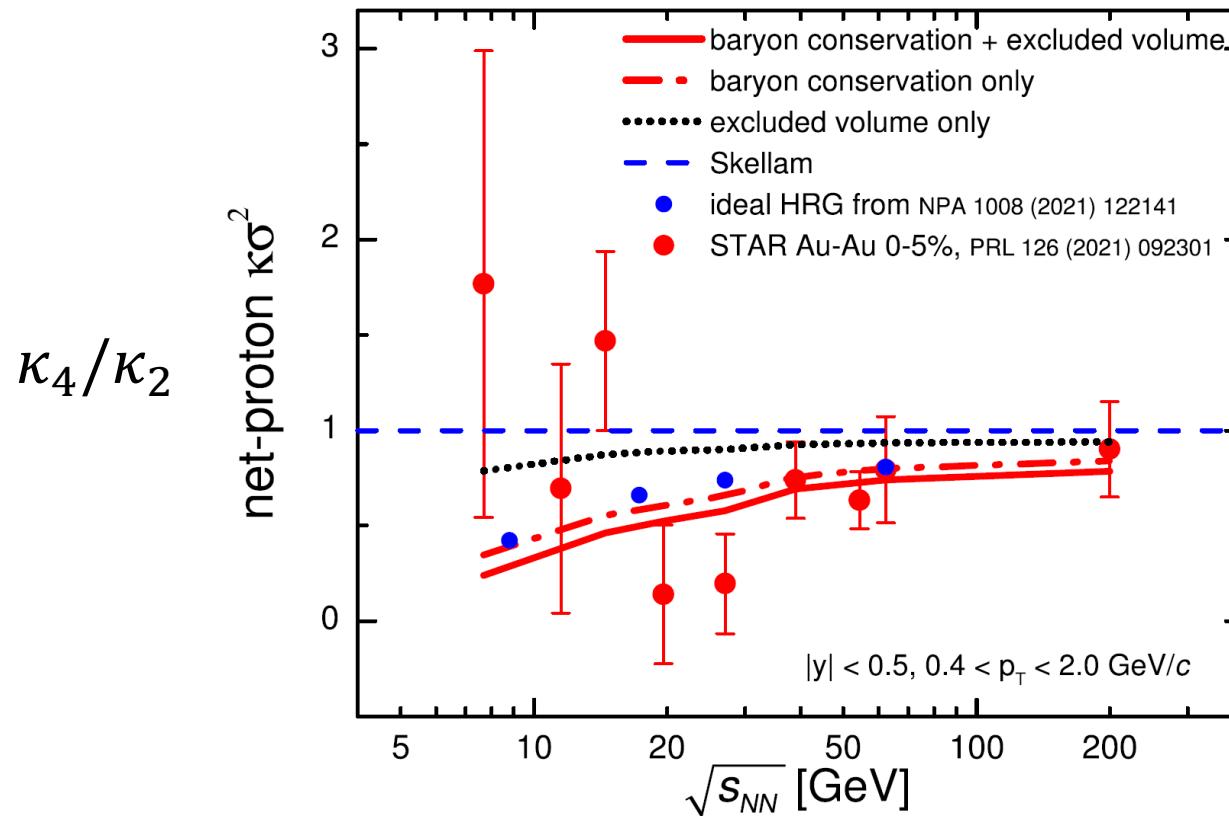
$$\kappa_4 = \langle N \rangle + 7\hat{C}_2 + 6\hat{C}_3 + \hat{C}_4$$

Symmetric vs. asymmetric rapidity bins (example)



Comparing experimental data points from different rapidity bins is misleading

STAR data vs. baryon conservation and excluded volume



Rapidity shape fluctuations might help

V.Vovchenko, V.Koch, C.Shen, PRC 105, 014904 (2022)

P.Braun-Munzinger, B.Friman, K.Redlich, A.Rustamov, J.Stachel, NPA 1008 (2021) 122141

AB, V.Koch, V.Skokov, PRC 87 (2013) 1, 014901

Conclusions

Long-range rapidity density fluctuations introduce correlations

Contribution to cumulants and factorial cumulants

Universal behavior for small fluctuations

Driven by two-particle correlations

Might be important

Backup

Cumulants with baryon conservation vs. cumulants without baryon conservation

V.Vovchenko, O.Savchuk, R.V. Poberezhnyuk,
M.I. Gorenstein, V.Koch, PLB 811, 135868 (2020)



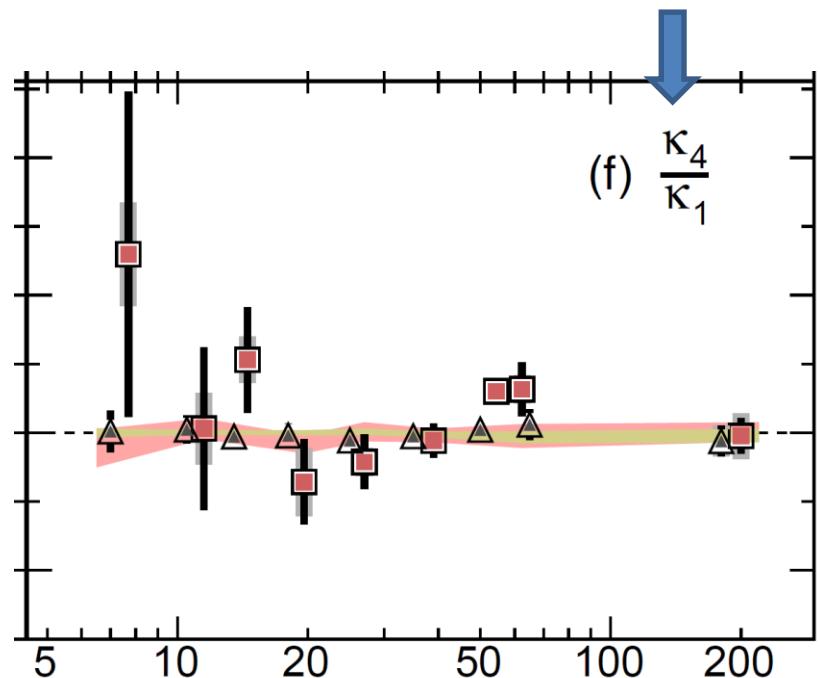
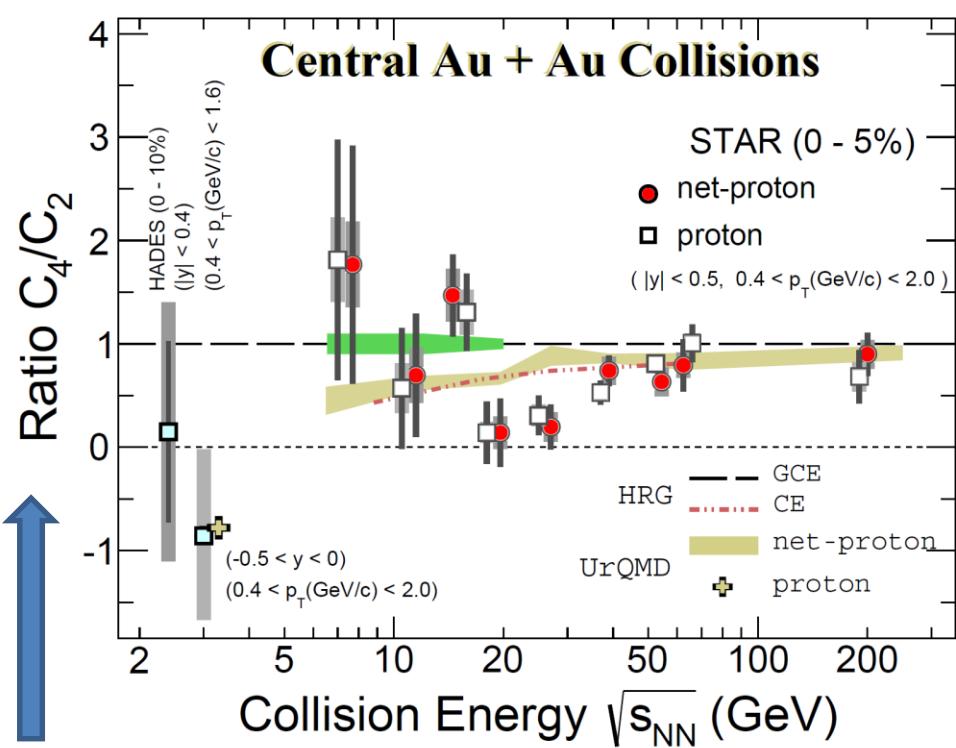
$$\kappa_4^{(1,B,\text{LO})} = f\bar{f} \left[\kappa_4^{(G)} - 3f\bar{f} \left(\kappa_4^{(G)} + \frac{(\kappa_3^{(G)})^2}{\kappa_2^{(G)}} \right) \right],$$

$$\begin{aligned} \kappa_4^{(1,B,\text{NLO})} &= \frac{1}{2} f\bar{f} \left\{ \frac{\kappa_3^{(G)} \kappa_5^{(G)} - \kappa_2^{(G)} \kappa_6^{(G)}}{(\kappa_2^{(G)})^2} \right. \\ &\quad \left. + 3f\bar{f} \left[\frac{2(\kappa_3^{(G)})^4 - 5\kappa_2^{(G)} (\kappa_3^{(G)})^2 \kappa_4^{(G)} + (\kappa_2^{(G)})^2 \kappa_3^{(G)} \kappa_5^{(G)}}{(\kappa_2^{(G)})^4} + \frac{(\kappa_4^{(G)})^2 + \kappa_2^{(G)} \kappa_6^{(G)}}{(\kappa_2^{(G)})^2} \right] \right\} \end{aligned}$$



M.Barej, AB, PRC 107 (2023) 3, 034914

Advertisement

kindly read κ_4/κ_2

Factorial cumulants vs cumulants

factorial
cumulant

$$\hat{C}_i = \frac{d^i}{dz^i} \ln \left(\sum_n P(n) z^n \right) \Big|_{z=1}$$

cumulant

$$\kappa_i = \frac{d^i}{dt^i} \ln \left(\sum_n P(n) e^{tn} \right) \Big|_{t=0}$$

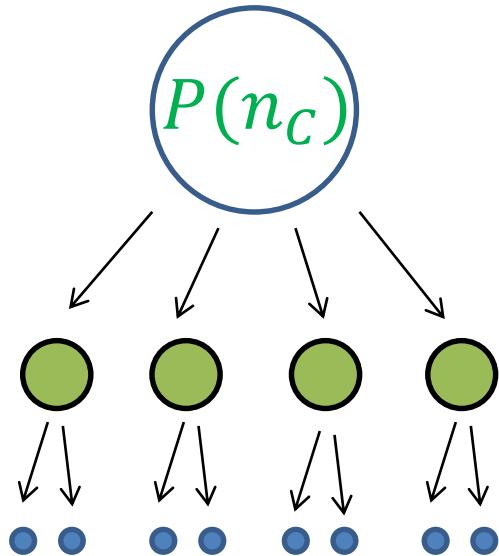
Poisson

$$\hat{C}_i = 0, \quad \kappa_i = \langle n \rangle$$

cumulants naturally appear
in statistical physics

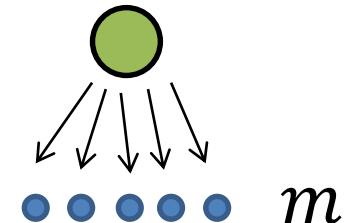
$$\ln(Z) = \ln \left(\sum_i e^{-\beta(E_i - \mu N_i)} \right)$$

Factorial cumulants – example



Poisson

m particle cluster



$$\hat{C}_2 \neq 0$$

$$\hat{C}_k = 0, k > 2$$

$$\hat{C}_{2,3,\dots,m} \neq 0$$

$$\hat{C}_k = 0, k > m$$

factorial
cumulants

$$\hat{C}_k = \frac{d^k}{dz^k} \ln \left(\sum_n P(n) z^n \right) \Big|_{z=1}$$