Thermalization and quark production in spatially homogeneous system of gluons

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in collaboration with

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Based on a work in progress and

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  Phys. Rev. D 55 (1997). Jalilian-Marian et al.
  Nucl. Phys. B 529 (1998). Kovchegov and Mueller

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- Our study uses the Boltzmann Equation in Diffusion Approximation (BEDA) as an alternative approach.

- The QCD Boltzmann equation at leading order:

$$(\partial_t + \mathbf{v} \cdot \nabla_{\mathbf{x}}) f^a = C^a_{2\leftrightarrow 2}[f] + C^a_{1\leftrightarrow 2}[f] \,, \quad f^a = \{f^g, f^q, f^{\bar{q}}\}$$

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$$f(\mathbf{p}) = f(p)$$

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- The thermalization can be studied following the time evolution of the screening mass, the jet quenching parameter and the effective temperature and net quark chemical potential[1].

$$m_D^2 = m_D^2[f] \qquad \hat{q} = \hat{q}[f]$$

$$T_*(t) \equiv \frac{\hat{q}}{2\alpha_s N_c m_D^2 \ln\frac{\langle p_t^2\rangle}{m_D^2}} \qquad \mu_* = \mu_*[f]$$

[1] All quarks are assumed to have identical distribution. In general each flavour would have its own μ_* associated.

- In diffusion approximation, the $2 \leftrightarrow 2$ collision kernel can be expressed as a Fokker-Planck equation plus an additional source term.

Phys. Lett. B 475 (2000). Mueller
Nucl. Phys. A 930 (2014). Blaizot, Wu, and Yan

$$C^a_{2\leftrightarrow 2} = \frac{1}{4}\hat{q}_a(t)\nabla_p \cdot \left[\nabla_p f^a + \frac{\mathbf{v}}{T^*(t)} f^a(1+\epsilon_a f^a)\right] + \mathcal{S}_a$$

$$\mathcal{S}_q = \frac{2\pi\alpha_s^2 C_F^2 \ln\frac{\langle p_t^2\rangle}{m_D^2}}{p}\left[\mathcal{I}_c f(1-F) - \bar{\mathcal{I}}_c F(1+f)\right],$$

$$\mathcal{S}_{\bar{q}} = \mathcal{S}_q\big|_{F\leftrightarrow\bar{F}}, \qquad \mathcal{S}_g = -\frac{N_f}{2C_F}(\mathcal{S}_q + \mathcal{S}_{\bar{q}}),$$

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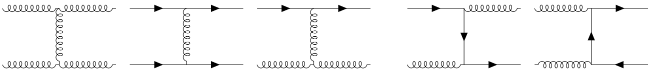
Phys. Lett. B 475 (2000). Mueller
Nucl. Phys. A 930 (2014). Blaizot, Wu, and Yan

Fokker-Planck term — Source term



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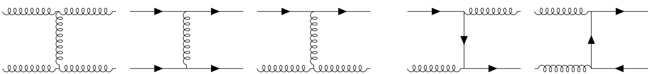
Fokker-Planck term Source term



- The gluon distribution function is known to diverge at small p, $f \propto 1/p$, for over-occupied systems, which is interpreted as the onset of Bose-Einstein Condensation (BEC).

Nucl. Phys. A 920 (2013). Blaizot, Liao, and McLerran

- The presence of BEC can be study numerically by choosing the appropriate boundary conditions with[2] $\dot{n}_c \propto (\lim_{p\to 0} pf - T_*)$.

Nucl. Phys. A 930 (2014). Blaizot, Wu, and Yan

[2] $n_c \equiv$ number density of the BEC.

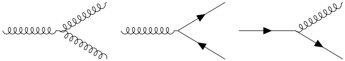
BEDA III. Inelastic collision kernel

- The $1 \leftrightarrow 2$ kernel can be computed in the deep LPM regime

Nucl. Phys. B 483 (1997). Baier et al.
Phys. Rev. D 78 (2008). Arnold and Dogan

$$C^a_{1\leftrightarrow 2} = \int_0^1 \frac{dx}{x^3} \sum_{b,c} \left[\frac{\nu_c}{\nu_a} C^c_{ab}(p/x; p, p(1-x)/x) - \frac{1}{2} C^a_{bc}(p; xp, (1-x)p) \right]$$

- The $C^a_{bc}(p; xp, (1-x)p)$ describes the collinear splitting $a \leftrightarrow bc$.
- The three possible processes involved are the three QCD interaction vertices.



- Will the BEC still appear in initially over-populated system after including inelastic collisions?

Rapid thermalization of the soft sector

- At small p, the $g \leftrightarrow gg$ and $g \leftrightarrow q\bar{q}$ are the dominant processes in the production of gluons and (anti)quarks, respectively.
- The distributions of gluons and quarks quickly fill a thermal distribution up to small soft momentum p_s

$$f^g(p) \approx \frac{T_*}{p} \qquad \text{for } p \lesssim p_g$$

$$f^q(p) \approx \frac{1}{e^{-\frac{\mu_*}{T_*}} + 1} \qquad \text{for } p \lesssim p_q$$

At early times, p_s is given by ($\mathcal{I}_c = \mathcal{I}_c[f]$)

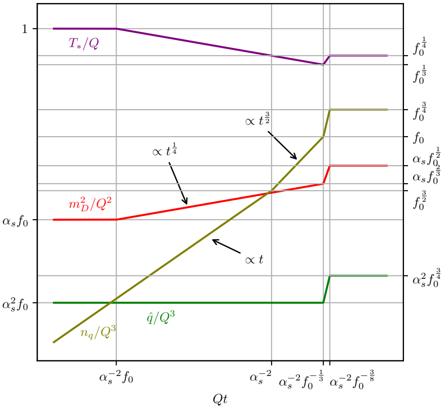
$$p_g \equiv (\hat{q}_A m_D^4 t^2/2)^{\frac{1}{5}} \qquad p_q \equiv [\alpha_s C_F \pi (\mathcal{I}_c + \bar{\mathcal{I}}_c) t]^{\frac{2}{5}} \, \hat{q}_F^{\frac{1}{5}}$$

- This behavior implies that $\dot{n}_c = 0$, so no BEC is observed as in the pure gluon case.

Nucl. Phys. A 961 (2017). Blaizot, Liao, and Mehtar-Tani
Physics Letters B 834 (2022). SBC, Salgado, and Wu

Three different stages for thermalization as in pure gluon scenario.

Physics Letters B 834 (2022). SBC, Salgado, and Wu



Parametric estimation for $f_0 \ll 1$

1. Soft gluon radiation and overheating.
   - T_* is almost constant since both m_D^2 and $\hat{q}$ are dominated by the hard sector.

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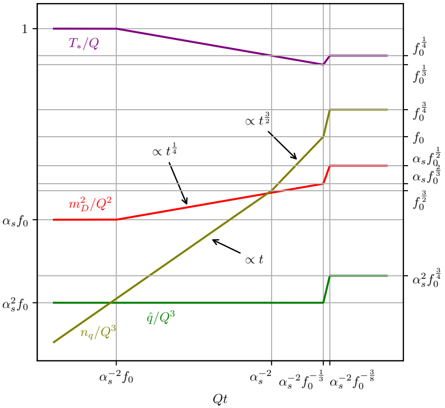
Physics Letters B 834 (2022). SBC, Salgado, and Wu



Parametric estimation for $f_0 \ll 1$

1. Soft gluon radiation and overheating.
   - T_* is almost constant since both m_D^2 and $\hat{q}$ are dominated by the hard sector.
2. Cooling and overcooling of soft gluons.
   - Soft gluons dominate the screening $\Rightarrow m_D^2 \uparrow \Rightarrow T_* \downarrow$.
   - $n_q \propto t$ lead by hard gluons, until $Qt \sim \alpha_s^{-2}$, when $n_q \propto t^{\frac{3}{2}}$ when the soft sector takes control.

Under-populated scenario I

Three different stages for thermalization as in pure gluon scenario.

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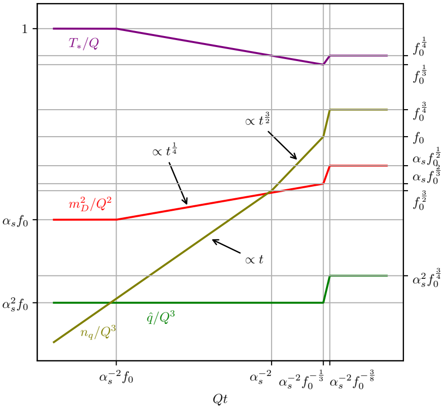

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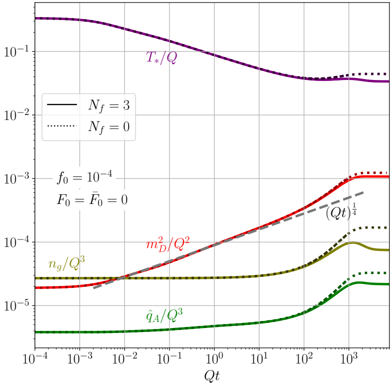
3. Reheating and mini-jet quenching in a QGP with T_*.
   - $\hat{q}$ receives dominant contribution from $g, q, \bar{q}$.
   - T_* increases until it reaches T_{eq}.

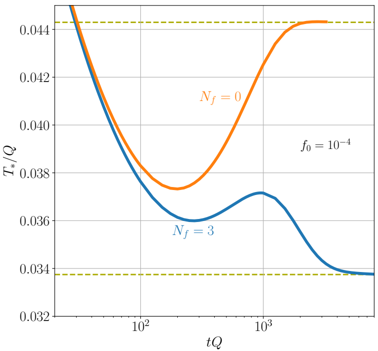


Numerical results for $f_0 = 10^{-4}$

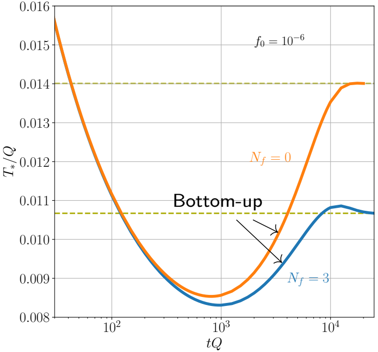
- Stage 1 and (early) stage 2 is easily identified since early times is insensitive to N_f.
- At later times, $N_f = 3$ deviate from $N_f = 0$ results.
- The macroscopic quantities follow the expected parametric behavior (except from the temperature at later times).
- Gluon number suffers an overshooting due to the quarks not playing a major role at early times of the evolution.

f, F and $\bar{F}$ are gluon, quark and antiquark distributions function. Initial condition is inspired by CGC: $f = f_0\theta(Q - p)$.

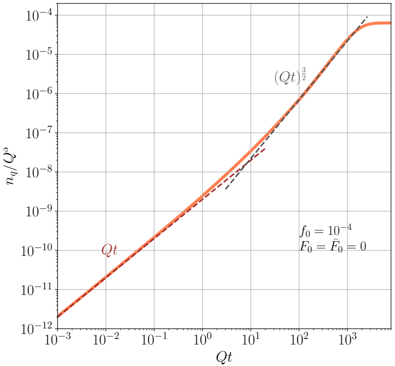
The temperature behavior at later times can be recovered in the limit $f_0 \ll 1$.



Effective temperature evolution for $f_0 = 10^{-4}$



Effective temperature evolution for $f_0 = 10^{-6}$

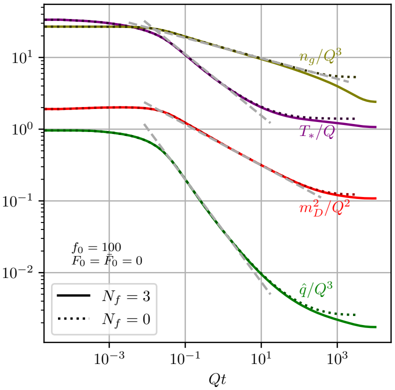


Quark number evolution for $f_0 = 10^{-4}$.

- At early times $g \leftrightarrow q\bar{q}$ dominates.
- At a certain time $Qt \sim \alpha_s^{-2}$, the quark production is accelerated due to the soft sector contributions.
- In the same time, $gg \leftrightarrow q\bar{q}$ joins to the relevant processes in quark production.

Two-stage thermalization, as in pure gluon scenario.

Physics Letters B 834 (2022). SBC, Salgado, and Wu

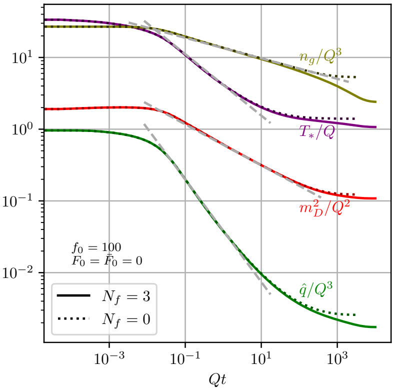


Numerical results for $f_0 = 100$

1. Soft gluon radiation and overheating.
   - T_* is almost constant since the soft gluons do not play an important role.

Two-stage thermalization, as in pure gluon scenario.

Physics Letters B 834 (2022). SBC, Salgado, and Wu



Numerical results for $f_0 = 100$

1. Soft gluon radiation and overheating.
    - T_* is almost constant since the soft gluons do not play an important role.
2. Momentum broadening and cooling (no overcooling)
    - T_* starts to decrease until it reaches thermal equilibrium.
    - All the quantities evolve according the universal scaling / self similar solution (dashed lines).

See also:

Phys. Rev. D 86 (2012). Kurkela and Moore
Phys. Rev. D 89.7 (2014). Abraao York et al.
Phys. Rev. D 86 (2012). Berges, Schlichting, and Sexty

- Unlike under-occupied scenario, fermionic contribution only becomes as important as bosonic at thermalization.

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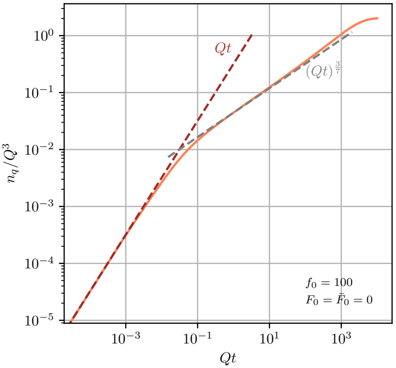

Quark number evolution for $f_0 = 100$

- At early times $g \leftrightarrow q\bar{q}$ dominates.
- From $Qt \sim \alpha_s^{-2}$, the quark production slow down due to the decrease of $\hat{q}$ and m_D^2 as typical momentum p increases.
- At the same time, $gg \leftrightarrow q\bar{q}$ joins to the relevant processes in quark production.

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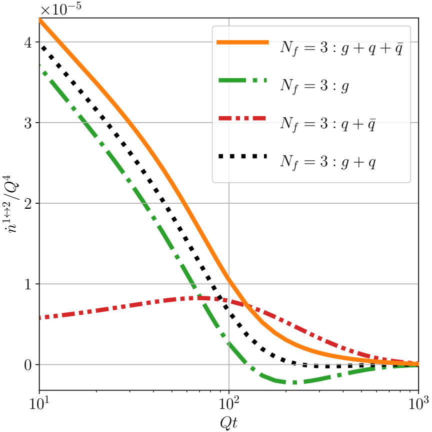
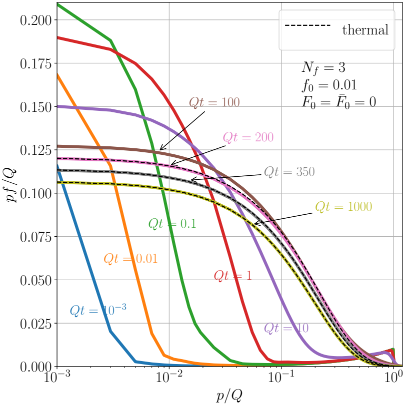
- At later times, the $g \leftrightarrow gg$ processes nearly establish detailed balance.
- The decrease in gluon number is mainly due to $gg \to q\bar{q}$ and $g \to q\bar{q}$.



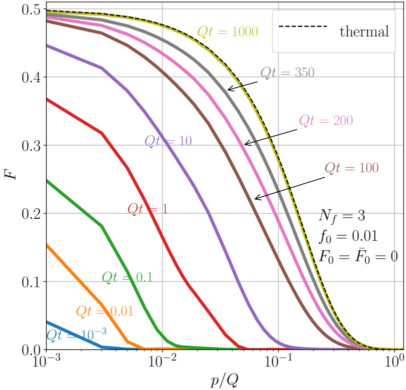
Variation in number density due to $1 \leftrightarrow 2$ processes for $f_0 = 0.01$.

- This consistent with the picture of the subsystem of gluons achieving thermal equilibrium among itself, while the quark sector still needs time to have a Fermi-Dirac profile.

Phys. Rev. Lett. 122 (2019). Kurkela and Mazeliauskas

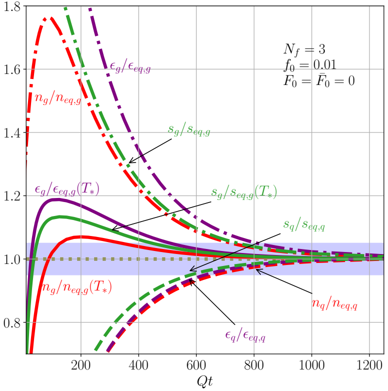


Distribution for gluons at different times.



Distribution for quarks at different times.

- This implies that gluons undergo the top-down thermalization in contrast with the bottom-up of the pure gluon scenario.



- This observation is true for all larger values of f_0 since in this cases gluon number always overshoot the thermal value.

Summary and conclusions

- The Boltzmann Equation in Diffusion Approximation (BEDA) provides a framework to study the thermalization of a system of quarks and gluons.
- The soft sector of gluons and (anti)quarks quickly achieves a thermal distribution due to inelastic processes $\Rightarrow$ no BEC.
- Both under and over-occupied limits have been parametrically understood in accordance with numerical simulations.
- Quark production, initially linear with time suffers a speed-up in the dilute limit, while it is slowed down in the highly occupied systems.
- Thermalization of gluons exhibits a top-down fashion as long as f_0 is not extremely small, delaying thermalization with respect to pure gluon scenario.

Thank you for your attention!

Back-up

- Jet quenching parameter

$$\hat{q}_a = 8\pi\alpha_s^2 C_a \ln\frac{\langle p_t^2\rangle}{m_D^2}\int\frac{d^3p}{(2\pi)^3}\left[N_c f\,(1+f)+\frac{N_f}{2}F(1-F)\right.$$
$$\left.+\frac{N_f}{2}\bar{F}(1-\bar{F})\right]$$

- Screening mass

$$m_D^2 = 8\pi\alpha_s\int\frac{d^3p}{(2\pi)^3}\frac{1}{|p|}\left(N_c f+\frac{N_f}{2}F+\frac{N_f}{2}\bar{F}\right)$$

- Integrals $\mathcal{I}_c$

$$\mathcal{I}_c = \int\frac{d^3p}{(2\pi)^3}\frac{1}{p}[f+F+f(F-\bar{F})], \qquad \bar{\mathcal{I}}_c = \mathcal{I}_c|_{F\leftrightarrow\bar{F}}$$

- Now we will have 3 coupled Boltzmann equations:

$$(\partial_t + \mathbf{v}\cdot\nabla_{\mathbf{x}})f^a = C^a_{2\leftrightarrow 2}[f] + C^a_{1\leftrightarrow 2}[f]\,, \quad f^a = \{f^g, f^q, f^{\bar{q}}\} \equiv \{f, F, \bar{F}\}$$

- The **elastic** kernel includes now an additional source term.

$$C^a_{2\leftrightarrow 2} = \frac{1}{4}\hat{q}_a(t)\nabla_{\mathbf{p}}\cdot\left(\nabla_{\mathbf{p}}f^a + \frac{\mathbf{v}}{T_*(t)}f^a(1\pm f^a)\right) + \mathcal{S}_a$$

$$\mathcal{S}_q = \frac{2\pi\alpha_s C_F^2 \ln\frac{\langle p_t^2\rangle}{m_D^2}}{p}\left[\mathcal{I}_c f\,(1-F) - \bar{\mathcal{I}}_c F\,(1+f)\right]$$

$$\mathcal{S}_{\bar{q}} = \mathcal{S}_q\big|_{F\leftrightarrow\bar{F}} \qquad \mathcal{S}_g = -\frac{N_f}{2C_F}\left(\mathcal{S}_q + \mathcal{S}_{\bar{q}}\right)$$

$$\mathcal{I}_c = \int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{p}\left[f + F + f(F-\bar{F})\right]\,, \quad \bar{\mathcal{I}}_c = \mathcal{I}_c\big|_{F\leftrightarrow\bar{F}}\,, \quad \frac{\mu_*}{T_*} = \ln\frac{\mathcal{I}_c}{\bar{\mathcal{I}}_c}$$

$$C^a_{1\leftrightarrow 2} = \int_0^1 \frac{dx}{x^3} \sum_{b,c} \left[\frac{\nu_c}{\nu_a} C^c_{ab}(p/x; p, p(1-x)/x) - \frac{1}{2} C^a_{bc}(p; xp, (1-x)p) \right],$$

where

$$C^a_{bc}(p; xp, (1-x)p) \equiv \frac{dI_{a\to bc}(p)}{dxdt} \mathcal{F}^a_{bc}(p; xp, (1-x)p)$$

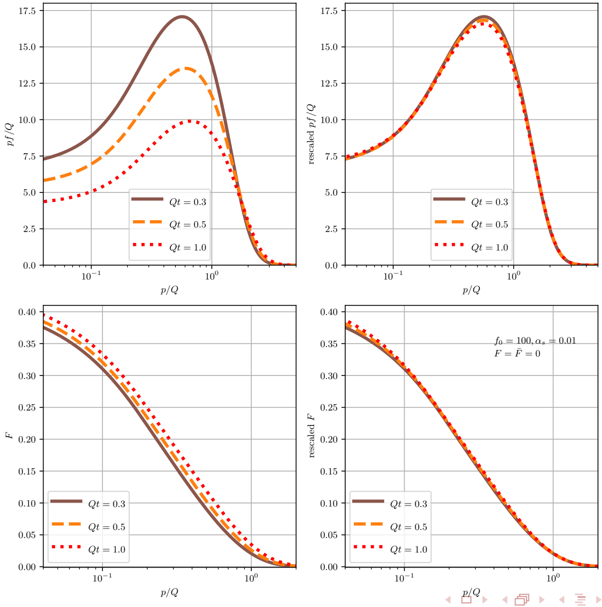
with

$$\mathcal{F}^a_{bc}(p; l, k) = f^a_p(1+\epsilon_b f^b_l)(1+\epsilon_b f^c_k) - f^b_l f^c_k (1+\epsilon_a f^a_p).$$

We use the splitting rate in the deep LPM regime [5, 2]

$$
\begin{aligned}
\frac{d^2 I_{g\to gg}(p)}{dxdt} &= \frac{\alpha_s N_c}{\pi}\frac{(1-x+x^2)^{5/2}}{(x-x^2)^{3/2}}\sqrt{\frac{\hat{q}_A}{p}},\\
\frac{d^2 I_{g\to q\bar{q}}(p)}{dxdt} &= \frac{\alpha_s}{4\pi}[x^2+(1-x)^2]\left[\frac{\frac{C_F}{C_A}-x(1-x)}{x(1-x)}\right]^{\frac{1}{2}}\sqrt{\frac{\hat{q}_A}{p}},\\
\frac{d^2 I_{q\to gq}(p)}{dxdt} &= \frac{d^2 I_{\bar{q}\to g\bar{q}}(p)}{dxdt} = \frac{\alpha_s C_F}{2\pi}\frac{1+(1-x)^2}{x}\left[\frac{1-x+\frac{C_F}{C_A}x^2}{x(1-x)}\right]^{\frac{1}{2}}\sqrt{\frac{\hat{q}_A}{p}},\\
\frac{d^2 I_{q\to qg}(p)}{dxdt} &= \frac{d^2 I_{\bar{q}\to \bar{q}g}(p)}{dxdt} = \frac{\alpha_s C_F}{2\pi}\frac{1+x^2}{1-x}\left[\frac{x+\frac{C_F}{C_A}(1-x)^2}{x(1-x)}\right]^{\frac{1}{2}}\sqrt{\frac{\hat{q}_A}{p}},
\end{aligned}
\tag{1}
$$

Self-similar solution for $f_0 = 100$ and $\alpha_s = 0.01$



Top-down thermalization for $f_0 = 100$ and $\alpha_s = 0.01$