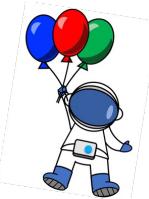


Inverse slope of the photon p_T spectrum and the QGP temperature profile

Jean-François Paquet & Steffen A. Bass

September 6, 2023

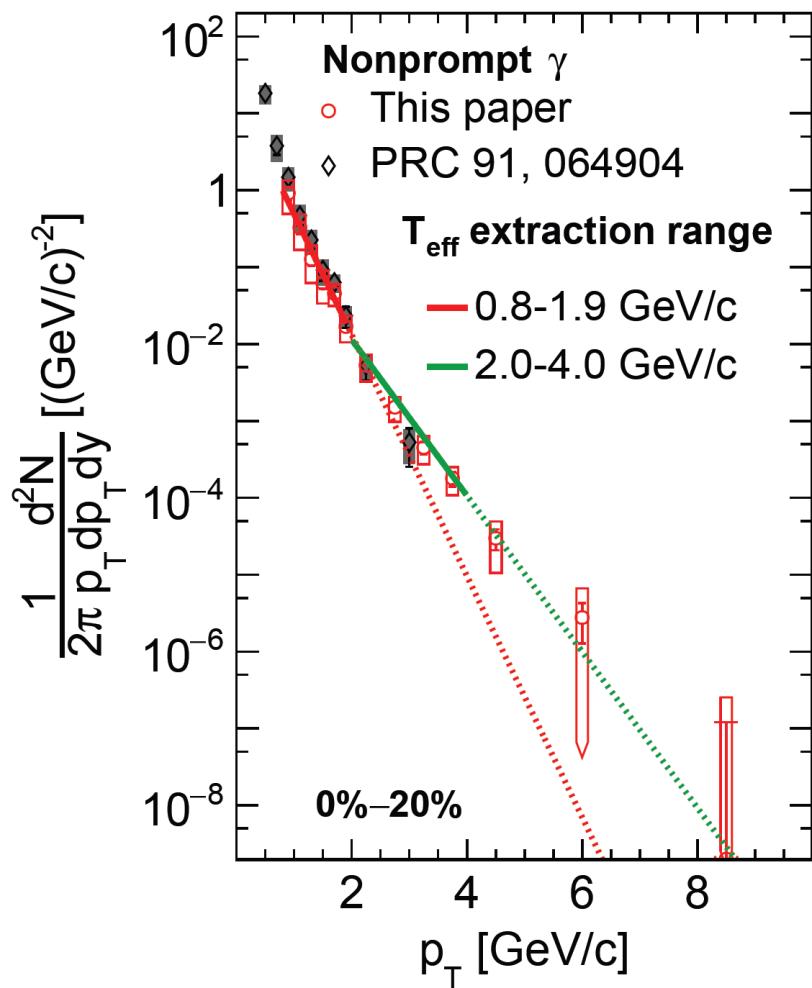


Quark Matter 2023



Photon p_T spectrum and inverse slope

Ref.: PHENIX Collaboration [arXiv:2203.17187]



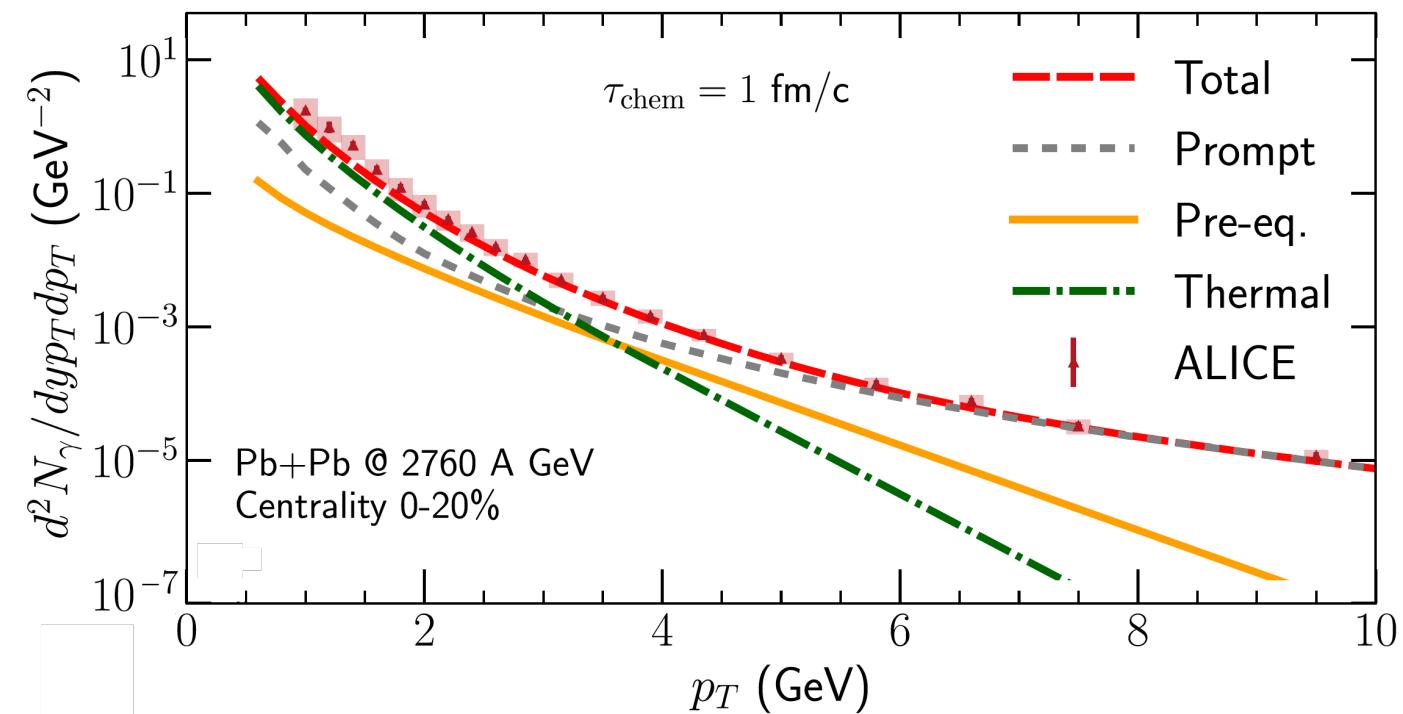
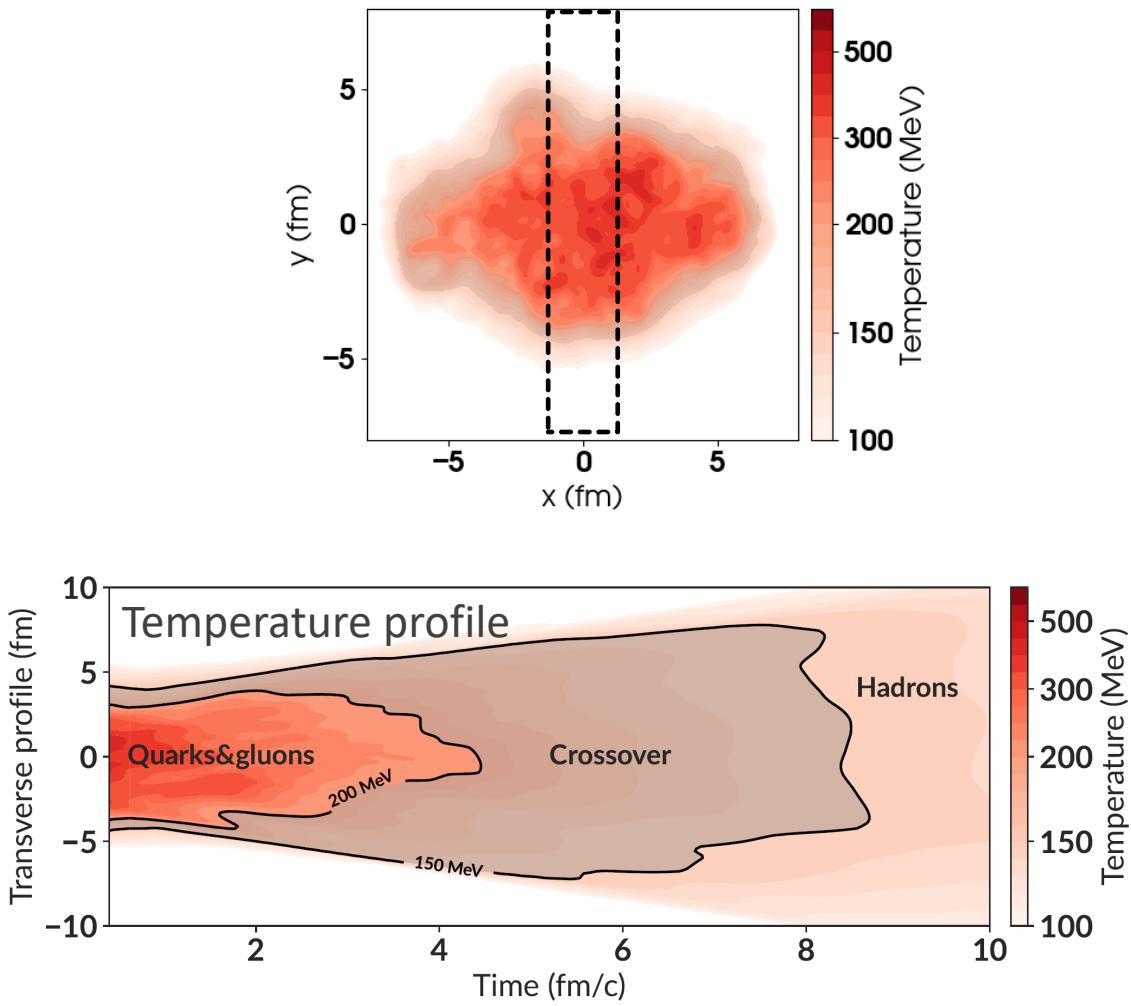
$$\frac{1}{2\pi p_T} \frac{dN_\gamma}{dp_T} \Big|_{y=0, p_{T,min} < p_T < p_{T,max}} \propto \exp\left(-\frac{p_T}{T_{eff}}\right)$$

centrality	T_{eff} (GeV/c)	T_{eff} (GeV/c)
	$0.8 < p_T < 1.9$ GeV/c	$2 < p_T < 4$ GeV/c
0%-20%	0.277 ± 0.017 $^{+0.036}_{-0.014}$	0.428 ± 0.031 $^{+0.031}_{-0.030}$
20%-40%	0.264 ± 0.010 $^{+0.014}_{-0.007}$	0.354 ± 0.019 $^{+0.020}_{-0.030}$
40%-60%	0.247 ± 0.007 $^{+0.005}_{-0.004}$	0.392 ± 0.023 $^{+0.022}_{-0.022}$
60%-93%	0.253 ± 0.011 $^{+0.012}_{-0.006}$	0.331 ± 0.036 $^{+0.031}_{-0.041}$

Results at the LHC by the ALICE Collaboration:

Centrality	T_{eff} (GeV) $0.9 < p_T < 2.1$ GeV	T_{eff} (GeV) $1.1 < p_T < 2.1$ GeV
0-20%	0.297	-
20-40%	-	0.410

Interpretation of the inverse slope

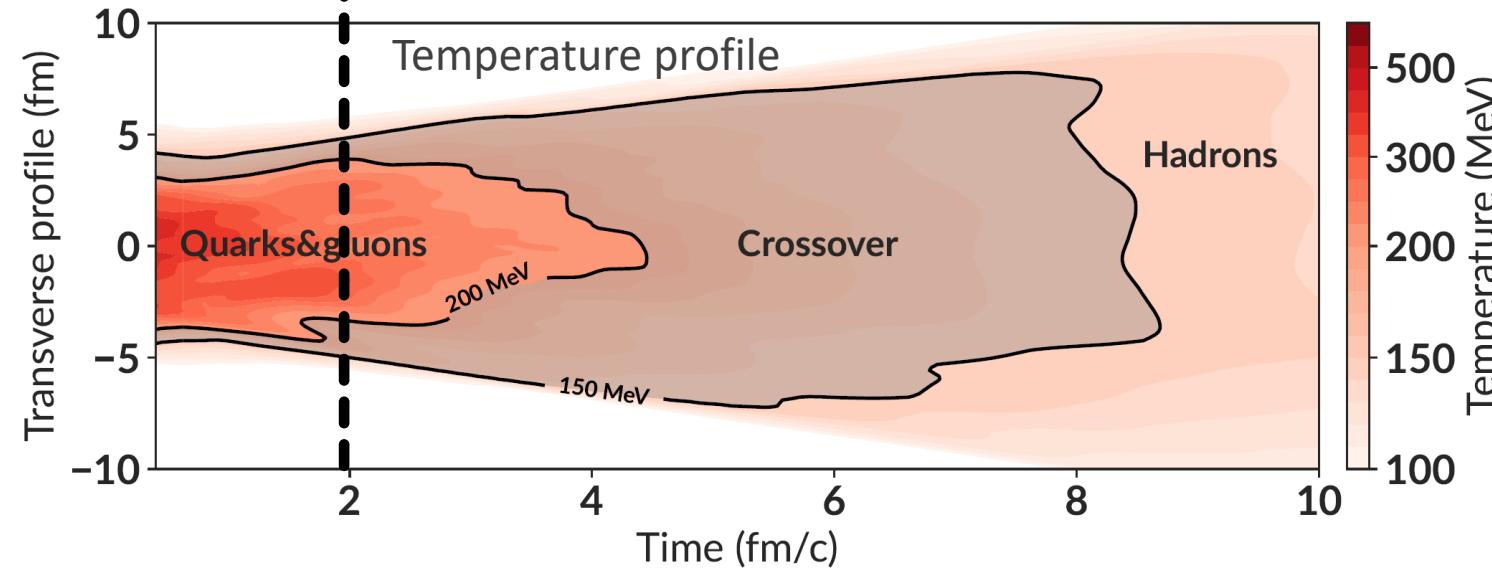


Spacetime profile of plasma

$$E \frac{dN_\gamma}{d^3p} = \int d^4X E \frac{d\Gamma_\gamma}{d^3p} (p_\mu u^\mu(X), T(X), \dots)$$

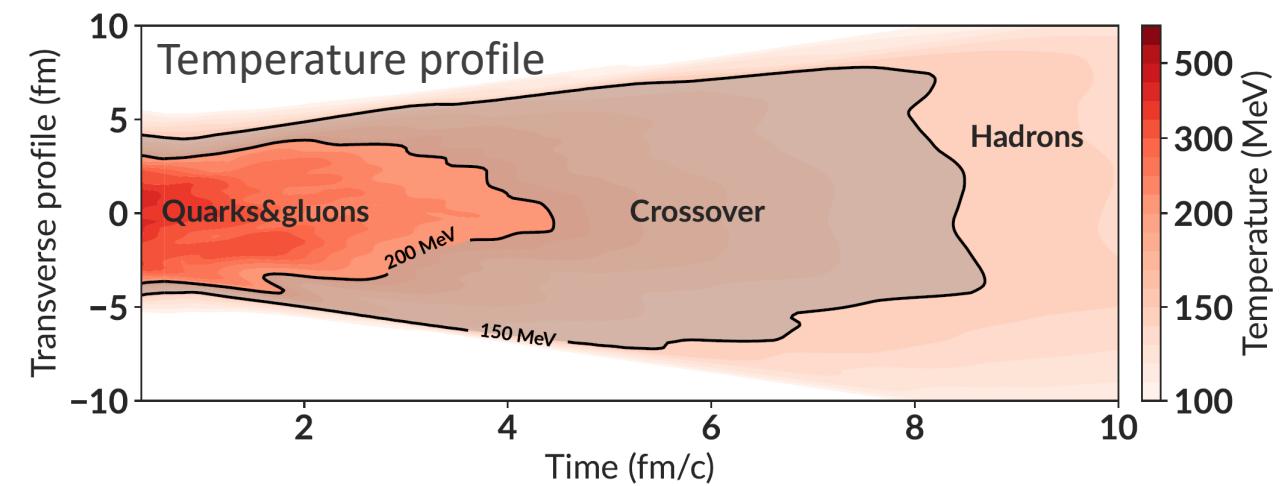
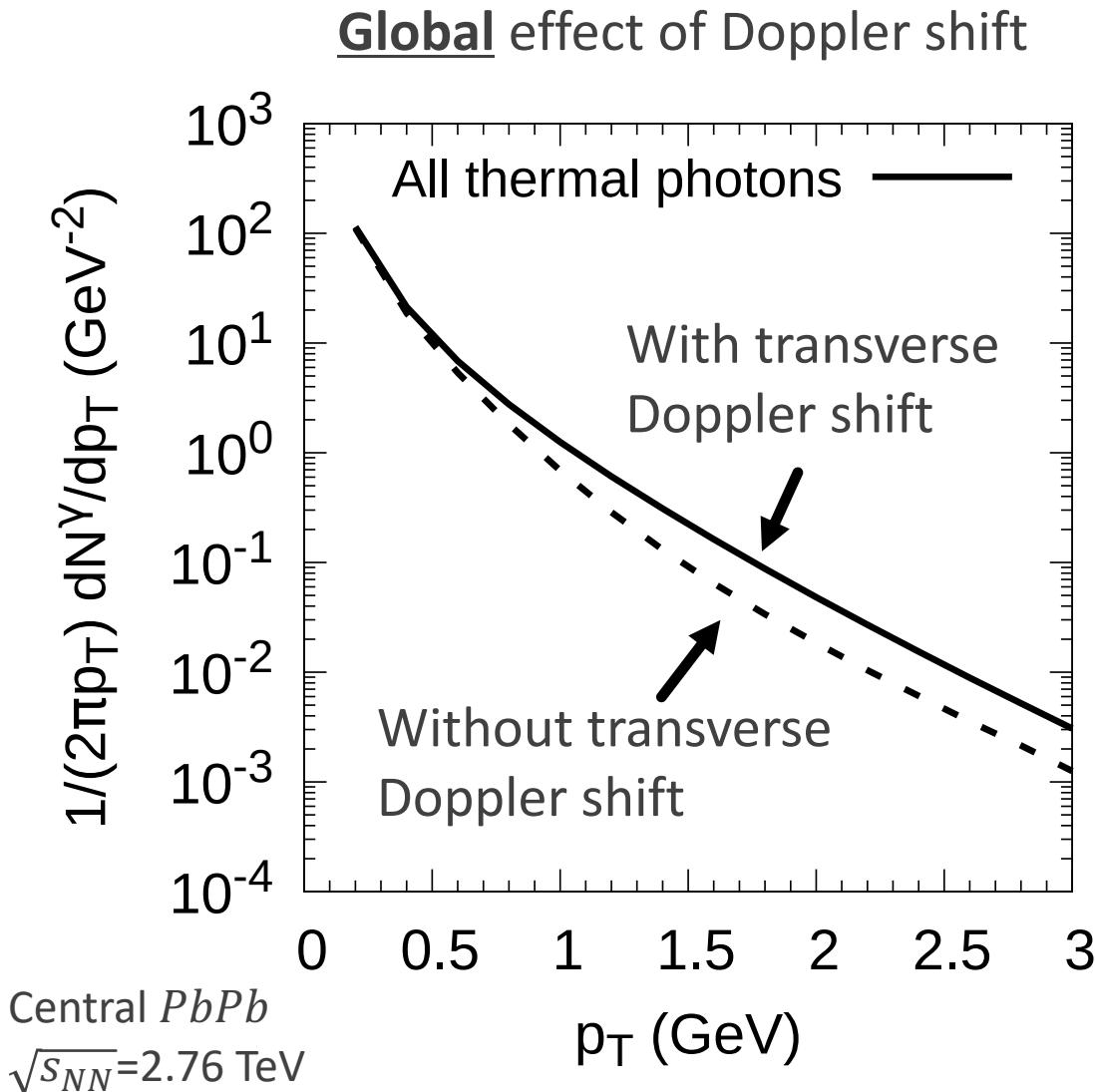
Photon emission rate

Longitudinal-expansion dominated Transverse-expansion dominated



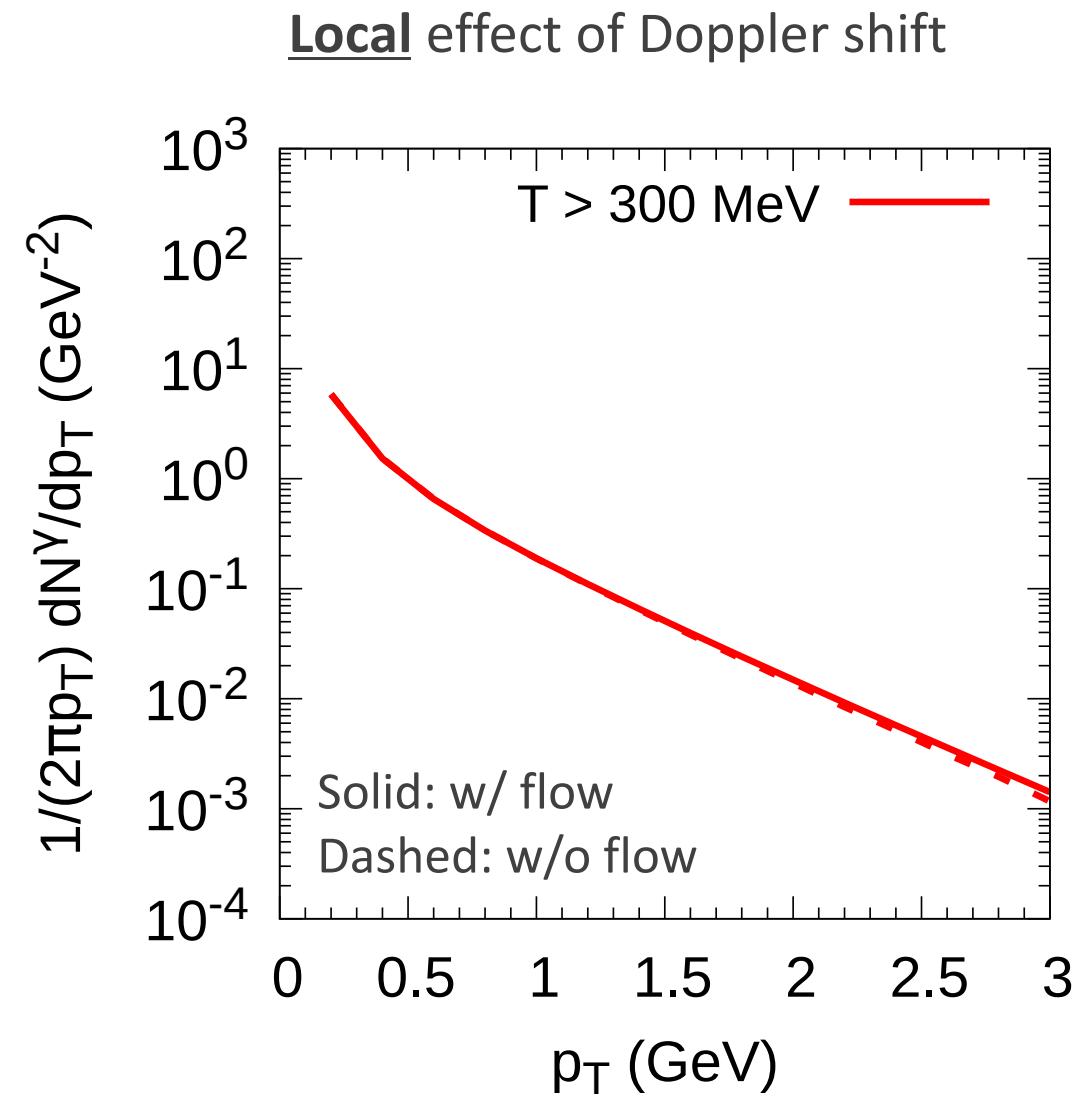
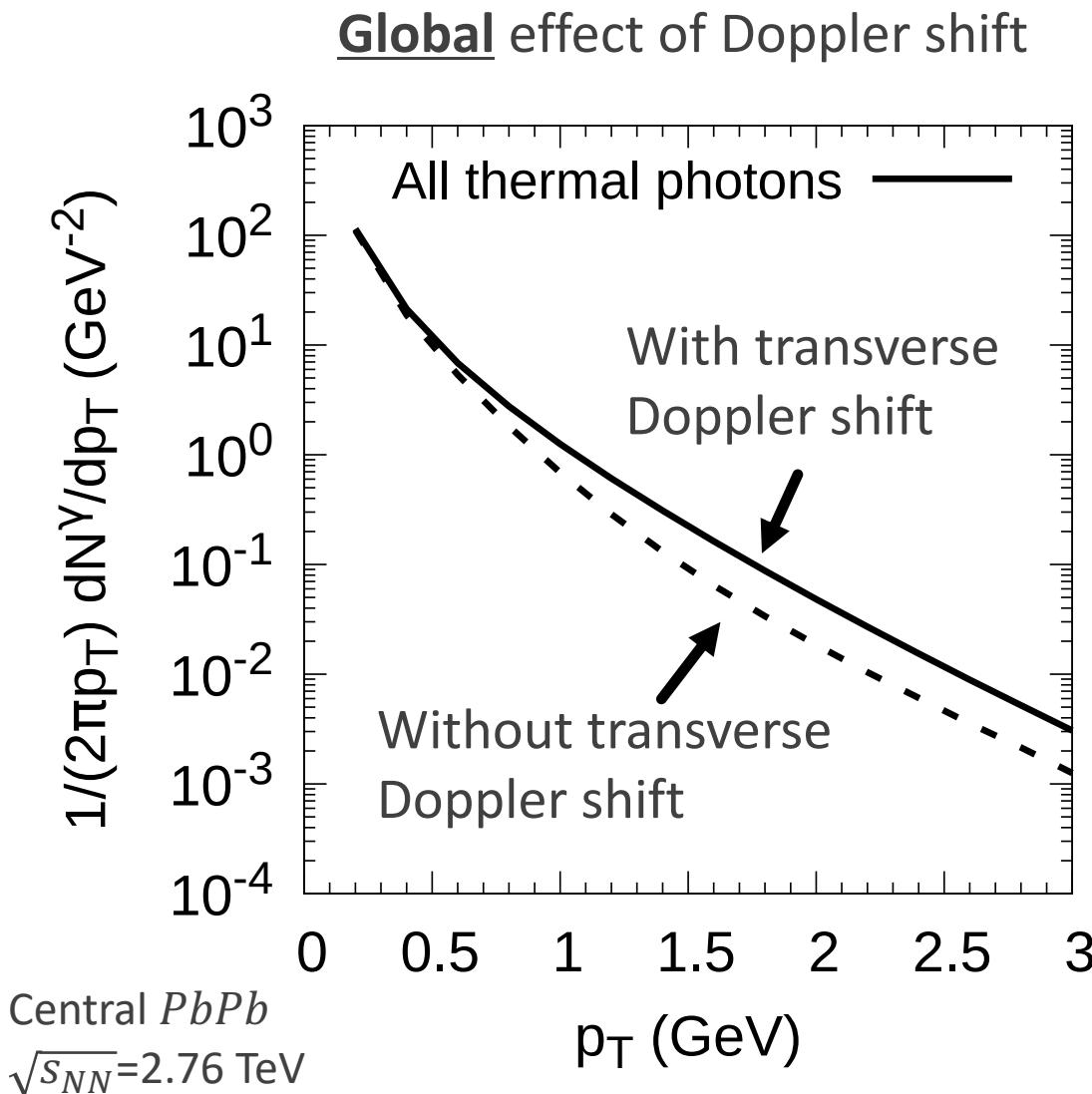
$$\begin{aligned}
 \frac{1}{2\pi p_T} \frac{dN_\gamma}{dp_T} \Big|_{y=0} &= \frac{1}{2\pi} \int_0^{2\pi} d\phi \left[\int d^4X E \frac{d\Gamma_\gamma}{d^3p} (p_\mu u^\mu(X), T(X), \dots) \right]_{y=0} \\
 &= \frac{1}{2\pi} \int_0^{2\pi} d\phi \left[\int d\eta_s d^2x_T d\tau \tau E \frac{d\Gamma_\gamma}{d^3p} \left(p_T \left(\cosh(\eta_s) \sqrt{1+u_\perp^2} - u_\perp \cos(\phi) \right), T, \dots \right) \right] \\
 &\approx \sqrt{2\pi} \int d^2x_T d\tau \tau \frac{1}{1 + \frac{p_T u_\perp}{T}} \sqrt{\frac{T}{p_T \sqrt{1+u_\perp^2}}} E \frac{d\Gamma_\gamma}{d^3p} \left(p_T \left(\sqrt{1+u_\perp^2} - u_\perp \right), T, \dots \right) \propto \exp \left(-\frac{p_T}{T_{eff}} \right)
 \end{aligned}$$

Effect of transverse flow in a realistic calculation

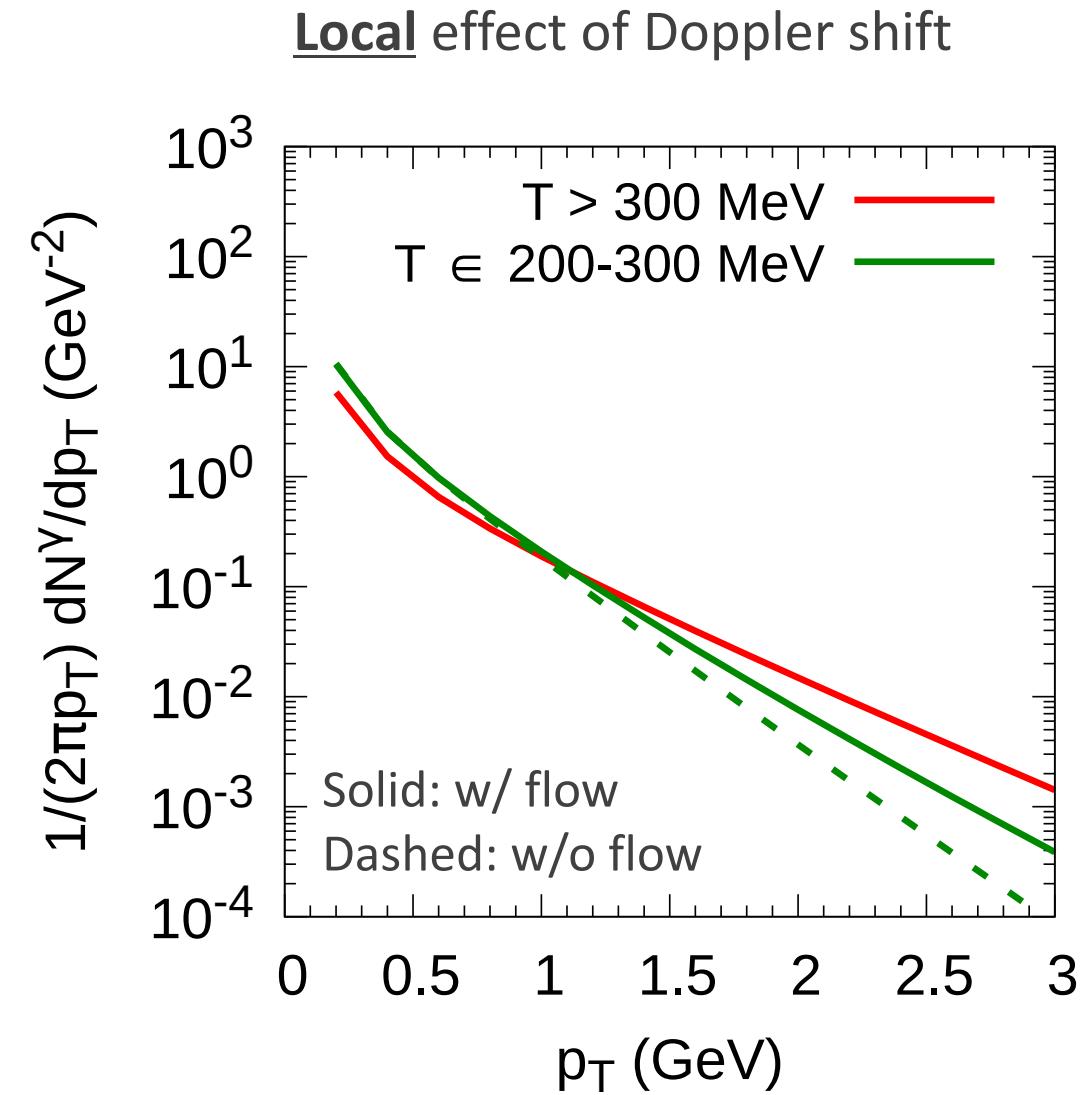
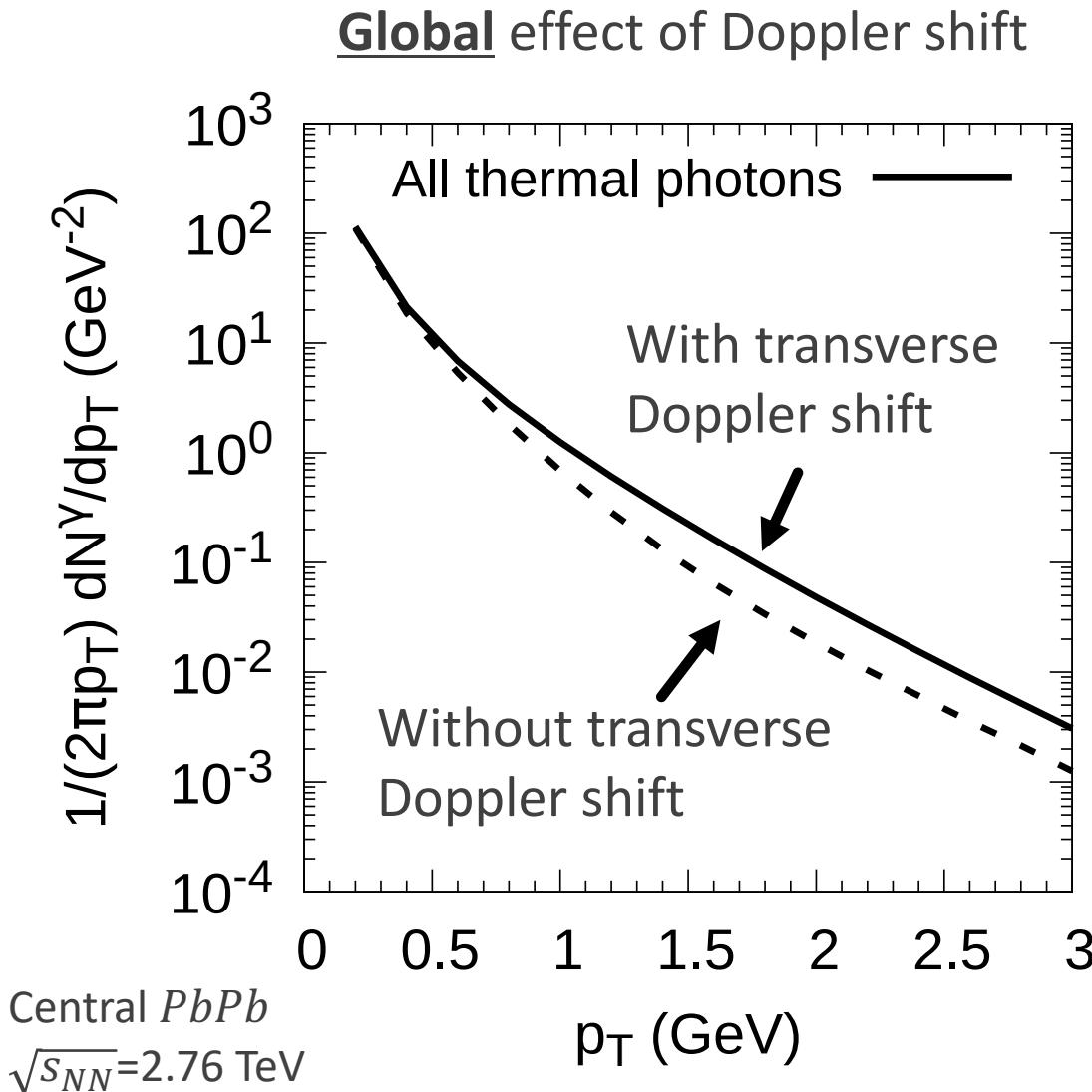


$$\frac{1}{2\pi p_T} \frac{dN_\gamma}{dp_T} \Big|_{y=0} = \frac{1}{2\pi} \int_0^{2\pi} d\phi \left[\int d^4X E \frac{d\Gamma_\gamma}{d^3p} (p_\mu u^\mu(X), T(X), \dots) \right]_{y=0}$$

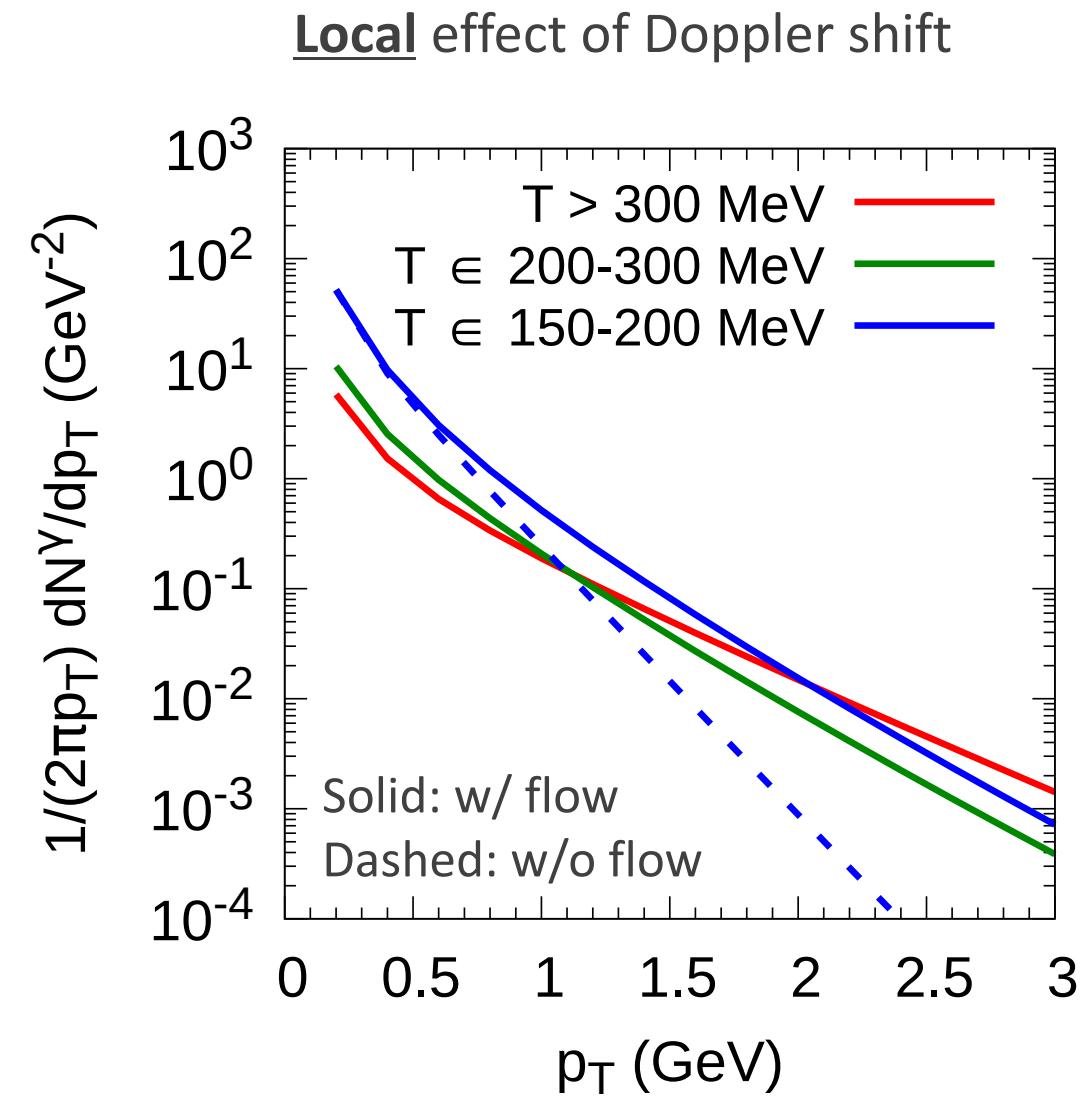
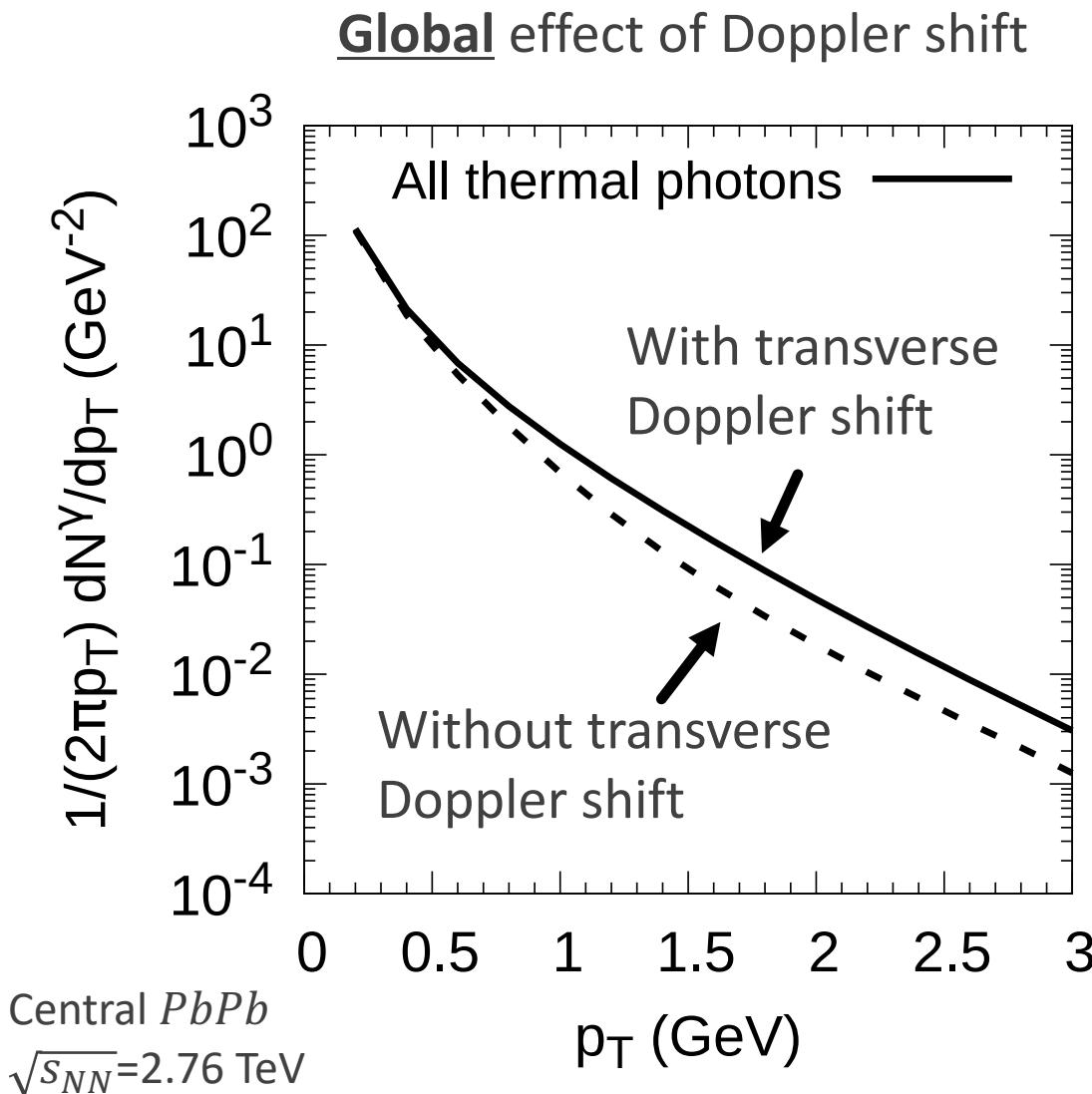
Effect of transverse flow in a realistic calculation



Effect of transverse flow in a realistic calculation

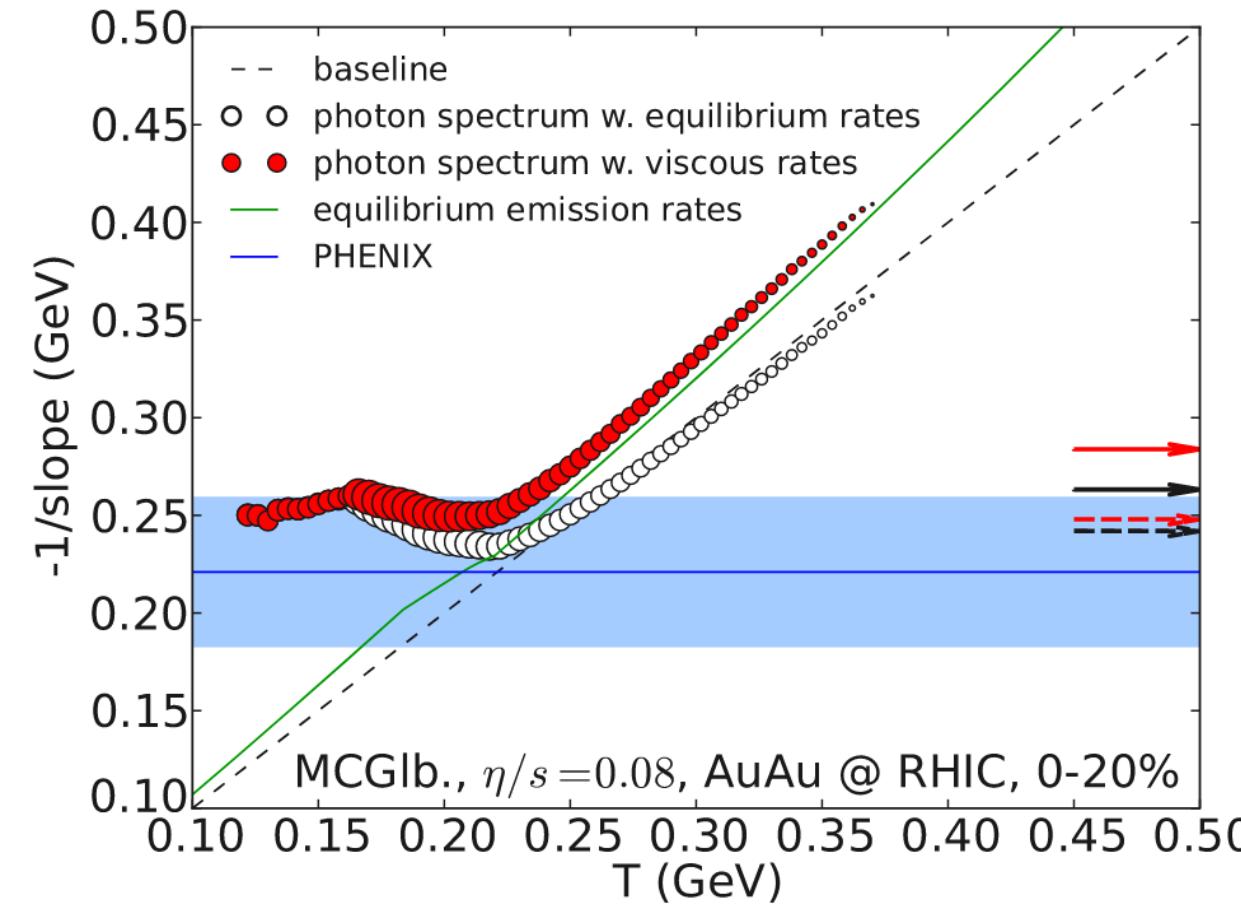


Effect of transverse flow in a realistic calculation



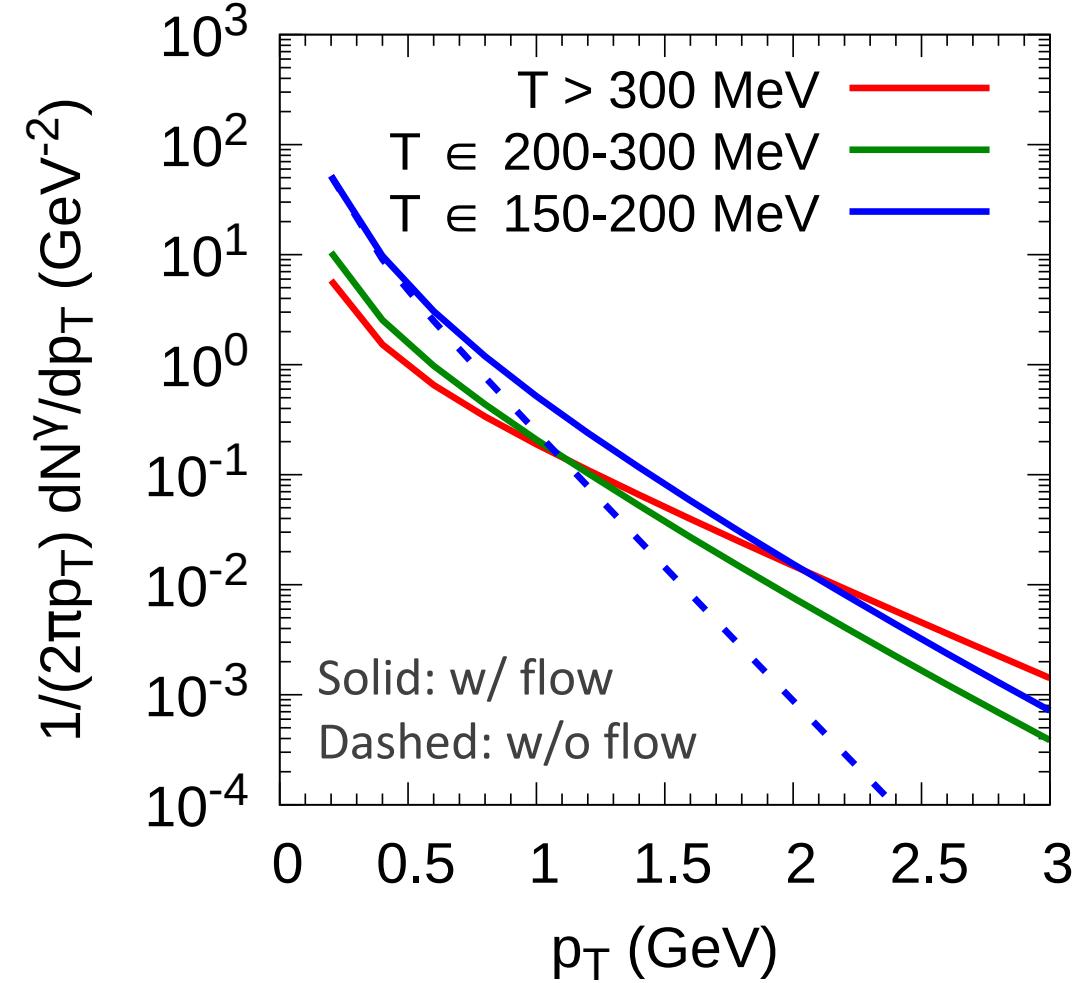
Effect of transverse flow in a realistic calculation

Local effect of Doppler shift

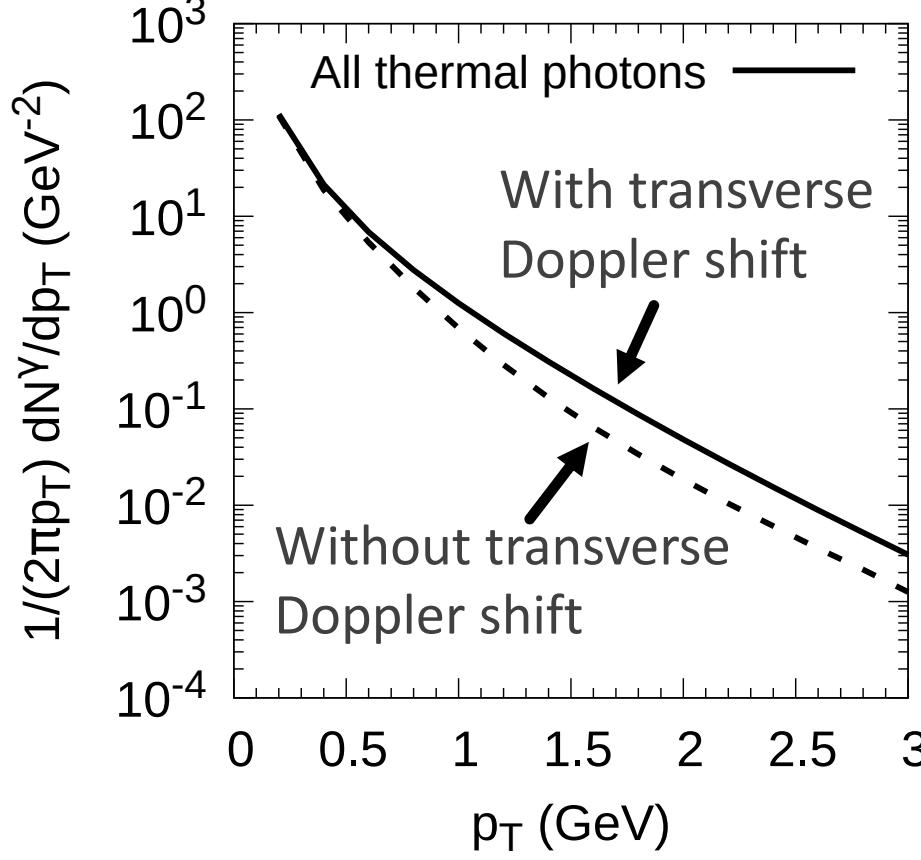


Ref.: Shen, Heinz, Paquet, Gale (2014) PRC;
See also van Hees, Gale, Rapp (2011) PRC

Local effect of Doppler shift

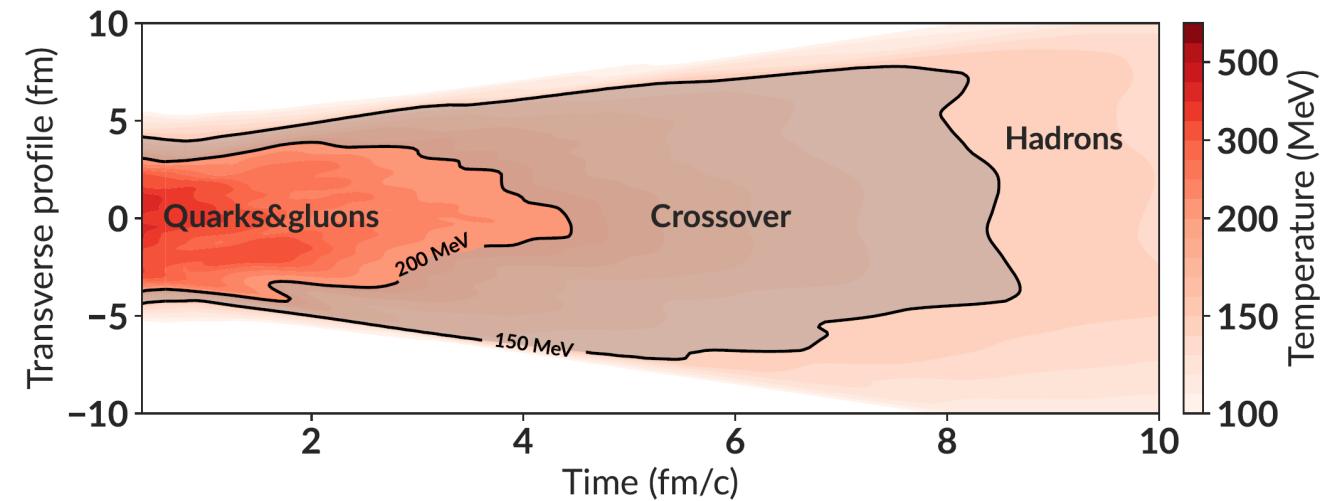


Effect of transverse flow in a realistic calculation

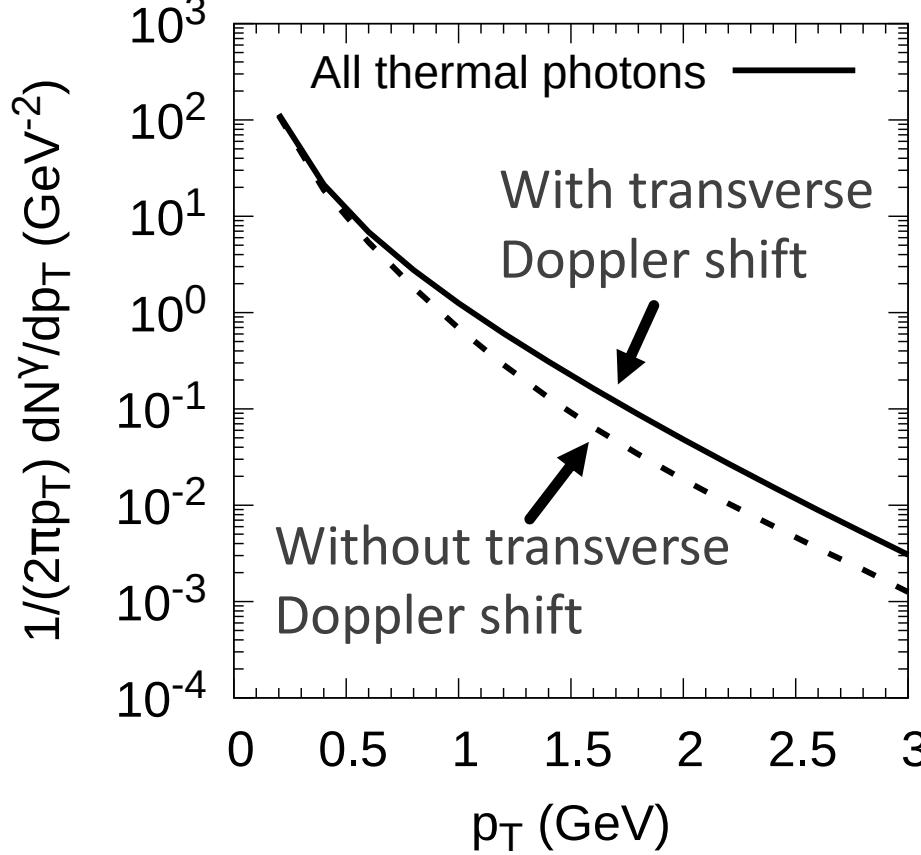


$$\frac{1}{2\pi p_T} \frac{dN_\gamma}{dp_T} = \frac{1}{2\pi} \int_0^{2\pi} d\phi \int d^4X E \frac{d\Gamma_\gamma}{d^3p} (p_\mu u^\mu(X), T(X), \dots)$$

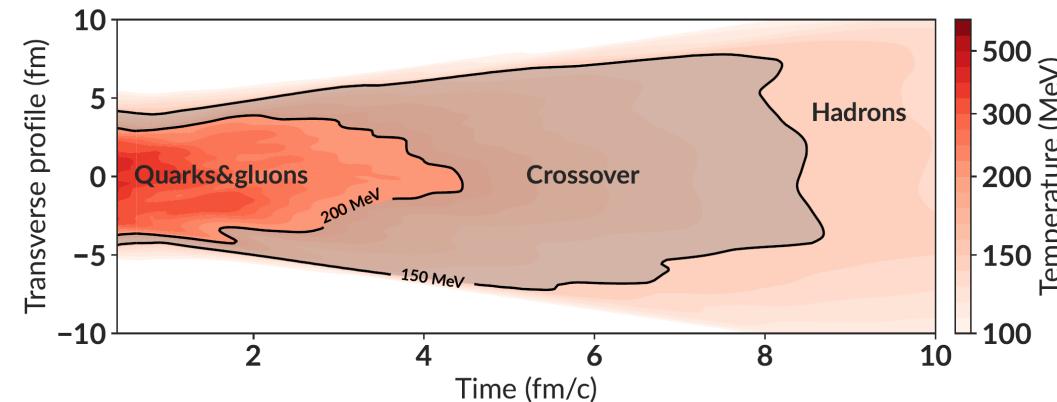
$$\approx \sqrt{2\pi} \int d^2x_T d\tau \tau \sqrt{\frac{T(\tau, \vec{x}_T)}{p_T}} E \frac{d\Gamma_\gamma}{d^3p} (p_T, T(\tau, \vec{x}_T))$$



Effect of transverse flow in a realistic calculation



$$\begin{aligned} \frac{1}{2\pi p_T} \frac{dN_\gamma}{dp_T} &= \frac{1}{2\pi} \int_0^{2\pi} d\phi \int d^4X E \frac{d\Gamma_\gamma}{d^3p} (p_\mu u^\mu(X), T(X), \dots) \\ &\approx \sqrt{2\pi} \int d^2x_T d\tau \tau \sqrt{\frac{T(\tau, \vec{x}_T)}{p_T}} E \frac{d\Gamma_\gamma}{d^3p} (p_T, T(\tau, \vec{x}_T)) \\ &= \sqrt{2\pi} \int_{T_{min}}^{T_0} dT \frac{dV_T}{dT} \sqrt{\frac{T}{p_T}} E \frac{d\Gamma_\gamma}{d^3p} (p_T, T) \end{aligned}$$

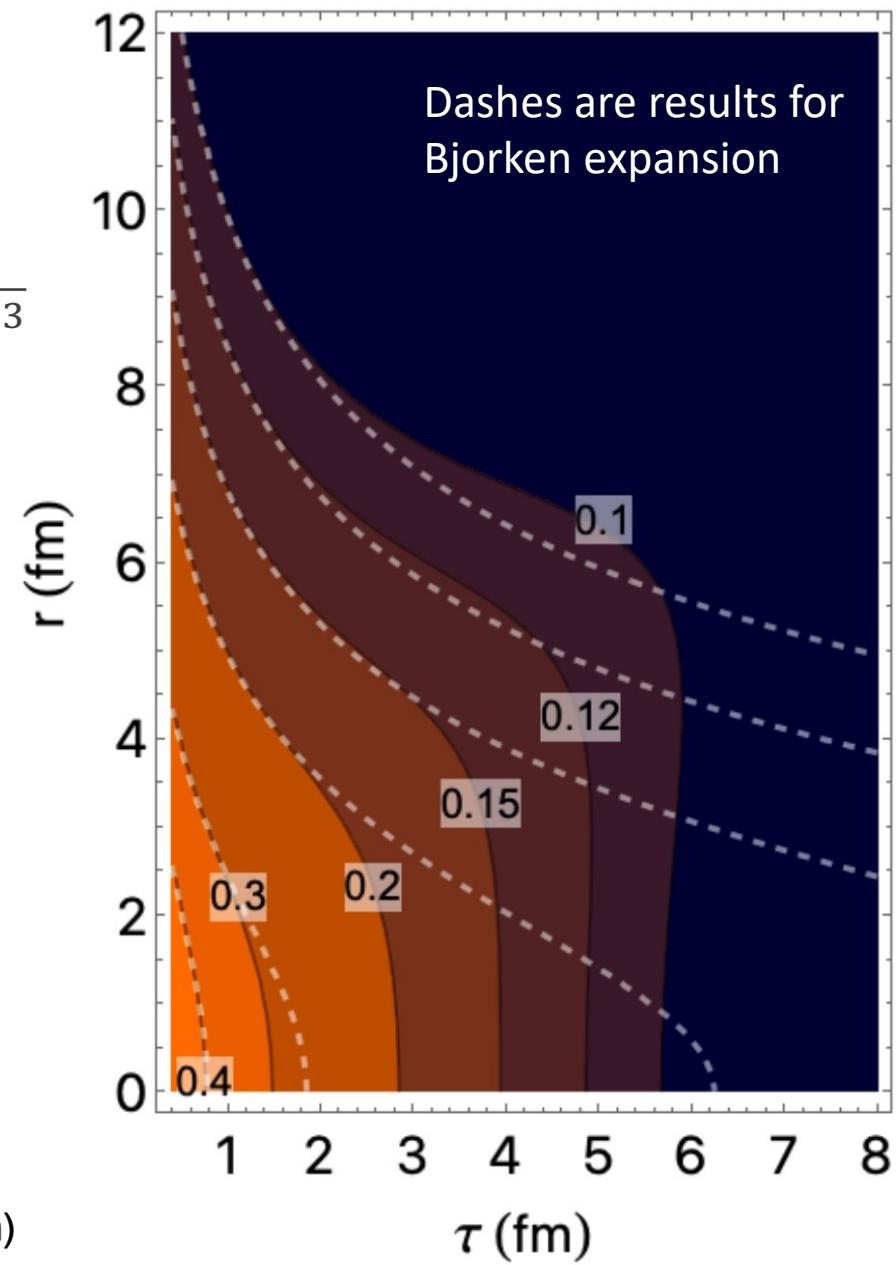
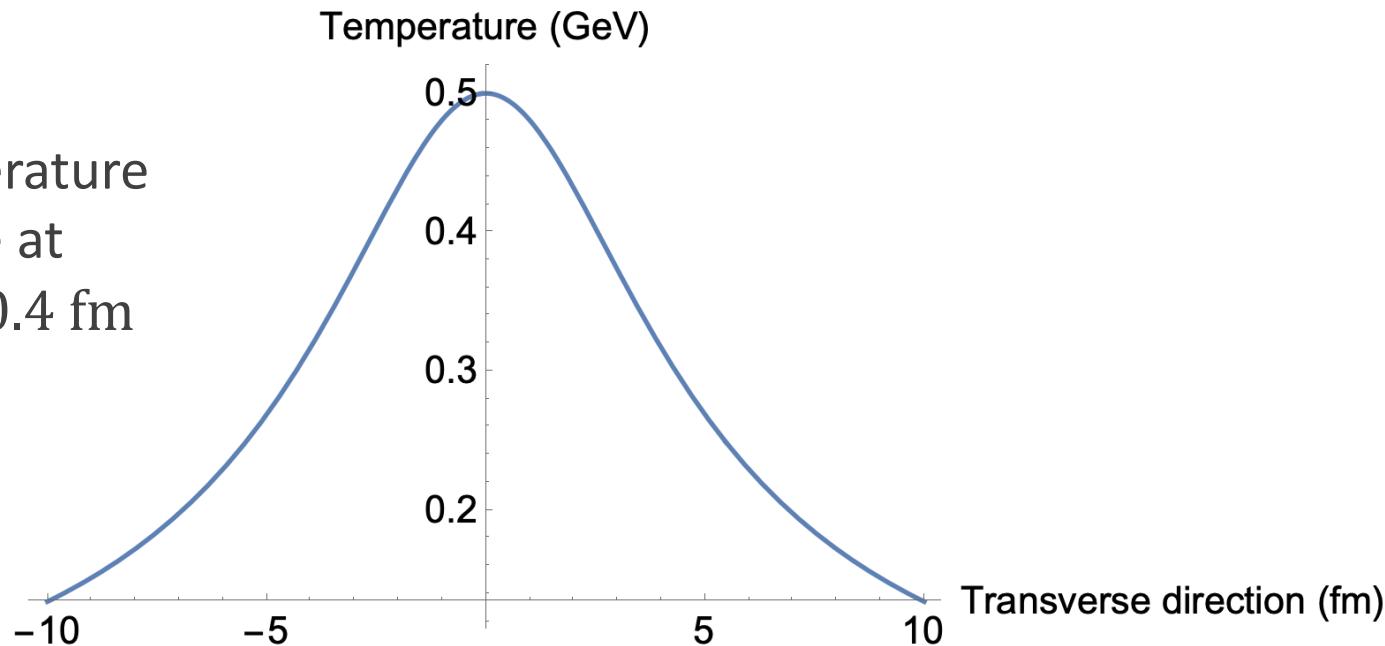


Inverse slope in Gubser hydro

$$T(\tau, r) = \left(\frac{T_0 \tau_0 (1 + q^2 \tau_0^2)}{2q \tau_0} \right) \frac{(2q\tau)^{\frac{2}{3}}}{\tau (1 + 2q^2(\tau^2 + r^2) + q^4(\tau^2 - r^2)^2)^{1/3}}$$

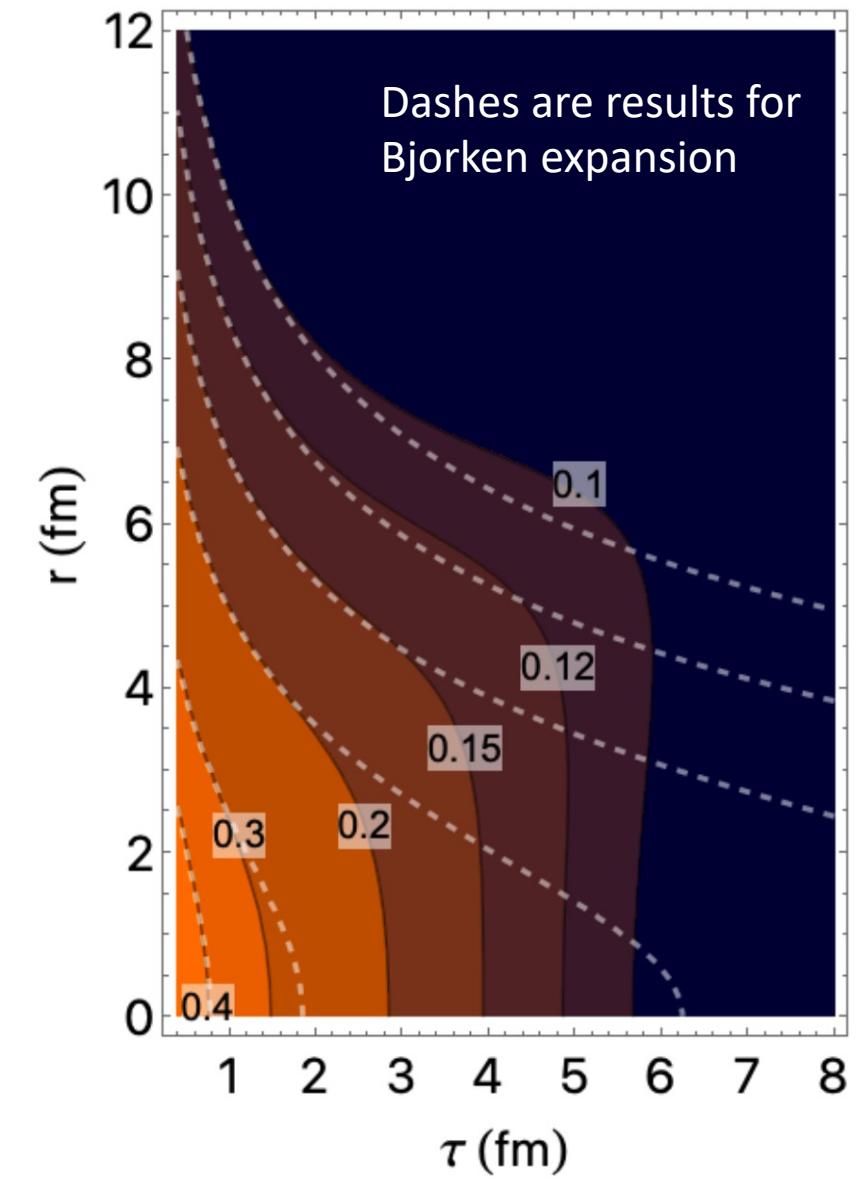
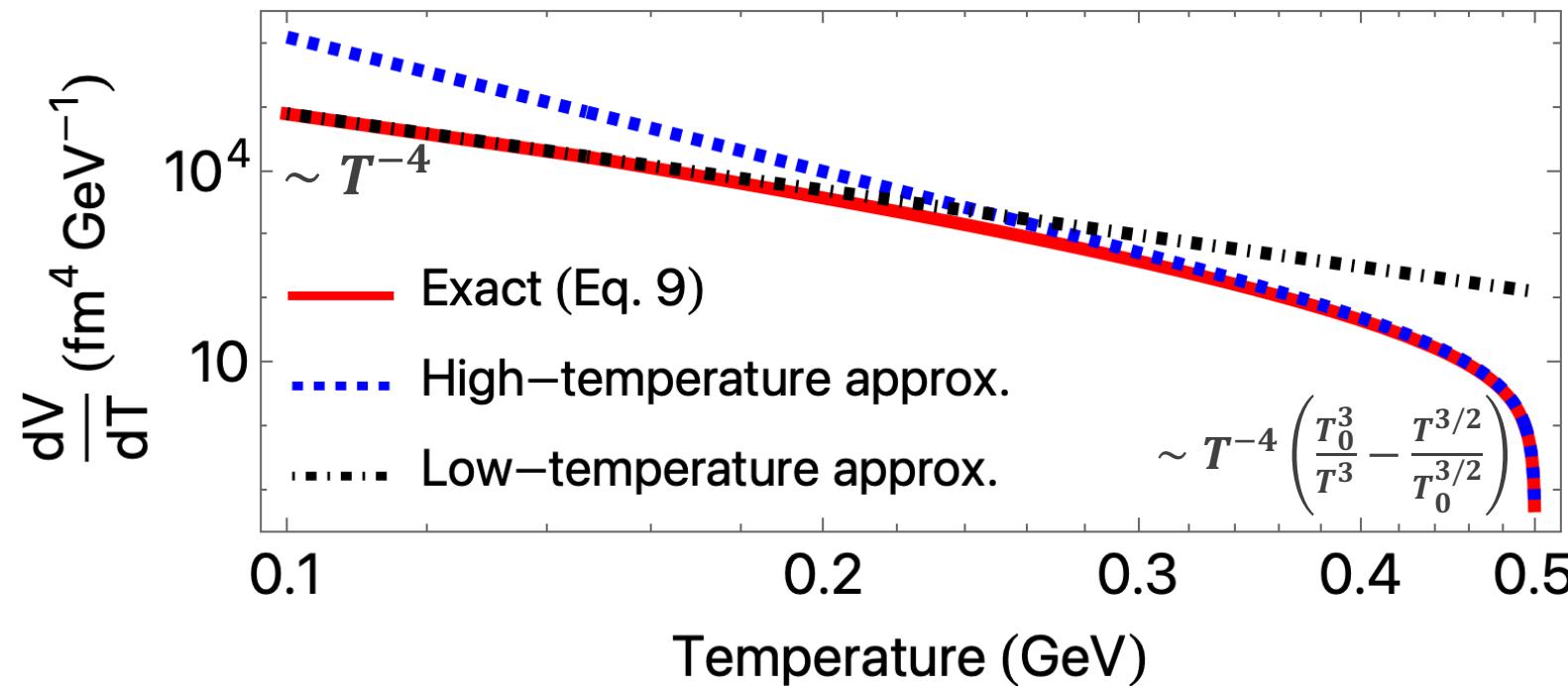
$$u^r(\tau, r) = \frac{2r q^2 \tau}{\sqrt{1 + 2q^2(\tau^2 + r^2) + q^4(\tau^2 - r^2)^2}}$$

Initial temperature profile at $\tau_0 = 0.4$ fm



Inverse slope in Gubser hydro

$$\frac{1}{2\pi p_T} \frac{dN_\gamma}{dp_T} \approx \sqrt{2\pi} \int_{T_{min}}^{T_0} dT \frac{\mathbf{dV}_T}{dT} \sqrt{\frac{T}{p_T}} E \frac{d\Gamma_\gamma}{d^3 p}(p_T, T)$$



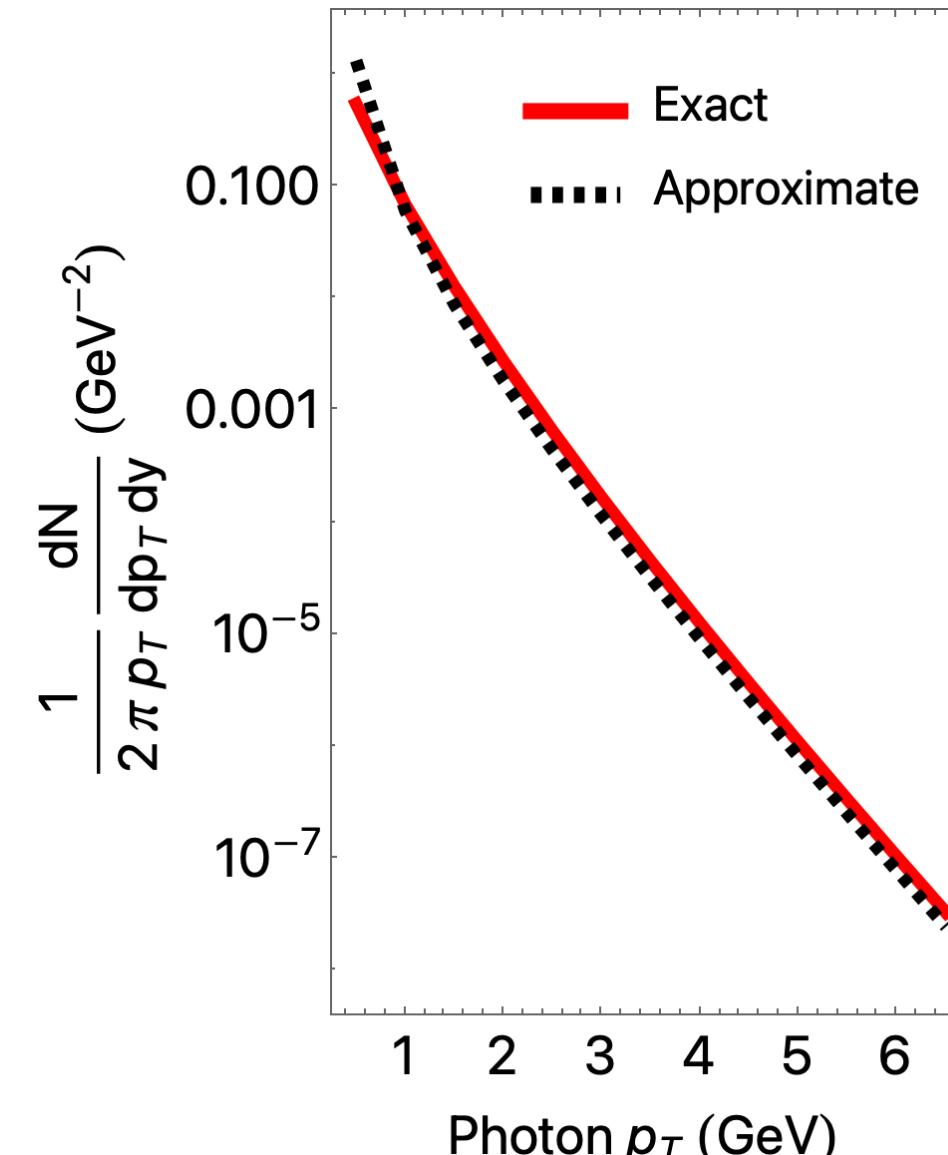
Inverse slope in Gubser hydro

$$\begin{aligned}
 \frac{1}{2\pi p_T} \frac{dN_\gamma}{dp_T} &\approx \sqrt{2\pi} \int d^2x_T d\tau \tau \sqrt{\frac{T}{p_T}} E \frac{d\Gamma_\gamma}{d^3p}(p_T, T) \\
 &= \sqrt{2\pi} \int_{T_{min}}^{T_0} dT \frac{dV_T}{dT} \sqrt{\frac{T}{p_T}} E \frac{d\Gamma_\gamma}{d^3p}(p_T, T) \\
 &\approx \frac{9\pi^2 \tau_0^2 (1+q^2 \tau_0^2)}{\sqrt{2} q^2} \left(\frac{T_0}{p_T}\right)^{5/2} E \frac{d\Gamma_\gamma}{d^3p}(p_T, T_0)
 \end{aligned}$$

(assuming $q\tau_0 \ll 1$ and $p_T > T_0$)

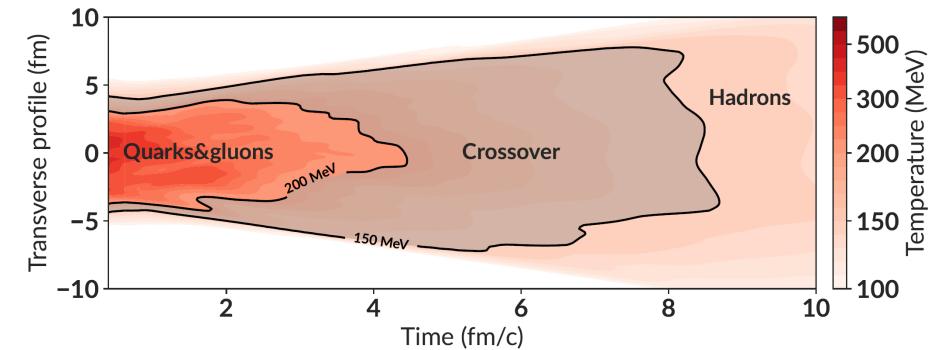
Inverse slope:

$$-\frac{1}{T_{eff}} \approx -\frac{1}{T_0} - \frac{5}{2} \frac{1}{p_T} + O\left(\frac{T_0}{p_T^2}\right)$$



Inverse slope in Bjorken hydrodynamics

$$\frac{1}{2\pi p_T} \frac{dN_\gamma}{dp_T} \approx \sqrt{2\pi} \int_{T_{min}}^{T_0} dT \frac{dV_T}{dT} \sqrt{\frac{T}{p_T}} E \frac{d\Gamma_\gamma}{d^3p}(p_T, T)$$

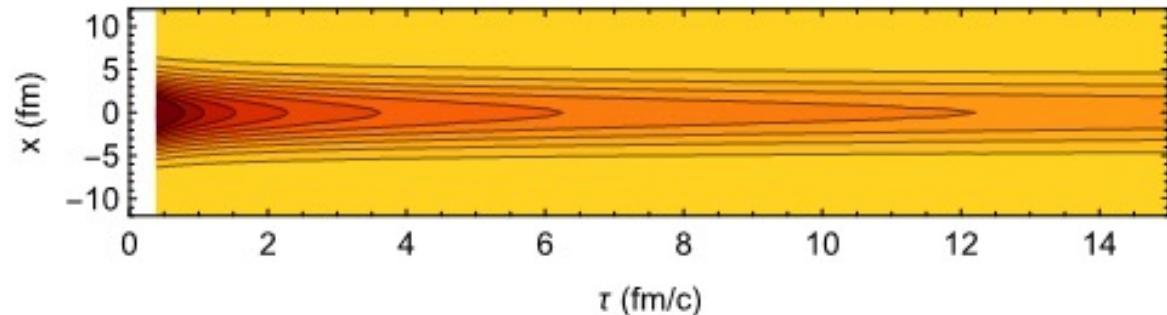
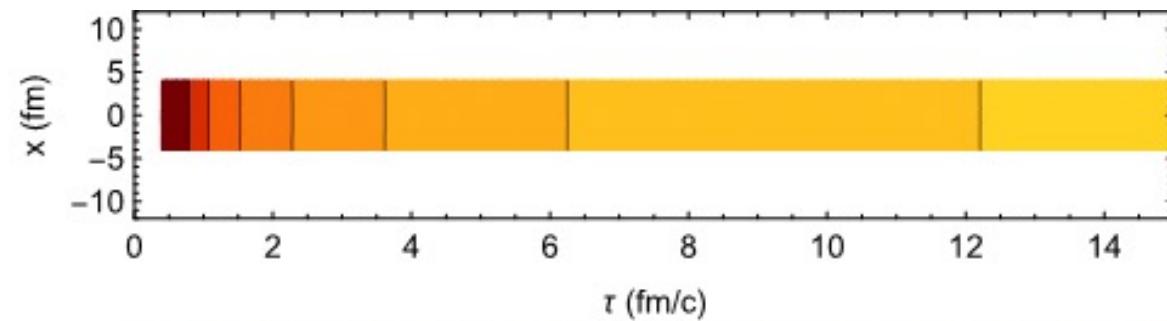


Bjorken hydrodynamics for longitudinal-dominated expansion: $T(\tau) = T_0(\tau_0/\tau)^{c_s^2}$

$$-\frac{1}{T_{eff}} \approx -\frac{1}{T_0} - \frac{3}{2} \frac{1}{p_T} + O\left(\frac{T_0}{p_T^2}\right)$$

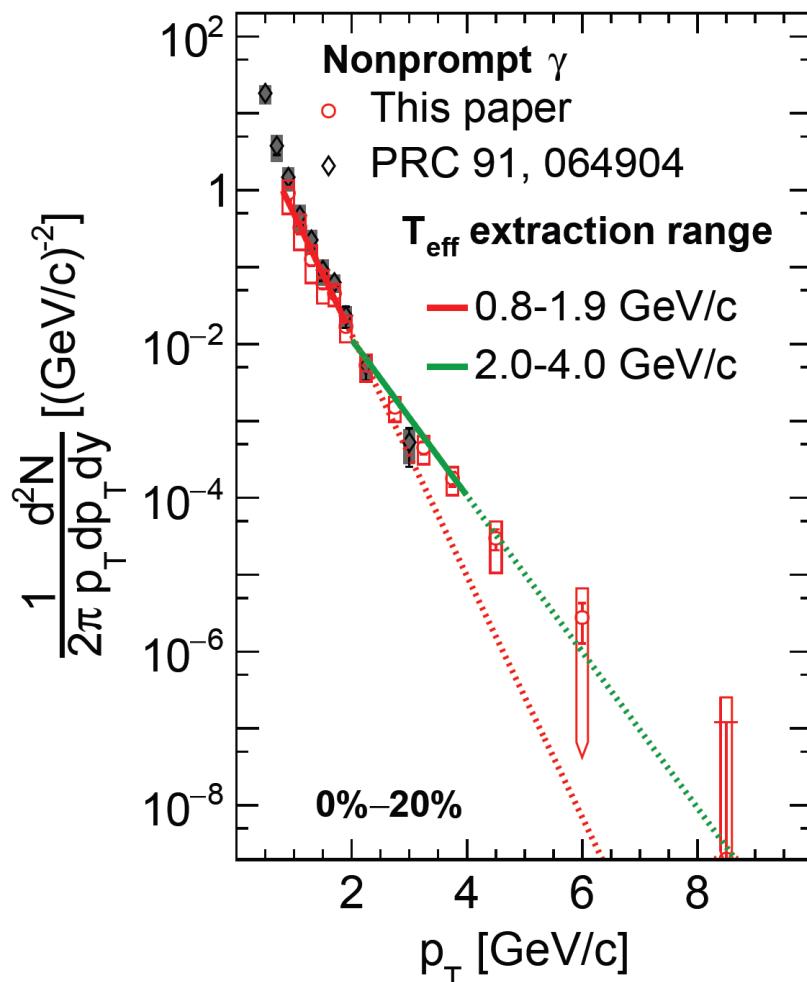


$$-\frac{1}{T_{eff}} \approx -\frac{1}{T_0} - \frac{5}{2} \frac{1}{p_T} + O\left(\frac{T_0}{p_T^2}\right)$$



Au-Au $\sqrt{s_{NN}} = 200 \text{ GeV}$, 0-20%

Ref.: PHENIX Collaboration [arXiv:2203.17187]



Caveats: other sources of photons (e.g. pre-equilibrium), viscosity, ...

p_T cut

T_{eff}

$$T_0 = \frac{T_{eff}}{1 - \frac{5}{2} \frac{T_{eff}}{p_T}}$$

$0.8 < p_T < 1.9 \text{ GeV}$

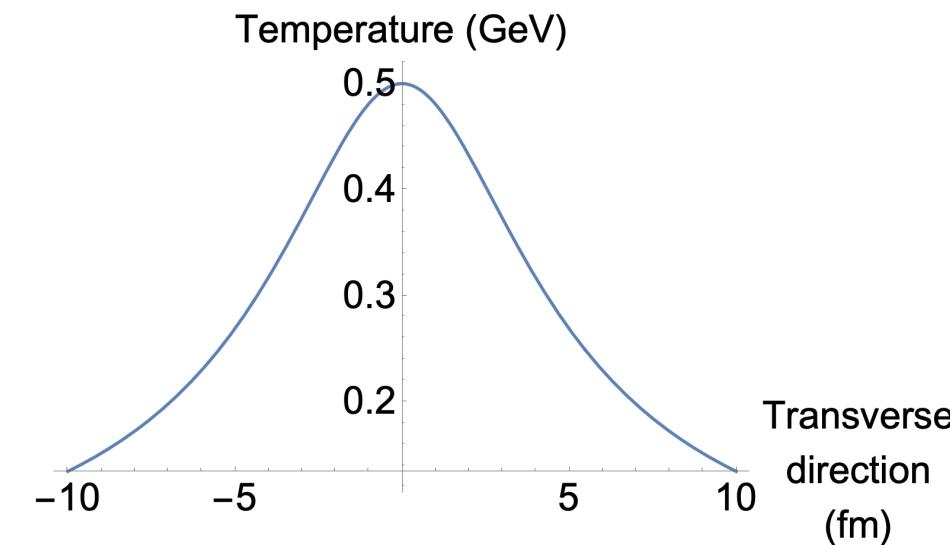
277 MeV

570 MeV

$2 < p_T < 4 \text{ GeV}$

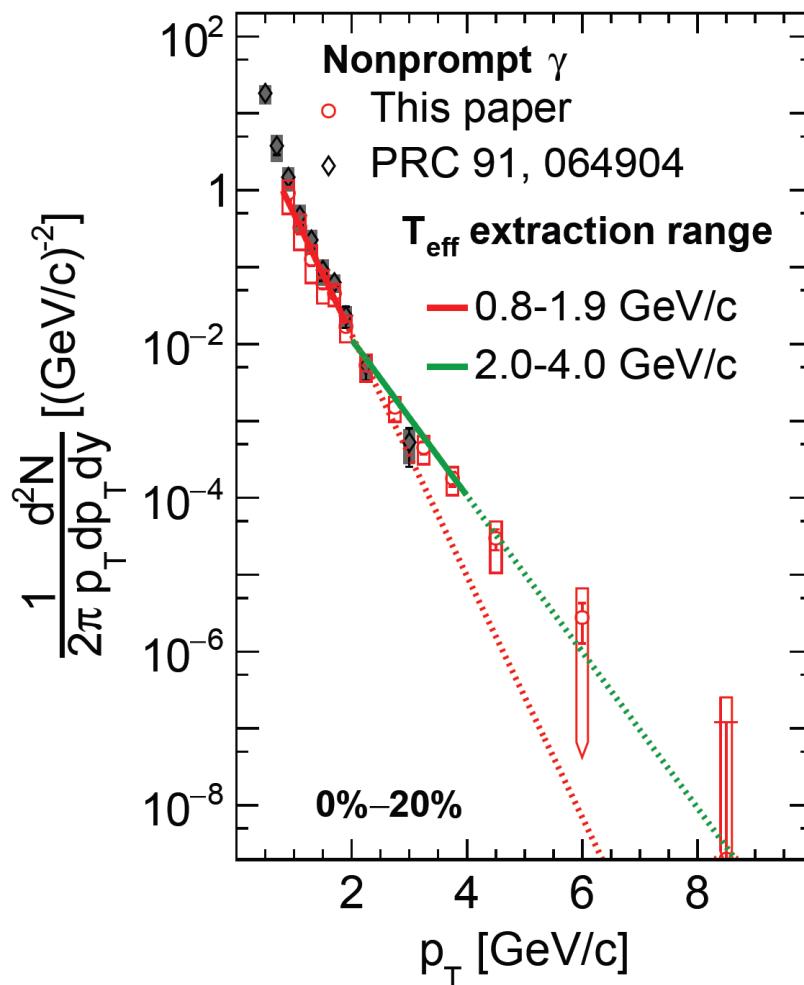
428 MeV

670 MeV



Au-Au $\sqrt{s_{NN}} = 200 \text{ GeV}$, 0-20%

Ref.: PHENIX Collaboration [arXiv:2203.17187]



Caveats: other sources of photons (e.g. pre-equilibrium), viscosity, ...

p_T cut	T_{eff}	$T_0 = \frac{T_{eff}}{1 - \frac{5}{2} \frac{T_{eff}}{p_T}}$
$0.8 < p_T < 1.9 \text{ GeV}$	277 MeV	570 MeV
$2 < p_T < 4 \text{ GeV}$	428 MeV	670 MeV

From hydro fit to hadronic data: $T_0 \approx 530 \text{ MeV}$

[from Gale, Paquet, Schenke, Shen (2022) PRC]

Partly explains large p_T -cut dependence of T_{eff}

Summary

- Doppler shift: large local effect but smaller global effect

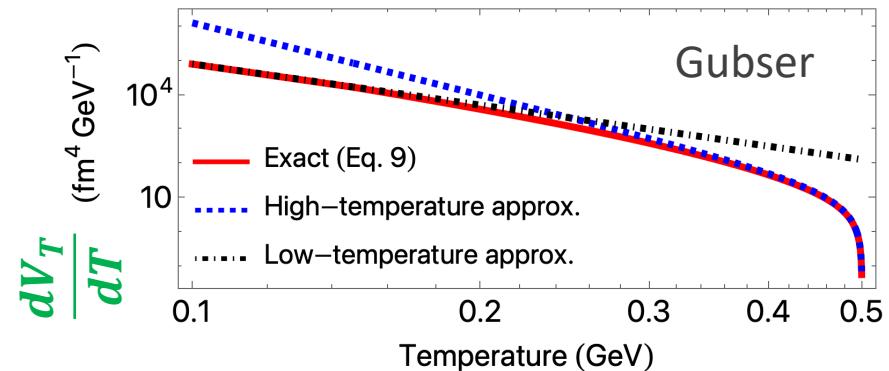
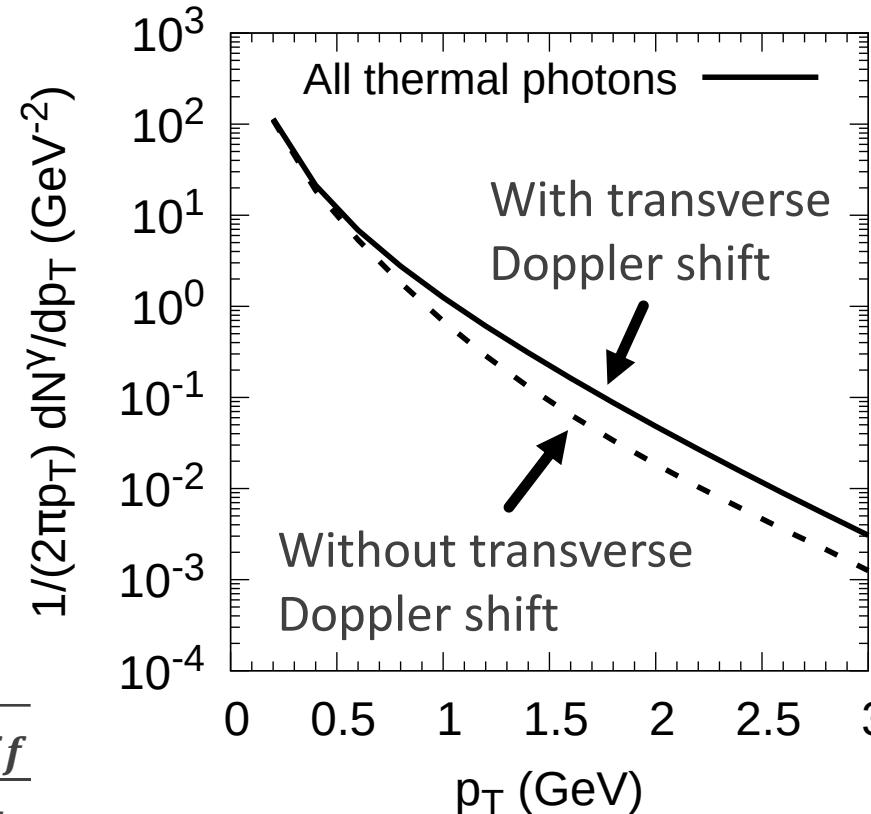
$$\frac{1}{2\pi p_T} \frac{dN_\gamma}{dp_T} \sim \int d^2x_T d\tau \tau \frac{1}{1 + \frac{p_T u_\perp}{T}} \sqrt{\frac{T}{p_T \sqrt{1+u_\perp^2}}} E \frac{d\Gamma_\gamma}{d^3p} \left(p_T \left(\sqrt{1+u_\perp^2} - u_\perp \right), \textcolor{violet}{T} \right)$$

- Transverse flow \sim not the main driver of the inverse slope

- Can study the inverse slope in simple models: $T_0 = \frac{T_{eff}}{1 - \frac{5}{2} \frac{T_{eff}}{p_T}}$

- Constrain temperature volume profile with photons (and dileptons)

$$\frac{1}{2\pi p_T} \frac{dN_\gamma}{dp_T} \approx \sqrt{2\pi} \int_{T_{min}}^{T_0} dT \frac{dV_T}{dT}(\textcolor{violet}{T}) \sqrt{\frac{T}{p_T}} E \frac{d\Gamma_\gamma}{d^3p}(p_T, \textcolor{violet}{T})$$

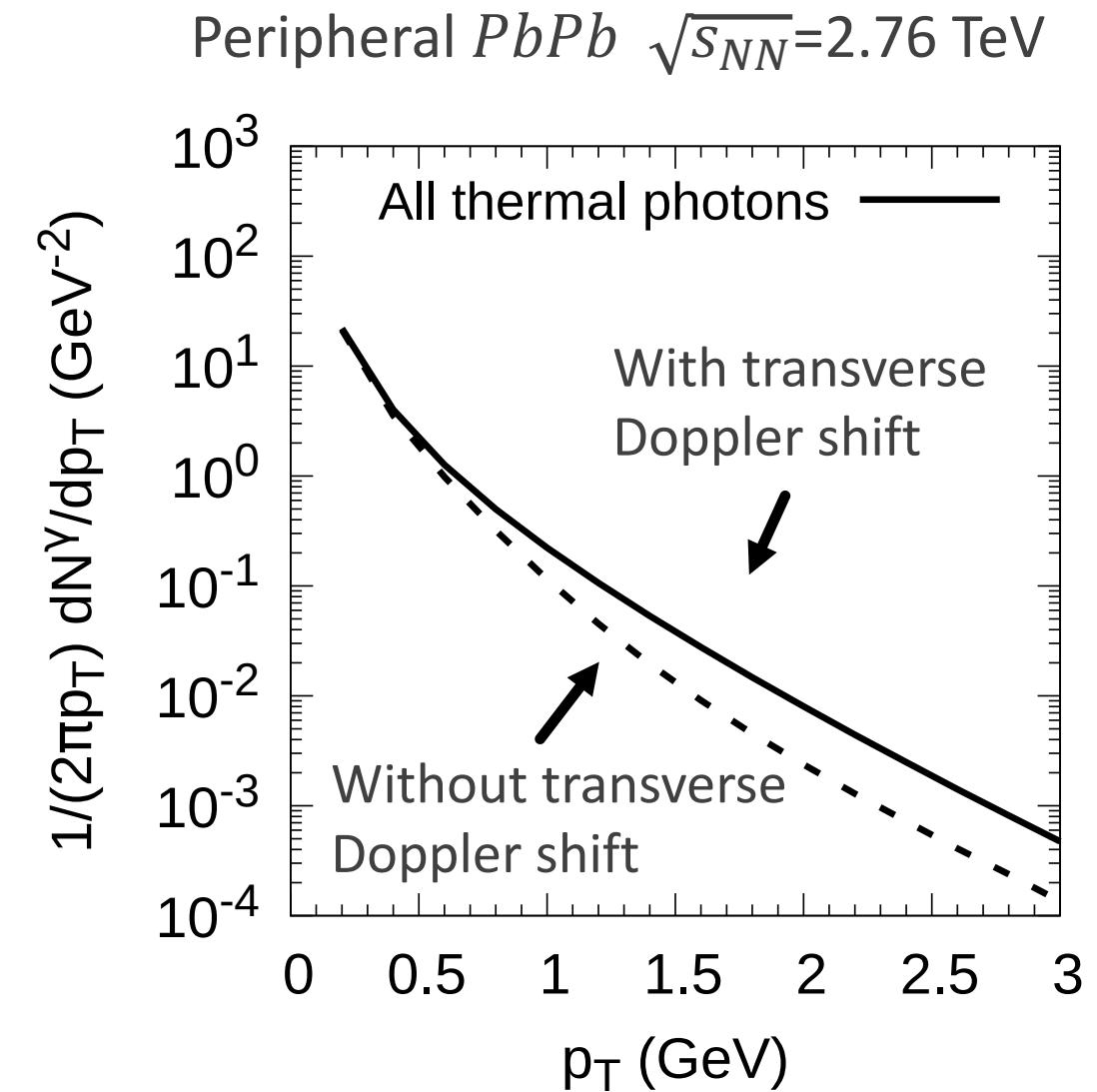
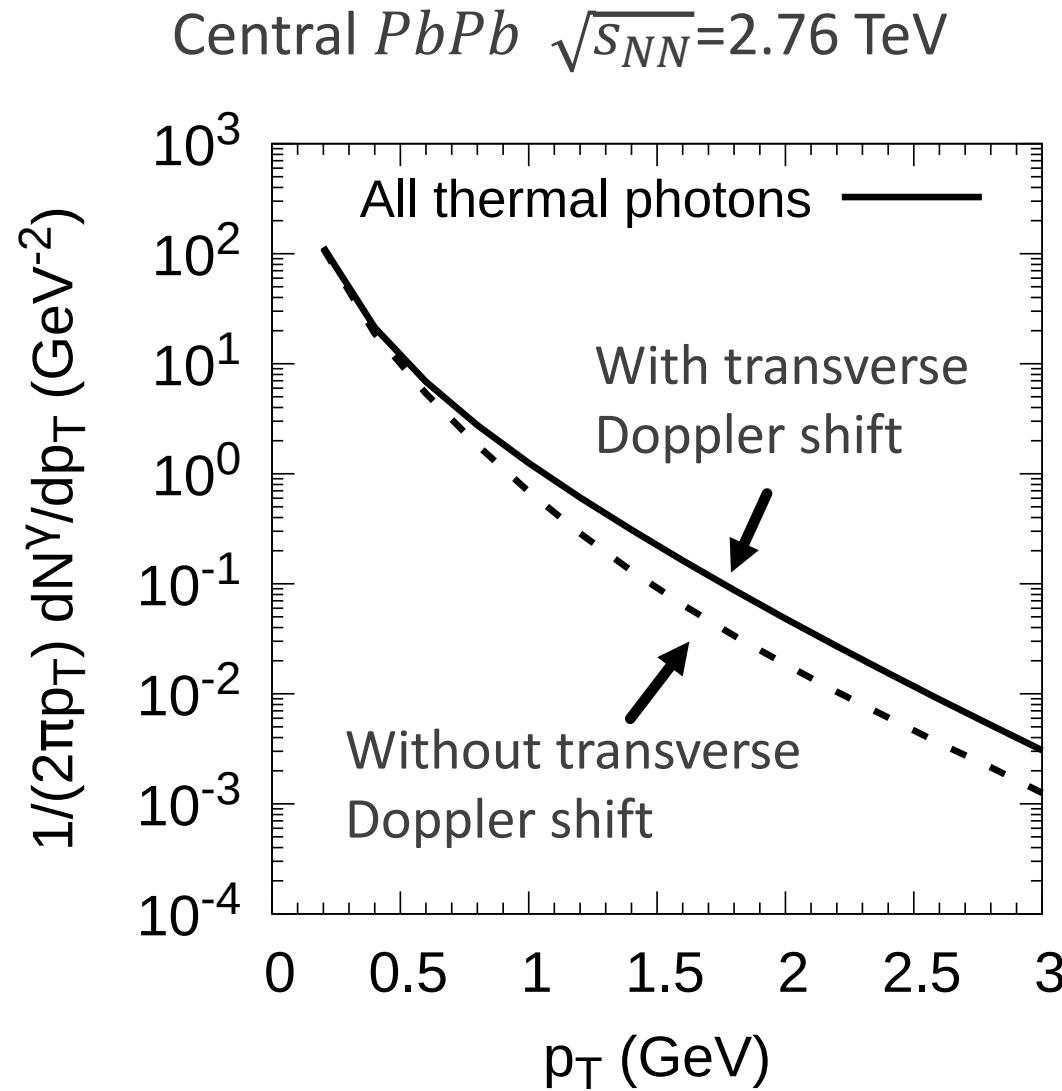




Questions

Backup

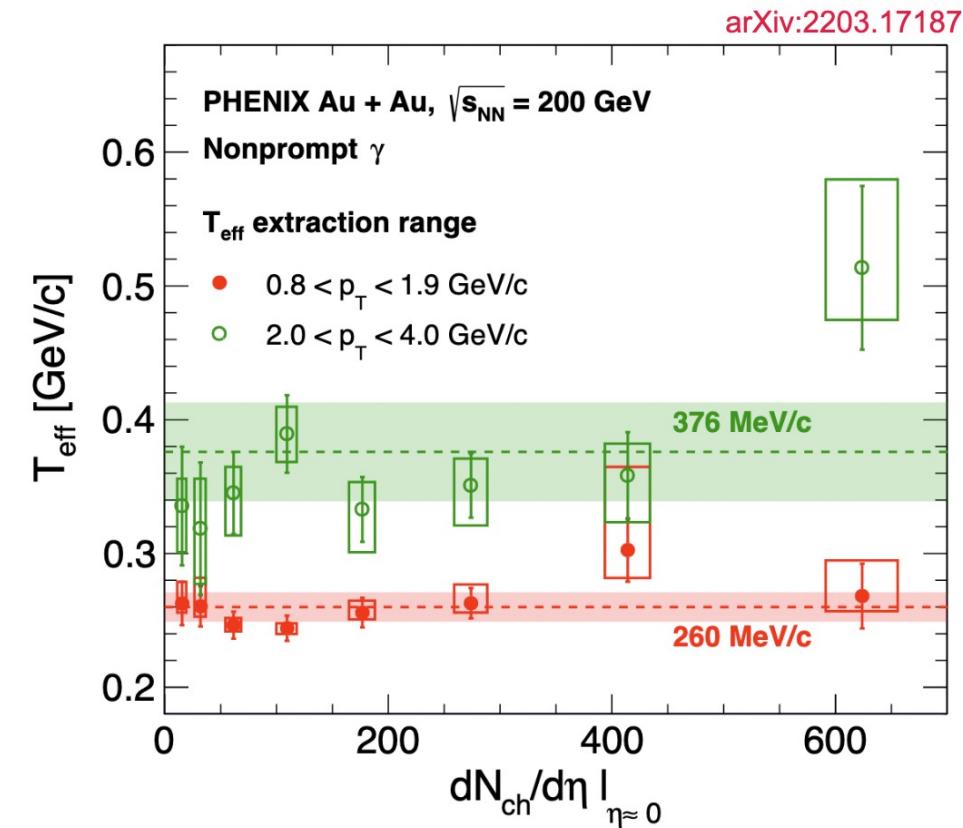
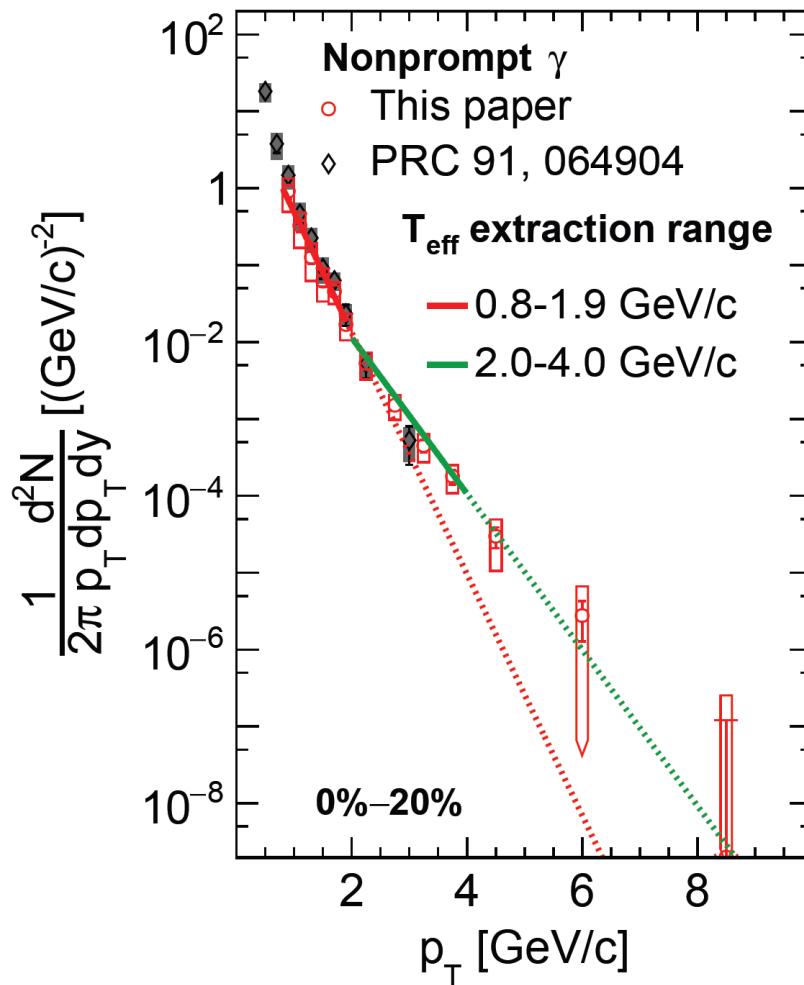
Effect of flow – central vs peripheral



Photon p_T spectrum and inverse slope

Vassu Doomra, Tuesday 3h30PM

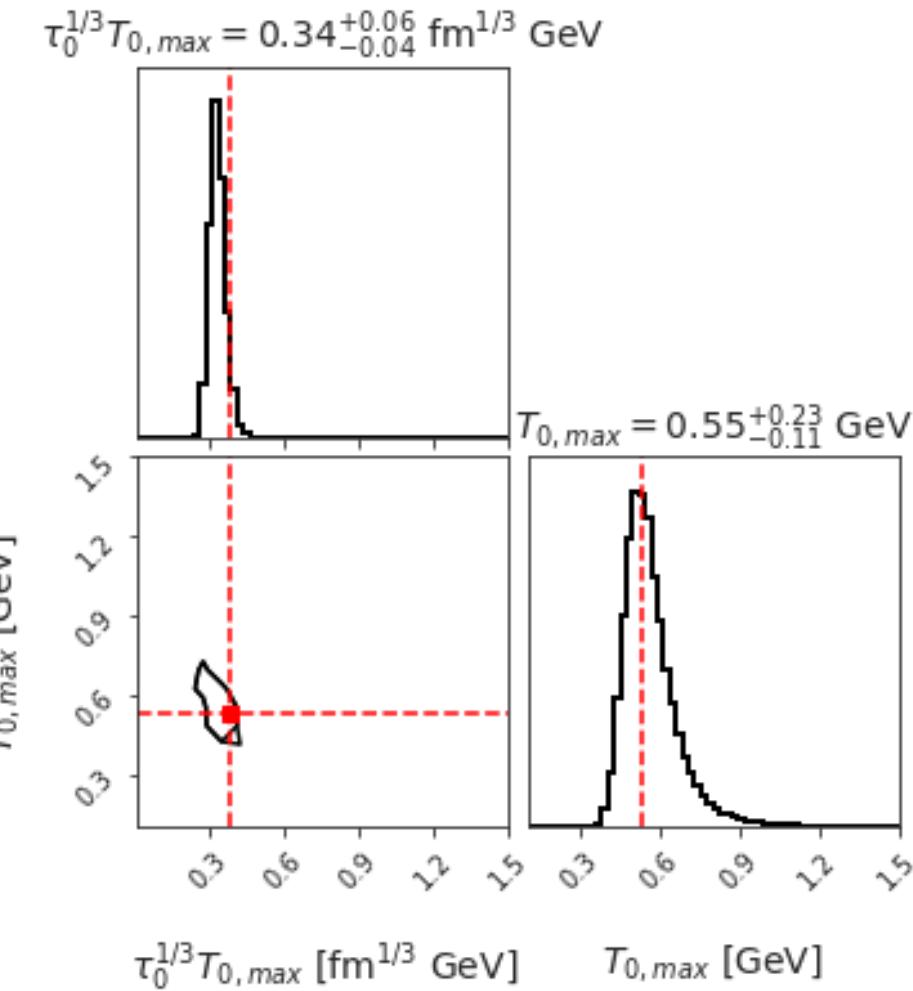
Ref.: PHENIX Collaboration [arXiv:2203.17187]



$$\frac{1}{2\pi p_T} \frac{dN_\gamma}{dp_T} \approx \sqrt{2\pi} \int_{T_{min}}^{T_0} dT \frac{dV_T}{dT} (\textcolor{violet}{T}) \sqrt{\frac{T}{p_T}} E \frac{d\Gamma_\gamma}{d^3p} (p_T, \textcolor{violet}{T})$$

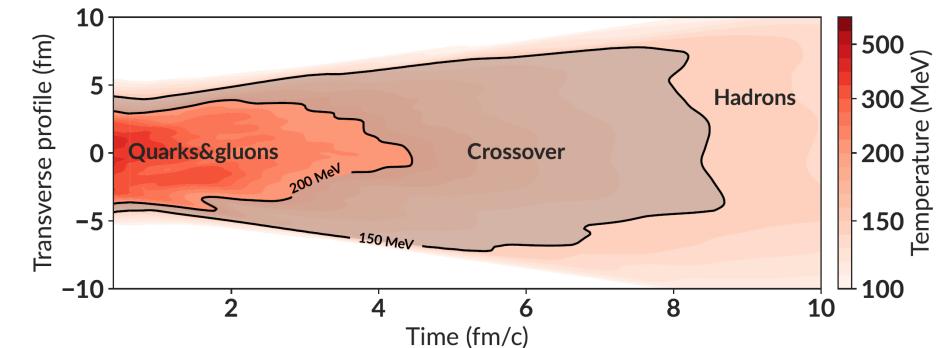
Comparing with spectrum (rather than T_{eff})

$$\begin{aligned} \frac{1}{2\pi p_T} \frac{dN_\gamma}{dp_T} = & -\frac{p_T}{T_0} + \frac{5}{2} \ln \left(\frac{T_0}{p_T} \right) \\ & + \ln \left((2\pi)^{\frac{3}{2}} \sigma_0^2 \tau_0^2 T_0^2 \right) \\ & + \ln \left(\frac{\exp(\frac{p_T}{T_0})}{T_0^2} E \frac{d\Gamma_\gamma}{d^3 p}(p_T, T_0) \right) \\ & + (\text{terms related to } c_s^{-2} \neq 1/3) \end{aligned}$$



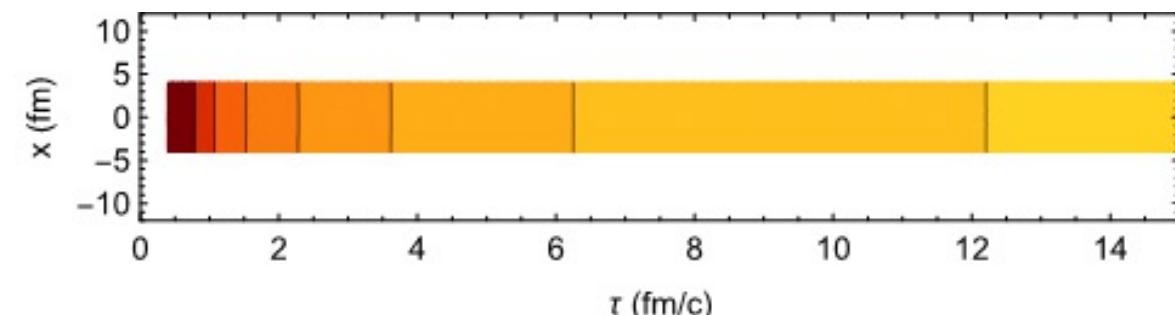
Inverse slope in Bjorken hydrodynamics

$$\frac{1}{2\pi p_T} \frac{dN_\gamma}{dp_T} \Big|_{y=0} \approx \sqrt{2\pi} \int_{T_{min}}^{T_0} dT \frac{dV_T}{dT} \sqrt{\frac{T}{p_T}} E \frac{d\Gamma_\gamma}{d^3p}(p_T, T)$$

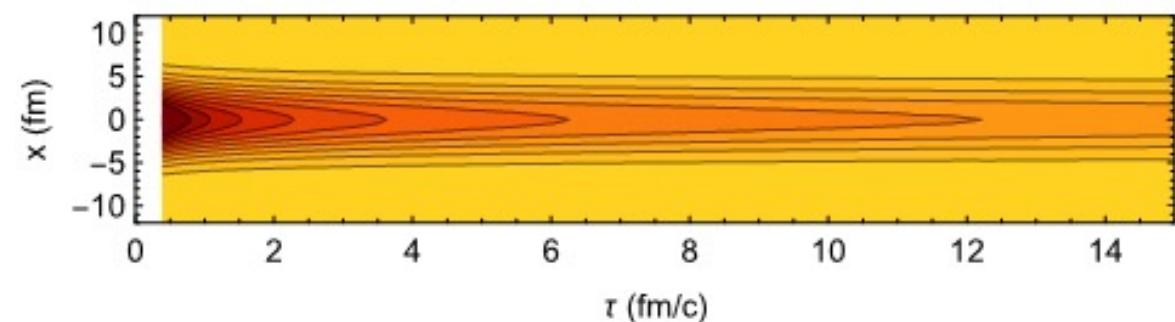


Bjorken hydrodynamics for longitudinal-dominated expansion: $T(\tau) = T_0 (\tau_0/\tau)^{c_s^2}$

→ Black disk approx: $T(\tau, r < \sigma) = T_0 \left(\frac{\tau_0}{\tau} \right)^{c_s^2}$

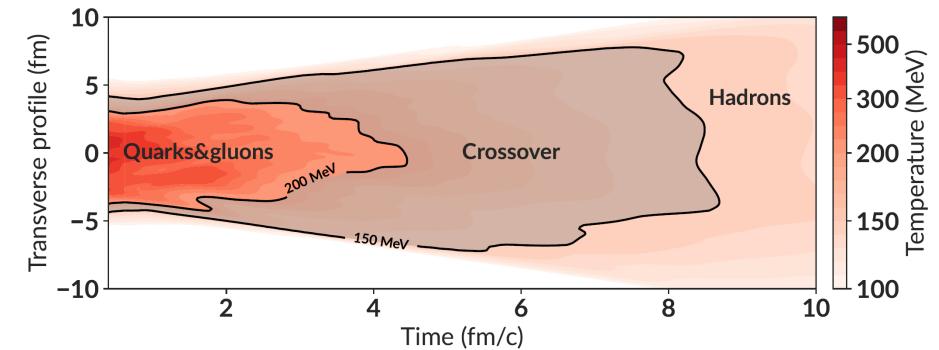


→ Gaussian approx: $T(\tau, r) = T_0 e^{-\frac{r^2}{2\sigma^2}} \left(\frac{\tau_0}{\tau} \right)^{c_s^2}$



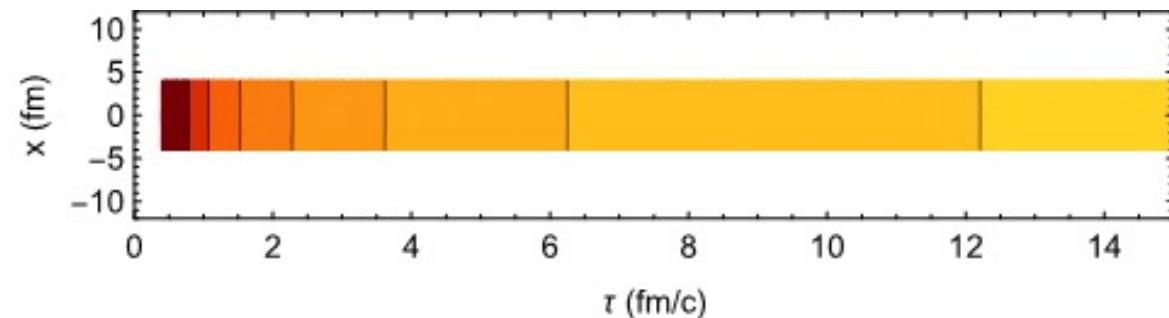
Inverse slope in Bjorken hydrodynamics

$$\frac{1}{2\pi p_T} \frac{dN_\gamma}{dp_T} \Big|_{y=0} \approx \sqrt{2\pi} \int_{T_{min}}^{T_0} dT \frac{dV_T}{dT} \sqrt{\frac{T}{p_T}} E \frac{d\Gamma_\gamma}{d^3p}(p_T, T)$$

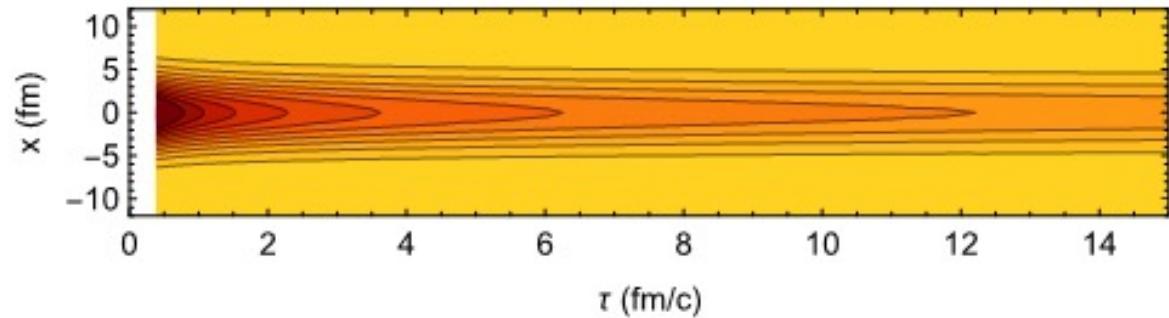


Bjorken hydrodynamics for longitudinal-dominated expansion: $T(\tau) = T_0(\tau_0/\tau)^{c_s^2}$

$$\ln \left(\frac{1}{E} \frac{dN_\gamma}{d^3p} \right) \approx -\frac{p_T}{T_0} + \frac{3}{2} \log \left(\frac{T_0}{p_T} \right) + O \left(\frac{T_0}{p_T} \right) \quad \leftarrow$$



$$\ln \left(\frac{1}{E} \frac{dN_\gamma}{d^3p} \right) \approx -\frac{p_T}{T_0} + \frac{5}{2} \log \left(\frac{T_0}{p_T} \right) + O \left(\frac{T_0}{p_T} \right) \quad \leftarrow$$



Inverse slope in Gubser hydro

$$\frac{dV_\perp}{dT} = \int_{\tau_0}^{\infty} d\tau \int_0^{\infty} dr r \tau 2\pi r \delta(T - T(\tau, r)). \quad (8)$$

For the Gubser temperature profile, the transverse volume per unit temperature can be evaluated analytically. Defining $v = q\tau_0$, the result is

$$\frac{dV_\perp}{dT} = \frac{\pi T_0^3 \tau_0^4 (1 + v^2)^2}{2v^3 T^4} F\left(\frac{T}{T_0}, v\right) \quad (9)$$

with

$$F = \begin{cases} \arcsin(\beta (\frac{\delta}{v})^{3/2}) - \arcsin(\beta) & \text{if } \frac{T}{T_0} > \frac{[v(1+v^2)^2]^{1/3}}{2^{2/3}} \\ \arccos(\beta) & \text{otherwise} \end{cases} \quad (10)$$

where

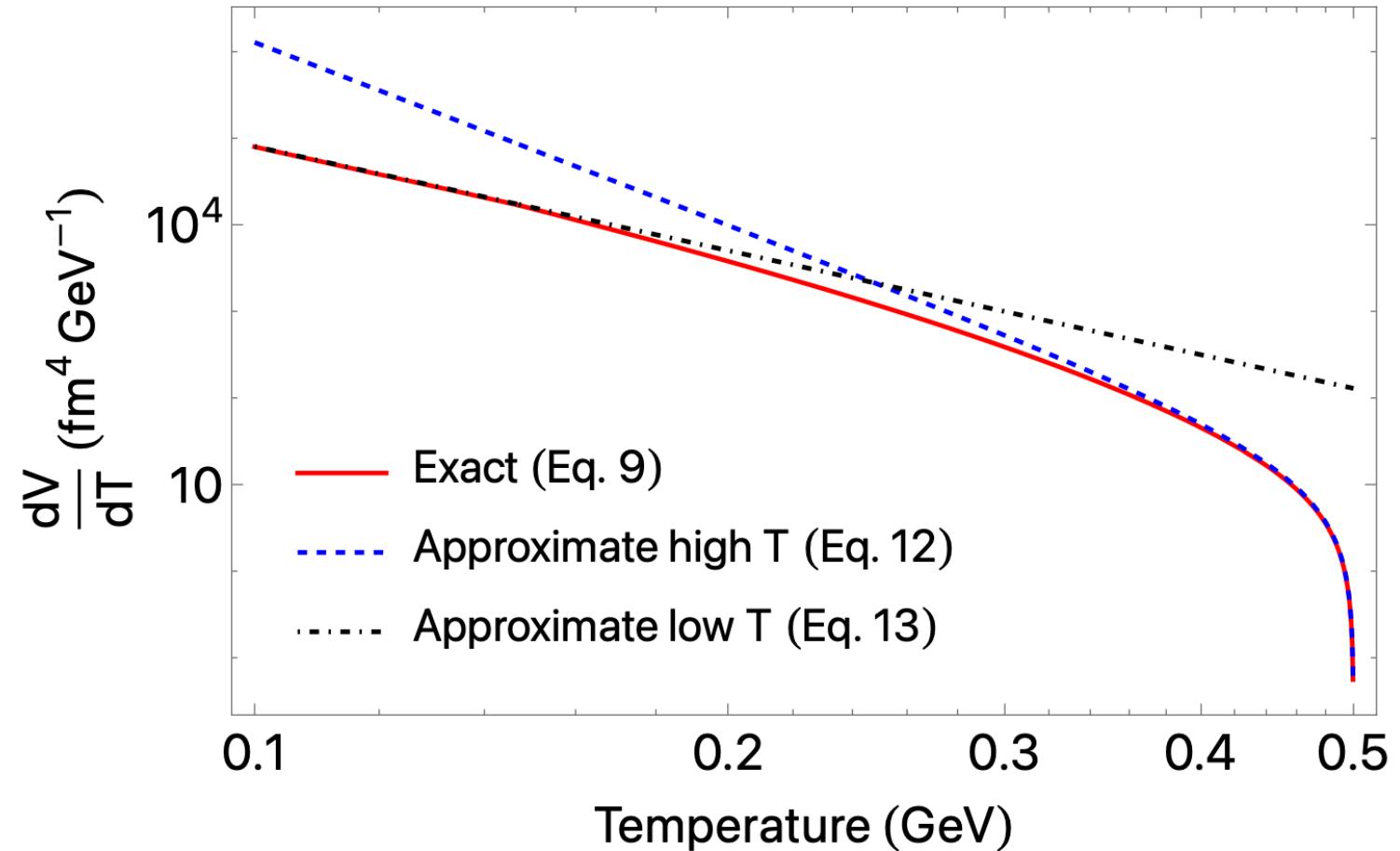
$$\beta = \frac{2v(T/T_0)^{3/2}}{1 + v^2}; \quad \delta (\delta^2 + 1)^2 = \frac{v(1 + v^2)^2}{(T/T_0)^3}; \quad \delta > 0. \quad (11)$$

At high temperature, and assuming $v = q\tau_0 \ll 1$, which is the case for the parameters used to obtain the temperature profile in Figure 1, Eq. 9 can be approximated by:

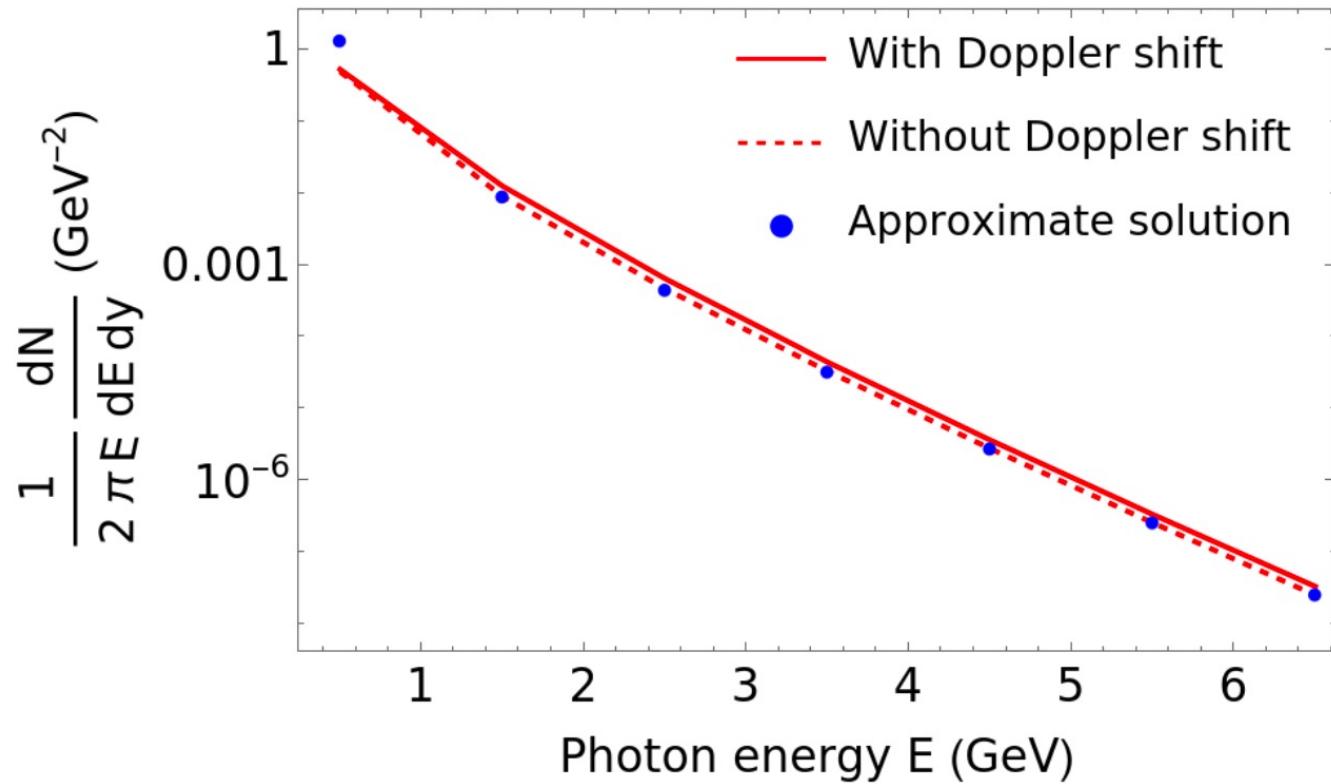
$$\frac{dV_\perp}{dT} \approx \frac{\pi T_0^3 \tau_0^4 (1 + v^2)^2}{2v^3 T^4} \left[2v \left(\frac{T_0^3}{T^3} - \left(\frac{T}{T_0} \right)^{3/2} \right) \right] \quad (12)$$

At low temperatures, Eq. 9 takes the simple form

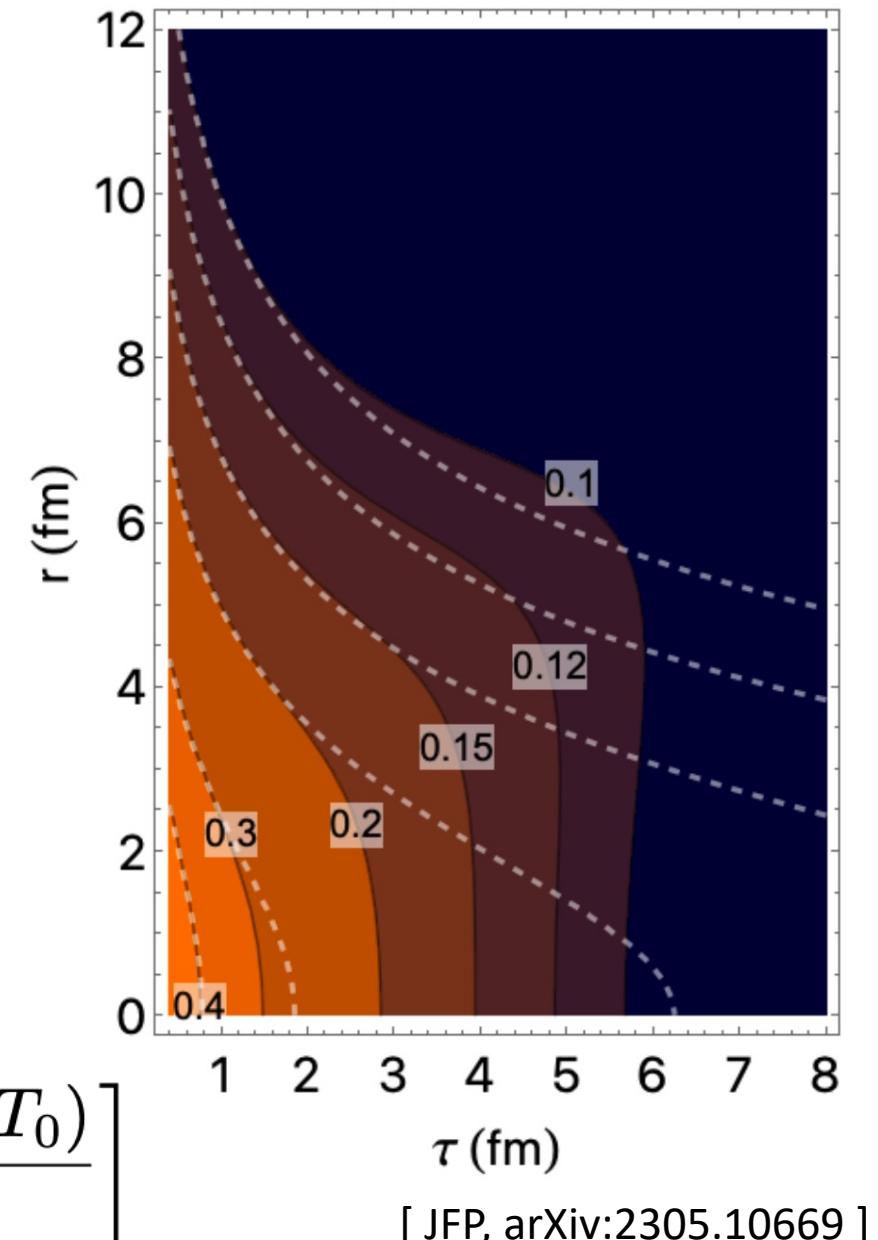
$$\frac{dV_\perp}{dT} \approx \frac{\pi T_0^3 \tau_0^4 (1 + v^2)^2 \pi}{2v^3 T^4} \frac{1}{2}. \quad (13)$$



Inverse slope in Gubser hydro



$$\frac{1}{2\pi E} \frac{dN}{dEdy_M} \approx \frac{9\pi^{3/2}\tau_0^2(1+q^2\tau_0^2)^2}{\sqrt{2}q^2} \left(\frac{T_0}{E}\right)^{5/2} \left[k \frac{d\Gamma_\gamma(E, T_0)}{d^3\mathbf{k}} \right]$$



[JFP, arXiv:2305.10669]

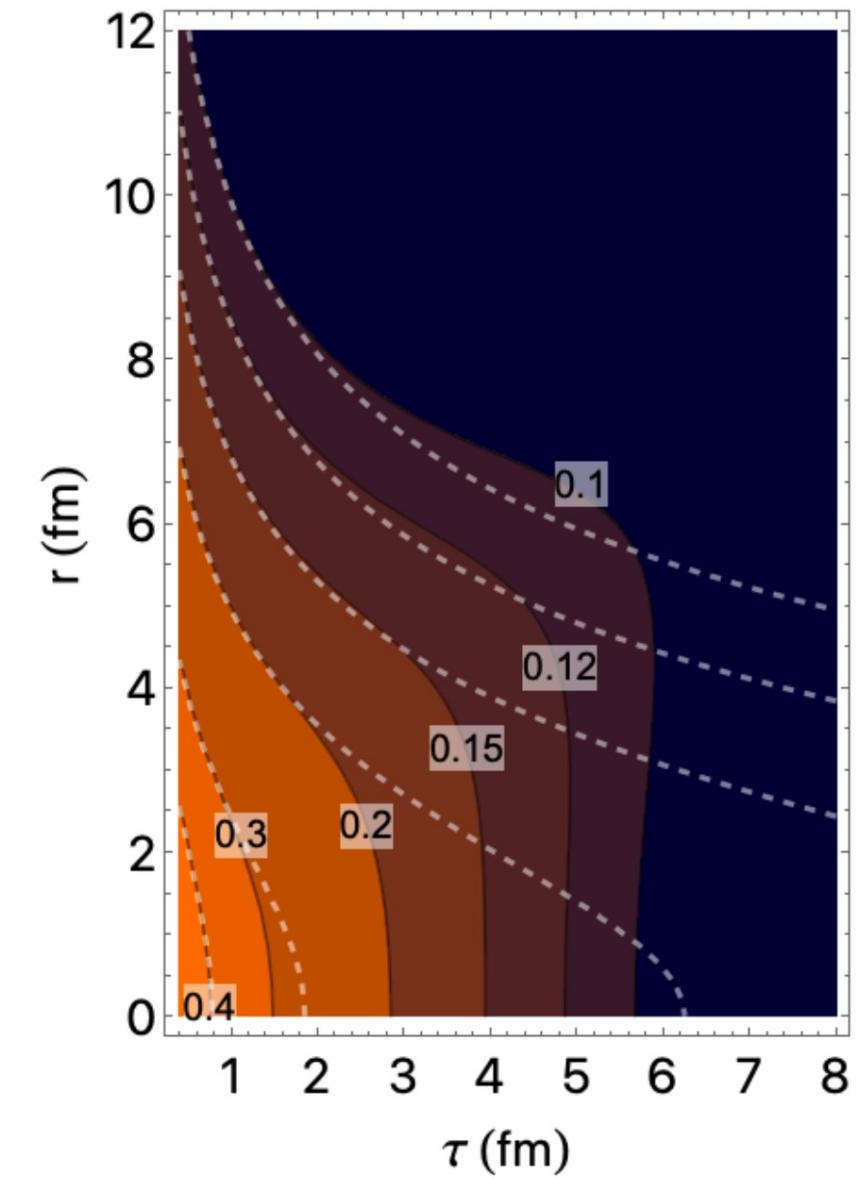
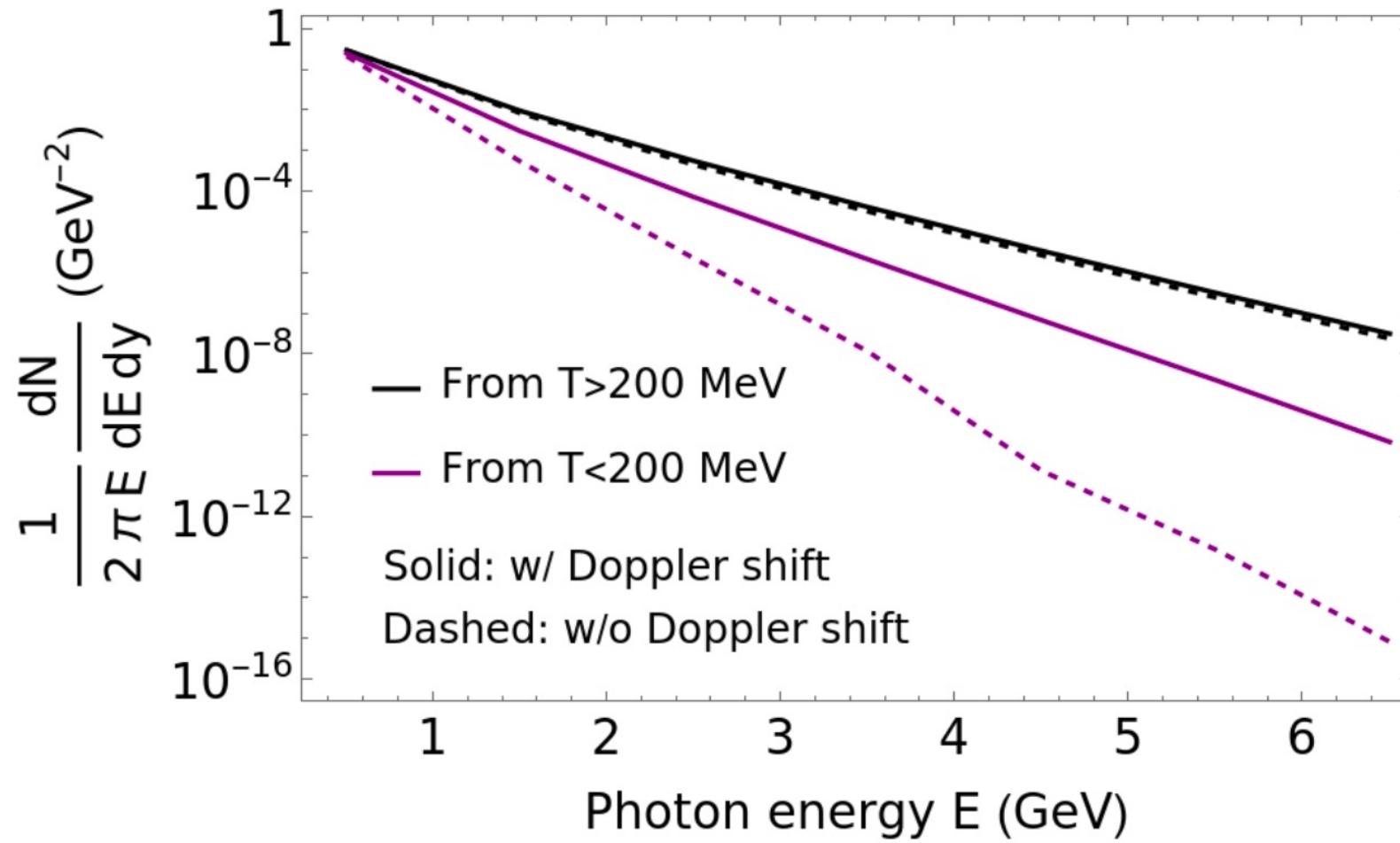
Inverse slope in Gubser hydro

$$\frac{d}{dE} \left[-\frac{E}{T_{\text{eff}}} \right] \approx \frac{d}{dE} \left[\frac{5}{2} \ln \left(\frac{T_0}{E} \right) + \ln \left[k \frac{d\Gamma_\gamma(E, T_0)}{d^3\mathbf{k}} \right] \right] \quad (16)$$

which, neglecting the non-exponential corrections to the rate, implies

$$T_{\text{eff}} \approx \frac{T_0}{1 + \frac{5}{2} \frac{T_0}{E}}. \quad (17)$$

Inverse slope in Gubser hydro

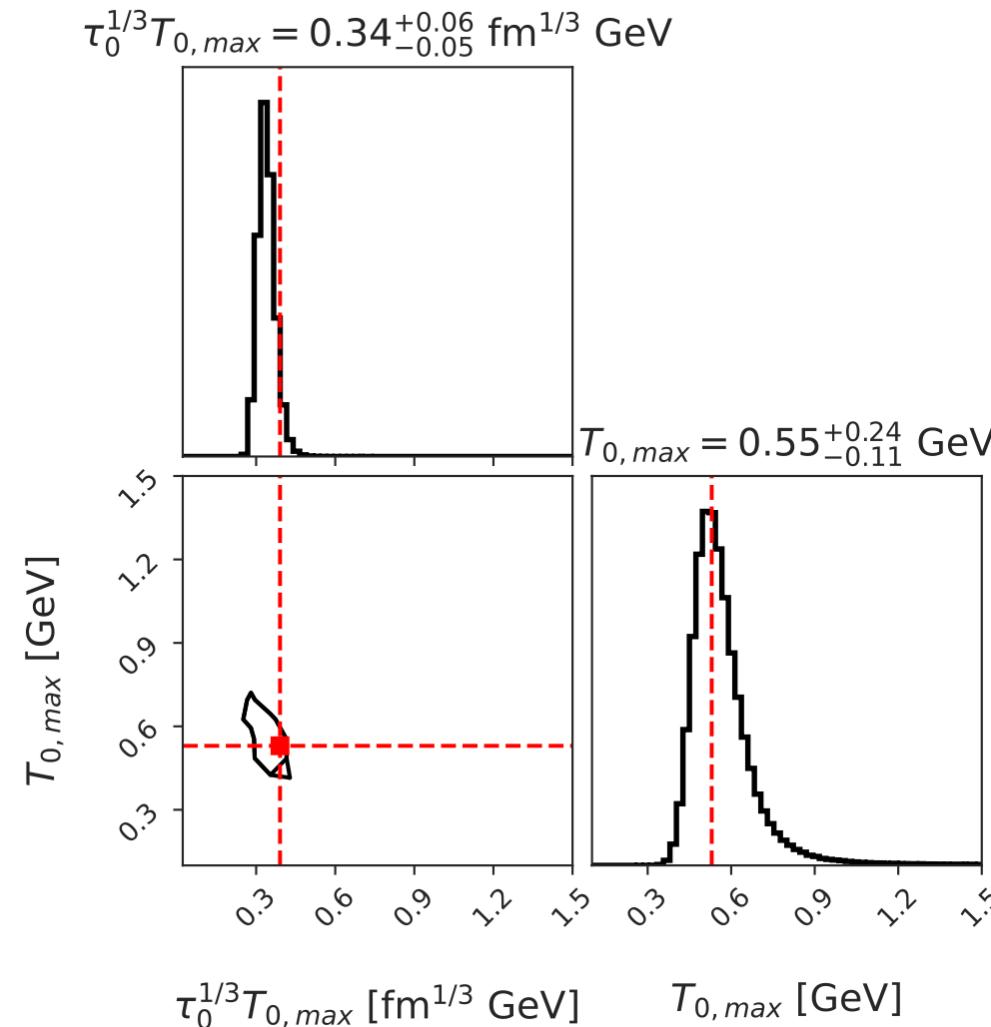


Paquet and Bass [arXiv:2205.12299]

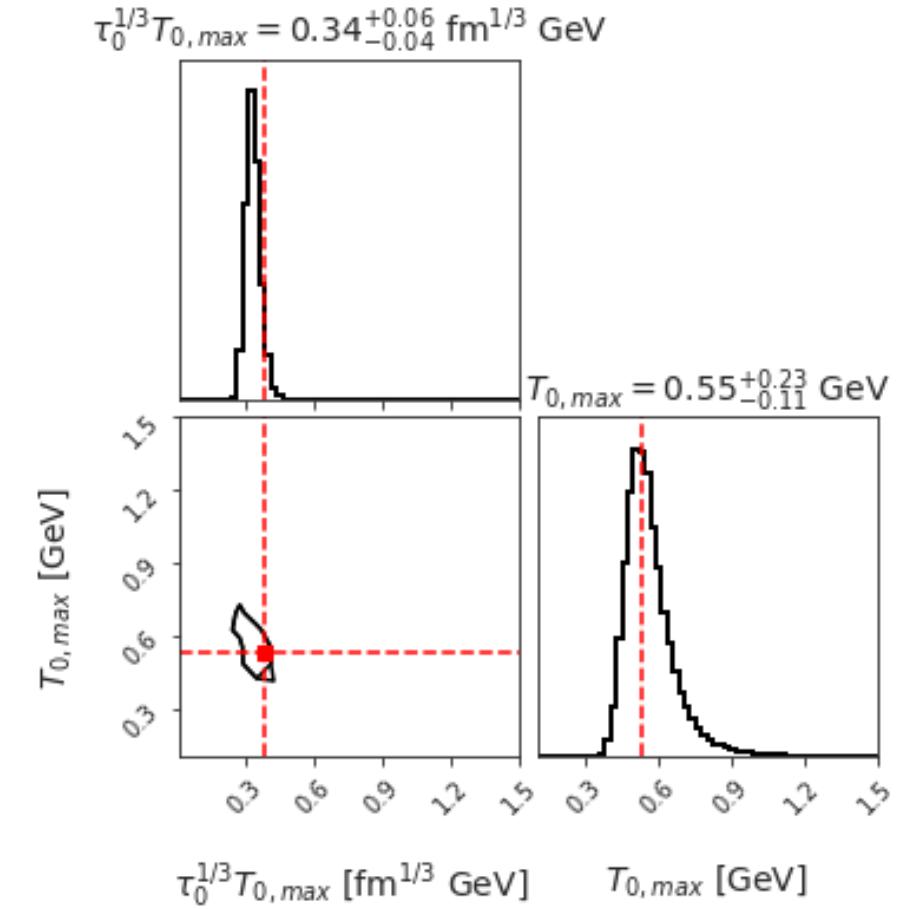
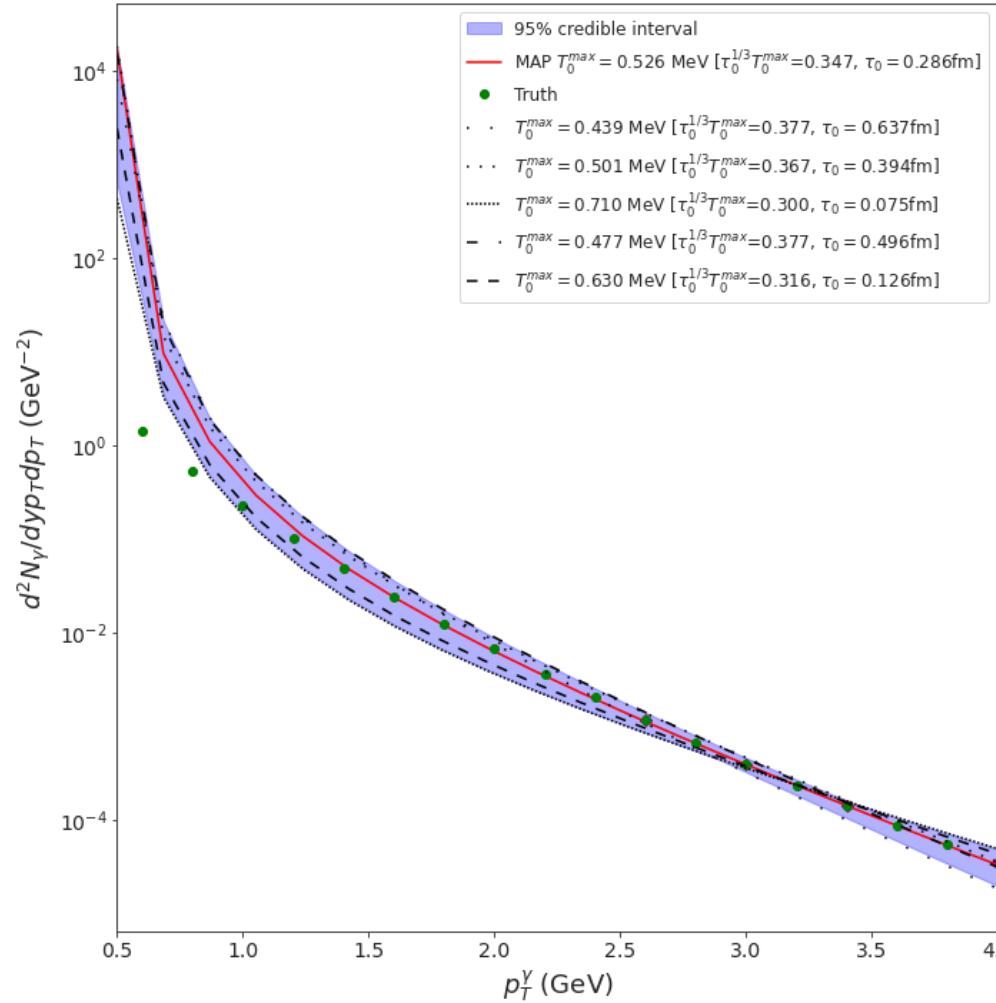
$$\begin{aligned}
 & \ln \left[\frac{1}{2\pi E} \frac{dN}{dEdy_M} \right] \approx -E/T_{0,\max} + \frac{5}{2} \ln \left(\frac{T_{0,\max}}{E} \right) \\
 & + \ln \left[(2\pi)^{3/2} \frac{\sigma_0^2 \tau_0^2}{(\hbar c)^4} T_{0,\max}^2 \right] + \ln \left[\frac{\exp(E/T_{0,\max})}{T_{0,\max}^2} k \frac{d\Gamma_\gamma(E, T_{0,\max})}{d^3k} \right] \\
 & + 2 \left[c_s^{-2} \left(\frac{T_{0,\max}}{1 + bT_{0,\max}/E} \right) - \bar{c}_s^{-2} \right] \frac{T_{0,\max}}{E} \\
 & + \ln \left(c_s^{-2} \left(\frac{T_{0,\max}}{1 + bT_{0,\max}/E} \right) \right) . \quad (10)
 \end{aligned}$$

Paquet and Bass [arXiv:2205.12299]

Comparison with
state-of-the-art
calculation of
thermal photons

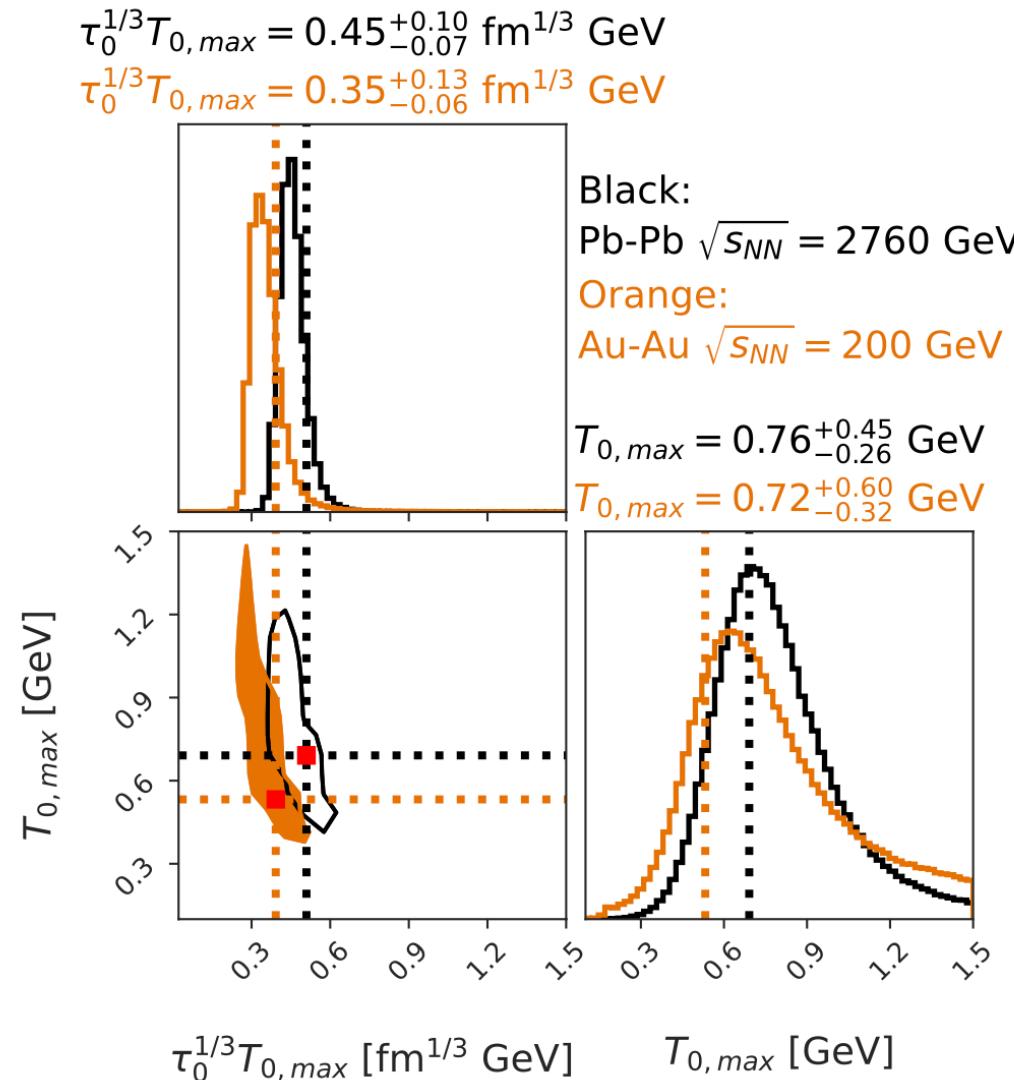


Comparison with model

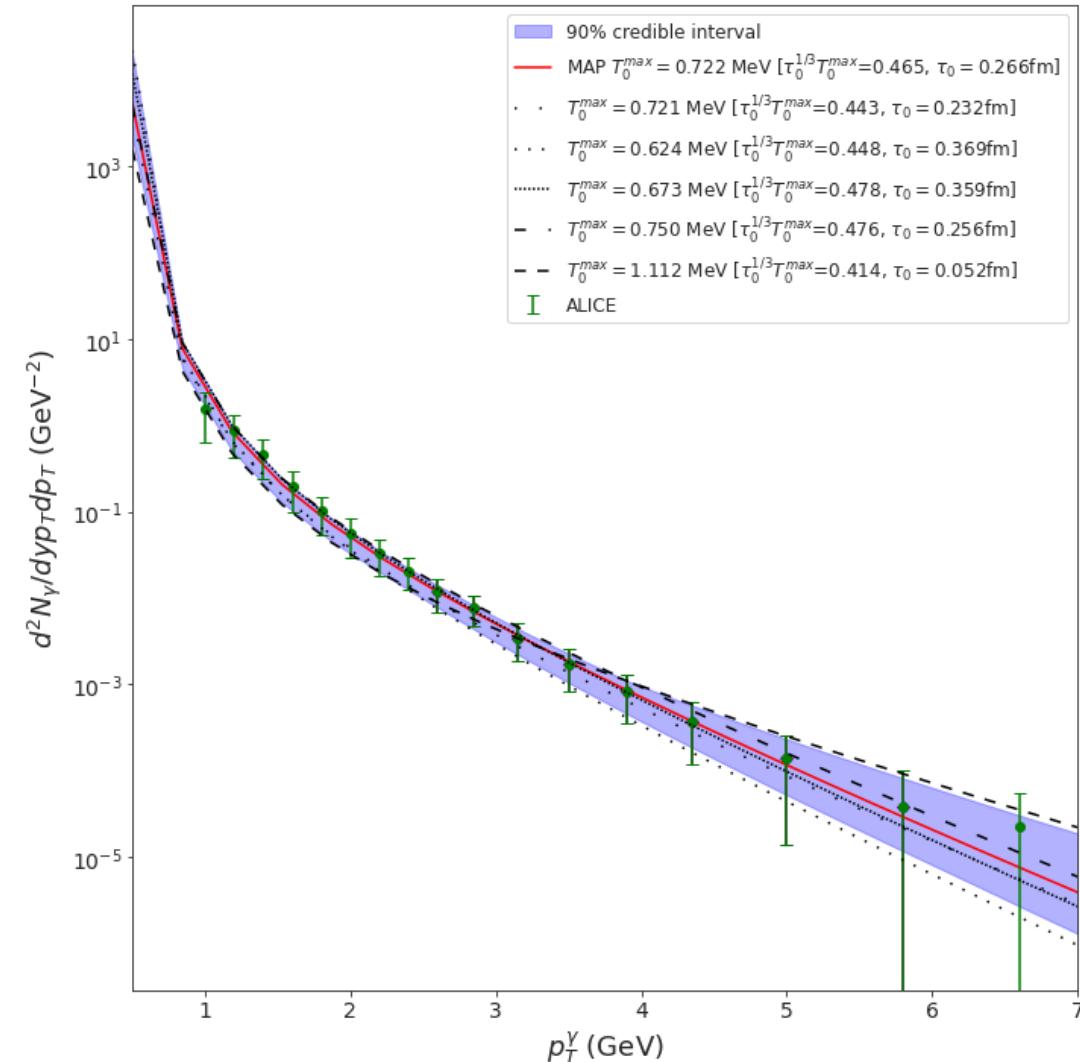
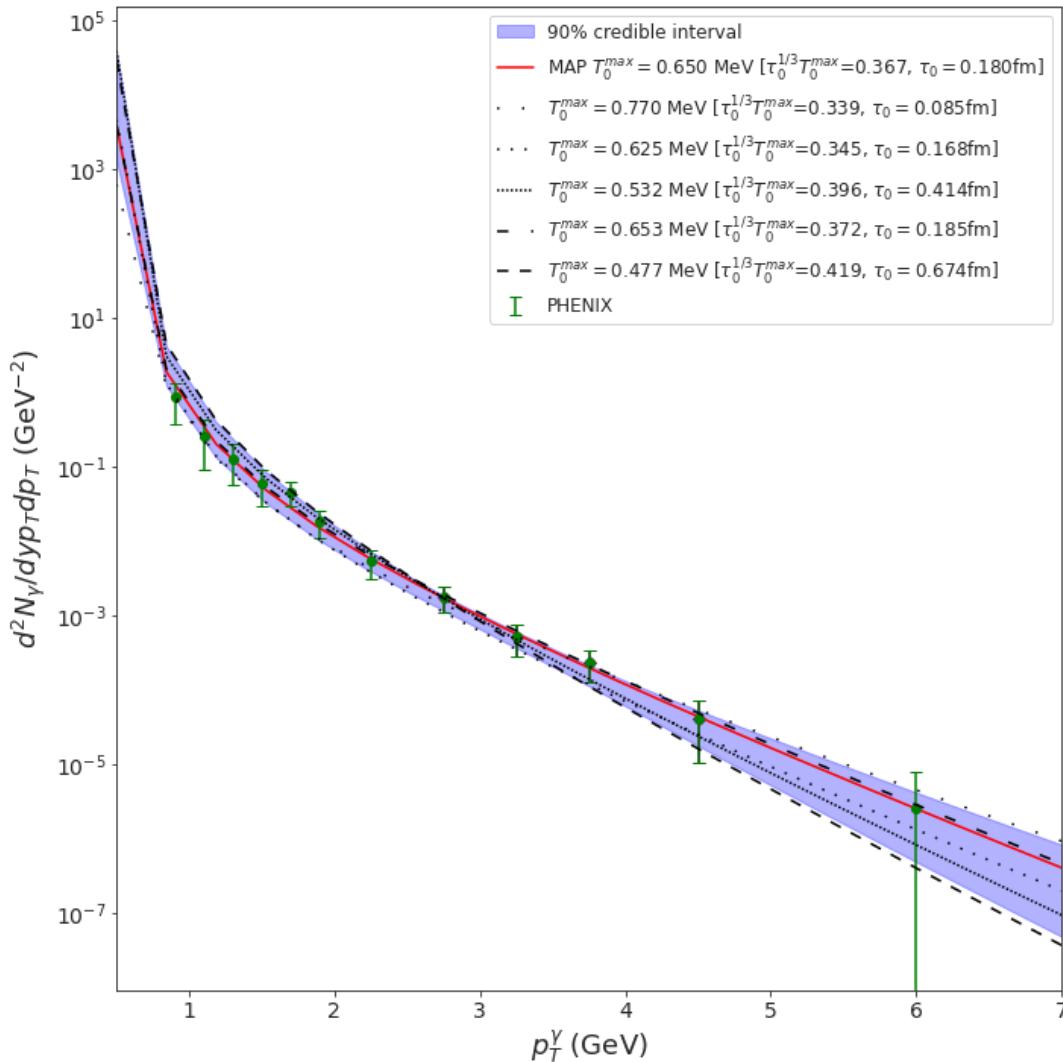


Paquet and Bass [arXiv:2205.12299]

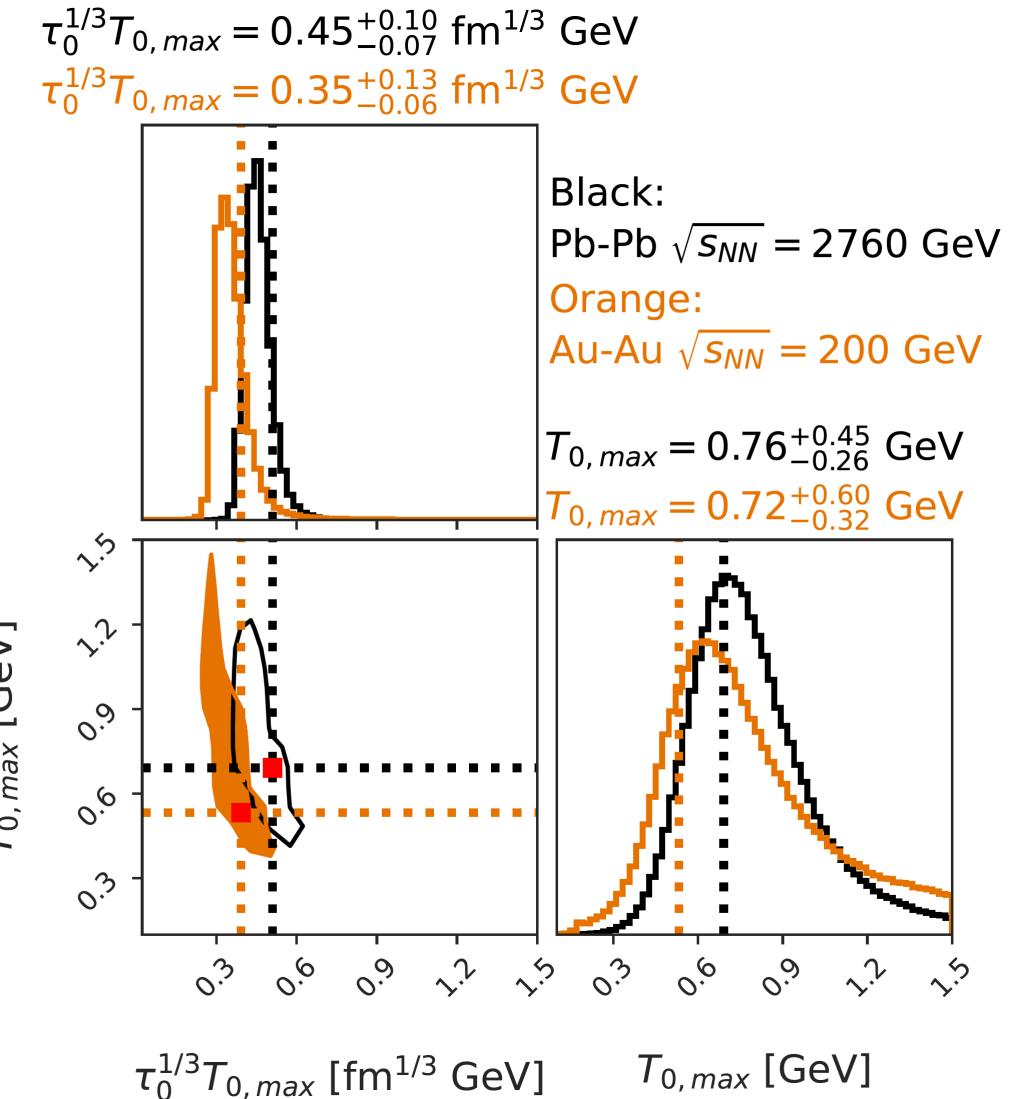
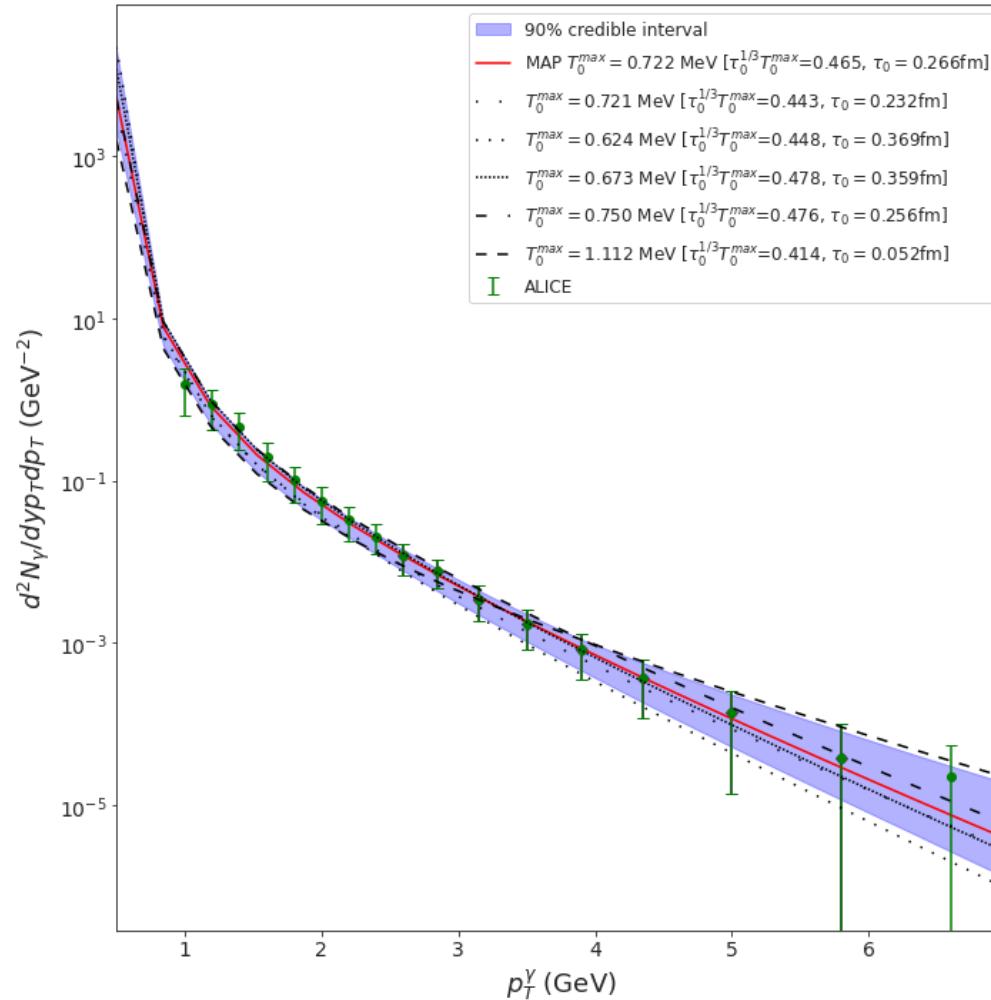
Comparison with
state-of-the-art
calculation of
thermal photons



Comparison with data

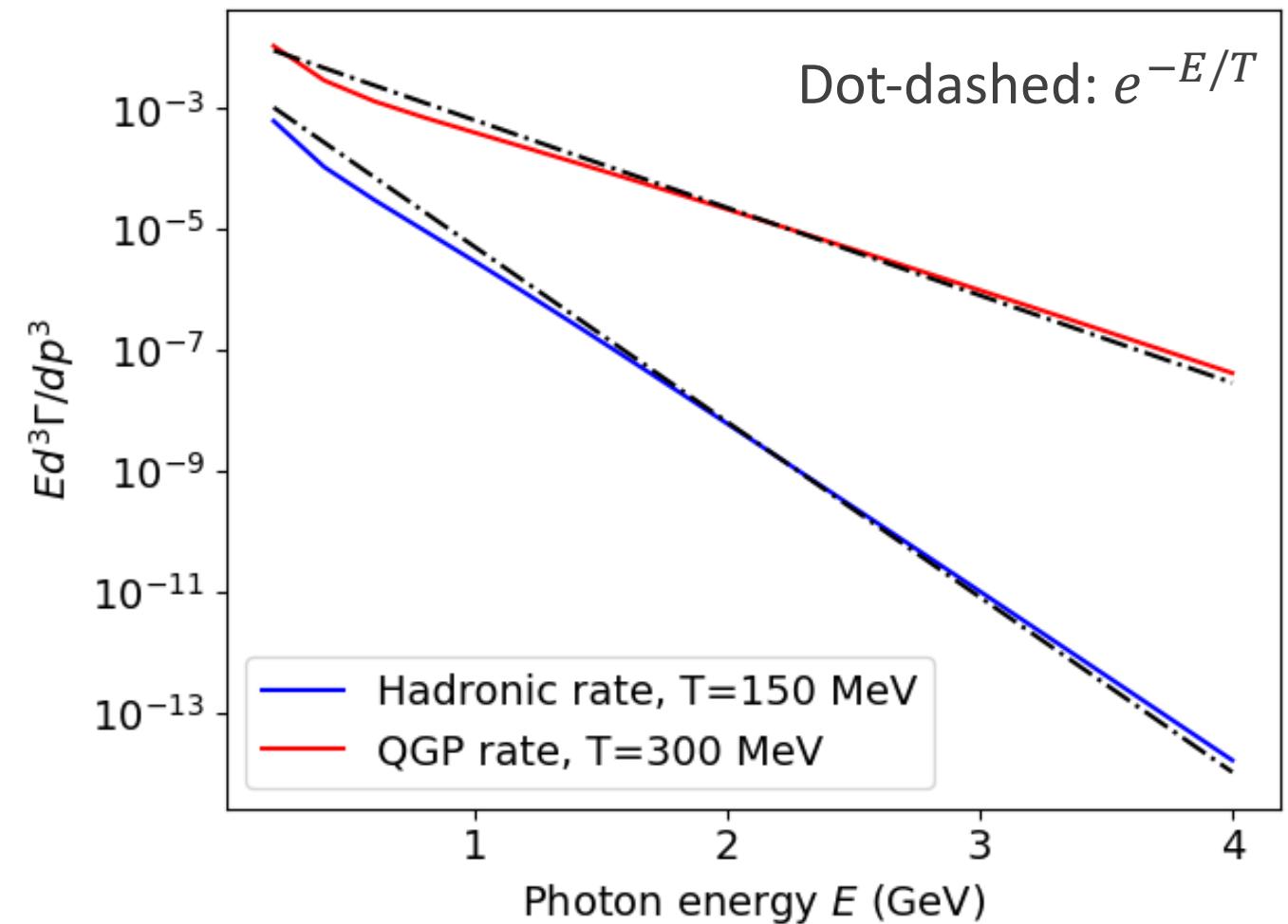


Comparison with data

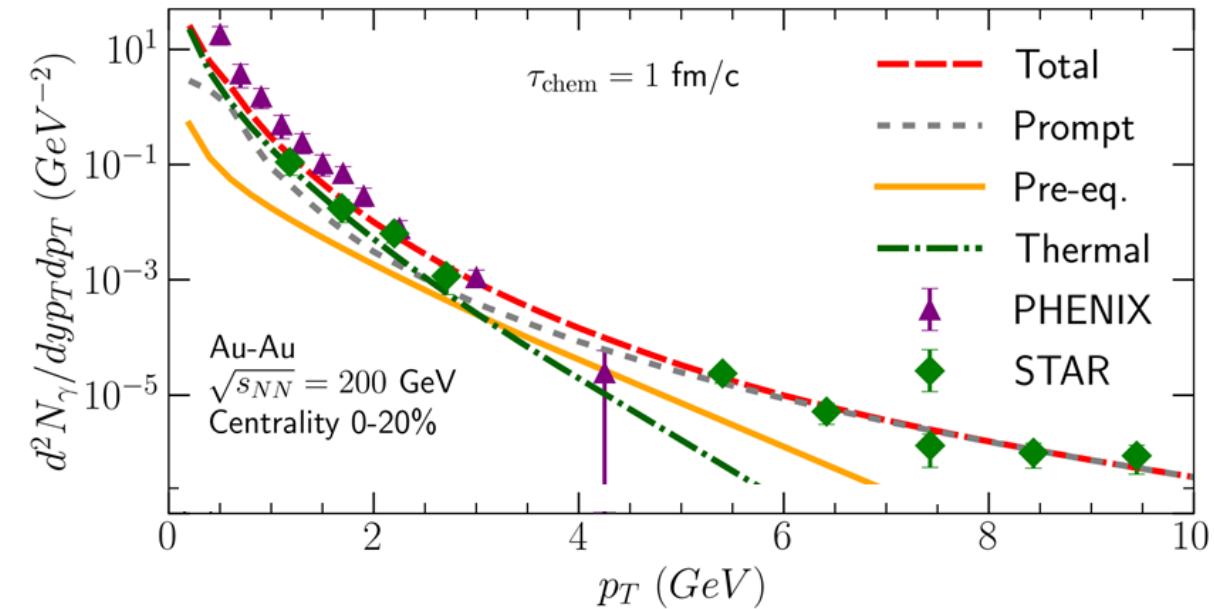
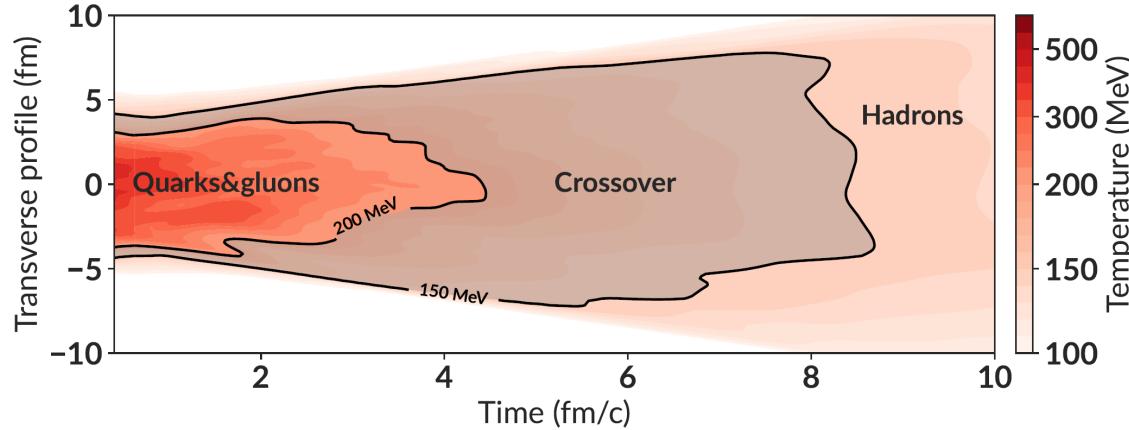


Energy dependence of photon emission rate

- Exact photon rate highly non-trivial
- But energy dependence overwhelmingly dominated by $\exp(-\frac{\text{energy}}{\text{temperature}})$

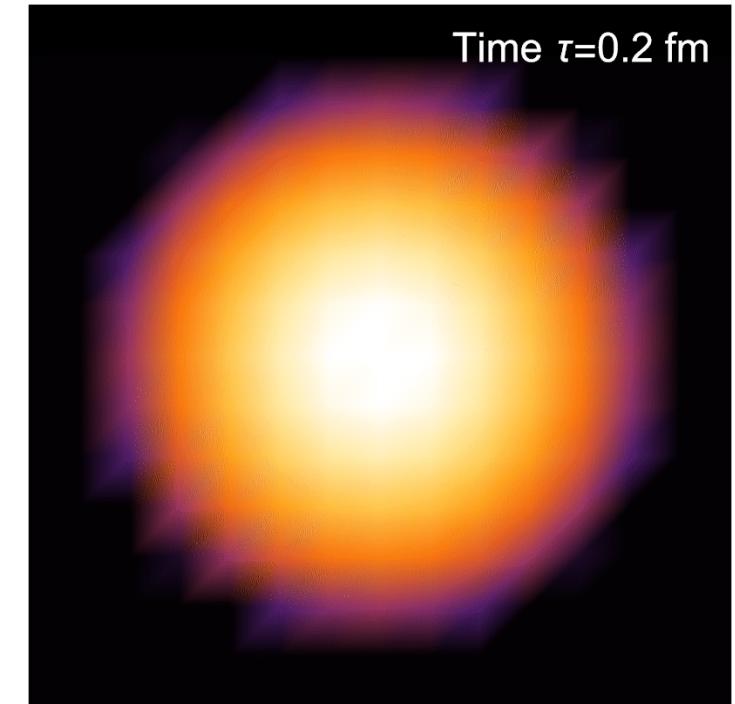
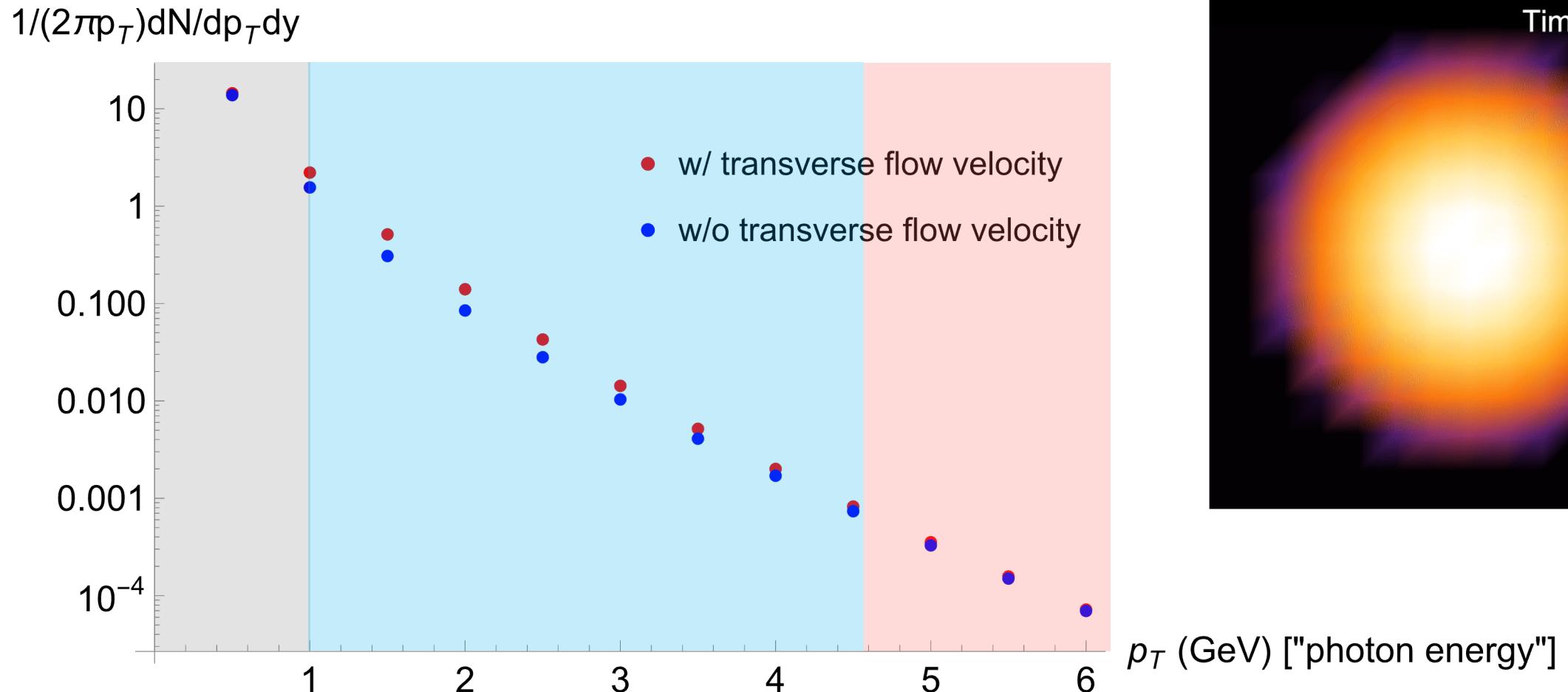


Thermal photon spectrum



$$\begin{aligned}
 \frac{1}{2\pi p_T} \frac{dN_\gamma}{dp_T} \Big|_{y=0} &= \frac{1}{2\pi} \int_0^{2\pi} d\phi \left[E \frac{dN_\gamma}{d^3p} \right]_{y=0} \\
 &= \frac{1}{2\pi} \int_0^{2\pi} d\phi \left[\int d^4X E \frac{d\Gamma_\gamma}{d^3p} (p_\mu u^\mu(X), T(X), \dots) \right]_{y=0} \\
 &\sim \frac{1}{2\pi} \int_0^{2\pi} d\phi \left[\int d^4X \exp(-p_\mu u^\mu(X), T(X)) \right]_{y=0}
 \end{aligned}$$

Hydrodynamic vs initial transverse flow



Profile

