

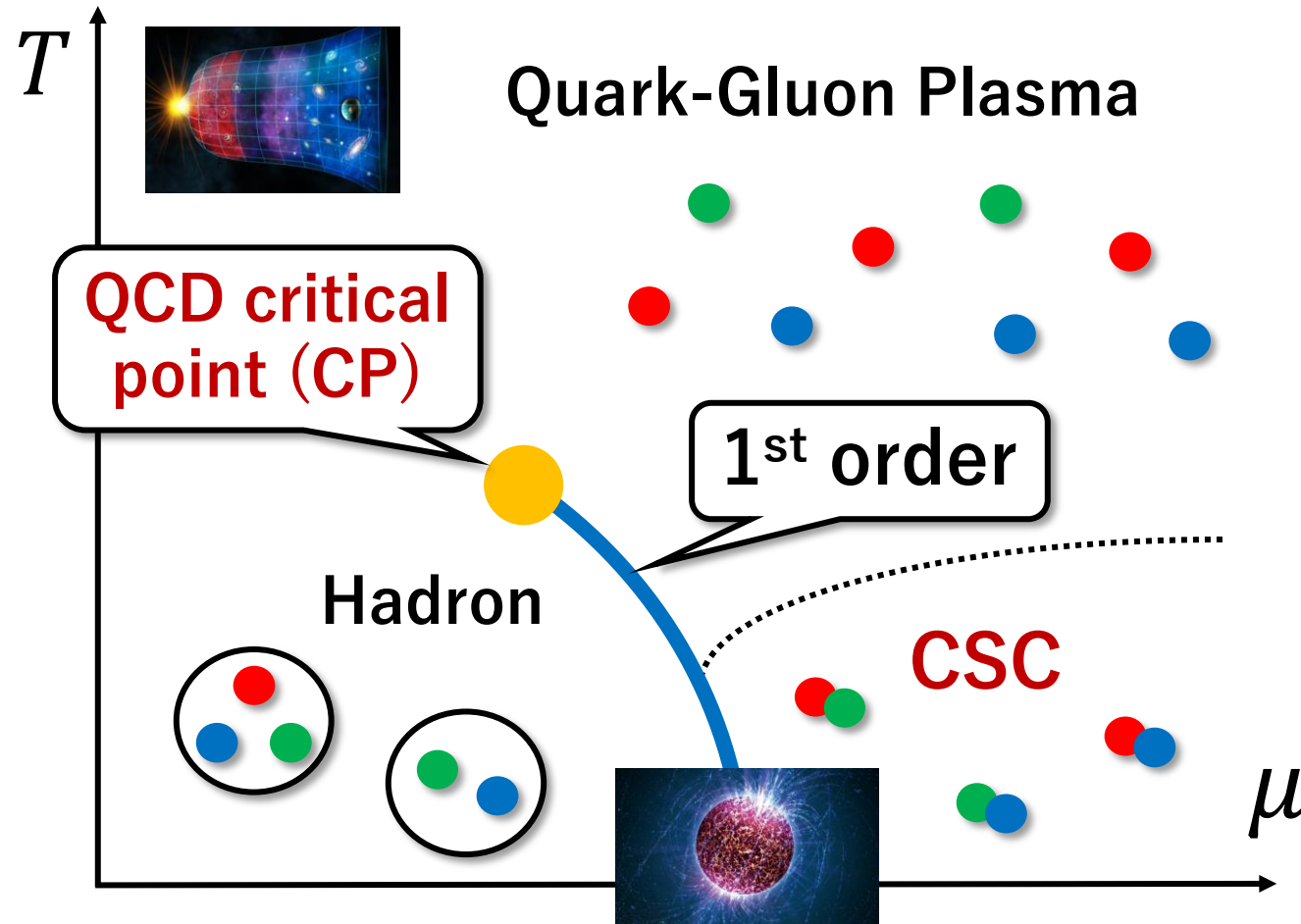
Electromagnetic probes for critical fluctuations of phase transitions in dense QCD

2201.00774, 2302.03191

Presenter : Toru Nishimura (Osaka University)

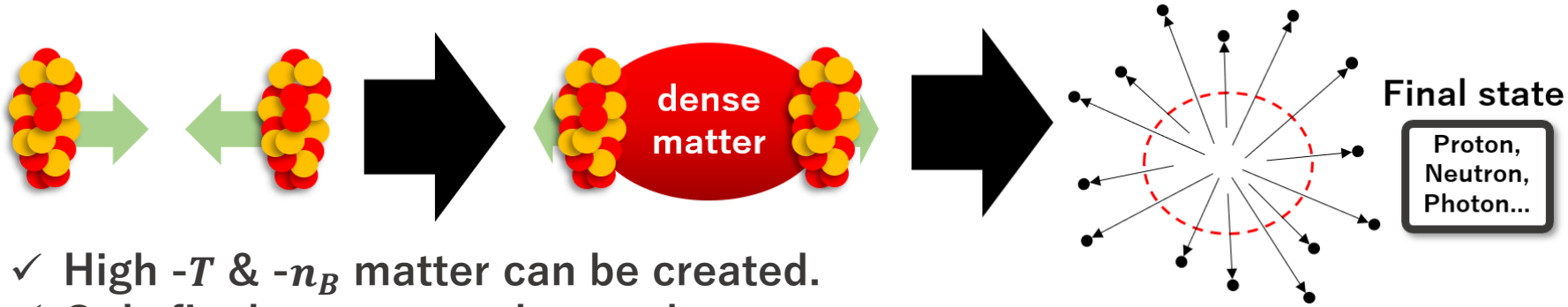
Collaborators : Masakiyo Kitazawa & Teiji Kunihiro

QCD phase diagram

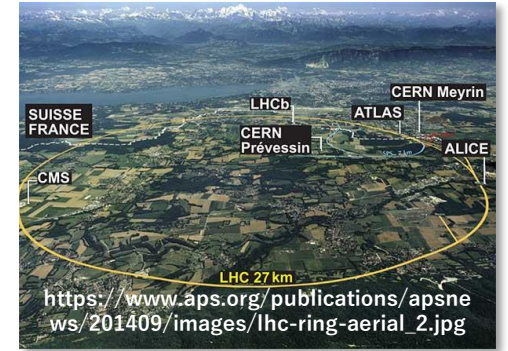


How do we understand this structure experimentally...?

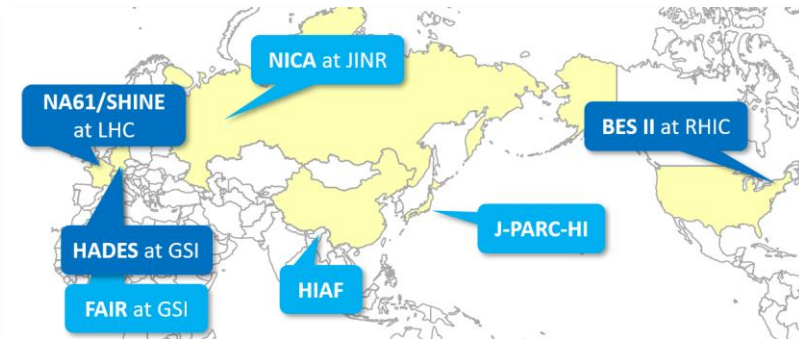
Heavy-ion collision (HIC)



- ✓ High $-T$ & $-n_B$ matter can be created.
- ✓ Only final states are observed.
- ✓ Various regions can be explored by changing collision energy.



High- n_B region is being actively explored

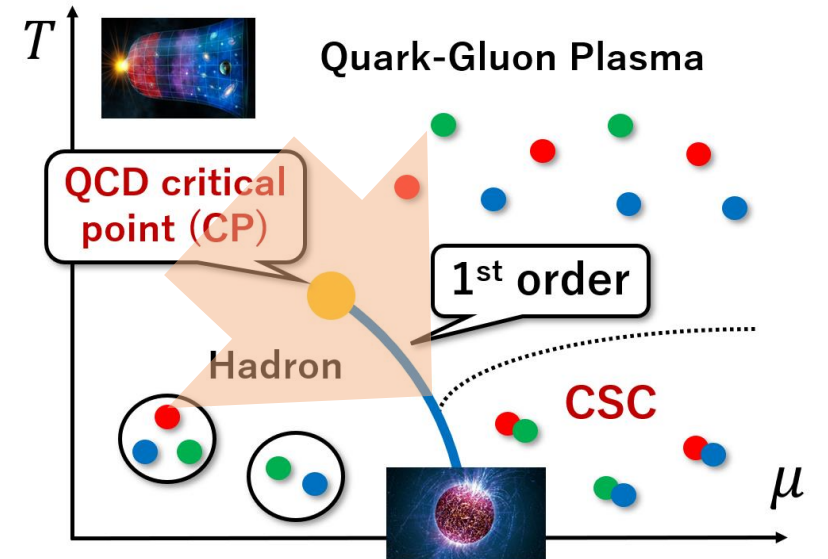


Ongoing

- BES II at RHIC
- NA61/SHINE at LHC
- HADES at GSI

Future

- FAIR at GSI
- NICA at JINR
- J-PARC-HI (planned)



**We focus on
2nd-order phase transition.**

Soft modes

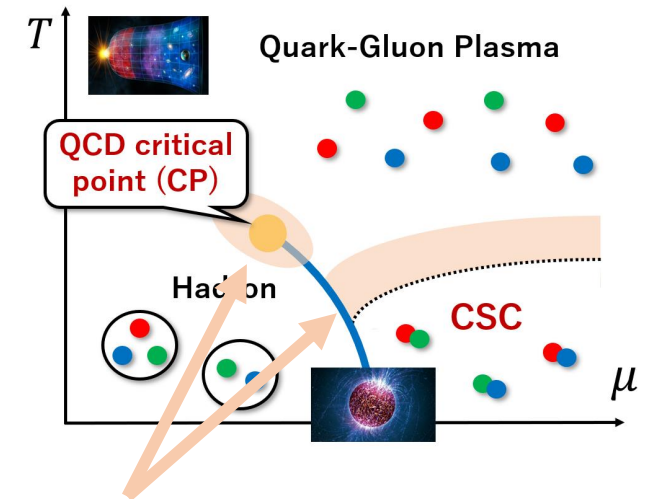
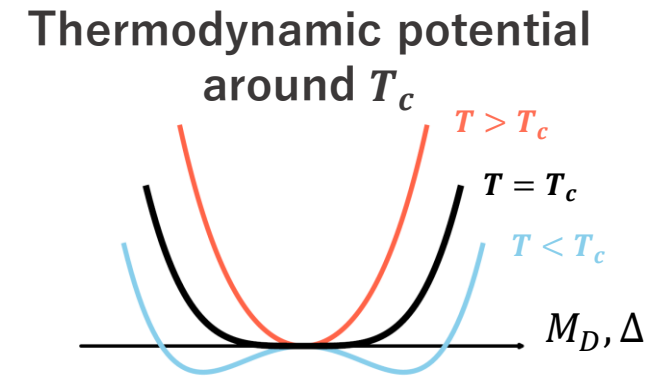
Quantum amplitude fluctuations of order parameters due to 2nd-order phase transitions

- CSC : fluctuation of diquark condensates
Kitazawa, Koide, Kunihiro, Nemoto (2005)
- QCD CP : fluctuation of chiral condensates
Fujii, Ohtani (2004)

- ✓ They develop prominently around the critical point or line.
- ✓ They are excited with low energy & momentum (< 300 MeV).

These soft modes can contribute significantly to observables.

If the contributions are observed in HIC, they will provide experimental signals of CSC & QCD CP.



Colored areas indicate the regions that the soft modes develop.

Dilepton production rate (DPR)

Dileptons don't have color charges.

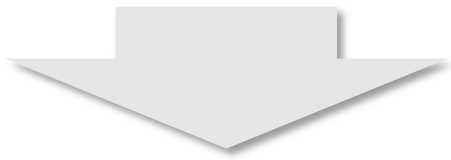
- They don't interact strongly.
- Once created, they go almost directly to the detector.
- We can obtain their initial information.

- ✓ **Observable in HIC**
- ✓ **Computable theoretically**

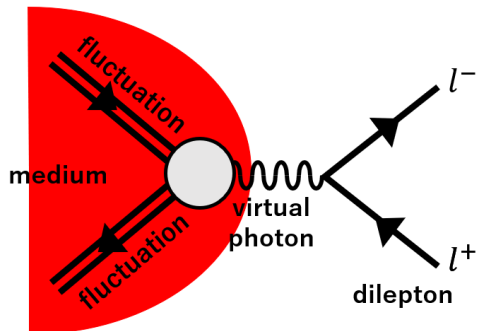
$$\frac{d^4\Gamma}{dk^4}(\mathbf{k}, \omega) = -\frac{\alpha}{12\pi^4} \frac{1}{\omega^2 - \mathbf{k}^2} \frac{1}{e^{\beta\omega} - 1} \rho(\mathbf{k}, \omega)$$

$$\rho(\mathbf{k}, \omega) = g_{\mu\nu} \text{Im}\Pi^{\text{R}\mu\nu}(\mathbf{k}, \omega)$$

Photon self-energy corresponding to the process we consider



Calculation of photon self-energy including soft modes is required.



- ✓ DPR is expected to enhance in the low invariant mass region due to the soft modes around the critical point or line.
- ✓ We construct the self-energy satisfying the Ward Identity for each soft mode.
 - We can respect the charge conservation laws.

Transport coefficients

Photon self-energy $\rho(\mathbf{k}, \omega) = g_{\mu\nu} \text{Im}\Pi^{R\mu\nu}(\mathbf{k}, \omega)$

Dilepton production rate

$$\frac{d^4\Gamma}{dk^4}(\mathbf{k}, \omega) = -\frac{\alpha}{12\pi^4} \frac{1}{\omega^2 - \mathbf{k}^2} \frac{1}{e^{\beta\omega} - 1} \rho(\mathbf{k}, \omega)$$

If the hydrodynamic picture is valid in the low energy-momentum region...

$$\rho(\mathbf{k}, \omega) = g_{\mu\nu} \text{Im}\Pi^{R\mu\nu}(\mathbf{k}, \omega) \simeq \frac{\sigma\omega(\omega^2 - \mathbf{k}^2)}{(\tau\omega^2 - D\mathbf{k}^2)^2 + \omega^2} + 2\frac{\sigma\omega}{\tau^2\omega^2 + 1} \quad \text{Kadanoff, Martin (1963)}$$

$$\Rightarrow \rho(\mathbf{0}, \omega) = -\sum_{i=1,2,3} \text{Im}\Pi^{Rii}(\mathbf{0}, \omega) = 3\frac{\sigma\omega}{\tau^2\omega^2 + 1}$$

Electric conductivity $\sigma = \frac{1}{3} \frac{\partial \rho(\mathbf{0}, \omega)}{\partial \omega} \Big|_{\omega=0}$

Relaxation time $\tau = \sqrt{-\frac{1}{3!} \frac{\frac{\partial^3}{\partial \omega^3} \rho(\mathbf{0}, \omega)|_{\omega=0}}{\frac{\partial}{\partial \omega} \rho(\mathbf{0}, \omega)|_{\omega=0}}}$

This analysis reveals differences of critical behaviors between CSC phase transition & QCD CP.

Summary of introduction

Motivation

We would like to discuss **the observabilities of CSC and QCD CP in heavy-ion collision (HIC) experiments.**

What to do

We investigate effects of **the respective soft modes** on

- the dilepton production rate (DPR)**
- the transport coefficients σ & τ**

TN, Kitazawa and Kunihiro : [2201.00774](#), [2302.03191](#)

TN, Kitazawa and Kunihiro : [In preparation](#)

➡ To do these analyses, we have to calculate the photon self-energy including the soft modes.

Formalism

- ✓ 2-flavor NJL model
- ✓ Soft modes
- ✓ Photon self-energy $\Pi^{\mu\nu}(k, \omega)$
- ✓ Approximation for calculation of $\Pi^{\mu\nu}(k, \omega)$

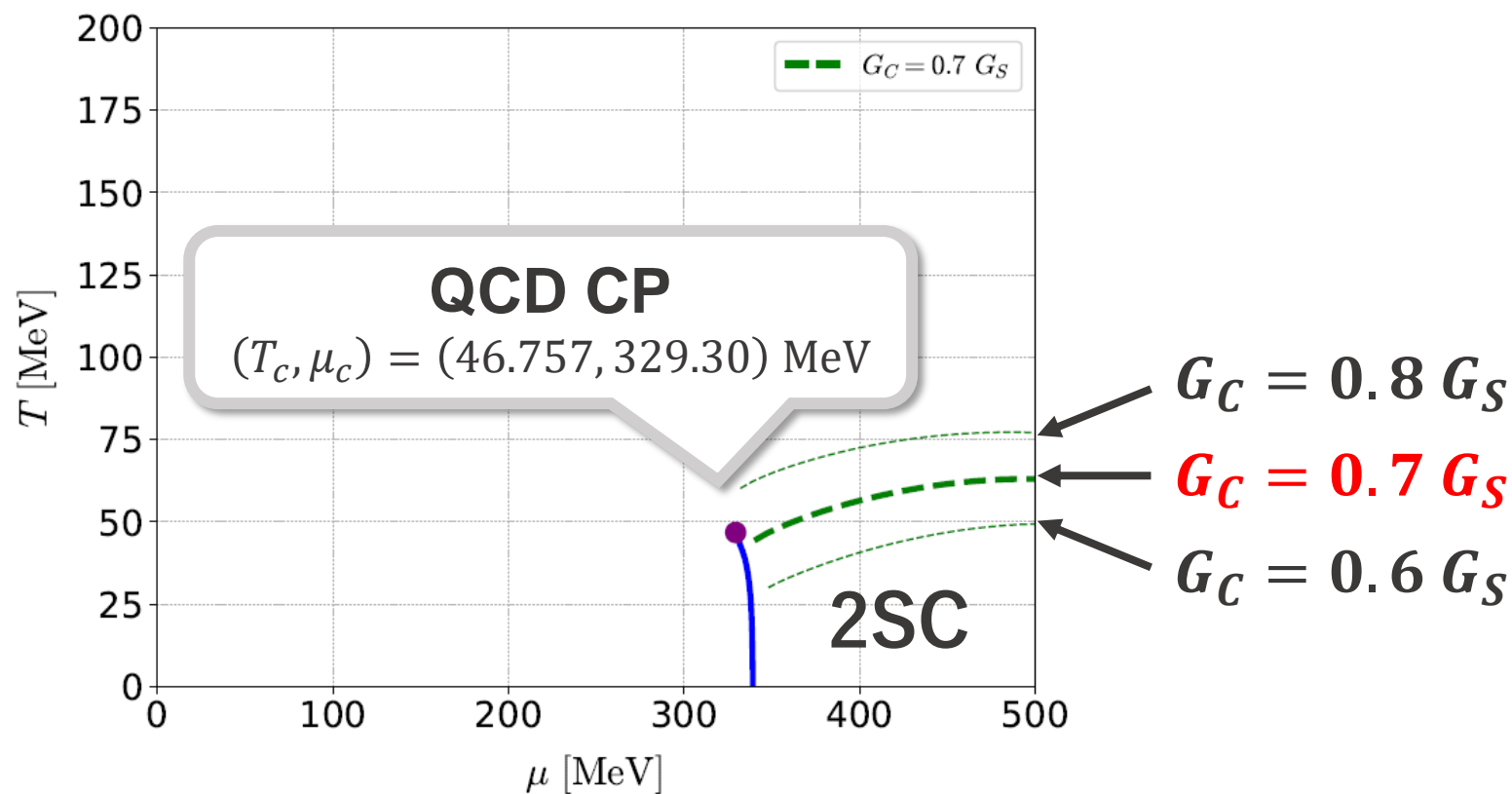
2-flavor NJL model

$$\mathcal{L} = \bar{\psi}i(\gamma^\mu\partial_\mu - m)\psi + \mathcal{L}_S + \mathcal{L}_C \quad \left\{ \begin{array}{l} \mathcal{L}_S = G_S[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\boldsymbol{\tau}\psi)^2] \\ \mathcal{L}_C = G_C(\bar{\psi}i\gamma_5\tau_2\lambda_A\psi^C)(\bar{\psi}^Ci\gamma_5\tau_2\lambda_A\psi) \end{array} \right.$$

Kitazawa, Koide, Kunihiro, Nemoto (2002)

($m = 5.5 \text{ MeV}$, $G_S = 5.50 \text{ MeV}$, $\Lambda = 631 \text{ MeV}$)

MFA

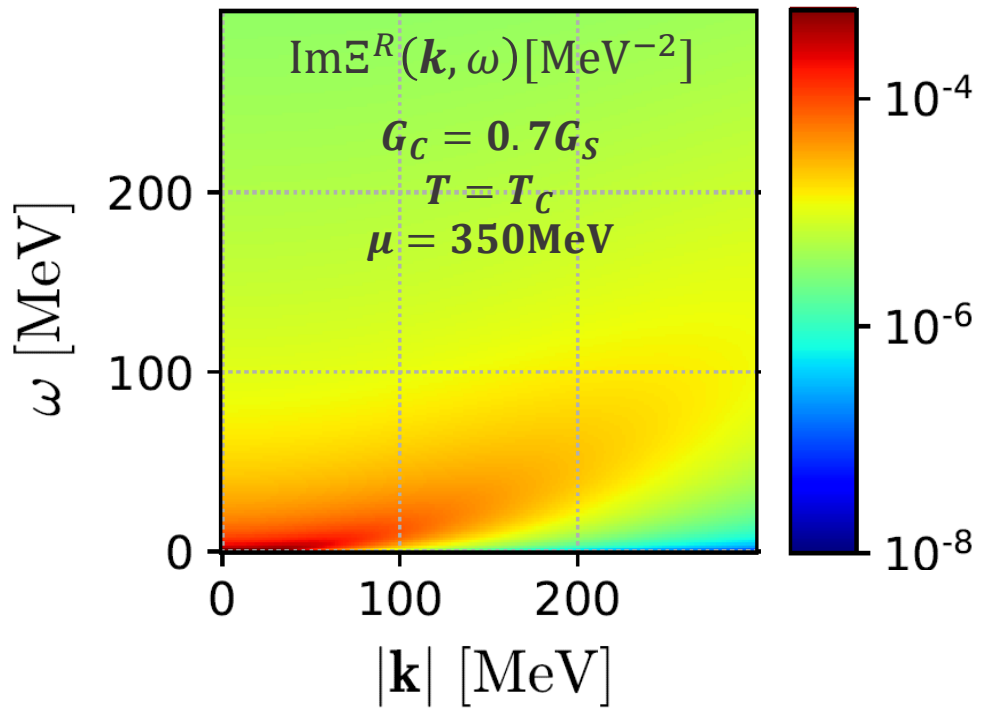


Soft modes (1/2)

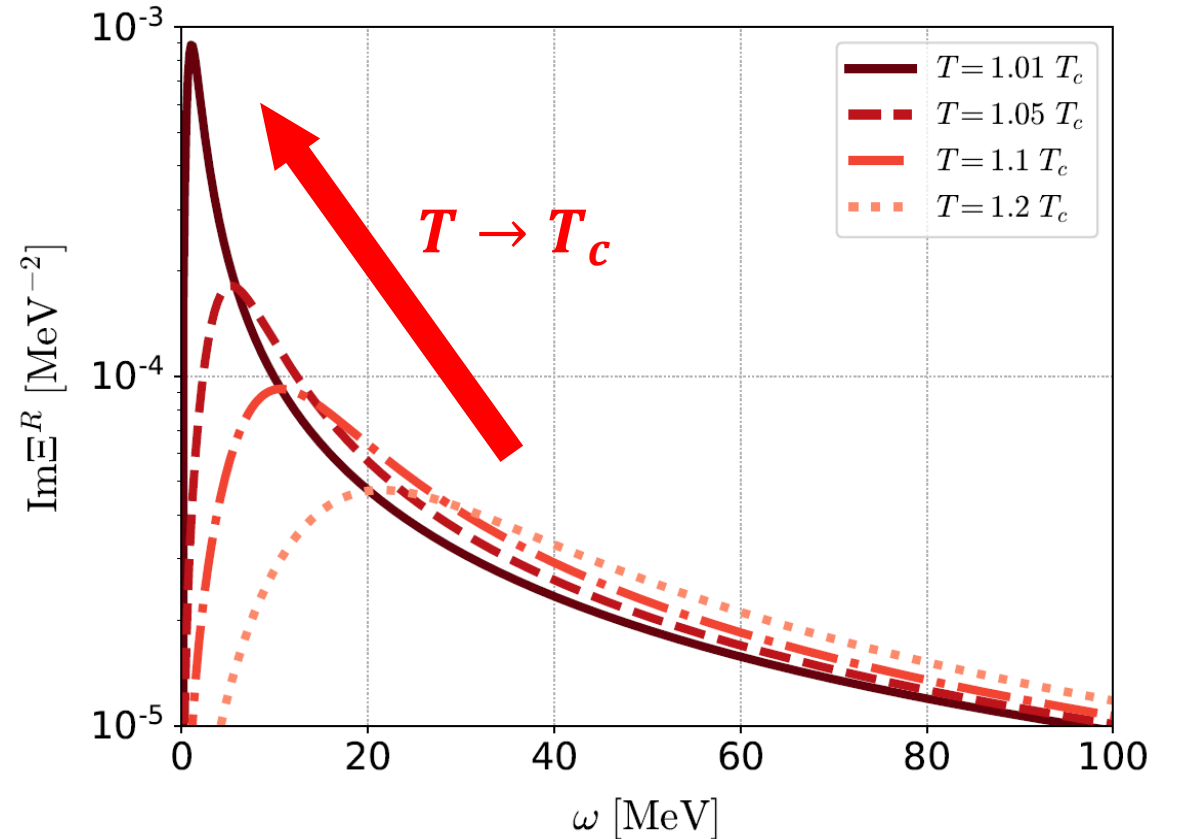
Diquark soft modes

$$\Xi(k, \omega) = \Rightarrow \Rightarrow \Rightarrow \text{T-matrix approx.}$$

$$= G_C + \text{loop} + \text{two loops} + \dots$$



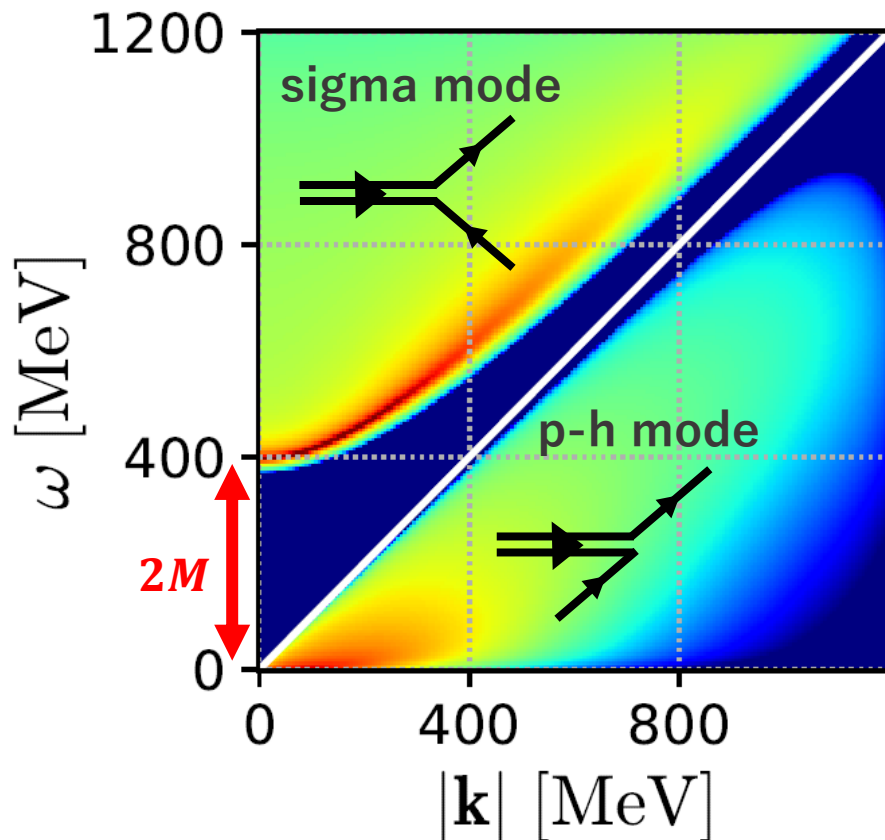
T -dependence ($|k| = 1\text{MeV}$)



As $T \rightarrow T_c$, the strength become bigger and excitation energy become smaller.

Soft modes (2/2)

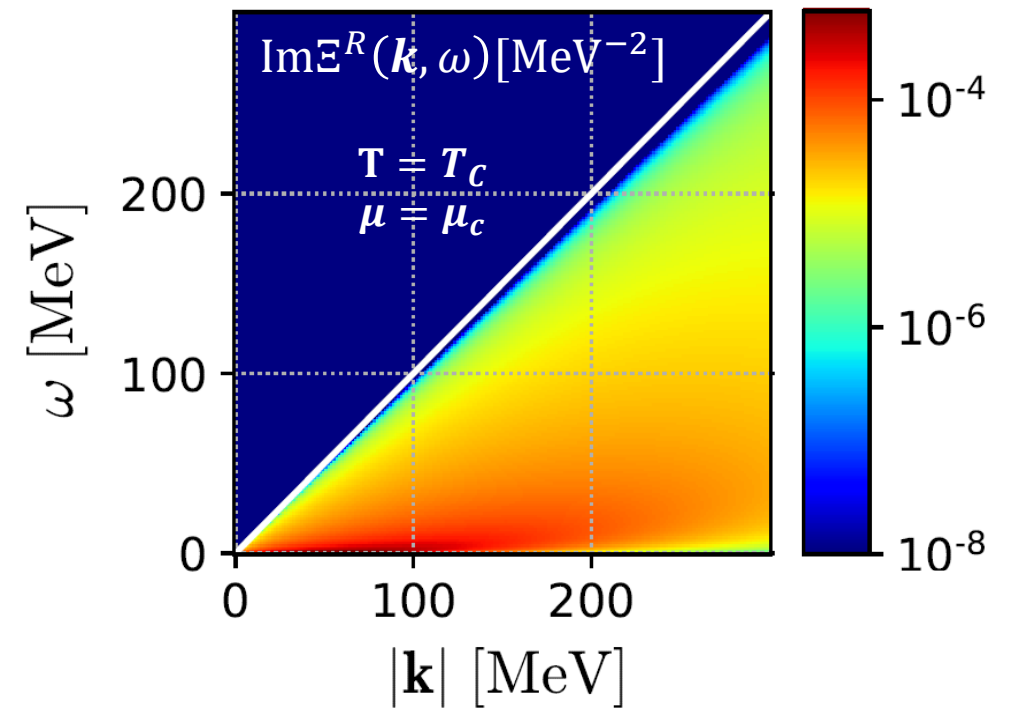
In our work, the soft mode of QCD CP is the particle-hole (p-h) mode.



Particle-hole soft modes

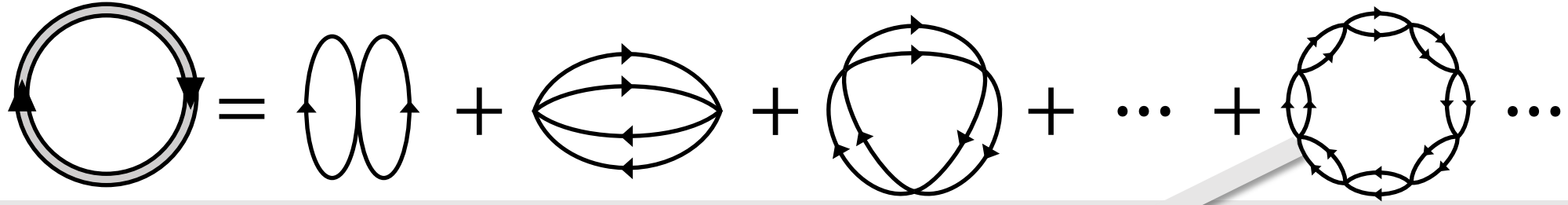
$$\Xi(k, \omega) = \text{diagram of a double line with an arrow} \quad \text{T-matrix approx.}$$

$$= G_S + \text{diagram of a loop} + \text{diagram of two loops} + \dots$$

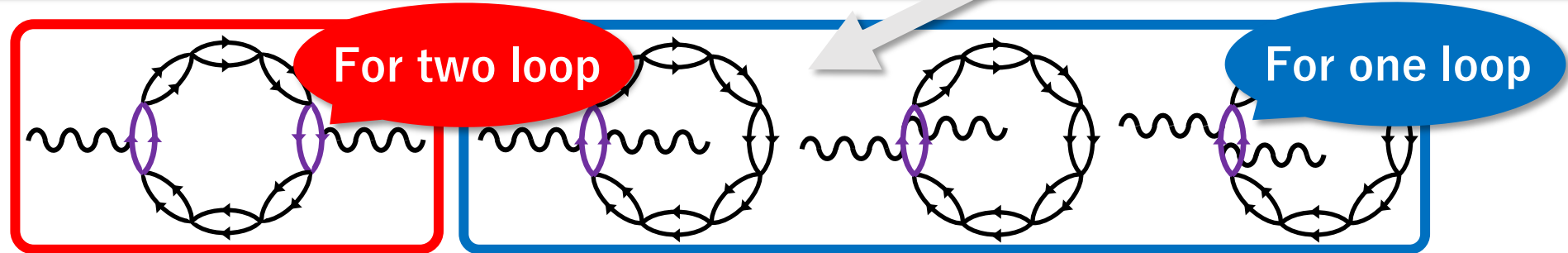


Photon self-energy $\Pi^{\mu\nu}(k, \omega)$ in case of diquark soft mode

Thermodynamic potential : One loop of the diquark soft mode



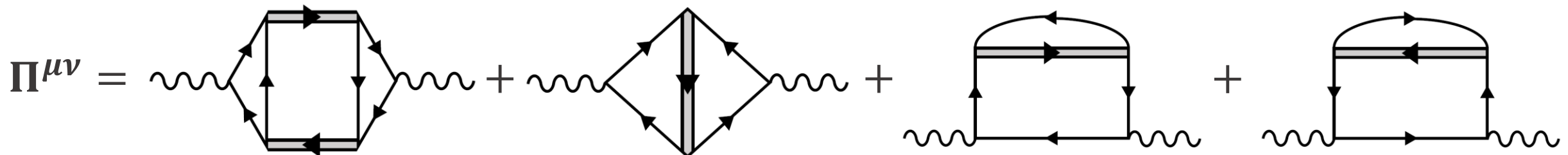
Attach
two
photons



Aslamazov-Larkin
(AL) term

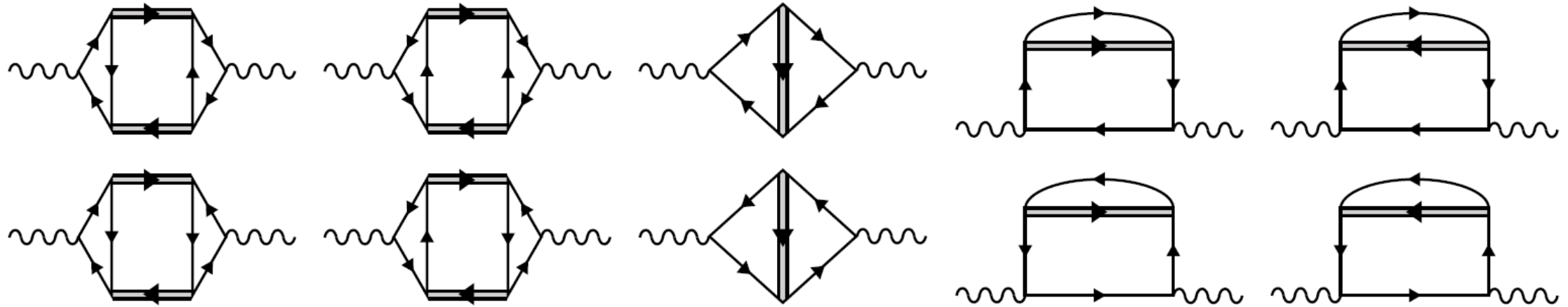
Maki-Thompson
(MT) term

Density of states
(DOS) term



Ward identity $k_\mu \Pi^{\mu\nu} = 0$ is satisfied.

Photon self-energy $\Pi^{\mu\nu}(k, \omega)$ in case of p-h soft mode



CSC \rightarrow QCD CP

Aslamazov-Larkin
(AL) term

Maki-Thompson
(MT) term

Density of states
(DOS) term

$$\Pi^{\mu\nu} = \text{AL term} + \text{MT term} + \text{DOS term} + \text{DOS term}$$

Ward identity of $\Pi^{\mu\nu}$ is satisfied.

Approximation for calculation of $\Pi^{\mu\nu}(k, \omega)$

W-I of $\Pi^{\mu\nu}$ is satisfied with this approximation!!

Propagator of soft modes

$$\Xi^R(q, \omega') = \frac{G_C}{1 + G_C Q^R(q, \omega')} = \frac{1}{A(q) + C(q)\omega'}$$

Thouless criterion : $1 + G_C Q^R(0, 0) = 0$ at $T = T_C$

Imaginary part of
MT and DOS term cancels.

$$\text{Im}(\text{MT} + \text{DOS}) = 0$$

Consistent with metallic SC !!!

Vertex of AL term

W-I of vertex

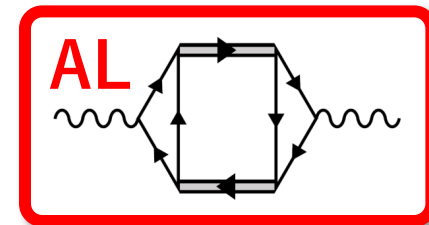
$$k_\mu \gamma^\mu \Xi(q, q+k) = \text{loop}(q+k) - \text{loop}(q)$$

$$k_\mu \Gamma^\mu(q, q+k) \propto \Xi^{-1}(q+k) - \Xi^{-1}(q)$$

Compare the lowest order terms of k and ω .

$$\Gamma^i(q, q+k) \propto \frac{A(q+k) - A(q)}{(q+k)^2 - q^2} (2q+k)^i$$

Only **AL term** is necessary to calculate the DPR.



Results

Dilepton production rate (DPR)

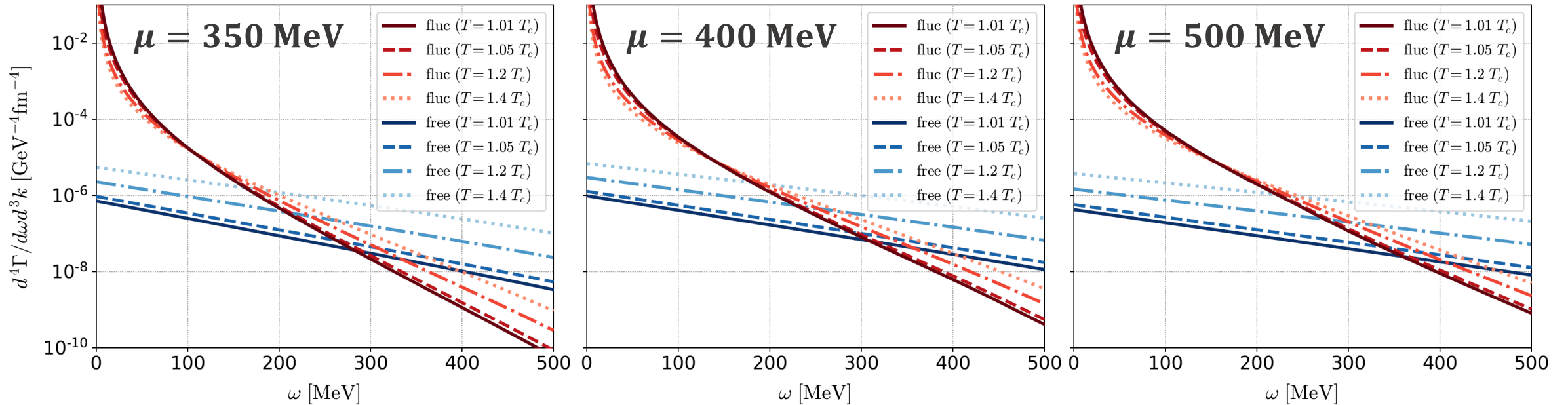
- ✓ Energy spectra at $k = 0$
- ✓ Invariant mass spectra

Transport coefficients

- ⋯ Analytic result : Divergence of transport coefficients

Contribution of diquark soft mode at $k = 0$ ($T > T_c$)

Red lines : Contribution of the soft modes
Blue lines : Contribution of the free quark gases

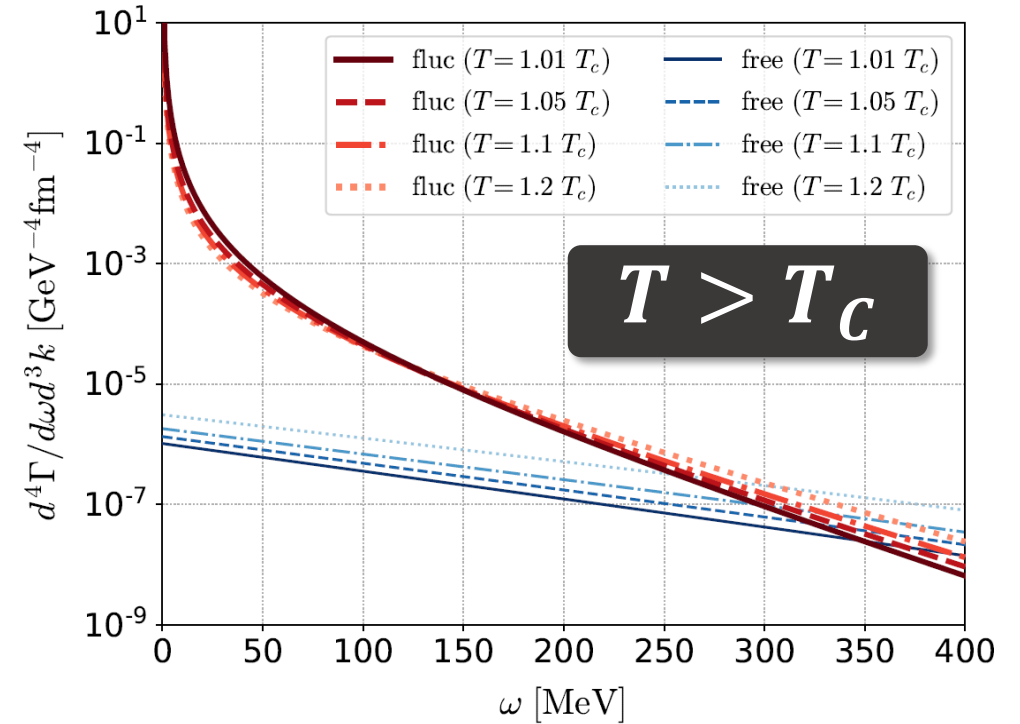
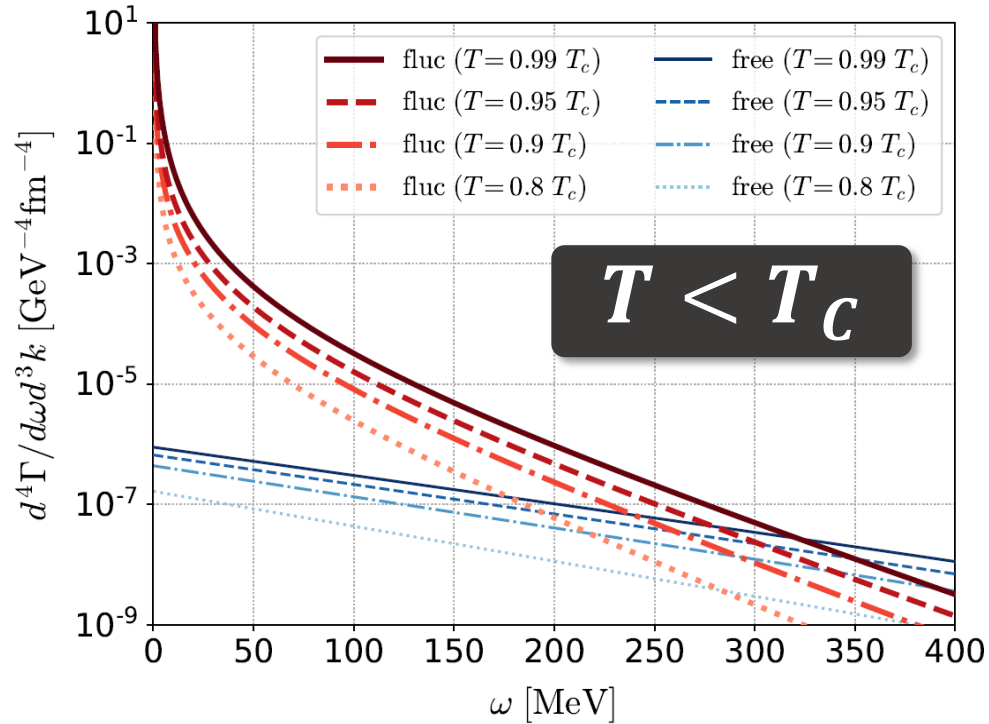


✓ The dilepton production rate is enhanced by the soft mode.

✓ As $T \rightarrow T_c$, the rate becomes bigger.

... This behavior is expected from the property of soft modes.

Contribution of p-h (QCD CP) soft mode at $k = 0$ ($\mu = \mu_c$)



Bigger as $T \rightarrow T_c$

Bigger as T is bigger

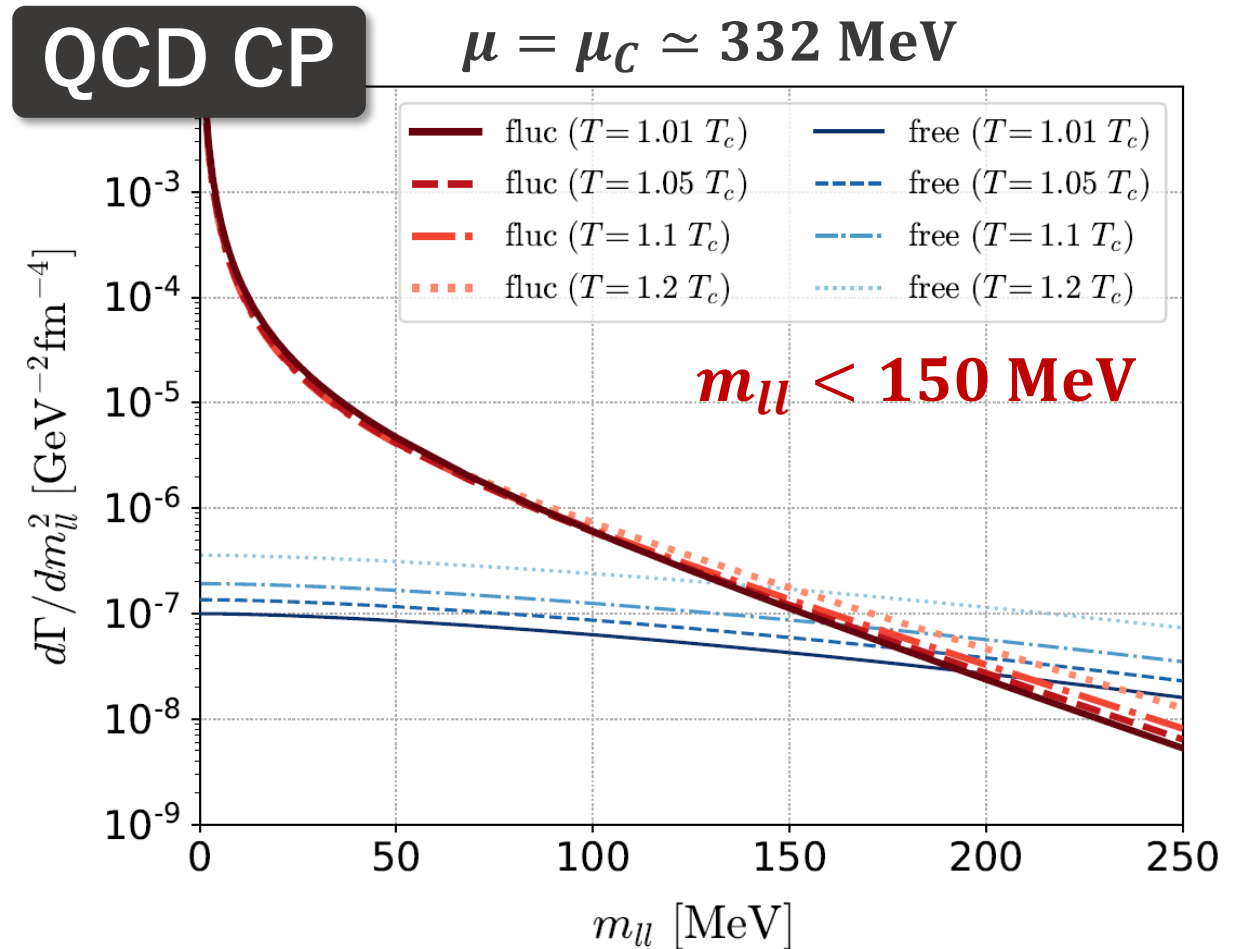
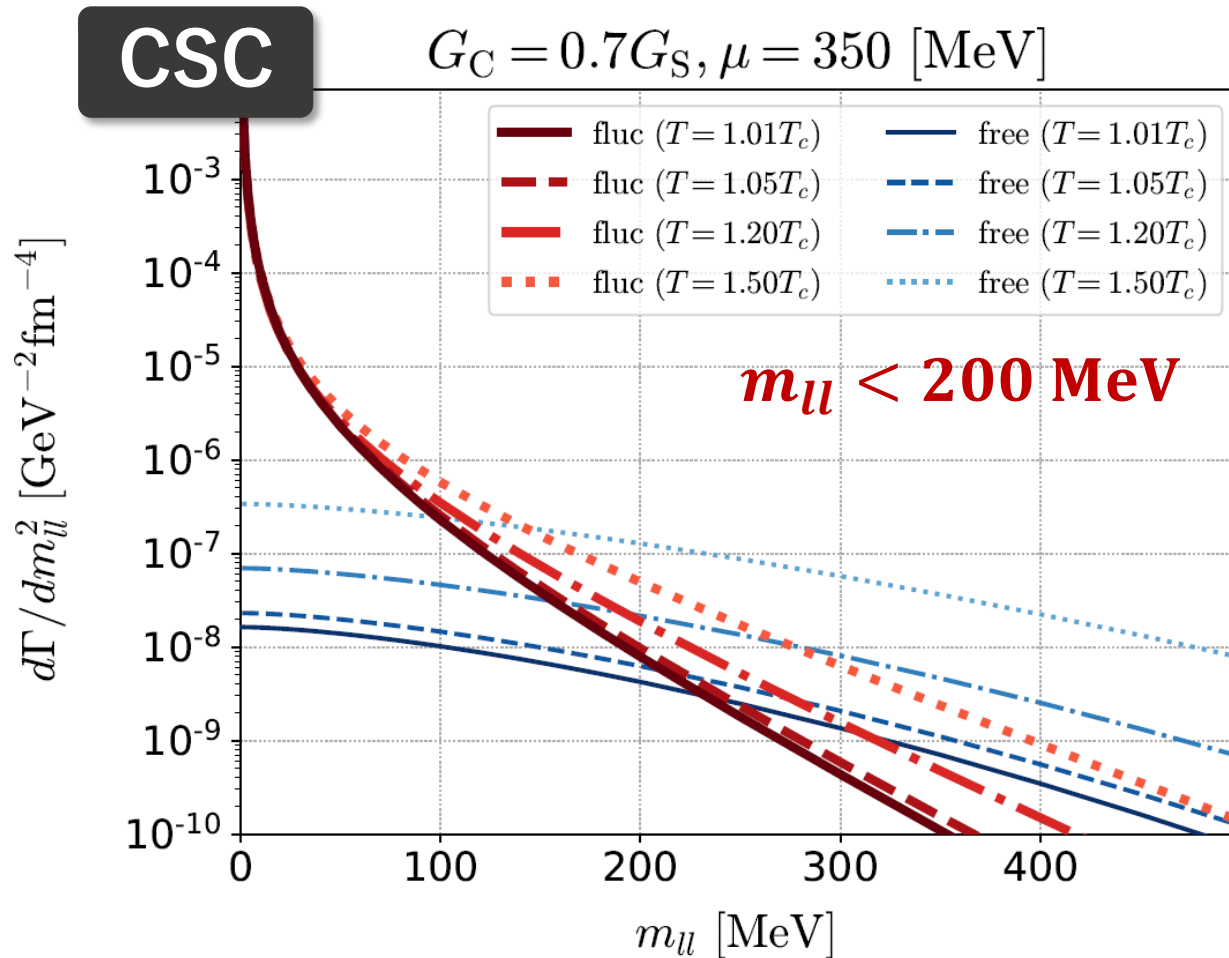
Competition between
contributions of soft modes and
kinematical (temperature) effects

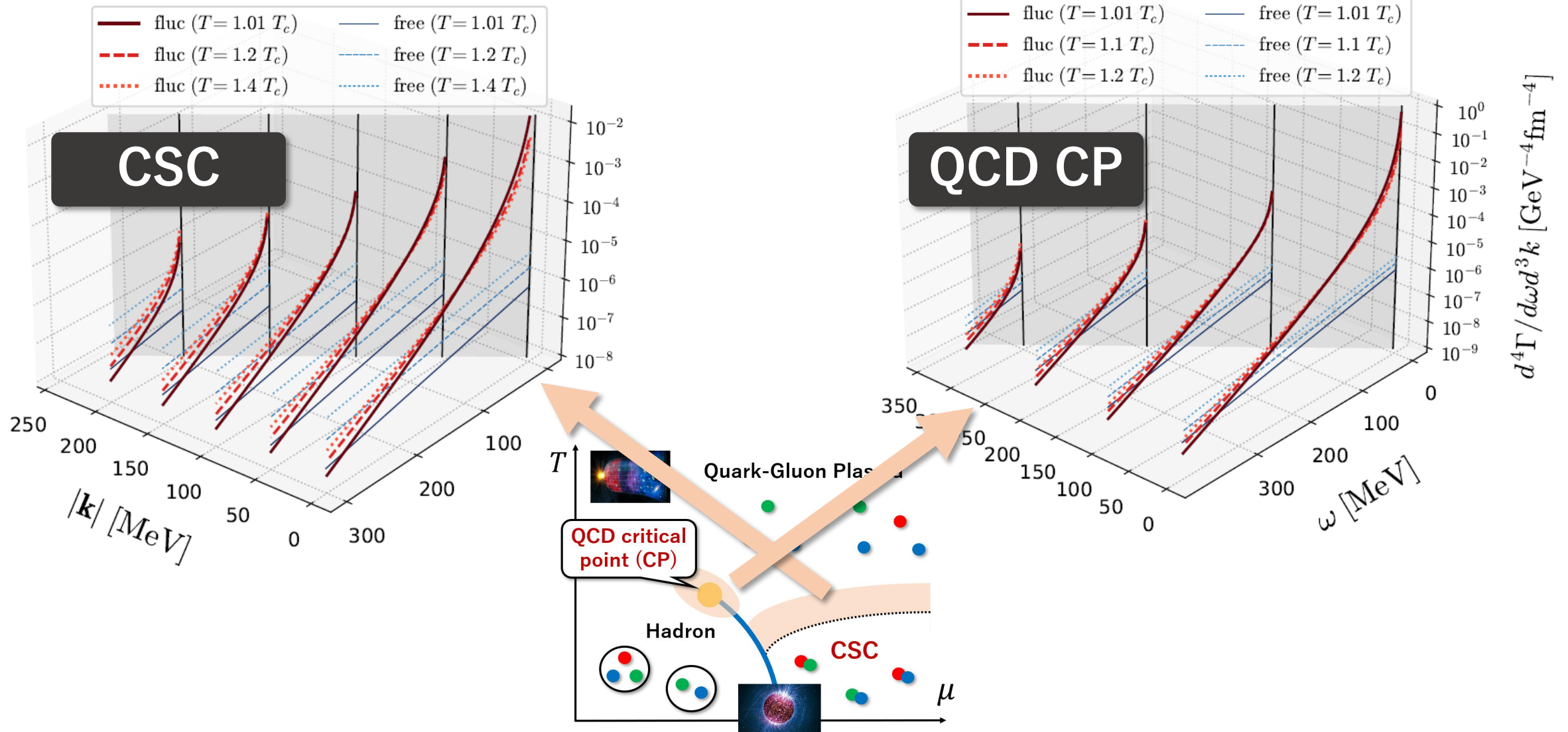
$$\frac{d^4\Gamma}{dk^4}(\mathbf{k}, \omega) = -\frac{\alpha}{12\pi^4} \frac{1}{\omega^2 - \mathbf{k}^2} \frac{1}{e^{\beta\omega} - 1} \rho(\mathbf{k}, \omega), \quad \rho(\mathbf{k}, \omega) = g_{\mu\nu} \text{Im}\Pi_{AL}^{R\mu\nu}(\mathbf{k}, \omega)$$

Invariant mass spectra

$$\frac{d\Gamma}{dM^2} = \int d^3k \frac{1}{2\omega} \frac{d^4\Gamma}{d^4k} \Big|_{\omega=\sqrt{k^2+M^2}}$$

If the enhancement is confirmed,
it may possibly give an experimental evidence
of the phase transition to CSC & QCD CP!





What is the fundamental difference between the two cases?
→ See the critical behavior of transport coefficients!

Transport coefficients

Electric conductivity $\sigma = \frac{1}{3} \frac{\partial \rho(\mathbf{0}, \omega)}{\partial \omega} \Big|_{\omega=0}$

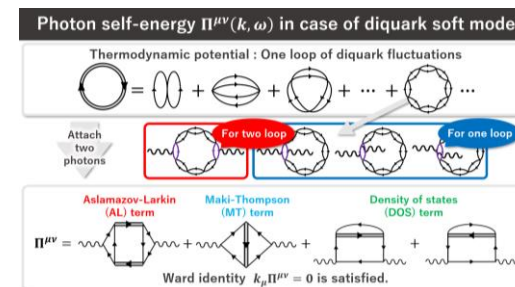
Relaxation time

$$\tau = \sqrt{-\frac{1}{3!} \frac{\partial^3 \rho(\mathbf{0}, \omega)}{\partial \omega^3} \Big|_{\omega=0}}$$

Only the AL term can contribute to σ and τ .

$$\rho(\mathbf{0}, \omega) = g_{\mu\nu} \text{Im} \Pi^{\text{R}\mu\nu}(\mathbf{0}, \omega) = g_{\mu\nu} \text{Im} \Pi_{\text{AL}}^{\text{R}\mu\nu}(\mathbf{0}, \omega)$$

$$\Pi_{\text{AL}}^{\mu\nu}(\mathbf{k}, \omega) = \text{[Diagram: A square loop with four fermion lines and two external photon lines]} = N \int \frac{d^4 q}{(2\pi)^4} \tilde{\Gamma}^\mu(q, q+k) \Xi(q+k) \tilde{\Gamma}^\nu(q+k, q) \Xi(q)$$



- This term includes the soft mode.
- The soft mode with $\omega = |k| = 0$ is divergent at the critical points.
- The respective transport coefficients are also **divergent** at the points.

Divergence of transport coefficients

Differences of the respective soft modes

	CSC	QCD CP
Soft mode $\Xi^R(\mathbf{q}, \omega')$	$\frac{1}{A + B\mathbf{q}^2 + C\omega'} \quad A \propto T - T_c ^1$	$\frac{1}{A + B\mathbf{q}^2 + \frac{C}{ \mathbf{q} }\omega'} \quad A \propto T - T_c ^{\frac{2}{3}}$



$$\sigma = \frac{1}{3} \frac{\partial}{\partial \omega} \rho(\mathbf{0}, \omega) \Big|_{\omega=0} = -\frac{1}{3} \sum_i \frac{\partial}{\partial \omega} \text{Im} \Pi_{AL}^{ii}(\mathbf{0}, \omega) \Big|_{\omega=0} \propto \int q^4 dq \int \frac{d\omega'}{\omega'} \text{Im} \Xi^R(\mathbf{q}, \omega') \frac{\partial}{\partial \omega'} \text{Im} \Xi^R(\mathbf{q}, \omega')$$

CSC

$$\sigma \sim \int q^4 dq \int d\omega' \left(\frac{1}{[A + Bq^2 + C_{\text{Im}}\omega']^2 + [C_{\text{Re}}\omega']^2} \right)^2 \sim \frac{1}{\sqrt{A}} \propto |T - T_c|^{-\frac{1}{2}}$$

QCD CP

$$\sigma \sim \int q^4 dq \int d\omega' \left(\frac{1}{[A + Bq^2]^2 q^2 + [C\omega']^2} \right)^2 \sim \frac{1}{A} \propto |T - T_c|^{-\frac{2}{3}}$$

- ✓ The divergence of transport coefficients is caused by soft modes.
- ✓ The divergence occurs with different exponents for each phase transition.

Divergence of transport coefficients

Differences of the respective soft modes

	CSC	QCD CP
Soft mode $\Xi^R(\mathbf{q}, \omega')$	$\frac{1}{A + B\mathbf{q}^2 + C\omega'} \quad A \propto T - T_c ^1$	$\frac{1}{A + B\mathbf{q}^2 + \frac{C}{ \mathbf{q} }\omega'} \quad A \propto T - T_c ^{\frac{2}{3}}$



σ	$ T - T_c ^{-\frac{1}{2}}$	$ T - T_c ^{-\frac{2}{3}}$
τ	$ T - T_c ^{-1}$	$ T - T_c ^{-1}$

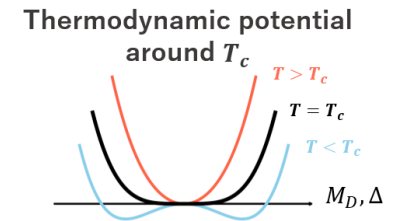
- ✓ The divergence of transport coefficients is caused by soft modes.
- ✓ The divergence occurs with different exponents for each phase transition.

Summary & Outlook

Summary

We calculated how the dilepton production rate & the associated transport coefficients are affected by the soft modes of the CSC & QCD CP.

Soft modes are quantum amplitude fluctuations of order parameters.



- The photon self-energy $\Pi^{\mu\nu}$ including the soft modes is satisfied with the Ward identity.
→ We can respect the charge conservation law.
- The DPR is enhanced due to the soft modes in the low-energy or low-mass region.
→ **Experimental evidence of the phase transition to CSC & the QCD CP!?**
- The transport coefficients diverge at critical temperatures due to soft modes.
→ **The divergence occurs at different exponents.**

Outlook

- Apply our works to dynamical simulations to obtain the amount of dilepton production.
- Investigate the case of $T < T_c$ for the CSC phase.
- Evaluate the effect of the vector coupling.

