



Sphaleron damping and anomalous transport in high- temperature QCD plasmas

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Overview

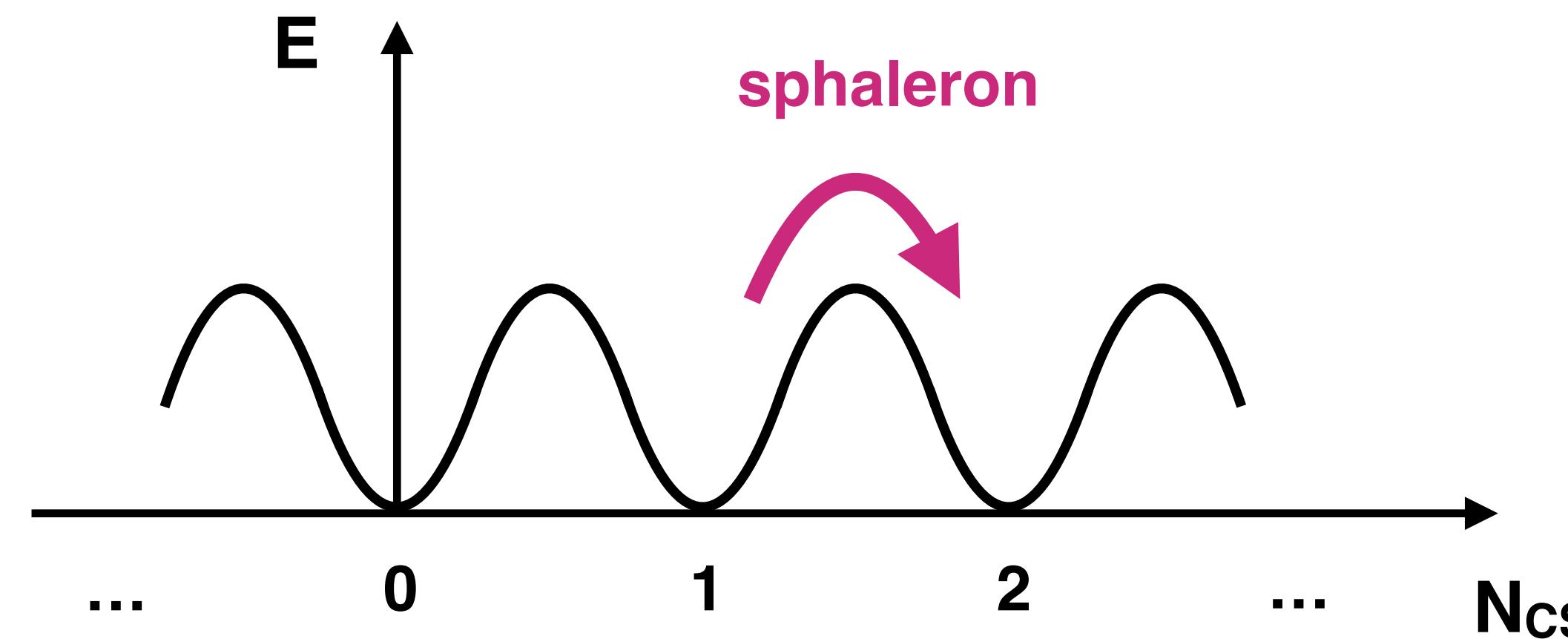
1. Motivation & Introduction
2. Hydrodynamic description of sphaleron damping
3. Charge separation and consequences
4. Conclusion and outlook

Topology in high T QCD

QCD has a topologically non-trivial vacuum structure

Transitions between sectors topologically nontrivial

In high T QCD plasmas, transitions are thermally activated by *sphaleron transitions*



$$N_{CS}(t_1) - N_{CS}(t_2) = \int_{t1}^{t2} dt \int d^3x \partial_\mu K^\mu$$

Axial charge and chiral phenomena

For an $SU(N_c) \times U(1)$ gauge theory coupled to N_f flavors of massless Dirac fermions, the axial charge is not conserved

$$\partial_\mu j_{A,f}^\mu = -\frac{(eq_f^2)N_c}{8\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu} - \frac{g^2}{16\pi^2} G_{\mu\nu}^a \tilde{G}_a^{\mu\nu}$$
$$\partial_\mu Q^\mu = \frac{e^2}{16\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu}$$
$$\partial_\mu K^\mu = \frac{g^2}{32\pi^2} G_{\mu\nu}^a \tilde{G}_a^{\mu\nu}$$

sphalerons generate/erase axial charge

Chiral anomaly leads to nontrivial modification of the hydro equations —> in a hot, dense medium like quark-gluon plasma, **quantum** chiral anomaly can be expressed **macroscopically**!

Macroscopic description of sphaleron damping

Over large time and distance scales, we can express the expectation value of the non-Abelian Chern-Simons current in terms of the sphaleron transition rate:

$$\left\langle \frac{g^2}{16\pi^2} G_{\mu\nu}^a \tilde{G}_a^{\mu\nu} \right\rangle = 4\Gamma_{\text{sph}} \sum_f \frac{\mu_{f,A}}{T}$$

In the presence of an axial charge imbalance, sphalerons exhibit a bias that tends to erase the imbalance

Schlichting and Sharma (2022);
McLerran, Mottola, and Shaposhnikov (1991)

Sphaleron rate is notoriously difficult to calculate, with current estimates from:

- weak coupling parametric estimate: $\Gamma_{\text{sph}} \propto \alpha_s^5 T^4$ Moore and Tassler (2011)
- quenched lattice calculation: $\Gamma_{\text{sph}} = (0.02 - 0.2)T^4$

Altenkort et al. (2021)

Objectives

Basic questions:

- How do sphaleron transitions influence chiral transport phenomena in QGP? (e.g. CME, CMW)
- What are the effects? Are these effects significant in heavy ion collisions?

Hydrodynamic description

General setup

We consider a single fermion-flavor, viscous relativistic fluid in d=3+1, $g^{\mu\nu} = (-1, 1, 1, 1)$

In local fluid rest frame, fluid velocity is defined by

$$u^2 = -1 \quad -u_\mu T^{\mu\nu} = \epsilon u^\nu$$

Consider two $U(1)$ currents:

$$j_V^\mu = \langle \bar{\psi} \gamma^\mu \psi \rangle$$

conserved

$$j_A^\mu = \langle \bar{\psi} \gamma^\mu \gamma_5 \psi \rangle$$

not conserved \rightarrow chiral anomaly, sphaleron transitions

Hydrodynamic description

Describe fluid with
constitutive relations and
conservation laws

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + Pg^{\mu\nu} + \tau^{\mu\nu}$$

$$j_V^\mu = n_V u^\mu + \nu_V^\mu$$

$$j_A^\mu = n_A u^\mu + \nu_A^\mu$$

$$\partial_\mu T^{\mu\nu} = eq_f F^{\nu\lambda} j_\lambda^V$$

$$\partial_\mu j_V^\mu = 0$$

$$\partial_\mu j_A^\mu = (eq_f)^2 C E^\mu B_\mu - 4\Gamma_{\text{sph}} \frac{\mu_A}{T}$$

Viscous corrections to transport

$$\tau^{\mu\nu} = -\eta\sigma^{\mu\nu} - \zeta\Delta^{\mu\nu}\partial \cdot u$$

$$\nu_V^\mu = -\sigma_{VV} \left(T\Delta^{\mu\nu}\partial_\nu \frac{\mu_V}{T} - eq_f E^\mu \right) - \sigma_{VA} T\Delta^{\mu\nu}\partial_\nu \frac{\mu_A}{T} + \sigma_{BV} eq_f B^\mu + \xi_V \omega^\mu$$

$$\nu_A^\mu = -\sigma_{AV} \left(T\Delta^{\mu\nu}\partial_\nu \frac{\mu_V}{T} - eq_f E^\mu \right) - \sigma_{AA} T\Delta^{\mu\nu}\partial_\nu \frac{\mu_A}{T} + \sigma_{BA} eq_f B^\mu + \xi_A \omega^\mu$$

Viscous corrections up to first order in external fields, gradients of hydrodynamic variables

Viscous corrections to transport

shear, bulk viscosity

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vector, axial conductivity

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$$\nu_V^\mu = -\sigma_{VV} \left(T \Delta^{\mu\nu} \partial_\nu \frac{\mu_V}{T} - e q_f E^\mu \right) - \sigma_{VA} T \Delta^{\mu\nu} \partial_\nu \frac{\mu_A}{T} + \sigma_{BV} e q_f B^\mu + \xi_V \omega^\mu$$

chiral vortical effect

$$\nu_A^\mu = -\sigma_{AV} \left(T \Delta^{\mu\nu} \partial_\nu \frac{\mu_V}{T} - e q_f E^\mu \right) - \sigma_{AA} T \Delta^{\mu\nu} \partial_\nu \frac{\mu_A}{T} - \sigma_{BA} e q_f B^\mu + \xi_A \omega^\mu$$

chiral magnetic effect

vector, axial conductivity

chiral electric separation effect

chiral separation effect

spin polarization

The diagram illustrates the decomposition of the shear stress tensor and transport coefficients into various physical effects. The shear stress tensor is given by $\tau^{\mu\nu} = -\eta \sigma^{\mu\nu} - \zeta \Delta^{\mu\nu} \partial \cdot u$, where η and ζ represent shear and bulk viscosity respectively. The transport coefficients are grouped into several effects: vector/axial conductivity (σ_{VV} and σ_{AV}), chiral electric separation (σ_{VA}), chiral magnetic effect (σ_{BV}), chiral separation effect (σ_{AA} and σ_{BA}), and spin polarization (ξ_V and ξ_A). Arrows point from the labels to the corresponding terms in the equations.

Viscous corrections up to first order in external fields, gradients of hydrodynamic variables

Entropy production

Dissipative effects contribute positively to entropy production in the system, while anomalous effects do not

Entropy production is quantified by the entropy current

$$\begin{aligned}\partial_\mu S^\mu = & -\frac{1}{T} \partial_\mu u_\nu \tau^{\mu\nu} - \nu_V^\mu \left(\partial_\mu \frac{\mu_V}{T} - \frac{eq_f}{T} E_\mu \right) - \nu_A^\mu \partial_\mu \frac{\mu_A}{T} \\ & - \frac{\mu_A}{T} \left((eq_f)^2 C E^\mu B_\mu - 4\Gamma_{\text{sph}} \frac{\mu_A}{T} \right) + \partial_\mu (D_B B^\mu + D_\omega \omega^\mu)\end{aligned}$$

Entropy current also constrains the anomalous transport coefficients

Son and Surowka (2009)

Hydrodynamic
excitations in a
charge-neutral plasma

Setting up linearized equations

To analyze hydrodynamic excitations, we collect the conservation laws, constitutive equations, and the transport coefficients to write the collection of hydrodynamic equations

We then linearize the equations around **static, charge-neutral** background **in equilibrium**:

$$u^\mu = (1, 0, 0, 0)$$

$$\text{finite } T \gg \mu_V$$

$$\text{background } \mu_V = 0, \mu_A = 0$$

Switch hydrodynamic variables:

$$\delta\epsilon, \delta n_{V,A}, \pi^i$$

Linearized hydrodynamic equations

$$\partial_t \delta\epsilon + i|\mathbf{k}| \pi_L = 0$$

sound waves

$$\partial_t \pi_L + i|\mathbf{k}| c_s^2 \delta\epsilon + \frac{4}{3} \gamma_\eta \mathbf{k}^2 \pi_L = 0$$
$$\partial_t \pi_{\perp B} + \gamma_\eta \mathbf{k}^2 \pi_{\perp B} = 0$$

diffusive shear mode

$$\partial_t \pi_{\perp\perp} + \gamma_\eta \mathbf{k}^2 \pi_{\perp\perp} - \sum_f i e q_f |\mathbf{k} \times \mathbf{B}| (D_V^f \delta n_{V,f}) = 0$$
$$\partial_t \delta n_{V,f} + D_V \mathbf{k}^2 \delta n_{V,f} + e q_f C \frac{i |\mathbf{k} \times \mathbf{B}|}{\chi_A} \delta n_{A,f} = 0$$
$$\partial_t \delta n_{A,f} + D_A \mathbf{k}^2 \delta n_{A,f} + e q_f C \frac{i |\mathbf{k} \times \mathbf{B}|}{\chi_V} \delta n_{V,f} = -\gamma_{\text{sph}} \sum_f \delta n_{A,f}$$

diffusive shear mode coupled to charge density modes

$$\gamma_{\text{sph}} = \frac{4\Gamma_{\text{sph}}}{\chi_A T}$$

Linearized hydrodynamic equations

In the single-flavor case, we rewrite the charge mode equations in matrix form,

$$M_{ab} = \begin{pmatrix} D\mathbf{k}^2 & ieq_f C \chi_A^{-1} \mathbf{k} \cdot \mathbf{B} \\ ieq_f C \chi_V^{-1} \mathbf{k} \cdot \mathbf{B} & D\mathbf{k}^2 + \gamma_{\text{sph}} \end{pmatrix}$$

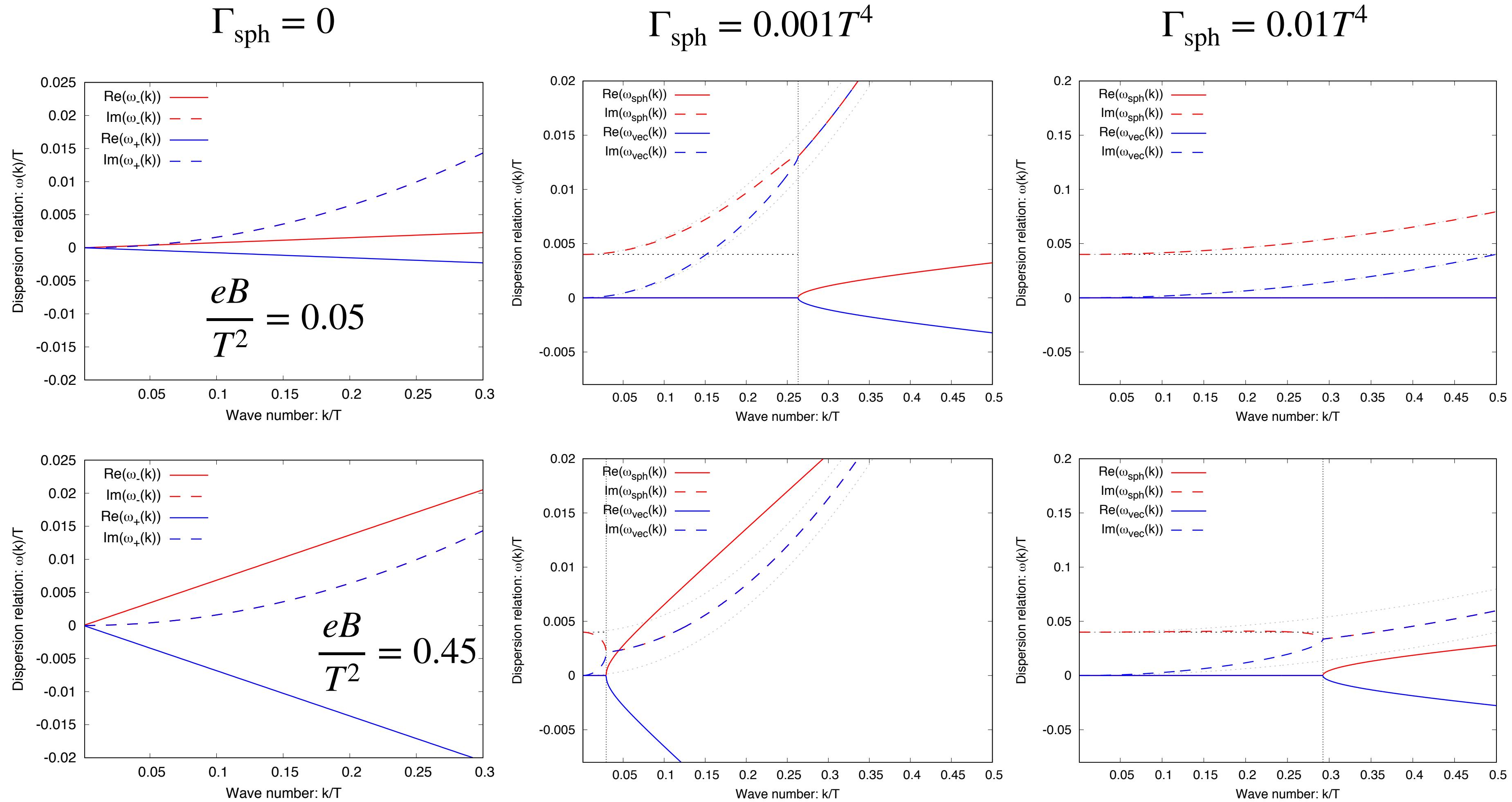
such that it solves the equation $\partial_t \phi_a + M_{ab} \phi_b = 0$

for $\phi_a = (\delta n_V, \delta n_A)$

For illustrative purposes, we use equation of state $c_s^2 = \frac{1}{3}$ and:

$$N_C = 3, \quad \chi_{V/A} = T^2, \quad D = (2\pi T)^{-1}$$

Dispersion relations, $N_f = 1$

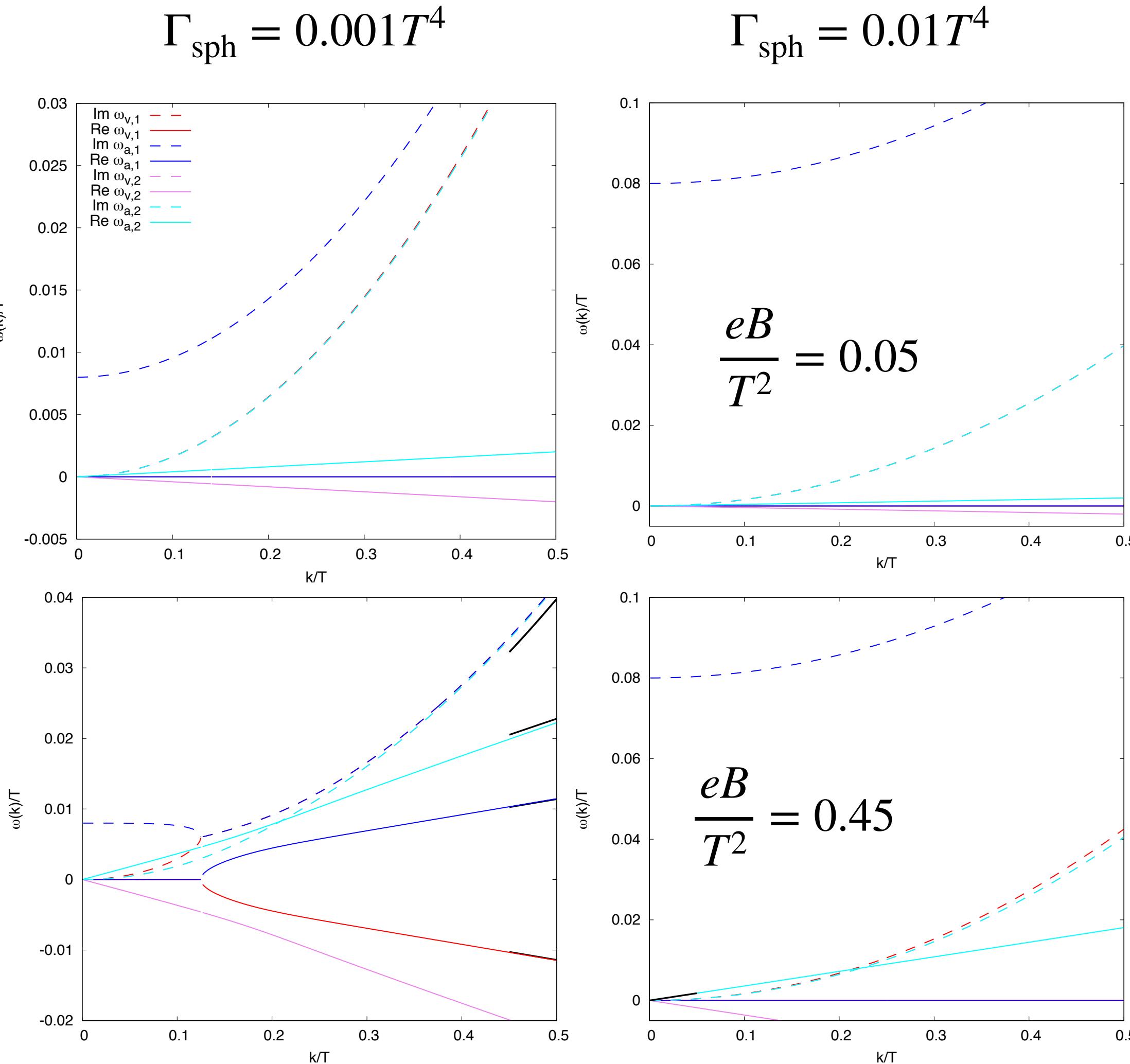


Inclusion of sphalerons leads to emergence of a characteristic wavenumber scale k_{CMW} , which signals the onset of the CMW

Dispersion relations, $N_f=2$

$$M_{ab}^{N_f=2} = \begin{pmatrix} M_{ab}^{N_f=1} \Big|_{q_f=q_u} & 0 & 0 \\ 0 & 0 & \gamma_{\text{sph}} \\ 0 & \gamma_{\text{sph}} & M_{ab}^{N_f=1} \Big|_{q_f=q_d} \end{pmatrix}$$

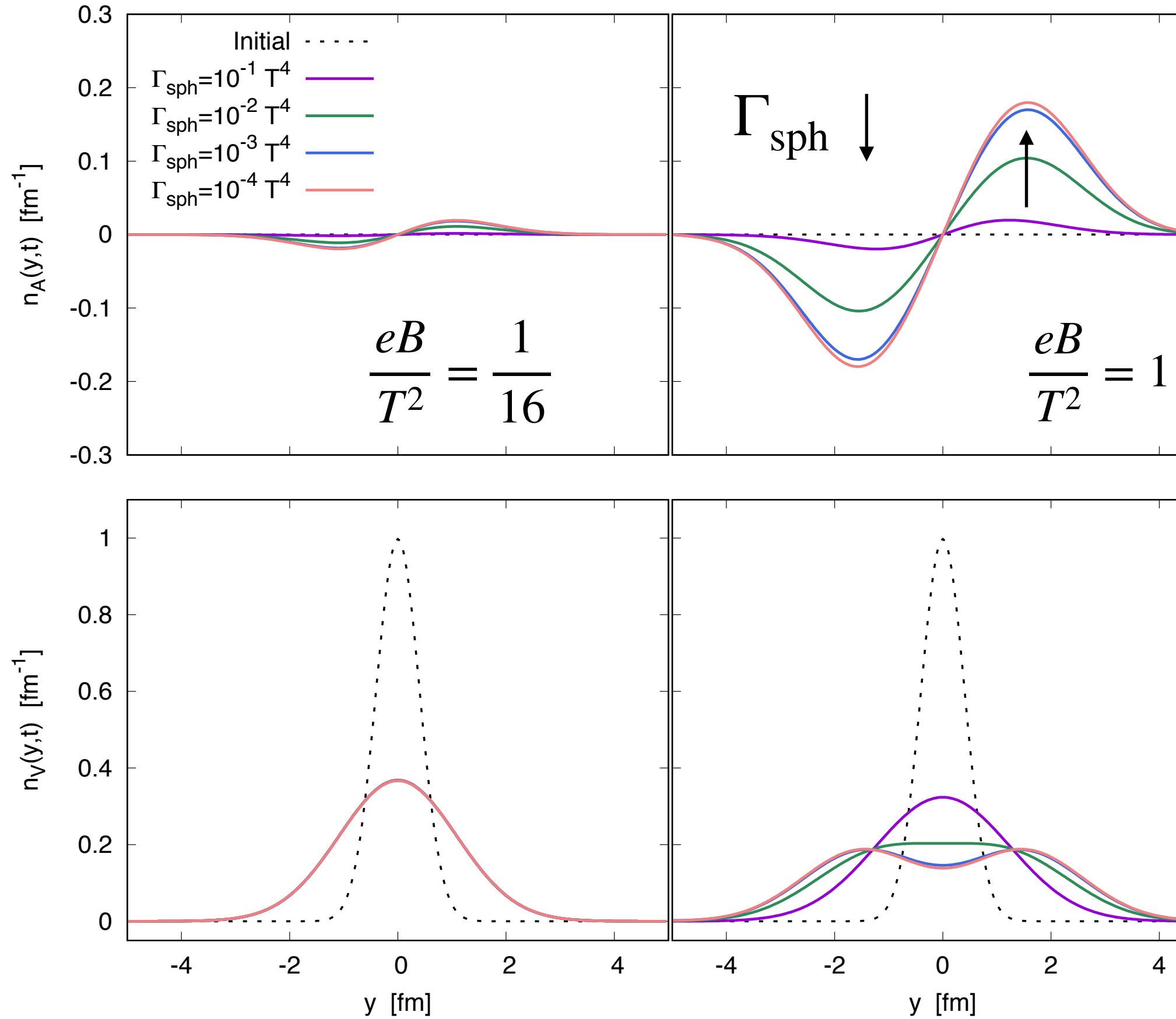
Sphaleron transitions couple quark flavor dynamics!



Charge diffusion: Numerical studies

Initial vector charge

Diffusion
insensitive to
 Γ_{sph}

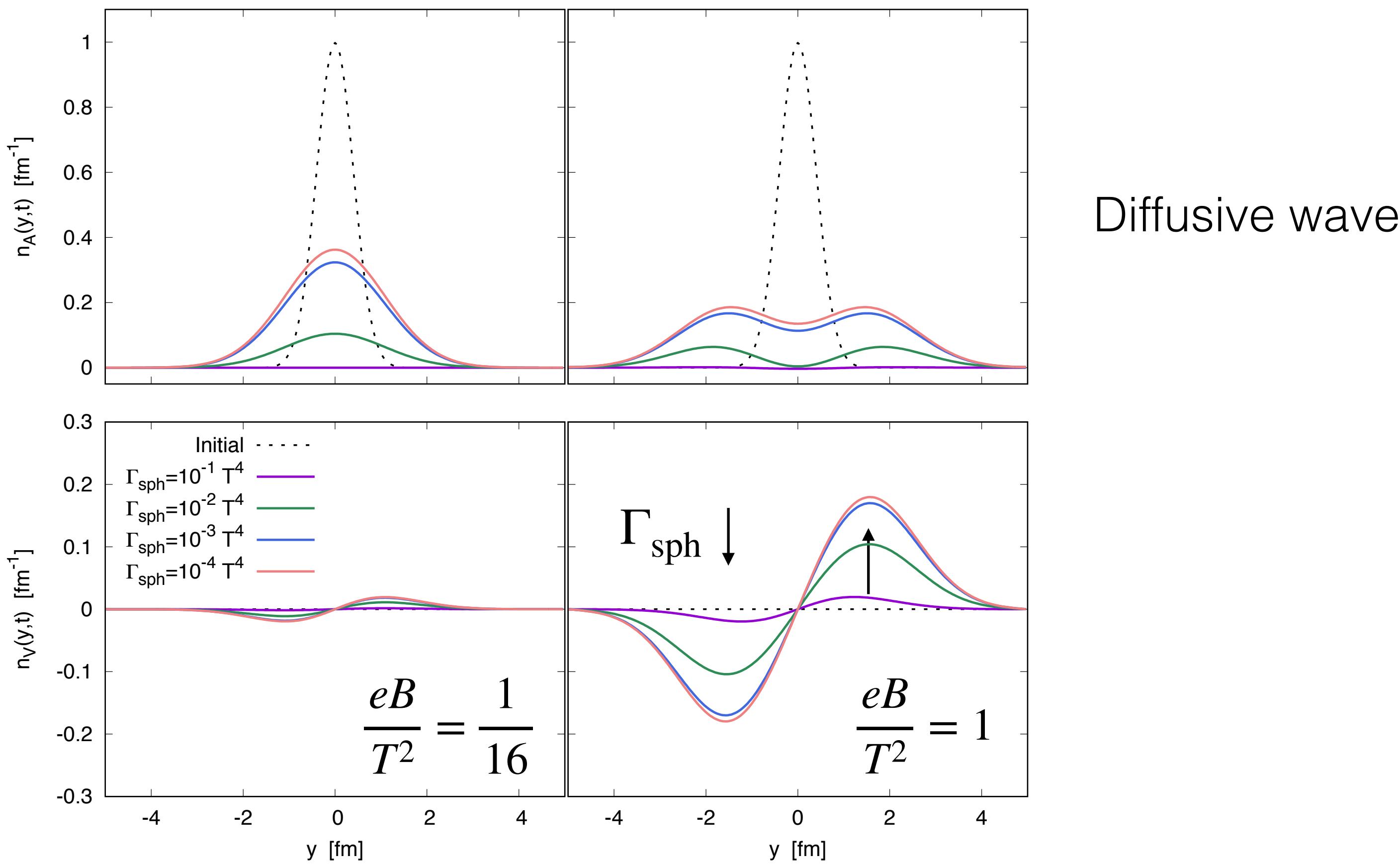


Diffusion and
diffusive wave

Sphaleron rate (if sufficiently large) impacts *vector charge* transport in
(strong) magnetic fields (*->possibility to see on the lattice?*)

Initial axial charge

Diffusion *very*
sensitive to
 Γ_{sph}



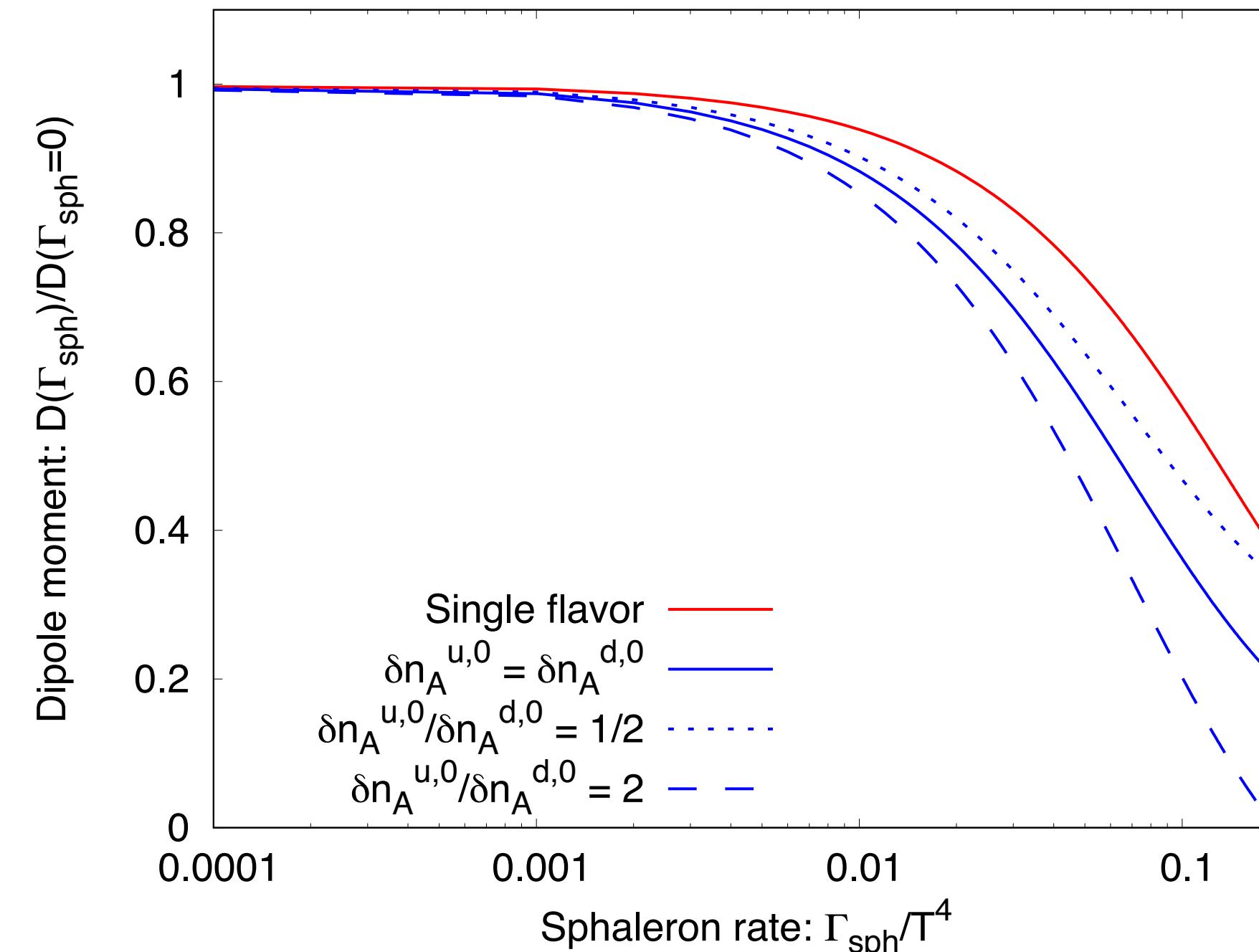
System with initial *axial charge* more sensitive to change in sphaleron rate

Quantifying charge separation

Charge separation is the indicator for chiral magnetic effect in heavy ion collisions

We can quantify charge separation via electric dipole moment

$$D(\mathbf{B}, t) = \int d^3x \frac{\mathbf{x} \cdot \mathbf{B}}{|\mathbf{B}|} \sum_f e q_f n_{V,f}(t, \mathbf{x})$$



Establishes a quantitative relationship between charge separation and sphaleron transition rate

Conclusions

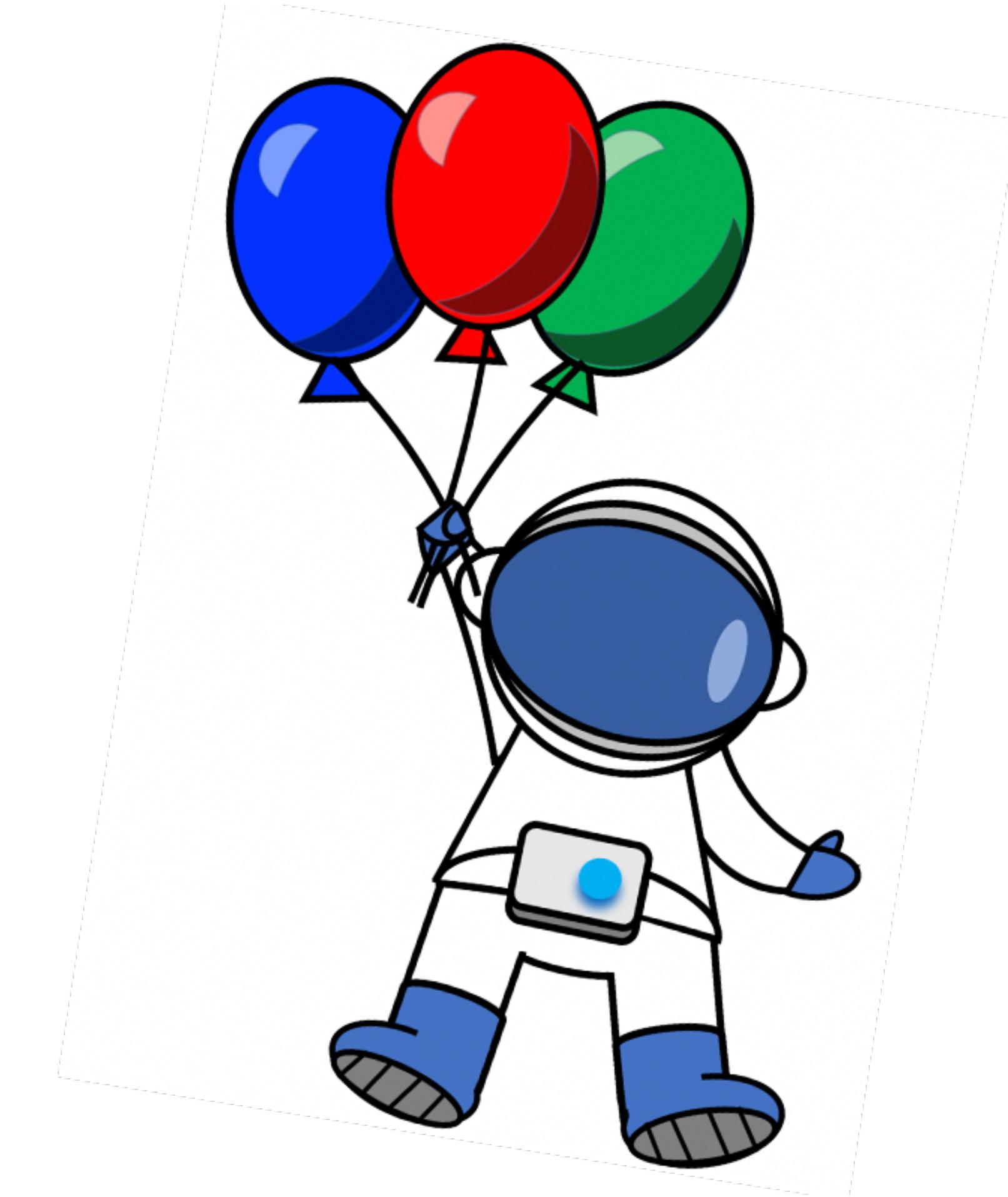
We modified the description of an anomalous relativistic plasma to include damping from sphaleron transitions

Inclusion of sphalerons leads to emergence of a characteristic wavenumber scale k_{CMW} that depends on the magnetic field strength: an indicator of chiral magnetic wave formation

Charge separation is very sensitive to sphaleron rate for both axial and vector charges

Suggests we might be able to constrain sphaleron rate experimentally with charge separation measurements

Thank you!



Backup slides

Transport coefficients

$$\sigma_{VB} = C \left(\mu_A - \frac{n_v \mu_A \mu_V}{\epsilon + P} \right)$$

$$\sigma_{AB} = C \left(\mu_V - \frac{n_A \mu_A \mu_V}{\epsilon + P} \right) + (eq_f)^{-1} \frac{\partial}{\partial \bar{\mu}_A} g(\bar{\mu}_A)$$

$$\xi_V = 2C \left(\mu_A \mu_V - \frac{n_V \mu_A \mu_V^2}{\epsilon + P} \right) + (eq_f)^{-1} g(\bar{\mu}_A)$$

$$\xi_A = C \left(\mu_V^2 - \frac{n_A \mu_A \mu_V^2}{\epsilon + P} \right) + (eq_f)^{-1} \bar{\mu}_V \frac{\partial}{\partial \bar{\mu}_A} g(\bar{\mu}_A) + \frac{\partial}{\partial \bar{\mu}_A} G(\bar{\mu}_A)$$

Diffusion on a 2-dim lattice

We solve the diffusion equation on a 2D spatial lattice (256^2) neglecting bulk expansion for a single-flavor system

We choose the following initial conditions:

- Initial charge width $\sigma = 0.4R_P$, $R_P = 1 \text{ fm}$
- System scaled with $T = 4T_C$, $T_C = 155 \text{ MeV}$
- Fix **B** in the y-direction
- Length of sides scaled to be 10 fm, $a_s=10/256$

Summary of results

For initial vector charge, vector charge diffuses and axial charge separates (and vice versa for initial axial charge)

System with initial axial charge more sensitive to change in Γ_{sph}

Charge separation for both depends on **sphaleron rate**: as sphaleron rate increases, the magnitude and distance of separation decreases

For physically realistic magnetic field strength, charge diffusion and charge separation are not as pronounced as they are for the larger magnetic field

Quantifying relationship between sphaleron rate and charge separation opens experimental chiral frontier to possibly constrain Γ_{sph}