Transverse-Momentum-Dependent (TMD) Factorization of Drell-Yan Process with Cold Nuclear Matter Effects

The 30th International Conference on Ultra-relativistic Nucleus-Nucleus Collisions, Houston, Texas, September 05, 2023

LA-UR-23-30044

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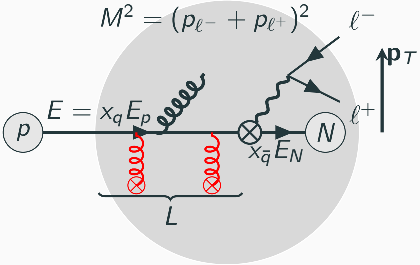
In collaboration with Ivan Vitev

Based on Ke, Vitev 2304.03302 and work in preparation.

Drell-Yan is a clean probe to study initial-state nuclear effects



Two types of nuclear effects:

- Dynamic: (1D/3D) modifications due to parton propagations in the cold nuclear matter (CNM).
- Intrinsic: (1D/3D) partonic structure of nucleon is different in a nucleus.

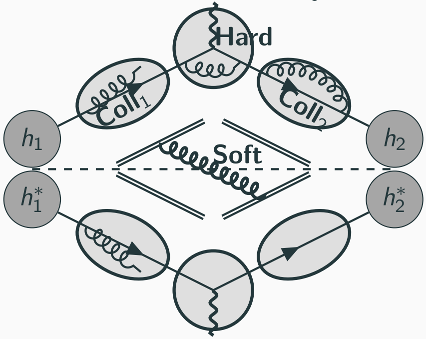
This works: How much can we understand TMD Drell-Yan in *pA* from dynamical effects?

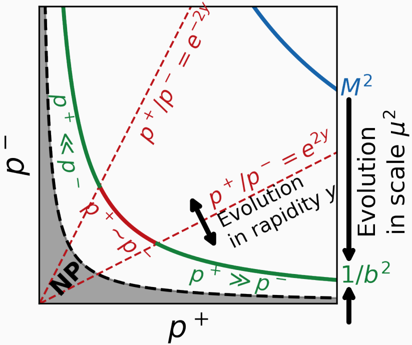
Theory framework: TMD factorization + $\mathrm{SCET}_{\mathrm{G}}$ (Soft Collinear Effective Theory with Glauber Gluon Ovaneceyan, Vitev JHEP06(2011)080).

TMD factorization of Drell-Yan in the vacuum

- When $\Lambda^2 \ll \mathbf{p}_T^2 \ll M^2$, the p_T differential DY cross-section is factorized into several sectors in the impact-parameter space ($b \longleftrightarrow p_T$) [The TMD Handbook, Boussarie et al. arXiv:2304.03302]
- Scale & rapidity evolution equations match the boundary of sectors.

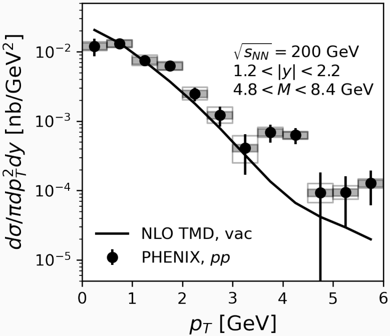
$$\frac{d\sigma}{dYdM^2 p_T dp_T} = H_{q\bar{q}}(M,\mu)\int_0^\infty bdbJ_0(p_T b)[C_{qi}\otimes f_{i/h_1}e^{-S}][C_{\bar{q}j}\otimes f_{j/h_2}e^{-S}](x_q, x_{\bar{q}}, \mathbf{b}, \mu, \zeta_q, \zeta_{\bar{q}})$$





NLO TMD calculation of $pp \to \mu^+\mu^-$

Good agreement at low p_T. At large p_T, one should match to fixed-order result (not included).



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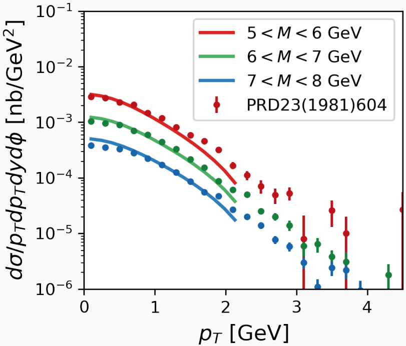
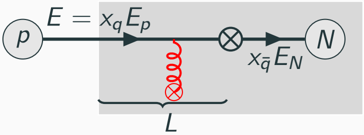


Non-perturbative inputs: CT18nlo collinear proton PDF $f_{q/p}$ PRD103(2021)014013, EPPS21 collinear nPDF EPJC82(2022)5, 413. TMD specific NP inputs from global analysis Sun, Isaacson, Yuan, Yuan, IJMPA33(2018)11, 1841006, and Echevarria, Kang, Terry JHEP01(2021)126.

LO in the CNM: pure collisional broadening



- Model the interaction with one scattering center

$$\Sigma_G(b) = g_s^2 C_F \int \frac{d^2\mathbf{q}}{(2\pi)^2} \frac{1}{(\mathbf{q}^2 + \xi^2)^2} \left(e^{i\mathbf{b}\cdot\mathbf{q}} - 1\right)$$

Quark broadening from multiple scatterings

$$f_{q/p} e^{-S} \to f_{q/p} e^{-S} e^{\rho_G L \cdot \Sigma_G(b)}$$

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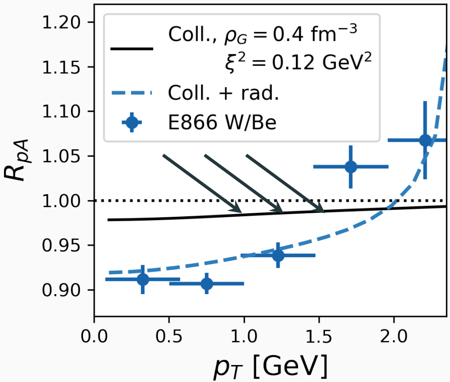

LO is inadequate.

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$$\Sigma_G(b) = g_s^2 C_F \int \frac{d^2\mathbf{q}}{(2\pi)^2} \frac{1}{(\mathbf{q}^2 + \xi^2)^2} \left(e^{i\mathbf{b}\cdot\mathbf{q}} - 1\right)$$

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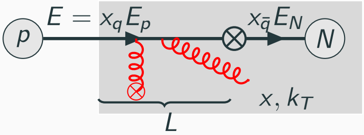
- CNM parameters determined from modified fragmentation functions in eA: $\rho_G \approx 0.4$ fm^{-3}, $\xi^2 \approx 0.12$ GeV^2 Ke, Vitev 2301.11940.

Radiative corrections: power counting in thin/dilute medium

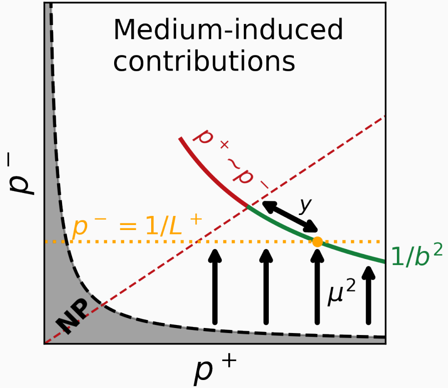


Consider a large separation of scales

$$\Lambda^2, \xi^2, \alpha_s \rho_G L \ll \mathbf{k}_T, E/L \ll M^2$$

Medium-induced modes, $\lambda = \frac{1}{\sqrt{EL}} \sim \frac{\mathbf{k}_T}{E}$

- Proton-collinear: $p^\mu \sim E(1, \lambda^2, \lambda)$, $e^{ip^- L^+}$ slowly varies, sensitive to medium size (the LPM effect).
- Soft: $p^\mu \sim E(\lambda, \lambda, \lambda)$. $e^{ip^- L^+}$ highly oscillating $\to 0$.



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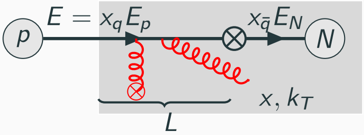

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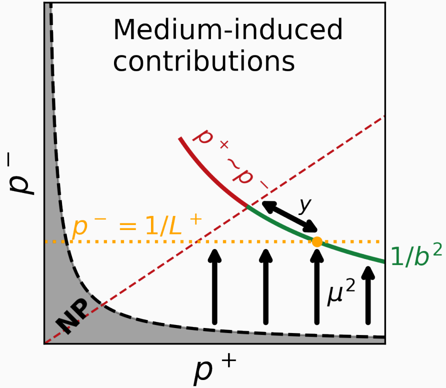
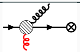
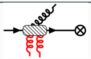
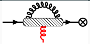
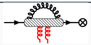
What's not included:

- No target evolution: ρ_G, ξ^2 are constant parameters.
- Ignore power correction to hard mode $\sim \xi^2/M^2$.



Radiative corrections: medium-induced collinear sector at NLO

$$C_{qq}^{\mathrm{med}}(x,b) = \int \frac{d^{2-2\epsilon}\mathbf{q}}{(2\pi)^{2-2\epsilon}} \frac{g_s^2 \rho_G L}{(\mathbf{q}^2+\xi^2)^2} \int \frac{d^{2-2\epsilon}\mathbf{k}}{(2\pi)^{2-2\epsilon}} g_s^2 C_F$$
$$\left\{ P_{qq}(x) \left[e^{i\mathbf{b}\cdot(\mathbf{k}-\mathbf{q})} \mathfrak{J}_{RR} + e^{i\mathbf{b}\cdot\mathbf{k}} \mathfrak{J}_{RV} \right] + \delta(1-x) \int_0^1 P_{qq}\, dx' \left[e^{-i\mathbf{b}\cdot\mathbf{q}} \mathfrak{J}_{VR} + \mathfrak{J}_{VV} \right] \right\}$$

$\mathfrak{J} = C_F J_F + C_A J_A$	Diagrams	$J_F(x,\mathbf{k},\mathbf{q})$	$J_A(x,\mathbf{k},\mathbf{q})$
RR		$\frac{1}{\mathbf{A}^2} + 2\frac{\mathbf{B}}{\mathbf{B}^2}\cdot\left(\frac{\mathbf{B}}{\mathbf{B}^2} - \frac{\mathbf{A}}{\mathbf{A}^2}\right)\phi_B$	$\frac{1}{\mathbf{C}^2} - \frac{\mathbf{A}\cdot\mathbf{C}}{\mathbf{A}^2\mathbf{C}^2} + \frac{\mathbf{B}}{\mathbf{B}^2}\cdot\left(\frac{\mathbf{A}}{\mathbf{A}^2} - \frac{\mathbf{C}}{\mathbf{C}^2}\right)\phi_B$
RV		$-\frac{1}{\mathbf{A}^2}$	$\frac{\mathbf{A}}{\mathbf{A}^2}\cdot\left(\frac{\mathbf{A}}{\mathbf{A}^2} - \frac{\mathbf{C}}{\mathbf{C}^2}\right)(\phi_A - 1)$
VR		$-2\frac{\mathbf{B}}{\mathbf{B}^2}\cdot\left(\frac{\mathbf{B}}{\mathbf{B}^2} - \frac{\mathbf{A}}{\mathbf{A}^2}\right)\phi_B - \frac{1}{\mathbf{A}^2}$	$-\frac{\mathbf{A}\cdot\mathbf{B}}{\mathbf{A}^2\mathbf{B}^2}\phi_B + \frac{\mathbf{A}\cdot\mathbf{D}}{\mathbf{A}^2\mathbf{D}^2}\phi_D$
VV		$\frac{1}{\mathbf{A}^2}$	$-\frac{1}{\mathbf{A}^2}\phi_A + \frac{\mathbf{A}\cdot\mathbf{C}}{\mathbf{A}^2\mathbf{C}^2}\phi_C$

$\mathbf{A} = \mathbf{k}$, $\quad \mathbf{B} = \mathbf{k} - (1-x)\mathbf{q}$, $\quad \mathbf{C} = \mathbf{k} - \mathbf{q}$, $\quad \mathbf{D} = \mathbf{k} + x\mathbf{q}$, $\quad \phi_{\mathbf{V}} = 1 - \mathrm{sinc}\left(\mathbf{V}^2 L/[2x(1-x)E]\right)$

Still complicated, but we can perform power expansion in $v = \frac{\xi^2}{E/L} \ll 1$.

1. Induced collinear divergences (after power expansion)

- Divergence absorbed into the collinear PDF: $C_{qq}^{\rm med} \otimes xf_{q/p}(x,\mu^2)$.

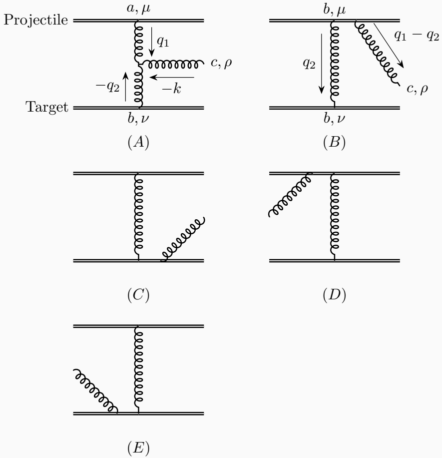
$$C_{qq}^{\rm med}(x,b) \supset \frac{\alpha_s^2 \rho_G L^2}{8E/L} \frac{P_{qq}^{(0)}(x)}{[x(1-x)]^{1+2\epsilon}} \cdot B(\frac{L}{b^2 2E},\epsilon) \left[\frac{\mu^2 L}{\chi E}\right]^{2\epsilon} (\cdots)$$

- Lead to the in-medium evolution of parton density via a set of PDEs Ke, Vitev 2301.11940. For flavor non-singlet, it encodes parton energy loss and $q \to g$ conversion

$$\left[\frac{\partial}{\partial\tau(\mu^2)} - 4C_F C_A \frac{\partial}{\partial z} + \frac{2C_F(1+\frac{C_F}{2C_A})}{z}\right] xf(\tau,z) = 0$$

- The natural scale determines the upper bound of evolution $\xi^2 < \mu^2 < \min(1/b^2, \chi E/L)$. **Small $1/b \sim p_T$ parton suffers less energy loss!**

2. Induced rapidity divergence



+ VR + RV + VV

- Rapidity divergence cancels among collinear & soft sectors. Lead to the BFKL evolution of the broadening factor. [Fleming PLB735(2014)266; Rothstein, Stewart, JHEP08(2016)025; Vaidya 2107.00029, 2109.11568]

$$\frac{\partial V_G(\mathbf{b}, y)}{\partial y} = \frac{\alpha_s C_A}{\pi^2} \left\{ \int_{|\mathbf{b}-\mathbf{b}'|<|\mathbf{b}|} d^2\mathbf{b}' \, \frac{V_G(\mathbf{b}') - V_G(\mathbf{b})}{|\mathbf{b}-\mathbf{b}'|^2} + \int_{|\mathbf{b}-\mathbf{b}'|>|\mathbf{b}|} d^2\mathbf{b}' \, \frac{V_G(\mathbf{b}')}{|\mathbf{b}-\mathbf{b}'|^2} \right\}, \quad \Sigma_G(b, y) = \frac{-1}{\nabla_b^2} V_G(b, y)$$

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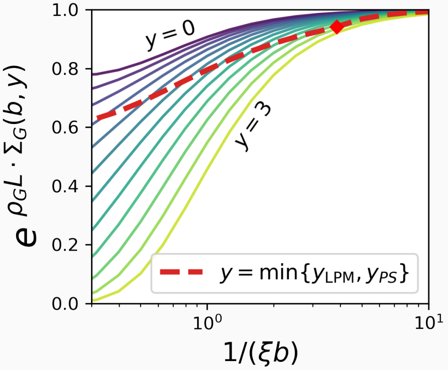

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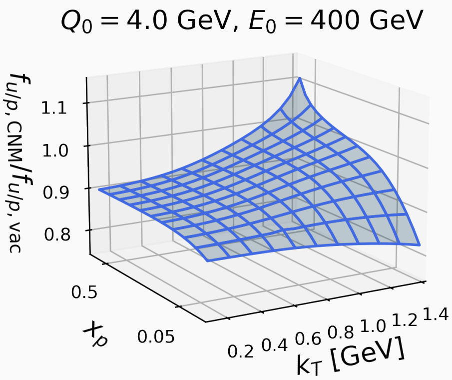
- Σ_G evolves rapidly, but the LPM suppression limits the range of evolution to $y = \min\{y_{\rm LPM},\ y_{PS}\}$

$$y_{\rm LPM} = \ln\left(1 + \frac{r_0'(A^{1/3} - 1)}{2b}\right), \quad y_{\rm PS} = \ln(Eb)$$

⋆ With initial condition

$$\Sigma_G(\mathbf{b}, 0) = g_s^2 C_F \int \frac{d^2\mathbf{q}}{(2\pi)^2} \frac{e^{i\mathbf{b}\cdot\mathbf{q}} - 1}{(\mathbf{q}^2 + \xi^2)^2}$$

Medium-evolved TMD distribution of incoming parton



$$C_{qq} \otimes f_{q/p} e^{-S} e^{\rho_G L \Sigma_G(b)}$$
$$\Longrightarrow \left(C_{qq} + C_{qq}^{\mathrm{med,finite}}\right) \otimes f'_{q/p} e^{-S} e^{\rho_G L \Sigma_G(b,y)}$$

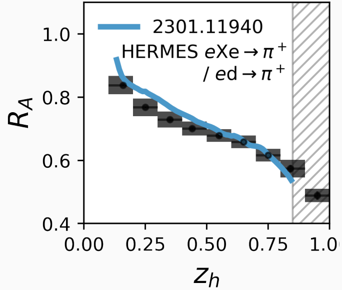
- $f'_{i/p}(x, \mu = 1/b)$ evolved with CNM correction.
- $\Sigma_G(b, y)$ BFKL evolved with LPM cut off.
- Finite correction $C_{qq}^{\mathrm{med,finite}}$ **not** included yet, important for scale uncertainty quantification.

△ At large x_q: a k_T broadening.

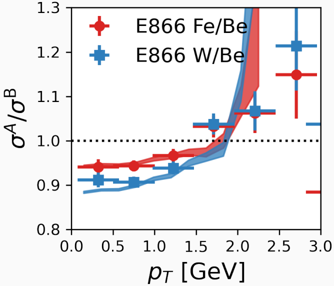
△ At small enough x_q, energy loss (increases with k_T) overcomes the broadening.

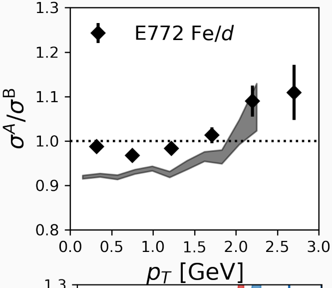
A consistent set of CNM input for DIS and DY

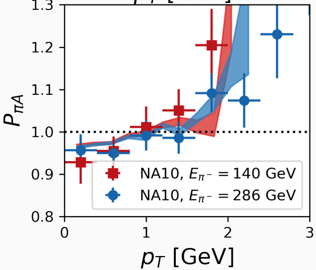
Nuclear effects in SIDIS



Nuclear effects in Drell-Yan p_T spectra







Same set of CNM inputs: $\rho_G = 0.4$ fm^{-3}, $\xi^2 = 0.12$ GeV

Uncertainty quantification is possible in the near future.

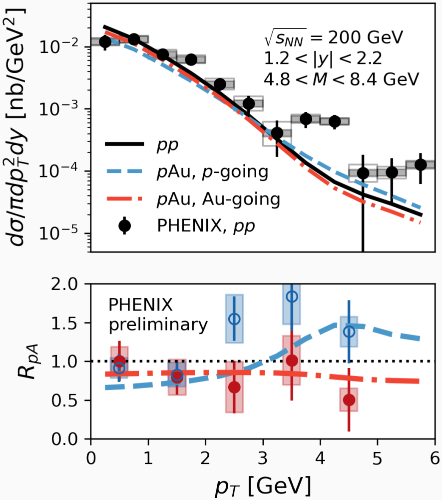
HERMES, NPB 780(2007)1-27

FNAL E866/NuSea Collab. PRL83(1999)2304-2307

E772, PRL64(1990)2479

NA10 Collab. PLB193(1987)373-375

For collider experiments



- p-going side: fractional energy loss is small, mostly broadening effect.
- Au-going side: broadening + large CNM energy loss (grows with k_T) $\Rightarrow$ R_{pA} flat/slight decreasing with p_T.
- Large exp uncertainty, very challenging measurement. Hope the sPHENIX experiment can update the measurement in p-Au or Au-Au.
- For LHC energy, see backup slides.

PHENIX pp: PRD99(2019)072003.

Summary

- Medium correction to TMD Drell-Yan using $\mathrm{SCET}_{\mathrm{G}}$.
- At NLO, opacity one, and leading power in $\xi^2 L/E$,
  - Induced collinear divergence $\Rightarrow$ medium evolved collinear PDF.
  - Induced rapidity divergence $\Rightarrow$ BFKL evolution of the broadening factor.
- Using the same CNM parameters from SIDIS in eA, dynamical calculations provide a good description of the p_T differential Drell-Yan data in pA.
- Future: generalization to TMD hadron productions in eA and pA.
  Improve the baseline calculation for searching jet quenching in pA e.g. Vitev's Talk on Wednesday, Symall Systems 11:20.
  Applied to the determination of intrinsic nuclear NP effects e.g. Alrashed, Anderle, Kang, Terry, Xing PRL129(2022)242001

Questions?

In-medium evolution of collinear PDF

- Medium-induced collinear divergences show up at endpoints of the emission spectra

$$\Delta F_{\mathrm{NS}}^{\mathrm{med}}(z) = \int_z^1 \frac{dx}{x} F_{\mathrm{NS}}\left(\frac{z}{x}\right) P_{qq}^{\mathrm{med}(1)}(x) \quad + \quad \text{virtual term.}$$

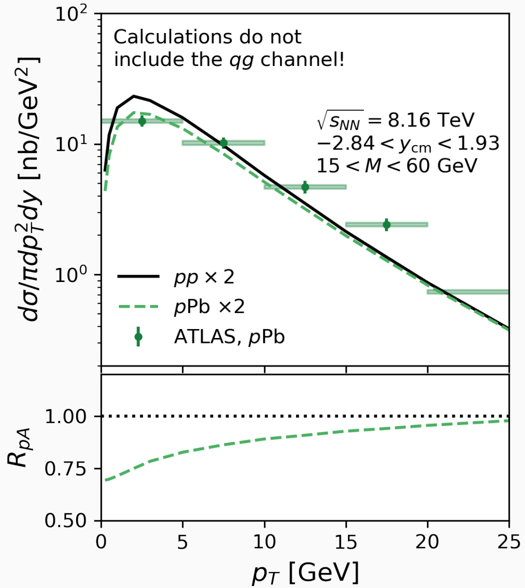
$$P_{qq}^{\mathrm{med}(1)}(x) = \frac{\alpha_s^2 B(\frac{Q^2 L}{2E})\rho_G L^2}{8E/L} \frac{P_{qq}^{vac(0)}(x)}{[x(1-x)]^{1+2\epsilon}} \cdot \left[\frac{\mu^2 L}{\chi z \nu}\right]^{2\epsilon} \cdot C_n \Delta_n(x)(1+\mathcal{O}(v))$$

- They can be regulated using dimension regularization ($d = 4 - 2\epsilon$),

$$\Delta F_{\mathrm{NS}}(z) = \frac{\alpha_s^2 B(\frac{Q^2 L}{2E})\rho_G L^2}{8E/L} \left(\frac{1}{2\epsilon} + \ln \frac{\mu^2 L}{\chi z \nu}\right) 2C_F \left[2C_A\left(-\frac{d}{dz} + \frac{1}{z}\right) + \frac{C_F}{z}\right] F_{\mathrm{NS}}(z) + \cdots$$

- For collinear observable μ^2 will be evolved to E/L.
  For TMD observables, it becomes $\min\{E/L, 1/b^2\}$

Compare to ATLAS Drell-Yan data in pPb



- Currently, we only included $q\bar{q} \to \gamma^* \to \ell^+\ell^-$ channel.
- A factor of 2 discrepancies with the ATLAS data.
- To improve:
  - Include $C_{qg} \otimes f_{g/p}$ contributions.
  - γ^*/Z exchange for high mass Drell-Yan.