Adiabatic hydrodynamization in the 'bottom-up' thermalization scenario

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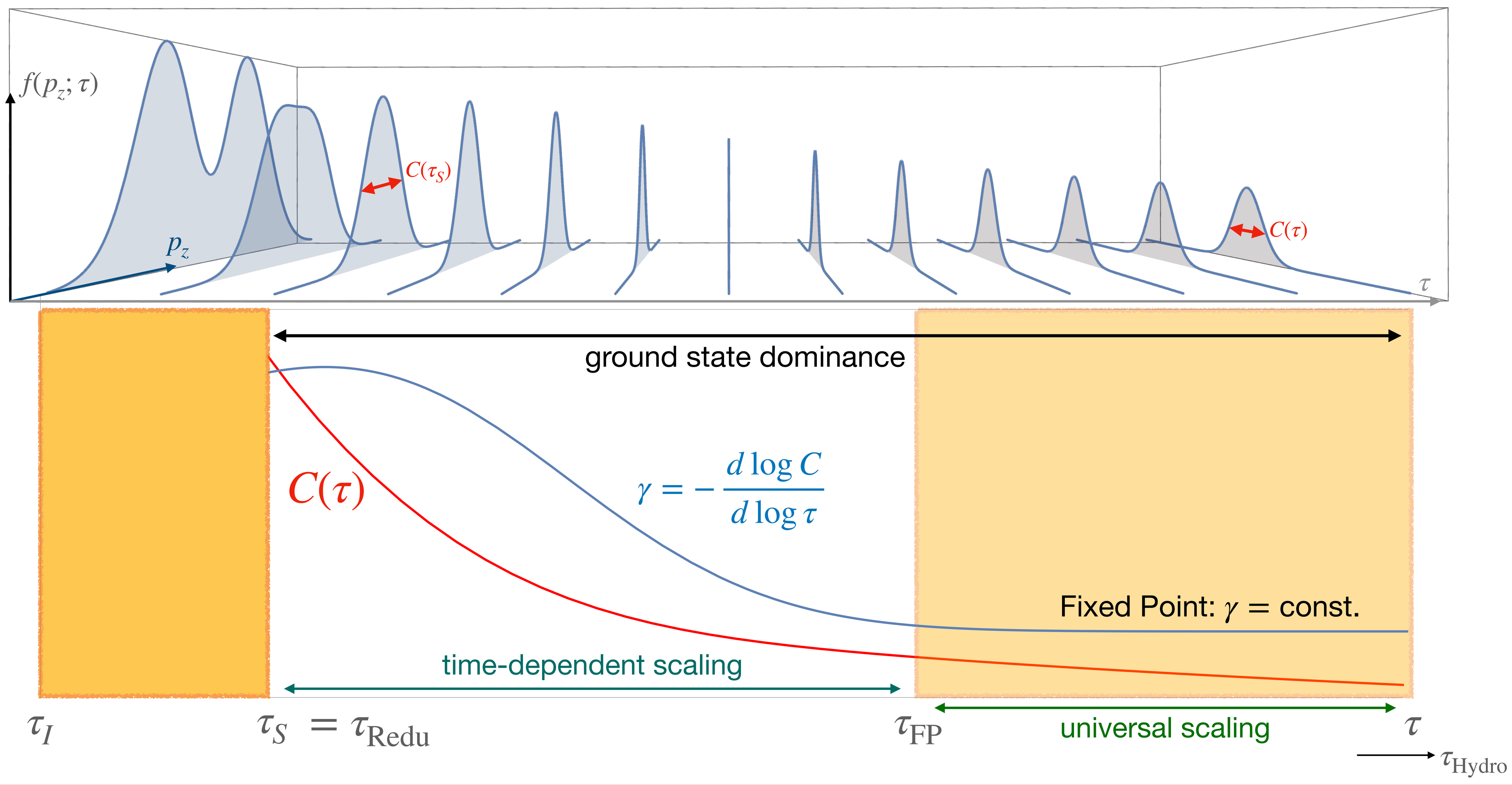
Typical time evolution of the gluon occupation number in a weakly-coupled Bjorken-expanding plasma at early times



If you want to learn more:

Summary

- The bottom-up thermalization scenario [1] has been central to our understanding of hydrodynamization in the effective kinetic theory of QCD.
- In practice, studying this process has required intensive numerical calculations, and emergent structure has been revealed by doing so.
- We now have an intuitive framework [2] to understand the bottom-up thermalization scenario, which explains:
  1. why this out of equilibrium gas of quarks and gluons quickly falls into an attractor solution, and
  2. the deviations from the original BMSS scaling exponents on the early-time attractor of the bottom-up scenario.

Adiabatic hydrodynamization

Consider a system described by the evolution equation

$$\partial_\tau|\psi\rangle = -H(\tau)|\psi\rangle\,. \tag{1}$$

$H(\tau)$ has instantaneous eigenstates $|n(\tau)\rangle$ with eigenvalues $E_n(\tau)$.

If adiabaticity $\frac{\dot a_n}{a_n} \ll |E_n - E_m|\ \forall m \neq n$ holds, then the ground state will dominate after a transient time:

$$|\psi\rangle = \sum_{n=0}^{\infty} a_n(\tau) e^{-\int^\tau E_n(\tau')d\tau'}|n(\tau)\rangle \approx a_0\, e^{-\int^\tau E_0(\tau')d\tau'}|0(\tau)\rangle\,, \tag{2}$$

thus reducing the number of active degrees of freedom of the system $\Longrightarrow$ We can understand the emergence of attractors!

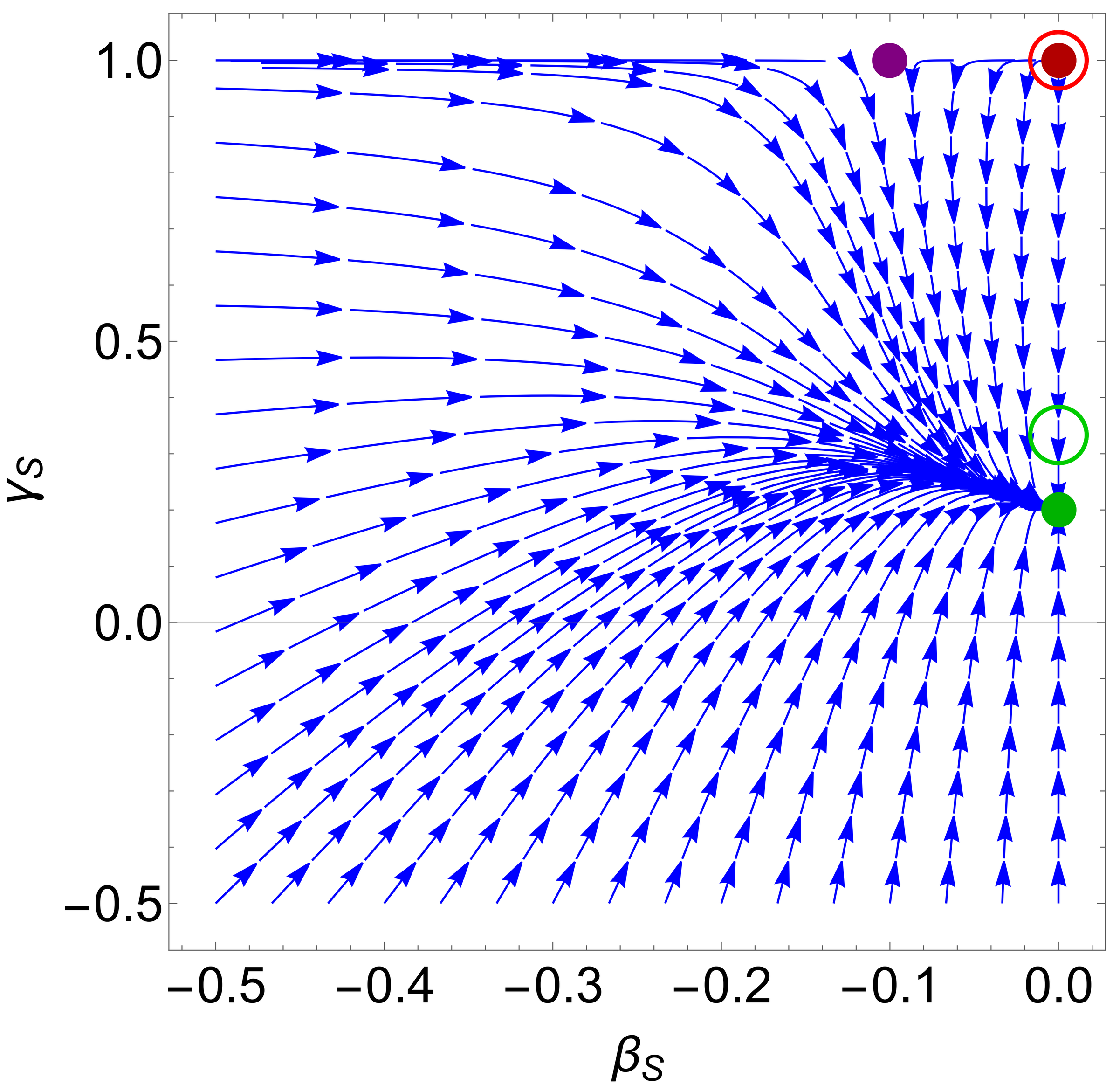
For more on this see Rachel Steinhorst's talk (Initial State session, Sep 5 5:10 pm), as well as [3].

Results: Scaling exponents

The attractor of this early-time theory has a scaling form:

$$f(p_\perp, p_z, \tau) = e^{\int^\tau \alpha(\tau')\,\mathrm{d}\ln\tau'} f_S\left(e^{\int^\tau \beta(\tau')\,\mathrm{d}\ln\tau'} p_\perp,\, e^{\int^\tau \gamma(\tau')\,\mathrm{d}\ln\tau'} p_z\right)\,. \tag{3}$$

This theory has the sum rule $\alpha = \gamma + 2\beta - 1$. See the figure below for the dynamical evolution of γ, β.



The above figure assumes $f \gg 1$ and $\dot\ell_{\mathrm{Cb}} = 0.4$, as explained in [2]. See [4] for an alternative approach on flow equations for early-time kinetic theory.

Early-time kinetic theory

We use the small-angle scattering approximation of the gluon collision kernel [5], neglecting number-changing processes. Furthermore, we assume that $\langle p_z^2\rangle \ll \langle p_\perp^2\rangle$ and $f \gg 1$, with which the kinetic theory simplifies to

$$\partial_\tau f - \frac{p_z}{\tau}\partial_{p_z} f = 4\pi\,\alpha_s^2 N_c^2 \ell_{\mathrm{Cb}}[f] I_a[f] \nabla_{\mathbf{p}}^2 f \tag{4}$$

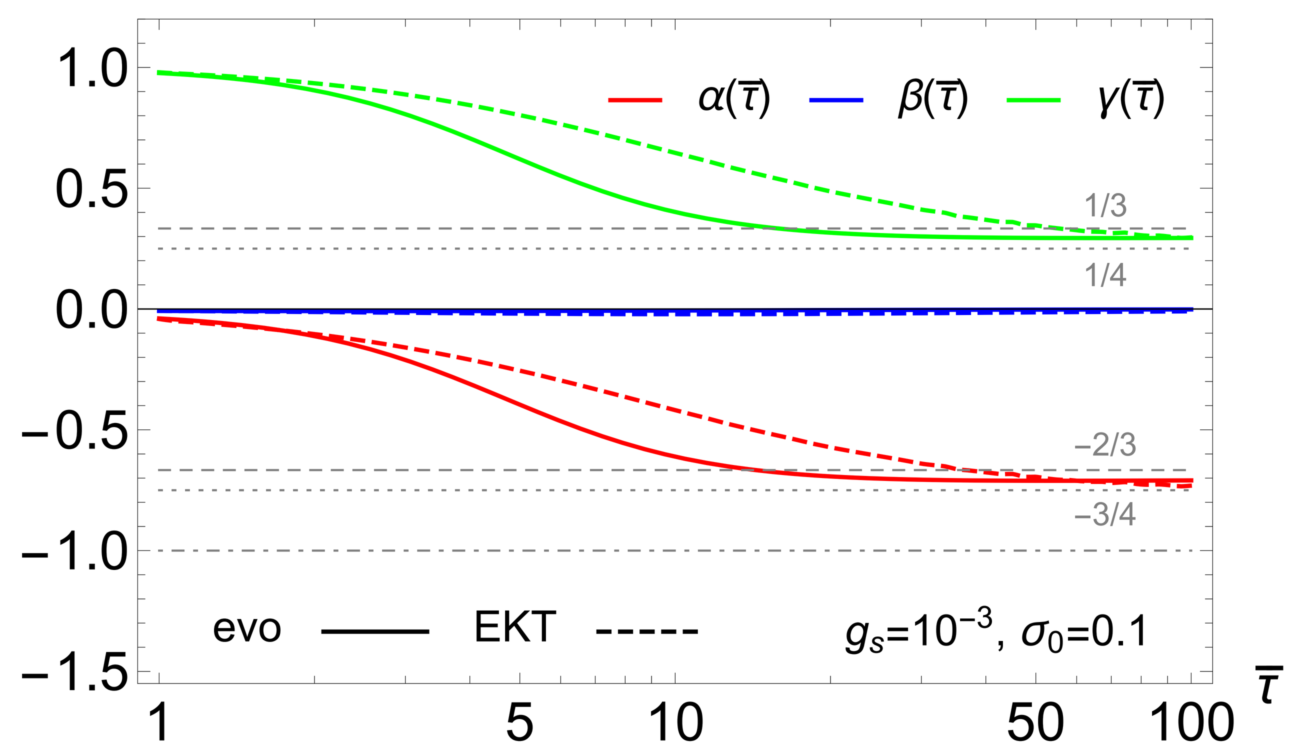
where $I_a[f] = \int_{\mathbf{p}}(1+f)f$, $\ell_{\mathrm{Cb}}[f] = \ln\left(\frac{p_{\mathrm{UV}}}{p_{\mathrm{IR}}}\right) \approx \frac{1}{2}\ln\left(\frac{\langle p_\perp^2\rangle}{m_D^2}\right)$.

We proceed by finding the instantaneous ground state of (4), and use it to solve for the scaling exponents of the attractor of the first stage of 'bottom-up' thermalization [1]. See [2] for details.

Results: Comparison with QCD effective kinetic theory

The scaling exponents provide a qualitative description of QCD effective kinetic theory and reproduce its late-time behavior [6].

Flow equations [2] (solid) versus QCD EKT [6] (dashed)



For these initial conditions, $f(\tau_I) = (\sigma_0/g_s^2)\exp\left(-(p_\perp^2 + \xi^2 p_z^2)/Q_s^2\right)$, both theories agree that $\gamma \approx 0.29$ at $\bar\tau = 100$. This logarithmic correction to the time dependence of the scaling exponents has been further verified more generally in [7].

References

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