Towards self-consistent spectral properties of quarks and mesons in a chiral quark model

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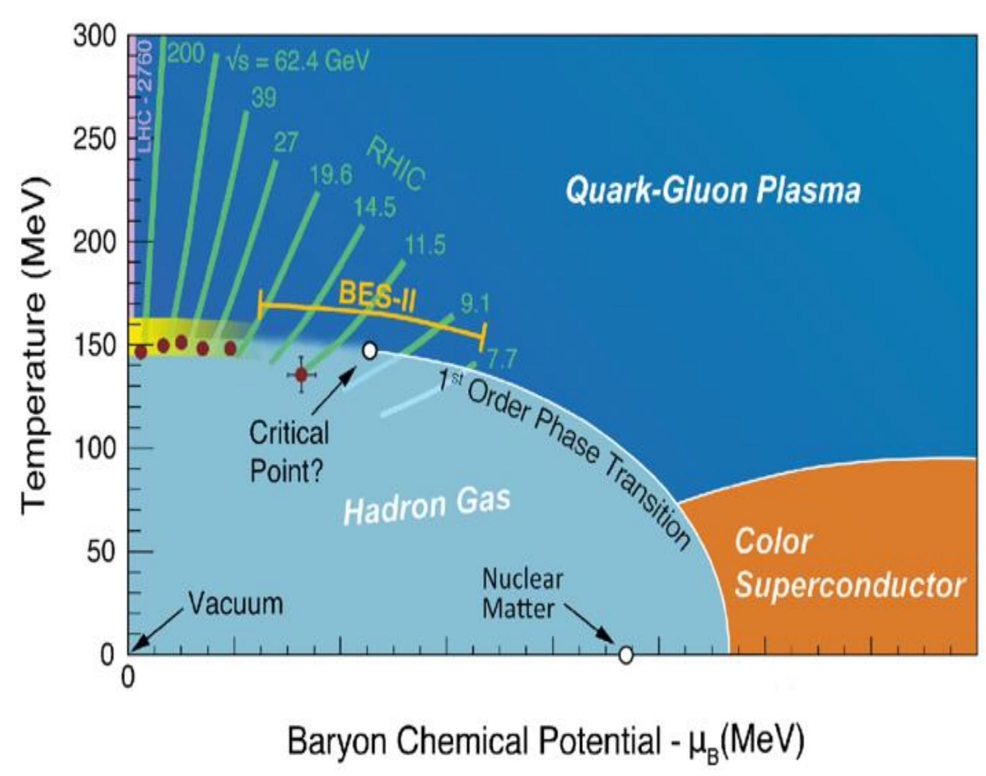
1. Motivation

- Explore the QCD phase diagram to understand whether a hot and dense system of nuclear matter displays critical phenomena.
- Lattice QCD simulations suggest an analytic crossover from hadronic matter to a hot quark gluon plasma at vanishing baryon chemical potential.
- To study the quasiparticle excitations near the chiral transition temperature, we require a self consistent approach to quarks and gluons.



2. Nambu - Jona-Lasinio Model

- The standard NJL model incorporates the chiral symmetry of two-flavor QCD, its breakdown and restoration at high T and μ. The Lagrangian of the NJL model is:

$$\mathcal{L}_{NJL} = \bar{q}\left(i\gamma^{\mu}\partial_{\mu} - \hat{m}\right)q + G_1[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2]$$

- The gluonic degrees of freedom are "integrated out" and replaced by a local four-point interaction of quark color currents. The thermodynamic potential for NJL is as follows:

$$\Phi = \frac{(m - m_0)^2}{4G_1} - 2N_c N_f \int_{\Lambda} \frac{d^3p}{(2\pi)^3} E_p - 2N_f T \int_{\Lambda} \frac{d^3p}{(2\pi)^3} 2e^{-\frac{E_p}{T}}$$

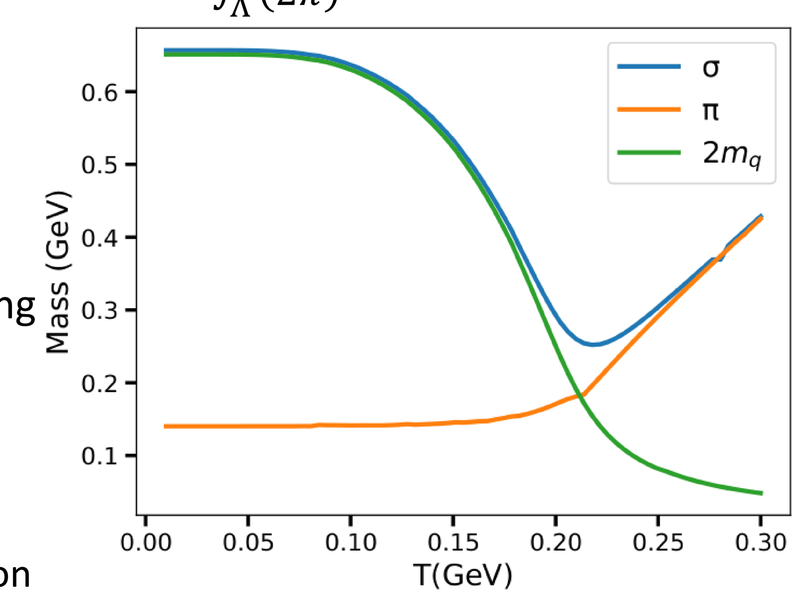
- Minimizing the thermodynamic potential with respect to mass gives the gap equation:

$$\frac{d\Phi}{dm} = 0$$

- The meson propagators in the NJL model in the ring approximation, obtained by the explicit solutions of the Dyson Schwinger equations, are as follows:

$$D_M(\omega, q) = \frac{-2G_1}{1 - 2G_1 \Pi_M(\omega, q)}$$

The pole of the propagator gives the mass of the meson $M = \pi, \sigma$ and Π_M is the meson polarization operator.



3. Pion Polarization Operator

- The quark propagator in the Hartree case is:

$$G(p_0, p) = \frac{1}{p_\mu\gamma^\mu - m_q + i\varepsilon\,\mathrm{sgn}(p_0)} \quad \text{and } G = \gamma^0 G_0 + \vec{\gamma}\frac{\vec{p}}{p}G_V + G_m$$

Using this, we get the three components of the quark propagator as follows:

$$G_0(p_0, p) = \frac{p_0}{p_0^2 - p^2 - m_q^2 + i\epsilon\,\mathrm{sgn}(p_0)},\ G_V(p_0, p) = \frac{p}{p_0^2 - p^2 - m_q^2 + i\epsilon\,\mathrm{sgn}(p_0)},\ G_m(p_0, p) = \frac{m_q}{p_0^2 - p^2 - m_q^2 + i\epsilon\,\mathrm{sgn}(p_0)}$$

The pion polarization operator is given as:

$$\Pi_P(\omega, \vec{q}) = \mathrm{T}\sum_{\omega_n}\int_P \mathrm{Tr}\,\{G(\omega_n, \vec{p})\Gamma_P G(\omega_n - \epsilon_k, \vec{p} - \vec{q})\Gamma_P\}$$

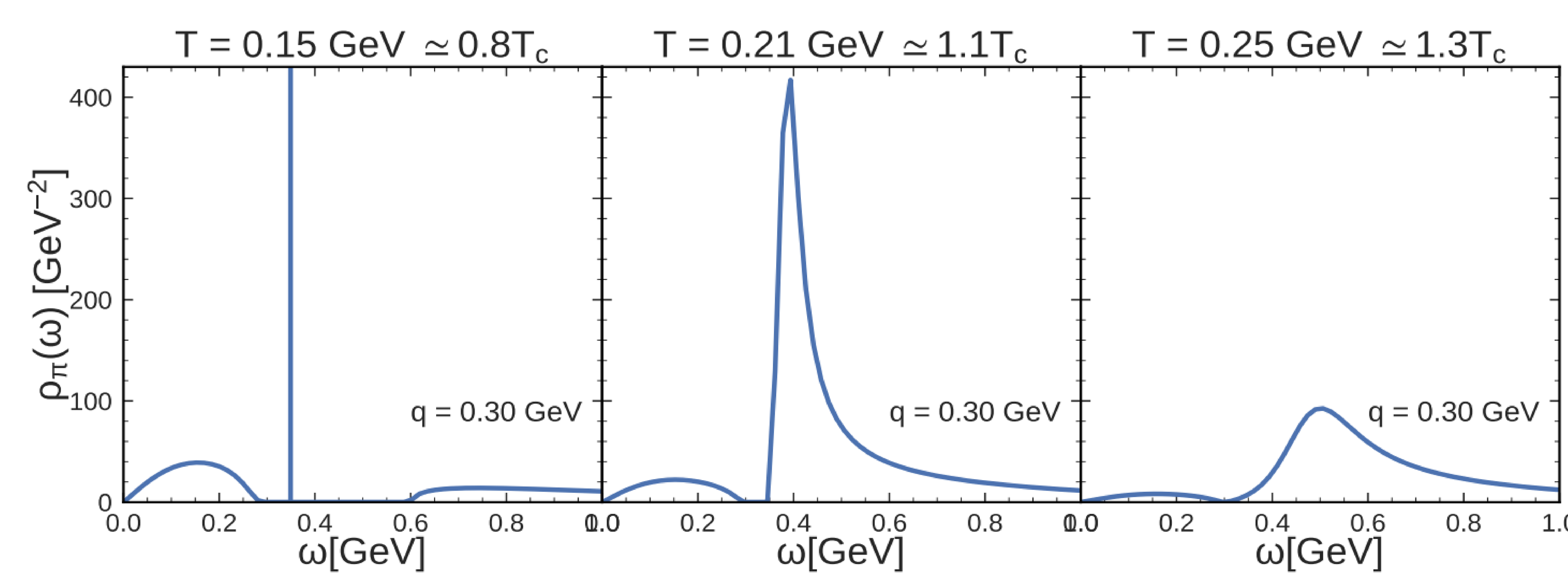
The imaginary part of the pion polarization operator in terms of quark propagator is as follows:

$$\mathrm{Im}\,\Pi(\omega, \vec{q}) = \int_{-\lambda}^{\lambda} dz \int_{-1}^{1} dx \int_0^{\lambda} \frac{N_c N_f dp}{\pi^3}\{p^2 \mathrm{Im}G_0(p,z)\mathrm{Im}G_0(p, z - \omega) - R\,\mathrm{Im}G_v(p,z)\mathrm{Im}G_v(p, z - \omega) - p^2\mathrm{Im}G_m(p,z)\mathrm{Im}G_m(p, z - \omega)\}f(T)$$

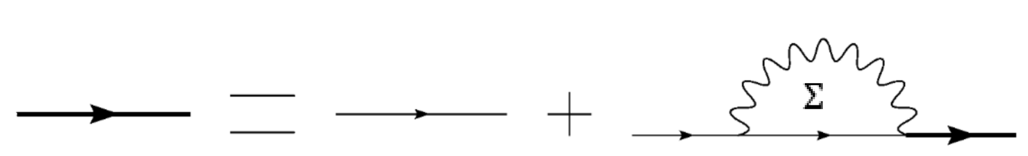
Here $f(T) = \tanh\left(\frac{\omega - z}{2T}\right) + \tanh\frac{z}{2T}$ and Λ is the momentum cutoff and $R = \frac{p(p^2 - pqx)}{\sqrt{p^2 + q^2 - 2pqx}}$

4. Pion Spectral Function and Landau Damping

- The pion spectral function as a function of energy at below, near and above the critical temperature of chiral phase transition.
- At $T = 0$, the pion would be a bound state and its spectral function would be a δ function.
- $T > T_c$: $m_\pi > 2m_q$ – decrease of the $\pi \to \bar{q}q$ threshold, pion dissociation.
- At $\omega < q$, interaction with quark heat bath leads to excitations in spacelike region (Landau damping).

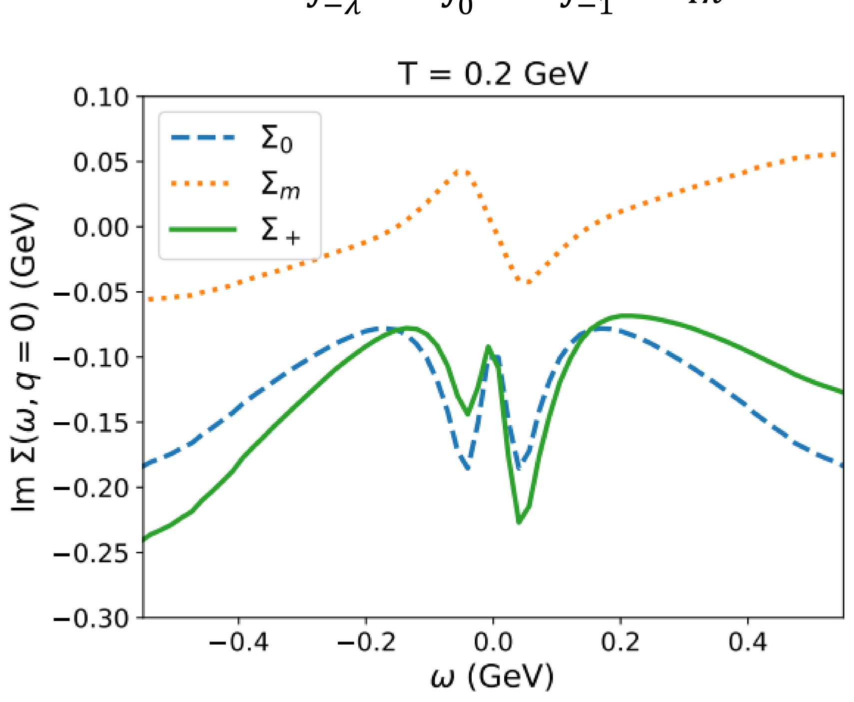


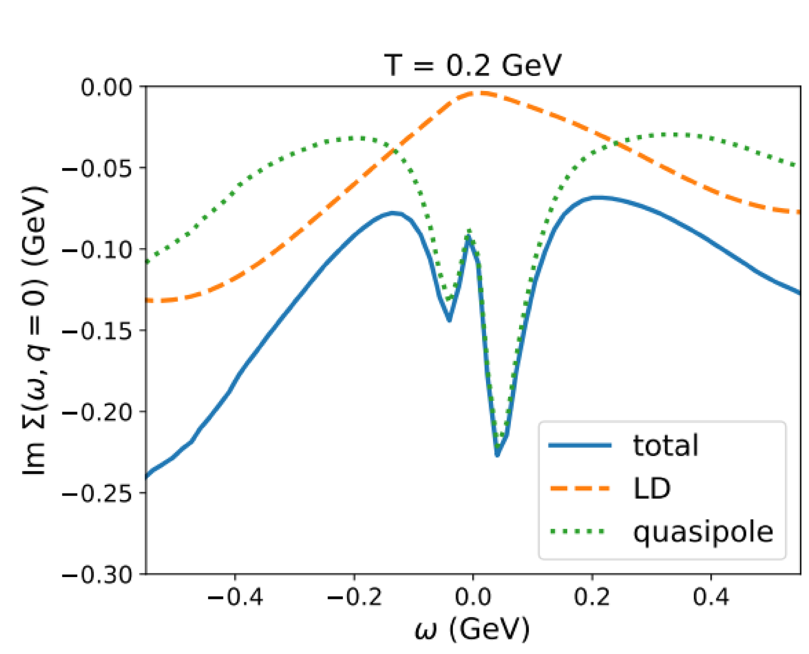
5. Quark Self Energies



The quark self energy is $\Sigma = \gamma_0\Sigma_0 + \vec{\gamma}\,\hat{p}\Sigma_V + \Sigma_m$

$$\mathrm{Im}\Sigma_0(q_0, \vec{q}) = \int_{-\lambda}^{\lambda} dp_0 \int_0^{\lambda} dp \int_{-1}^{1} dx \frac{-p^2}{4\pi^3}(1 - f(p_0, T) + n(q_0 - p_0, T))\mathrm{Im}G_0(p_0, p)\mathrm{Im}D(q_0 - p_0, \vec{p} - \vec{q})$$

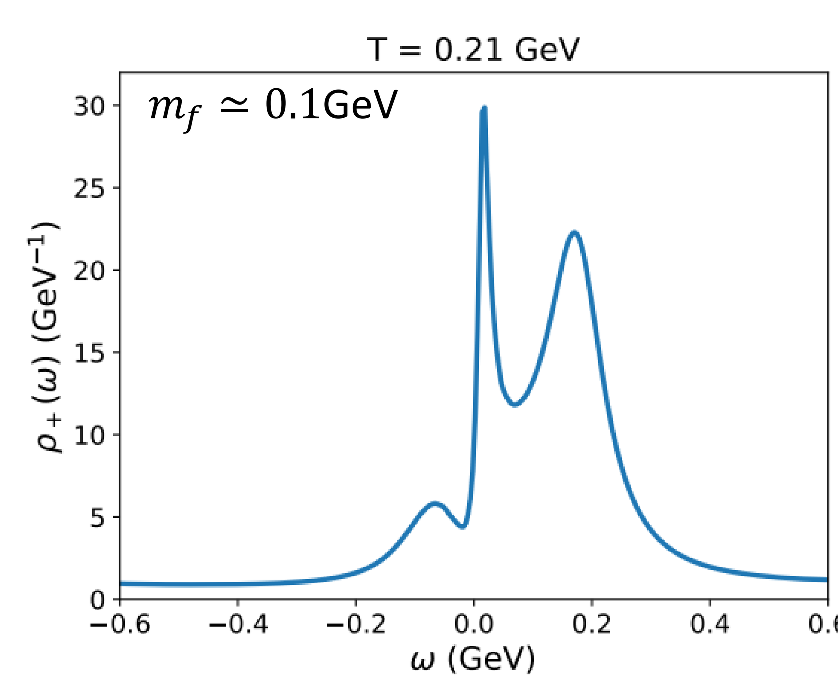


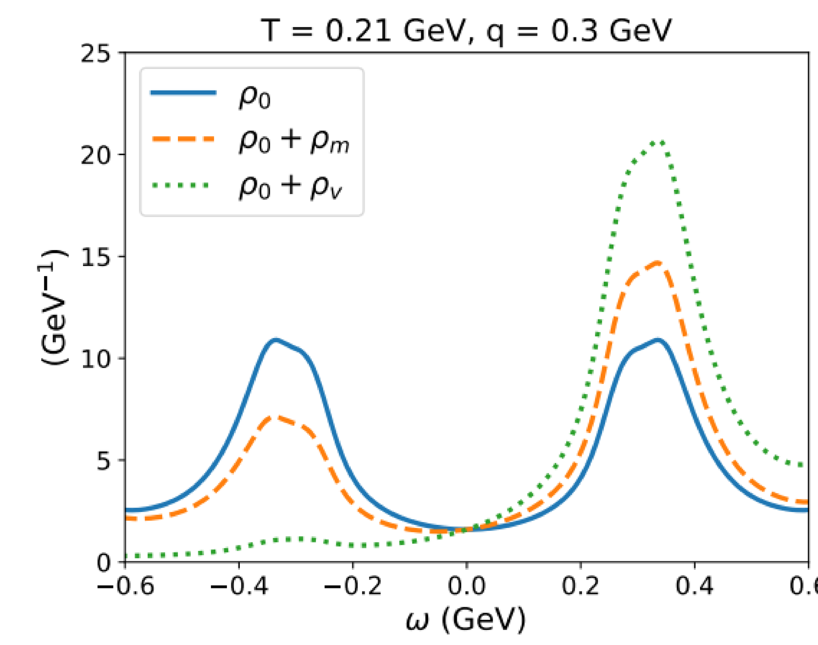


- Σ_0 is symmetric, Σ_m is anti-symmetric.
- $\Sigma_+ = \Sigma_0 + \Sigma_m$.
- Near m_f, Landau damping provides large contribution to the quark width.

6. Quark Spectral Functions:

$$\rho = \gamma_0\rho_0 + \vec{\gamma}\,\hat{p}\rho_V + \rho_m$$





- ρ_+ is obtained from the three components at vanishing external momentum, where the ρ_V component goes to zero, and hence $\rho_+ = \rho_0 + \rho_m$.
- For ρ_+, near the critical temperature, we observe the three-peak structure of the quark spectral function.
- At finite external momentum, ρ_+ doesn't have direct physical meaning.
- For defining fermion spectral properties, we will need all the three components of ρ.

7. Conclusion and Future goals

- Inclusion of quark self energy in the NJL model due to interaction with mesonic correlations leads to significant change of quark spectral properties. [5]
- Significant part of quark width comes from interaction of quarks with zero modes (Landau damping).
- We have confirmed the three-peak structure of the quark spectral function right above T_c[2].
- We have set up the formalism for future self consistent calculation of quark spectral function at arbitrary temperature, constituent quark mass and external momentum.

8. References

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9. Acknowledgements

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