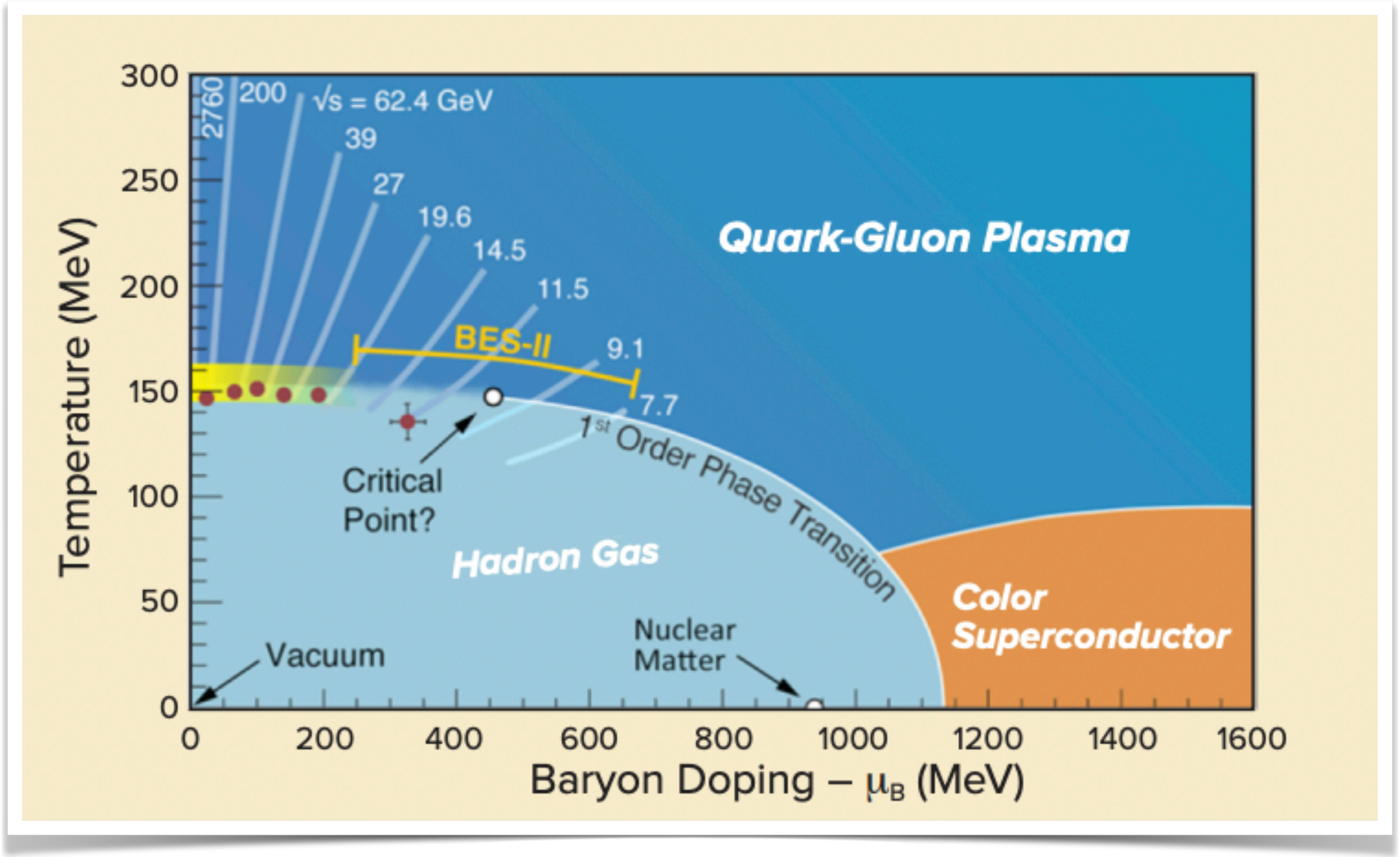
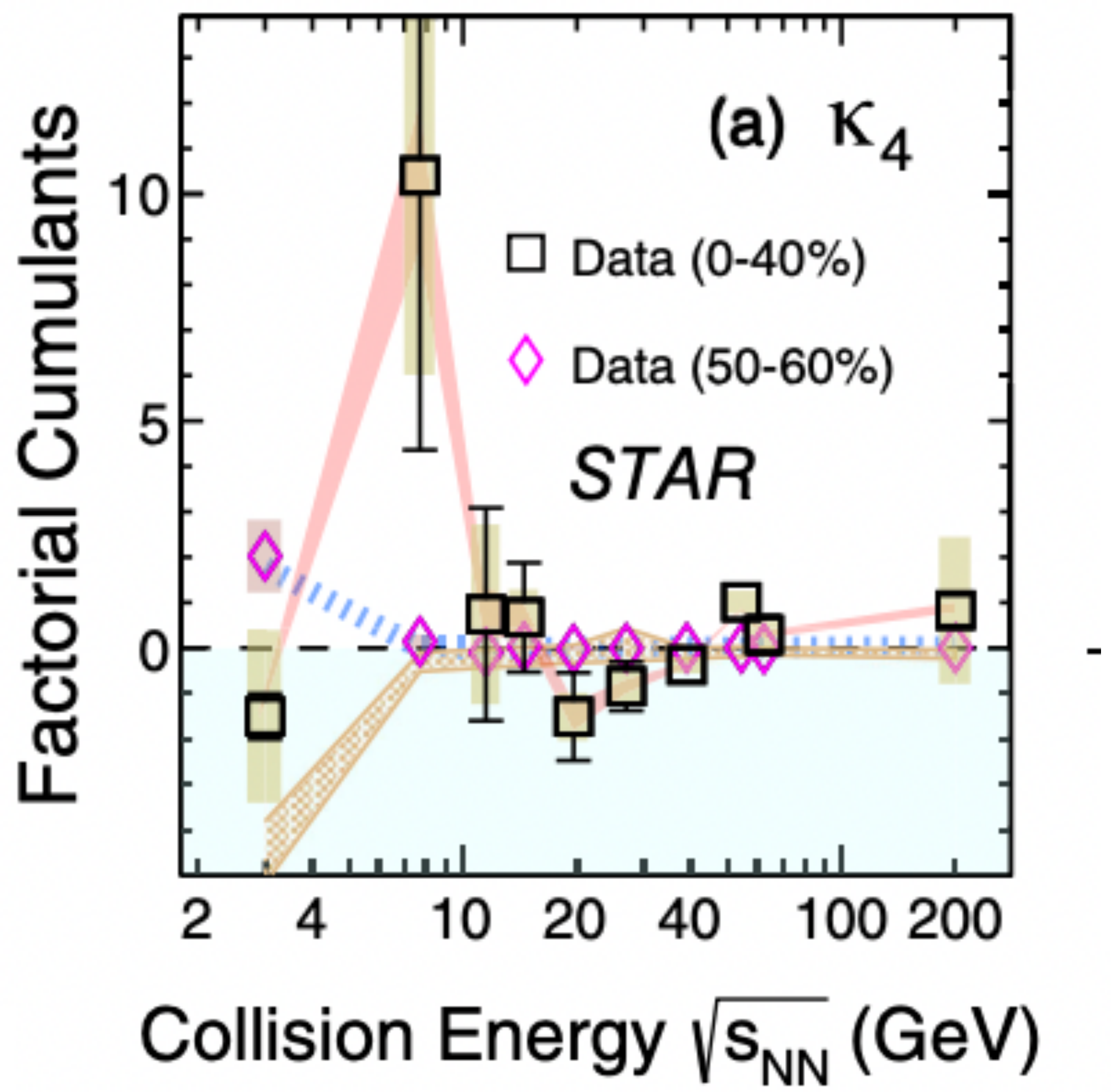
Critical expectations : Non-Gaussian cumulants of Particle multiplicity near the critical point

Speaker : Maneesha Pradeep, University of Maryland at College Park
With Jamie Karthein, Krishna Rajagopal, Misha Stephanov, Yi Yin

Is there a critical point in the QCD phase diagram?





Fourth factorial cumulants for proton multiplicity

Intriguing results from Beam Energy Scan - I (STAR: PRL 130, 082301 (2023))

If there is a CP in the QCD phase diagram,

- Fluctuations are enhanced near the critical point - how ***large*** are the cumulants of particle multiplicities in ***equilibrium***?
- How much do the fluctuations grow within the ***short time*** the fireball spends in the critical region before it freezes out?
- Does the enhancement in the e-b-e correlation functions survive till ***freeze-out***?

Qualitative answers and order of magnitude estimates today and updates on ongoing work

Estimating cumulants of particle multiplicities near the critical point in heavy-ion collision

Ongoing…

- Test in ***simplified*** scenarios
- Identify the most ***consequential*** parameters/***time scales*** in the paradigm
- Identify possible ***challenges***
- Make ***estimates***

BEST EoS

↓

Hydro+ evolution

↓

ME freezeout

Future….

- Simulate realistic scenarios
- Bayesian analysis to make comparisons to experiment

This talk :

- Updates on estimation of cumulants of proton multiplicity in equilibrium
- Effects of critical slowing down (and conservation)
- General method to freeze-out fluctuations : Maximum entropy freeze-out

Estimating cumulants of particle multiplicity near the critical point in equilibrium

The equilibrium estimates are made using a model in which the *interaction of the hadrons with a scalar field in the 3D Ising universality class* modifies the mass of the hadrons

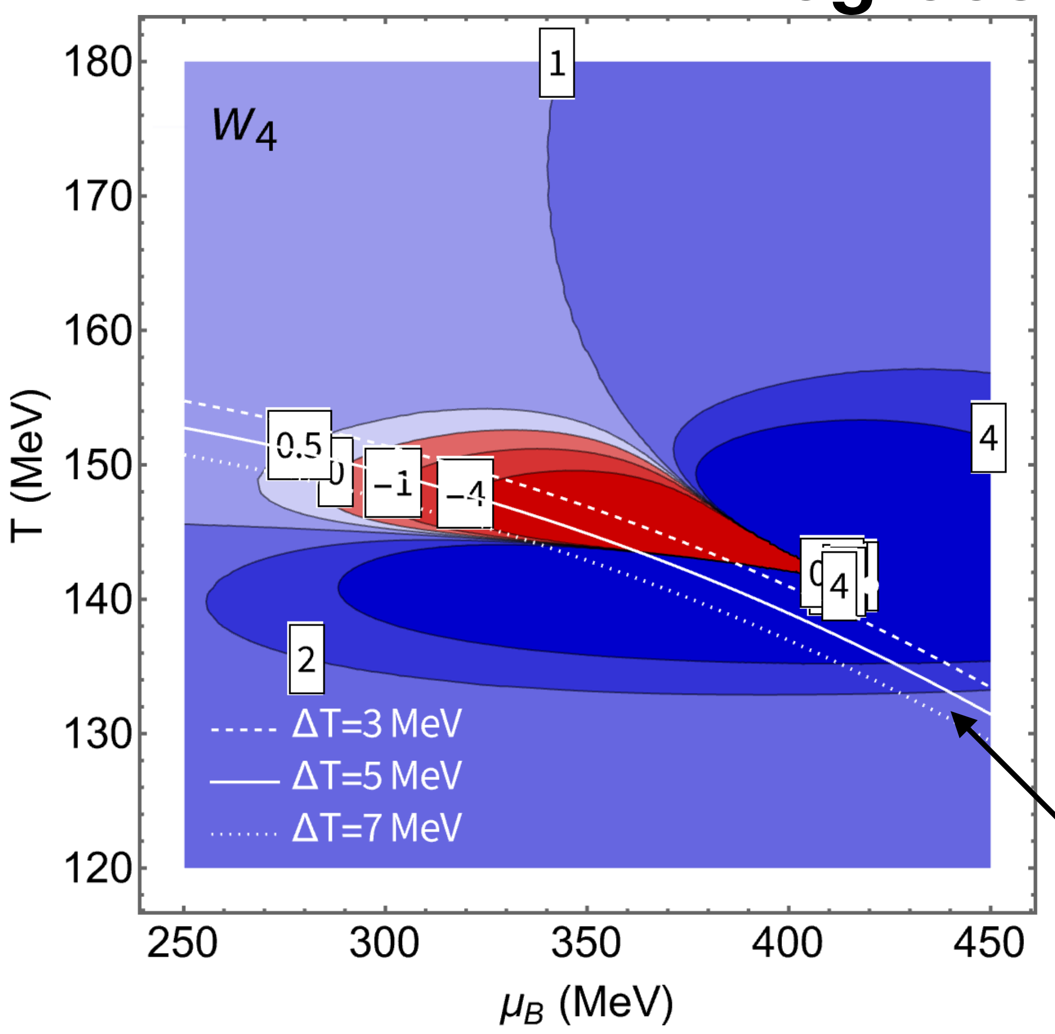
This model has already been used by *Athanasiou, Rajagopal and Stephanov in 2010* to make equilibrium estimates.

Updates on equilibrium estimates for the non-Gaussian cumulants of proton multiplicities

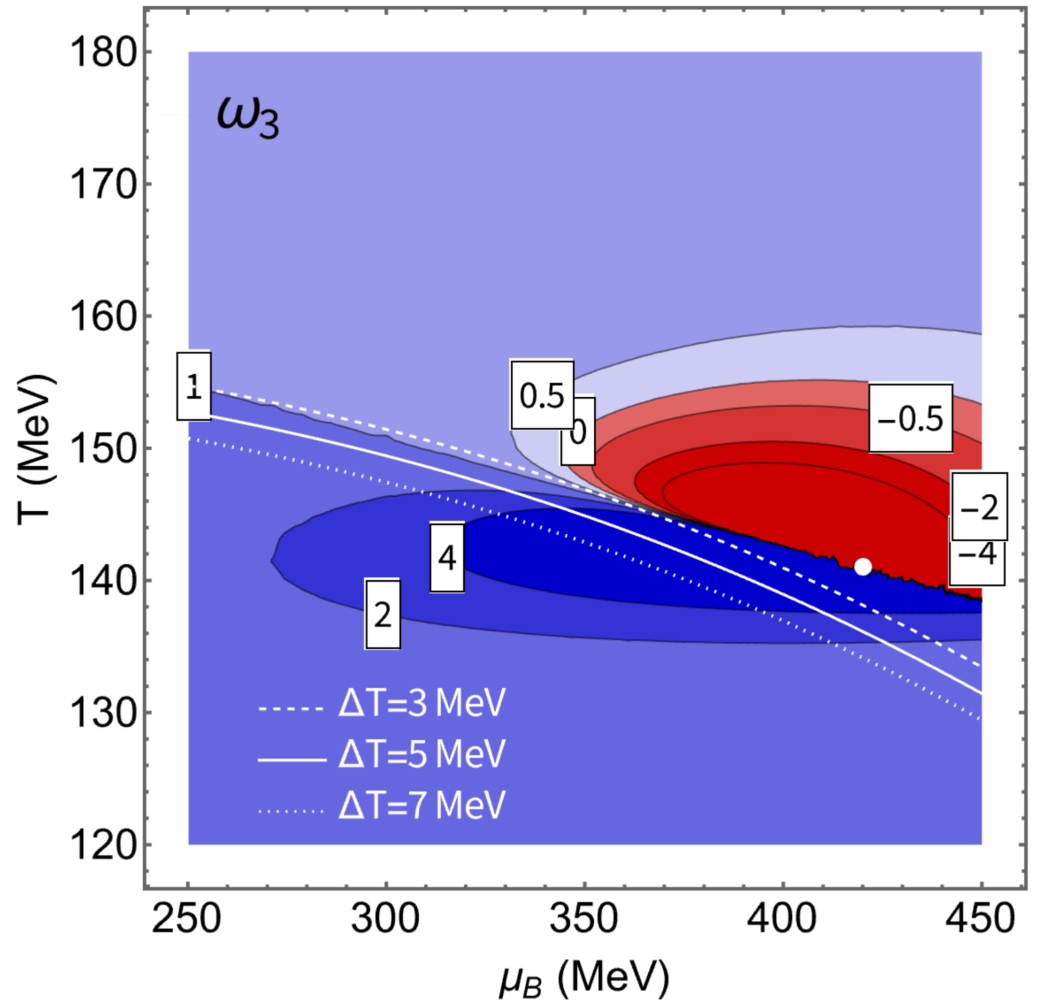
- ***Updates*** to Athanasiou et al, 10 : **Karthein, MP, Rajagoal, Stephanov, Yin (in preparation)**
  - Slope of ***phase boundary*** from lattice **Borsanyi et al.,20**
  - ***Mixing*** between h and r variables **Rehr and Mermin, 1973**
  - In the small quark mass limit, the ***slope difference*** between Ising axes is small **MP,Stephanov, 19**
  - 3D Ising Correlation Length from epsilon expansion to second order **Zinn-Justin + new**
  - Higher point correlations from ***3D Ising model*** **Parotto et al, 18, Karthein et al, 21**
  - Updated freeze-out parametrization **Andronic et al, 18**
  - To implement : Use coupling constants from the equation of state from maximum entropy principle **MP, Stephanov, 22**

Equilibrium estimates for the non-Gaussian cumulants of proton multiplicities

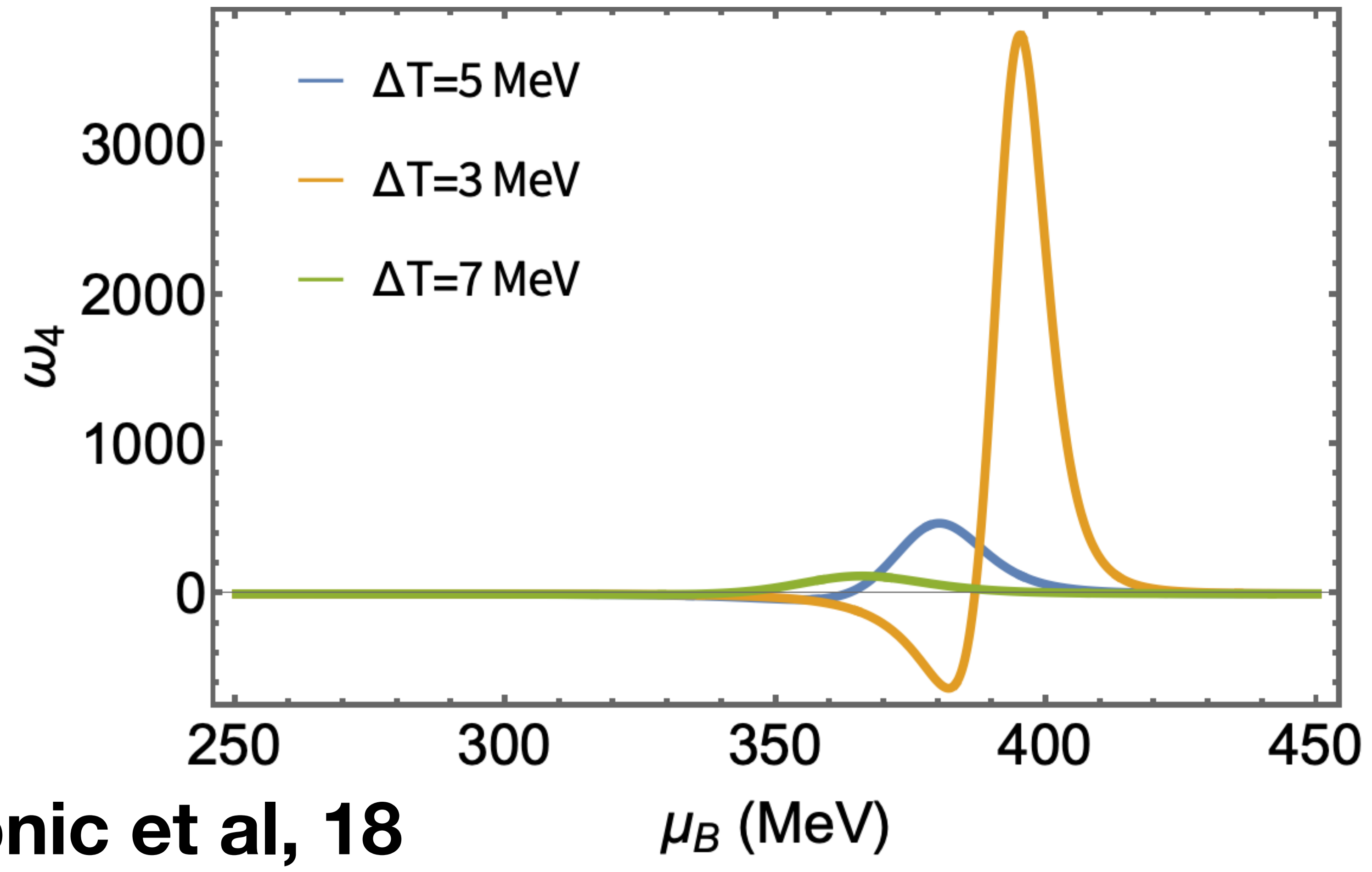
Slope difference between Ising axes - 10 Degrees

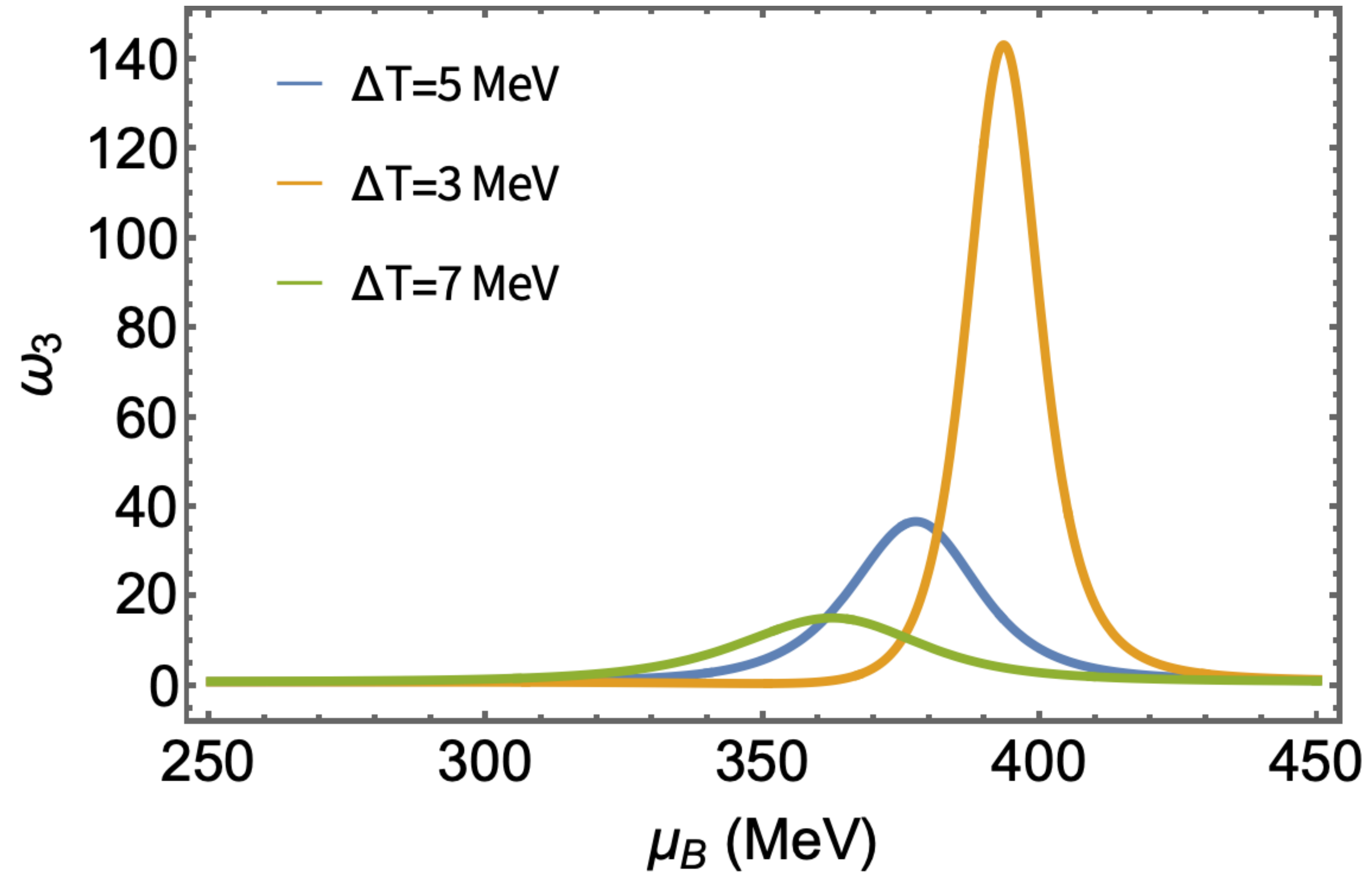


Andronic et al, 18



$$\omega_n = \frac{C_n}{C_1}$$





$$\mu_c = 420\,\mathrm{MeV}\,,\; T_c = 141\,\mathrm{MeV}$$

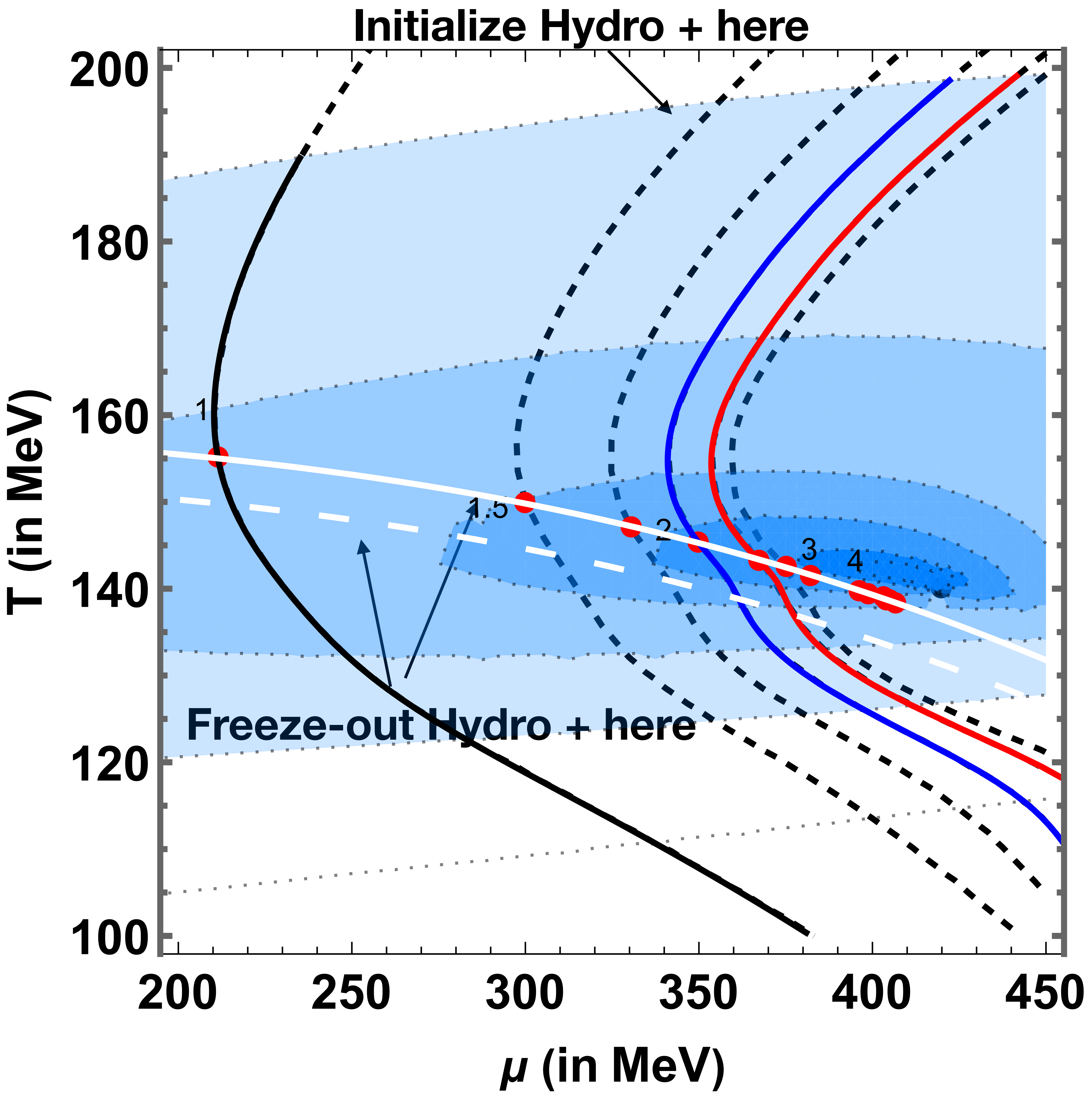
$$\Delta T = T_c - T_f(\mu_c)$$

$$w = 8, \rho = 0.2$$

- Can be ***large*** enough to have ***observable*** consequences
- Sensitive to the ***proximity of freeze-out*** to the phase boundary
- Sensitive to non-universal mapping parameters

Karthein, MP, Rajagoal, Stephanov, Yin (in preperation)

Dynamical evolution of background and fluctuations



- Evolution along ***isentropes*** from ***BEST EoS*** **Parotto et al., 18, Karthein et al, 22**
- ***Hydro+*** to evolve correlation functions of entropy per baryon **Yin, Stephanov, 17**
- Initialized on a constant correlation length curve away from CP
- ***Relaxation equations*** for the Wigner transform of the hydrodynamic correlation functions

$$\partial_t W_n = -n\gamma q^2 \left[W_n - F[W_{n-1}, \ldots W_2]\right]$$

Xin An's talk at 12:40 pm today in New Theory Session

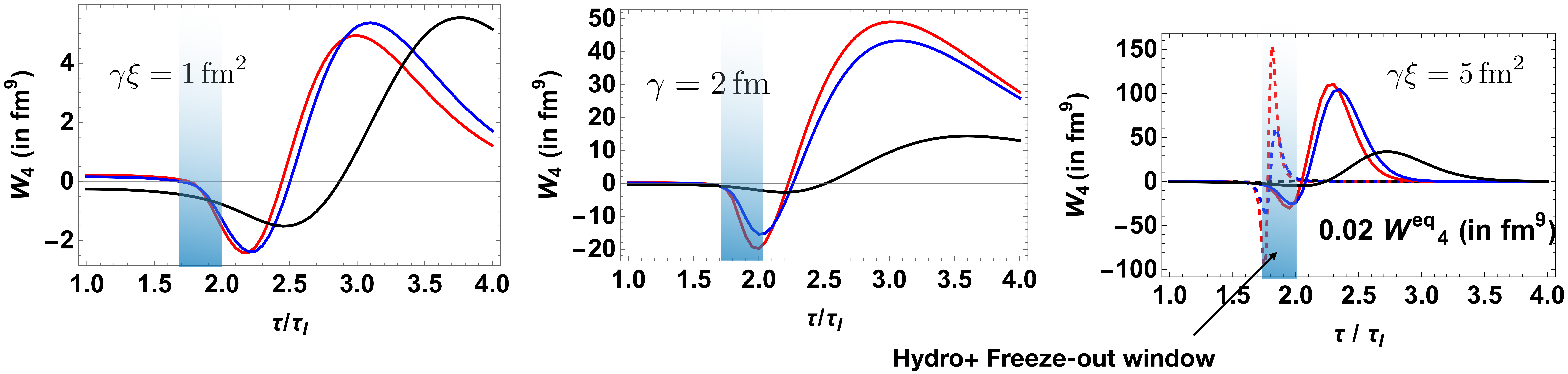
$$\gamma q^2 \sim \xi^{-1} q^2$$

Critical slowing down

Karthein, MP, Rajagoal, Stephanov, Yin (in preperation)

Four point correlations along the evolution trajectories - critical slowing down

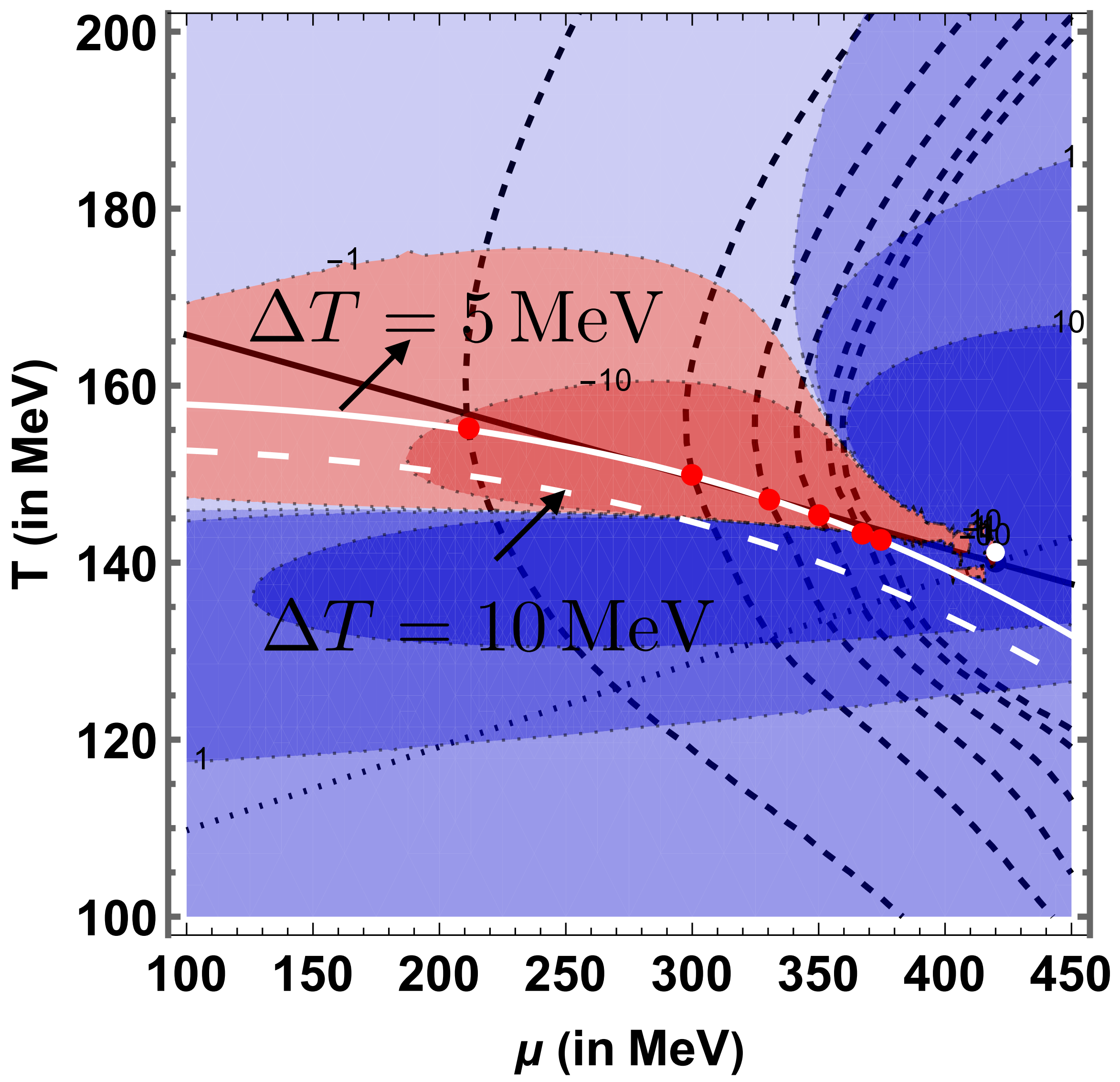
$q = 0.44\,\mathrm{fm}^{-1}$ Low q modes dominate the contribution to freeze-out due to ***thermal smearing*** $q \sim (v_{\mathrm{thermal}}\,\tau_f)^{-1}$

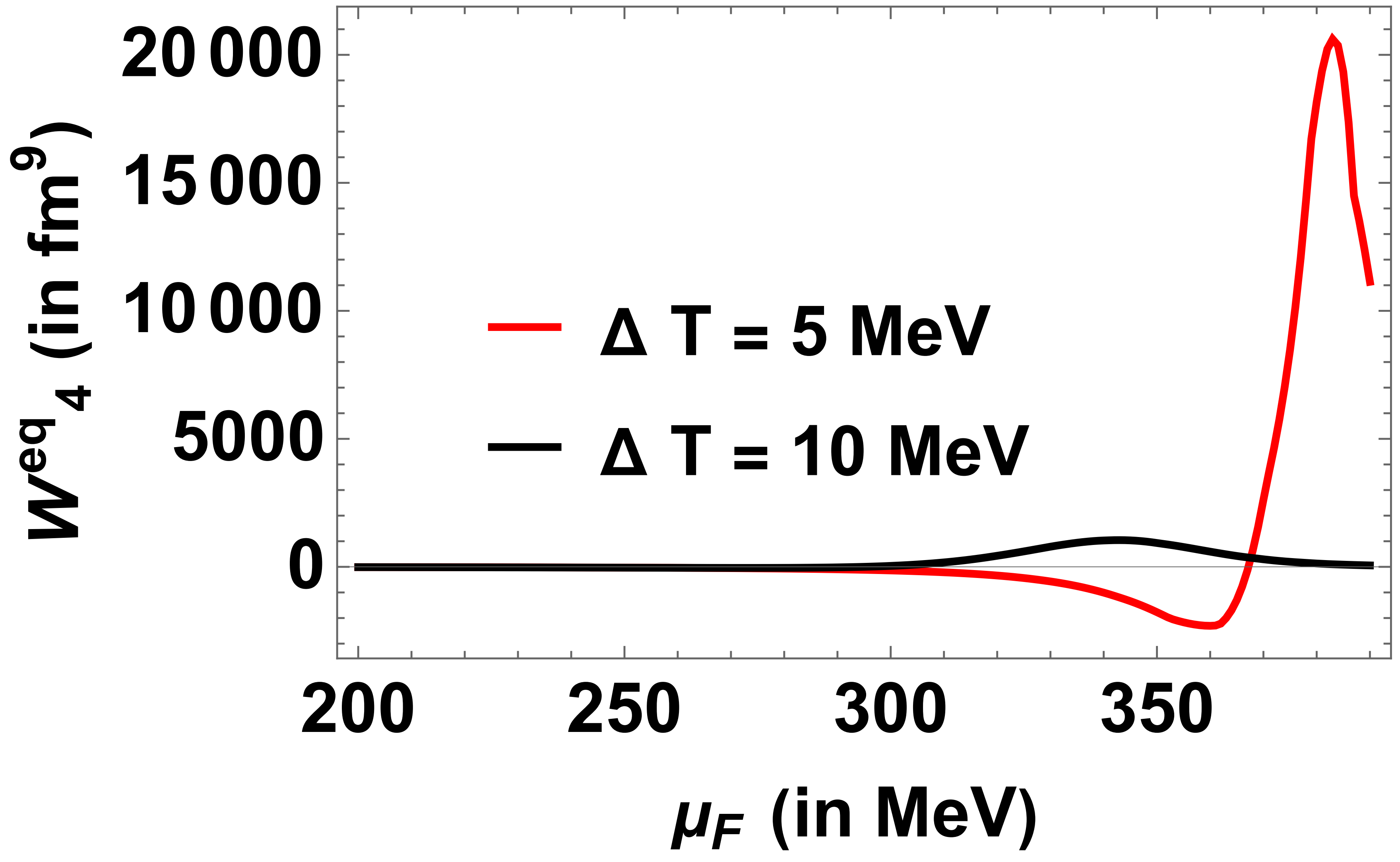


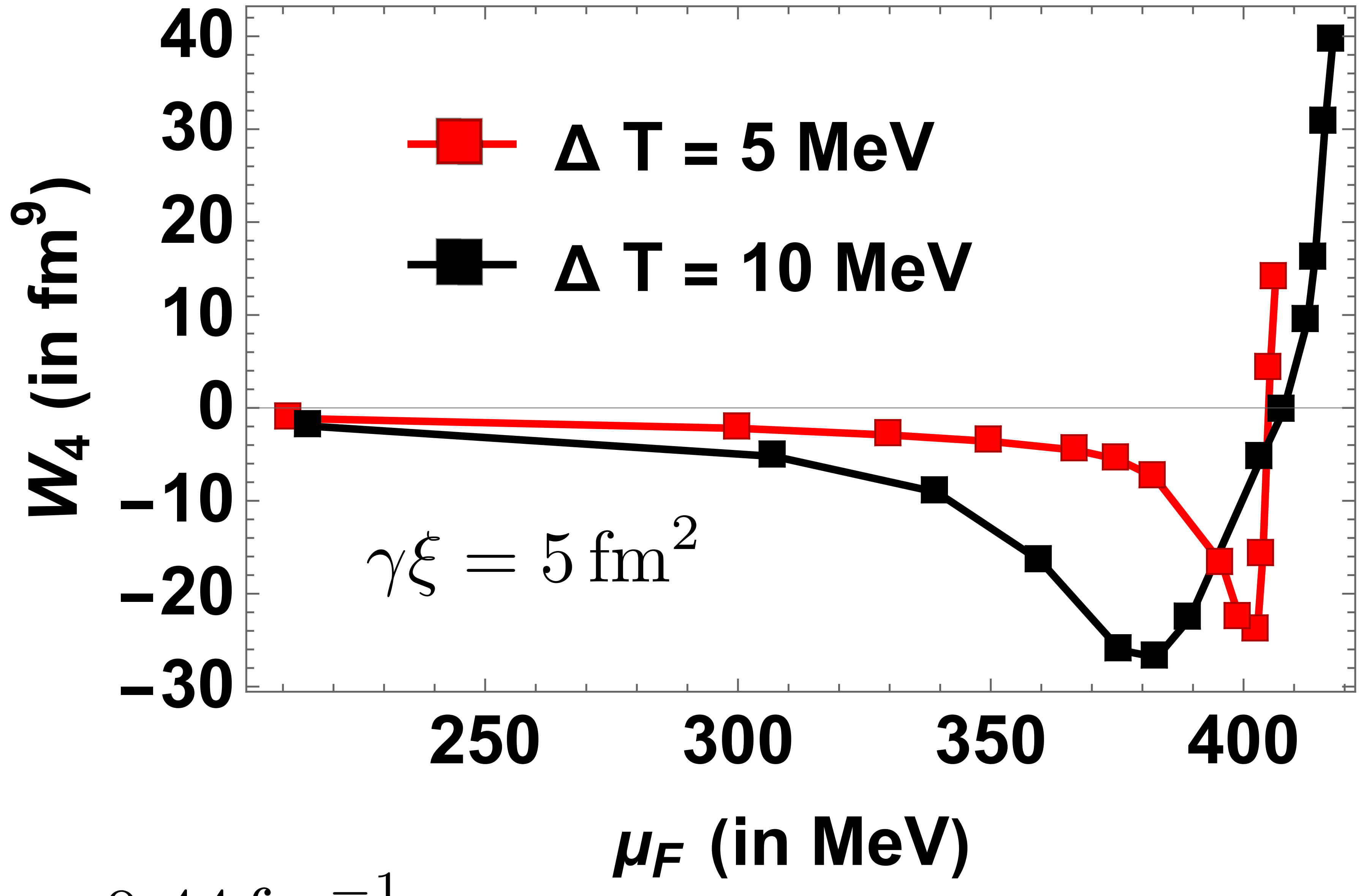
- Suppression of the peak relative to equilibrium value is stronger for higher cumulants
- The sign of the correlation functions may also suffer a lag due to slowing down
- The memory effects are retained for a much longer time well past the fireball's exit from the critical region.

Karthein, MP, Rajagoal, Stephanov, Yin (in preparation)

Four point correlations along the freeze-out curve







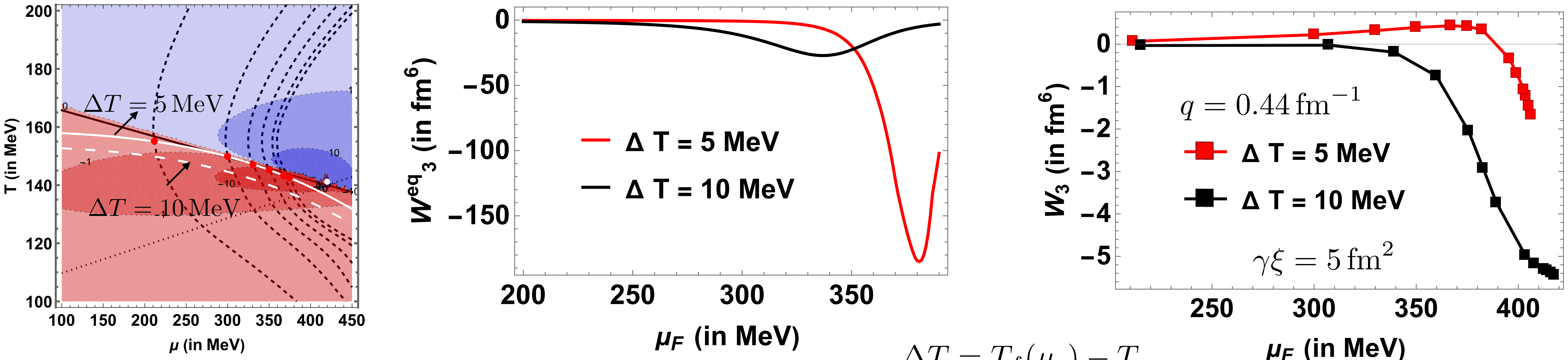
$$\Delta T = T_c - T_f(\mu_c)$$

$$q = 0.44\,\mathrm{fm}^{-1}$$

- The peak and dip in a dynamical scenario along the freeze-out curve is suppressed due to conservation and critical slowing down
- The change of sign is delayed due to critical slowing down
- The peak and dip are shifted to larger chemical potentials than expected in equilibrium
- Longer the time to freeze-out, more prominent critical effects

Karthein, MP, Rajagoal, Stephanov, Yin (in preparation)

Three point correlations along the freeze-out curve



$$\Delta T = T_f(\mu_c) - T_c$$

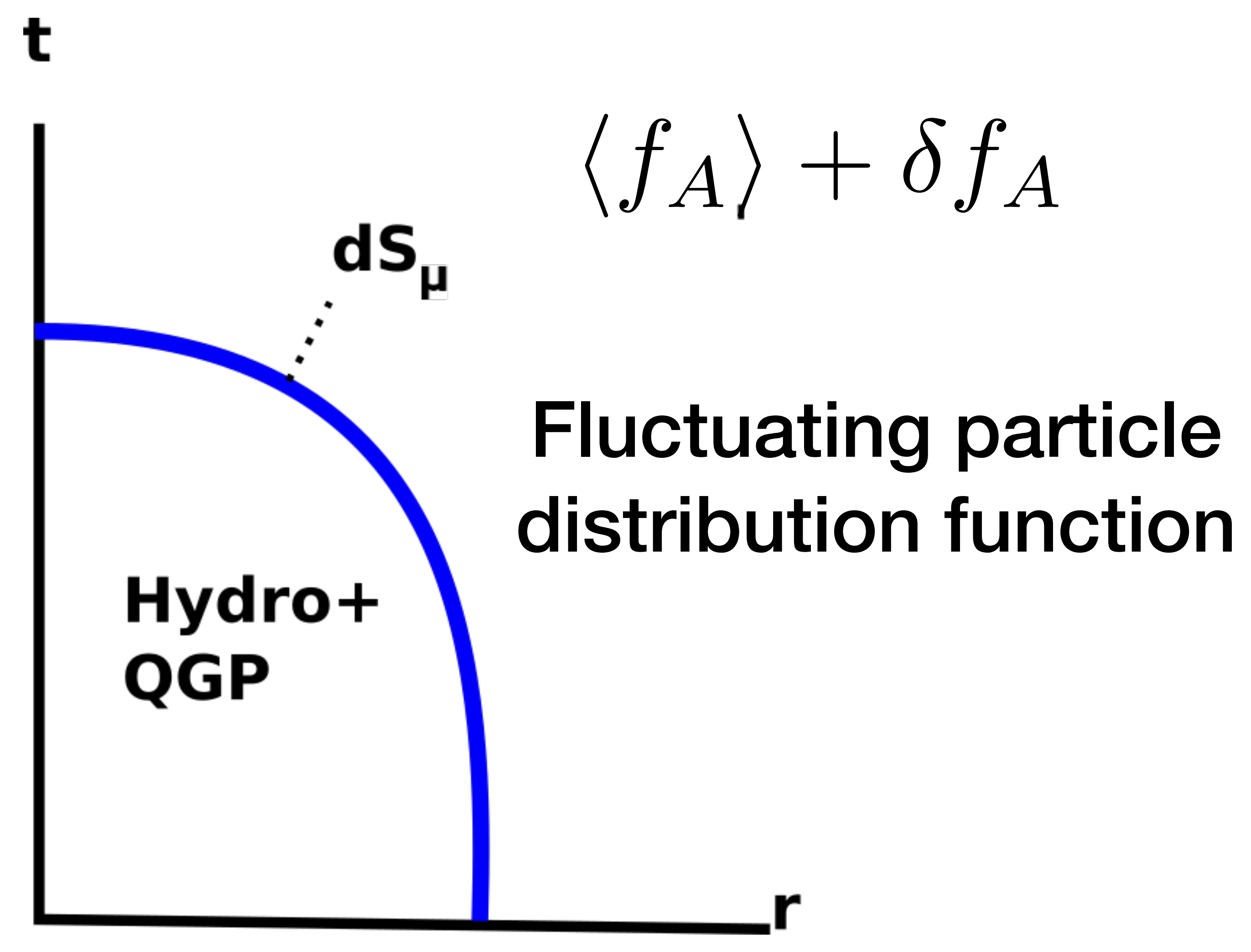
- Qualitatively similar observations for three point function, although the suppression relative to equilibrium value is lesser by an order of magnitude

Karthein, MP, Rajagoal, Stephanov, Yin (in preparation)

Maximum entropy freeze-out

MP,Stephanov, 22

Infinitely many ensembles of free streaming particles whose energy-momentum and charge density correlations match with hydrodynamic description.



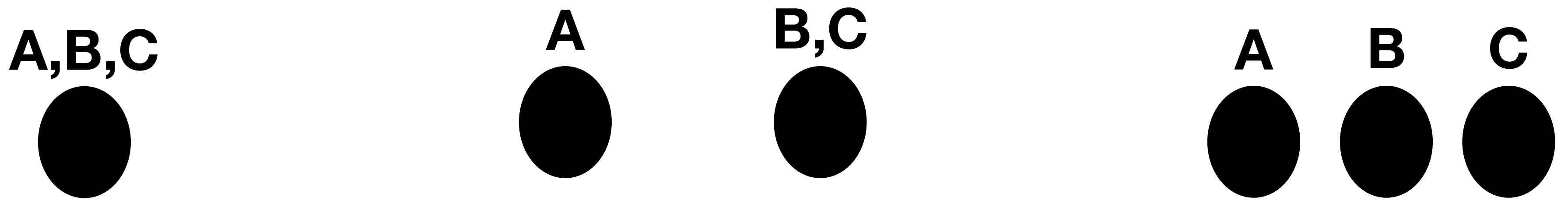
Which is the ***most probable?***

The one which ***maximizes the entropy*** of the fluctuating particle distribution function, subject to the constraints of the matching conditions.

MP,Stephanov, 22

- Generalization of Cooper-Frye freeze-out to freeze-out n-point correlations of hydrodynamic densities
- Leads to natural generalization of factorial cumulants (IRCs, or irreducible relative cumulants)
- IRCs subtracts the baseline correlations for any given reference distribution

$$\widehat{\Delta} G_{ABC\ldots} = F_{abc\ldots}^{\text{Baseline EoS}} \widehat{\Delta} H^{abc\ldots}$$



$$G_{ABC} = \bar{G}_{ABC} + 3\widehat{\Delta} G_{AD} \left(\bar{G}_2^{-1} \bar{G}_3\right)_{DBC} + \widehat{\Delta} G_{ABC}$$

ME freeze-out is currently being employed to make estimates for cumulants of particle multiplicities in simplified settings.

Summarizing & Looking forward

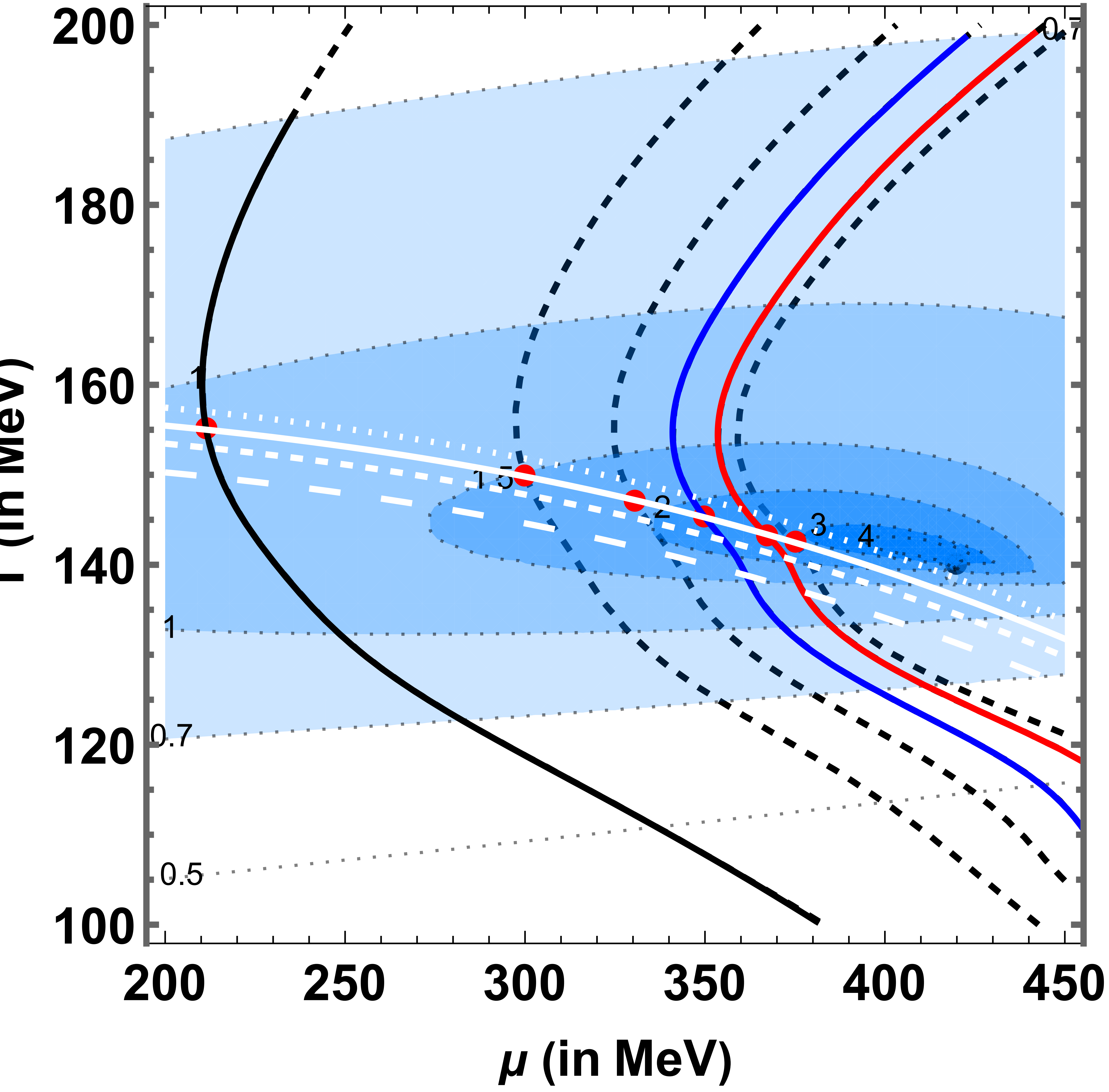
- Equilibrium ratios of third and fourth cumulant can be of the order of hundreds and thousands
- Suppression in the peak relative to equilibrium can be 1-3 orders of magnitude
- Magnitude and sign of cumulants at freeze-out are sensitive to the value of relaxation rate
- Increasing relaxation rate and delaying freeze-out time have similar effect
- The change of sign is shifted to larger freeze-out chemical potentials
- Generalized prescription for freezing out fluctuations is ready -ME freeze-out
- Looking forward : Make quantitative estimates for cumulants of various particle multiplicities

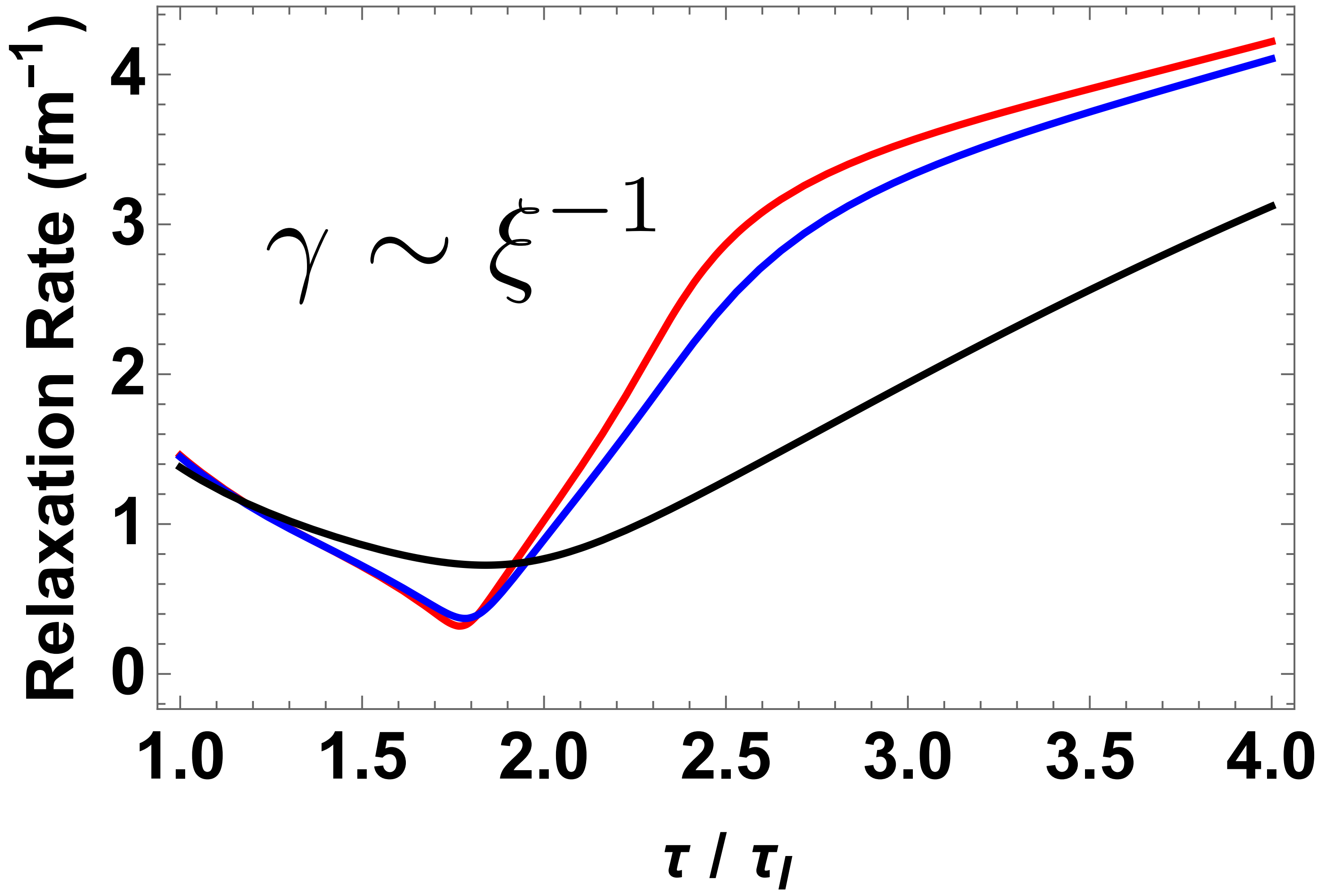
Thank you for your attention!

Additional Slides

- Relaxation rates along trajectories
- Critical slowing down : Three point correlations
- Sensitivity to location of freeze-out :Three point correlations
- Suppression due to conservation : Effect on lower q modes
- More on Maximum entropy freeze-out

Dynamical evolution of background and fluctuations





Entropy per baryon

$$W_n = \mathrm{WignerTransform}\left[\langle \delta \hat{s}^n \rangle\right]$$

$$\partial_t W_n = -n\gamma \left[W_n - F[W_{n-1}, \ldots W_2]\right]$$

$$F[W_{n-1}, \ldots W_2] \overset{\text{in eq}}{\rightarrow} W_n^{\text{eq}}$$

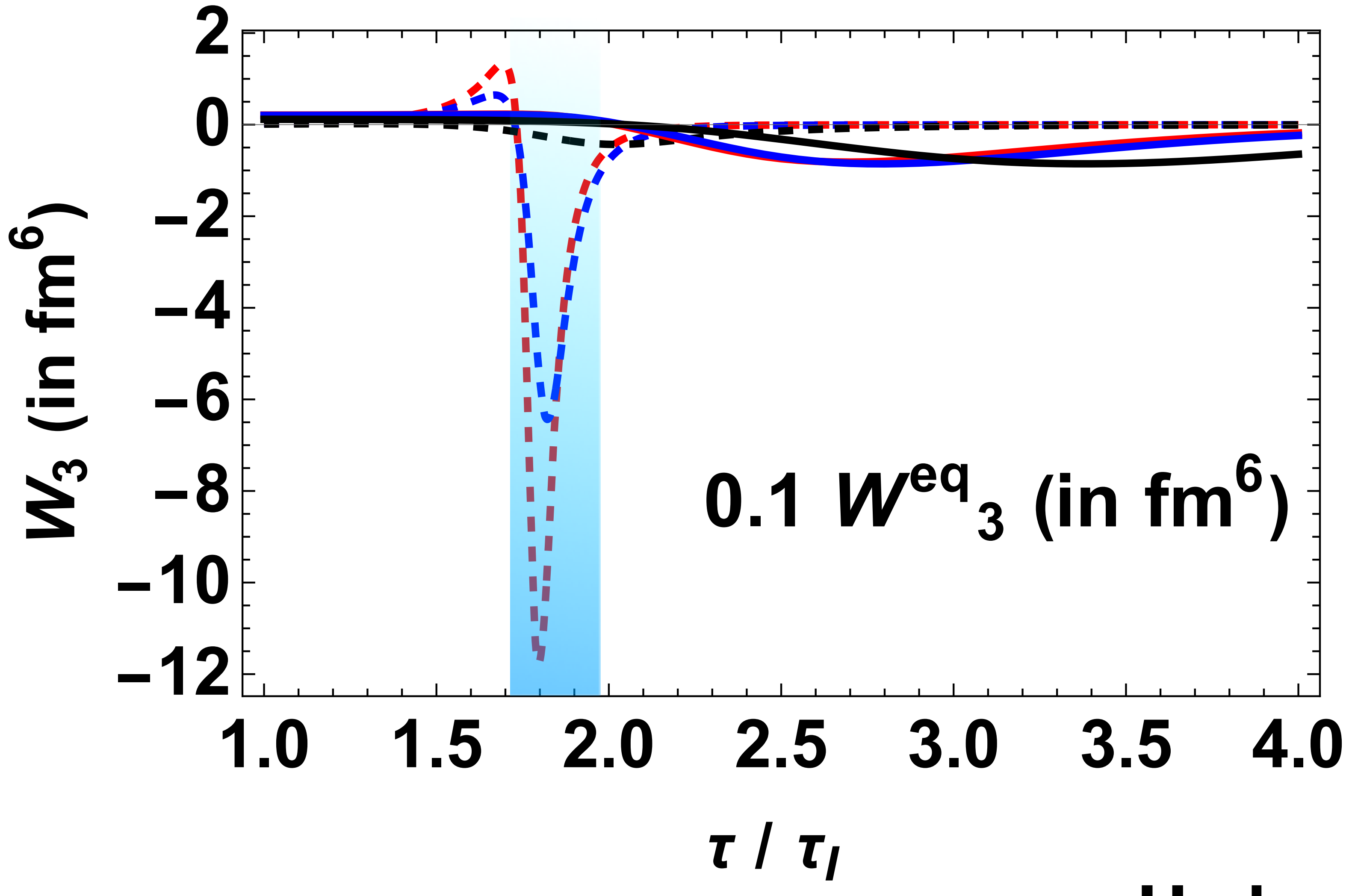
$$\gamma q^2 \sim \xi^{-1} q^2$$

Xin An's talk at 12:40 pm today in New Theory Session

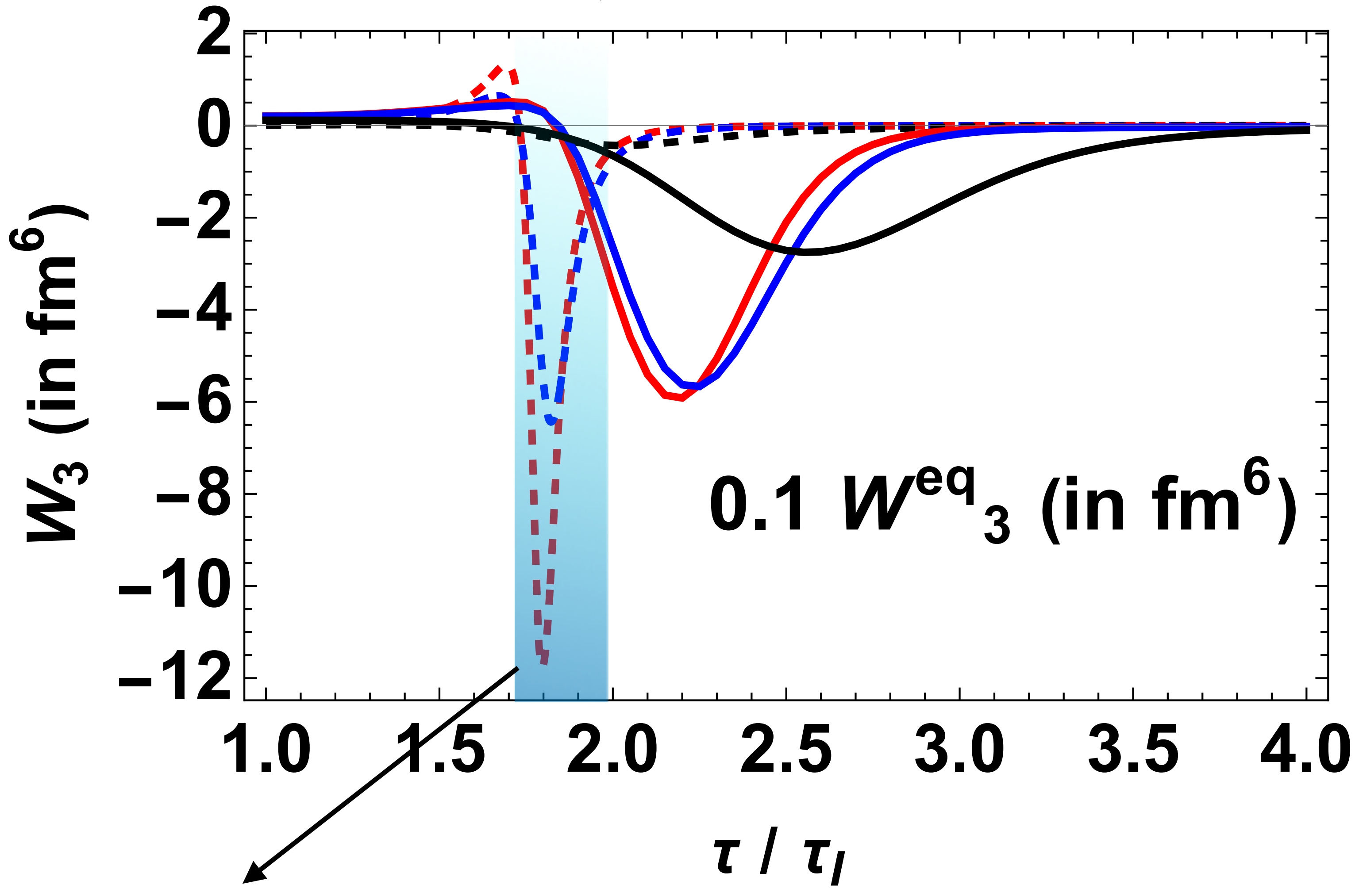
Critical slowing down : Three point correlations

$q = 0.44\,\mathrm{fm}^{-1}$ Low q modes dominate the contribution to freeze-out due to thermal smearing $q \sim (v_{\mathrm{thermal}}\,\tau_f)^{-1}$

$\gamma\xi = 1\,\mathrm{fm}^2$



$\gamma\xi = 5\,\mathrm{fm}^2$



Hydro+ Freeze-out window

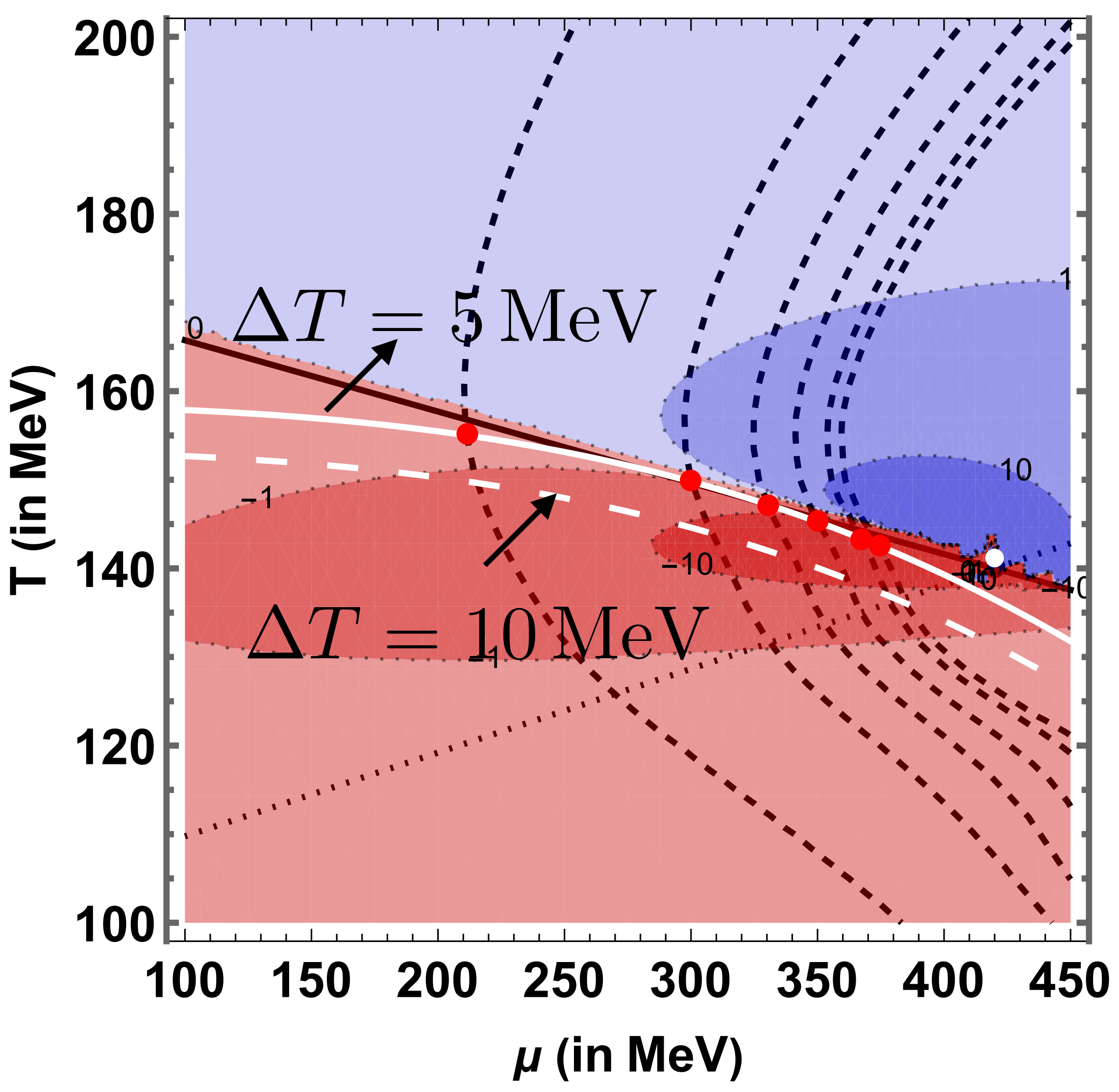
$$\partial_t W_3 = -3\gamma q^2 \left(W_3 - \left(\frac{W_2}{W_2^{\mathrm{eq}}}\right)^2 W_3^{\mathrm{eq}} \right)$$

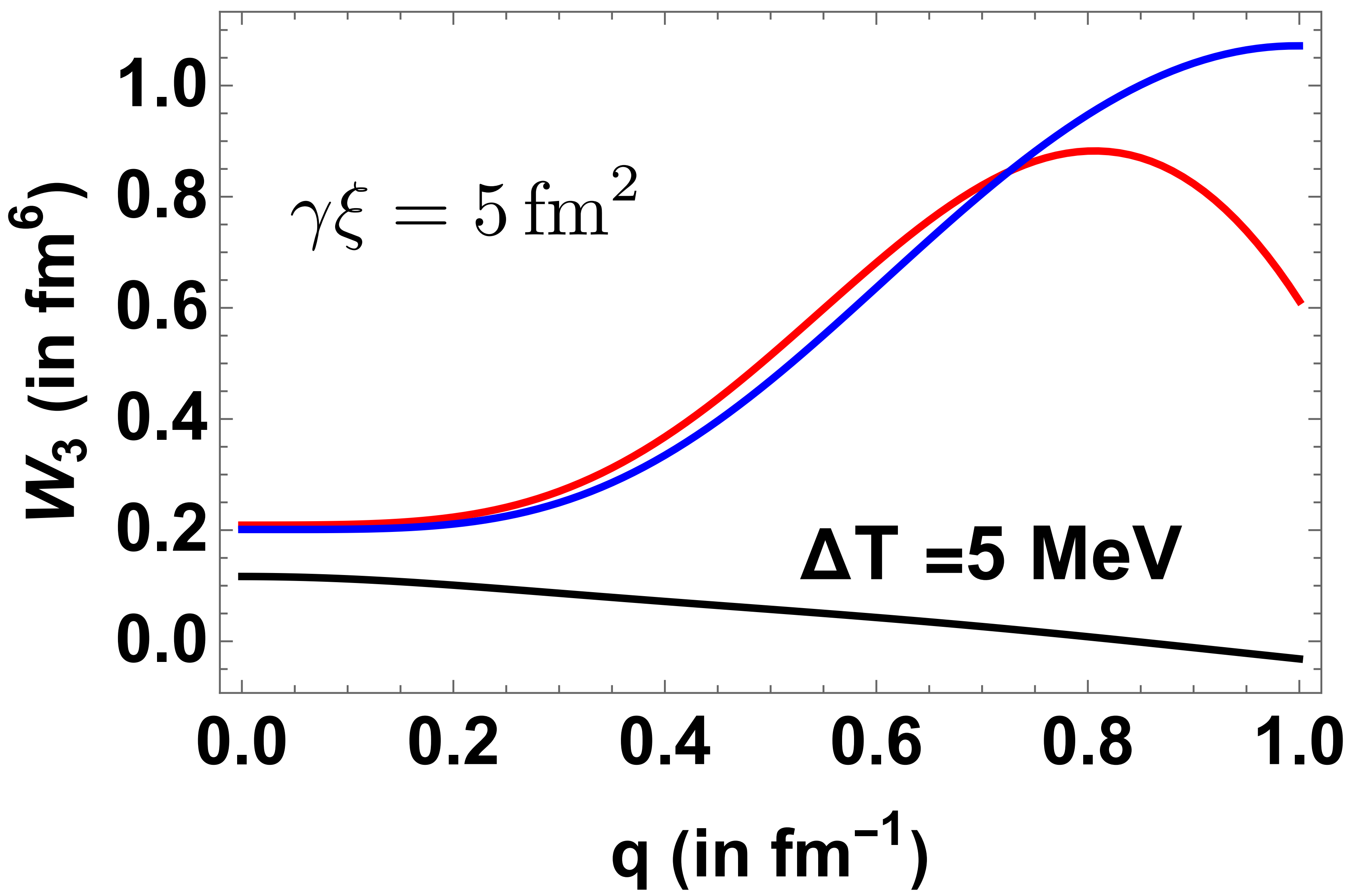
$$\partial_t W_2 = -2\gamma q^2 \left(W_2 - W_2^{\mathrm{eq}} \right)$$

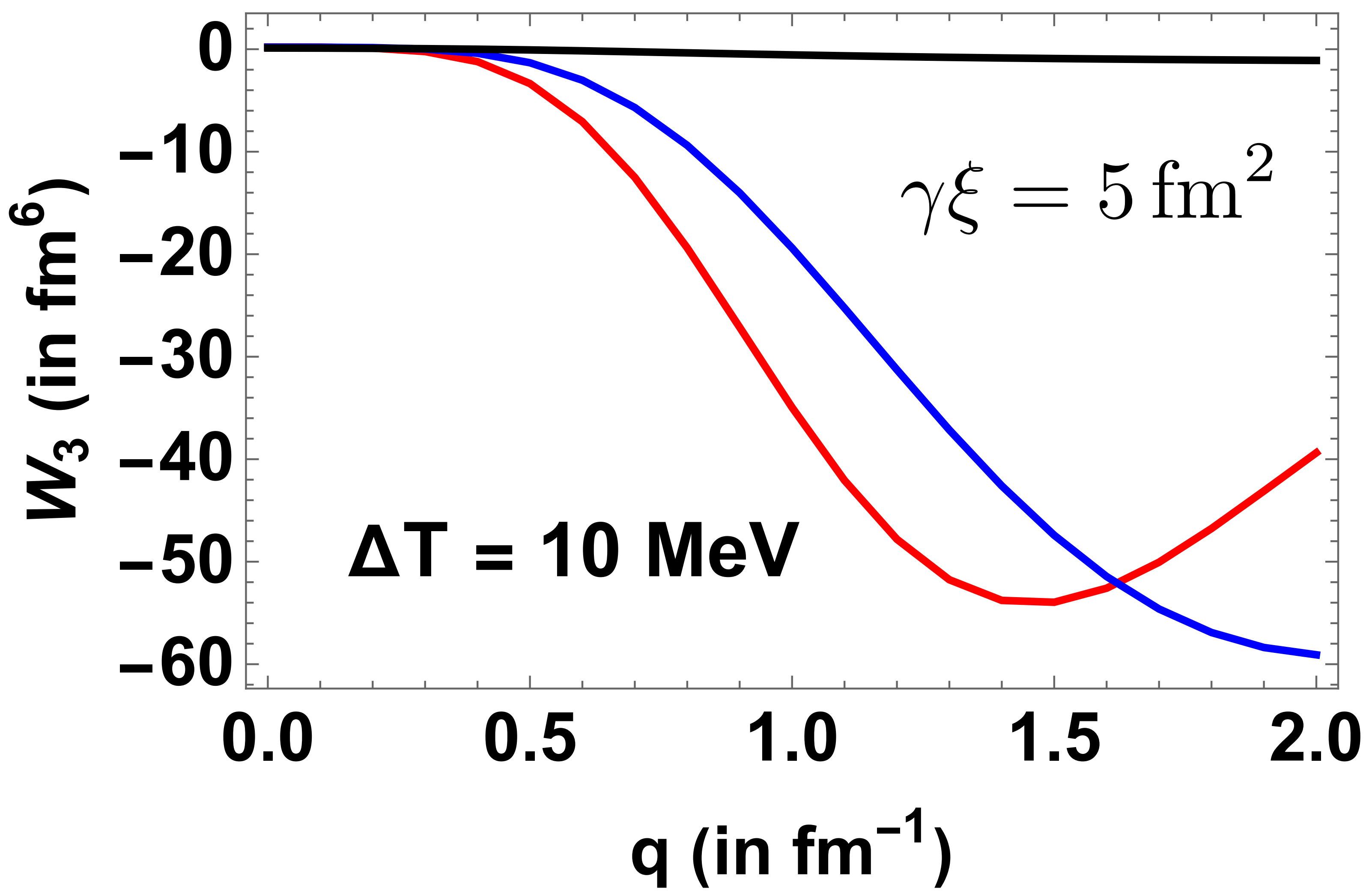
An, Basar, Stephanov and Yee , 19, 21, 22
Sogabe and Yin, 21

- Suppression of the peak in a dynamical scenario depends strongly on the relaxation rate
- The memory effects are retained for a much longer time well past the fireball’s exit from the critical region.

Suppression due to baryon number and energy conservation : Effect on low Q modes



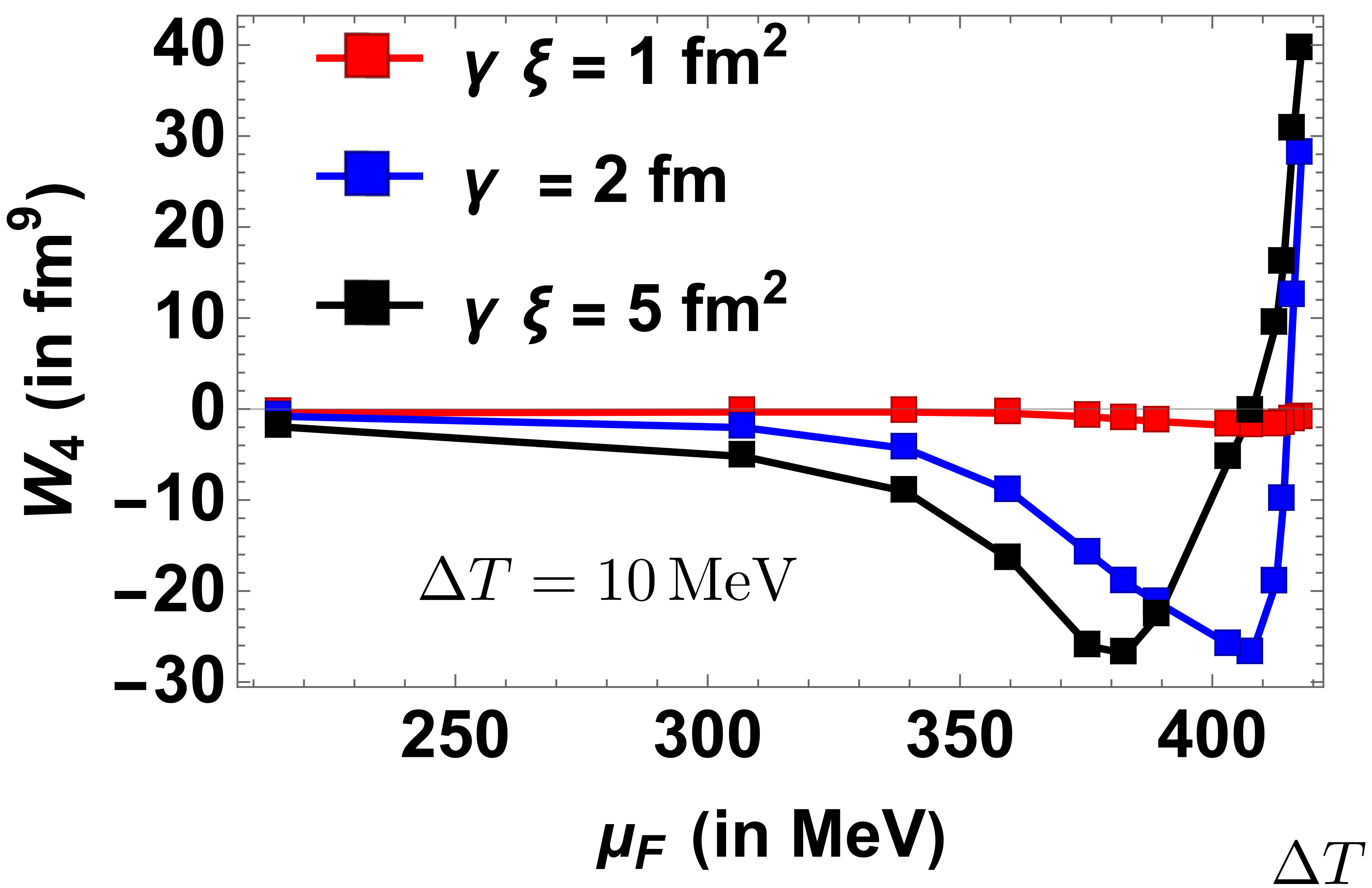


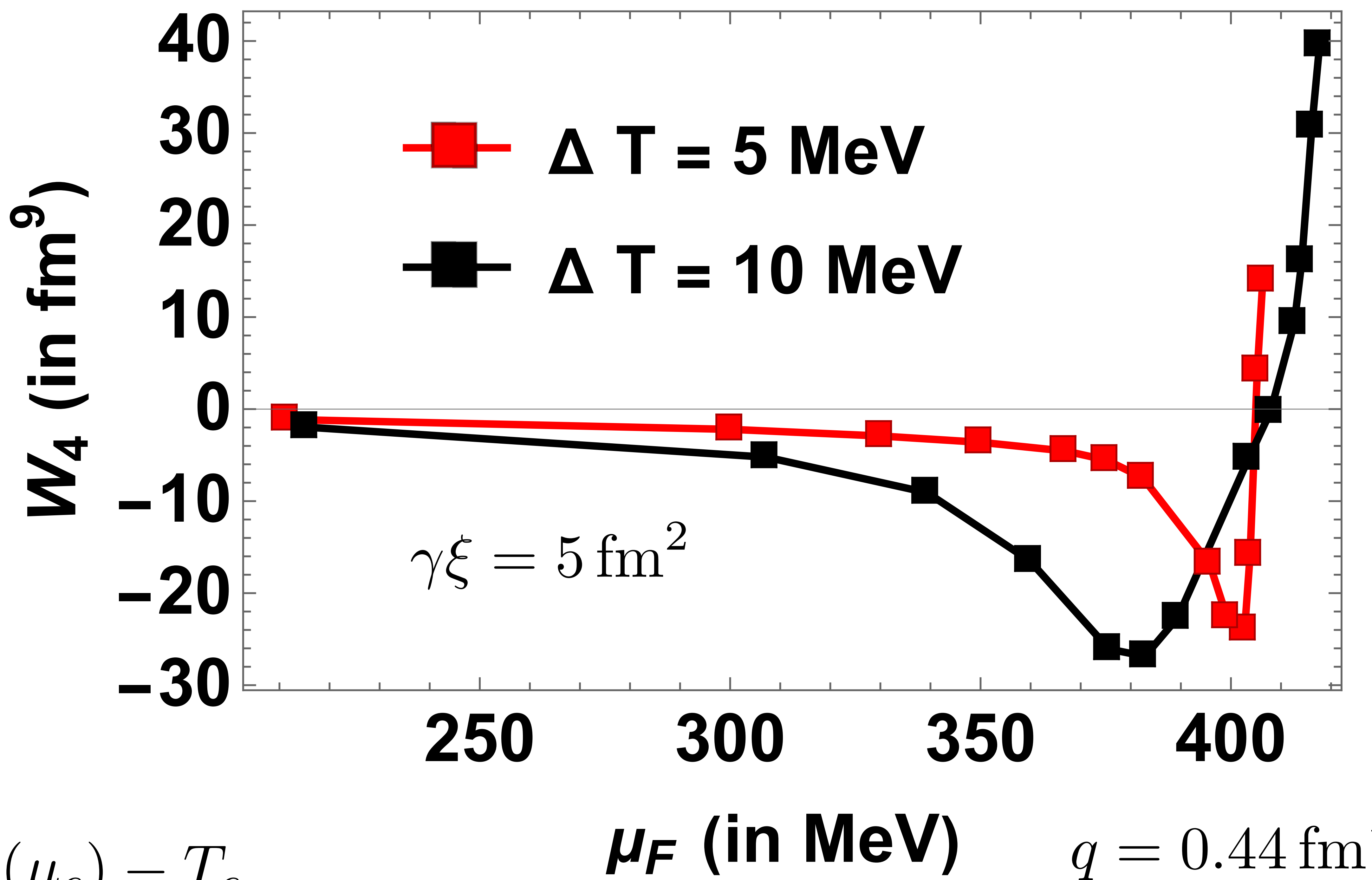


Snapshot at freeze-out

- Low q modes are suppressed more due to effects of charge and energy conservation - Relaxation rate $\sim q^2$
- Low q modes dominate the contribution to freeze-out due to thermal smearing - $q \sim (v_{\text{thermal}}\, \tau_f)^{-1}$
- We are working on implementing freeze-out by Taylor expansion of W_n in q^2

Interplay of various factors which control the magnitude of correlation functions of hydrodynamic densities at freeze-out





$\Delta T = T_f(\mu_c) - T_c$

$q = 0.44\,\mathrm{fm}^{-1}$

- Closer the **freeze-out location to CP**, lesser is the time for fluctuations to grow $\equiv$ Slower **relaxation rate** and freeze-out being further away from CP

Freeze-out of higher point fluctuations

In the hydrodynamic limit, when the Knudsen number is small :

General freeze-out prescription (linearized)

$$P_A = \begin{bmatrix} p^\mu_A \\ q_A \end{bmatrix}$$

$$\widehat{\Delta} G_{AB\ldots} = \widehat{\Delta} H_{ab\ldots} \boxed{(\bar{H}^{-1} P \bar{G})^a_A (\bar{H}^{-1} P \bar{G})^b_B \ldots},$$

Irreducible relative cumulants (IRCs)

Polynomial in P_{A} expressible in terms of quantities known from EoS

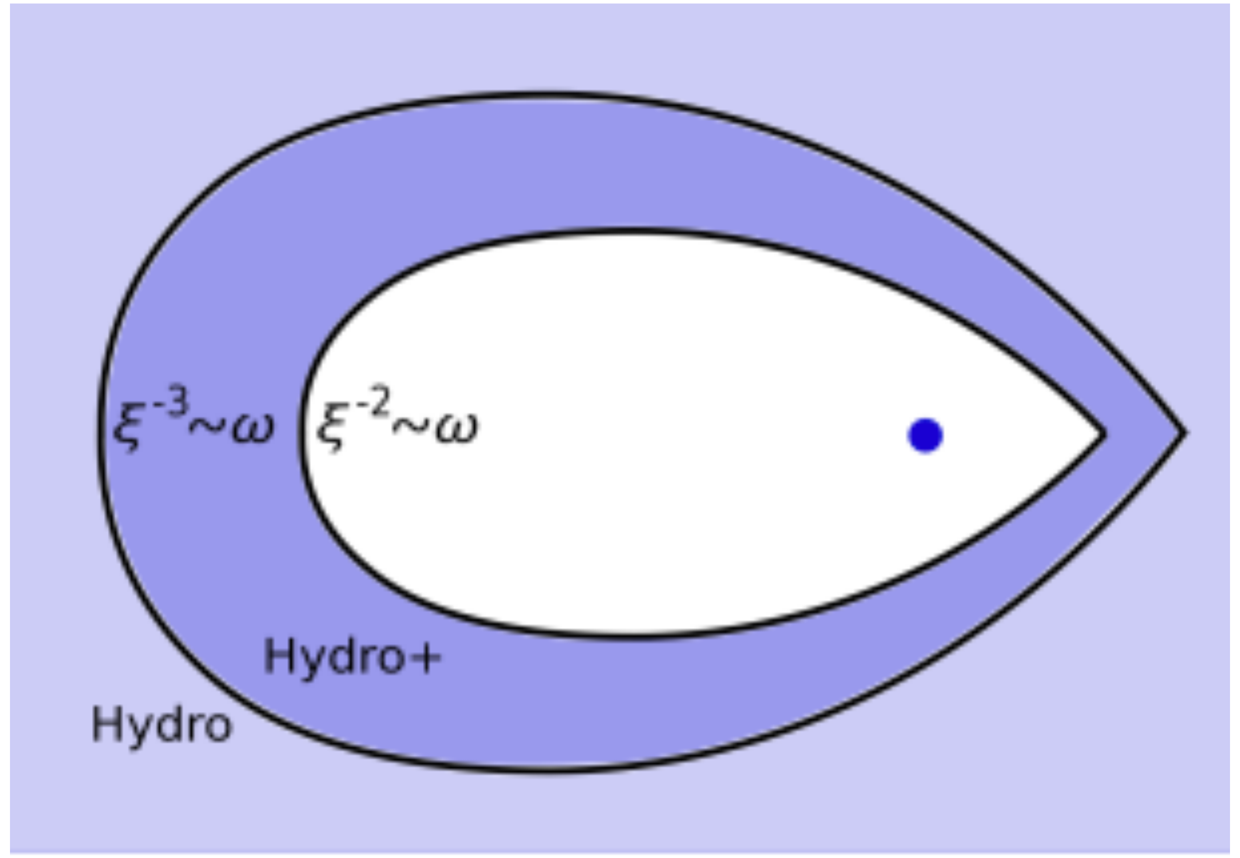
For general non-linear freeze-out prescription, refer MP, Stephanov, 22

Application : Freeze-out near the critical point

- Near the CP : Critical slowing down -> Relaxation to equilibrium is infinitely slow.

- The fluctuations of $\hat{s} \equiv s/n$ which relaxes parametrically as $\Gamma \sim \xi^{-3}$ is the slowest non-hydrodynamic mode

- Focus on a regime where only correlations of $\hat{s}$ are out of equilibrium - Hydro+



Application to Hydro+

Applying *maximum-entropy freeze-out* to a *Hydro+* simulation

where there is only one mode which is singular and out of equilibrium:

$$\Delta G_{AB} = \left(\frac{n_c}{\bar{c}_p T_c}\right)^2 \left[E_A - \frac{w_c}{n_c} q_A\right] \left[E_B - \frac{w_c}{n_c} q_B\right] f_A f_B \Delta \langle \delta\hat{s}\delta\hat{s}\rangle$$

$$\Delta G_{AB} = \widehat{\Delta} H_{ab} (\bar{H}^{-1} P \bar{G})^a_A (\bar{H}^{-1} P \bar{G})^b_B$$

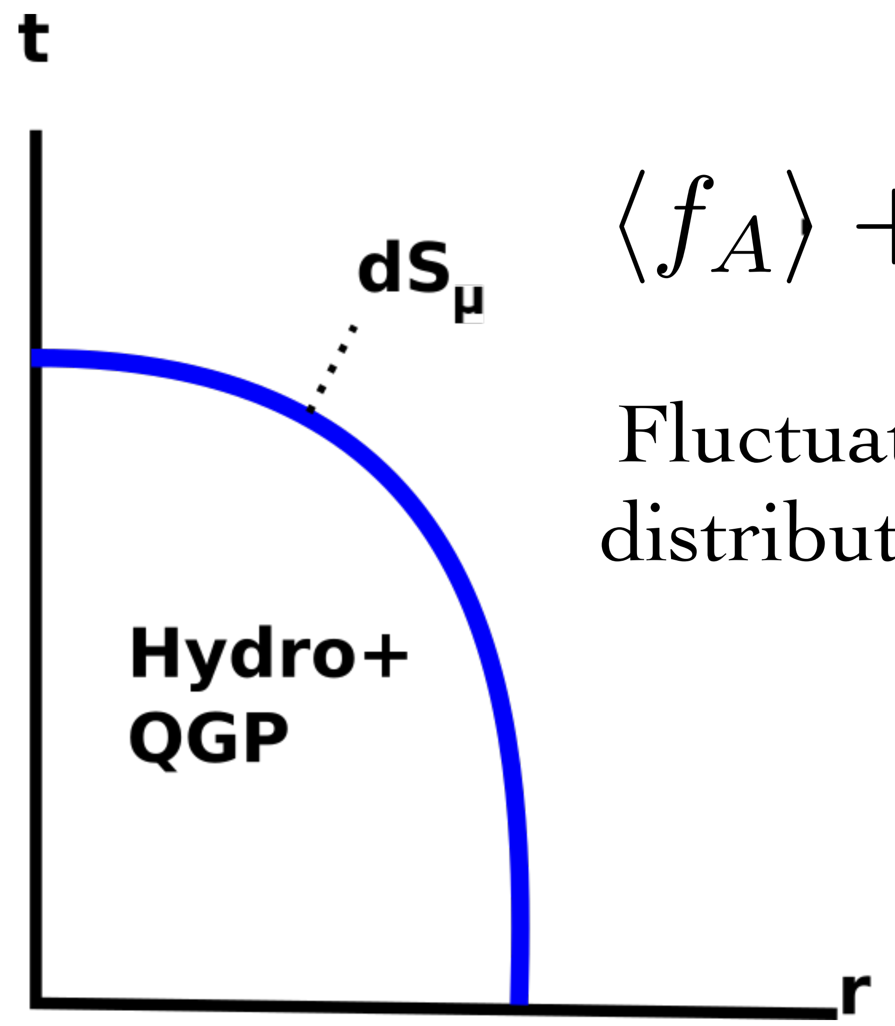
$$\widehat{\Delta} H_{\hat{s}\hat{s}} = \Delta \langle \delta\hat{s}\delta\hat{s}\rangle \ , \ \widehat{\Delta} H_{pp} = \widehat{\Delta} H_{p\hat{s}} = \widehat{\Delta} H_{pu^\mu} = \widehat{\Delta} H_{\hat{s}u^\mu} = \widehat{\Delta} H_{u^\nu u^\mu} = 0$$

n_c & w_c Baryon density and enthalpy at the critical point

$\bar{c}_p$ Specific heat of HRG in equilibrium

Now, we compare this to a previously used freeze-out prescription for critical fluctuations

Freeze-out prescription based on EFT near critical point



$\langle f_A \rangle + \delta f_A$

Fluctuating particle distribution function

We incorporate the effects of critical fluctuations via the modification of particle masses due to their interaction with the critical sigma field

$$\delta m_A \approx g_A \sigma$$ **Stephanov, Rajagopal, Shuryak, 1999**

Fluctuating particle distribution function

$$f_A = \langle f_A \rangle + g_A \frac{\partial \langle f_A \rangle}{\partial m_A} \sigma$$

$$\langle \sigma \rangle = 0, \; \langle \sigma(x_+) \sigma(x_-) \rangle = Z^{-1} \langle \delta \hat{s}(x_+) \delta \hat{s}(x_-) \rangle$$

MP, Rajagopal, Stephanov, Yin, 22

Freeze-out of Gaussian fluctuations near the critical point

Unknowns!

$$\Delta G_{AB} \equiv \langle \delta f_A \delta f_B \rangle = \frac{g_A g_B}{Z T^2} \frac{m_A}{E_A} \frac{m_B}{E_B} f_A f_B \langle \delta \hat{s} \delta \hat{s} \rangle$$

$$\Delta \langle \delta N_A \delta N_B \rangle_\sigma = d_A \, d_B \int D p_A \int D p_B \int (dS \cdot p_A) \int (dS \cdot p_B) \Delta G_{AB}$$

Deviations from baseline
(critical+dynamical effects)

$$\langle \delta N_A \delta N_B \rangle = \langle N_A \rangle \, \delta_{AB} + \Delta \langle \delta N_A \delta N_B \rangle_\sigma$$

Poisson (or more generally, baseline) contribution

MP, Rajagopal, Stephanov, Yin, 22

Maximum-entropy freeze-out

$$\Delta G_{AB} = \left(\frac{n_c}{\bar{c}_p T_c}\right)^2 \left[E_A - \frac{w_c}{n_c} q_A\right] \left[E_B - \frac{w_c}{n_c} q_B\right] f_A f_B \Delta \left\langle \delta\hat{s}\delta\hat{s} \right\rangle$$

Agrees with the prescription obtained using the *EFT with sigma field*:

$$\Delta G_{AB} = \frac{g_A g_B}{Z T^2} \frac{m_A}{E_A} \frac{m_B}{E_B} f_A f_B \Delta \left\langle \delta\hat{s}\delta\hat{s} \right\rangle$$

if g_As have a specific energy dependence

MP,Stephanov, 22

Hydrodynamic fluctuations → **MAXIMUM ENTROPY FREEZE-OUT** → Cumulants of particle multiplicities

Mixed correlations between event by event multiplicities of pions and low energy protons can become negative near CP.

$$\Delta G_{p\pi} \approx \left(\frac{n_c}{\bar{c}_p T_c}\right)^2 \overbrace{\left[-\frac{w_c}{n_c}\right]}^{<0} E_\pi f_p f_\pi \Delta \langle \delta\hat{s}\delta\hat{s}\rangle < 0$$

ME freeze-out is currently being employed to make estimates for cumulants of particle multiplicities in simplified settings.