Bayesian analysis of nontrivial features in the c_s^2 inside neutron stars in light of astrophysical and pQCD constraints

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Bayesian analysis: 2309.02345, D. Mroczek, M. C. Miller (Maryland), J. Noronha-Hostler and N. Yunes (UIUC/ICASU)
pQCD constraints: In preparation, D. Mroczek, R. Somasundaram (Syracuse/Los Alamos), I. Tews (Los Alamos), M. C. Miller (Maryland), J. Noronha-Hostler and N. Yunes (UIUC/ICASU), C. Drischler (Ohio U.), J. Margueron (Lyon)

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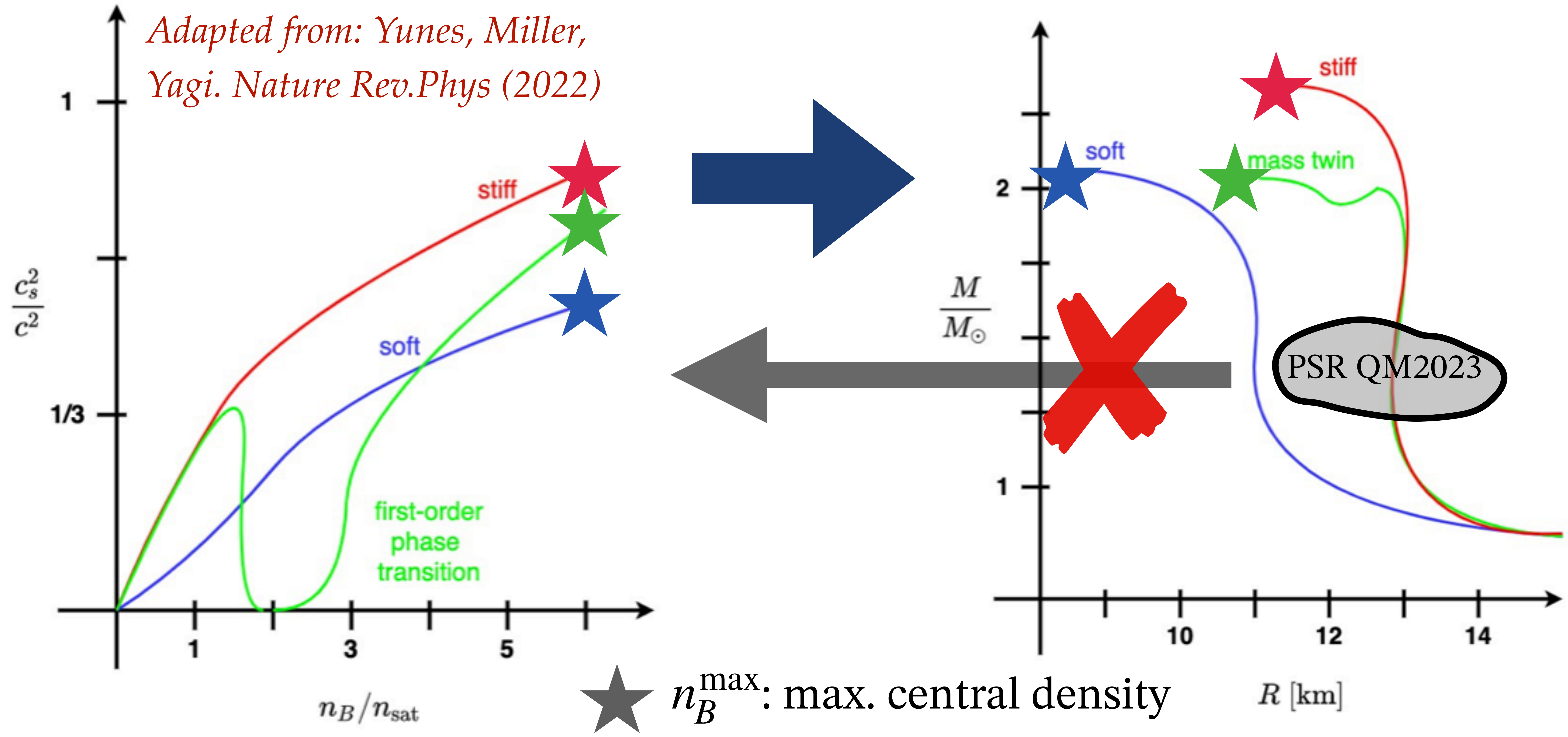
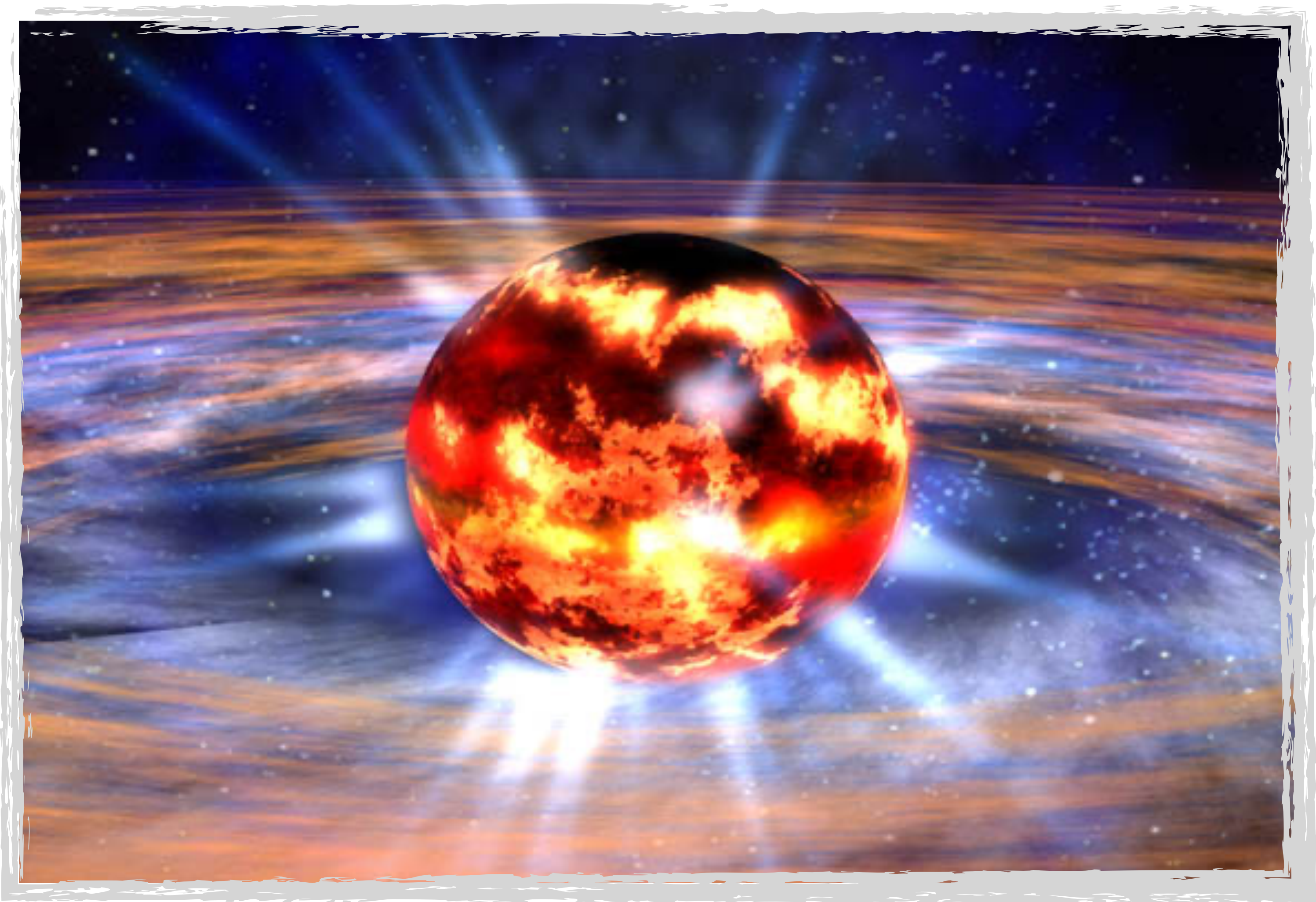
Learning the T=0, high n_B EoS from neutron stars

- Neutron stars have macroscopic properties that we can measure.
- For stable, slowly-rotating stars, these observables depend only on the EoS.

From any EoS → M-R, Λ-M sequence

- We have reliable mass (M), radius (R), and tidal deformability (Λ) measurements.





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Bayesian statistics and choosing a prior

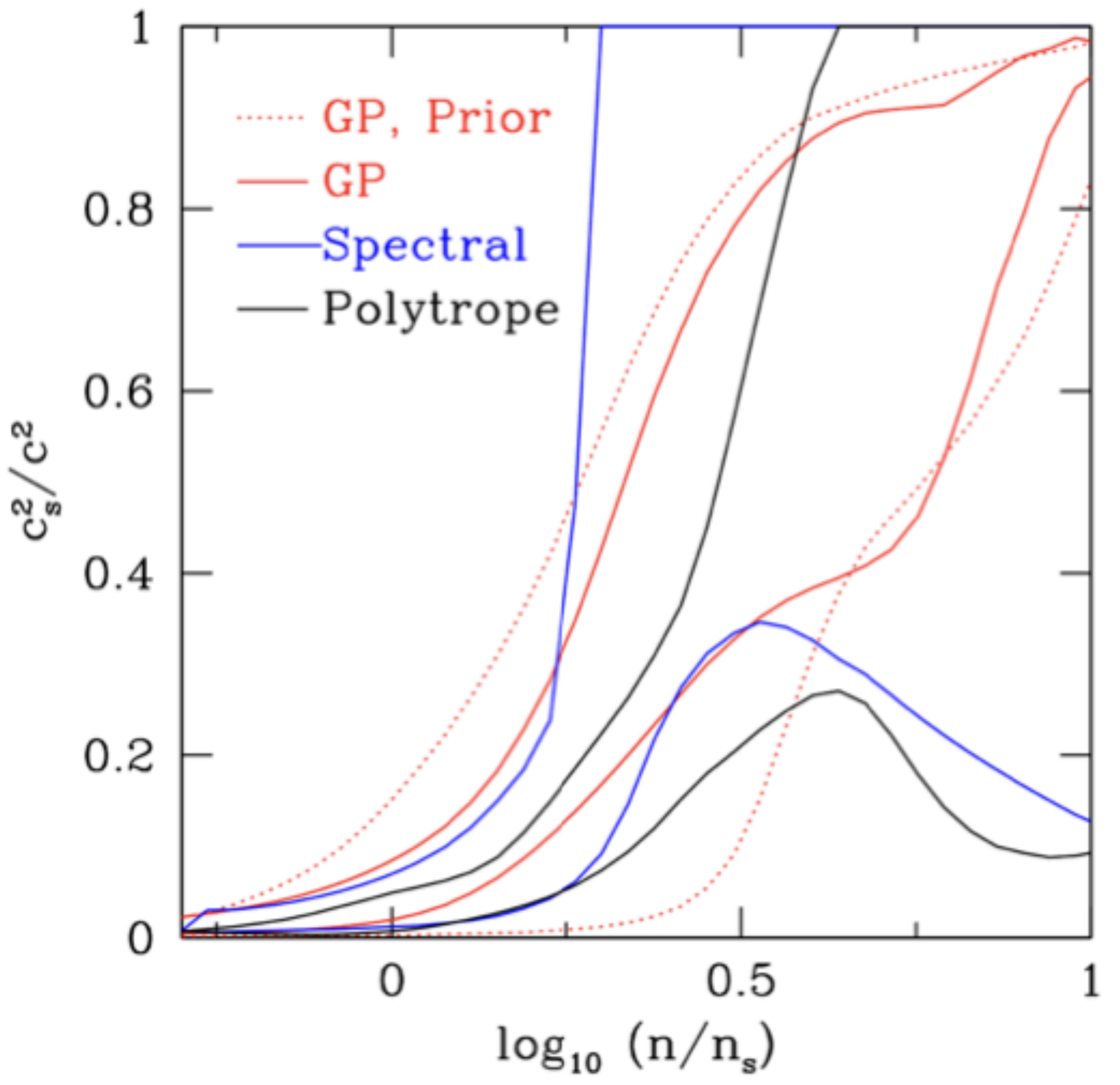
$$p_k = \frac{q_k \mathscr{L}_k}{\int q_l \mathscr{L}_l dl}$$

p_k: posterior, q_k: prior, $\mathscr{L}_k$: likelihood

$\longrightarrow$ model evidence

Infinitely many possible EoS:
How do we account for all possibilities?

Prior dependence: test different priors



Miller et al. AJL (2021)

- Model-agnostic approaches are common

→ Gaussian processes (GPs):

EoS modeled via: $\phi(x) = \log(1/c_s^2 - 1)$, **stable and causal**

$$\phi \sim \mathcal{N}(\mu_i, \Sigma_{ij})$$

Collection of functions, behavior specified by a mean and covariance kernel

Squared-exponential is a common choice:

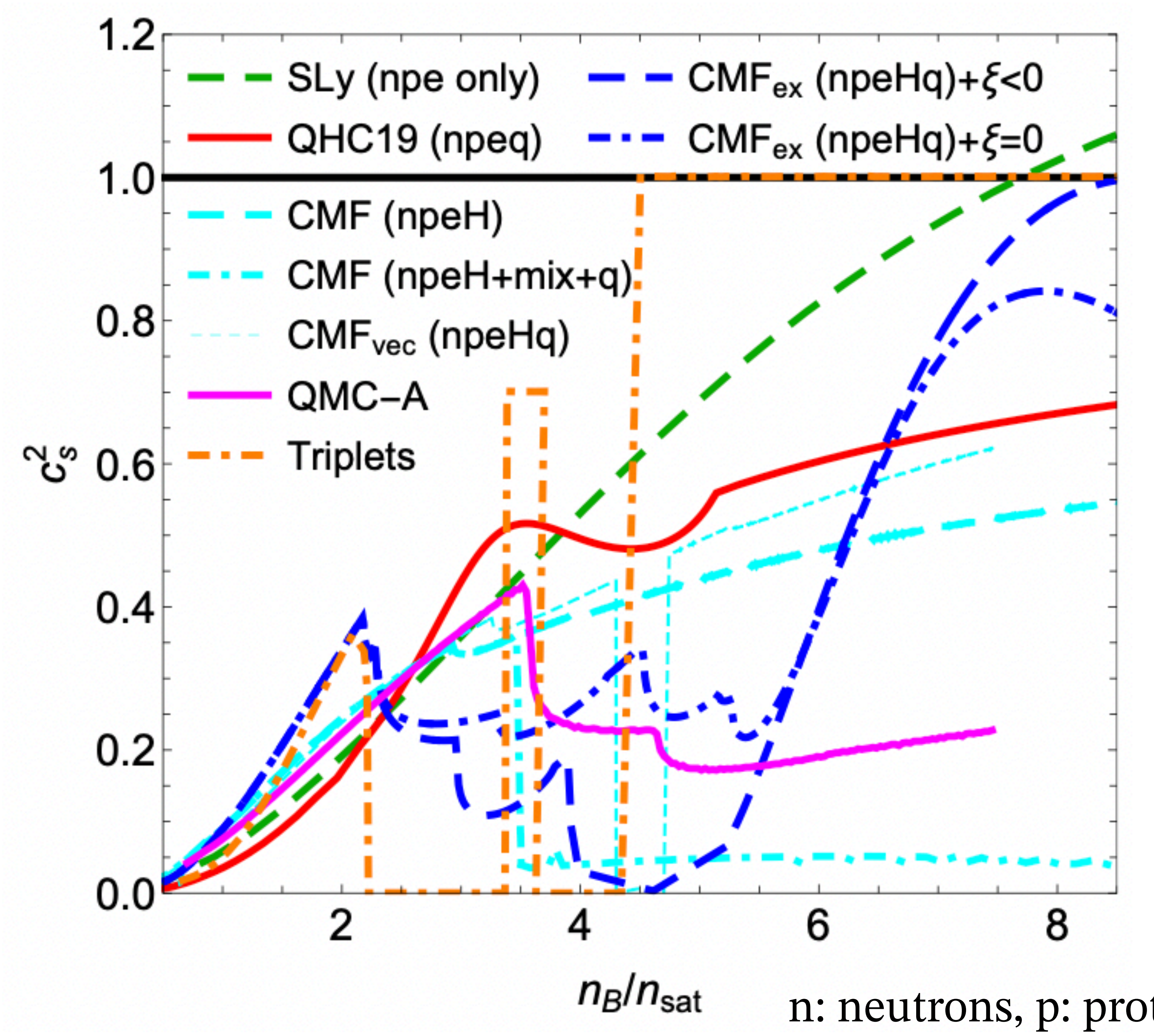
$$K_{se}\left(x_i, x_j\right) = \sigma^2 \exp\left[-\left(x_i - x_j\right)^2 / 2\ell^2\right]$$

ℓ: correlation length

σ: correlation strength

Influence of exotic degrees of freedom on the EoS from nuclear physics models

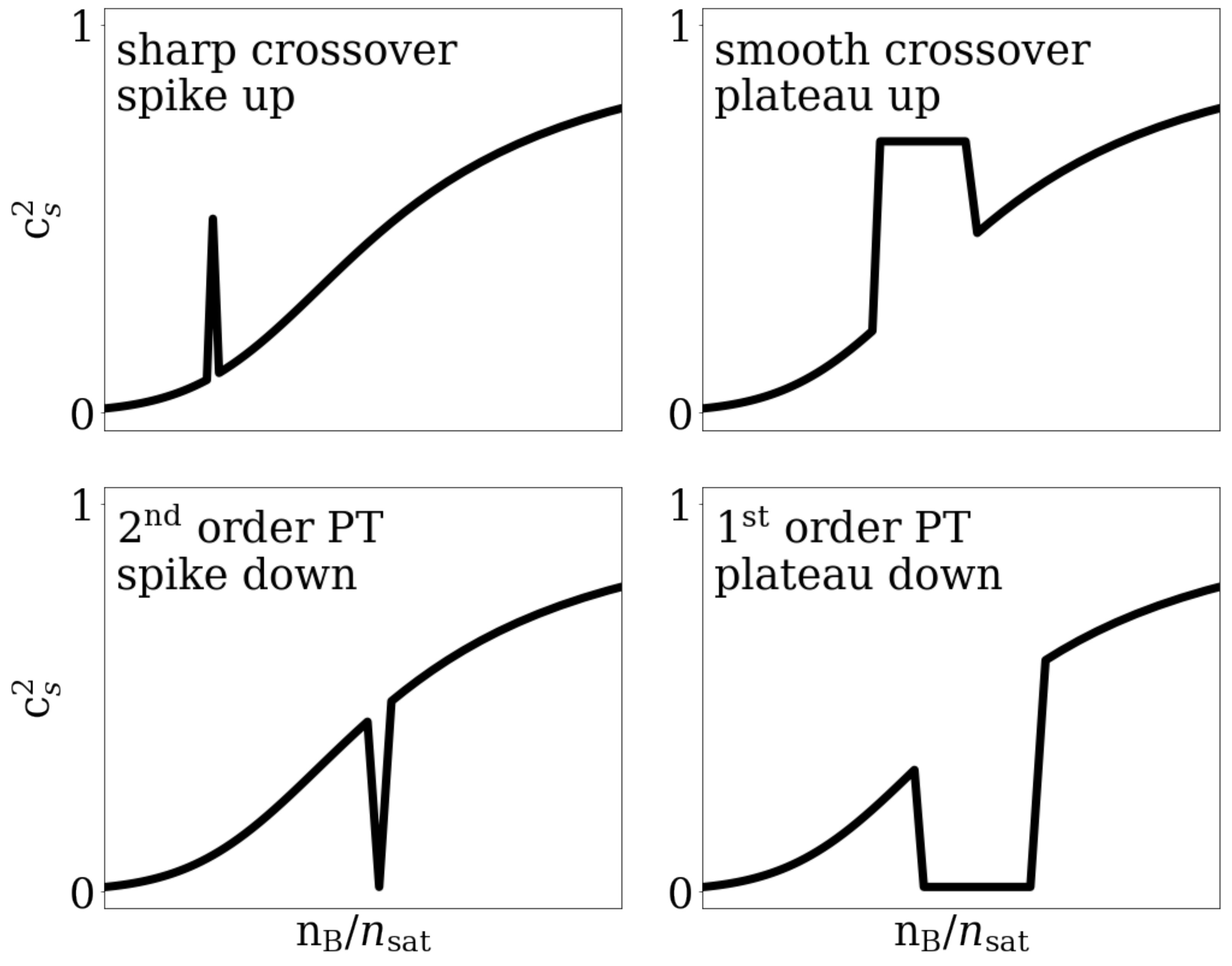
Multi-scale correlations characterize the onset of exotic phases



from: Tan et al. PRD (2022), see for refs.

n: neutrons, p: protons, e: electrons, q: quarks, H: hyperons

Modeling **non-trivial structure in c_s^2** from a phenomenological perspective



Physically-motivated long + short/ medium length correlations in n_B

Bayesian friendly!

→ systematic study + model comparison

D. Mroczek et al. 2309.02345

Are there nontrivial features in the c_s^2 inside neutron stars?

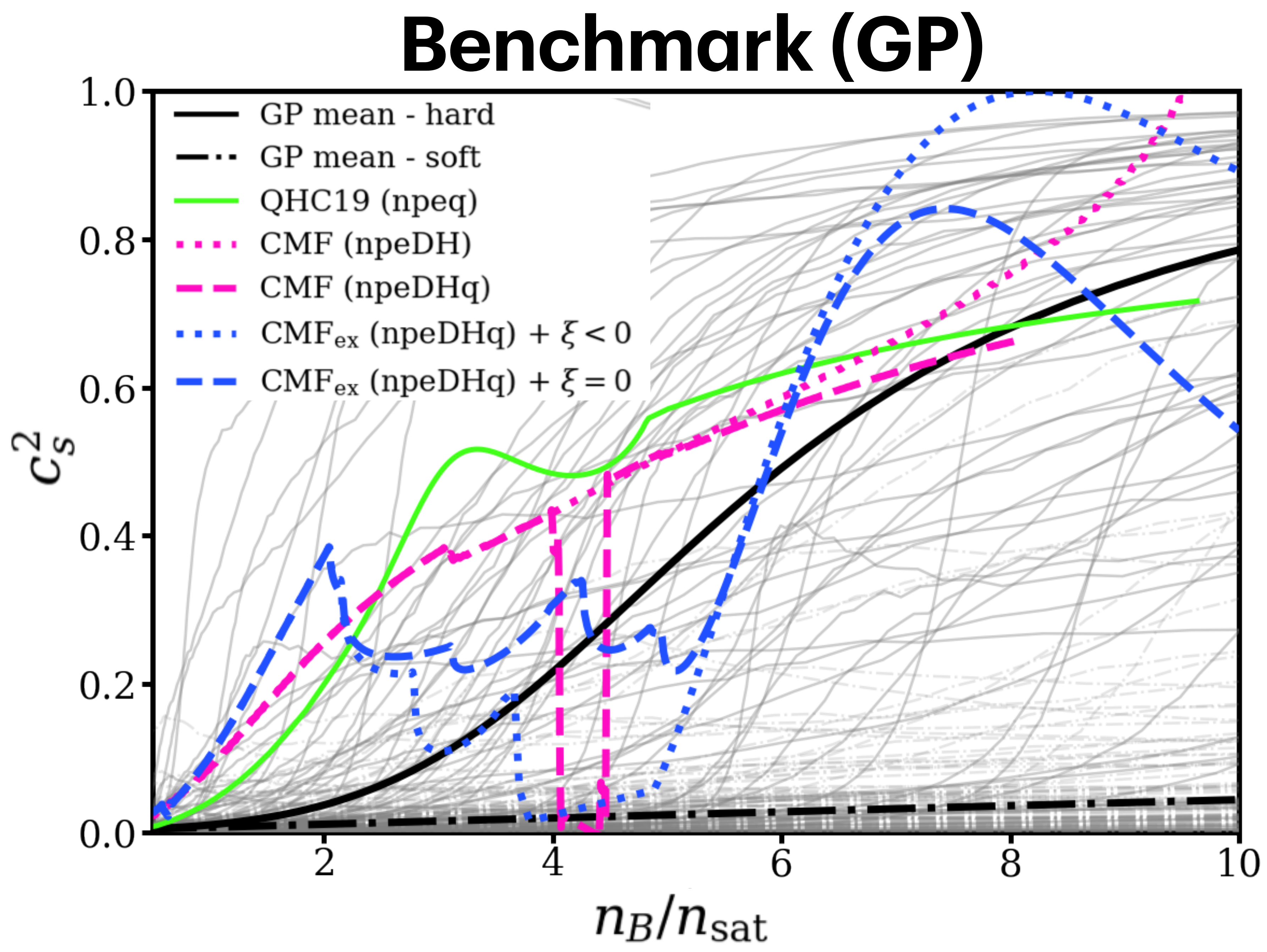
$$p_k = \frac{q_k \mathscr{L}_k}{\int q_l \mathscr{L}_l dl}$$

↗ model evidence ($\mathscr{E}$): quantifies level of support of the data for a given model

Bayesian model comparison:

$$\text{Bayes factor } K = \frac{\mathscr{E}_{\text{benchmark}}}{\mathscr{E}_{\text{structure}}}$$

- Benchmark model in gray: GP with long-range correlations fixed across all densities
- Modified GP (mGP): multi-scale correlations → emergence of exotic degrees of freedom



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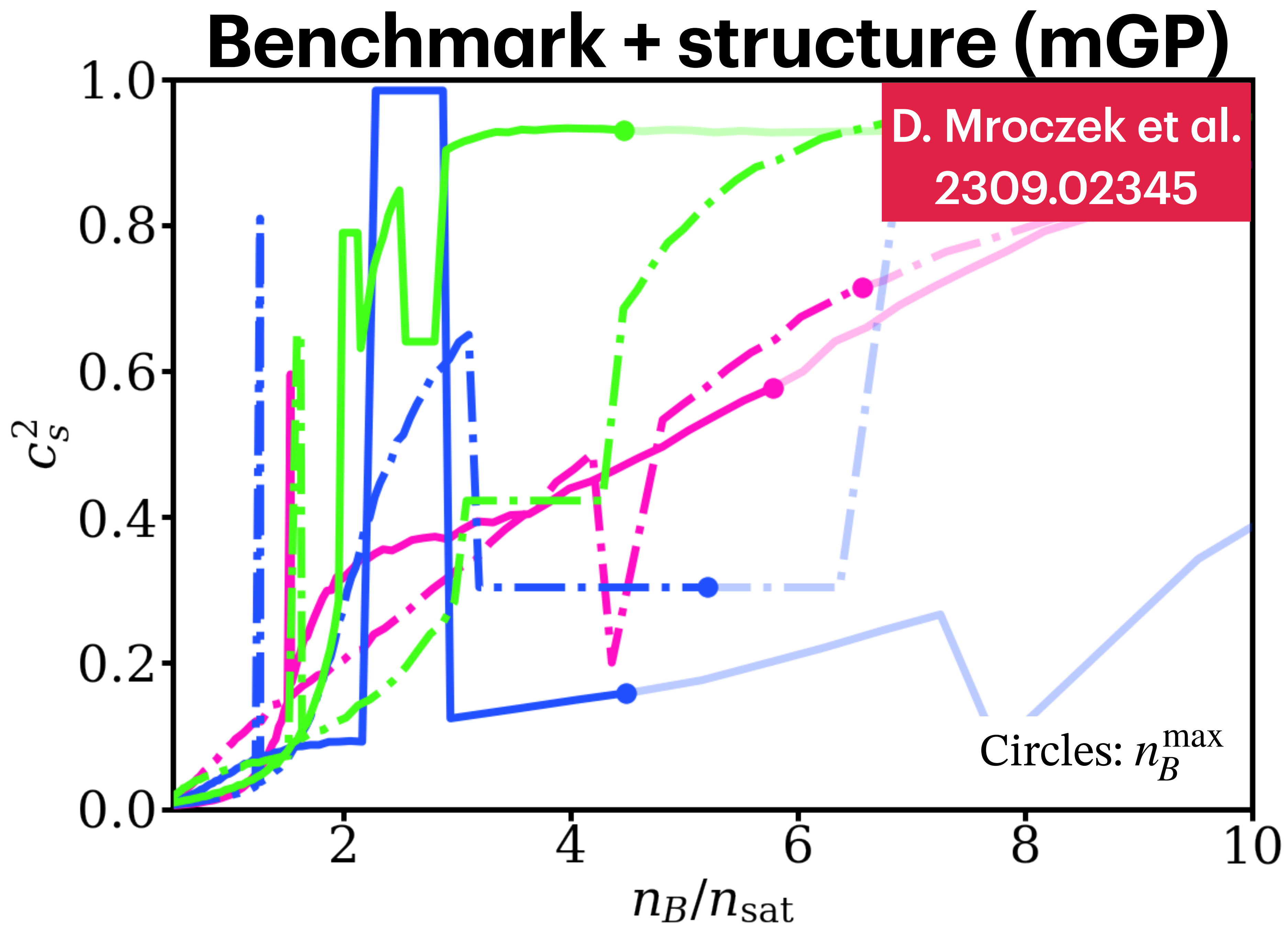


*more on pQCD later

Astrophysical and theoretical constraints

Astrophysics

- $\mathbf{M_{max}}$: 3 highest measured NS masses from Shapiro-delay measurements (~2.0 $M_\odot$)
- $\mathbf{\Lambda}$: GW170817 and GW190425 ($\tilde{\Lambda}, M_{ch}, M_1, M_2$)
- **M-R:** NICER (IL/MD) PSR J0740 + 6620, PSR J0030 + 0451

See Mroczek et al. 2309.02345 for refs.

Low-energy

Symmetry energy:
$E_{sym} = 32 \pm 2$ MeV

Tsang et al. PRC (2012)

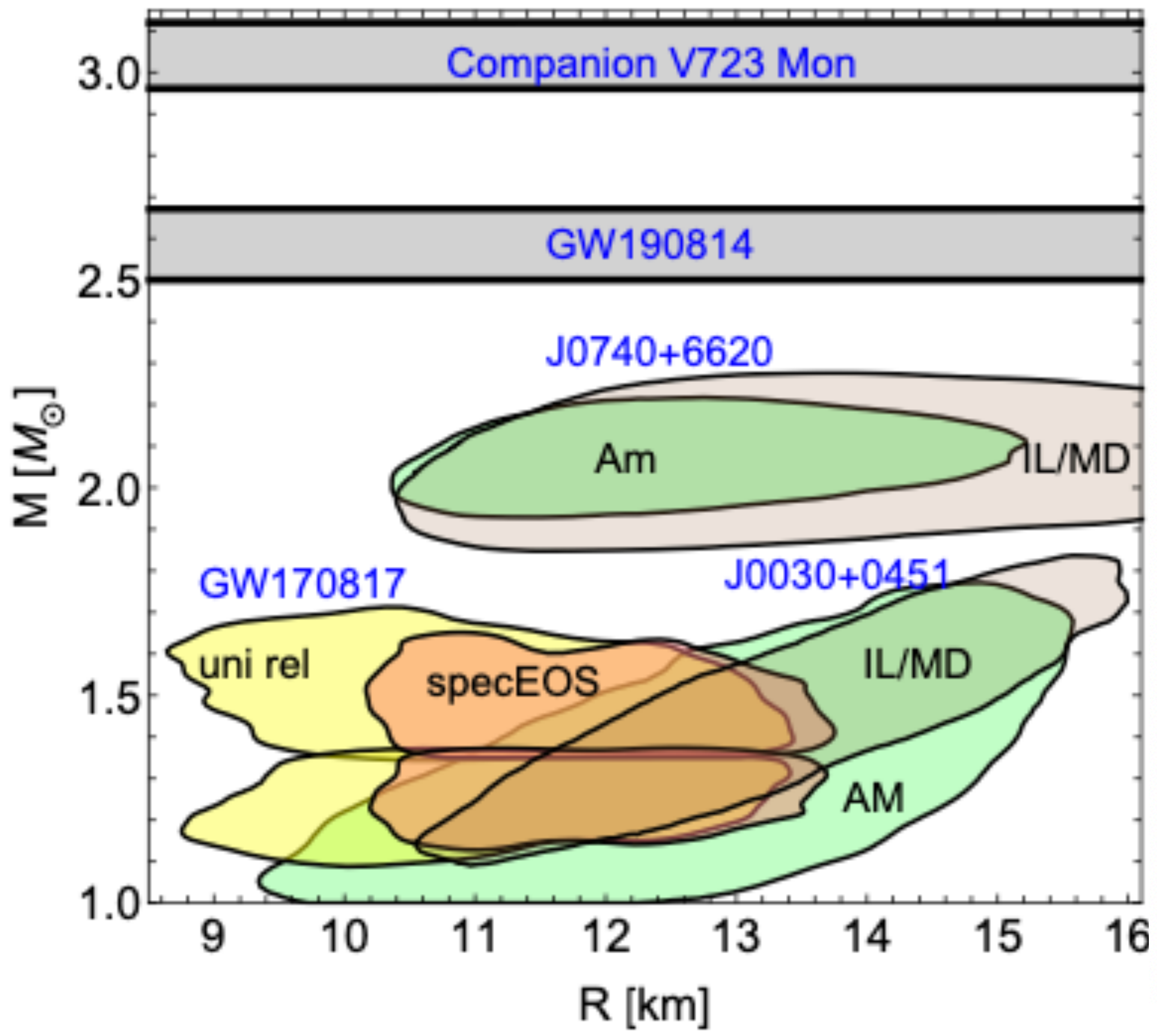
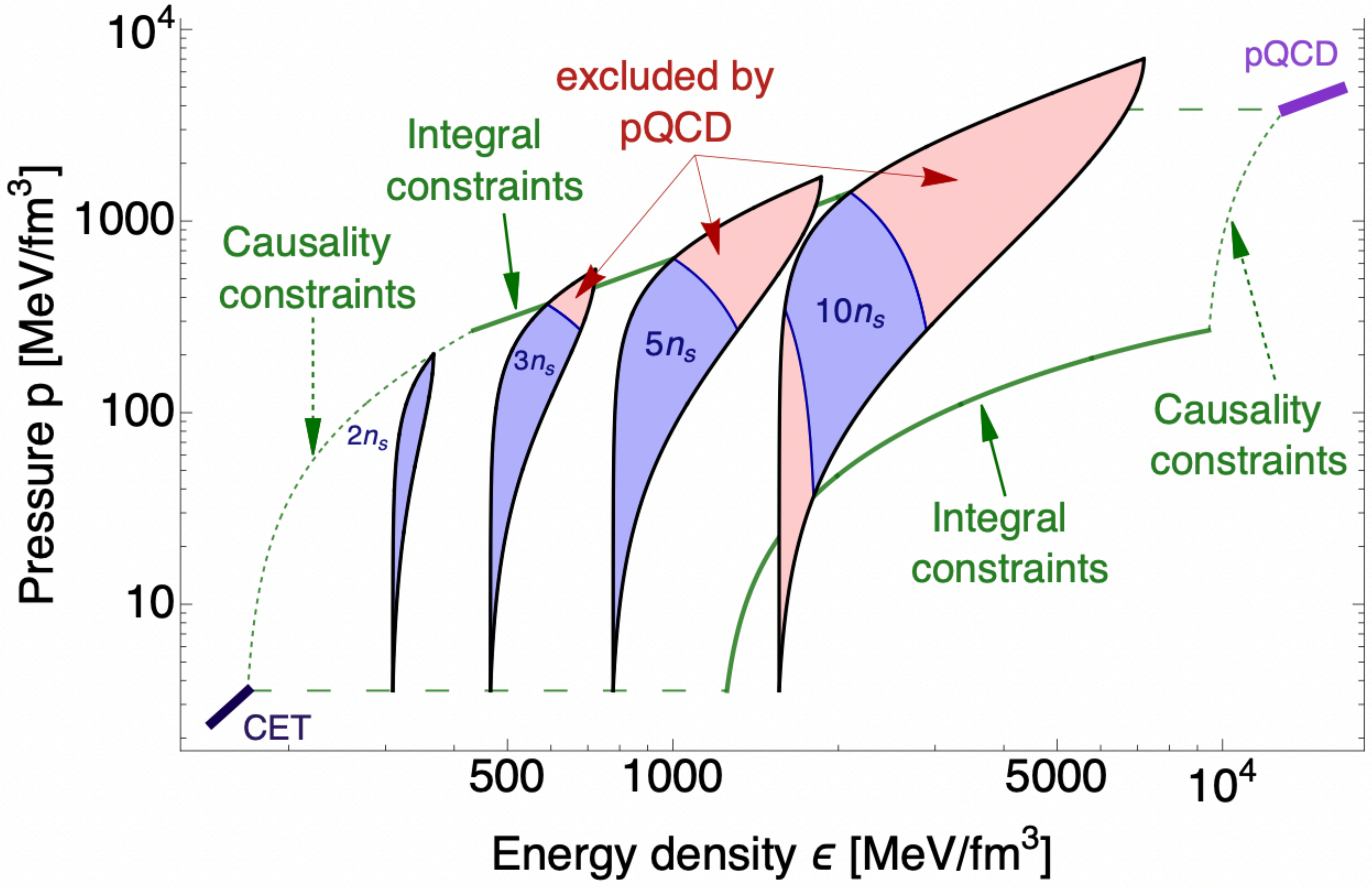


Fig. from: Tan et al. PRD (2022), see for refs.



pQCD constraints: Komoltsev, Kurkela PRL (2022), Gorda et al. Astrophys. J. (2023)

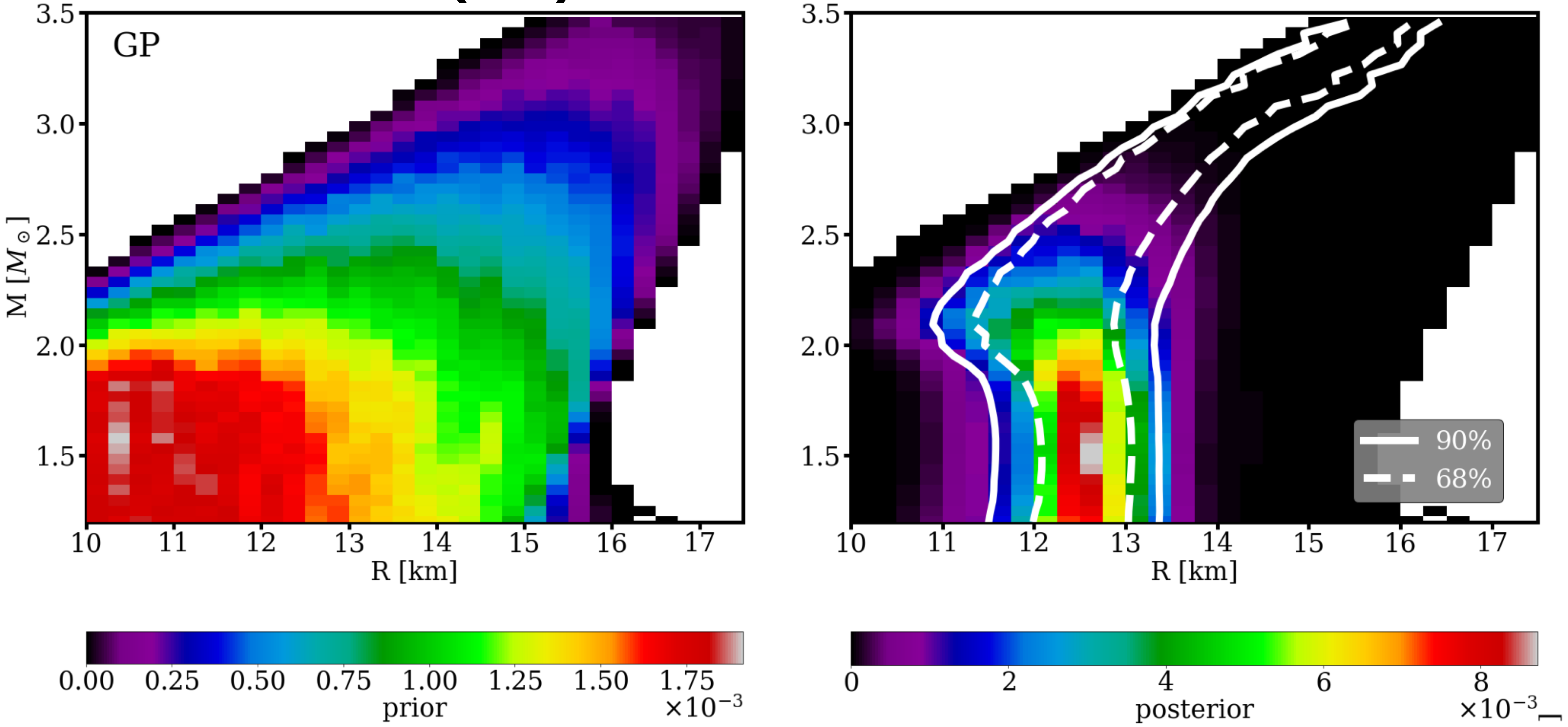
pQCD*

- partial N3LO results, propagated using causality, stability, and integral constraints down to $\mathbf{n_B^{max}}$ for each EoS.
- Truncated expansion uncertainty accounted for with scale-averaging.

pQCD results: Gorda et al. PRL 127 (2021) and PRD 104 (2021)

Are M-R posteriors sensitive to structure in $c_s^2(n_B)$?

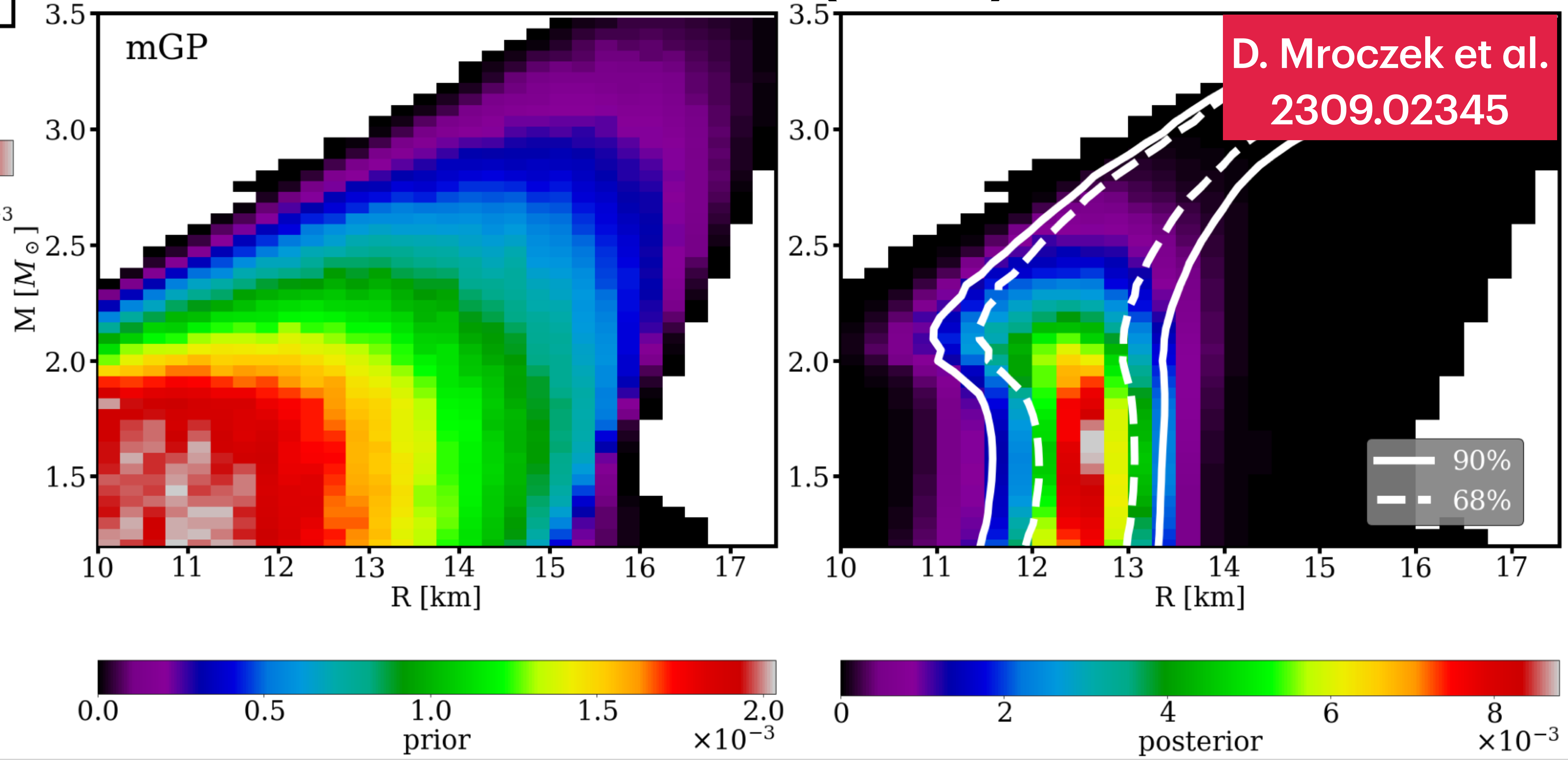
Benchmark (GP)



Why check M-R priors and posteriors?

- Diverse neutron star EoS prior = broad prior in M-R
- Sanity check: can we reproduce measurements when we assume nontrivial features in $c_s^2(n_B)$?

Benchmark + structure (mGP)



We find **no differences** in the prior or posterior probability distributions between benchmark (GP) and benchmark + structure (mGP)

Are $c_s^2(n_B)$ posteriors sensitive to structure in $c_s^2(n_B)$?

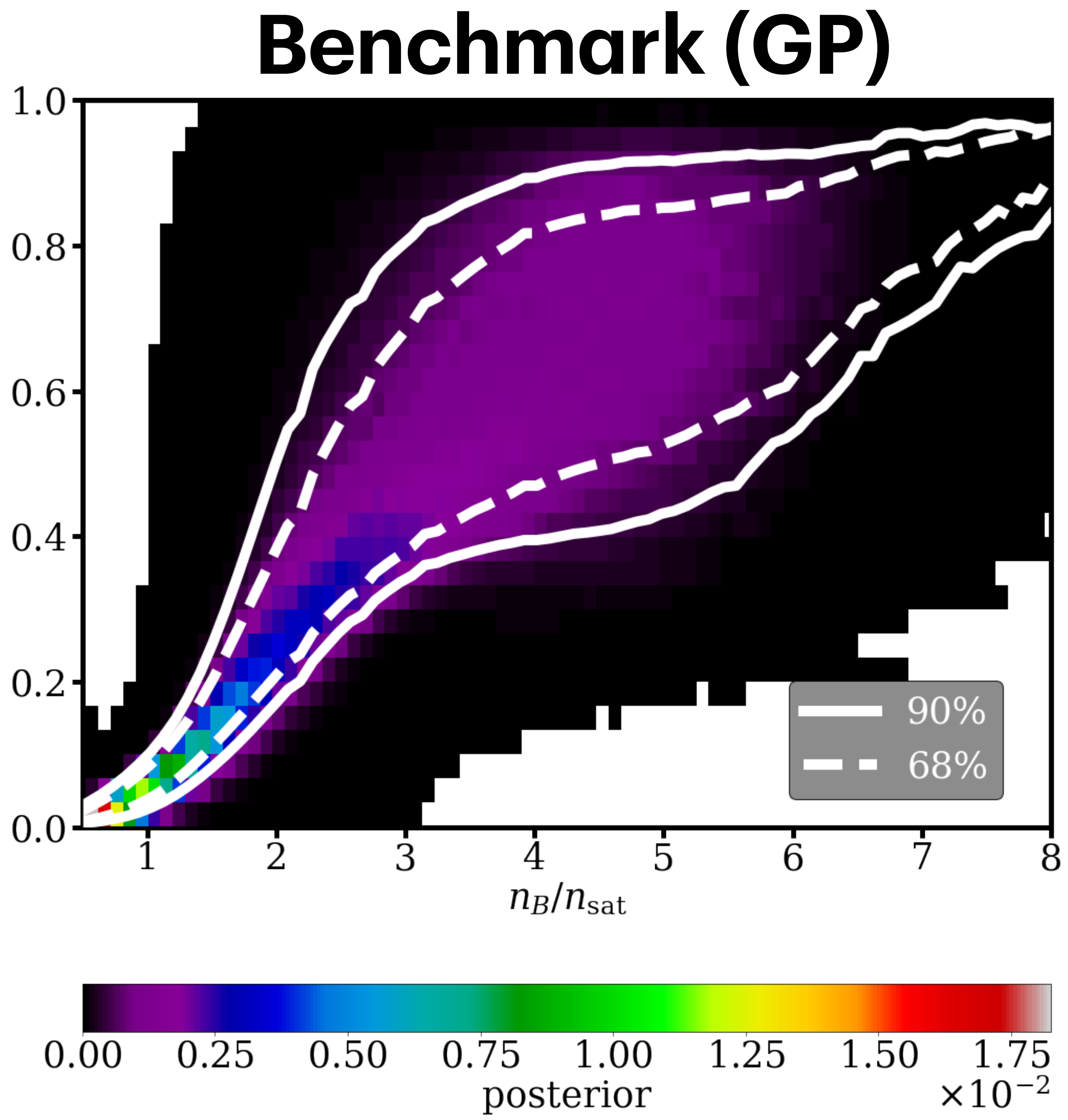
D. Mroczek et al. 2309.02345

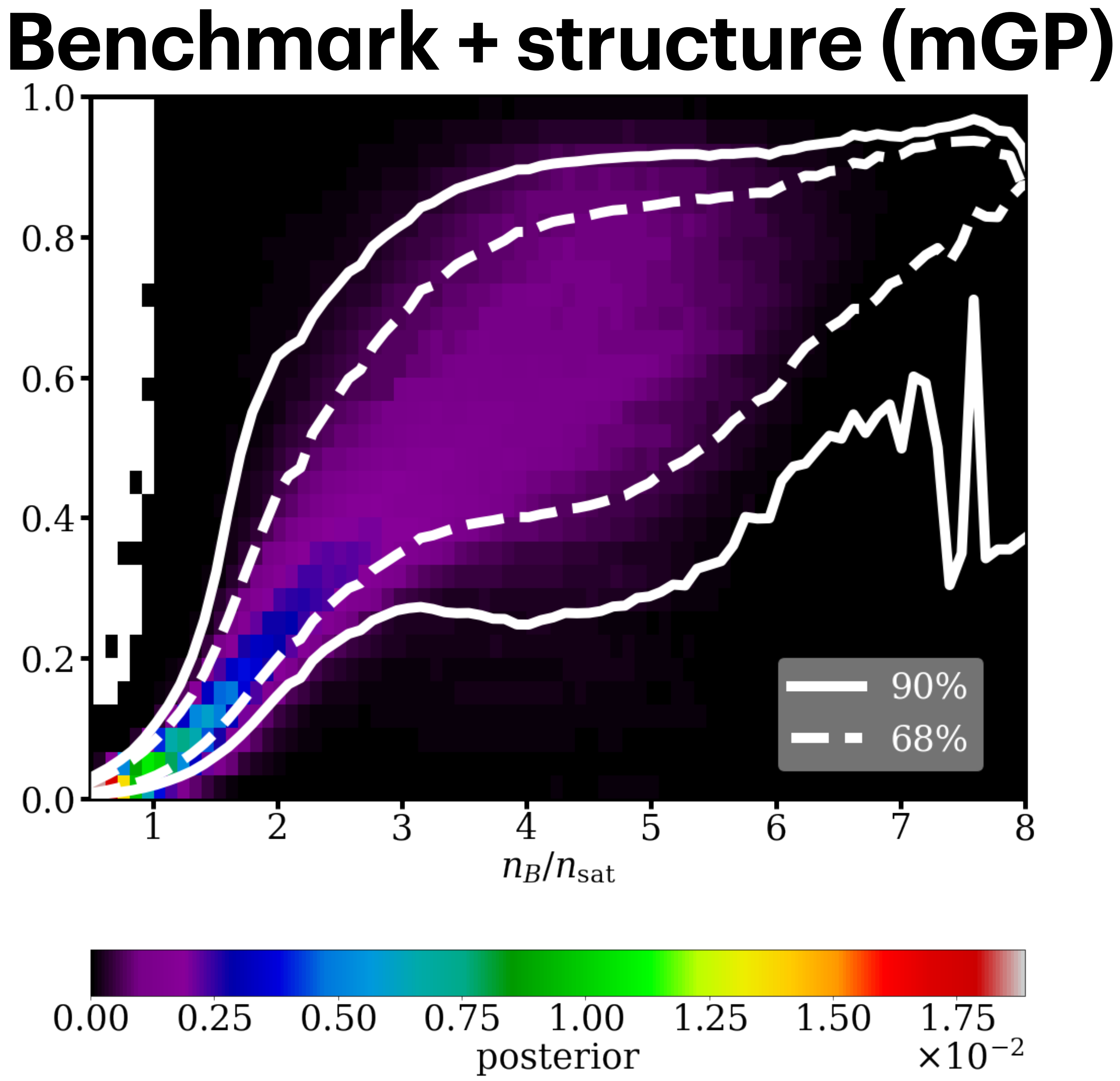
Constraints affect priors differently:

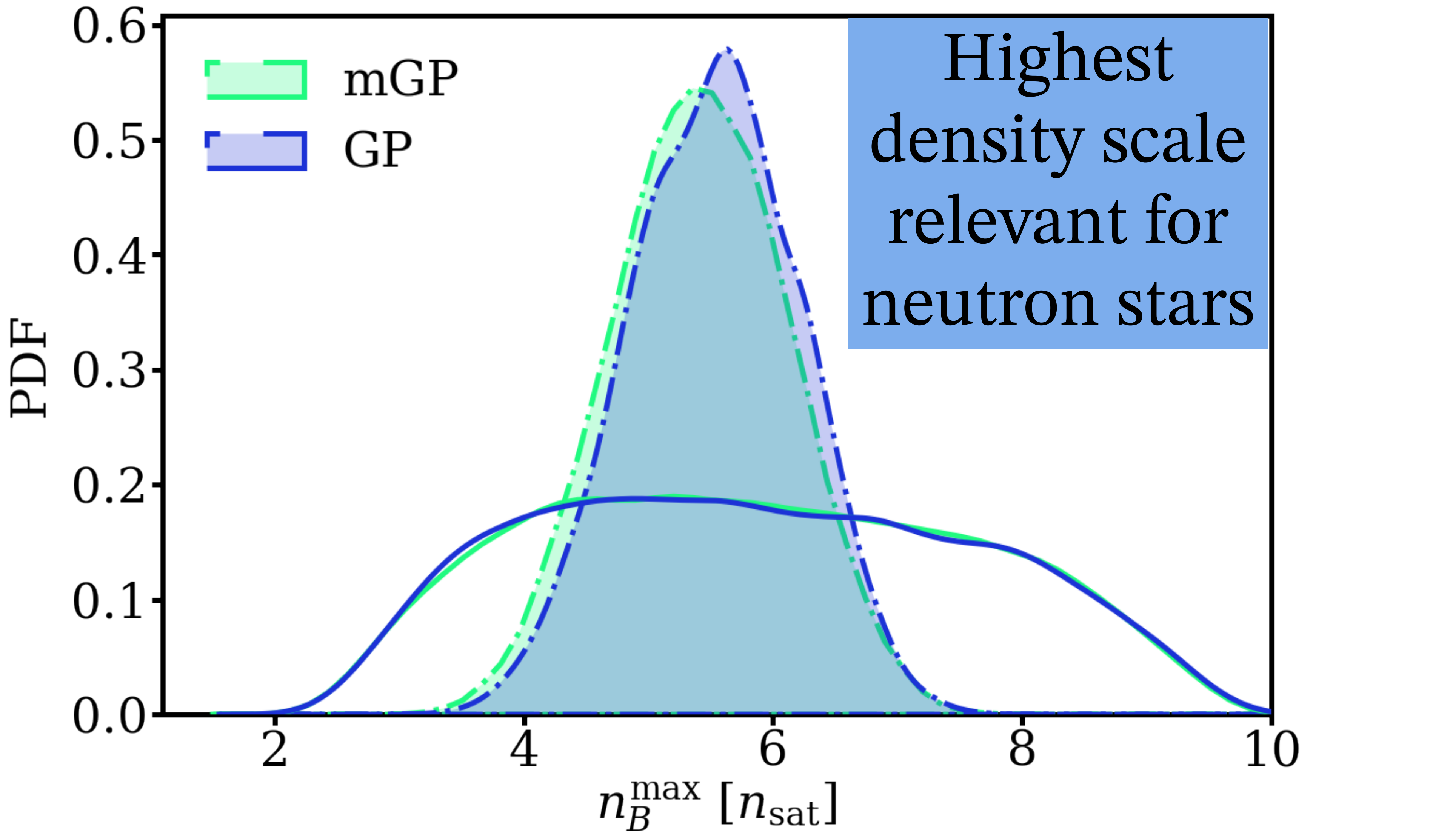
Long-range correlations → **tighter** c_s^2 posterior
Structure → **broader** c_s^2 posterior

- EoS are shown up to n_B^{max}

→ credibility bands are correlated with posterior for n_B^{max}







- Constraints favor $n_B^{max} \sim 5-7\ n_{sat}$

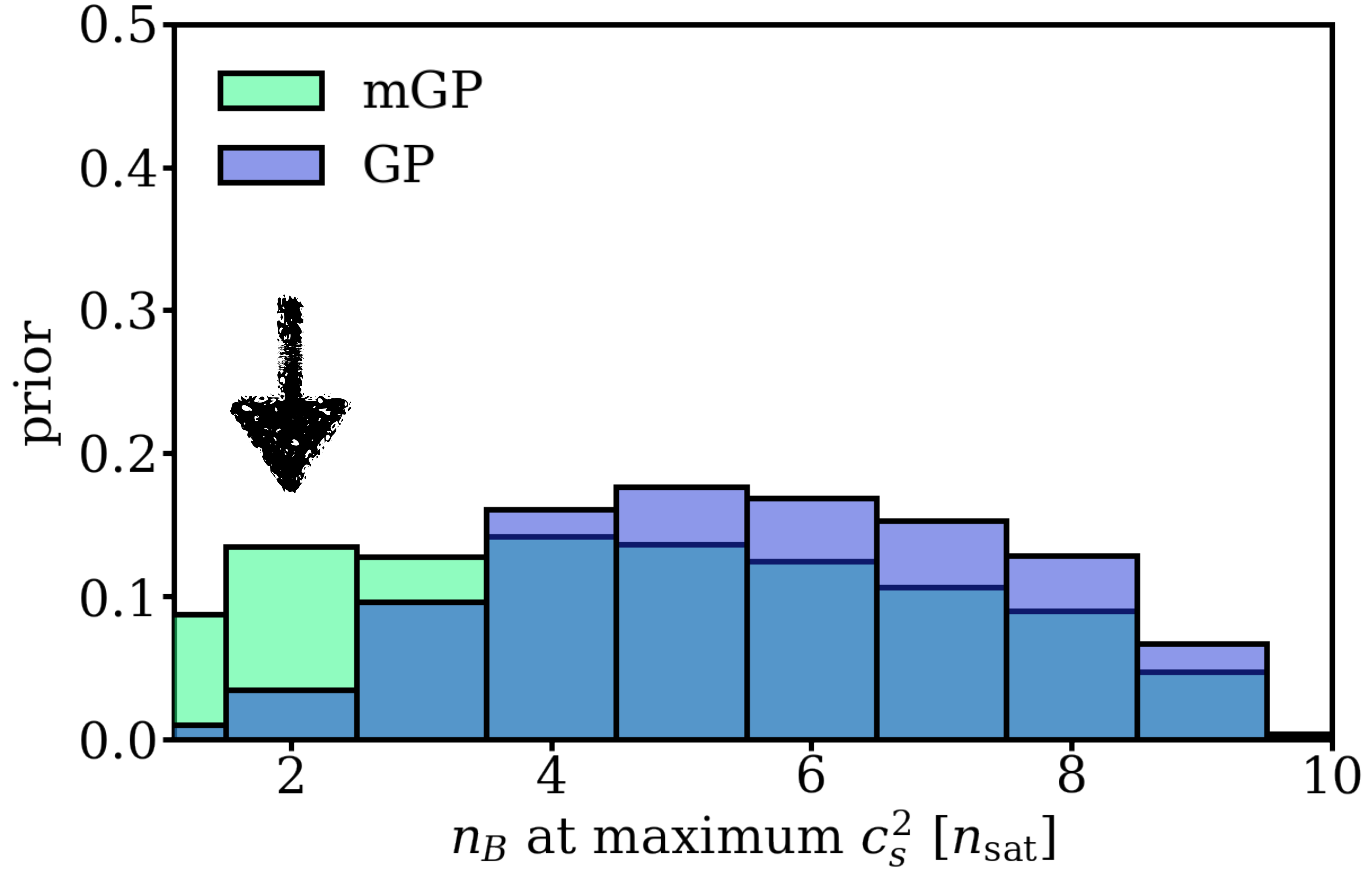
Does $c_s^2(n_B)$ display a peak within neutron star densities?

Bump in c_s^2: softening of the EoS signaling crossover to new degrees of freedom.

→ **global maximum in c_s^2 that occurs within neutron star densities**

Prior

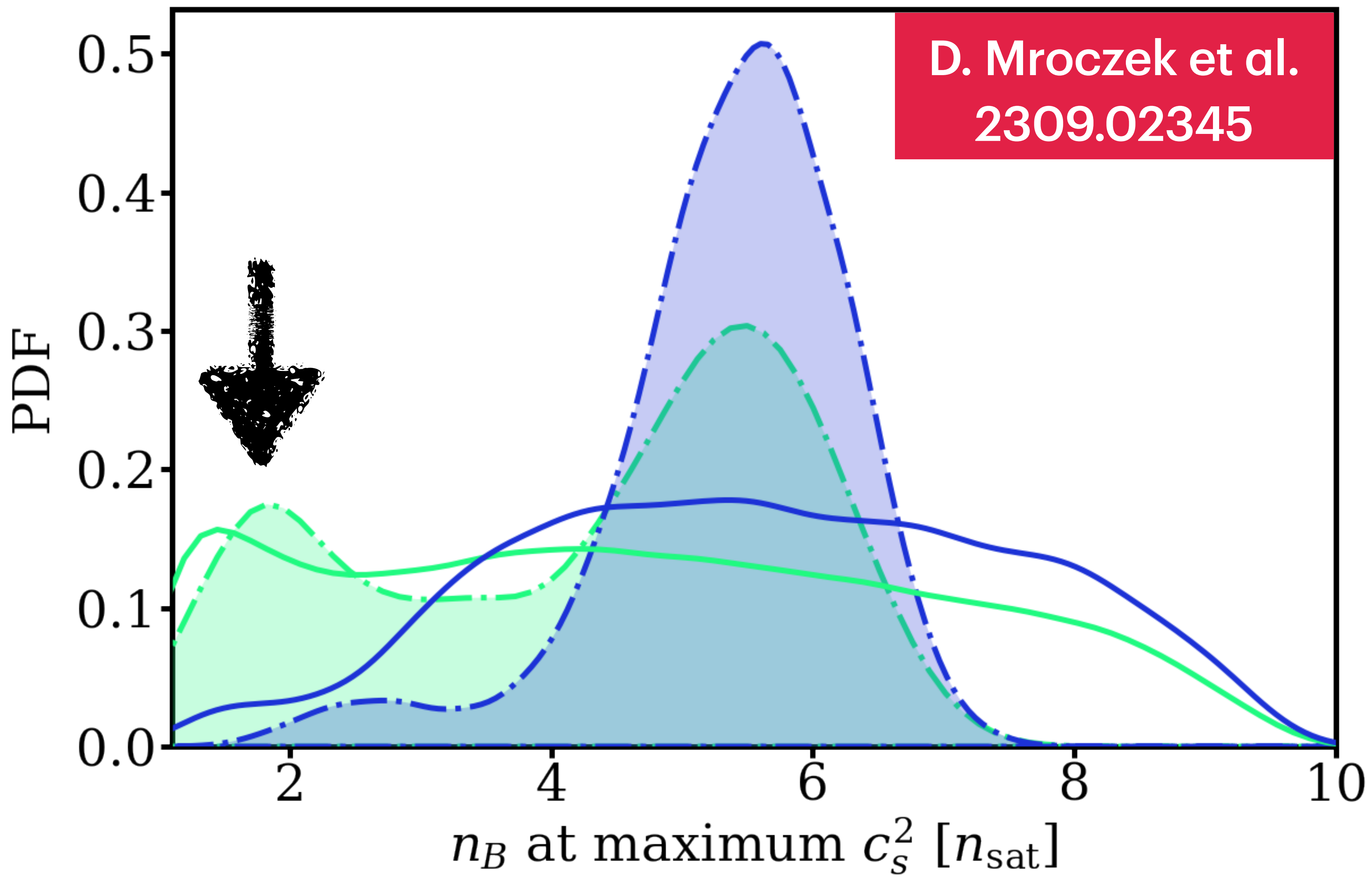
Multi-scale correlations allow for a bump **before** 3 n_{sat}



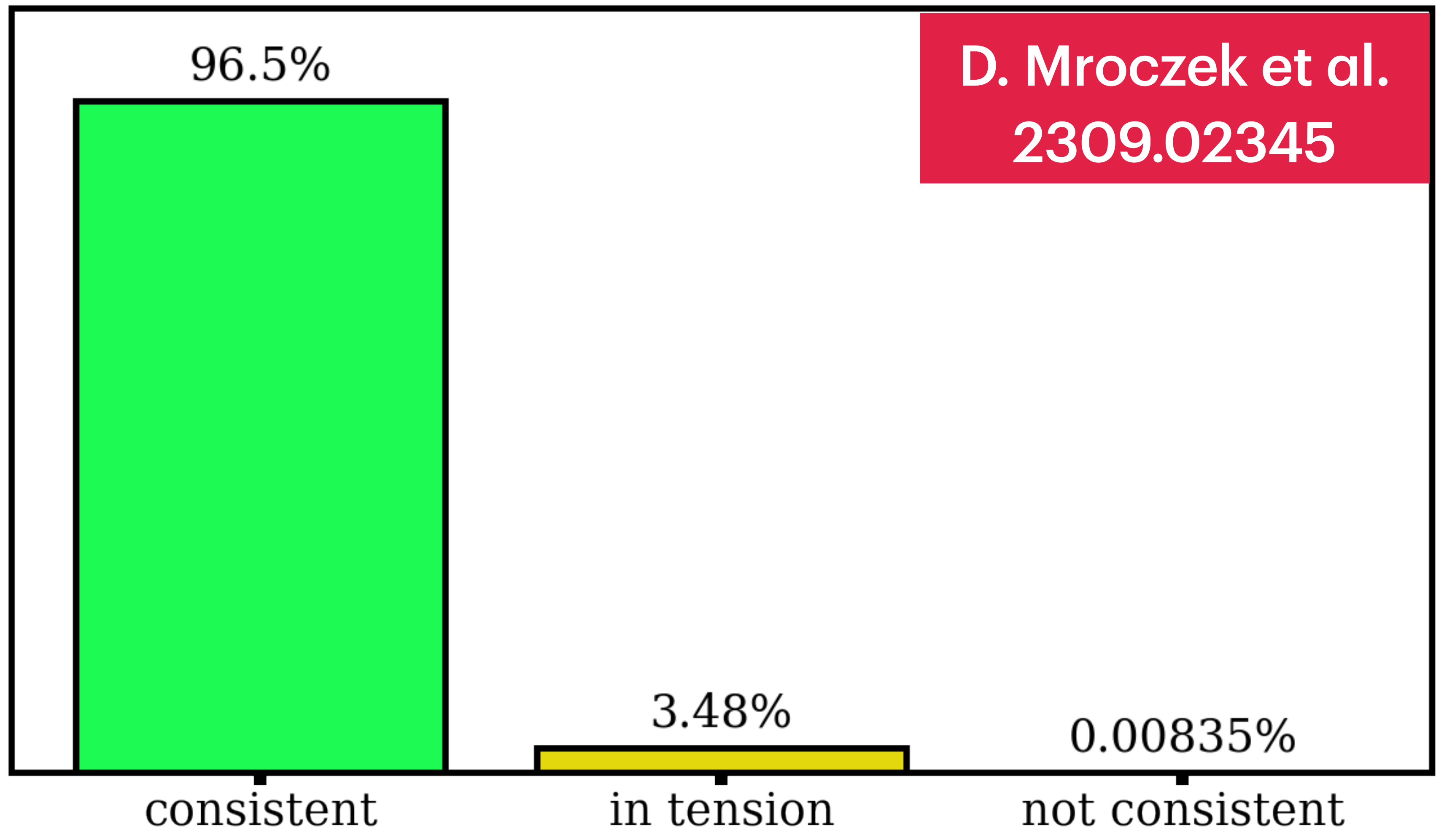
Posterior

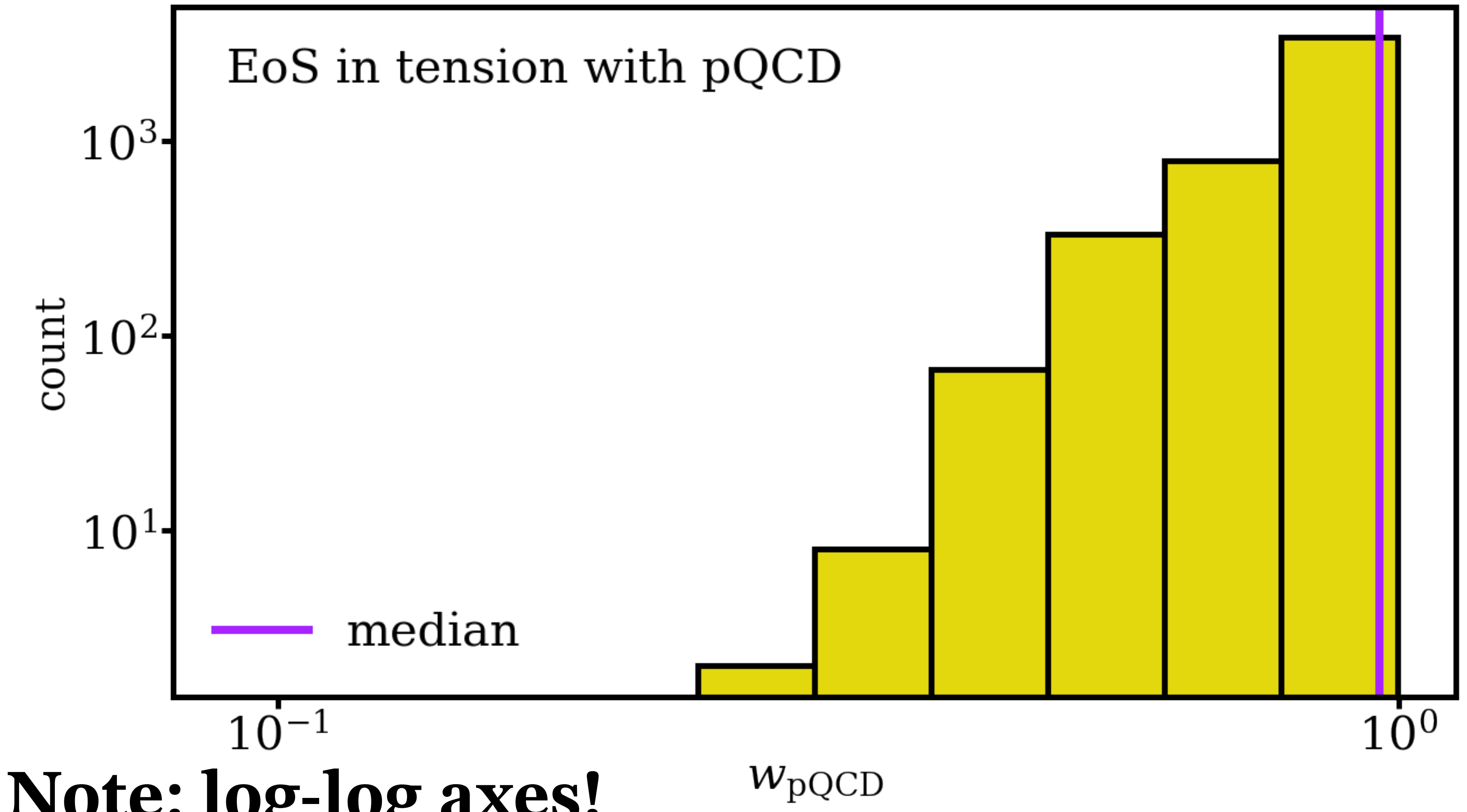
Benchmark (GP): c_s^2 peak near n_B^{max} → **monotonic** $c_s^2(n_B)$

Benchmark + structure (mGP): **bump allowed** $\sim 2-3\ n_{sat}$



What is the impact of pQCD constraints applied at $n_B^{\max}$?





Note: log-log axes!

pQCD results must be averaged over an unphysical scale due to uncertainty around missing higher-order terms.

- Adopt the scale-averaging approach in Gorda et al. (Astrophys. J., 2023)

$w_{\text{pQCD}} = 0$: not consistent with any results considered

$0 < w_{\text{pQCD}} < 1$: in tension with at least one result considered

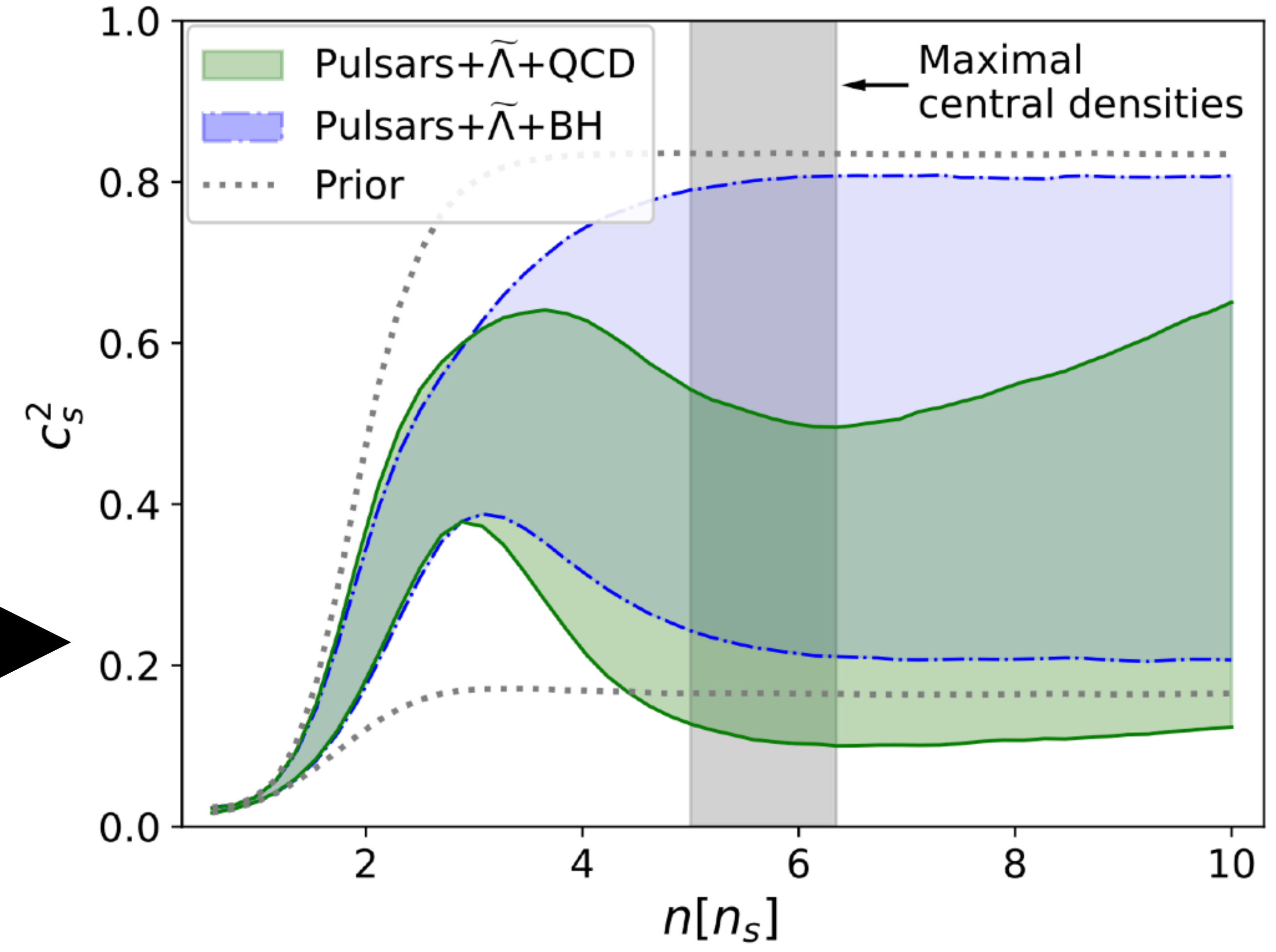
$w_{\text{pQCD}} = 1$: consistent with all results considered

pQCD constraints applied at $n_B^{\max}$ do not affect EoS posteriors

BUT

they seem to have a significant impact when applied 10 n_{sat}

Why?



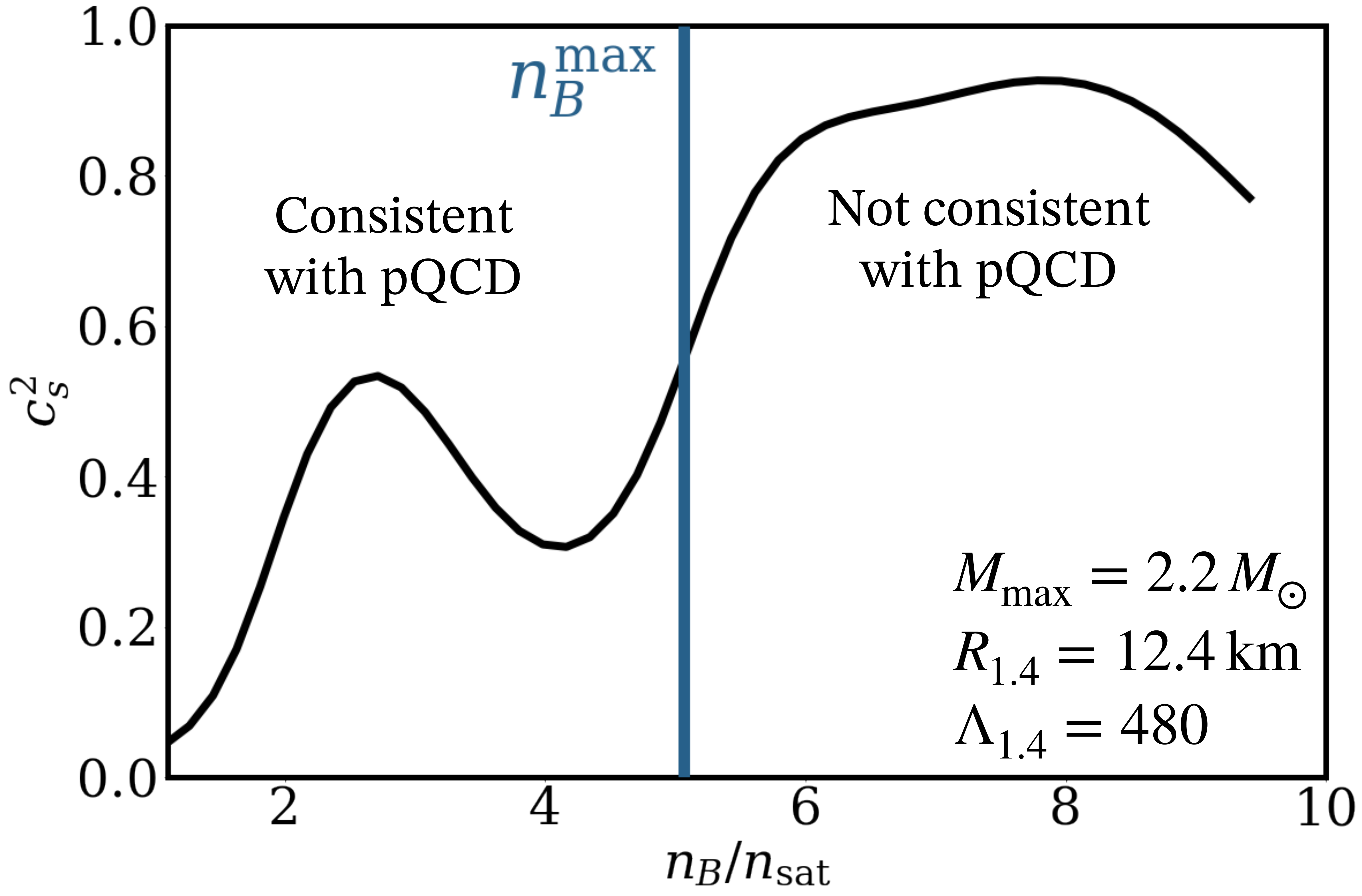
Gorda et al. Astrophys. J. (2023)

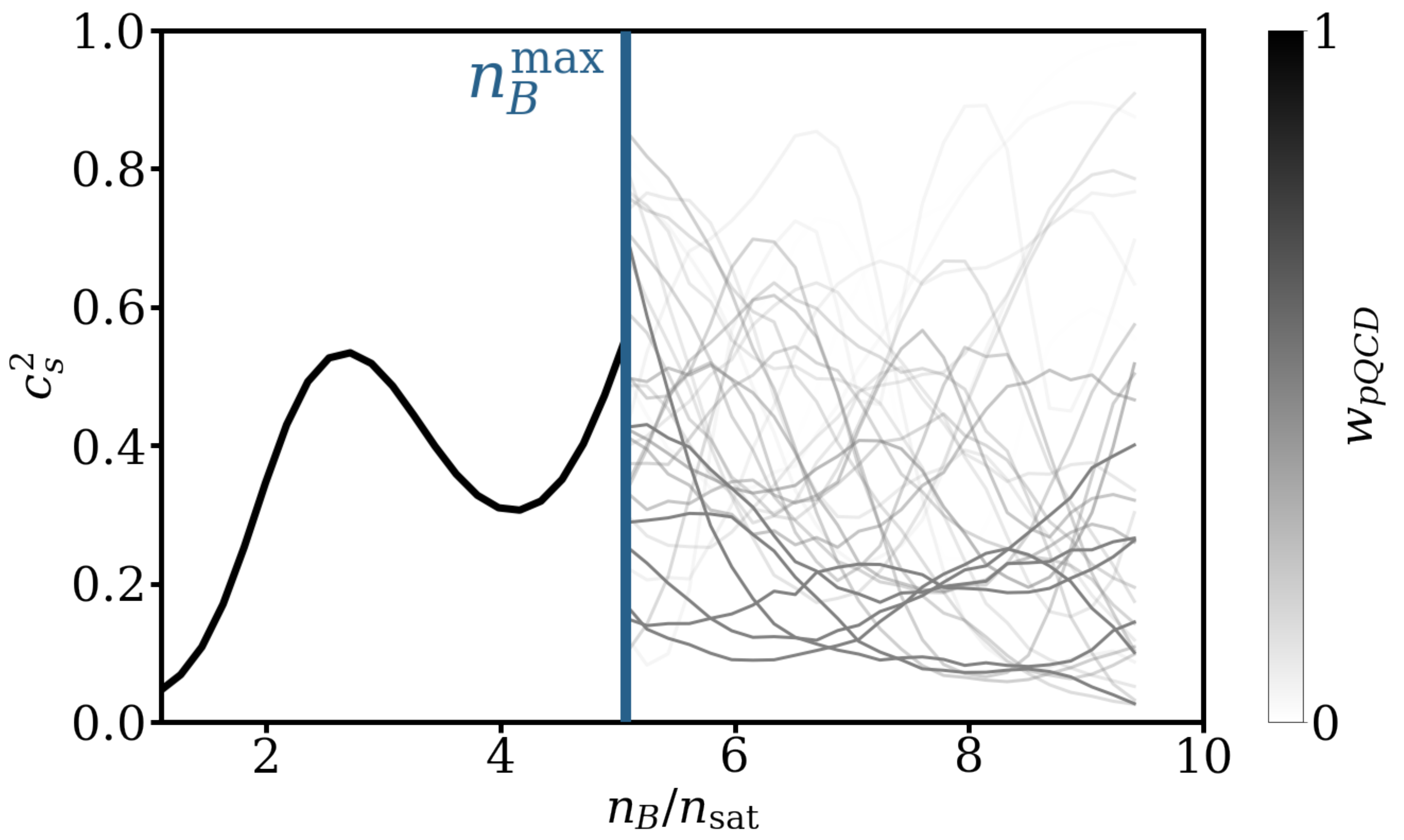
The role of pQCD beyond NS densities

In prep: D. Mroczek, R. Somasundaram, I. Tews, M.C. Miller, J. Noronha-Hostler, N. Yunes, J. Margueron

Method: build extensions of a strong candidate EoS beyond $n_B^{\max}$, use pQCD input at 10 $n_{\rm sat}$

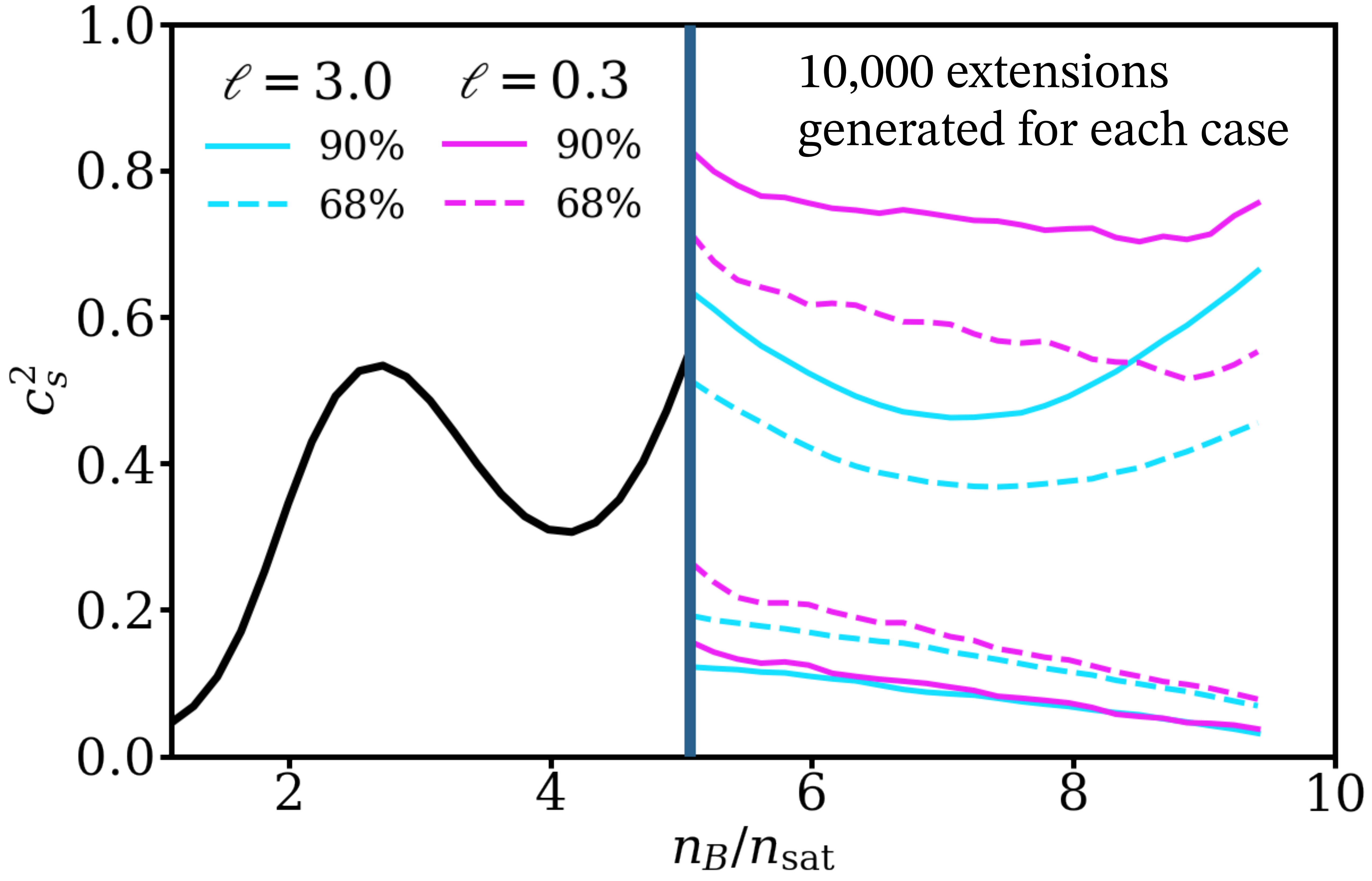
Goal: Learn about prior dependence of pQCD constraints between $n_B^{\max}$ and 10 $n_{\rm sat}$





Short-range correlations reduce sensitivity to pQCD constraints

We tested **long- and short-range** correlations across densities (units of n_{sat})



- EoS **must soften** (68% credibility) when **long-range** correlations are present.
- **NOT** the case when **short-range** correlations are assumed.

EoS posteriors from pQCD constraints applied beyond n_B^{max} are **model dependent**

How EoS posteriors are affected by pQCD constraints applied beyond n_B^{max} can be studied systematically with multi-scale correlations.

In prep: D. Mroczek, R. Somasundaram, I. Tews, M.C. Miller, J. Noronha-Hostler, N. Yunes, J. Margueron

Are there nontrivial features in the c_s^2 inside neutron stars?

We find a **Bayes factor of K = 1.5** between GP and mGP → current constraints do not favor either model.

Physical interpretation: multi-scale correlations and nontrivial features in $c_s^2(n_B)$ inside neutron stars are **not ruled out** by current constraints, but **neither are they required**.

Summary and outlook

- Nuclear physics models predict **nontrivial features** in c_s^2 and **multi-scale correlations** across densities when **exotic degrees of freedom are present.**
- Introduced **modified Gaussian processes** as novel approach for **modeling nontrivial features in** c_s^2.
- Performed a fully Bayesian analysis including astrophysical, low-energy, and pQCD constraints.
- **Multi-scale correlations** important for **searches for a crossover** within NS densities.
- **pQCD** applied **beyond central densities** realized in neutron stars leads to **model-dependent constraints** — pQCD applied at n_B^{max} currently does not affect EoS posteriors.