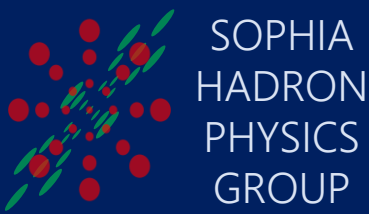


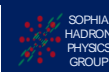
Dynamics of causal hydrodynamic fluctuations in an expanding system

Shin-ei Fujii, Tetsufumi Hirano
Sophia Univ.



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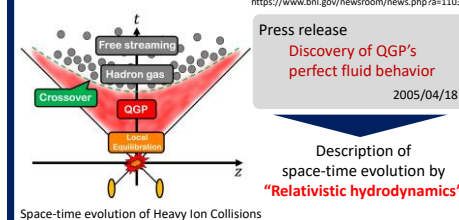
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1. Introduction



2. Formalisms

Perturbative expansion around the Bjorken's solution

$$\mu_{\text{Bj}} = (\cosh s, 0, 0, \sinh s), \quad \tau = \frac{1}{2} \ln \left(\frac{t+z}{t-z} \right): \text{coordinate rapidity}$$

Small deviations

$$\mu \rightarrow (\cosh(s + \delta s), 0, 0, \sinh(s + \delta s))$$

$$\rightarrow \delta \mu + \delta s \text{ etc.} \quad \delta \mu: \text{energy density}, \quad \delta s: \text{proper time}$$

Energy-momentum conservation

Balance equation for Background (0th order perturbation)
Balance equation for Fluctuation (1st order perturbation)

for arbitrary constitutive equations for ϵ and Π

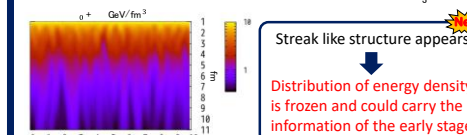
Background: $-\partial_s \left(\epsilon + \Pi \right) = 0$ (Bjorken equation)

Fluctuation: $-\partial_s \left(\delta \epsilon + \delta \Pi \right) + \frac{1}{\tau} \left(\delta \epsilon + \delta \Pi \right) + \frac{1}{2} \left(\delta \epsilon + \delta \Pi \right) = 0$

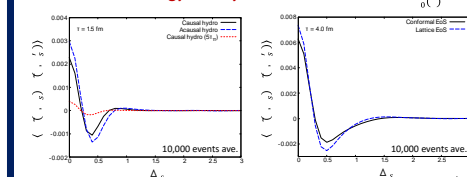
3. Results

Space-time evolution of energy density

Initial condition: $\epsilon_0(\tau=1 \text{ fm}) = 10 \text{ GeV/fm}^3$, $\epsilon_0(\tau=1 \text{ fm}) = 0 \text{ GeV/fm}^3$, $\epsilon_0(\tau=1 \text{ fm}) = \frac{4}{3} \text{ GeV/fm}^3$

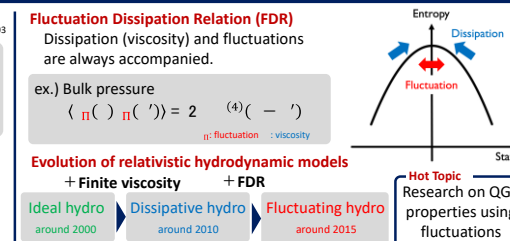


Correlations of energy density fluctuations



4. Summary

- We developed a framework which deals with causal hydrodynamic fluctuations in 1-dimensional expanding system.
- We observed streak like structure through time evolution of energy density caused by a freeze of distribution.
- We found behaviors of correlations of thermodynamic variables and particles are closely related with the properties of the medium.



Causal constitutive equations

Israel-Stewart equation + noise $(1 + \pi) = \frac{4}{3} + \pi$

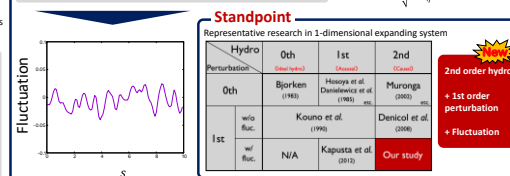
Perturbative expansion

Background: $(1 + \pi) = \frac{4}{3}$

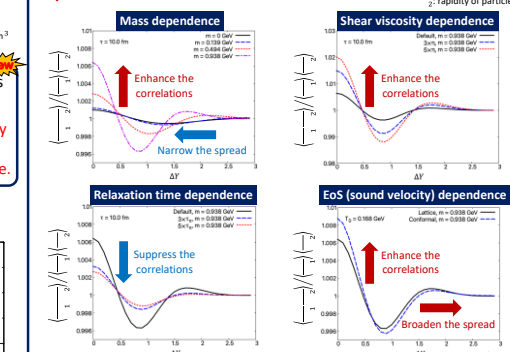
Fluctuation: $(1 + \pi) = \frac{4}{3} + \pi$

Fluctuations

FDR for shear stress
 $\langle \pi(s, s') \pi(s', s') \rangle = \frac{1}{2} \frac{\partial \epsilon}{\partial s} \exp \left(-\frac{(s-s')^2}{2\tau} \right)$



2-particle correlations



- Heavier hadrons are good probes of correlations.
 - Shear viscosity and relaxation time work oppositely.
 - Correlations include the information of EoS.
- Extract the properties of the medium from 2-particle correlations!

Introduction

QGP created in the HIC is successfully described by
“Relativistic Viscous Hydrodynamics”

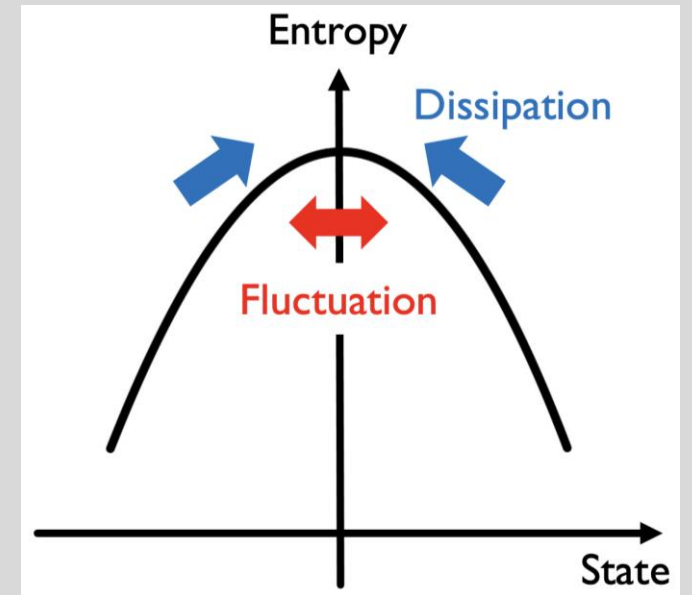
Fluctuation Dissipation Relation (FDR)

Dissipation (viscosity) and fluctuations are always accompanied.

Ex.) FDR for shear stress tensor

$$\left\langle \xi_{\pi}^{\mu\nu}(x) \xi_{\pi}^{\alpha\beta}(x') \right\rangle = 4\eta T \delta^4(x - x') \Delta^{\mu\nu\alpha\beta}$$

$\xi_{\pi}^{\mu\nu}$: fluctuation η : viscosity



Purpose

To see the effect of hydrodynamic fluctuations on observables

Perturbative expansion around the Bjorken's solution

$$u_{\text{Bj}}^\mu = (\cosh \eta_s, 0, 0, \sinh \eta_s) \quad \eta_s = \frac{1}{2} \ln \left(\frac{t+z}{t-z} \right): \text{coordinate rapidity}$$



Small deviations

$$u^\mu \rightarrow \left(\cosh(\eta_s + \delta y(\tau, \eta_s)), 0, 0, \sinh(\eta_s + \delta y(\tau, \eta_s)) \right)$$

$$e \rightarrow e_0(\tau) + \delta e(\tau, \eta_s) \quad \text{etc.}$$

e : energy density

$\tau = \sqrt{t^2 - z^2}$: proper time



Energy-momentum conservation

Background

$$\frac{d}{d\tau} e_0 + \frac{1}{\tau} (w_0 + \Pi_0 - \pi_0) = 0 \quad (\text{Bjorken equation})$$

Fluctuation

$$\frac{\partial}{\partial \tau} \left(\delta y (w_0 + \Pi_0 - \pi_0) \right) + \frac{1}{\tau} \frac{\partial}{\partial \eta_s} \left(\delta y (w_0 + \Pi_0 - \pi_0) \right) + \frac{1}{\tau} \left(\delta w + \delta \Pi - \delta \pi \right) = 0$$

$w = e + p$: enthalpy density p : hydrostatic pressure $\pi \equiv \pi^{00} - \pi^{33}$: shear stress Π : bulk pressure

Perturbative expansion around

$$u_{\text{Bj}}^\mu = (\cosh \eta_s, 0, 0, \sinh \eta_s)$$

η_s



Small deviations

$$u^\mu \rightarrow (\cosh(\eta_s + \delta y(\tau, \eta_s)), 0, 0, \sinh(\eta_s + \delta y(\tau, \eta_s)))$$

$$e \rightarrow e_0(\tau) + \delta e(\tau, \eta_s) \quad \text{etc.}$$



Energy-momentum conservation

Causal constitutive equations + noise

η : shear viscosity

τ_π : relaxation time

Background

$$\left(1 + \tau_{\pi 0} \frac{d}{d\tau}\right) \pi_0 = \frac{4\eta_0}{3\tau}$$

Fluctuation

$$\left(1 + \tau_{\pi 0} \frac{\partial}{\partial \tau}\right) \delta \pi = -\frac{\delta \tau_\pi}{\tau_{\pi 0}} \left(\frac{4\eta_0}{3\tau} - \pi_0\right) + \frac{4\eta_0}{3\tau} \frac{\partial}{\partial \eta_s} \delta y + \frac{4\delta \eta}{3\tau} + \xi_\pi$$

noise term



Background

$$\frac{d}{d\tau} e_0 + \frac{1}{\tau} (w_0 + \Pi_0 - \pi_0) = 0 \quad (\text{Bjorken equation})$$

Fluctuation

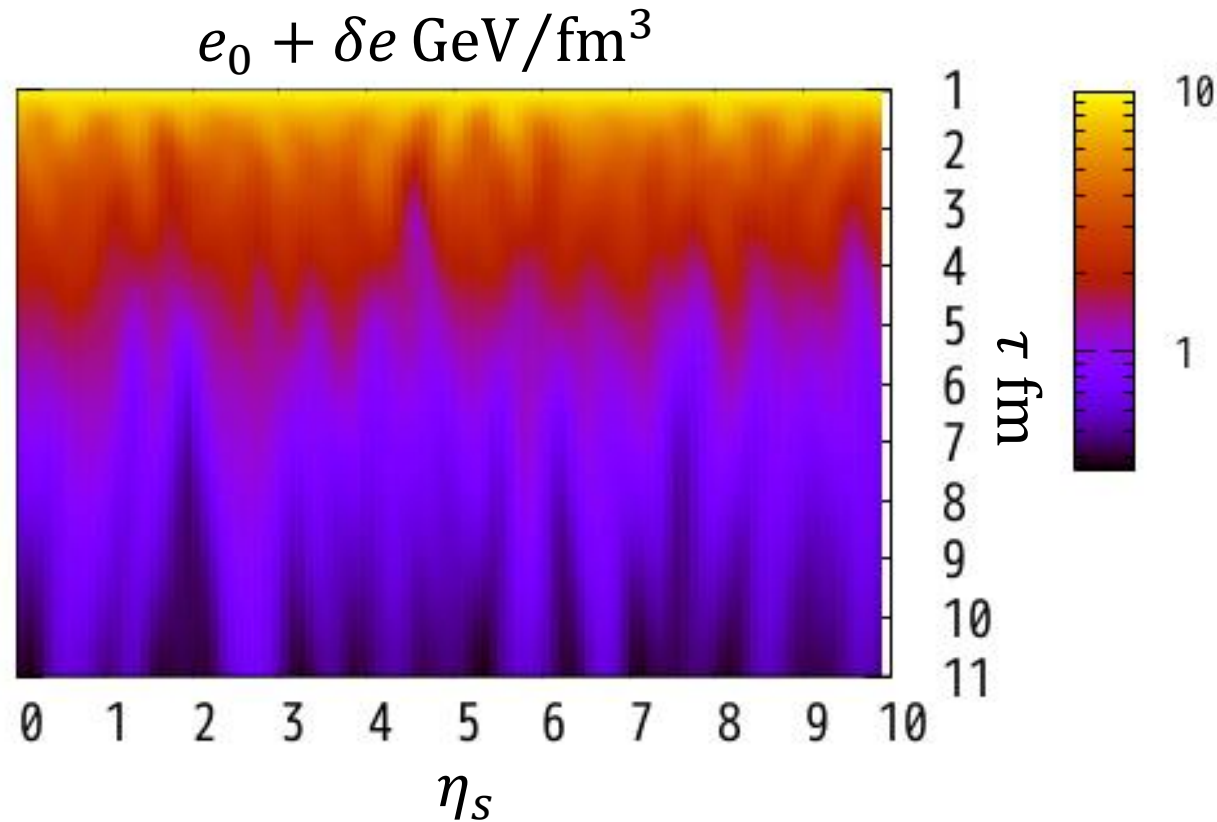
$$\frac{\partial}{\partial \tau} \left(\delta y (w_0 + \Pi_0 - \pi_0) \right) + \frac{1}{\tau} \frac{\partial}{\partial \eta_s} \left(\delta y (w_0 + \Pi_0 - \pi_0) \right) + \frac{1}{\tau} \left(\delta w + \delta \Pi - \delta \pi \right) = 0$$

$w = e + p$: enthalpy density p : hydrostatic pressure $\pi \equiv \pi^{00} - \pi^{33}$: shear stress Π : bulk pressure

Results

Space-time evolution of energy density

Initial conditions: $e_0(\tau = 1 \text{ fm}) = 10 \text{ GeV/fm}^3$, $\delta e(\tau = 1 \text{ fm}) = 0 \text{ GeV/fm}^3$, $\pi_0(\tau = 1 \text{ fm}) = \frac{4\eta}{3\tau} \text{ GeV/fm}^3$



Distribution is frozen



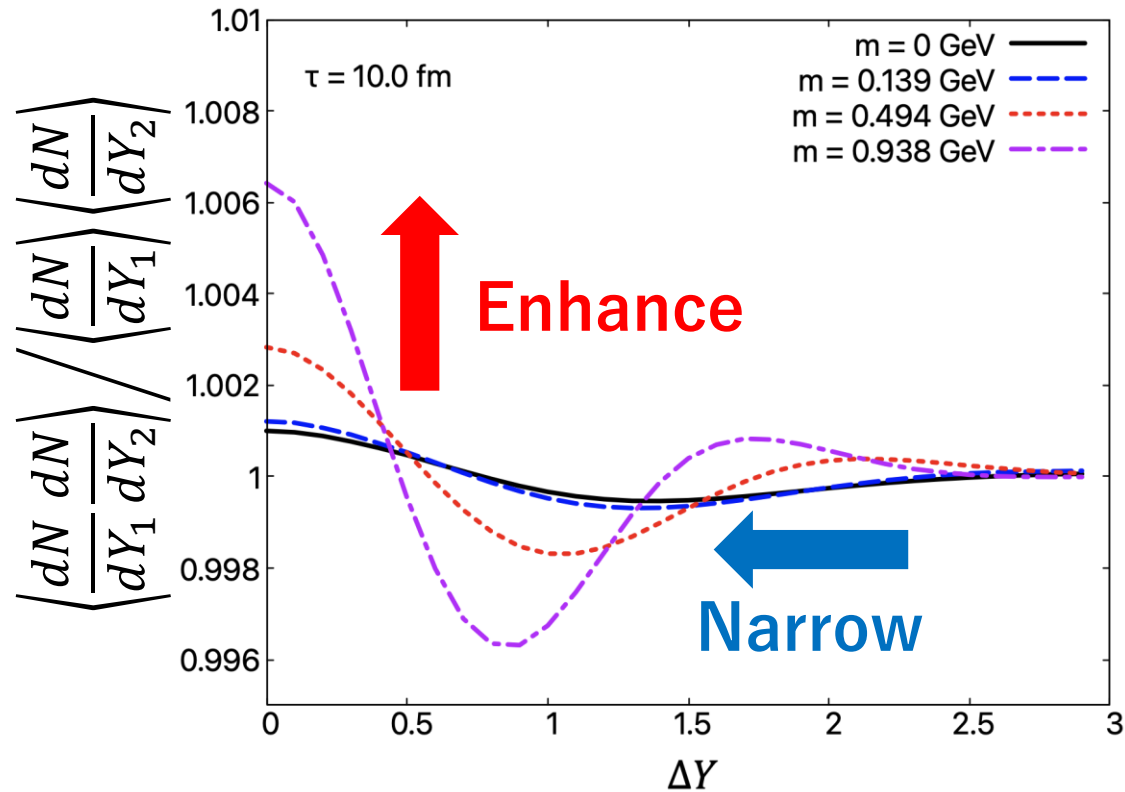
Carry the information of early stage

Results

2 particle correlations

Y_1 : rapidity of particle 1
 Y_2 : rapidity of particle 2

Mass dependence



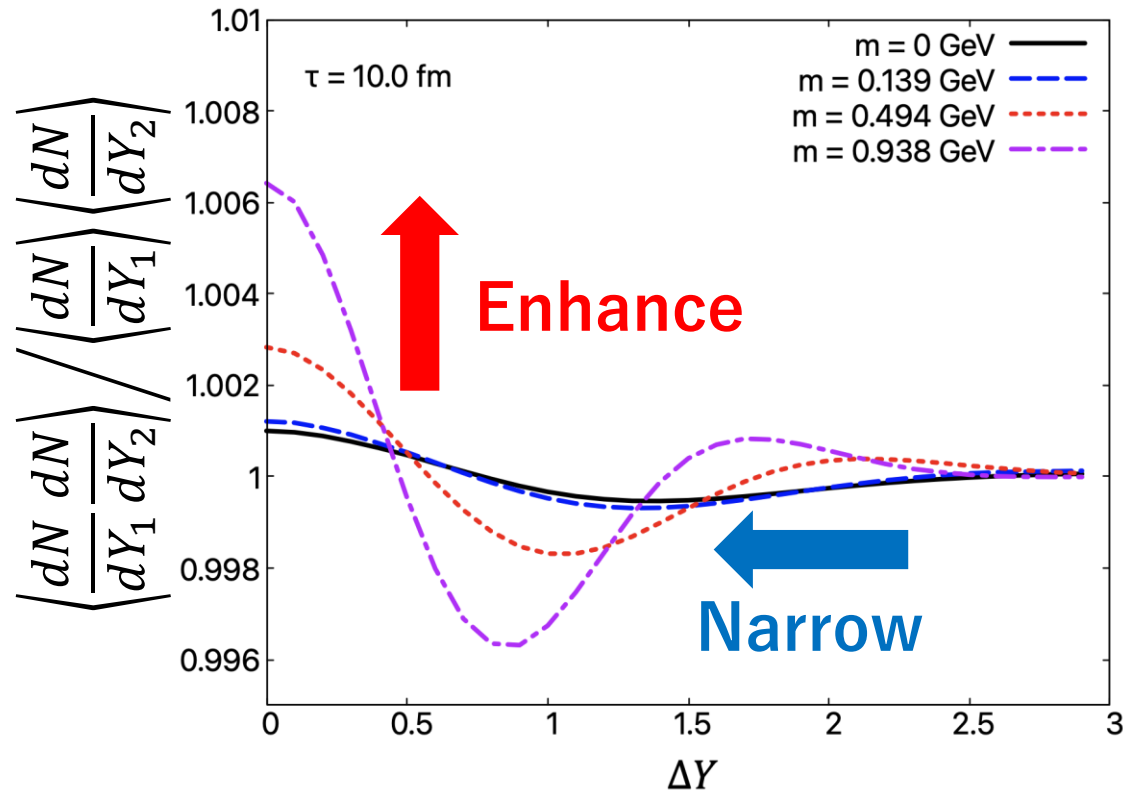
Heavier hadrons are good probes of correlations

2 particle correlations

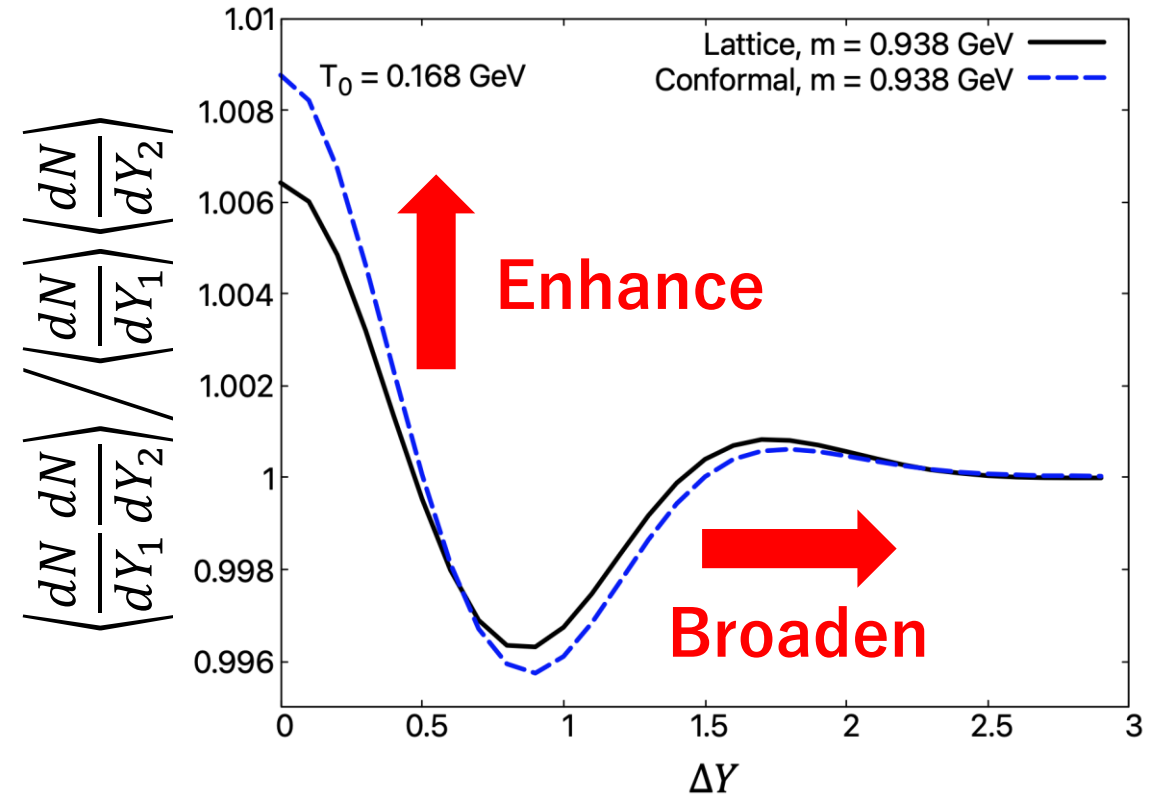
Y_1 : rapidity of particle 1

Y_2 : rapidity of particle 2

Mass dependence



EoS (sound velocity) dependence



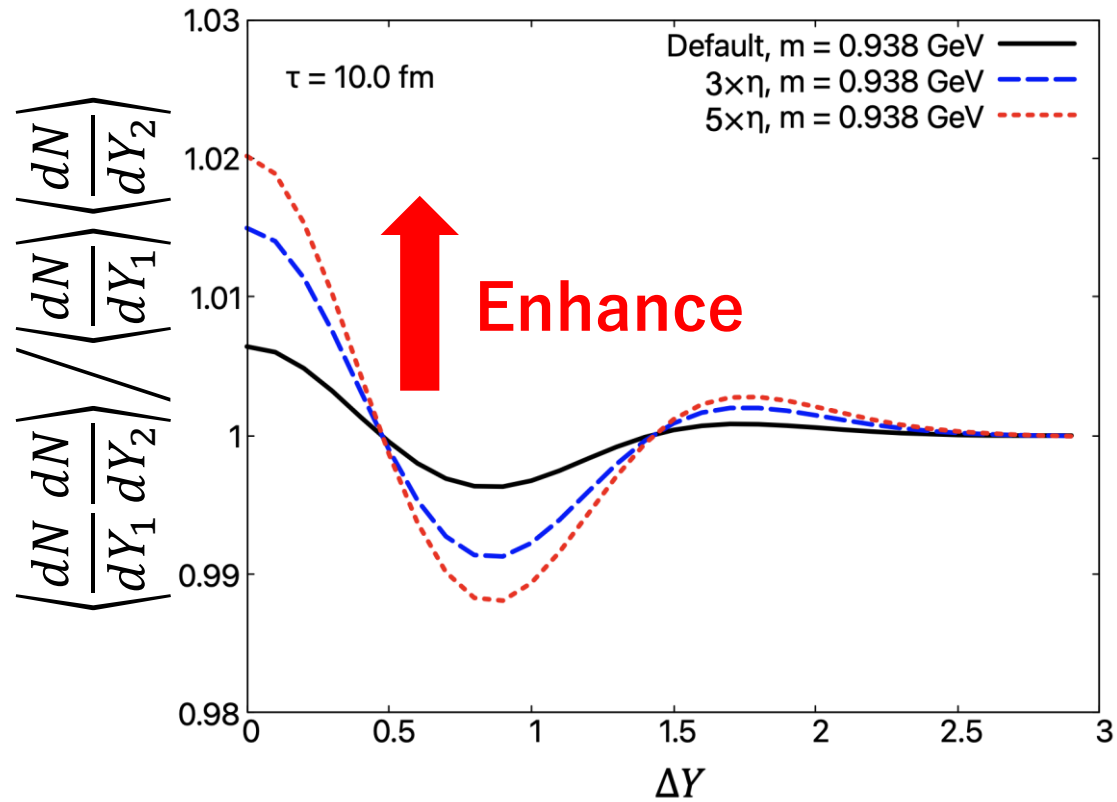
Correlations include information of EoS

Results

2 particle correlations

Y_1 : rapidity of particle 1
 Y_2 : rapidity of particle 2

Shear viscosity dependence



Viscosity \rightarrow Enhance

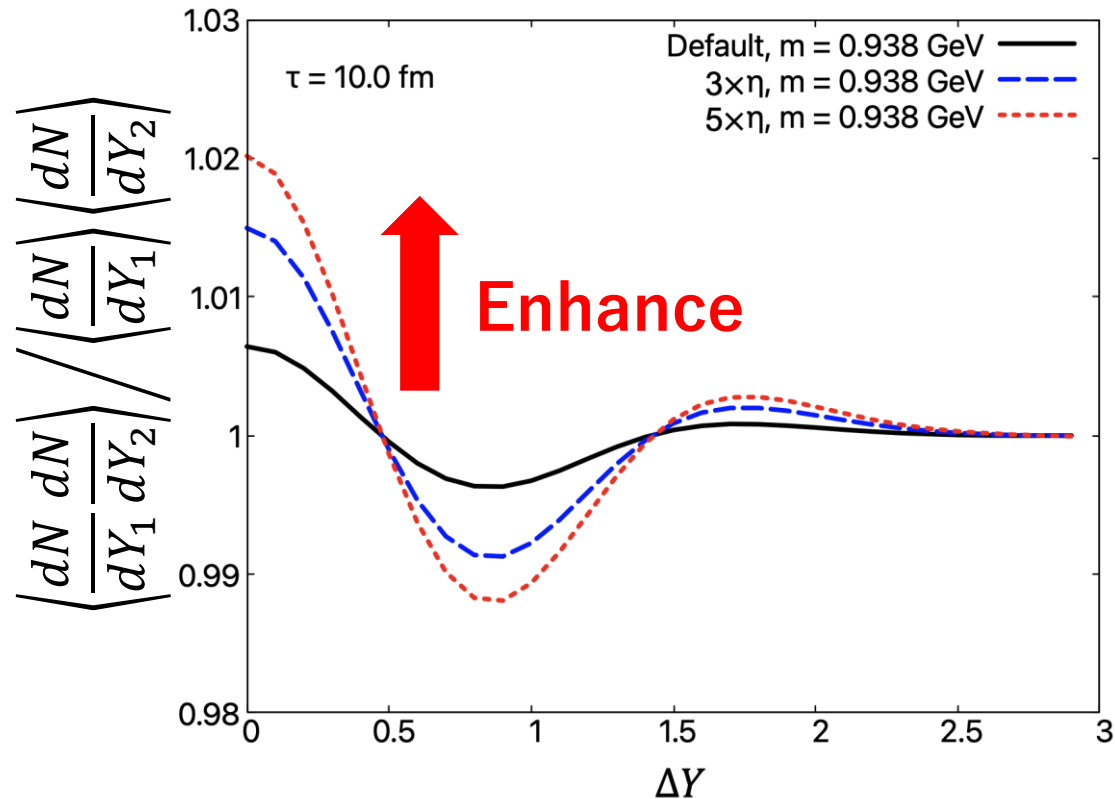
Results

2 particle correlations

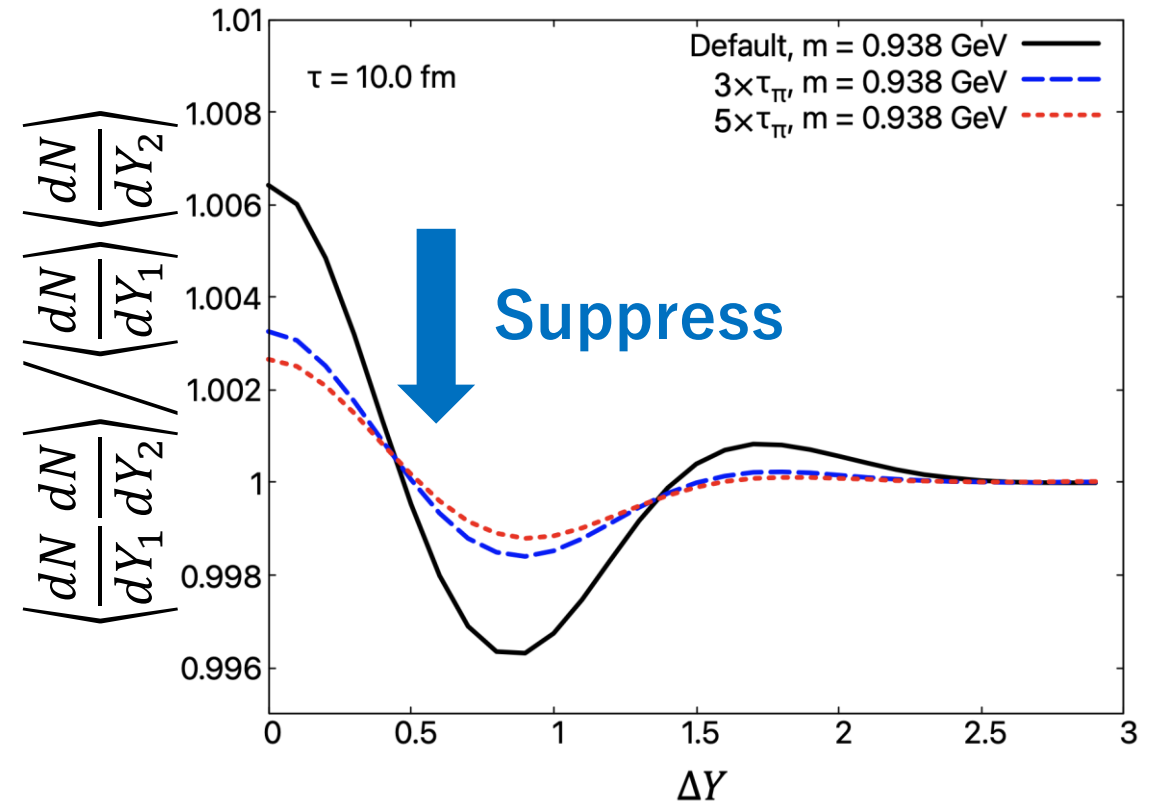
Y_1 : rapidity of particle 1

Y_2 : rapidity of particle 2

Shear viscosity dependence



Relaxation time dependence



Viscosity \rightarrow Enhance

Relaxation time \rightarrow Suppress

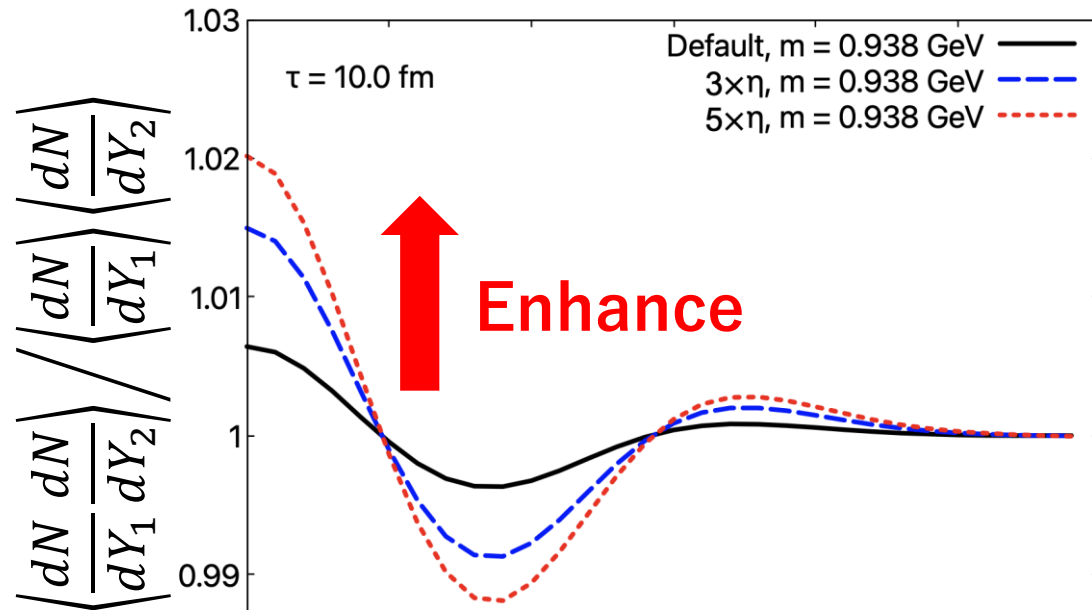
Results

2 particle correlations

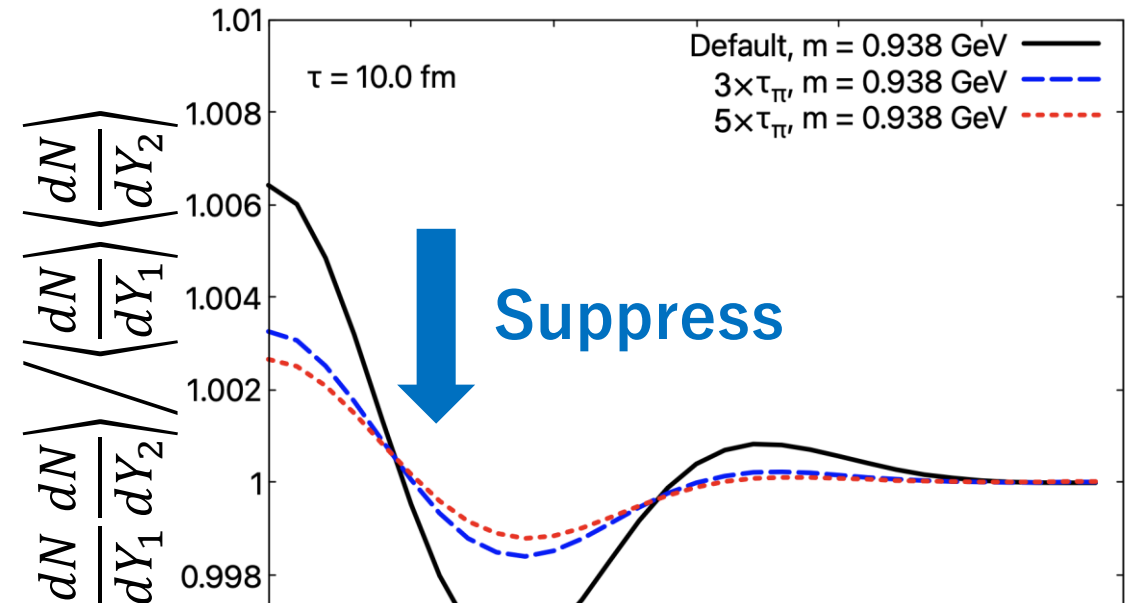
Y_1 : rapidity of particle 1

Y_2 : rapidity of particle 2

Shear viscosity dependence



Relaxation time dependence



Summary

New framework of **causal hydrodynamic fluctuations** in (1+1)D system

➡ Hydrodynamic fluctuations provide a **multidimensional analysis**