QUARKONIUM SPIN ALIGNMENT IN A VORTICAL MEDIUM

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Abstract

We examine in detail the mass, lifetime and spin structure of quarkonium in a rotating vortical medium, where the quark spin is not necessarily aligned with the vortex. After justifying this set-up in terms of spin hydrodynamics, and outlining the expected dependence of spin and vorticity, we examine the mass, lifetime and spin density matrix of quarkonium. Our analysis implies a novel distillation-based mechanism for spin-alignment generation as well as experimental probes of spin-vorticity nonequilibrium.

Introduction

- The advent of the study of vorticity in heavy ion collisions added a potentially new arena where quarkonium could be used.
- Quarkonium can be formed early in the colision and can survive throughout the quark gluon plasma evolution.
- If spin and vorticity are not in equilibrium, this lack of equilibration can be imprinted on the density matrix's measurable off-diagonal elements $\rho_{0,\pm 1}, \rho_{\pm 1,\mp 1}$.
- For phenomenological consequences, see Poster 54 on *Chirality Section*, by Kayman Gonçalves.

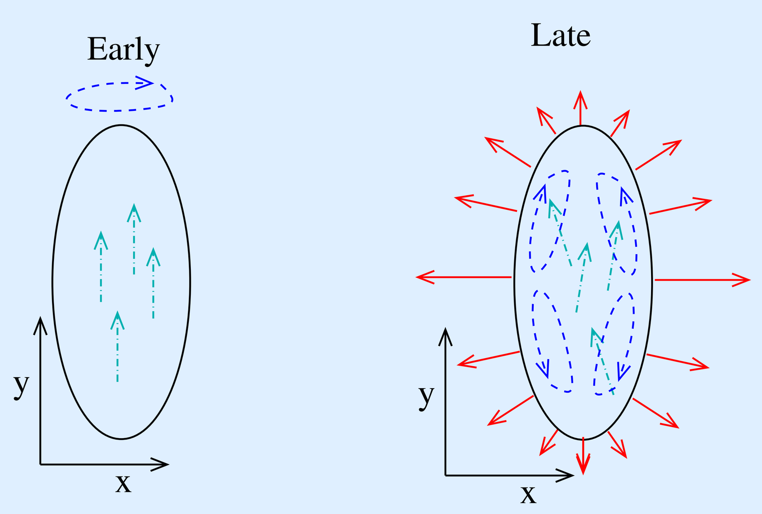

Figure: Blue dashed arrows represent vorticity, cyan dot-dashed ones polarization, and red solid ones flow. The misalignment of spin in the right panel is due to the lack of equilibrium between vorticity and polarization.

Quarkonium states in rotating reference frames

The Schrödinger Equation

Quarkonium can be viewed as solutions to a Schrödinger equation with heavy quark wave functions moving around a QCD potential.

$$\mathcal{H} = \sum_{i=1,2}\left[\frac{(\boldsymbol{p}_i - m_i\boldsymbol{\omega}_i\times\mathbf{r}_i)^2}{2m_i} - \frac{m_i}{2}(\boldsymbol{\omega}_i\times\mathbf{r}_i)^2 - \boldsymbol{\omega}_i\cdot\mathbf{S}_\mathrm{i}\right] + V(|\mathbf{r}_1 - \mathbf{r}_2|)$$

$$V(r) = br - \frac{\alpha_\mathrm{eff}}{r} \qquad C = \oint \mathbf{v}\cdot d\mathbf{l}$$

Mass and Vorticity

From the binding energy in the rotating frame we can obtain the quarkonium invariant mass

$$E_{n,l,m} = \frac{3b}{\delta} - \frac{2\mu\left(\alpha_\mathrm{eff} + \frac{3b}{\delta^2}\right)^2}{\left[(1+2n) + \sqrt{1 + 4l(l+1) + \frac{4\mu\, m_j\, C}{\pi} + \frac{8\mu b}{\delta^3}}\right]^2} \qquad M = 2m_\mathrm{q} + E_{n,l,m}$$

Vorticity and Melting

Using a semiclassical analysis with $\langle p\rangle \sim 1/r$ and $\langle p^2\rangle \sim 1/r^2$

$$E(r) = \frac{1}{2\mu r^2} - m_j\omega + V(r) \qquad V(r) = -\alpha_\mathrm{eff}\frac{\mathrm{e}^{-r/\lambda_\mathrm{D}}}{r}$$

and the circulation theorem,

$$E(r) = \left[\frac{1}{2\mu} - \frac{m_j C}{2\pi}\right]\frac{1}{r^2} - \frac{\alpha_\mathrm{eff} e^{-r/\lambda_D}}{r}$$

The bound state is defined when the energy has a minimum, so we can write:

$$\frac{dE(r)}{dr} = 0 \quad\Longrightarrow\quad f(\tilde{r}) \equiv \tilde{r}\,(1+\tilde{r})\,e^{-\tilde{r}} = \frac{1}{\alpha_\mathrm{eff}\lambda_D}\left[\frac{1}{\mu} - \frac{m_j C}{\pi}\right], \quad \tilde{r} = r/\lambda_D$$

The maximum value of $f(\tilde{r})$ is 0.840 at $\tilde{r} = 1.92$.

$$\frac{1}{\alpha_\mathrm{eff}\lambda_D}\left[\frac{1}{\mu} - \frac{m_j C}{\pi}\right] > 0.840 \qquad \lambda_D = \sqrt{\frac{2}{9\pi\alpha_\mathrm{eff}}}\frac{1}{T} \qquad T_\mathrm{melt} = 0.84\sqrt{\frac{2\alpha_\mathrm{eff}}{9\pi}}\left[\frac{1}{\mu} - \frac{m_j C}{\pi}\right]^{-1}$$

Density matrix elements and vorticity

We can write the density matrix operator on the energy basis

$$\hat{\rho} = \mathrm{e}^{-\beta\hat{\mathrm{H}}} \qquad \beta = \frac{1}{T}$$

Now we will make a rotation to the lab frame

$$\hat{\rho}^r = U(\theta_r,\phi_r)\hat{\rho}U^{-1}(\theta_r,\phi_r)$$

$$\rho^r_{m,m'} = \sum_{m'',m'''} e^{i(m'''-m)\phi_r} d^j_{m,m''}(\theta_r)\rho_{m'',m'''}\left[d^j_{m''',m'}(\theta_r)\right]^{-1}, \qquad \rho_{m'',m'''} = \frac{1}{Z}e^{\beta E_{m'''}}\delta_{m'',m'''}$$

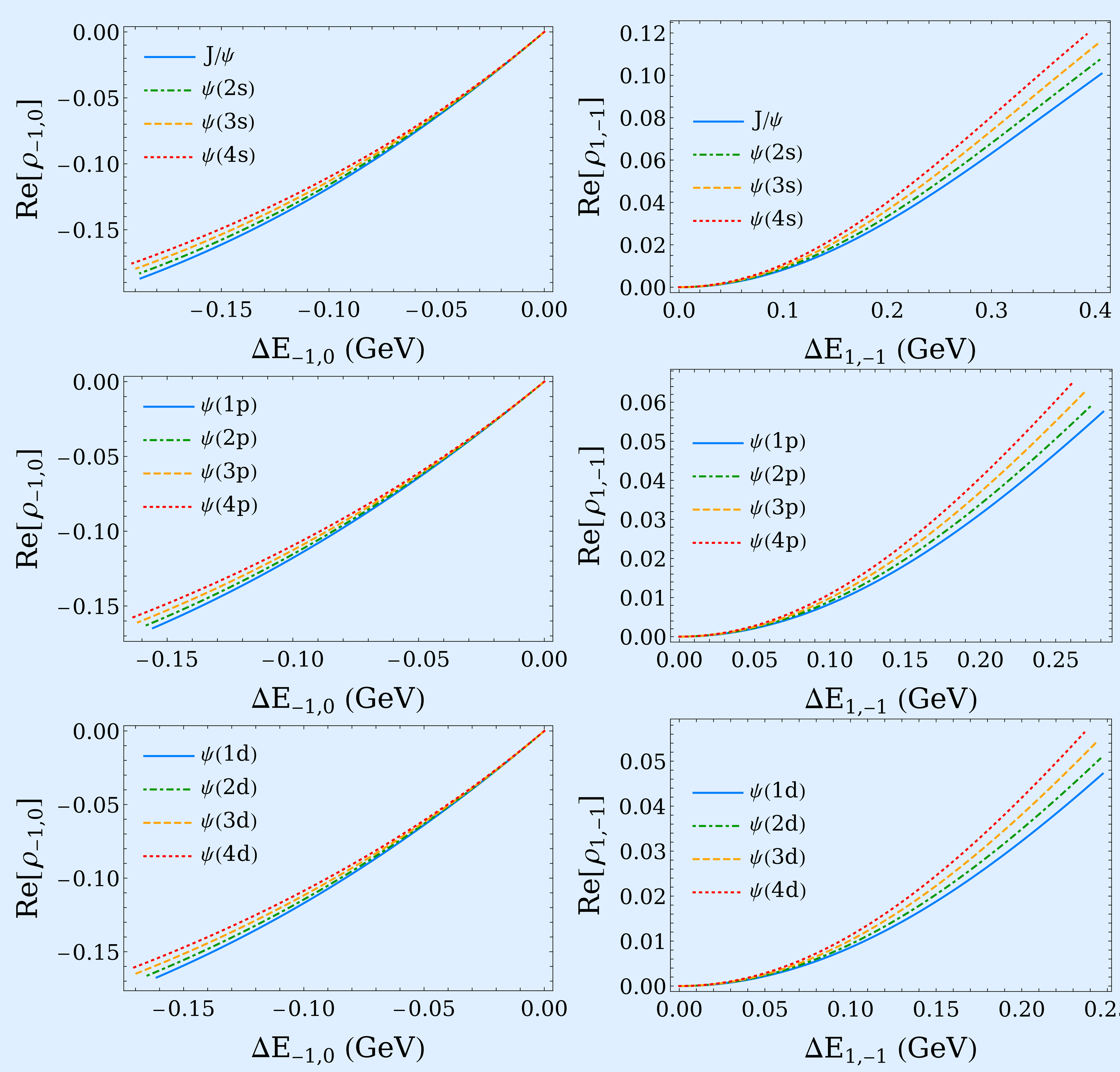

Figure: The off-diagonal density matrix components versus invariant mass shift.

Such an observation would indicate spin-vorticity non-equilibrium.

Conclusions

- Quarkonium is a useful probe for non-equilibrium between vorticity and polarization.
- This model is good enough to get a physical intuition of the problem of linking spin-vorticity nonequilibrium to the quarkonium state in a rotating frame — and, respectively, the quarkonium state to experimental observables.
- Our formalism suggests experimental observables probing how binding energy and melting probability respond to rotation.

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