Houston, we have a problem : Causality !

Thiago Siqueira Domingues, Matthew Luzum, Tiago Nunes da Silva, Renata Krupczak, Jean-François Paquet, Jorge Noronha, Jacquelyn Noronha-Hostler

The effects of hydrodynamic causality conditions on Bayesian Analysis

The hydrodynamic regime of validity

Question :

- Why do we trust hydrodynamics as a faithful representation of Quantum Chromodynamics ?

Relativistic viscous Hydrodynamics

- Hydrodynamics is an essential tool for the phenomenological modeling of ultrarelativistic heavy-ion collisions
- It is traditionally expected to describe the behavior of systems close to thermodynamic local equilibrium
- Equations of motion for a set of dynamical variables (Landau frame definition)

$$\partial_\mu T^{\mu\nu} = 0, \tag{1}$$
$$T^{\mu\nu} = \epsilon u^\mu u^\nu - (p + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

- Israel-Stewart-like theories are theories that avoid acausality and instability assuming that dissipative currents ($\pi^{\mu\nu}$ and Π)obey nonlinear relaxation equations, derived using the DNMR formalism :

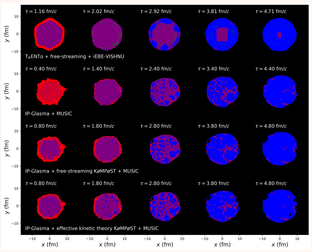
$$\tau_\Pi \dot{\Pi} + \Pi = -\zeta\theta - \delta_{\Pi\Pi}\Pi\theta + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu}\ , \tag{2}$$

$$\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} - \delta_{\pi\pi}\pi^{\mu\nu}\theta + \varphi_7 \pi_\alpha^{\langle\mu}\pi^{\nu\rangle\alpha} - \tau_{\pi\pi}\pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\alpha} + \lambda_{\pi\Pi}\Pi\sigma^{\mu\nu}. \tag{3}$$

- These relaxation equations describe how the dissipative currents relax to their relativistic Navier-Stokes limits within relaxation time scale τ_π and τ_Π.

Problems :

- Recently-derived nonlinear causality conditions [1] implemented in numerical simulations [5] showed large acausal regimes especially at early times



- It remains unclear why hydrodynamics is applicable even in systems with large local deviations from equilibrium as nucleus-nucleus collisions

Nonlinear causality conditions

- These causality conditions are algebraic inequalities to ensure causality holds in the full nonlinear regime
- They are a set of **necessary** and **sufficient** constraints which apply to the Israel-Stewart-like equations of motion
- They involve all the transport coefficients and viscous currents Π and 0, Λ_i the eigenvalues of π^μ_ν.

Linear causality Condition

We first examine the nonlinear causality constraints linearized around local equilibrium ($\pi^{\mu\nu} = 0$ and $\Pi = 0$) :

$$c_s^2 + \frac{4}{3}\frac{1}{b_\pi} + \frac{(\frac{1}{3} - c_s^2)^2}{b_\Pi} \leq 1 \tag{4}$$

- b_π, b_Π are relaxation factors related to τ_π, τ_Π, respectively.
- $c_s^2 = c_s^2 = dp/d\epsilon$, is the equilibrium speed of sound squared.
- This linearized analyses miss the effects from the other coefficients, which contribute to the nonlinear evolution and it says nothing about the nonlinear regime.

We need a more systematic and global way to investigate causality violation $\Rightarrow$ A Bayesian analysis.

Bayesian Analysis

- Based on Bayes's theorem $P(A|B) = \frac{P(B|A)P(A)}{P(B)}$
- Prior distribution $P(A)$, our prior knowledge $\Rightarrow$ Causality information takes place here !
- Likelihood $P(B|A)$ to constrain parameters using experimental data
- Posterior distribution of model parameters $P(A|B) \propto P(B|A)P(A)$
- Maximum a Posteriori (MAP) parameter : the point in parameter space which maximizes the posterior

Acausal Bayesian parameter range

Current Bayesian analyses [2-4], present a large acausal prior range, even in the near-equilibrium regime :

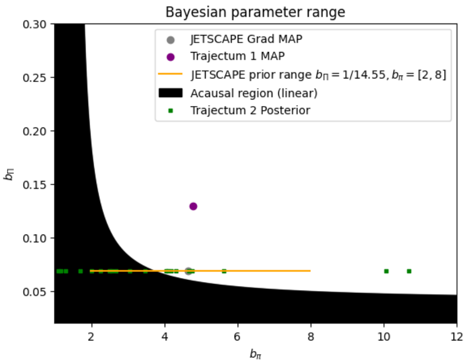
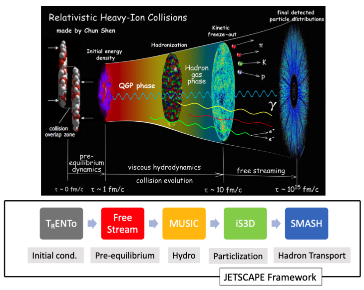


Figure 1. The prior range for Trajectum with a yellow line representing the JETSCAPE one.

We need to fix this prior range and see how it affects the JETSCAPE Bayesian parameter estimation !

JETSCAPE Framework

- Multistage dynamical simulations
- 17-dimensional parameter space



We use specific **parametrizations** for the shear and bulk relaxation times :

$$T\tau_\pi(T) = b_\pi \frac{\eta}{s}(T) \tag{5}$$

$$\tau_\Pi = b_\Pi \frac{\zeta}{(1/3 - c_s^2)^2(\epsilon + p)} \tag{6}$$

where b_π is a model parameter and $b_\Pi = 1/14.55$.

Linearized analysis

- It affects only b_π, since $b_\Pi = 1/14.55$.
- The linear causality condition requires that $b_\pi \geq 3.705$ and we use this value to limit the prior range [3.705, 8] explored for b_π in our parameter estimation.
- We compared the original prior range [2, 8] with the range imposed by the linearized analysis.
- **Experimental data** : in this study we included the ALICE data for Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$

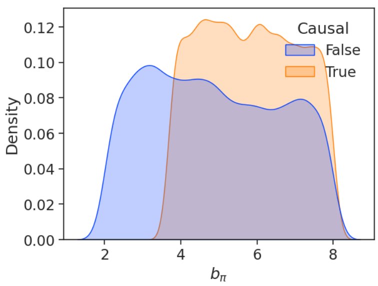


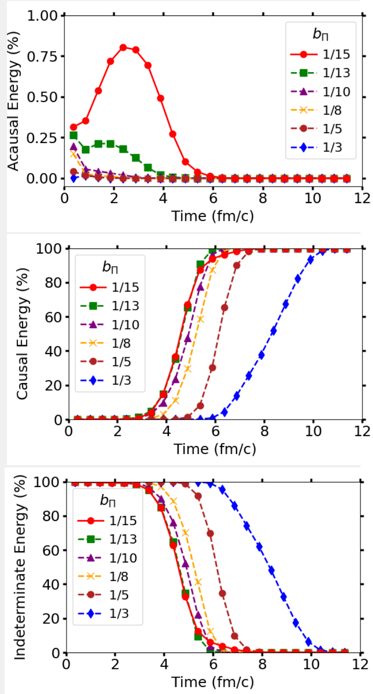
Figure 2. Effects of imposing the linear causality condition on b_π posterior distribution

Nonlinear Causal Analysis

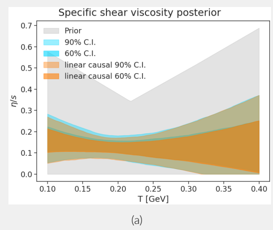
We simulated Pb-Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV

The effects of causality on Bayesian analysis

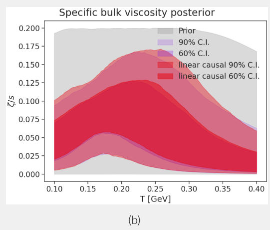
Nonlinear causality conditions could impose more restrictions into parameter space



Effects of imposing the linear causality condition



(a)



(b)

Conclusions

- We imposed a linear causality condition as a theoretical knowledge into JETSCAPE bayesian analysis
- It doesn't affect this Bayesian parameter estimation
- The Nonlinear causality conditions could impose stronger restrictions into the allowed parameter space

References

[1] Fábio S. Bemfica, Marcelo M. Disconzi, Vu Hoang, Jorge Noronha, and Maria Radosz. Nonlinear constraints on relativistic fluids far from equilibrium. *Phys. Rev. Lett.*, 126:222301, Jun 2021.

[2] JETSCAPE Collaboration. Multisystem bayesian constraints on the transport coefficients of qcd matter. *Phys. Rev. C*, 103:054904, May 2021.

[3] Govert Nijs and Wilke van der Schee. Predictions and postdictions for relativistic lead and oxygen collisions with the computational simulation code trajectum. *Phys. Rev. C*, 106:044903, Oct 2022.

[4] Govert Nijs, Wilke van der Schee, Umut Gürsoy, and Raimond Snellings. Bayesian analysis of heavy ion collisions with the heavy ion computational framework trajectum. *Phys. Rev. C*, 103:054909, May 2021.

[5] Christopher Plumberg, Dekrayat Almaalol, Travis Dore, Jorge Noronha, and Jacquelyn Noronha-Hostler. Causality violations in realistic simulations of heavy-ion collisions. *Physical Review C*, 105(6), jun 2022.

- **In preparation** : Two papers about the effects on JETSCAPE bayesian analysis constructing a prior distribution informed by nonlinear causality conditions