

Using multi-particle correlations to estimate fluctuations in jet and rare probe azimuthal anisotropies

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Rare Probe Anisotropies

The study of Jets, heavy flavor quarks, and other signals created in heavy-ion collisions is indispensable to the study of QCD, and the Quark Gluon Plasma (QGP) which with they interact. The azimuthal yield of these particles (referred to as POI, with angles ψ_i) around a symmetry axis Ψ_n can be decomposed into Fourier components, v'_n :

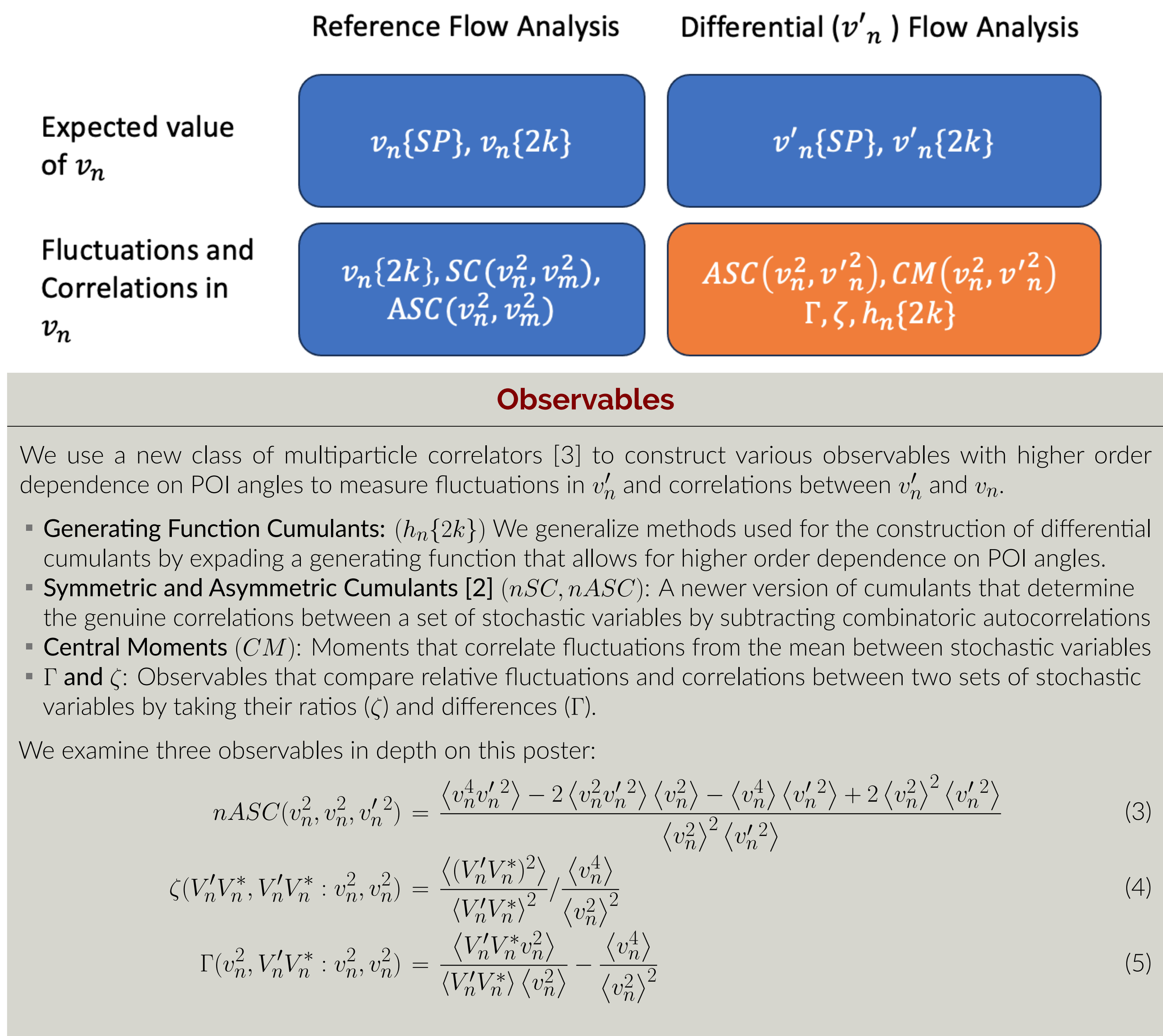
$$v'_n \equiv \left\langle e^{in(\psi - \Psi'_n)} \right\rangle = \left\langle \cos[n(\psi - \Psi'_n)] \right\rangle \quad (1)$$

Due to limited statistics, measurements of v'_n typically rely on measurements of v_n : the fourier coefficients of the yield of an inclusive set of low p_T particles with angle ϕ .

$$v_n \equiv \left\langle e^{in(\phi - \Psi_n)} \right\rangle = \left\langle \cos[n(\phi - \Psi_n)] \right\rangle \quad (2)$$

These differential flow measurements have proven effective at determining the event-by-event average $\langle v'_n \rangle$. To determine fluctuations in path length dependent Jet energy loss, or fluctuations in other rare probes, we introduce an extension of this method to determine fluctuations in v'_n and correlations between v'_n and v_n^2 . We then present a toy model to demonstrate the sensitivity of these observables, and their capacity to identify fluctuations in v'_n and v_n in which we:

- Evaluate multivariate moments of $v_n'^2$, v_n^2 , and $V_n'V_n^* = v_n'v_n e^{in(\Psi_n' - \Psi_n)}$ using multiparticle correlators
- Use a bivariate copula distribution to model flow fluctuations and correlations between v'_n and v_n
- Evaluate a variety of observables using the bivariate v'_n and v_n distribution
- Demonstrate the sensitivity of the various observables to the fluctuations and correlations in v'_n and v_n



Fluctuation Toy Model

- We consider distributions of v_n and v'_n to be parametrized identically, but with different parameter values producing different amounts of fluctuations in their distributions.
- The models we choose are the **Elliptic Power distribution**, $P(v_n : \epsilon_0, \alpha)$ and the **Gaussian distribution** $P(v_n : \mu, \sigma)$. To demonstrate their feasibility as models for v'_n , their fits to Glauber Eccentricity data are shown in Fig. (1).
- Finally, we use a gaussian copula model to form a joint distribution of v'_n and v_n . This parametrization allows us to directly parametrize the pearson correlation coefficient $\rho(v'_n, v_n)$. The complete bivariate distributions are visible in Fig. (2).

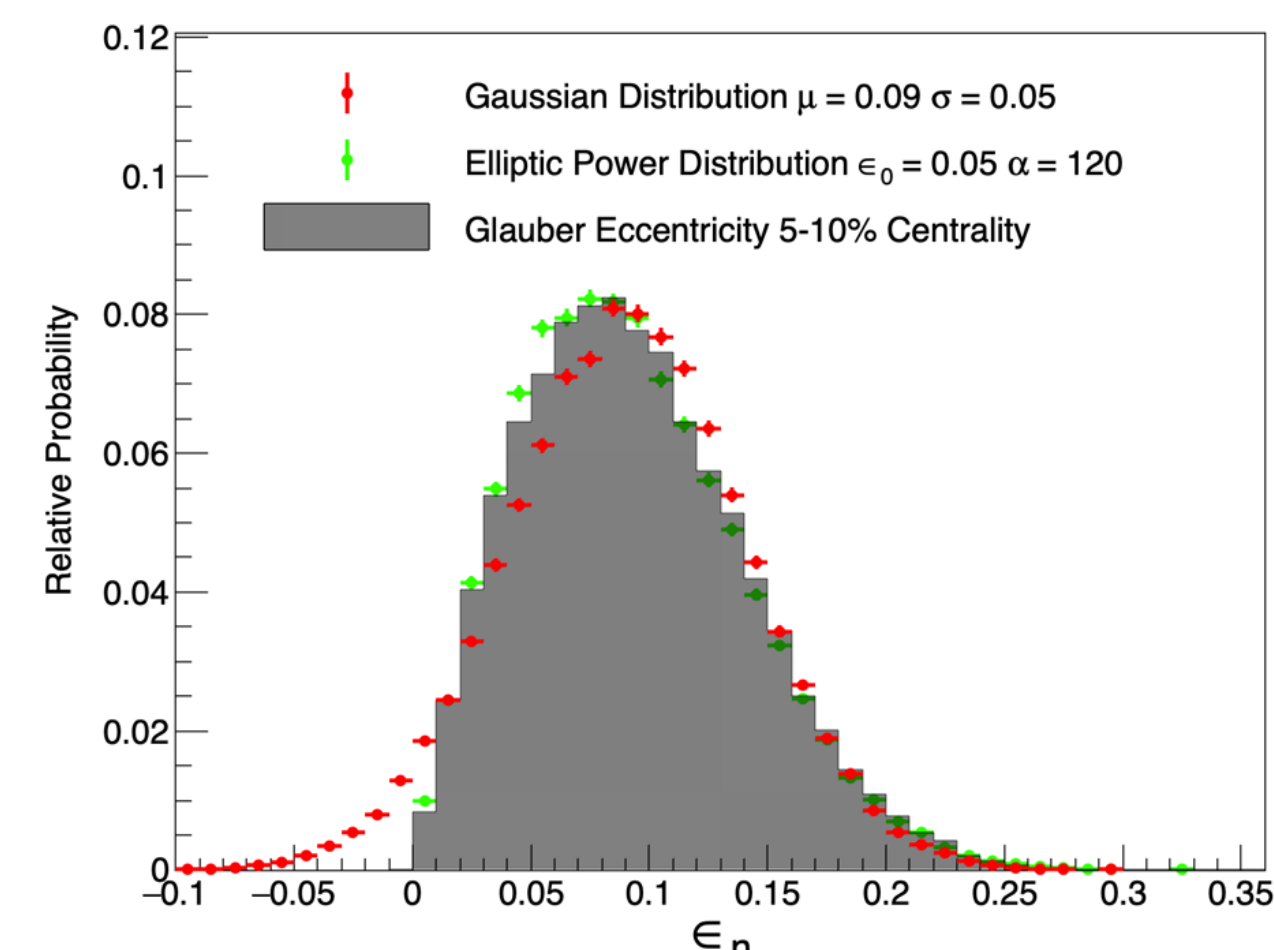


Figure 1. Elliptic Power distribution and Gaussian distribution fit to Glauber ϵ_2 data from 5-10% centrality.

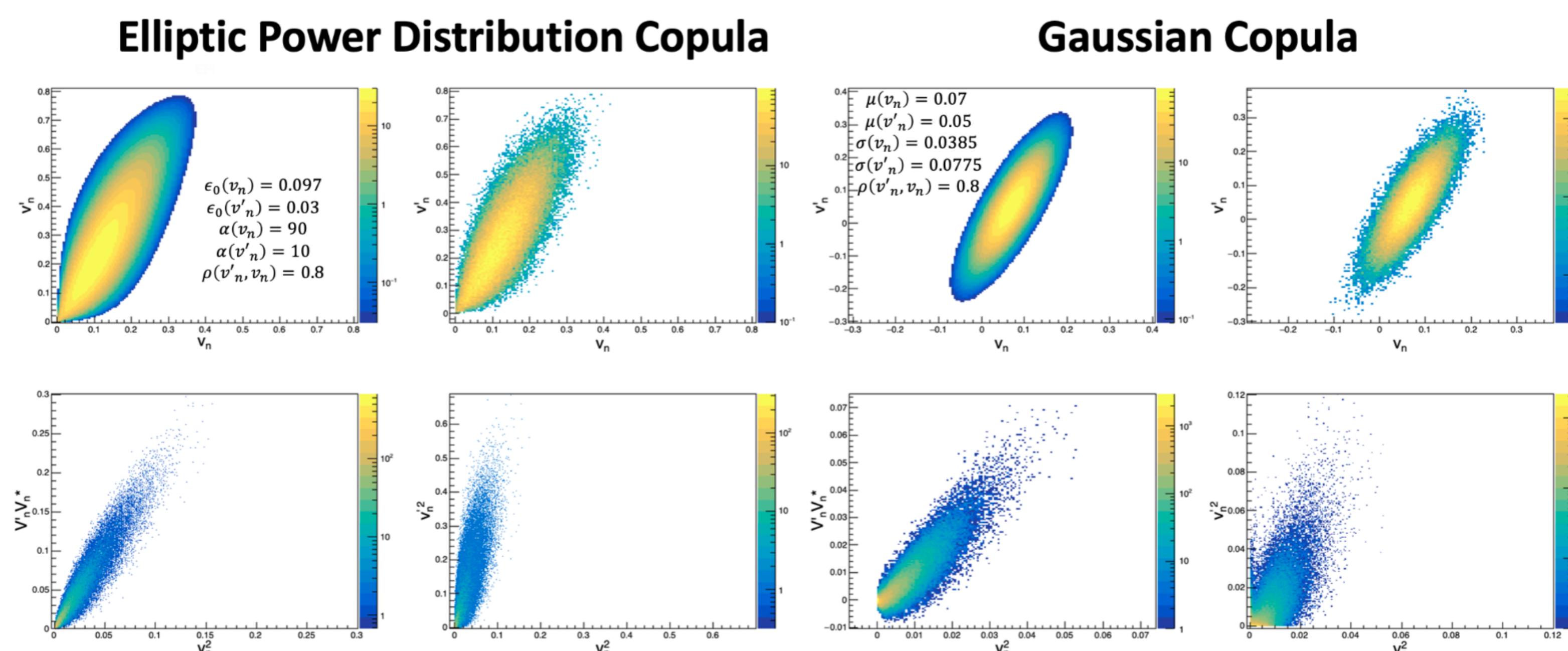
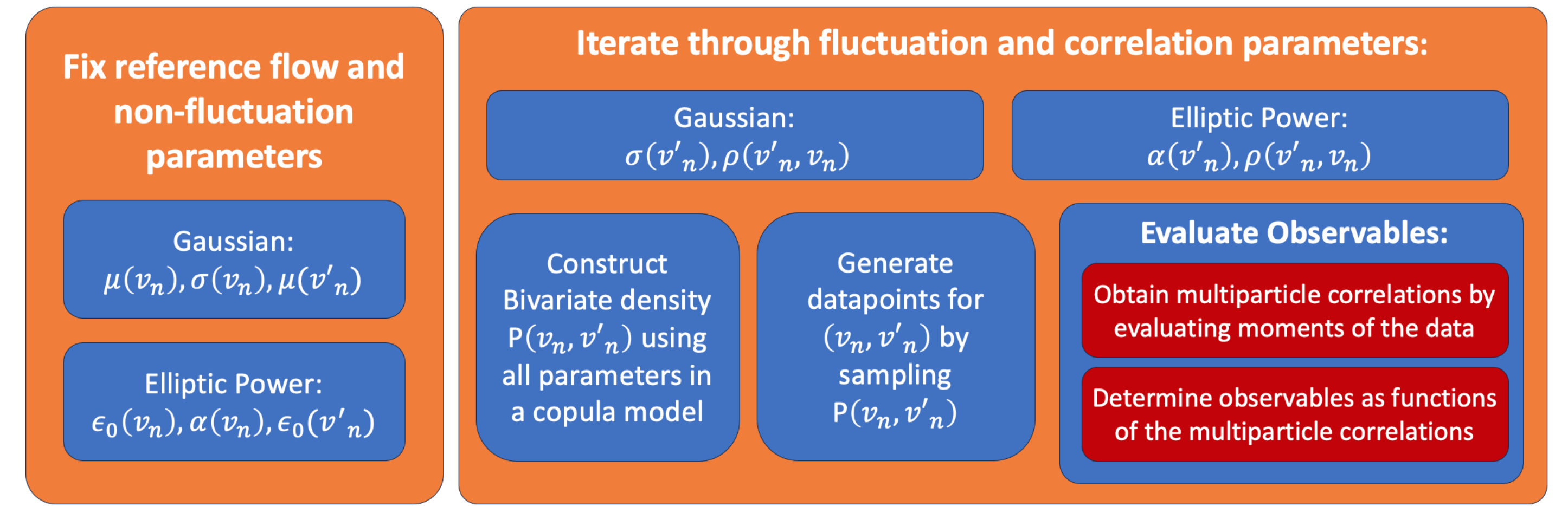


Figure 2. Bivariate Copula Models with Elliptic Power marginal distributions (left) and Gaussian marginal distributions (right). The actual distribution, Monte-Carlo sampled data, and distributions for $v_n^2, v_n'^2$, and $V_n'V_n^*$ are shown for each bivariate distribution, demonstrating a correspondance between correlations in v'_n and v_n , to correlations between $v_n'^2$ and v_n^2 , as well as $V_n'V_n^*$ and v_n^2

Isolating Fluctuations

- Each bivariate distribution uses two parameters for $P(v'_n)$, two parameters for $P(v_n)$, and ρ .
- For a given parametrization, we fix the parameters that can be deduced using existing observables: parameters for $P(v'_n)$, and most strongly related to $\langle v'_n \rangle$
- Then we iterate through the remaining parameters: $\rho(v'_n, v_n)$ and the parameter governing fluctuations in v'_n . At each point in this 2D phase space we evaluate each observable by sampling the bivariate distribution.



Results

We plot the observables in Eqs. (3,4,5) as a function of ρ and their relative spread (RS) defined as $RS = \sigma(v'_n)\mu(v_n)/\sigma(v_n)\mu(v'_n)$, to determine their sensitivity to fluctuations in v'_n relative to fluctuations in v_n without being biased by $\langle v'_n \rangle$ or $\langle v_n \rangle$.

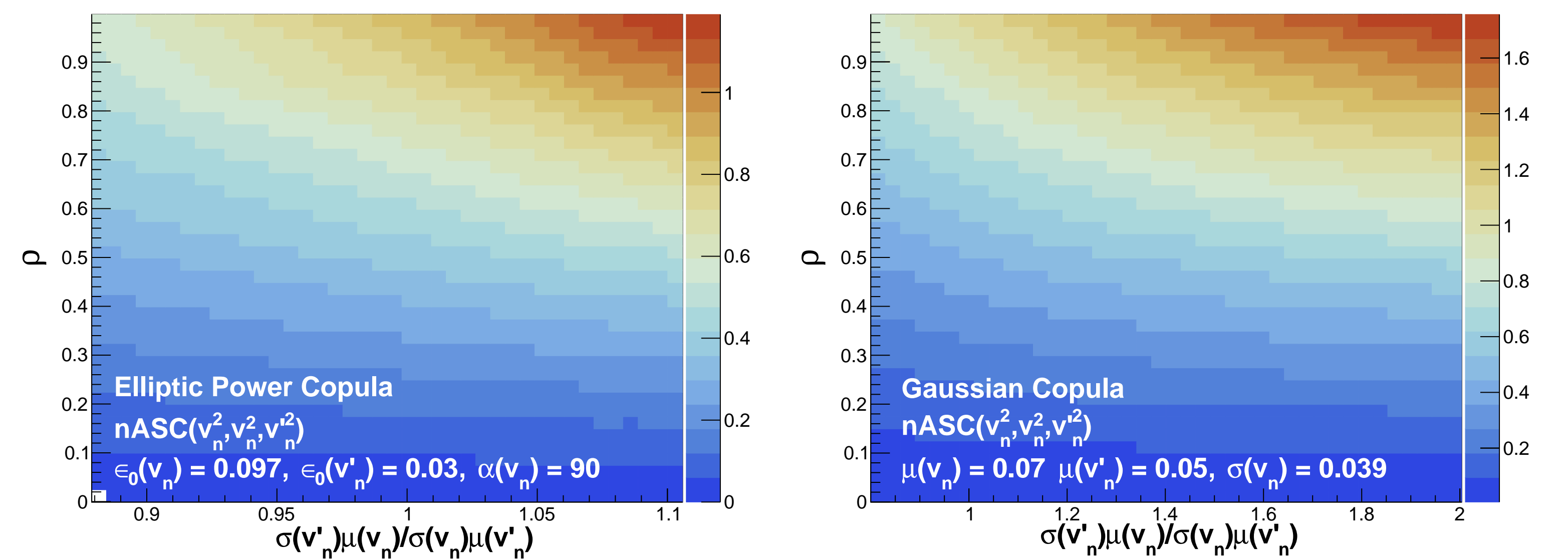


Figure 3. The normalized Asymmetric Cumulant $nASC(v_n^2, v_n'^2, v_n'^2)$ represents a correlation between v_n^4 and $v_n'^2$, or a correlation between a departure of $v_n'^2$ from $\langle v_n'^2 \rangle$ with a squared departure of v_n^2 from $\langle v_n^2 \rangle$. It shows strong dependence on ρ , and weak dependence on RS

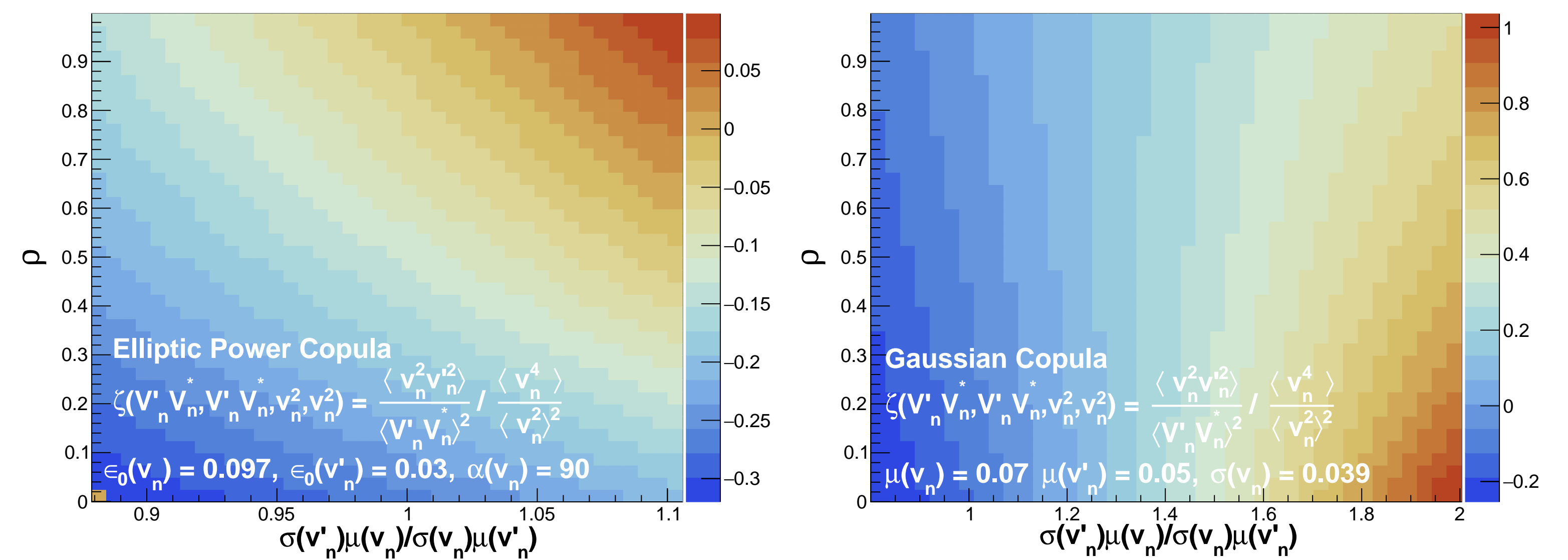


Figure 4. The observable $\zeta(V_n'V_n^*, V_n'V_n^* : v_n^2, v_n^2)$ is plotted as a function of ρ and RS. While the results for each distribution look disparate, considering the range of RS for the Elliptic Power case indicates a similar behavior displayed by both plots. It's clear that this quantity is most sensitive to RS, especially at lower ρ values.

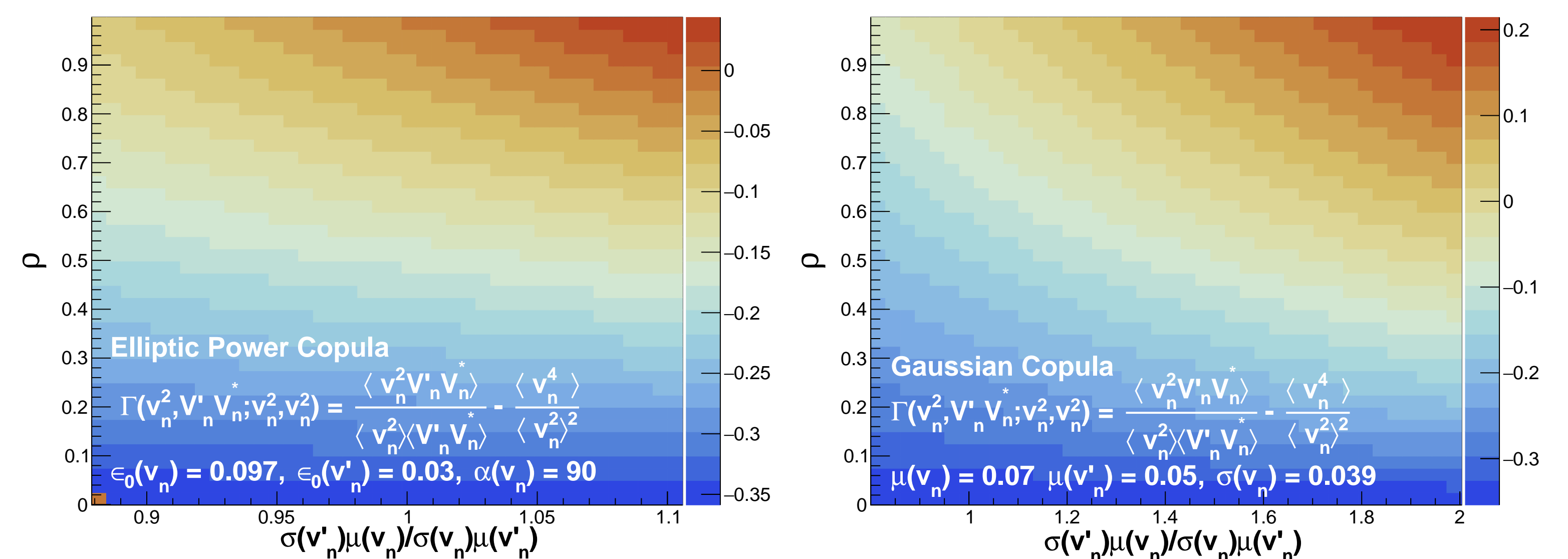


Figure 5. $\Gamma(v_n^2, V_n'V_n^* : v_n^2, v_n^2)$ identifies the difference between the normalized covariance of $V_n'V_n^*$ with v_n^2 , and the variance of v_n^2 . Positive values for this quantity can be found in [1], suggesting large values of ρ and or RS.

Conclusions

The observables shown here can discern second order fluctuations in v'_n , and correlations between v_n and v'_n under bivariate copula flow correlation models. It is clear that under these v_n distributions, $\zeta(V_n'V_n^*, V_n'V_n^* : v_n^2, v_n^2)$ is a solid proxy for RS, and that $nASC(v_n^2, v_n'^2, v_n'^2)$ has strong sensitivity to ρ . A measurement of any one of these observables can be used to constrain ρ and RS to a range of level curves apparent in Figs. (3,4,5). Additionally, at any point on these plots where their level curves don't intersect, two or more of these observables can be used in tandem to uniquely identify the RS and ρ for an event-by-event distribution of v_n and v'_n .

References

- [1] Barbara Betz, Miklos Gyulassy, Matthew Luzum, Jorge Noronha, Jacquelyn Noronha-Hostler, Israel Portillo, and Claudia Ratti. Cumulants and nonlinear response of high p_T harmonic flow at $\sqrt{s_{NN}} = 5.02$ TeV. *Phys. Rev. C*, 95(4):044901, 2017. doi:10.1103/PhysRevC.95.044901.
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